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No 56
August 2000

WORKING PAPER SERIES: FINANCE & ACCOUNTING

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No 56
August 2000

ISSN 1434-3401

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[***] The authors would like to thank Klaus Bender, Paul Kleindorfer, Howard Kunreuther, Petra Lenz and Nadia Masri for helpful comments. Of course all errors and omissions remain the responsibility of the authors. Project support from an academic fellowship granted by the German Academic Exchange Service (DAAD) under the *Gemeinsames Hochschulsonderprogramm III von Bund und Ländern* (Richter) is gratefully acknowledged.

Catastrophe Index-Linked Securities and Reinsurance as Substitutes

Abstract

The use of catastrophe bonds (cat bonds) implies the problem of the so called basis risk, resulting from the fact that, in contrast to traditional reinsurance, this kind of coverage cannot be a perfect hedge for the primary's insured portfolio. On the other hand cat bonds offer some very attractive economic features: Besides their usefulness as a solution to the problems of moral hazard and default risk, an important advantage of cat bonds can be seen in the presumably lower transaction costs compared to (re)insurance products. Insurance coverage usually incurs costs of acquisition, monitoring and loss adjustment, all of which can be reduced by making use of the financial markets. Additionally, cat bonds are only weakly correlated with market risk, implying that in perfect financial markets these securities could be traded at a price including just small risk premiums. Although these aspects have been identified in economic literature, to our knowledge there has been no publication so far that formally addresses the trade-off between basis risk and transaction cost. In this paper, therefore, we introduce a simple model that enables us to analyze cat bonds and reinsurance as substitutional risk management tools in a standard insurance demand theory environment. We concentrate on the problem of basis risk versus transaction cost, and show that the availability of cat bonds affects the structure of optimal reinsurance contract design in an interesting way, as it leads to an increase of indemnity for small losses and a decrease of indemnity for large losses.

Keywords: Insurance, Financial Markets, Decision Making und Risk

JEL-Classification: G10, G22, D81

Introduction

Damages inflicted by natural catastrophes in recent years have accounted for economic losses of a size heretofore unknown.[1] During this period, one could detect an increasing frequency of catastrophic events as well as an increase in the average amount of loss per event; the latter largely stemming from the geographic concentration of values in catastrophe-prone areas. As examples of such extreme catastrophes, reference is usually made to earthquake hazards in Tokyo or California as well as hurricanes in Florida. The estimated loss potential (PML) of any of these risks seemingly shows the capacity limits of traditional insurance markets. For instance, estimations of insured losses after a major earthquake in the San Francisco area amount to approximately \$100 billion; on the other hand, balance sheets of the U.S. property liability insurance industry show a cumulative surplus of about \$300 billion.[2]

These "capacity gaps" in the industry[3] have been at the heart of many discussions among insurance economists and practitioners in the recent past, largely aimed at the development of possible solution strategies involving the financial markets. Contributions can be expected, if, for example, the issuance of marketable insurance-linked securities was able to attract additional capacity from investors who are not otherwise related to the insurance industry. In practice, rudiments of this kind can be observed in various forms since 1992, even though they have yet to reach a significant market share.[4]

To summarize these arguments, the existence of insurance-linked securitization is normally explained by its ability to (partly) close the capacity gap of the insurance supply, especially in terms of reinsurance.[5] This line of reasoning is, however, not entirely convincing, since additional capacities could also be acquired by means of extending the level of insurers' (primaries' and reinsurers') equity capital.

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[1] See e.g. Cummins/Lewis/Phillips (1999).

[2] See e.g. Cholnoky/Zief/Werner/Bradistilov (1998) or Cummins/Doherty/Lo (1999).

[3] For an approach to measure the (re)insurance markets' capacity for covering catastrophe risks see Doherty/Cummins/Lo (1999).

[4] See Swiss Re (1999).

In order to explain the increasing relevance of insurance-linked securities, we therefore have to consider their special features.[6] In comparison to traditional reinsurance cover, insurance risk securitization has to include elements that provide specific advantages for covering certain risks. They have to be analyzed in detail to enhance the understanding of these securities' importance and possible usefulness. Surprisingly, insurance economists so far have shown only limited attempts to do this kind of research. Our paper therefore is an endeavor to shed light on the specific advantages and disadvantages of a certain type of insurance securitization, namely the issuance of catastrophe bonds (or cat bonds, for short).

A cat bond is a contract between an issuer and an investor. The investor puts up an amount of cash at the beginning of the coverage period; this is held in escrow until either a catastrophe occurs or the coverage period ends. The issuer offers a certain coupon payment exceeding the risk-free rate at the end of the period, provided that no catastrophe occurs, and returns both principal and interest to the investor. In the event of a catastrophe, the investor will receive no coupon payment, and some or all of their principal may go to the issuer.[7]

Many papers regarding insurance-linked securities are primarily concerned with the classification and detailed explanation of different possibilities for composing and designing alternative risk transfers. Most of the literature concerned with cat bonds and reinsurance as substitutional risk management instruments is mainly descriptive. Economic analyses in this area have been of a more qualitative nature. The following differences between cat bonds and (re)insurance coverage were identified.[8]

Compared to traditional reinsurance, indexed cat bonds exhibit highly imperfect risk allocation, since they are based on stochastic variables which are not identical with the losses to be covered.

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5 See e.g. Swiss Re (1996), Kielholz/Durrer (1997).

6 See also Jaffee/Russel (1997), who argue that the insurance industry's problems in covering catastrophe risks are caused by the institutional framework, since it limits the incentives for holding sufficiently large amounts of liquid capital, which would be needed to spread such risks over time.

7 For the structure of recent cat bonds see Doherty (1997a), for the case of non-indexed cat bonds see also Bantwal/Kunreuther (1999).

8 See Doherty (1997a), Doherty (1997b), Froot (1997).

To be of any use they have to be correlated with those losses, but usually cannot be a perfect hedge. Thus a buyer of index-linked coverage always has to face the so-called basis risk.

An important advantage of cat bonds, on the other hand, can be seen in the presumably low transaction costs related to this kind of coverage in comparison to insurance or reinsurance products. Insurance coverage usually incurs considerable costs of acquisition, monitoring and loss adjustment, all of which can be reduced or spared by making use of the financial markets. Furthermore, cat bonds are only weakly correlated with market risk, implying that in perfect financial markets these securities could be traded at a price including just small risk premiums.[9]

Moreover, while the insured is, more or less, usually in a position to influence the loss distribution, index-linked coverage can be based on an underlying stochastic which cannot be controlled or heavily influenced by the buyer. Thus indexed cat bonds provide a solution for the problem of moral hazard, which can be reduced in insurance contracts only by incorporating monitoring or coinsurance provisions.

Finally, in contrast to catastrophe reinsurance contracts, cat bonds are not subject to default risk. While a catastrophic event could influence a reinsurer's ability to compensate the primary, this problem can be avoided by cat bonds: The issuer of a cat bond hedges loss payments without credit risk since his obligation to pay interest and/or principal to the investors is forgiven when the bond is triggered. Roughly speaking, an ex ante collected amount of money is available in this case.

To our knowledge there has been no literature so far which – in light of the above-mentioned advantages and disadvantages of both instruments – deals with modeling the simultaneous demand for (re)insurance and index-linked catastrophe risk coverage as independent risk management tools.[10] With this paper therefore we try to take a first step in this direction by

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[9] See e.g. Litzenberger/Beaglehole/Reynolds (1996) or Lewis/Davis (1998). The profitability of insurance-linked securities traded on financial markets so far significantly exceeded the risk free interest rate. This is usually explained by pointing out that high returns were necessary to attract investors to this kind of transactions.

[10] This paper is related to Doherty/Richter (2000), who also introduce a model to formally address the attractiveness of a joint use of insurance and index-linked coverage. The authors consider the case that insurance can be used to insure the basis risk, which means the policyholder can purchase a separate policy – called gap insurance – to cover the difference between the index-linked coverage and the actual loss. So, the reinsurance contract is, in contrast to our model, defined in a way that the indemnity directly depends on the realization of basis risk. The analysis concentrates mainly on the trade-off between basis risk and moral hazard. Using mean variance to

tackling the trade-off between basis risk on the one hand and higher transaction cost on the other.[11] For this purpose we consider the case of a primary insurer facing a catastrophic risk that endangers his insured portfolio. To cover the risk there are two possible opportunities: He can buy traditional reinsurance as well as coverage provided by the issuance of an indexed cat bond. The index which serves as a trigger mechanism could be a measure for the extent of a natural disaster, like earthquakes or hurricanes, in a certain area, or the insurance industry's cumulative losses for that area.

Since coverage based on this kind of index can only serve as an imperfect hedge, it raises the problem of basis risk for the primary insurer. On the other hand it is cheaper, a fact that we incorporate in our model by assuming that the index-linked coverage is sold at an actuarially fair rate,[12] while reinsurance premiums exceed expected losses.

Using the described framework the paper is organized as follows: In section 2 we first introduce our model and then consider the case of a primary who has to choose exclusively the optimal index-linked coverage. We derive the very plausible result that the primary will buy more coverage the better the cat bond is linked to his insured portfolio, i.e. the less basis risk he would have to face. The more interesting case of cat bonds as well as reinsurance being available to cover the catastrophe risk will be considered in section 3. We show that the availability of cat bonds changes the structure of the optimal indemnity function in an interesting way: it leads to an increase of indemnity for small losses and a decrease of indemnity for large losses. In section 4 we summarize and discuss our results.

The model

Cat bonds are – as was mentioned above – an imperfect hedging tool, implying basis risk for the primary insurer. It seems suitable to measure the basis risk by the covariance of the primary's

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describe the primary's preferences, and assuming a risk neutral reinsurer, it is shown that combining the two hedging tools might extend the possibility set and by that means lead to efficiency gains.

[11] Moral hazard in the reinsurance relationship thus is not considered in this paper.

[12] The assumption that index-linked coverage is supplied at an actuarial fair rate is not necessary for our results, but it simplifies the argumentation. Crucial in this context is only that for a given price of reinsurance the alternative coverage is not prohibitively expensive.

actual losses and the index, and then to use a mean variance approach to analyze the demand for cat bonds. This kind of analysis, however, has certain disadvantages: First, catastrophe risk, as considered here, is usually represented by considerably skew distributions for which mean variance is problematic. Furthermore, most of the literature on insurance demand theory, which this paper is aimed to link with, is based on an expected utility approach. Therefore, we introduce a different approach to measure basis risk, that enables us to study the interaction between cat bonds and reinsurance in a simple expected utility model.

A risk-averse[13] primary insurer faces stochastic losses X from an insured portfolio. He considers buying index-linked coverage A, which would be triggered with probability $\overline{p}$. Since this kind of product is usually defined discretely, we can – without major loss of generality – concentrate on the simple case of a stochastic variable with only the two possible outcomes 0 and A. The primary receives the payment if an exogenous trigger variable Y which is correlated with X reaches a certain level $\overline{y}$.

The correlation between X and Y is expressed by means of the conditional probabilities

(1) $$p(x) := P\{Y \geq \overline{y} \mid X = x\}$$

(if a certain outcome is not specified we also write $p(X)$).

With these definitions clearly: $\overline{p} = E[p(X)]$.[14]

Consider, for a moment, the following problem: Without any further restrictions, construct an index-linked product with two possible outcomes that is optimal in terms of risk allocation. This product would have to be designed in such a way that the payment A is triggered with probability $p(x) = 0$ for losses up to a certain level, but that it is triggered with certainty if X reaches or exceeds this level. This is due to the feature of decreasing marginal utility, which characterizes a risk-averse decision-maker's von Neumann-Morgenstern utility function.

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[13] For a motivation of risk averse behavior in entrepreneurial decisions see e.g. Nell/Richter (1996).

[14] In the following $E[\cdot]$ always denotes the expectation with regard to the distribution of X.

A situation like this, however, is conceivable only if the coverage can be tied directly to X. But then the product would suffer exactly the same moral hazard problems as traditional reinsurance. Since we want to concentrate on instruments that eliminate especially these problems by connecting the coverage to an exogenous index, the situation mentioned above can just be seen as a limiting case for our analysis.

The other extreme case is an index-linked coverage which turns out to be completely useless in terms of risk allocation: If the conditional trigger probability $p(x)$ does not depend on x ($p(x) \equiv \overline{p}$), the primary cannot reduce the risk from his portfolio by issuing a cat bond. So he would simply worsen his situation by buying additional risk.

Naturally, general statements about the function $p(x)$ cannot be made. To keep our argument as general as possible, we only assume that $p(x)$ vanishes for sufficiently small x, and that on the other hand $p(x) = 1$ for sufficiently large losses, and finally that there is an area where the trigger probability is strictly between 0 and 1 and increasing. To formalize this, we say that potential levels of loss $x_1 < x_2$ exist such that

(2)
$$\begin{array}{ll} p(x) = 0 & x \le x_1 \\ 0 < p(x) < 1 & x_1 < x < x_2 \\ p(x) = 1 & x \ge x_2 \end{array} .$$

$p(x)$ is assumed to be differentiable with:

(3)
$$p'(x) > 0 \quad \text{for } x_1 < x < x_2 .$$

Fig. 1 shows an exemplary shape of $p(x)$.

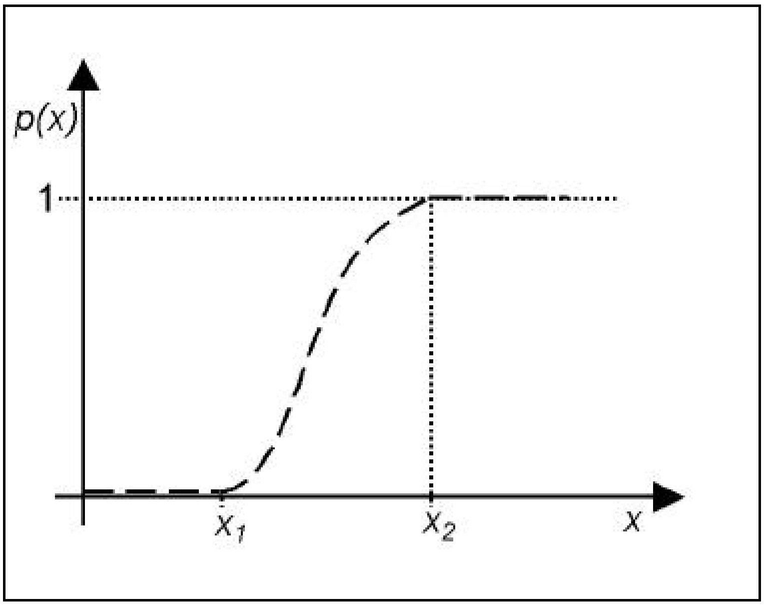

Fig. 1

The demand for cat bonds

To become familiar with the model employed in this paper we first analyze the demand for index-linked coverage for the case that reinsurance is not available. We especially want to investigate the impact of a change in basis risk.

We assume that the index-linked coverage does not cause any transaction cost, and that it is sold on a competitive market at a rate that equals the expected payment. u_1 denotes the (three times continuously differentiable) primary's utility function. It is characterized by $u_1' > 0$ and $u_1'' < 0$, since the primary is risk averse.

The optimal index-linked coverage in this framework is a solution to the following optimization problem:

(4) $$\max_A \quad E[p(X) \cdot u_1(W_1 - X - \overline{p} \cdot A + A) + (1 - p(X)) \cdot u_1(W_1 - X - \overline{p} \cdot A)].$$

As a first order condition for an interior solution we get

(5) $\quad \overline{p} \cdot E[(1-p(X)) \cdot u_1'(W_1 - X - \overline{p} \cdot A)] = (1-\overline{p}) \cdot E[p(X) \cdot u_1'(W_1 - X - \overline{p} \cdot A + A)]$

Considering the above-mentioned case that in contrast to our assumptions *p(x)* is independent of *x*, we see that such a cat bond cannot be attractive because in this situation condition (5) would be

(6) $\quad \overline{p} \cdot (1-\overline{p}) \cdot E[u_1'(W_1 - X - \overline{p} \cdot A)] = \overline{p} \cdot (1-\overline{p}) \cdot E[u_1'(W_1 - X - \overline{p} \cdot A + A)],$

implying $A = 0$.

To analyze the impact of a change, namely a reduction, in basis risk on the optimal coverage, we examine the consequences of *ceteris paribus* varying the function *p(x)* towards the above-mentioned situation where the index-linked coverage can be tied directly to *X*. We keep the unconditional trigger probability $\overline{p}$ constant and consider a transformation of the conditional trigger probability function that shifts the probability weight to higher values of *x*. More precisely, we consider the effect of replacing *p(x)* by a function $\tilde{p}(x)$ with the properties (2), (3),

(7) $\quad E[\tilde{p}(X)] = E[p(X)] = \overline{p},$

and

(8) $\quad \tilde{p}(x) \leq p(x) \ \forall x \leq x_3$ und $\tilde{p}(x) \geq p(x) \ \forall x \geq x_3$.

for an $x_3 \in (x_1, x_2)$. To exclude trivial cases we assume

(9) $\quad P\{X = x : \tilde{p}(x) \neq p(x)\} > 0$.

The idea behind this is that in our setting a product with the same unconditional trigger probability, one that is less likely to be triggered for low levels of actual losses but more likely to be triggered for higher losses, means a better fit to the primary's portfolio.

Proposition 1

A reduction in basis risk as defined in (7), (8), and (9) implies an increase in the optimal amount of index-linked coverage A.

Proof: see appendix.

This result is clearly plausible: All other things equal, the primary will buy the more index-linked coverage the better it fits for compensating the losses from his original risk, i.e. the better the hedge is.

The optimal risk management mix

We now turn to the analysis of a simultaneous decision on index-linked coverage and reinsurance. As mentioned, we assume that the issuance of indexed cat bonds is cheaper than traditional reinsurance. According to our model cat bonds are traded in a perfect market, and their price equals the expected pay-out. Reinsurance contracting incurs additional costs, which we restrict to the (implicit) cost arising from the reinsurer's risk-aversion. However, results similar to those in this paper can be derived by incorporating other types of costs.

The reinsurance premium is denoted by P_2, $I(x)$ denotes the indemnity function and u_2 the concave (and three times continuously differentiable) reinsurer's utility function.

We derive Pareto-optimal solutions according to:

$$\begin{aligned}(10)\quad \max_{I(\cdot),A}\ & a \cdot E[p(X) \cdot u_1(W_1 - X - \bar{p} \cdot A - P_2 + A + I(X)) \\ & + (1 - p(X)) \cdot u_1(W_1 - X - \bar{p} \cdot A - P_2 + I(X))] + b \cdot E[u_2(W_2 + P_2 - I(X))]\end{aligned}$$

Using the Euler-Lagrange equation the following first order conditions can be derived[15]

[15] If the reinsurer is risk-neutral, an optimal solution is given by $I(x)=x$ and $A=0$. This solution is fairly plausible since under these circumstances reinsurance would be more attractive than index-linked coverage. It could be sold at the same rate and would (of course) not cause any basis risk.

$$
(11)\quad \begin{aligned} & a \cdot p(x) \cdot u_1'(W_1 - x - \overline{p} \cdot A - P_2 + A + I(x)) \\ & + a \cdot (1 - p(x)) \cdot u_1'(W_1 - x - \overline{p} \cdot A - P_2 + I(x)) \; = \; b \cdot u_2'(W_2 + P_2 - I(x)) \qquad \forall\, x \end{aligned}
$$

or

$$
(12)\quad \frac{p(x) \cdot u_1'(W_1 - x - \overline{p} \cdot A - P_2 + A + I(x)) + (1 - p(x)) \cdot u_1'(W_1 - x - \overline{p} \cdot A - P_2 + I(x))}{u_2'(W_2 + P_2 - I(x))} = \frac{b}{a} \quad \forall\, x,
$$

showing the well-known result, that in a Pareto-optimum the marginal rate of substitution is constant.

As a point of reference we compare the optimization problem without the possibility of index-linked coverage:

$$
(13)\qquad \max_{I(\cdot)} \quad a \cdot E[u_1(W_1 - X - P_2 + I(X))] + b \cdot E[u_2(W_2 + P_2 - I(X))]
$$

This means we assume the same reinsurance budget for this case as for the situation with cat bonds, which enables us to concentrate on the implications the availability of index-linked coverage has on the *structure* of an ideal reinsurance contract. We do not analyze the impact on the budget spent on reinsurance.

Let $I_0^*(\cdot)$ denote the solution of (13) and $I_I^*(\cdot)$ the optimal indemnity function from (10). The following results can be derived:

Proposition 2

For sufficiently large levels of losses x $I_I^*(x)$ is smaller than $I_0^*(x)$. In particular, $p(x) = 1$ implies $I_I^*(x) < I_0^*(x)$. On the other hand, $I_I^*(x)$ does not assign less indemnity to every level of the loss. For sufficiently small x, particularly where $p(x) = 0$, $I_I^*(x) > I_0^*(x)$. If the primary's preferences are represented by utility functions with constant or decreasing absolute risk aversion, $I_I^*(x) > I_0^*(x)$ for $p(x) \leq \overline{p}$.

Proof: see appendix.

That the optimal reinsurance indemnity for small losses is larger in a situation where index-linked coverage is available, compared to the model without cat bonds, can be explained quite easily: for small x the effect prevails, that the cost of the index-linked product increases the marginal utility of the reinsurance coverage.

To find out more about the optimal indemnity function we consider the slope of $I_I^*(x)$. Applying the implicit function theorem to (11) we get

(where $W_A := W_1 - x - \overline{p} \cdot A - P_2 + A + I_I^*(x)$, $W_B := W_1 - x - \overline{p} \cdot A - P_2 + I_I^*(x)$,

and $W_C := W_2 + P_2 - I_I^*(x)$)

(14)
$$\frac{dI_I^*(x)}{dx} = \frac{a \cdot p(x) \cdot u_1''(W_A) + a \cdot (1-p(x)) \cdot u_1''(W_B)}{a \cdot p(x) \cdot u_1''(W_A) + a \cdot (1-p(x)) \cdot u_1''(W_B) + b \cdot u_2''(W_C)} - \frac{a \cdot p'(x) \cdot [u_1'(W_A) - u_1'(W_B)]}{a \cdot p(x) \cdot u_1''(W_A) + a \cdot (1-p(x)) \cdot u_1''(W_B) + b \cdot u_2''(W_C)}.$$

The first expression in (14) is positive and smaller than 1, the second is negative only if $A > 0$. Note that the optimal indemnity function can be decreasing, especially if the function $p(x)$ is very steep.

Concerning the comparison of the optimal indemnity functions for the cases with, and respectively without cat bonds, an interesting result can be derived for a certain class of utility functions:

Proposition 3

If the primary's and the reinsurer's preferences are represented by CARA utility functions with risk aversion coefficients a (primary) and b (reinsurer), the slope of the optimal indemnity function in a market with cat bonds is given by

(15)
$$\frac{dI_I^*(x)}{dx} = \frac{a}{a+b} - \frac{p'(x) \cdot [1 - e^{-a \cdot A}]}{(a+b) \cdot [p(x) \cdot e^{-a \cdot A} + (1-p(x))]}$$

$I_I^*(x)$ and $I_0^*(x)$ are parallel for $x \in [0, x_1)$ and $x \in [x_1, \infty)$. Elsewhere $I_I^*(x)$ is less steep than $I_0^*(x)$.

Proof: see appendix.

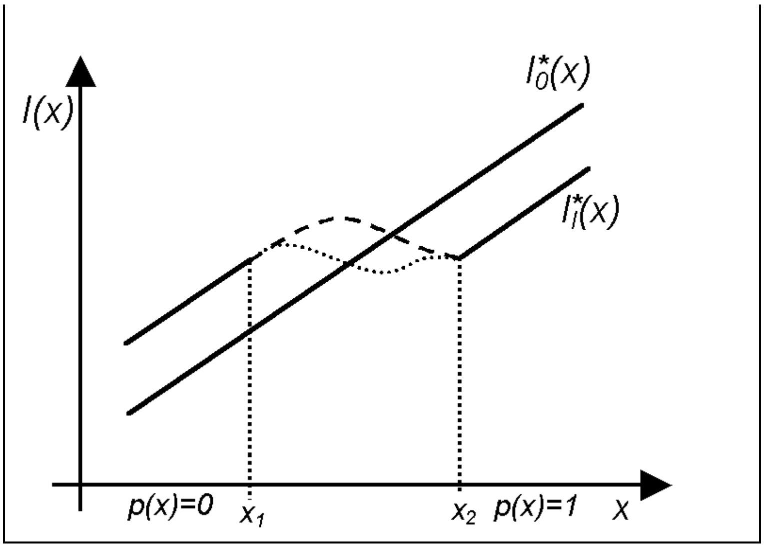

Fig. 2

We can also derive an explicit formulation for the connection between the optimal indemnity functions:[16]

(16) $$I_I^*(x) = I_0^*(x) + \frac{a}{a+b} \cdot \overline{p} \cdot A + \frac{\ln[\, p(x) \cdot e^{-a \cdot A} + (1 - p(x))]}{a+b}$$

The difference between $I_I^*(x)$ and $I_0^*(x)$ is

(17) $$\left| I_I^*(x) - I_0^*(x) \right| = \frac{a}{a+b} \cdot \overline{p} \cdot A$$

where $p(x) = 0$, and

(18) $$\left| I_I^*(x) - I_0^*(x) \right| = \frac{a}{a+b} \cdot (1 - \overline{p}) \cdot A$$

[16] To show this we use (11) and (25) (appendix).

for $p(x)=1$.

Our central result is that the existence of catastrophe index-linked securities affects the structure of the reinsurance demand: the coverage and thus the indemnity payments increase for small losses and decrease for large losses. The optimal mix of risk allocation instruments thus entails that smaller losses are mainly covered by reinsurance contracts, whereas larger losses are instead covered by index-linked securities. This confirms the assessment often stated by insurance practitioners, that cat bonds are mainly useful for covering extremely large losses.

This result was derived by assuming the same reinsurance budget in the case with as well as in the case without indexed cat bonds. Thus we compared the optimal structure of the reinsurance contracts in both situations, but we did not make a statement about the total effect the availability of index-linked coverage has on the amount of reinsurance indemnity for different levels of losses. Of course one would expect a decrease in the demand of reinsurance measured in terms of the reinsurance premium when indexed cat bonds are introduced. It should be kept in mind, though, that one conclusion from our analysis is the following: depending on whether the latter effect predominates, there might be areas where the optimal reinsurance coverage is raised after index-linked coverage comes into play.

Conclusion

In this paper we consider two important alternatives a primary insurer has for covering catastrophic risks: contracting reinsurance or buying index-linked coverage. We analyze the optimal mix of these instruments. We show that there are strong interdependencies, because both means influence each other heavily with respect to their efficiency.

Clearly, the demand for cat bonds can only be explained via imperfections in the reinsurance market, since cat bonds always result in a basis risk for the insurer. The demand for index-linked coverage cannot be advantageous if reinsurance coverage is offered at fair prices in a market with complete information and without default risk. This implies that factors such as transaction cost, moral hazard, and/or credit risk in the reinsurance contracts are a conditio sine qua non for the attractiveness of cat bonds.

Concentrating on transaction cost aspects and treating basis risk in a way that allows for expected utility analysis, we show that, given a certain reinsurance budget, the existence of catastrophe index-linked securities changes the structure of the demand for reinsurance: the indemnity payments increase for small losses and decrease for large losses. The optimal mix of risk allocation instruments thus entails that small losses are mainly covered by reinsurance contracts, while large losses are instead covered by catastrophe index-linked securities. The explanation for this result is that cat bonds imply an additional stochastic element. The parameters of the optimal reinsurance contract therefore change: the coverage for small losses, which imply a small probability that the cat bond is triggered, increases, while the coverage for large losses, which imply a large probability that the cat bond is triggered, is reduced.

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Appendix

Proposition 1

A reduction in basis risk as defined in (7), (8), and (9) implies an increase in the optimal amount of index-linked coverage A.

Proof:

(5) can be reformulated as

$$\text{(19)}\quad \begin{aligned} &\overline{p}\cdot E[u_1'(W_1 - X - \overline{p}\cdot A)] \\ &\quad = E[p(X)\cdot\{(1-\overline{p})\cdot u_1'(W_1 - X - \overline{p}\cdot A + A) + \overline{p}\cdot u_1'(W_1 - X - \overline{p}\cdot A)\}]. \end{aligned}$$

If – starting from an optimal solution – the trigger probability function is transformed according to (7), (8), and (9), the marginal utility levels for large amounts of losses are weighed more heavily. Since u_1' is strictly decreasing, we get:

$$\text{(20)}\quad \begin{aligned} &E[p(X)\cdot\{(1-\overline{p})\cdot u_1'(W_1 - X - \overline{p}\cdot A + A) + \overline{p}\cdot u_1'(W_1 - X - \overline{p}\cdot A)\}] \\ &\quad < E[\tilde{p}(X)\cdot\{(1-\overline{p})\cdot u_1'(W_1 - X - \overline{p}\cdot A + A) + \overline{p}\cdot u_1'(W_1 - X - \overline{p}\cdot A)\}] \end{aligned}$$

and therefore

$$\text{(21)}\quad \overline{p}\cdot E[(1-\tilde{p}(X))\cdot u_1'(W_1 - X - \overline{p}\cdot A)] < (1-\overline{p})\cdot E[\tilde{p}(X)\cdot u_1'(W_1 - X - \overline{p}\cdot A + A)].$$

In order to fulfill condition (5) again after the variation of $p(\cdot)$, A has to be increased.

QED

Proposition 2

For sufficiently large levels of losses x $I_I^*(x)$ is smaller than $I_0^*(x)$. In particular, $p(x)=1$ implies $I_I^*(x) < I_0^*(x)$. On the other hand, $I_I^*(x)$ does not assign less indemnity to every level of the loss. For sufficiently small x, particularly where $p(x)=0$, $I_I^*(x) > I_0^*(x)$. If the primary's *preferences are represented by utility functions with constant or decreasing absolute risk aversion,* $I_I^*(x) > I_0^*(x)$ *for* $p(x) \leq \overline{p}$.

Proof:

I_I^* is defined as an optimal reinsurance indemnity function according to

(22)
$$\max_{I(\cdot),A} \; a \cdot E[p(X) \cdot u_1(W_1 - X - \overline{p} \cdot A - P_2 + A + I(X)) + (1 - p(X)) \cdot u_1(W_1 - X - \overline{p} \cdot A - P_2 + I(X))] + b \cdot E[u_2(W_2 + P_2 - I(X))]$$

yielding the first order conditions (11) and (12).

Furthermore we derive

(23)
$$a \cdot E[p(X) \cdot (1 - \overline{p}) \cdot u_1'(W_1 - X - \overline{p} \cdot A - P_2 + A + I(X)) - (1 - p(X)) \cdot \overline{p} \cdot u_1'(W_1 - X - \overline{p} \cdot A - P_2 + I(X))] = 0.$$

$I_0^*(\cdot)$ is the solution of:

(24)
$$\max_{I(\cdot)} \; a \cdot E[u_1(W_1 - X - P_2 + I(X))] + b \cdot E[u_2(W_2 + P_2 - I(X))]$$

The optimal indemnity function with regard to (24) is defined by

(25)
$$\alpha \cdot u_1'(W_1 - x - P_2 + I(x)) = \beta \cdot u_2'(W_2 + P_2 - I(x)) \qquad \forall\, x.$$

For a given level of losses x $I_I^*(x)$ will be smaller than $I_0^*(x)$, if the left hand side of (11) is smaller than the left hand side of (25), both evaluated at $I_0^*(x)$. This condition is obviously fulfilled for sufficiently large values of x, respectively $p(x)$.

For values of x with $p(x) = 0$, the left hand side in (11) is

(26)
$$a \cdot u_1'(W_1 - x - \overline{p} \cdot A - P_2 + I(x)).$$

This expression (for $A > 0$) exceeds $a \cdot u_1'(W_1 - x - P_2 + I(x))$, such that $I_I^*(x) > I_0^*(x)$.

In the case of constant or decreasing absolute risk-aversion ($\Rightarrow u_1''' > 0$), we can derive the result $I_I^*(x) > I_0^*(x)$ for x with $p(x) \le \overline{p}$ from the convexity of u_1':

$$
\begin{aligned}
& a \cdot p(x) \cdot u_1'(W_1 - x - \overline{p} \cdot A - P_2 + A + I(x)) + a \cdot (1 - p(x)) \cdot u_1'(W_1 - x - \overline{p} \cdot A - P_2 + I(x)) \\
(27) \quad & \ge a \cdot \overline{p} \cdot u_1'(W_1 - x - \overline{p} \cdot A - P_2 + A + I(x)) + a \cdot (1 - \overline{p}) \cdot u_1'(W_1 - x - \overline{p} \cdot A - P_2 + I(x)) \\
& > a \cdot u_1'(W_1 - x - P_2 + I(x)).
\end{aligned}
$$

QED

Proposition 3

If the primary's and the reinsurer's preferences are represented by CARA utility functions with risk aversion coefficients a (primary) and b (reinsurer), the slope of the optimal indemnity function in a market with cat bonds is given by

$$(28) \qquad \frac{dI_I^*(x)}{dx} = \frac{a}{a+b} - \frac{p'(x) \cdot [1 - e^{-a \cdot A}]}{(a+b) \cdot [\,p(x) \cdot e^{-a \cdot A} + (1 - p(x))]}$$

$I_I^*(x)$ and $I_0^*(x)$ are parallel for $x \in [0, x_1)$ and $x \in [x_1, \infty)$. Elsewhere $I_I^*(x)$ is less steep than $I_0^*(x)$.

Proof:

If index-linked coverage is not available or not attractive, as a well known result from the theory of optimal risk-sharing we get that $I_0^*(x)$ is a linear function:[17]

$$(29) \qquad \frac{dI_0^*}{dx} = \frac{a}{a+b}$$

[17] See e.g. Arrow (1963). The fundamental work on the features of Pareto-optimal risk-sharing rules goes back to Borch (1960). See also Wilson (1968), Borch (1968), Gerber (1978), Raviv (1979) or Bühlmann/Jewell (1979).

Now consider again the indemnity function for the situation with index-linked coverage. Dealing with constant absolute risk-aversion, we can, without loss of generality, use the utility functions $u_1(W) = -\frac{1}{a} \cdot e^{-a \cdot W}$ and $u_2(W) = -\frac{1}{b} \cdot e^{-b \cdot W}$.[18] For this specific case (14) is of the form

(30)

$$\frac{dI_I^*(x)}{dx} = \frac{-a \cdot a \cdot e^{-a \cdot I_I^*(x)} \cdot e^{-a \cdot (W_1 - x - \overline{p} \cdot A - P_2)} \cdot [p(x) \cdot e^{-a \cdot A} + (1 - p(x))]}{-a \cdot a \cdot e^{-a \cdot I_I^*(x)} \cdot e^{-a \cdot (W_1 - x - \overline{p} \cdot A - P_2)} \cdot [p(x) \cdot e^{-a \cdot A} + (1 - p(x))] - b \cdot b \cdot e^{b \cdot I_I^*(x)} \cdot e^{-b \cdot (W_2 + P_2)}}$$

$$- \frac{a \cdot p'(x) \cdot e^{-a \cdot I_I^*(x)} \cdot e^{-a \cdot (W_1 - x - \overline{p} \cdot A - P_2)} \cdot [e^{-a \cdot A} - 1]}{-a \cdot a \cdot e^{-a \cdot I_I^*(x)} \cdot e^{-a \cdot (W_1 - x - \overline{p} \cdot A - P_2)} \cdot [p(x) \cdot e^{-a \cdot A} + (1 - p(x))] - b \cdot b \cdot e^{b \cdot I_I^*(x)} \cdot e^{-b \cdot (W_2 + P_2)}}.$$

From (11) follows

(31) $$a \cdot e^{-a \cdot I_I^*(x)} \cdot e^{-a \cdot (W_1 - x - \overline{p} \cdot A - P_2)} \cdot [p(x) \cdot e^{-a \cdot A} + (1 - p(x))] = b \cdot e^{b \cdot I_I^*(x)} \cdot e^{-b(W_2 + P_2)},$$

such that (30) can be simplified to

(32) $$\frac{dI_I^*(x)}{dx} = \frac{a}{a+b} - \frac{p'(x) \cdot [1 - e^{-a \cdot A}]}{(a+b) \cdot [p(x) \cdot e^{-a \cdot A} + (1 - p(x))]}.$$

By comparing (29) and (32) we see that $I_I^*(x)$ and $I_0^*(x)$ are parallel if $p'(x)$ vanishes. This is the case for $x \in [0, x_1)$ and $x \in [x_1, \infty)$.

QED

[18] See e.g. Pratt (1964), or Bamberg/Spremann (1981).