SECURITY ISSUANCE AND INVESTOR INFORMATION

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Abstract

Although the commoditisation of illiquid asset exposures through securitisation facilitates the disciplining effect of capital markets on the risk management, private information about securitised debt as well as complex transaction structures could possibly impair the fair market valuation. In a simple issue design model without intermediaries we maximise issuer proceeds over a positive measure of issue quality, where a direct revelation mechanism (DRM) by profitable informed investors engages endogenous price discovery through auction-style allocation preference as a continuous function of perceived issue quality. We derive an optimal allocation schedule for maximum issuer payoffs under different pricing regimes if asymmetric information requires underpricing. In particular, we study how the incidence of uninformed investors at varying levels of valuation uncertainty and their function of clearing the market effects profitable informed investment. We find that the issuer optimises own payoffs at each valuation irrespective of the applicable pricing mechanism by awarding informed investors the lowest possible allocation (and attendant underpricing) that still guarantees profitable informed investors appropriate higher profit than uninformed types. Any reservation utility by issuers lowers the probability of information disclosure by informed investors and the scope of issuers to curtail profitable informed investment.

Keywords: asset securitisation, security design, security issue, direct revelation mechanism, asymmetric information, auction model, asset securitisation, reservation utility

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Abstract

Private information about securitised debt as well as complex transaction structures could possibly impair the fair market valuation. In a simple issue design model without intermediaries we maximise issuer proceeds over a positive measure of issue quality, we derive an optimal allocation schedule for maximum issuer payoffs for endogenous price discovery under different pricing regimes if asymmetric information requires underpricing. In particular, we study how the incidence of uninformed investors at varying levels of valuation uncertainty and their function of clearing the market effects profitable informed investment. We find that the issuer optimises own payoffs at each valuation irrespective of the applicable pricing mechanism by awarding informed investors the lowest possible allocation (and attendant underpricing) that still guarantees profitable informed investment. Under uniform pricing the composition of the investor pool ensures that informed investors appropriate higher profit than uninformed types. Any reservation utility by issuers lowers the probability of information disclosure by informed investors and the scope of issuers to curtail profitable informed investment.

I. INTRODUCTION

Asset securitisation refers to the growing tendency of substituting capital markets for intermediaries in channelling external funds to efficient uses of economic activity. Recently it has been touted as a viable and expedient risk management and refinancing method. It allows issuers to convert existing or future cash flows from pooled asset exposures ("reference portfolio") into marketable debt securities as commoditised structured claims, which blend default risk and asset pricing features of securitised assets (mostly mortgages, consumer debt, trade receivables and corporate loans) and the merchantability of fixed income securities. Secured debt, such as asset-backed securities (ABS), registers as a safer claim than unsecured debt under the pecking order theory (Myers, 1977; Leland, 1998), mainly because it derives its value from repayment on a scrutinisable asset portfolio insulated from overall issuer performance. At the same time, the inherent asset transformation of securitisation challenges the traditional value proposition of financial intermediation by separating asset origination and risk management as two distinctive components in external finance. Despite its efficiency-enhancing effect as a diversified source of liquid funds, securitisation falls short of mitigating incomplete capital allocation in financial markets. The complex nature of securitisation engenders valuation uncertainty and possible non-verifiability of trading motives due to imperfect information dissemination. Asymmetric information between issuers and investors suggests that issuers have superior information about the true asset value, so that investors in securitised assets would reasonably command external price discounting to compensate for ex ante moral hazard as regards the deliberate misrepresentation of securitised asset quality and adverse selection by rational investor expectations à la Akerlof (1970). Issuers usually retain

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¹ Rational investors would expect to be offered only poor deals in securitisation markets under asymmetric information. If the investment choice is conditional on the level of investor information, uninformed investors assume to partake in a disproportionately large number of poor transactions once better informed investors have picked off most if not all profitable deals. Asymmetric information might also arise from (i) incentives of biased loan selection at the time the asset composition of the portfolio is determined (ex ante moral hazard) and (ii) reduced monitoring of asset exposure after securitisation (ex post moral hazard). See Jobst (2003) for a detailed review of the information economics of asset securitisation.

the most junior claim in a transaction (credit enhancement) as ex ante reservation utility to mitigate these agency costs of asymmetric information (DeMarzo and Duffie, 1997).

In this chapter, we present a general issue design, which demonstrates how valuation uncertainty and credit enhancement might affect both the incentive structure of investors and issuer payoff of security issuance. A low incidence of informed investors suggests an auction-style allocation mechanism with price discounting ("underpricing") as a feasible model design for the optimal choice of pricing and allocation under valuation uncertainty. Our proposed model introduces a new argument for optimal security issuance under asymmetric information without intermediaries in keeping with the "winner's curse" problem. Although our framework of optimal security issuance relies on the conventional allocation-based argument of IPO underpricing due to asymmetric information between issuers and investors in keeping with the "winner's curse" problem (Rock, 1986), our simple one-period approach goes beyond the rationing of uninformed investors as the main determinant of underpricing. In a general auction-style design, we maximise issuer payoffs conditional on price discounting needed to guarantee profitable informed investment over a positive measure of issue quality for a given degree of valuation uncertainty about securitised assets. As opposed to Rock (1986), where underpricing compensates uninformed investors for being rationed by informed demand across all states of profitable investment, we explain underpricing to be jointly determined by both an auction-style share allocation to informed investors and the degree of uninformed investment associated with valuation uncertainty. It is not the rationing of uninformed investors, but the allocation preference by informed investors, which guides our thinking about underpricing and how it relates to the optimisation problem of issuer proceeds. We treat the level of allocation as a strategic choice variable, which allows issuers to extract information about the actual quality of the security issue through revealed allocation preference by informed investors in a direct revelation mechanism (DRM).2 DRM endogenises price discovery in an auction-style allocation preference as a continuous function of perceived issue quality. Informed investors accept some allocation as a continuous function of their beliefs about the actual issue valuation and reveal their valuation

to uninformed investors only if a known price-quantity schedule implies profitable investment.³ The acceptance set of profitable informed investment qualifies an optimal allocation schedule for maximum issuer payoffs at varying degrees of valuation uncertainty and different pricing regimes. Issuers maximise issue payoffs at a positive measure of issue quality for an allocation that ensures participation by informed investors. The price discovery of actual issue quality conditional on some acceptance set of informed investors allows issuers to price the residual allocation to uninformed investors to clear the market. In particular, we study how the incidence of uninformed investors at varying levels of valuation uncertainty affects the utility from informed investment if the offering price is set to be either the same for both types of investors (uniform pricing) or higher for uninformed investors (discriminatory pricing). The residual allocation to uninformed investors and the incentive of informed investors to subscribe to DRM at any issue quality - as long as some allocation yields positive payoff - curtail the ability of informed investors to optimise own payoffs from disclosing their beliefs under the profitability condition of DRM. Under uniform pricing, the incidence of investor types associated with the degree of valuation uncertainty further conditions the propensity of informed investors to participate. As an extension to the existing underpricing paradigm, we add credit enhancement to the model as some reservation utility in the form of fractional investor repayment, which sanctions the scope of profitable informed investment.⁴

² Due to private information informed investors have superior knowledge about the actual quality of the security issue, whose valuation uncertainty is indicated by the precision measure of the private signal received by informed investors.

³ The option value of informed investment increases (decreases) the higher (lower) the valuation uncertainty and the lower (higher) the precision of investor beliefs, which implies that more investors become informed as information gathering about the true value of the transaction becomes more profitable. An increase in the number of informed investors raises the rational expectation of uniformed investors to be allocated shares in a disproportionately large number of unprofitable (bad) deals ("winner's curse dilemma"). Uninformed investors will require sufficient underpricing to compensate for *ex ante* valuation uncertainty ("*ex ante* uncertainty hypothesis") as agency cost of adverse selection. Also informed investors would only commit to profitable, underpriced investments. If the size of the overall investor pool is kept unchanged, the altered composition of the investor pool due to a larger share of informed investors at higher valuation uncertainty changes the prices both types of investors would be prepared to pay.

⁴ If issuers retain some reservation utility the resultant fractional repayment increases demands on the minimum issue quality.

We find that issuers maximise own payoffs and derive an optimal solution to the design problem if their allocation to informed investors remains large enough to elicit "truth telling" in return for profitable investment, irrespective of the pricing regime (uniform or discriminatory). A higher allocation to informed investors means that a larger portion of the transaction is subject to underpricing, which in turn reduces overall issue payoffs. The presence of an unknown number of uninformed investors only matters as a participation constraint of optimal allocation under uniform pricing, which requires an adjustment of the allocation choice to still guarantee profitable informed investment. Increased uninformed investment demand at lower valuation uncertainty limits the utility of informed investment. Thus, the composition of the investor pool ensures that informed investors⁵ appropriate higher relative profit than uninformed types. We find that issuers maximise payoffs under uniform pricing by keeping the actual quality of the transaction, valuation uncertainty and any reservation utility as low as possible. This rule of action establishes an "efficient frontier" of allocation choices, which implies higher individual net payoff from informed investment relative to uninformed investment.

The rest of the chapter is structured as follows. The chapter begins with a review of the literature, linking stylised facts about asset securitisation with information processing under asymmetric information in matters pertinent to efficient security issuance in securitisation markets. In the next sections we present a simple issue design model without intermediaries, where a *direct revelation mechanism* (DRM) determines the optimal allocation choice for maximum issuer payoffs at varying degrees of valuation uncertainty and different pricing regimes – assuming asymmetric information requires "winner's curse"-type underpricing and uninformed investment demand clears the market. With information processing by informed investors taking a critical role in security issuance, we first derive an acceptance set of profitable informed investment, which prescribes an optimal allocation schedule for a perceived issue quality. We then determine expected issuer proceeds if informed investors maximise their payoffs within this acceptance set according to a fixed price-quantity schedule. In particular, we study how the incidence of uninformed investors at varying levels of valuation

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⁵ Informed investors can infer valuation uncertainty and the incidence of uninformed investment from the precision of

uncertainty impacts the utility from informed investment under uniform pricing conditions. Subsequently, we introduce endogenous price discovery through auction-style allocation preference as a continuous function of perceived issue quality (in keeping with a fixed price-quantity schedule) within the acceptance set of profitable informed investment to derive maximum issuer net payoffs. Finally, we provide a numerical illustration of the relationship between perceived issue quality and net issuer proceeds contingent on the degree of valuation uncertainty (see section V). The chapter concludes with a summary of significant findings and recommendations.

II. LITERATURE REVIEW AND EMPIRICAL REASONING

The design problem of security issuance under asymmetric information and valuation uncertainty has been extensively studied in past research on the underwriting process and investor behaviour in stock markets.⁶ However, so far the well-understood economic rationale behind the alignment of asset pricing and share allocation choices to investor incentives has not been transposed into related areas of external finance, such as asset securitisation. Asset securitisation represents a cost-efficient and flexible structured finance instrument to convert illiquid present or future asset claims of varying maturity and quality into tradable debt securities by re-packaging and diversifying receivables into securitisable asset portfolios (*liquidity transformation* and *asset diversification*).⁷ Transactions typically involve reference portfolios of one or more (fairly illiquid) asset exposures, from which stratified positions (or tranches) with different seniority are created, reflecting

their private signal, which qualifies the allocation schedule of profitable investment.

⁶ See Welch and Ritter (2002) for a recent overview of the literature in this regard.

Asset securitisation initially started as a way of depository institutions, non-bank finance companies and other corporations to explore new sources of asset funding either through moving assets off their balance sheet or raising cash by borrowing against balance sheet assets ("liquifying"). In the meantime, securitisation goes a long way in advancing two main objectives: (i) to curtail balance sheet growth and realise certain accounting objectives and balance sheet patterns, and/or (ii) to reduce economic cost of capital as a proportion of asset exposure and ease regulatory capital requirements (by lower bad debt provisions) to manage risk more efficiently. Most commonly, a balanced mix of both objectives and further operational and strategic considerations determine the type of securitisation – traditional or synthetic – in the way issuers envisage securitisation as a method to shed excessive asset exposures.

different degrees of investment risk.⁸ The existing literature in securitisation primarily focuses on the implications of potential agency costs arising from adverse selection and moral hazard sanctioned by capital market investors.

In securitisation, issuers and/or investors tend to retain some of the securitised asset exposure and/or provide other means of structural support to build investor confidence in the quality of their security issue. Frequently, such risk sharing agreement between issuers and investors comes in the form of an equity-like claim⁹ on the expected losses of the securitised assets in the effort to limit agency costs of asymmetric information due to inherent valuation uncertainty. These information problems associated with the lack of external verifiability of securitised assets and the risk-sharing arrangements between issuers and investors are common considerations in existing security design models. We reconcile existing approaches to model the information structure of investors and partial asset retention by issuers as crucial elements to security issuance under asymmetric information. In order to specify (i) information processing of informed investors as "truth tellers" in an auction-style allocation choice under asymmetric information and (ii) how valuation uncertainty affects the degree of underpricing, we amalgamate previous findings from (i) economic models with multiple equilibrium outcomes from information processing and coordination games, (ii) security design model of

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⁸ These positions may take the form of fully/partially funded asset-backed securities or unfunded derivatives.

⁹ The structural risk sharing arrangement between issuers and investors through subordination, which concentrates most default loss in the most junior tranche, also entails leveraged investment due to the difference of tranche sizes across different levels of seniority. Tranches with little or no subordination are more affected by the mean and volatility of default losses (expected and unexpected losses) (Gibson, 2004), i.e. their ratio of relative tranche losses to relative portfolio losses is higher than for more senior tranches. So we would expect an ever greater effect of adverse selection from valuation uncertainty on leveraged exposures in securitised asset portfolios. Issuers and investors might also be faced with the prospect of high trading cost (Duffie and Gârleanu, 2001) associated with a small market volume of outstanding issues, liquidity premium to the agency cost from adverse selection.

¹⁰ Early models suggest signalling (DeMarzo and Duffie, 1997; Leland and Pyle, 1977) as a means to curb investor uncertainty, where sellers of a security issues convey the value of the security by their willingness to partake in the risk as they retain a portion of the issue. Riddiough (1997) takes a slightly different twist on risk sharing. He proposes a theoretical model of asset-retention as an effort choice by issuers to mitigate external price discounting as agency cost of

debt contracts with partial repayment and (iii) auction-style solutions to IPO mechanisms. In order to determine how informed investors process private information we resort to the concept of adjusted investor beliefs in a coordination game setting proposed by Morris and Shin (2000) in the context of bank runs, where the discrepancy between the indeterminacy of beliefs and the objective assessment could lead to suboptimal economic outcomes.¹¹ In particular, we adopt the definition of a precision measure of private signals to specify informed investment decisions as a basis of a *direct revelation mechanism* (DRM). Second, we borrow the optimal design of lending contracts with partial repayment from Inderst and Müller (2002) in order to derive the first-best condition of optimal informed investment if a reservation utility associated with *credit enhancement* reduces expected payoffs from investment. This approach is in stark contrast to many erroneous accounts in the literature, which regard credit enhancement as a signalling device¹² Finally, we resort to the rich literature about IPO underpricing (Malakhov, 2003; Welch and Ritter, 2002; Myerson, 1981) of corporate share issues as the theoretical basis for the specification of an *optimal security auction* under asymmetric information with maximum issuer payoffs. We rule out all but asymmetric information from the list of researched explanations for IPO underpricing, ¹³ as most of the legal and strategic considerations of alternative explanatory approaches

rational investor beliefs about superior information¹⁰ about the securitised asset risk held by non-recourse single-purpose entities in conventional securitisation structures.

¹¹ In their view multiple equilibria assume that economic outcomes result from actions motivated by the beliefs of individuals. However, any indeterminacy of beliefs, although these beliefs themselves are rationale and consistent with fundamental economic features, yields quite different states of affairs, which might not be perfectly in a nod to what would be deemed appropriate judging by the underlying information to start with.

¹² Since credit enhancement compensates for the rating shortfall between the rating quality and the desired rating quality of the transaction (as a completely discretionary choice), the level of credit enhancement cannot increase information transparency as a signalling device.

¹³ In classical IPO models issuers offer new shares at a selling price below fair market value ("underpricing") due to one or more of the following factors: (i) asymmetric information, (ii) institutional and systemic constraints, (iii) strategic considerations, and (iv) ownership and control. However, individual characteristics of national stock markets and disparate statutory regulations limit how these factors might actually explain the reasons for discounted IPOs. Besides asymmetric information other main reasons for underpricing are defined as: (i) legal risk of violations against securities laws ("lawsuit hypothesis"), price support and book building as a mechanism of information revelation could explain high levels of underpricing as investors require significant compensation for systemic uncertainty and institutional constraints by means of underpricing; (ii) pricing and/or explicit rationing bias give rise to restrictions on ownership and

do not apply to securitisation. Asymmetric information models suggest a positive correlation between *ex ante* valuation uncertainty and underpricing. The "winner's curse" problem is one of the asymmetric information models, whose economic reasoning for IPO underpricing seems to be most in tune with empirical observations about the workings of securitisation markets. The "winner's curse" problem postulated by Rock (1986) implies that asymmetric information about the actual issue quality entails adverse selection of investor as regards share allocation, where informed investors benefit from better information. Since the information advantage of informed investors carries higher gross payoffs as the degree of valuation uncertainty rises, higher informed investment demand in the composition of the investor pool entails a higher degree of underpricing (to maintain the participation incentive of investors). Hence, higher gross payoffs from informed investment exacerbate the "winner's curse" problem. Uninformed investors would rationally believe that they receive a disproportionately high allocation of transactions of poor quality.

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It is commonplace to argue that securitisation markets are notorious for weak information disclosure about underlying reference portfolios, intricate auditing standards and legal uncertainty surrounding the estimation of expected investor return and the complex enforcement of restrictive covenants and redemption criteria. These contingencies and information constraints impede efficient asset pricing and hinder full understanding of the fundamental risk involved in securitisation transactions. Low market liquidity of securitisation

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control; (iii) strategic considerations ("manager's strategic underpricing explanation"), where underpricing occurs as an agency cost that results from strategic considerations by managers to benefit from higher expected shareholding value at lock-up expiration if underpricing creates an information momentum, which shifts the demand curve for the issued shares outwards (Aggarwal et al., 2002). Hence, managers trade-off substantial underpricing against a maximisation of personal wealth when they have their first opportunity to sell shares.

¹⁴ Since informed investors condition their decision to request some allocation on positive payoff, this allocative benefit results in underpricing and increases in valuation uncertainty. Hence, the benefit from generating private information production is similar to investment in a call option on the IPO with the offering price as strike price and the valuation of the issue as the underlying asset price. The call option reflects the degree of underpricing. As the option value increases with uncertainty about the underlying valuation, more investors become informed.

¹⁵ Some empirical studies confirming the winner's curse problem on the basis of allocation rates of IPO issues include Koh and Walter (1989), Levis (1990), Keloharju (1993) as well as Amihud et al. (2003).

¹⁶ See also Rutledge (2004) on the frequently decried absence of widespread standardisation in securitisation markets.

instruments suggests substantial valuation uncertainty.¹⁷ In the presence of disintegrated capital markets, the low degree of informed investment could provide grounds for discounted offerings to compensate for investment risk. So, the adaptation of asymmetric information models of IPO pricing has intuitive appeal. Moreover, participants in securitisation markets¹⁸ learn about allocation rates, which award all agents regardless of their size the same chance of placing a successful bid. Consequently, the "winner's curse" problem seems a plausible cause to underpricing of securitisation transactions.

III. MODEL

We tender a security (issue) design model, where a single monopolistic issuer of securitised claims maximises his proceeds through an optimal allocation that is incentive compatible with informed investment demand. The model describes a simplified issuing process in a simplified securitisation market consisting of one issuer without endowment¹⁹ and two discrete types of investors, with competition limited to investors only. The issuer offers securitised claims to outside investors at some selling price after having sounded out the perceived issue quality by taking initial quantity orders from sophisticated investors on the basis of a commonly understood pricing scheme. The total number of claims is set to unity. We distinguish between two discrete types of buyers: informed investors I (e.g. large institutional investors, banks, hedge fund managers) and uninformed investors $\theta \in \Psi = \left[1, \overline{\theta}\right]$ (e.g. retail investors), whose types are defined by nature ex ante as measures of informed and uninformed demand. Informed investors act as quasi-market makers and

¹⁷ Substantial liquidity risk and rent seeking from information advantage has confined most investment in securitisation markets to "buy-and-hold" strategies by large and well-informed institutional investors, insurance companies, banks and other financial institutions; yet evidence about the degree of uninformed investment remains inconclusive for loss of empirical observations.

¹⁸ The securitisation market consists of two types of investors: individual investors and institutional investors. While the majority of investors, which mostly invest in high-volume issue tranches with high seniority (such as big insurance companies), could be regarded as uninformed, the small portion of institutional and private investors function is informed and invests in junior and riskier. As senior tranches outweigh lower rated tranches by far in notional volume, uninformed investor claim a sizeable part of investment demand in securitisation markets.

¹⁹ i.e. funds generated from the issue accrue irrespective of other assets the issuer might hold on his books.

price setters during initial placement, before uninformed investors clear the market after price discovery by informed types. The probability of being an informed or uninformed investor is proportional to the incidence of types, where $I/(I+\theta)$ is the probability of being informed. The distribution of uninformed investment θ and the total number of informed investors I is common knowledge. Informed investors have sufficient funds to buy the entire transaction (or as much as available). The same applies to the total number of uninformed investors. In keeping with Rock (1986) we assume uniform informed investment, where each informed investor can be allocated more than one share (i.e. varying quantity orders). Uninformed investors can only buy at most one share each and have sufficient funds to buy the entire issue at any valuation irrespective of the offering price. If informed investors decide to buy (at some pricing schedule based on allocation), we anticipate rationing of uninformed investors in the sense of the "winner's curse" adverse selection problem in Rock (1986).²⁰ All agents in the model are assumed to be risk-neutral. The issue valuation r is a random variable $r \sim N(\tilde{r}, \alpha^{-1})$ with precision α . The issuer does not know the realisation of uninformed investment θ and offers the transaction with promised repayment $c(r) \in C = [0,1]$ to informed investors $i \in I$ at a fixed price-quantity schedule. Informed investors learn about the actual valuation by gathering precise but not perfect information about the quality of the issue before they tender a bid.²¹ They observe the realisation of valuation r as a i.i.d. private (and costless)²² noisy signal $\zeta = r + \varepsilon$, where

²⁰ See also section II for a brief review of the rationale of underpricing in the context of initial public offerings of stocks (IPO).

²¹ This superior capability of interpreting the investment risk of securitised exposures in a more informative way could be interpreted in several ways. Informed investment by large brokerage firms or other financial institutions with expert knowledge, either within or outside the issuer's industry, could stem from their own expertise in originating and monitoring credit risk and structured risk (i.e. market and asset liquidity, interest and currency volatility as well as organisational risk of asymmetric information in lending relationships), such as credit risk analysis (Boot and Thakor, 2000). Similarly, Inderst and Müller (2002) suggest that also gathering new information about macroeconomic facts, such as market growth and product demand, effecting the outcome of issue performance might help improve the accuracy of risk assessment. Both arguments indicate that informed investors are able to extract private information about the actual issue quality and update their beliefs accordingly.

²² Inderst and Müller (2002) point out two prime inefficiencies associated with the information production through noisy signals: (i) misclassification of the actual valuation r, so that the action of informed investment after observing signal s

 $\varepsilon \sim N(0, \beta^{-1})$ and $f(\varsigma) = \exp\left[-(\varsigma - r)^2/(2\beta^{-2})\right]/(\sqrt{2\pi}\beta^{-1})$. Due to perfect information sharing all informed investors form uniform beliefs about the actual issue valuation on aggregate; however, we rule out information extraction by means of simple cross-reporting (Crémer and McLean, 1988).²³ Informed investors adjust their beliefs ς about realisation r with non-decreasing contractual repayment $\varepsilon(r)$ to the weighted measure²⁴ $s = (\alpha \tilde{r} + \beta \varsigma)/(\alpha + \beta)$ with $s \in S \equiv [0,1]$.²⁵ They have an incentive to participate only if the noisy signal ς of private information is sufficiently accurate, so that precision measure $\gamma = (\alpha^2(\alpha + \beta))/\beta(\alpha + 2\beta)$ of the private signal received by informed investors satisfies $\gamma \le 2\pi$ (see section IV.B). The precision also indicates the degree of valuation uncertainty.

Our design problem maximises issuer payoffs contingent on an efficient rule of action, which prescribes a particular allocation preference of informed investors with belief s to obtain positive payoffs for a given price-quantity schedule. Informed investors request some allocation $0 \le q(s) \le 1$ if and only if the fixed price-quantity schedule of general property $p(s) = q(s)^a$ $(0 \le a \le 1)$ implied by an auction-style allocation preference as a continuous function of perceived issue quality yields profitable investment E(v(r)|s) > p(s), where $0 \le r < p(s)$ and $\theta > \overline{r}$. The acceptance set of allocation choices associated with profitable informed

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would constitute either overpriced investment or forgone profitable investment; and (ii) mismatch of actual efforts taken by informed investors and required effort level for appropriate risk analysis (Manove et al., 2001). In order to remedy these inefficiencies, for simplicity we consider (i) the information content of the signal fixed and (ii) the effort of risk analysis essentially costless (instead of the proposition of a marginal cost associated with the signal).

²³ In contrast, uninformed investors behave quasi-atomistically, so their allocation implies forgone informed investment, given sufficient availability of investment funds by both categories of investors.

²⁴ Assuming that uncertainty about the valuation r would otherwise eliminate private signals ς unless they were sufficiently precise, informed investors adjust their subjective beliefs ς about the expected returns by the degree of perceived accuracy of private information.

²⁵ The acceptance set of adjusted beliefs for profitable informed investment is adapted from the work by Morris and Shin (2000) on the indeterminacy of beliefs as a source of co-ordination failure. Their model of bank runs is based on a

investment formalises a direct revelation mechanism (DRM). The issuer allocates the residual portion of the transaction to uninformed investors at the same (i.e. uniform) or a higher (i.e. discriminate) offering price. Uninformed investors are unaware of the realisation of both r and θ . If the uniformed price is still lower than fair market price, passive uninformed investment demand clears the market. We attribute no additional function to uninformed investors. If informed investors do not appropriate any profit for a given issue quality, they refrain from disclosing information about actual issue quality through an acceptable allocation level. Without allocation to informed investors, everybody receives zero payoffs. Hence, our issue design model relies on efficient allocation as the only strategic choice variable to (i) maximise issuer payoffs under optimal information extraction from informed investors and (ii) ensure their as price setters of uninformed investment demand. Set

IV. OPTIMAL ISSUING PROCESS AND ALLOCATION

Our basic model framework of optimal security issuance relies on the conventional allocation-based argument of IPO underpricing due to asymmetric information between issuers and investors in keeping with the "winner's curse" problem (Rock, 1986). However, our approach goes beyond the rationing of uninformed investors as the main determinant of underpricing. In a general auction-style design, we maximise issuer proceeds conditional on price discounting needed to guarantee profitable informed investment over a positive

Bayes Nash equilibrium of an imperfect information game. In our case, we treat each realisation of perceived valuation as a continuum of varying investment decisions by informed investors in a one-shot game.

²⁶ This issue process requires waiting to be the dominant strategy of uninformed investors if the appellation of being informed is limited only to those investors who can adjust their beliefs about actual issue quality based on the realisation of signal ς . So no uninformed can pretend to be informed by definition.

²⁷ Since any allocation of claims will only take place if informed investors decide to participate, all poor transactions are singled out through this direct revelation mechanism, and, hence, have no effect on the optimal allocation and pricing schedule of the issuing process. This implies that issuers would not be able to solicit any investment demand unless a true market valuation (as some "seal of approval") has been sought from informed investors.

²⁸ Only the proportion of informed investment is common knowledge, and both types of investors have sufficient funds on aggregate to theoretically buy the entire transaction.

measure of issue quality for a given degree of valuation uncertainty about securitised assets reflected in the composition of the investor pool. In extension to the "winner's curse" problem, we derive a sustainable equilibrium solution for an optimal issuing process with endogenous price discovery, in which the allocation choice satisfies informed investment demand as a continuous function of perceived issue quality. At the same time, issuers are able to extract maximum surplus from informed investors in a *direct revelation mechanism* (DRM).

Before we present an auction-style allocation choice to derive maximum issuer payoffs under uniform and discriminatory pricing, we solve the optimisation problem of informed investors within an efficient acceptance set of adjusted beliefs about actual issue quality (see section B), which prescribes a profile of profitable allocation choices at a fixed price-quantity schedule. We firs derive expected issuer returns under uniform and discriminatory pricing if informed investors were granted optimal allocation (see section C). Then we introduce an auction-style allocation preference as a continuous function of perceived issue quality within the acceptance set of profitable informed investment, which allows issuers to maximise own payoffs by extracting information surplus from price discovery through DRM by informed investors (Malakhov, 2003; Myerson, 1981) (see section D). Let us now revisit the fundamental rationale of the Rock IPO model, before we derive the acceptance set of optimal informed investment and an allocation schedule under DRM, which maximises profitable informed investment at a fixed price-quantity schedule.

A. The Rock (1986) model revisited

The aforementioned *ex ante* rationing problem of uninformed investors for an issuing process of "good deals" at a fixed price offering equates to the widely known "winner's curse" problem of IPOs in equity markets. According to Rock (1986), less privileged investors are crowded out by investors with superior information about the true value of the issue, who would only invest if shares priced at their expected value or lower, else they withdraw from the market in response to an observed bad quality of the IPO shares. This argument

explains why issuers would need to discount uniform offering price below fair market value in order to compensate uniformed investors for a "lemons problem" (Akerlof, 1970) of share allocation. Most shares allocated to uninformed issuers are "overpriced" compared to shares desired by informed investors. So underpricing accommodates the rational expectation that a disproportionately large share of "bad deals" are allocated to uninformed investment demand. Uninformed investors receive a full allocation of all shares only for overpriced issues (with informed investment being limited to "good deals"). A simplified version of the Rock model in Biais et al. (2002) conveys the essence of the "winner's curse" dilemma of issuers.^{29,30}

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In line with Rock (1986) an issuer offers a total number of shares at uniform price p, where all informed investors (with individually varying quantity orders) demand at most I shares, whilst each of θ uninformed buyers are allocated at most one share. This assumption reflects allocative benefits associated with better information about the actual issue quality, where only individual allocation of shares to uninformed investors matters to model an optimal allocation schedule in the presence of investor rationing. Informed investors request I shares on aggregate if the IPO is a "good deal", i.e. the market valuation v of the issue is larger than the offering price p. If v < p informed investors abstain from investing and leave all shares to uninformed investors. Consequently, higher overall informed demand and associated rationing of investors for "good deals" results in a "winner's curse problem" – uninformed investors receive a disproportionately large amount of shares in "bad deals" if their bids are successful. Hence, uninformed investors would expect a "price discount" proportional to the rationing rate, so that $\pi_U = E[\tau(v-p)] > 0$, where the rationing rate $\tau = 1/(I+\theta)$ if v > p, else $\tau = 1/\theta$. Since the covariance of τ and v is positive, it follows that E(v) > p. Informed investors have the weaker pricing condition $\pi_I = E[v-p] > 0$.

³⁰ Note that the participation incentive of informed investors to engage in information production represents a call option on the actual value of the IPO, which they will only exercise (by requesting shares in the IPO) if the underlying expected value exceeds the offering price (as strike price). The value of the option held by informed investors increases with valuation uncertainty. More investors become informed as higher information asymmetry between issuers and investors increases the option value, which exacerbates the "winner's curse problem". Higher uncertainty also implies that a declining fraction of uninformed investors suffers from higher chances of being allocated a disproportionately large amount of shares in "bad deals". Empirical evidence of IPOs suggests that the degree of asymmetry seems to be correlated with the size of the issue. The larger the issue the higher the chances of professional management and transparency, so more information about the true valuation reduces the degree of asymmetric information.

B. Optimisation problem of informed investors

Since price discovery in our DRM is contingent on profitable informed investment, we first derive the acceptance set of allocation choices that generate positive net payoffs at a fixed price-quantity schedule for eligible (i.e. sufficiently precise) beliefs about actual issue quality. At this stage we represent uniform informed investment demand by *one* informed investor. Informed investor belief s about the true issue quality is associated with an absolutely continuous distribution function $G_s(r)$ of valuation $r \in R$ with positive conditional density $g_s(r) > 0$ continuous in the interior of S, where $g_{s'}(r)/g_s(r)$ strictly increases for all $r \in R$, given any pair of signals $(s,s') \in \Omega(s)$ with s' > s [Monotone Likelihood Ratio Property (MLRP)]. The conditional and unconditional expected return of the issue at valuation r is defined as $\mu_s = \int_R rg_s(r)dr$ and $\int_S f(s)g_s(r)ds$ respectively. Given a repayment contract³¹ s(r) with f(s) > 0, we re-specify expected investor return as

$$u_{s}(r) = \int_{\mathbb{R}} c(r) g_{s}(r) dr. \tag{1}$$

If the noisy signal ς is deemed to be sufficiently precise, informed investors would only request an allocation $0 \le q(s) \le 1$ as a continuous function of updated investor beliefs s, where the associated offering price implies positive payoff for $p(s) \le u_s(r)$, which is binding at the optimum.³² In order to devise a rule of

³¹ Fractional repayment arises if issuers retain some expected return ("first loss provision"/"credit enhancement") as a positive effort choice to guarantee residual claims over and above full payment on issued securities. We follow the credit decision approach by Maskin and Tirole (1990 and 1992) in modelling the specification of the overall repayment level to investors.

³² This specification restricts the specification of repayment in Inderst and Müller (2002), where informed investment maximise gross payoff for a menu $m \in M$ of possible repayment contracts $c_m(r) \in C$, to a single repayment contract. In keeping with Innes (1990) as well as Marzo and Duffie (1999) we assume that repayment is non-decreasing in investment returns. In lending relationships borrowers could realise *ex post* arbitrage gains by borrowing cash to boost expected

action for optimal informed investment, we need to specify a lower bound of informed investor belief s with associated conditional investor payoff $u_s(r)$ to yield profitable investment.

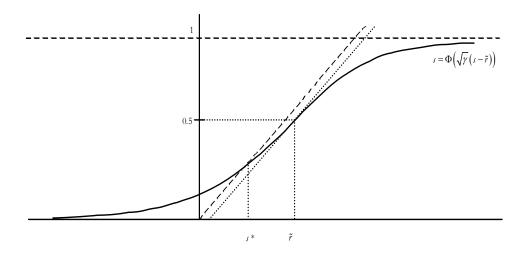


Fig 1. Cumulative distribution function of updated investor beliefs.

Informed investors adjust their beliefs about the realisation r with contractual repayment s(r) to the weighted measure $s = (\alpha \tilde{r} + \beta \varsigma)/(\alpha + \beta)$ with $s \in S = [0,1]$ upon observing noisy signal ς . The distribution functions F(s) and $F(\varsigma)$ with f(s) > 0 and $f(\varsigma) > 0$ are absolutely continuous and common knowledge. Informed investors consider ς sufficiently accurate only if precision measure $\gamma = (\alpha^2(\alpha + \beta))/\beta(\alpha + 2\beta)$ is small enough to satisfy $\gamma \le 2\pi$, so that (weighted) signal s meets critical value s^* as unique solution to the cumulative distribution function $s^* = \Phi(\sqrt{\gamma}(s^* - \tilde{r}))$, where $\Phi(.)$ denotes a standard normal distribution.³³ Higher precision (at a low γ) reflects lower valuation uncertainty of informed investor belief s about the realisation of r. The critical level s^* (see Fig. I) is obtained at the intersection of the c.d.f. of $\Phi(.)$ with the 45 degree line, which divides the indeterminate region [0,1] around its mid point. The critical level s^* diverges to the left from mean \tilde{r} the less precise the signal.

future cash flows and qualify for some lending criteria if contractual repayment generated from an investment project was to decrease over some subset of realised project payoffs (Innes, 1990).

³³ In this set-up we ignore the co-ordination problem of several agents in Morris and Shin (2000).

Conversely, if the signal becomes less noisy, s^* approximates \tilde{r} at $\Phi(.) = 0.5$. As noise becomes negligible in the limit, the curve of $\Phi(.)$ flattens out, and γ and s^* tend to zero and 0.5 respectively. Once signal $s > s^*$ passes muster as sufficiently precise private information, informed investors consider a profitable allocation level that satisfies $u_s(r) \ge p(s)$. At s^* the utility of private information from noisy signal ς is zero and non-random. The expectation of r is only conditional on s^* , which is s^* itself. Since noise β of signal s is independent of r, informed investor are uniformly indifferent at s^* in expectation of valuation r.

Since all eligible signals $s > s^* \in S$ of sufficient precision belong to the absolutely continuous distribution function $G_s(r)$ and each allocation level is subject to a fixed price-quantity schedule with the general property $p(s) = q(s)^a$ ($0 \le a \le 1$), by monotonicity we obtain an optimisation problem with a simple crossing property and an unconstrained maximum. Provided that informed investors only disclose their private information if their allocation generates positive net payoff, we define two cases of the relationship between (implied) offering price and expected investment return: $u_s(r) < p(s)$ and $u_s(r) \ge p(s)$, which rules out the trivial case of either positive or negative signals for all levels of adjusted investor beliefs s about valuation $r \in R$. Since we assume the margin of indifference to divulge private information to be a zero-probability event, we include the case $u_s(r) = p(s)$ of zero payoffs from informed investment in the acceptance set as boundary condition. Note that the repayment level restricts the acceptance set of profitable informed investment.³⁴

2.

³⁴ With full repayment (i.e. no restriction on conditional return from valuation r by some repayment contract), we would need to distinguish the less restrictive conditions $\mu_s < p(s)$ and $\mu_s > p(s)$. This consideration reflects the repayment choice in securitisation – the lower the quality of securitised assets, the higher the level of required credit support as reservation utility and the lower repayment from the realised portfolio value as higher expected default reduces expected returns from the securitised asset pool.

Lemma. The acceptance set of informed investors for repayment c(r) is defined by $\Omega(s) = \left\{ s \in S \middle| s \geq s * \land s \geq \underline{s} \text{ s.t. } u_s(r) \geq p(s) \right\} \subseteq S \text{ with cut-off signal } \underline{s} \in [0,1] \text{ with zero profit from informed investment at } u_{\underline{s}}(r) = p(\underline{s}). \text{ Unless } u_s(r) < p(s) \text{ with } \underline{s} = 1, \underline{s} \geq s * \text{ is unique and informed investment occurs for all } s \geq \underline{s} \geq s * > 0.$

Based on Lemma, we can derive the net payoff from optimal informed investment for allocation choice q(s) and conditional return $u_s(r)$ with $g_s(r) > 0$ for each belief s within the acceptance set $\Omega(s)$. Informed investors derive the first best solution of their optimisation problem by requesting allocation $0 \le q_I(s) \le 1$ for payment of offering price $p_I(s)$, which maximises the concave objective function

$$U(s)_{I} = \max_{q(s)} \int_{\Omega(s)} q_{I}(s) \left(u_{s}(r) - p_{I}(s)\right) f(s) ds, \qquad (2)$$

where the optimal allocation choice $q_I^*(s) = \sqrt[q]{u_s(r)/(a+1)}$ implies $p_I^*(s) = q_I^*(s)^a = u_s(r)/(a+1)$ under the general property of a fixed price-quantity schedule. Note that non-decreasing repayment $\varepsilon(r)$ yields surplus $\int_{\Omega(s)} q_I^*(s) (\mu_s - u_s(r)) f(s) ds$ as reservation utility from $\int_{\Omega(s)} q_I^*(s) (\mu_s - p_I^*(s)) f(s) ds$ before repayment at valuation r. Since informed investors optimise net payoff $U(s) \ge 0$ over acceptance set $\Omega(s)$ the probability of profitable informed investment for all eligible private signals is illustrated as the shaded area in Fig. II for the distribution function $F(\varepsilon(r))$ and F(s) of expected conditional return and adjusted belief, where the expectation of realisation r for precision measure γ is exactly s^* . Hence, this probability measure reflects the chances of private information about the actual issue to be sufficiently accurate for consideration of profitable investment within acceptance set $\Omega(s)$ in Lemma. We will revisit this interim

observation at a later stage of our analysis (see section D) when we were to represent issuer payoff over the entire range of $r \in [0,1]$.

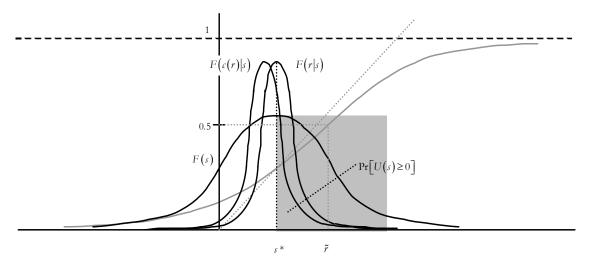


Fig II. Probability of profitable informed investment given updated investor belief.

C. Issuer payoffs under optimal informed investment

For illustrative purposes, we first determine issuer payoffs for our issue design problem with price discovery through first-best informed investment at optimal allocation $q_I^*(s)$ within the acceptance set under *uniform* and *discriminatory* pricing, ³⁵ with informed investors acting as price setters for uninformed investment demand. Under *uniform* pricing, both informed and uninformed investors pay the same offering price, which creates straightforward incentive compatibility. All investors obtain positive payoff with certainty, with uninformed investors being rationed at a rate of $\tau = 1/(q_I^*(s) + \theta)$. With total issue volume set to unity, complete allocation at uniform price $p_I^*(s)$ generates issuer payoff

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³⁵ Note that by restricting ourselves to solving the design problem for maximum informed investor payoff, we deliberately disregard valuation uncertainty and the associated composition of the investment demand as a determinant of the optimal allocation choice by the issuer to achieve a sustainable equilibrium outcome.

$$E(\Pi)_U = \int_{\Omega(s)} p_I^*(s) f(s) ds = \int_{\Omega(s)} \left(u_s(r) / (a+1) \right) f(s) ds = \int_{\Omega(s)} u_s(r) f(s) ds, \tag{3}$$

where informed investors obtain $U(s)_I$ in (2). Since the remainder, $1-q_I^*(s)$, is tendered to uninformed investors at the same offering price to clear the market, they each receive expected net payoff

$$U(s)_{U} = \frac{1}{\theta} \int_{\Omega(s)} \left(1 - q_{I}^{*}(s)\right) \left(u_{s}(r) - p_{I}^{*}(s)\right) f(s) ds.$$

$$\tag{4}$$

Issuers can increase their expected issue payoff $E(\Pi)_U$ through a minimum allocation of claims at a slightly discounted offering price within acceptance set $\Omega(s)$. A low value of $u_s(r)$ further limits the absolute measure of underpicing. However, uniform pricing could weaken incentives of informed investors to engage in price discovery for an efficient allocation choice, as net payoff $U(s)_I$ of informed investors might even be smaller than individual payoff $U(s)_U$ of uninformed investors at high valuation uncertainty. If $(1-q_I^*(s))/\theta \ge q_I^*(s) \Leftrightarrow \theta \le (1-q_I^*(s))/q_I^*(s)$ informed investors³⁶ may choose to misrepresent their type for a given expected conditional return $u_s(r)$. Only a high incidence of uninformed investors associated with low valuation uncertainty preserves informed investment demand for efficient allocation choices in $\Omega(s)$ under uniform pricing.

Note that higher valuation uncertainty is reflected in lower precision (i.e. a high γ measure, see section B) of informed investor belief s about the realisation of r, which decreases acceptance set $\Omega(s)$ as the critical value s^* increases. If $U(s)_I$ were to be kept constant, a higher allocation $q_I^*(s)$ associated with a smaller range of

³⁶ For the determination of this threshold of uninformed investment demand, we maintain the assumption of uniform informed investment behaviour, such that our comparative statics are only influenced by the number of informed investors in relation to positive net payoffs from investment.

profitable allocation choices leaves a smaller residual allocation $1-q_I^*(s)$ to uninformed investors θ . Since uninformed investors are limited to one share each, higher rationing at lower $\tau = 1/(q_I^*(s) + \theta)$ leaves a smaller number of uninformed investors θ in the investor pool, who might possibly claim $U(s)_U \ge U(s)_I$.³⁷

Proposition 1 [Valuation uncertainty and acceptance set]. Lower valuation uncertainty increases the acceptance set $\Omega(s)$ and decreases both the optimal allocation to informed investors and underpricing. Lower (higher) valuation uncertainty also implies a higher (lower) incidence of uninformed investors.

Proposition 2 [Uniform pricing]. Under uniform pricing the issuer extracts most informed investor surplus by keeping the perceived valuation and valuation uncertainty as low as possible within acceptance set $\Omega(s)$ according to Proposition 1, while preventing misrepresentation by informed investors.

Alternatively, issuers might have discretion in tendering the residual allocation to uninformed investors at an offering price higher than the offering price $p_I^*(s)$ implied by a first-best allocation to informed investors. Since both types of investors act independently, *discriminatory* pricing in favour of informed investors allows the issuer to extract more surplus from investors, while it eliminates the danger of misrepresentation by informed investors. Discriminatory pricing can satisfy the incentive compatibility constraint $U(s)_I \ge U(s)_U$ invariant to the incidence of uninformed investors. The issuer allocates the proportion $0 \le q_I^*(s) \le 1$ of the issue to informed investors at price $p_I^*(s)$. The remainder $q_U(s) \le 1 - q_I^*(s)$ is offered to uninformed

³⁷ At the same time, we could also argue this aspect from the perspective of underpricing in line with the IPO underpricing model by Rock (1986). Valuation uncertainty represents an (implicit) "outside option", where uninformed issuers would expect higher underpricing associated with a higher rationing rate for higher levels of valuation uncertainty, which increases the option value. Lower valuation uncertainty implies higher levels of market information about the true issue quality and lower discounting of the uniform offering price as uninformed investors would assume lower chances of being outsmarted by informed investors.

investors to clear the market. The maximum offering price $p_U(s)$ the issuer can charge to uninformed investors is

$$p_{U}^{*}(s) = \max \left\{ p_{I}^{*}(s), u_{s}(r) - \frac{q_{I}^{*}(s)}{q_{U}(s)} (u_{s}(r) - p_{I}^{*}(s)) \theta \beta \right\},$$
 (5)

which solves inequality

$$\beta U(s)_{I} \geq U(s)_{U} \Leftrightarrow \beta \int_{\Omega(s)} q_{I}^{*}(s) \left(u_{s}(r) - p_{I}^{*}(s)\right) f(s) ds \geq \frac{1}{\theta} \int_{\Omega(s)} q_{U}(s) \left(u_{s}(r) - p_{U}(s)\right) f(s) ds , \tag{6}$$

where fraction $0 < \beta \le 1$ denotes the multiple of the payoff received by all informed investors at allocation $q_t^*(s)$ to the maximum net payoff of each uninformed investor at allocation $q_U(s) \le 1 - q_t^*(s)$. The measure β becomes binding if informed investors expect β -times higher informed payoff than individual uninformed investment payoff, which requires $\partial \theta / \partial \beta = -1$. Thus, expected issuer payoff under discriminatory pricing would be

$$E(\Pi)_{D} = \int_{\Omega(s)} p_{I}^{*}(s) f(s) ds + \int_{\Omega(s)} q_{U}(s) (p_{U}^{*}(s) - p_{I}^{*}(s)) f(s) ds = \int_{\substack{a \to 0, \\ q_{U}(s) = 1 - q_{I}^{*}(s)}} \int_{\Omega(s)} u_{s}(r) f(s) ds$$
(7)

Proof of equation (7). See Appendix.

Since $p_U^*(s) \ge p_I^*(s)$, the issuer could extract more surplus from uninformed investors, so that expected issuer gross payoffs under discriminatory pricing satisfies

$$E(\Pi)_{D} \geq E(\Pi)_{U} \Leftrightarrow \int_{\Omega(s)} \underbrace{\left[(1+a) - \sqrt[d]{\frac{u_{s}(r)}{1+a}} (2+a(1-\theta\beta)) \right]}_{\geq 1} u_{s}(r) f(s) ds \geq \int_{\Omega(s)} u_{s}(r) f(s) ds , \quad (8)$$

within the range for all $0 \le a \le 0$. Only in the limit of $a \to 0$, when the selling price equals unity, would issuers be indifferent between both pricing regimes.

Proposition 3 [Discriminatory pricing]. Discriminatory pricing allows issuers to charge uninformed investors a higher offering price than informed investors to achieve separation. Higher relative payoff of informed investors (regardless of the degree of uninformed investment demand by Proposition 1) completely eliminates the incentive of misrepresentation. The issuer extracts most informed investor surplus by keeping the valuation as low as possible within acceptance set $\Omega(s)$.

D. Optimal allocation for maximum issuer payoffs

The ability of issuers to achieve complete allocation within acceptance set $\Omega(s)$ of profitable informed investment under different pricing regimes indicates the importance of the incidence of investor types in our issue design problem. However, the residual allocation to uninformed investors and the incentive of informed investors to participate in DRM at any issue quality – as long as some allocation yields positive payoff – curtail the ability of informed investors to optimise own payoffs by disclosing their beliefs. So far, we have not recognised the allocation level as a strategic choice variable of issuers. In the following section we derive the conditions for maximum expected issuer payoffs in an auction-style issuing process under uniform and discriminatory pricing, where the issuer's allocation choice satisfies the acceptance set $\Omega(s)$. In line with the general notion of a fixed price-quantity schedule in the previous section, we now derive the offering price from an auction-style allocation choice of informed investors as a continuous function of adjusted beliefs about the actual issue quality. We also assume multiple informed investors to compare individual investor payoffs similar to our approach in section C.

Under discriminatory pricing issuers discount their allocation to informed investors and solve the allocation choice for optimal (gross) payoffs by offering the residual allocation to uninformed investors at a fair (market) price. This implies zero net payoffs from uninformed investment while completely denying informed investors incentives of misrepresenting themselves as uninformed types (with relative benefits of price discovery increasing in $\beta \to 0$). Since the issue mechanism depends on the participation of informed investors for an allocation choice within the acceptance set $\Omega(s)$, the issuer chooses to discount the issue for $p(s) < u_s(r)$ at unit offering price p(s)/q(s) and acceptable allocation $0 \le q(s) \le 1$ according to the fixed price-quantity schedule.³⁸ In extension to the previous section, we model the allocation choice as a continuos function of investor beliefs about the true issue quality to represent the fixed price-quantity schedule. The remainder 1-q(s) is tendered to all uninformed investors at the offering price $p(s) = u_s(r)$, so that

$$E(\Pi)_{D} = \max_{p(s),q(s)} \int_{\Omega(s)} \left(\frac{p(s)}{q(s)} q(s) + u_{s}(r) (1 - q(s)) \right) f(s) ds$$

$$= \max_{p(s),q(s)} \int_{\Omega(s)} \left(p(s) + u_{s}(r) (1 - q(s)) \right) f(s) ds.$$
(9)

Under uniform pricing the issuer offers the same selling price to both types of investors at individual allocation rates of q(s)/I and $(1-q(s))/\theta$ respectively to maximise expected payoff

³⁸ The variables p(s) and q(s) are used as shorthand to denote the offering price and the allocation to informed investors. For simplicity we drop the index for the investor type from the notation in the remainder of the chapter, as the allocation to uninformed investors is not a strategic parameter choice and follows from the price-quantity schedule of informed investors.

$$E(\Pi)_{U} = \max_{q(s), p(s)} \int_{\Omega(s)} \left(\frac{p(s)}{q(s)} q(s) + \frac{p(s)}{q(s)} (1 - q(s)) \right) f(s) ds = \max_{q(s), p(s)} \int_{\Omega(s)} \frac{p(s)}{q(s)} f(s) ds .^{39}$$
(10)

We solve the above optimisation problem in (9) and (10) for both pricing regimes by means of a DRM auction model adapted from Myerson (1981), where the issuer maximises own payoffs over a positive measure of issue quality through an allocation choice within an acceptance set of profitable informed investment. Each allocation level of the acceptance set relies on a fixed price-quantity schedule implied by an auction-style allocation preference as a continuous function of perceived issue quality. This implies an offering price that satisfies the following participation and incentive constraints:

$$U(s)_{I} \ge 0 \Leftrightarrow q(s) \left(u_{s}(r) - \frac{p(s)}{q(s)} \right) = q(s)u_{s}(r) - p(s) \ge 0$$
(PC)

$$q(s)u_s(r) - p(s) \ge q(\hat{s})u_s(r) - p(\hat{s}) \quad \forall s, \hat{s}, r$$
 (IC₁)

$$U(s)_{I} \ge U(s)_{U} \Leftrightarrow u_{s}(r) \frac{q(s)}{I} - \frac{p(s)}{q(s)} \ge u_{s}(r) \frac{1 - q(s)}{\theta} - \frac{p(s)}{q(s)}$$

$$\Leftrightarrow q(s) \ge \frac{I(1 - q(s))}{\theta} \Leftrightarrow 1 \ge q(s) \ge \frac{I}{I + \theta},$$
(IC₂)

where $g_s(r) > 0$ is strictly continuous. IC₂ applies only to uniform pricing, ensuring that the proposed allocation-based direct information revelation awards informed investors higher individual net payoff.⁴⁰ We

³⁹ Note that if we wanted to represent issuer payoff over the entire range of $r \in [0,1]$, we would need to adjust our maximisation problem by the probability of informed investment to occur (see section IV.B).

 $^{^{40}}$ IC₂ implies a higher (lower) allocation to informed investors in response to a higher (lower) number of informed investors relative to the number of informed investors associated with high (low) uncertainty. For efficient price discovery under uniform pricing, knowledge about θ (as a determinant of the allocation schedule) registers as a critical factor. We know from section III that only the distribution of uninformed types is commonly known. However, if

consider the allocation choice q(s) a continuous function of investor belief $s \in \Omega(s)$. From rewriting IC₁ and PC above (see Proof Theorem 1) we obtain an alternative definition of non-decreasing and absolutely continuous $U(s)_I \ge 0$ with $U'(s)_I = q(s)$

$$U(s)_{I} = U(\underline{s})_{I} + \int_{\Omega(s)} q(s) ds.$$
(11)

Combining PC and (11) with $U(\underline{s})_I = 0$ (see Lemma) yields the "allocation-based" offering price

$$p(s) = q(s)u_s(r) - \int_{\Omega(s)} q(s)ds, \qquad (12)$$

Theorem 1 and 2 follow from substituting (12) in equations (9) and (10) respectively.

Theorem 1 [Discriminatory pricing]. The issuer maximises own payoff under discriminatory pricing by solving $\max_{q(s)} \int_{\Omega(s)} \left(u_s(r) - \int_{\Omega(s)} q(s) ds \right) f(s) ds \text{ for } s \in \Omega(s), \text{ where } 0 \le q(s) \le 1 \text{ is non-decreasing.}$

Proof of Theorem 1. See Appendix.

informed investors could estimate θ valuation uncertainty, issuer payoffs would decrease in the precision of investor knowledge about θ as IC₂ would become more restrictive. The lack of information about the presence of uninformed investors adds inefficiency to the maximisation problem of issuers in Theorem 2 (see section IV.D). Chances are that informed investors would be more inclined to misrepresent themselves under uniform pricing unless they can claim higher net payoffs as they refine their investment decision. Given a precision measure $\gamma \in \Gamma$ from absolutely continuous $F(\gamma)$ with $f(\gamma) > 0$, informed investors might infer the realisation of uninformed investment $\theta \in \Psi \equiv \left[\underline{\theta}, \overline{\theta}\right]$ from the accuracy of their noisy signal ς . Conditioning $F(\gamma)$ on $g_{\gamma}(\theta) > 0$ yields the conditional number of uninformed as $E_{\gamma}(\theta) = \int_{0}^{2\pi} f(\gamma)g_{\gamma}(\theta)d\theta$, with the unconditional number of uninformed investors $E_{\gamma}(\theta) = \int_{0}^{2\pi} f(\gamma)g_{\gamma}(\theta)d\gamma$. In keeping with MLRP any pair $(\gamma, \gamma') \in \Gamma$ with $\gamma' > \gamma$ the ratio $g_{\gamma'}(\theta)/g_{\gamma}(\theta)$ is strictly increasing in θ for all $\theta \in \Psi$.

Theorem 2 [Uniform pricing]. The issuer maximises own payoff under uniform pricing by solving $\max_{q(s),p(s)} \int_{\Omega(s)} \left(u_s(r) - \int_{\Omega(s)} q(s) ds / q(s)\right) f(s) ds \text{ for } s \in \Omega(s), \text{ where allocation } q(s) \in [I/(I+\theta),1] \text{ is non-decreasing.}$

Proof of Theorem 2. See Appendix.

The issuer can mitigate underpricing and optimise the proposed issue design at the lowest possible allocation q(s) to informed investors within acceptance set $\Omega(s)$. We now derive the optimal range of allocation choices to maximise issuer payoff, with some underpricing required for profitable participation based on their private information about actual issue quality.

Corollary 1 [Discriminatory pricing]. Under discriminatory pricing and full allocation the issuer can extract investor surplus only up to $E(\Pi)_D \equiv \max_{q(s)} \int_{\Omega(s)} \left(u_s(r) - \int_{\Omega(s)} q(s) ds\right) f(s) ds \ge \int_{\Omega(s)} u_s(r) f(s) ds - \varepsilon$, which implies allocation $q_{\varepsilon}(s) \in \left[\sqrt[3]{6\varepsilon}, 1\right]$ to satisfy informed investment demand according to Lemma at discount $\varepsilon > 0$.

Proof of Corollary 1. See Appendix.

Corollary 2 [Uniform pricing]. Under uniform pricing and full allocation the issuer can extract investor surplus only up to $E(\Pi)_U \equiv \max_{q(s),p(s)} \int_{\Omega(s)} \left(u_s(r) - \int_{\Omega(s)} q(s) ds / q(s)\right) f(s) ds \ge \int_{\Omega(s)} u_s(r) f(s) ds - \varphi$ s.t. $1 - q(s) \le \theta / (I + \theta)$, which implies allocation $q_{\varepsilon}(s) \in \left[\sqrt[3]{6I/(I + \theta)}, 1\right]$ to satisfy informed investment demand according to Lemma at discount to $\varphi > 0$.

Proof of Corollary 2. See Appendix.

Corollary 1 verifies previous findings about higher sustainability of the proposed issue design under price discrimination, when only little allocation to informed investors suffices to induce price discovery by informed investors through an allocation preference and overall investor surplus ε invariant to uninformed investment demand. *Discriminatory* pricing allows issuers to extract the most investor surplus from informed investors, who might otherwise misrepresent themselves as uninformed types if $p(s) = p_U(s) \le u_s(r)$ under *uniform* pricing. This case requires a lower (higher) incidence of uninformed investors associated with a higher (lower) valuation uncertainty to coincide with a higher (lower) allocation to informed investors, so that each informed investor receives a higher individual payoff than uninformed investors (IC2), given overall investor surplus φ . Corollary 2 shows that the optimal rule of action of the issuer in the case of uniform pricing prescribes an allocation choice based primarily on the incidence of types rather than the degree of underpricing (see also section C).

V. DISCUSSION

In the course of the above analysis we saw that the prospect of informed investors to obtain positive payoffs from DRM-based disclosure of their private information about the true issue quality via allocation preference is fundamental to our issue design process. The acceptance set of profitable informed investment qualifies the optimal allocation schedule for maximum issuer payoffs from endogenous price discovery at varying degrees of valuation uncertainty and pricing regimes. Issuers maximise their payoffs over a positive measure of issue quality if the fixed price-quantity schedule implied by an auction-style allocation preference as a continuous function of perceived issue quality yields profitable informed investment. Moreover, a contractually predefined repayment level would restrict the acceptance set of perceived issue quality due to lower payoff to be appropriated by investors. We find that issuers would strictly prefer discriminatory over uniform pricing. Issuers can extract most surplus from informed investors as "truth tellers" by offering only marginal positive net payoff ("underpricing") through a certain allocation choice. The residual allocation to uninformed

investors and the incentive of informed investors to subscribe to DRM at any issue quality – as long as some allocation yields positive payoff – curtails the ability of informed investors to optimise own payoffs from disclosing their beliefs. So uninformed investment demand implicitly strengthens the position of issuers to maximise their payoffs under any pricing regime. Under *uniform* pricing, price discovery by informed investors is only sustainable if both the incidence of investor types and the allocation choice translate into higher individual profit of each informed investor relative to uninformed investors. Informed investors require higher underpricing under uniform pricing to obtain higher relative payoffs than uninformed investors in return for private information disclosure.⁴¹ Hence, uniform pricing generates (even) lower expected issuer payoffs than discriminatory pricing the higher the valuation uncertainty. Issuers would generally prefer a small (high) allocation to informed (uninformed) investors at low (high) valuation uncertainty to maximise own payoffs under either pricing regime. Again, the presence of uninformed investors, depending on the degree of valuation uncertainty contributes to the optimisation of issuer payoffs. The higher the incidence of uninformed investors, the lower the degree of underpricing due to the profitability constraint of informed investors under uniform pricing.

If we were to rule out price discrimination as a suitable pricing regime due to statutory provisions in securitisation markets, further analysis of our issue design model begs the question how the (strategic) allocation choice conditional on valuation uncertainty changes expected issuer payoffs under uniform pricing. Our preliminary findings in Corollary 1 and 2 suggest that higher informed investment demand associated with more valuation uncertainty and higher perceived issue quality always reduces issuer payoffs irrespective of the pricing regime – though the effect is larger under uniform pricing. We consider a numerical solution to illustrate optimal issuer payoffs under uniform pricing at varying allocation levels.

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⁴¹ This implies a low option value of informed investment from valuation uncertainty and a high precision of adjusted investor beliefs s at the limit $s \to \tilde{r}$.

In Fig. III we approximate net issuer payoff under uniform pricing in a quasi-closed form solution of Theorem 2, where the allocation choice to informed investors for $U(s)_{I}' = q(s)$ (see section IV.D) is a continuous function of perceived issue quality for all $s \in \Omega(s)$ (see Lemma in section IV.B). We obtain conditional investment return $u_s(r)$ from repayment $\varepsilon(r)$ at $s \in \Omega(s)$ by assuming the precision measure $\gamma \to 0$ (i.e. belief s becomes noiseless) to model how investor belief s translates into a corresponding realisation r according to MLRP of $G_s(r) > 0$ (see section IV.B). We set the discrete allocation level commensurate with the incidence of investors in accordance with $q(s) > I/(I + \theta)$ of IC₂. (see section IV.D) The cut-off signal is assumed to be $q(s) = \underline{s} = \{0.25; 0.5; 0.75\}$ for simplicity. The issuer retains a reservation utility in the form of credit enhancement so that constant repayment c(r) = 0.9. For illustrative purposes we also show net issuer payoff for full repayment, c(r) = r, at q(s) = 0.25 and cut-off signal $\underline{s} = 0.15$ (scaled to conditional expected return $u_s(r)$ on the x-axis of Fig. III). As we traverse different degrees of valuation uncertainty – proxied by the minimum discrete allocation level q(s) according to Theorem 2 (see section IV.D) – we find that optimal allocation to informed investors as a strategic choice variable to maximise issuer payoffs is contingent upon the valuation of conditional return $u_s(r)$. Once more informed investors participate at higher valuation uncertainty – so that only a high allocation q(s) satisfies IC₂ – higher valuation will engender higher issuer payoff. Conversely, we maximise issuer payoff only if lower issue valuation entails a matching reduction in valuation uncertainty. Fig. III represents optimal issue payoffs as an "efficient frontier" of deterministic allocation levels for given conditional expected return for all levels of issuer beliefs about actual issue quality. We derive a positively concave function as solution to the DRM design problem of issue payoffs if valuation uncertainty is continuous. The curvature is induced by continuous allocation preference $\int_{\Omega(s)} q(s) ds$ (see Proof of Theorem 1), which drains issuer profits as higher perceived issue quality increases informed investment demand in excess of q(s). This situation follows the basic routine of our model. If the allocation choice is not commensurate to informed investment demand contingent on perceived

issue quality, issuers cannot achieve optimal issue payoffs. We also observe that the reservation utility from partial repayment c(r) limits the acceptance set $\Omega(s)$ of eligible perceived issue quality. Fig. III also shows the efficiency loss associated with forgone net issue payoffs due to the reservation utility from repayment c(r) as the shaded area between the payoff curves at allocation q(s) = 0.25 for full repayment r and repayment c(r) respectively.

Both the comparative perspective of both pricing regimes and the graphical representation of issuer payoffs in Fig. III reveal two main insights into the mechanics of our model under uniform pricing. First, only high uninformed investment demand associated with low valuation uncertainty allows issuers to satisfy IC₂ at low valuation, while higher valuation uncertainty requires higher valuation for issuer payoff to remain the same. Second, we find that lower expected repayment facilitates higher valuation at lower (valuation) uncertainty to generate the same net issuer payoff.

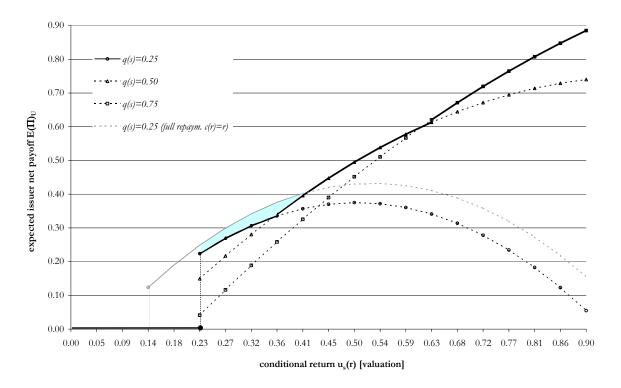


Fig. III. Approximated optimal issuer payoffs under uniform pricing at varying levels of valuation uncertainty.

VI. CONCLUSION

Securitisation markets are marred by problems of asymmetric information between market makers with superior knowledge about securitised asset exposures and uninformed investment demand, where issuers frequently sound out a fair market price from sophisticated investors before they issue new securities. The potential effects of this market configuration on price formation, however, have mostly been acknowledged in the academic and professional literature as agency costs of "winner's curse"-type underpricing.

In the course of the above analysis, we addressed this issue in a general allocation-based, auction-style issue design based on price discovery by informed investors. We presented a basic model framework of optimal security issuance in the spirit of the conventional, allocation-based argument of IPO underpricing due to asymmetric information between issuers and investors. However, our approach did not reason underpricing on the grounds of the "winner's curse" problem. Instead of compensating rationed uninformed investors, price discounting in our general issue design ensured profitable informed investment over a positive measure of issue quality to maximise issuer proceeds. We formalised a direct revelation mechanism (DRM) with a fixed pricequantity schedule, which endogenised price discovery in an auction-style allocation preference as a continuous function of perceived issue quality. Our thinking was mainly guided by sustainable allocation-based price discovery, assuming that a monopolistic issuer can only solicit "truth telling" from informed investors if their allocation choice yields profitable investment. The resultant acceptance set of efficient allocation choices qualified maximum issuer payoffs at varying degrees of valuation uncertainty and pricing regimes. With uninformed investment demand clearing the market, we studied how the incidence of uninformed investors at varying levels of valuation uncertainty affects the utility of informed investment especially under uniform pricing. Hence, we explored underpricing as jointly determined by profitable allocation by informed investors and the incidence of uninformed investment demand. We also conditioned profitable informed investment on some exogenous repayment level to account for structural support mechanisms in securitisation markets.

We found that - irrespective of the applicable pricing mechanism - the issuer maximises own payoffs at the lowest possible allocation (within the acceptance set of efficient allocation choices) that still implies profitable informed investment. Although discriminatory pricing yields higher issuer payoffs, our evidence suggests that issuers could mitigate forgone net payoffs under uniform pricing by maintaining low valuation uncertainty at moderate levels of issue quality to induce a high presence of uninformed investors. Uninformed investment demand implicitly strengthens the position of issuers to maximise own payoffs, mainly because it lowers the degree of underpricing needed to satisfy the profitability constraint of informed investors. Under uniform pricing, the issuer needs to ensure that the composition of the investor pool allows informed investors to appropriate higher individual profit (than uninformed types). Otherwise, they might be inclined to request no allocation at all (i.e. misrepresent themselves as uninformed investors) due to insufficient profitability from price discovery in DRM. Any reservation utility from partial repayment carried an efficiency loss and required a higher issue valuation. The degree of valuation uncertainty critically mattered only under uniform pricing, where an altered incidence of investor types required an adjustment of the allocation choice to still guarantee profitable informed investment at the highest possible level of issuer payoffs. Since a higher (lower) allocation to informed investors at higher (lower) valuation uncertainty and a lower (higher) incidence of uninformed investors implies higher (lower) underpricing, we would expect the minimisation of valuation uncertainty to be the dominant strategy for each level of valuation at the margin (cf. second moment of payoff curve in Fig. III). The issuer maximised payoffs under uniform pricing by following an "efficient frontier" of allocation choices across all states of issue quality, where the amount of implied investment induced information disclosure by informed investors as a continuous function of perceived issue valuation. Nonetheless, informed investors never receive an allocation that maximises their own payoffs from investment unless high valuation uncertainty rules out any uninformed investment demand.

Overall this chapter represents a first attempt to reason underpricing on the grounds of a strategic allocation choice by issuers to maximise own payoffs by engaging informed investors in profitable price discovery of actual issue quality. The coincidence of valuation uncertainty and the allocation choice for a certain level of

perceived issue quality seems to be a prime consideration for optimal issuer payoffs under asymmetric information. While our approach might be overly parsimonious in many respects, we have restricted our issue design to include the reservation utility from a pre-defined level of repayment as the only element pertinent to securitisation markets. Hence, the general tenor of our model invites a more specialised adaptation of our findings to different asset types and entertains the need for more refined modelling of intricate security design features of asset-backed securities, such as the impact of option clauses, loss subordination and payment structures. Also the possible relaxation of several exogenous assumptions in our model design, such as the repayment level and uniform informed investment, warrants further theoretical investigation.

VII. REFERENCES

- Aggarwal, R. K., Krigman, L. and K. L. Womack (2002), "Strategic IPO Underpricing, Information Momentum and Lockup Expiration Selling," *Journal of Financial Economics*, Vol. 66, 105-37.
- Akerlof, G. A. (1970), "The Market for Lemons: Quality Uncertainty and the Market Mechanism," *Quarterly Journal of Economics*, Vol. 84, 488-500.
- Amihud, Y., Hauser, S. and A. Kirsh (2003), "Allocations, Adverse Selection, and Cascades in IPOs: Evidence from the Tel Aviv Stock Exchange," *Journal of Financial Economics*, Vol. 68, 137-58.
- Basle Committee on Banking Supervision (2004a). *Changes to the Securitisation Framework*. Basle Committee Publications No. 105 (30 January) (available at http://www.bis.org/publ/bcbs105.htm).
- Basle Committee on Banking Supervision (2004b). *International Convergence of Capital Measurement and Capital Standards: Credit Risk Securitisation Framework*. Basle Committee Publications No. 107 (11 May) (available at http://www.bis.org/publ/bcbs107.htm).
- Basle Committee on Banking Supervision (2002), "Second Working Paper on Asset-Backed Securitisation",
 Bank for International Settlements (BIS), October.
- Basle Committee on Banking Supervision (2001), "First Working Paper on Asset-Backed Securitisation",

 Bank for International Settlements (BIS), October.
- Biais, B., Bossaerts, P., and J. C. Rochet (2002), "An Optimal IPO Mechanism," Review of Economic Studies, Vol. 69, 117-46.
- Boot, A. W. A. and A. V. Thakor (2000), "Can Relationship Banking Survive Competition?" *Journal of Finance*, Vol. 55, 679-713.
- Crémer, J. and R. McLean (1988), "Full Extraction of the Surplus in Bayesian and Dominant Strategy Auctions," *Econometrica*, Vol. 56, 1247-57.
- DeMarzo, P. and D. Duffie (1999), "A Liquidity-Based Model of Security Design," *Econometrica*, Vol. 67, No. 1, 65-99.
- Duffie, D. and N. Gârleanu (2001), "Risk and Valuation of Collateralised Debt Obligations," Financial Analysts Journal, Vol. 57, No. 1, 41-59.

- Gibson, M. (2004), "Understanding the Risk of Synthetic CDOs," Working Paper, Division of Research and Statistics, Federal Reserve Board, Washington, D.C.
- Inderst, R. and H. M. Müller (2002), "Credit Risk Analysis and Security Design," Working Paper S-CDM-02-13, Stern School of Business, Salomon Center (November).
- Jobst, A. (2003), "Collateralised Loan Obligations (CLOs): A Primer," The Securitization Conduit, Vol. 6, Nos.
 1-4.
- Keloharju, M. (1993), "The Winner's Curse, Legal Liability, and the Long-run Price Performance of Initial Public Offerings in Finland," *Journal of Financial Economics*, Vol. 34, 251-77.
- Koh, F. and T. Walter (1989), "A Direct Test of Rock's Model of Pricing of Unseasoned Issues," *Journal of Financial Economics*, Vol. 23, 251-72.
- Leland, H. (1998), "Agency Cost, Risk Management and Capital Structure," *Journal of Finance*, Vol. 52, 1213-43.
- Leland, H. and D. Pyle (1977), "Information Asymmetries, Financial Structure, and Financial Intermediation," *Journal of Finance*, Vol. 32, 371-87.
- Levis, M. (1990), "The Winner's Curse Problem, Interest Costs, and the Underpricing of Initial Public Offerings," *Economic Journal*, Vol. 100, 76-89.
- Malakov, A. (2003), "The Role of Uninformed Buyers in Optimal IPO Mechanisms," Working Paper, Kellogg School of Management, Northwestern University.
- Manove, M., Padilla, A. J., and M. Pagano (2001), "Collateral vs. Project Screening: A Model of Lazy Banks," Rand Journal of Economics, Vol. 32, 726-44.
- Milgrom, P.R. (1981), "Good News and Bad News: Representation Theorems and Applications," *Bell Journal of Economics*, Vol. 12, 380-91.
- Morris, S. and H. S. Shin (2000), "Rethinking Multiple Equilibria in Macroeconomic Modelling," NBER Macroeconomics Annual Conference 2000.
- Myers, S. C. (1977), "Determinants of Corporate Borrowing," Journal of Financial Economics, Vol. 5, 147-75.
- Myerson, R. (1981), "Optimal Auction Design," Mathematics of Operations Research, Vol. 6, 58-73.

- Riddiough, T. J. (1997), "Optimal Design and Governance of Asset-Backed Securities," *Journal of Financial Intermediation*, Vol. 6, No. 2, 121-52.
- Rock, K. (1986), "Why New Issues are Underpriced," Journal of Financial Economics, Vol. 15, 187-212.
- Rutledge, A. (2004), "ABS Analysis A Rigorous Approach," The RMA Journal (December), 26-29.
- Welch, I. and J. Ritter (2002), "A Review of IPO Activity, Pricing and Allocations," Yale ICF Working Paper No. 02-01.
- Zweig, P. L. (2002), "Asset-Backed Securities," in: Henderson, D. R. (ed.). *The Concise Encyclopedia of Economics*.

 The Library of Economics and Liberty, Liberty Fund Inc. (available at http://www.econlib.org).

VIII. APPENDIX: PROOFS

Proof of equation (7).

q.e.d.

Proof of Theorem 1. In keeping with the standard logic of the optimal auction model by Myerson (1981) we can re-write IC and PC in section IV.D in order to substitute the pricing component of the optimisation problem as an allocation-based profitability constraint. We re-write (IC₁) in terms of \hat{s} and \hat{r} as

$$U(\hat{s})_{I} \ge 0 \Leftrightarrow q(\hat{s})u_{s}(r) - p(\hat{s}) \ge q(s)u_{s}(\hat{r}) - p(s) \quad \forall s, \hat{s}, r, \hat{r}.$$
 (IC₁')

Combining IC₁' with IC₁ for $U(s)_I - U(\hat{s})_I \ge 0$ yields

$$q(\hat{s}) \le \frac{U(s)_I - U(\hat{s})_I}{u_s(\hat{r}) - u_s(\hat{r})} \le q(s), \tag{14}$$

which implies $U(s)_I' = q(s)$ for $u_s(\hat{r}) \to u_s(r)$ with continuous q(s). Hence, we can derive $U(s)_I = U(\underline{s})_I + \int_{\Omega(s)} q(s) ds$, where the assessment of cut-off signal \underline{s} yields zero payoff of informed investors set. Combining IC₁ and PC to

$$U(\underline{s})_{I} + \int_{\Omega(s)} q(s)ds = q(s)u_{s}(r) - p(s) \ge 0$$
(15)

yields the allocation-based offering price

$$p(s) = q(s)u_s(r) - \int_{\Omega(s)} q(s)ds, \qquad (16)$$

where $U(\underline{s})_I = 0$ as optimal mechanism for non-decreasing and absolutely continuous $U(s)_I \ge 0$. Substituting equation (16) into the optimisation problem in (9) yields

$$E(\Pi)_{D} = \max_{q(s)} \int_{\Omega(s)} \left(p(s) + u_{s}(r) (1 - q(s)) \right) f(s) ds \qquad p(s) = q(s) u_{s}(r) - \int_{\Omega(s)} q(s) ds$$

$$= \max_{q(s)} \int_{\Omega(s)} \left(q(s) u_{s}(r) - \int_{\Omega(s)} q(s) ds + u_{s}(r) (1 - q(s)) \right) f(s) ds \qquad (17)$$

$$= \max_{q(s)} \int_{\Omega(s)} \left(u_{s}(r) - \int_{\Omega(s)} q(s) ds \right) f(s) ds.$$

q.e.d.

Proof of Theorem 2. Analogous to the Proof of Theorem 1, the optimal price-quantity schedule under uniform pricing hinges only on the continuous allocation q(s) to informed investors for $U(s)_t \ge 0$. We substitute $p(s) = q(s)u_s(r) - \int_{\Omega(s)} q(s)ds$ into the optimisation problem and obtain

$$E(\Pi)_{D} = \max_{q(s), p(s)} \int_{\Omega(s)} \frac{p(s)}{q(s)} f(s) ds \Leftrightarrow \max_{q(s), p(s)} \int_{\Omega(s)} \frac{q(s)u_{s}(r) - \int_{\Omega(s)} q(s) ds}{q(s)} f(s) ds$$

$$\Leftrightarrow \max_{q(s), p(s)} \int_{\Omega(s)} u_{s}(r) - \frac{\int_{\Omega(s)} q(s) ds}{q(s)} f(s) ds$$
(18)

s.t. IC₂ to prevent informed investors from misrepresenting their type.

q.e.d.

Proof of Corollary 1. Since $U(s)_I' = q(s)$ for $u_s(\hat{r}) \to u_s(r)$ (see Proof of Theorem 1) of profitability constraint $U(s)_I \ge 0$ for $s \in \Omega(s)$, let us assume that some investor surplus $\varepsilon > 0$ (which implies $u_s(r) > u_{\underline{s}}(r)$ for profitable informed investment in Lemma) as upper bound of "underpricing" involves allocation $q_{\varepsilon}(s) = A_{\varepsilon}(u_s(r) - u_{\underline{s}}(r))$ (with $A_{\varepsilon} \in]0,1]$) so that the issuer appropriates payoff $E(\Pi)_D = \int_{\Omega(s)} \left(u_s(r) - \left(\int_{\Omega(s)} q_{\varepsilon}(s) ds\right)\right) f(s) ds = \int_{\Omega(s)} u_s(r) f(s) ds - \varepsilon$ (see Theorem 1) under discriminatory pricing (see Theorem 1). Issuers minimise the amount of underpricing

$$\int_{\Omega(s)} \left(\int_{\Omega(s)} q(s) ds \right) f(s) ds \le \int_{\Omega(s)} \left(\int_{\Omega(s)} q_{\varepsilon}(s) ds \right) f(s) ds = \frac{A_{\varepsilon}}{6} \left(u_{s}(r) - u_{\underline{s}}(r) \right)^{3} = \varepsilon, \tag{19}$$

which yields $A_{\varepsilon}(u_s(r) - u_{\underline{s}}(r)) = q_{\varepsilon}(s) = \sqrt[3]{6\varepsilon}$ as the optimal allocation schedule for investor surplus $\varepsilon > 0$ with issuer payoff $E(\Pi)_D = \int_{\Omega(s)} u_s(r) f(s) ds - \varepsilon$ under discriminatory pricing and full allocation.

q.e.d.

Proof of Corollary 2. Analogous to the Proof of Corollary 1 we assume that for all $s \in \Omega(s)$ informed investor surplus $\varphi > 0$ is associated with allocation $q_{\varphi}(s) = A_{\varphi}(u_s(r) - u_{\underline{s}}(r))$ (with $A_{\varphi} \in]0,1]$), which entails issuer payoff $E(\Pi)_D = \int_{\Omega(s)} \left(u_s(r) - \left(\int_{\Omega(s)} q_{\varphi}(s) ds / q_{\varphi}(s)\right)\right) f(s) ds = \int_{\Omega(s)} u_s(r) f(s) ds - \varphi$ under uniform pricing (see Theorem 2), where issuers minimise the amount of "underpricing"

$$\int_{\Omega(s)} \left(\int_{\Omega(s)} q(s) \, ds \, \middle/ q(s) \right) f(s) ds \le \int_{\Omega(s)} \left(\int_{\Omega(s)} q_{\varphi}(s) \, ds \, \middle/ q(s) \right) f(s) ds = \varphi. \tag{20}$$

Rewriting L.H.S. of (20) with IC₂ generates inequality

$$\int_{\Omega(s)} \left(\int_{\Omega(s)} q(s) ds / q(s) \right) f(s) ds \ge \frac{I}{(I+\theta)} \int_{\Omega(s)} \frac{1}{q(s)} f(s) ds, \qquad (21)$$

and rewriting R.H.S. of (20) yields

$$\int_{\Omega(s)} \left(\int_{\Omega(s)} q_{\varphi}(s) ds / q(s) \right) f(s) ds = \frac{A_{\varphi}}{6} \left(u_{s}(r) - u_{\underline{s}}(r) \right)^{3} \int_{\Omega(s)} \frac{1}{q(s)} f(s) ds = \varphi. \tag{22}$$

Combining both equations above generates inequality

$$\frac{I}{\left(I+\theta\right)}\int_{\Omega(s)}\frac{1}{q(s)}f(s)ds \leq \frac{A_{\varphi}}{6}\left(u_{s}(r)-u_{\underline{s}}(r)\right)^{3}\int_{\Omega(s)}\frac{1}{q(s)}f(s)ds \Leftrightarrow \frac{6I}{\left(I+\theta\right)} \leq A_{\varphi}\left(u_{s}(r)-u_{\underline{s}}(r)\right)^{3}, (23)$$

which yields $A_{\varphi}(u_s(r) - u_{\underline{s}}(r)) = q_{\varphi}(s) = \sqrt[3]{6I/(I+\theta)}$ as the optimal allocation schedule for investor surplus $\varphi > 0$ with issuer payoff $E(\Pi)_U = \int_{\Omega(s)} u_s(r) f(s) ds - \varphi$ under uniform pricing and full allocation.

q.e.d.