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Search for Short Lived Particles in High Multiplicity Environment

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Abstract. A method of statistical selection of short lived particles in high multiplicity nucleus–nucleus collisions is discussed.

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1 Introduction

High energy nuclear collisions will produce many thousands charged particles in the central rapidity interval usually covered by tracking detectors. Among the measured charged hadrons are those which are:

- directly produced in the process of strong interaction between two nuclei,
- products of decay of resonances during the hadronization process and
- products of weak decays of heavy hadrons.

The characteristic distance between emission points of particles originating from the first two processes is of the order 1 to 100 fm; this distance can be measured only in the statistical way using particle interferometry. However, the characteristic distance between weak decay point and the collision region is often well measurable using recent experimental technics.

In this note we discuss a possible method of statistical selection of weakly decaying particles in the experimental case when the characteristic decay distance is of the same order as experimental resolution of its measurement. In this case the main problem is to make efficient rejection of background due to the particles originating directly from the interaction.

2 General Description

We consider here two body decay of a given particle and assume that momenta and charges of both decay products are measured by the experimental set up. Typical examples are neutral strange or charm particle decays into charged particles (V topology) which can be easily measured.

The basic points of the method are the following:

- for each combination of two tracks coming from the same interaction, which are potential candidates for products of the decay the hypothetical decay point is fitted and the kinematical fit is done.
- values of cut–variables are calculated for each pair and the probability density function (pdf) for accepted pairs from the same interaction in the cut–space (see below) is obtained ('raw signal' pdf),

- the 'raw background' pdf in the cut–space is calculated using combinations of tracks from different events,
- the acceptance region in the cut–space is selected using 'raw background' pdf, expected 'signal' and 'background' multiplicities and number of collisions,
- the 'signal' in the acceptance region is calculated as a difference between 'raw signal' and 'raw background' pdfs multiplied by a 'raw signal' average multiplicity.

The important element of the method is an introduction of the cut–space, using which transparent and well defined method for the selection of the best acceptance region (cuts) for the given experimental case is developed. The procedure should be used separately for different $y - p_T$ regions. The weighting factors, due to applied cuts in the cut–space necessary to obtain final results are trivial.

3 Cut–variables

In this Section we describe a set of five cut–variables which are later used to select the best acceptance region for separation of 'signal' from the 'background'. The cut–variables are defined in a specific way, which grantees several important features of these variables calculated for the 'signal' (real V decays):

- the variables are independent from each other,
- the variables are independent of kinematical properties of the decaying V particle (eg. y or p_T),
- the 'signal' pdf in cut–variables is independent of an experimental resolution,
- the 'signal' pdf is uniform; the variables range between 0 and 1.
- the cuts in the cut–variables do not affect acceptance in the physically important variables eg. rapidity or transverse momentum.

In the following a specific set of 5 cut–variables is described.

1. The cumulative χ^2 of the decay vertex fit.
   The cumulative χ^2 variable of the vertex fit is defined as:

$$C_V = \int_0^{\chi^2} f(u,\nu)du, \tag{1}$$

   where $f(u,\nu)$ is chi–square probability density function for ν degrees of freedom. The χ^2 value for each decay is calculated by decay vertex fit procedure.

2. The cumulative χ^2 of the kinematical fit.
   The cumulative χ^2 variable is defined as:

$$C_K = \int_0^{\chi^2} f(u,\nu)du, \tag{2}$$

   where $f(u,\nu)$ is chi–square probability density function for ν degrees of freedom. The χ^2 value for each decay is calculated using standard method of Least Squares estimation with constrains (kinematical fit). In a case of measured momenta vectors of both decay products and polar, θ_V, and azimuthal, ϕ_V, angles of the decay point (defined in the spherical system with the origin in the interaction point) the number of degrees of freedom is $\nu = 3$. In the limit when there is no information on θ_V and ϕ_V $\nu = 1$.

3. The polar angle of the decay product.
   This cut–variable is defined as:

$$C_{\theta^*} = 0.5(cos\theta^* + 1), \tag{3}$$

   where θ^* is an angle between momentum vector of positive (or negative) decay product and V momentum vector calculated in the V center of mass system. The values obtained from the kinematical fit are used for the calculation.

4. The azimuthal angle of the decay product.
   This cut–variable is defined as:

$$C_\phi = \frac{\phi}{2\pi} \tag{4}$$

   where ϕ is an angle between a projection of the beam direction and the projection of the momentum vector of the positive (or negative) decay product onto plane perpendicular to the V direction. The values obtained from the kinematical fit are used for calculations.

5. The cumulative life time.
   The cumulative life time cut–variable is defined as:

$$C_\tau = \int_0^{R(\tau)} P_{p_V}(R')dR', \tag{5}$$

   where R is a distance between measured V decay point and the interaction point, τ is a V life time in the V center of mass system and $P_{p_V}(R)$ is a probability density function of decay at the distance R calculated for a V with momentum p_V. In the case of an ideal measurement of the decay distance R one gets:

$$P_{p_V}(R) = P^I_{p_V}(R) = Ae^{-\frac{\tau(R)}{\tau_0}} = Ae^{-\frac{R}{R_0}}, \tag{6}$$

   where $R_0 = p_V\tau_0/m_V$ and A is normalization factor. If the decay distance measurement is not an ideal one we get:

$$P_{p_V}(R) = \int_0^{\infty} P^I_{p_V}(R')\rho(R,R')dR', \tag{7}$$

   where $\rho(R,R')$ is a probability density that decay at a distance R' is measured as a decay at the distance R. The values obtained from kinematical fit are used for the calculations.

The choice of the given set of cut–variables is not unique, especially various definitions of angular variables are possible. The final selection depends on the experimental situation. The variables should be defined in order to maximize nonuniformity of the 'raw background' pdf.

4 Cut-space and Acceptance Region

We define a 5–D cut–space as a set of points:

$$\mathbf{C} = (C_V, C_K, C_{\theta^*}, C_\phi, C_\tau), \tag{8}$$

From the definitions of C_i given in previous Section follows that:

$$V = \int\int\int\int dC_V dC_K dC_{\theta^*} dC_\phi dC_\tau = \int dV = 1. \tag{9}$$

For the 'signal' pdf we obtain:

$$\rho_s(\mathbf{C}) = const(\mathbf{C}) = 1 \tag{10}$$

where $\rho_s(\mathbf{C})$ is a 'signal' pdf at point $\mathbf{C}$.

The 'background' pdf, $\rho_b(\mathbf{C})$, is obviously nonuniform and therefore the ratio of the 'signal' to the 'background' depends on the point $\mathbf{C}$ or on the subspace selected in the cut–space.

The average number of 'signal' events (averaged over all collisions) in the subspace of the volume V^{acc} around any point $\mathbf{C}$ is given by:

$$< n_s^{acc} >= V^{acc} < n_s >, \tag{11}$$

where $< n_s >$ is an average number of all 'signal' events. eg. all charged decays for which both decay products were measured.

The corresponding number of 'background' events is given by:

$$< n_b^{acc} >=< n_b > \int_{V^{acc}} dV \rho_b = \overline{\rho}_b^{acc} V^{acc} < n_b >, \tag{12}$$

where $< n_b >$ is an average number of 'background' events and $\overline{\rho}_b^{acc}$ is an average 'background' density in the subspace V^{acc}.

However, the average number of 'background' events and the 'background' pdf can not be directly measured due to unknown contamination of 'signal' events. The 'background' pdf, $\rho_b(\mathbf{C})$, can be safely estimated by 'raw background' pdf, $\rho_{rb}(\mathbf{C})$, obtained from combinations of tracks from different events providing that $< n_s ><<< n_b >$ and that V decay products have similar $y - p_T$ distribution to that obtained for tracks originating from the main vertex.

The only multiplicity which is measured is 'raw signal' multiplicity i.e. the average multiplicity of all track pairs, which are accepted as candidates for products of V decay, $< n_{rs} >=< n_b > + < n_s >$. If the total 'signal' multiplicity is much smaller than total 'background' multiplicity one can safely write:

$$< n_b >\approx< n_{rs} > . \tag{13}$$

The above approximation allows to calculate 'signal' multiplicity in the given acceptance (V^{acc}):

$$< n_s^{acc} >\equiv< n_{rs}^{acc} > - < n_b^{acc} >\approx< n_{rs}^{acc} > -\overline{\rho}_{rb} V^{acc} < n_{rs} >, \tag{14}$$

providing that the uncertainty of the $< n_s^{acc} >$ due to the used approximation (Eq. 13) is much smaller than $< n_s^{acc} >$ i.e.:

$$< n_s^{acc} >>> \frac{< n_s >}{< n_b >} < n_b^{acc} > \tag{15}$$

or consequntly

$$\overline{\rho}_{rb}^{acc} << 1. \tag{16}$$

Therefore in the region of cut–space in which the last inequality is satisfied the 'signal' multiplicity can be expressed as:

$$< n_s >=< n_{rs} > (\overline{\rho}_{rs}^{acc} - \overline{\rho}_{rb}^{acc}). \tag{17}$$

For a given number of collisions, N_{ev}, the 'signal' can be extracted from the 'background' in the subspace V^{acc} when the statistical uncertainty of the $< n_s >$ is much smaller than $< n_s >$. Assuming that $< n_{rs} >$, $\overline{\rho}_{rs}^{acc}$ and $\overline{\rho}_{rb}^{acc}$ are independent one gets:

$$\sigma(< n_s >) = [(\sigma(< n_{rs} >)(\overline{\rho}_{rs}^{acc} - \overline{\rho}_{rb}^{acc}))^2 + (< n_{rs} > \sigma(\overline{\rho}_{rs}^{acc}))^2 + (< n_{rs} > \sigma(\overline{\rho}_{rb}^{acc}))^2]^{1/2} \tag{18}$$

Obviously the second term dominates in the above expression (for central collisions $< nrs >$ is approximately fixed and therefore $\sigma(< n_{rs} >)$ is relatively small even for small number of collisions; number of background combinations increarses like N_{ev}^2 whereas number of signal like N_{ev} and therefore $\sigma(\overline{\rho}_{rb}^{acc})$ is $\sqrt{N_{ev}}$ times smaller than $\sigma(\overline{\rho}_{rs}^{acc})$. Neglecting the first and the last terms in the Eq. 18 we get:

$$\sigma(< n_s >) =< n_{rs} > \sigma(\overline{\rho}_{rs}^{acc}). \tag{19}$$

Let us try to estimate $\sigma(\overline{\rho}_{rs}^{acc})$. From definition we have:

$$\overline{\rho}_{rs}^{acc} = \frac{n_{rs}^{acc}}{n_{rs} V^{acc}}. \tag{20}$$

Due to the fact that $n_{rs}^{acc} << n_{rs}$ we can assume Poissonian flactuations of n_{rs}^{acc} and therefore we get:

$$\sigma \overline{\rho}_{rs}^{acc} = \frac{\sqrt{n_{rs}^{acc}}}{n_{rs} V^{acc}}. \tag{21}$$

Substituting $n_{rs}^{acc} = n_{rs}\ \overline{\rho}_{rs}^{acc}\ V^{acc}$ one gets:

$$\sigma \overline{\rho}_{rs}^{acc} = \sqrt{\frac{\overline{\rho}_{rs}^{acc}}{V^{acc}}} \frac{1}{\sqrt{N_{ev} < n_{rs} >}}. \tag{22}$$

Therefore the minimum number of events neccesary for signal–background separation is given by:

$$N_{ev} >> \frac{< n_{rs} >}{< n_s >^2} \frac{\overline{\rho}_{rs}^{acc}}{V^{acc}} \tag{23}$$

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