Ω, J/ψ and ψ' Production in Nuclear Collisions and Quark Gluon Plasma Hadronization

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Abstract

The transverse mass spectra of Ω, J/ψ and ψ' in Pb+Pb collisions at 158 A·GeV are studied within a hydrodynamical model of the quark gluon plasma expansion and hadronization. The model reproduces the existing data with the common hadronization parameters: temperature $T = T_H \cong 170$ MeV and average collective transverse velocity $\overline{v}_T \cong 0.2$.

The possibility of observing the quark gluon plasma (QGP) in nucleus-nucleus (A+A) collisions motivated an extensive experimental program. Rich experimental results on hadron spectra and multiplicities are now available for a broad range of collision energy and for various colliding systems [1]. The anomalies observed [2] in the energy dependence of pion and strangeness production indicate [3] that in central collisions of heavy nuclei the threshold for QGP creation during the early stage of the reaction is in the region of low CERN SPS energies (≈ 40 A·GeV).

The equilibrium hadron gas (HG) model describes remarkably well the hadron multiplicities measured in A+A collisions at top SPS [4] and RHIC [5] energies, where the creation of QGP is expected. The extracted hadronization temperature parameter is similar for both energies $T_H = 170 \pm 10$ MeV. This is close to an estimate of the temperature T_C for the QGP–HG transition obtained in Lattice QCD simulations at zero baryonic density (see e.g. [6]). One may therefore argue that the QGP created in high energy heavy ion collisions hadronizes into an (approximately) locally equilibrated HG and the chemical composition of this HG is weakly affected by rescattering during the expansion of the hadronic system [7].

In our previous paper [8] we formulated the hypothesis that the kinetic freeze–out of J/ψ and ψ' mesons takes place directly at hadronization and that those mesons therefore carry information on the flow velocity of strongly interacting matter just after the transition to the HG. Based on the measured J/ψ and ψ' spectra in Pb+Pb collisions at 158 A·GeV [9] and using the hypothesis of the statistical production of charmonia at hadronization [10, 11, 12] we extracted a mean transverse collective flow velocity of hadronizing matter: $\overline{v}_T \cong 0.2$.

The effect of the rescattering in the hadronic phase was recently studied within a "hydro + cascade" approach [13, 14]. A+A collisions are considered there to proceed in three stages: hydrodynamic QGP expansion ("hydro"), transition from QGP to HG and the stage of hadronic rescattering and resonance decays ("cascade"). The change from "hydro" to "cascade" modelling takes place at $T = T_C$, where the spectrum of hadrons leaving the surface of the QGP–HG transition is taken as input for the subsequent cascade calculations. The results of Refs. [13, 14] suggest that the transverse momentum (p_T) spectrum of Ω may be weakly affected during the cascade stage even for central Pb+Pb collisions at the top SPS energy. This is because of the small hadronic cross section and large mass of the Ω hyperon [15]. The corresponding calculations for charmonia are not yet performed within this model.

Thus we are faced with an intriguing problem: if the above considerations for charmonia [8] and Ω [13, 14] are correct, their p_T spectra should be simultaneously reproduced using the same hydrodynamic parameters, T_H and $\overline{v}_T$. In this letter we demonstrate that such a description is indeed possible. The transverse mass m_T ($m_T = \sqrt{p_T^2 + m^2}$, where m is a particle rest mass) spectra around midrapidity in Pb+Pb at the SPS (158 A·GeV) were recently measured for Ω by WA97 Collaboration [16, 17] and for J/ψ and ψ' by NA50 Collaboration [9]. These spectra will be the subject of the present analysis.

Assuming kinetic freeze–out of matter at constant temperature T, the transverse mass spectrum of i-th hadron species (with mass m_i) in cylindrically symmetric and longitudinally boost invariant fluid expansion can be approximated as [18]:

$$\frac{dN_i}{m_T dm_T} \;\propto\; m_T \int_0^R r dr \; K_1\left(\frac{m_T \cosh y_T}{T}\right) \; I_0\left(\frac{p_T \sinh y_T}{T}\right) \;, \tag{1}$$

where $y_T = \tanh^{-1} v_T$ is the transverse fluid rapidity, R is the transverse system size, K_1 and I_0 are the modified Bessel functions. The spectrum (1) is obtained under assumption that the freeze-out occurs at constant longitudinal proper time $\tau = \sqrt{t^2 - z^2}$, where t is the time and z is the longitudinal coordinate. Thus the freeze–out time t is independent of the transverse coordinate r [18]. The analysis of the numerical calculations of Ref. [14] shows that the latter is approximately fulfilled. The quality of the approximation made gets better for considered here heavy particles and small transverse flow velocities because a possible deviation from Eq. (1) is proportional to $p_T^2 v_T/(2 m_T T)$ and thus it decreases with increasing particle mass at constant p_T.

In order to calculate (1) the function $y_T(r)$ has to be given. A linear flow profile, $y_T(r) = y_T^{max} \cdot r/R$, is often assumed in phenomenological fits [18]. The numerical calculations of Ref. [14] justify this assumption. For heavy hadrons analysed in this work the condition $m_i >> T$ is always satisfied and, therefore, the asymptotic form for large arguments of $K_1(x) \sim x^{-1/2} \exp(-x)$ can be used in Eq. (1). At SPS energies typical values of v_T are small ($v_T^2 << 1$) and consequently $\cosh y_T \cong 1 + \frac{1}{2} v_T^2$ and $\sinh y_T \cong v_T$.

The experimental m_T–spectra are usually parametrised by a function[1]:

$$\frac{dN_i}{m_T dm_T} \;\propto\; \sqrt{m_T} \; \exp\left(-\,\frac{m_T}{T_i^*}\right) \;, \tag{2}$$

[1] It corresponds formally to neglecting the transverse flow ($v_T \equiv 0$ in Eq. (1)), but introducing instead an "effective" temperature.

where the inverse slope T_i^* is extracted from the fit to the data. However, when Eq. (2) is considered as an approximation of Eq. (1), the inverse slopes T_i^* should depend on both m_i and p_T. The limiting cases of T_i^* behaviour at low and high p_T can be easily studied using the small and large argument asymptotic of the modified Bessel function I_0 in Eq. (1):

$$T_i^*(p_T \to 0) \;=\; \frac{T}{1 \;-\; \frac{1}{2}\,\overline{v}_T^2\;(m_i/T \;-\; 1)} \;\approx\; T \;+\frac{1}{2}\;m_i\;\overline{v}_T^2\;, \tag{3}$$

$$T_i^*(p_T \to \infty) \;\equiv\; T^* \;=\; \frac{T}{1 \;-\; v_T^{max} \;+\; \frac{1}{2}\;(v_T^{max})^2}\;, \tag{4}$$

where the average velocity $\overline{v}_T$ in Eq. (3) is defined as $\overline{v}_T^2 \;=\; \int_0^R rdrv_T^2(r)/\int_0^R rdr$. The maximum velocity v_T^{max} in Eq. (4) is related to $\overline{v}_T$ as $\overline{v}_T^2 = (v_T^{max})^2/2$ provided a linear flow profile is assumed, $v_T(r) = v_T^{max}\cdot r/R$. Note that T^* in Eq. (4) is equivalent to the well known "blue shifted" temperature, $T[(1+v_T^{max})/(1-v_T^{max})]^{1/2}$ (see e.g. [14, 18]), calculated for $(v_T^{max})^2 << 1$. The shape of the "high–p_T" tail ($p_T >> m_i$) of the m_T distribution (1) is "universal", i.e. T^* given by Eq. (4) is independent of particle mass m_i. On the other hand, the inverse slopes T_i^* (3) at "low–p_T" are strongly dependent on m_i. Two remarks are appropriate here. First, for heavy particles like Ω and J/ψ the term $\frac{1}{2}m_i\overline{v}_T^2/T$ in Eq. (3) is not small compared to one thus the second (approximate) equality in this equation is violated. Second, a condition of the validity of Eq. (3), $p_T\overline{v}_T << T$, is too restrictive for heavy hadrons, e.g. for $T \cong 170$ MeV and $\overline{v}_T \cong 0.2$ discussed below it leads to $m_T - m_i << 0.3/m_i$ GeV$^2/c^4$. This means that Eq. (3) is valid for the values of $m_T - m_i$ which are much smaller than 0.2 GeV/c^2 for Ω and than 0.1 GeV/c^2 for J/ψ.

Summarising: (a) none of the asymptotic regimes (3) or (4) can be clearly seen in the experimental m_T spectra, i.e. neither "low–p_T" ($m_T - m_i << 0.3/m_i$ GeV$^2/c^4$) nor "high–p_T" ($m_T - m_i >> m_i$) approximations are useful ones (at least for studying the available m_T spectra of Ω and charmonia); (b) fitting the experimental m_T spectrum of i–th hadron species by Eq. (2) one finds in fact the "average inverse slopes" which depend not only on particle mass m_i, but also on the $m_T - m_i$ interval covered in a given experiment (see also Ref. [14], where T_i^* have been discussed separately for $m_T - m_i < 0.6$ GeV/c^2 and for 0.6 GeV/c$^2 < m_T - m_i < 1.6$ GeV/c^2).

For small values of v_T relevant for our discussion a good approximation of Eq. (1) at $m_T - m_i < m_i$ can be obtained by substituting the v_T distribution in Eq. (1) by its average value $\overline{v}_T$ and by using large argument K_1 asymptotic:

$$\frac{dN_i}{m_T dm_T} \;\propto\; \sqrt{m_T}\;\exp\left(-\;\frac{m_T(1+\frac{1}{2}\overline{v}_T^2)}{T}\right)\;I_0\left(\frac{p_T\overline{v}_T}{T}\right)\;. \tag{5}$$

We checked numerically that the values of the parameter $\overline{v}_T$ extracted from the fits (see below) to Eqs. (1) and (5) assuming a linear velocity profile differ by about 5%.

We turn now to the test of our hypothesis of the kinetic freeze–out of J/ψ, ψ' and Ω occurring directly at hadronization i.e. at $T = T_H = 170$ MeV. The m_T–spectra of these hadrons are measured around midrapidity [9, 16] for Pb+Pb collisions at 158 A·GeV. The fit to these data performed using Eq. (5) with $T = T_H = 170$ MeV yields

$\overline{v}_T = 0.194 \pm 0.017$ and $\chi^2/dof = 1.3$. The value of $\overline{v}_T$ varies by ∓ 0.016 when T_H is changed within its uncertainty ± 10 MeV. Note that T and $\overline{v}_T$ parameters are anti–correlated. A surprisingly good agreement (see Fig. 1) of our model with the data on m_T–spectra serves as a strong support of the hypothesis of statistical nature of J/ψ and ψ' production [10] and their kinetic freeze–out occurring directly at hadronization [8].

The dependence of the J/ψ and ψ' transverse mass spectrum on the centrality (quantified by the neutral transverse energy E_T) of Pb+Pb collisions at 158 A·GeV was also measured by NA50 Collaboration [9]. An increase of $\langle p_T \rangle$ and $\langle p_T^2 \rangle$ from peripheral collisions to the most central collisions can be explained, within our approach, by an increase of the model parameter $\overline{v}_T$ with E_T. Note that the increase of mean flow velocity and consequently an increase of $\langle p_T \rangle$ and $\langle p_T^2 \rangle$ with a centrality of the collision as well as an increase of $\langle p_T \rangle$ and $\langle p_T^2 \rangle$ with particle mass m_i are characteristic features of hydrodynamics. In contrast to J/ψ and ψ' mesons the m_T–spectra of the Drell–Yan pairs (dileptons with invariant mass $M > 4.2$ GeV/c^2) do not show this type of hydrodynamical behaviour. The values of $\langle p_T \rangle$ and $\langle p_T^2 \rangle$ [9] for the Drell–Yan pairs are smaller than those for J/ψ and ψ' and do not change significantly with E_T.

The kinetic freeze–out parameters of pions were extracted from the analysis of single– and two–pion spectra measured for central Pb+Pb collisions at 158 A·GeV by the NA49 Collaboration [19, 20, 21]. The results are: $T_f \cong 120$ MeV and $\overline{v}_T \cong 0.55$, i.e. they are very different than those obtained here from the analysis of heavy hadron spectra. In fact we checked that the parameters obtained for pions lead to the m_T spectra of heavy hadrons which strongly disagree with the data. A decrease of temperature and an increase of transverse flow velocity with time is a general property of expanding systems. The different kinetic freeze–out times of heavy hadrons and pions allow us to follow the expansion history of the hadron gas phase created in Pb+Pb collisions at 158 A·GeV. The freeze–out points of heavy hadrons and pions extracted from the data are plotted in Fig. 2 defining the path of the expanding hadron system in the $T - \overline{v}_T$ plane.

In conclusion, the m_T–spectra of Ω, J/ψ and ψ' produced in Pb+Pb collisions at 158 A·GeV are analysed within the hydrodynamical model of the QGP expansion and hadronization. The spectra are in agreement with the hypothesis of kinetic freeze–out of these heavy hadrons occurring directly after the transition from the quark gluon plasma to the hadron gas. A mean collective transverse flow of hadronizing matter of $\overline{v}_T \cong 0.2$ is extracted from the fit to the spectra using temperature $T_H \cong 170$ MeV fixed by the analysis of hadron multiplicities [4]. This result together with a previously obtained parameters of pion freeze–out ($T_f \cong 120$ MeV and $\overline{v}_T \cong 0.55$) allow for the first time to establish the history of the expanding hadron matter created in nuclear collisions.

In the RHIC energy range the temperature parameter is approximately the same $T = T_H \cong 170$ MeV [5], whereas the transverse hydrodynamic flow at $T = T_H$ is expected to be stronger. The model predictions for the m_T–spectra of Ω and charmonia at the RHIC energies will be presented elsewhere.

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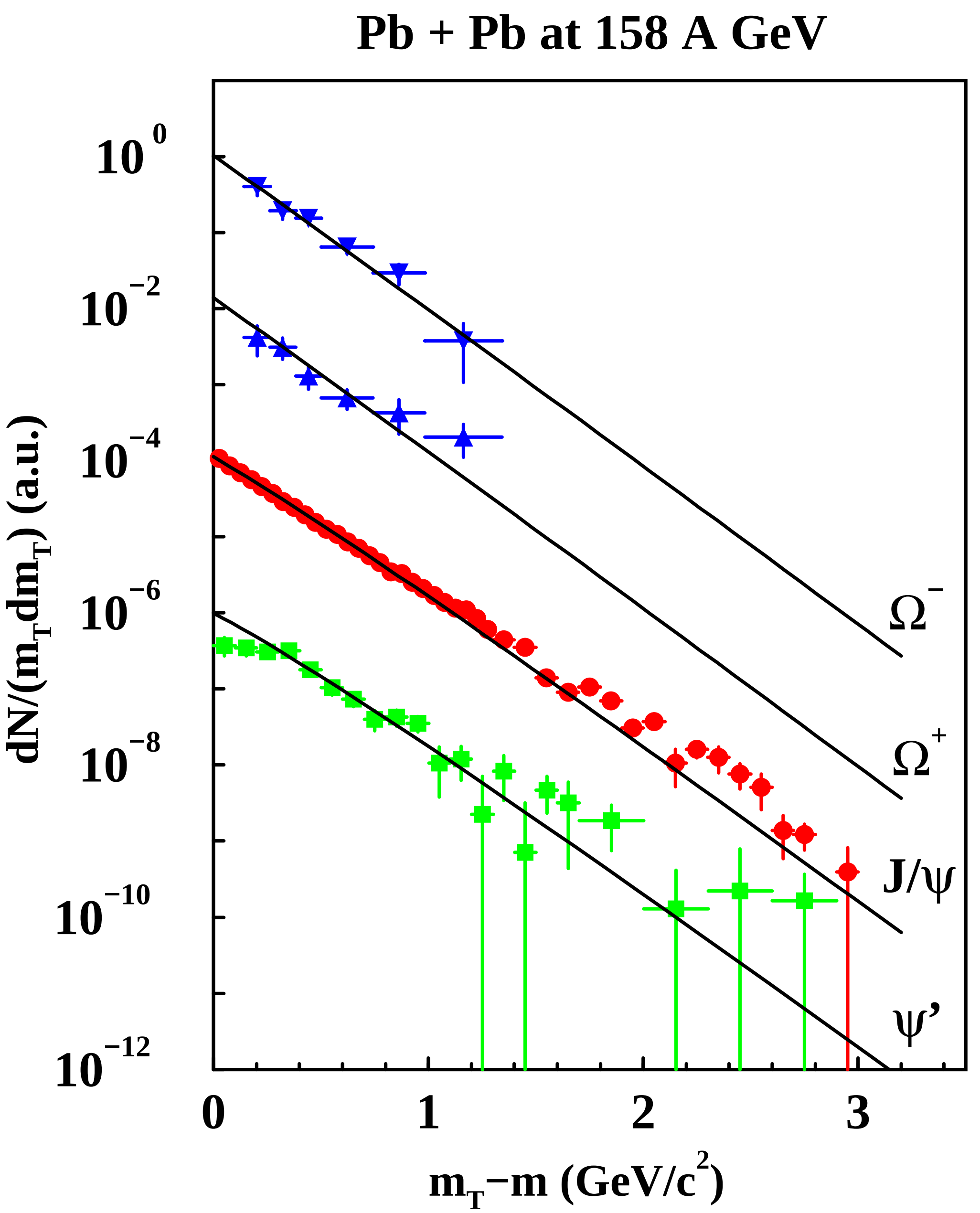

Figure 1: The transverse mass spectra of Ω^- (triangles down) and Ω^+ (triangles up) [16] as well as J/ψ (circles) and ψ' (squares) [9] produced in Pb+Pb collisions at 158 A·GeV. The solid lines indicate a prediction of model (5) assuming kinetic freeze–out of heavy hadrons directly after hadronization of expanding quark gluon plasma. The freeze–out parameters are: $T = 170$ MeV and $\overline{v}_T = 0.194$.

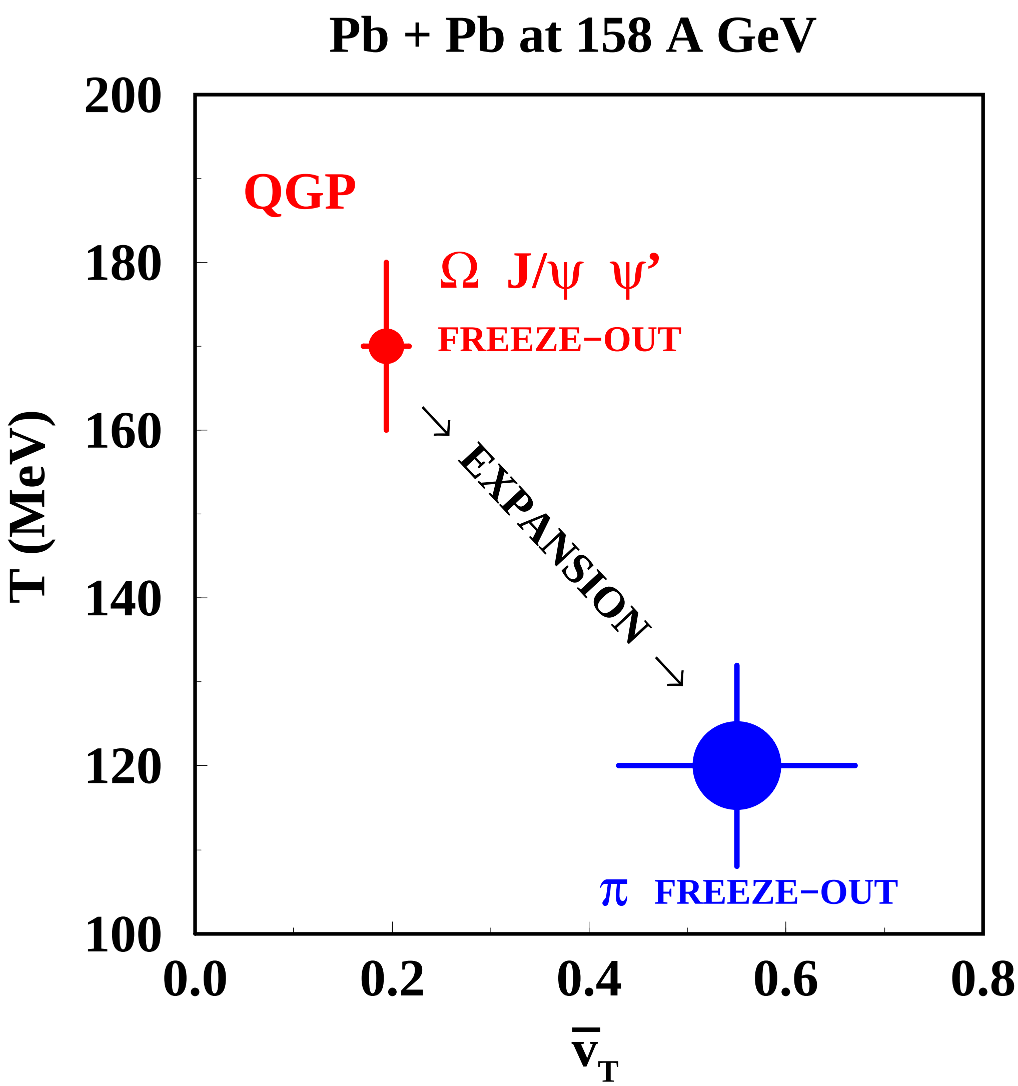

Figure 2: The expansion history of strongly interacting matter created in Pb+Pb collisions at 158 A·GeV. The points indicate the temperature T and the mean transverse flow velocity $\overline{v}_T$ of matter at the time of Ω, J/ψ and ψ' freeze–out (upper point) and at the time of pion kinetic freeze–out (lower point).