Power Law in Hadron Production

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In high energy p($\overline{\mathrm{p}}$)+p interactions the mean multiplicity and transverse mass spectra of neutral mesons from η to Υ ($m \cong 0.5 \div 10$ GeV/c^2) and the transverse mass spectra of pions ($m_T > 1$ GeV/c^2) reveal a remarkable behaviour: they follow, over more than 10 orders of magnitude, the power–law function: $C \cdot m_{(T)}^{-P}$. The parameters C and P are energy dependent, but similar for all mesons produced at the same collision energy. This scaling resembles that expected in the statistical description of hadron production: the parameter P plays the role of a temperature and the normalisation constant C is analogous to the system volume. The fundamental difference is, however, in the form of the distribution function. In order to reproduce the experimental results and preserve the basic structure of the statistical approach the Boltzmann factor $e^{-E^*/T}$ appearing in standard statistical mechanics has to be substituted by a power–law factor $(E^*/\Lambda)^{-P}$.

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It is well established that basic properties of hadron production in high energy collisions approximately follow simple rules of statistical mechanics. A principal assumption of statistical models of strong interactions [1] states that the single particle energy distribution in the local rest frame of hadronizing matter follows a Boltzmann form:

$$\frac{dn}{d\mathbf{p}^*} \sim \exp\left(-\frac{E^*}{T}\right) , \tag{B.1}$$

where $E^* = \sqrt{m^2 + \mathbf{p}^{*2}} = m_T \cdot \cosh(y^*)$ is the hadron energy ($m_T \equiv \sqrt{m^2 + p_T^2}$ is the transverse mass and y^* is the local hadron rapidity) and T is a common (for all particles) temperature parameter usually extracted by a comparison with the experimental data. In the fixed rest frame (e.g. in the c.m. frame of the colliding particles) the energy E^* equals to $m_T \cdot \cosh(y - y')$, where y and y' are the rapidities of considered particle and the 'hadron fluid elements' (fireballs), respectively.

Integration of hadron distribution along the collision axis (i.e. over y and y' with an arbitrary rapidity distribution function, $f(y')$, of fireballs) leads to an approximately (for $m_T >> T$) exponential form of transverse mass spectrum [1]:

$$\frac{dN}{m_T dm_T} \sim \exp\left(-\frac{m_T}{T}\right) . \tag{B.2}$$

According to this approach the final state hadrons should obey this distribution providing that collective transverse motion of hadronizing matter and contributions from resonance decays are neglected. Further integration over transverse mass yields an expression for the mean hadron multiplicity, $N(m)$, which is also (for $m >> T$) governed by the Boltzmann factor:

$$N(m) \sim \exp\left(-\frac{m}{T}\right) . \tag{B.3}$$

The exponential distributions (B.2,B.3) are confirmed in high energy particle collisions by numerous experimental results on p_T spectra in the transverse momentum region $p_T \leq 2$ GeV/c and on hadron yields at $m \leq 2$ GeV/c^2 [2] (a correction for contributions from resonance decays is necessary for a detailed comparison with data). For a given hadron species, h, the normalisation factors in Eqs.(B.2, B.3) are proportional to a volume parameter V (a sum of the proper volume of 'hadron fluid elements'), which has to be common for all hadrons, following the assumption of statistical production. Additional factors are degeneracy factor $g = (2j + 1)$, where j is the particle spin, and a chemical factor $\exp(\mu_h/T)$, which accounts for material conservation laws in grand canonical approximation. The chemical potential μ_h equals to zero for neutral hadrons (i.e. for hadrons with zero values of the conserved charges). Consequently, for neutral hadrons the exponential distributions (B.2, B.3) with a common temperature parameter and a common volume parameter (multiplied by the degeneracy factors g) are expected to be valid.

However, in high energy p+p interactions the measurements of π^0 meson spectra in a high p_T ($p_T >$ 2 GeV/c) domain show a strong deviation from the exponential shape. In this region the distribution follows a power–law dependence [3]. In the standard, QCD–based, approach this behaviour is attributed to the hard parton scattering [4]. A strong violation of the Boltzmann behaviour in the high mass region is also seen in the case of mean hadron multiplicities. The hadron yields from the η-meson ($m \cong 0.55$ GeV/c^2) to the J/ψ-meson ($m \cong 3.1$ GeV/c^2) follow [5] approximately an exponential law (B.3). This is, however, not true any more for the Υ meson ($m \cong 9.5$ GeV/c^2). The experimental ratio of Υ to J/ψ mean multiplicity at 800 GeV/c is [6]

$$\frac{\langle\Upsilon\rangle}{\langle J/\psi\rangle} \approx 10^{-3} ,$$

whereas Boltzmann's exponential law predicts:

$$\frac{\langle\Upsilon\rangle}{\langle J/\psi\rangle} \approx \exp\left(-\frac{m_\Upsilon - m_{J/\psi}}{T}\right) \approx 10^{-16}$$

for typical value of $T \cong 0.170$ GeV. Thus the standard statistical model underpredicts the experimental data by more than 10 orders of magnitude!

In this letter we discuss a possibility to extend the statistical approach of hadron production in elementary hadronic interactions to the high (transverse) mass domain. Based on experimental data we postulate that the energy spectrum in the local rest frame of hadronizing matter is given by a power–law distribution:

$$\frac{dn}{d\mathbf{p}^*} \sim \left(\frac{E^*}{\Lambda}\right)^{-P} , \tag{P.1}$$

instead of the exponential Boltzmann distribution (B.1). We assume further that the remaining structure of the statistical approach is unchanged. Two new parameters appear in the proposed statistical power–law model: a scale parameter Λ and an exponent P, both are assumed to be common for all hadrons. We do not attempt here to introduce proper canonical treatment of material conservation laws needed for description of charged hadrons in small systems [7,2]. Therefore, we limit our analysis to the case of neutral mesons only. Further on, in order to be able to neglect effects of large resonance widths in the statistical treatment we consider only narrow mesonic states.

Integration of Eq. (P.1) over longitudinal motion results in a power–law transverse mass distribution:

$$\rho(m_T) \equiv \frac{dN}{g\ m_T^2\ dm_T} = C \cdot m_T^{-P} , \tag{P.2}$$

where C is a normalisation parameter in which we absorbed the dependence on the scale (Λ) and volume (V) parameters. As both Λ and V are assumed to be common for all hadrons, so does the normalisation parameter C.

The integration of Eq. (P.2) over transverse mass yields:

$$\rho(m) \equiv \frac{P-3}{g\ m^3} \cdot N(m) = C \cdot m^{-P} , \tag{P.3}$$

where $N(m)$ is the hadron multiplicity.

Several nontrivial predictions of the statistical power–law model follow from Eqs. (P.2) and (P.3):

- both transverse mass spectra and hadron yields should obey a power–law behaviour; the power P in the m_T–distribution, $\rho(m_T)$ (P.2), and in the hadron yield formula, $\rho(m)$ (P.3), should be the same,
- the normalisation constant C also should be the same for both m_T–distribution (P.2) and hadron yield spectrum (P.3),
- the power P and the normalisation constant C should be universal (equivalent to the temperature and volume parameters in the standard statistical approach), i.e. they are the same for different hadron species.

In order to test these predictions we plot in Fig. 1 experimental results on m_T–spectra and hadron yields for p+p interactions at $\sqrt{s} = 27 \div 30.8$ GeV as a function of m_T and m, respectively. Within the statistical power–law model, results on both $\rho(m)$ vs m and $\rho(m_T)$ vs m_T should follow the same dependence: $C \cdot m_{(T)}^{-P}$.

Full dots in Fig. 1 indicate results on $\rho(m)$ calculated for the mean multiplicity of hadrons: η, ω, ϕ [8], $J/\psi, \psi'$ and Υ [6], whereas triangles indicate the data on m_T–spectra, $\rho(m_T)$, of neutral pions [10]. The measurements of $J/\psi, \psi'$ and Υ [6] were performed for p+S interactions at 800 GeV/c. The extrapolation to p+N interactions was done assuming the $A^{0.92}$ dependence on nuclear mass number. The measured energy dependence of midrapidity yield [9] was used to calculate multiplicities at $\sqrt{s} \approx 30$ GeV. The π^0 measurements are done at central rapidities only. In order to calculate the rapidity integrated distributions needed for comparison with the model, we use the following approximation: $dN/dm_T = d^2N/(dm_T dy) \cdot \Delta y$, where $\Delta y = 1$. Measurements of the transverse momentum spectra of π^0 mesons performed by several experiments at the ISR [10] differ typically by a factor of about 2. This together with similar uncertainty introduced by the extrapolation procedure for rapidity integrated distributions, yield a relatively large systematic error on the absolute normalisation of π^0 spectra. An additional bias of a similar magnitude is possible due to a missing correction for contributions from resonance decays. A more detailed analysis is obviously needed in the future. Nevertheless, we observe a surprising agreement over more than 10 orders of magnitude of the experimental results with the expectations following from the statistical power–law model. Typical deviations of the experimental points from the universal power–law dependence are of about factor 2, which is a similar magnitude as the systematic errors estimated above. In the power–law fits to the data the statistical and 50% systematic errors were added in squares. The values of the parameter P ($\cong 10$) and the noramlization constant C resulting from the separate fits to the m_T spectra and hadron multiplicities are similar (for details see Table 1). The solid line in Fig. 1 indicates a fit to the multiplicity data.

As seen from Fig. 1 the power–law model (P.2, P.3) works reasonably well even for intermediate values of $m_{(T)}$ ($0.5 \div 2$ GeV/c^2), where the exponential distributions (B.2,B.3) are normally assumed, e.g. the power–law function (P.3) seems to describe also the data for η and ϕ multiplicities. This fact requires a further study.

In order to check whether the observed power–law m_T–scaling is valid also for higher collision energies we compiled data on J/ψ, ψ' [11], $\Upsilon(1s)$, $\Upsilon(2s)$ and $\Upsilon(3s)$ [12] in p+$\overline{\text{p}}$ interactions at $\sqrt{s} = 1800$ GeV (the highest energy accessible today). The p_T distributions for the quarkonia are measured at midrapidity and they allow to calculate $\rho(m_T)$ spectra in this region. The resulting distributions are indicated by open symbols in Fig. 1 together with the line showing the power–law fit ($P \cong 8$, for details see Table 1) to these data. It is seen that the distributions of quarkonia produced in p+$\overline{\text{p}}$ interactions at $\sqrt{s} = 1800$ GeV follow approximately the same power–law function. The relative comparison of data for different quarkonia may be biased by a systematic error of about 50% due to the use of midrapidity spectra instead of spectra integrated over all rapidities. Therefore, as previously, in the fit the square of the error was calculated as a sum of the squares of statistical and systematic (50%) errors.

There are no data on π^0 meson production in p+$\overline{\text{p}}$ interactions at $\sqrt{s} = 1800$ GeV. However, there are measurements of π^0 spectra performed close to midrapidity ($y^* \approx 1.4$) at $\sqrt{s} = 540$ GeV which extend up to $p_T \cong 40$ GeV/c [13]. The resulting $\rho(m_T)$ distribution is shown in Fig. 2. The fit of the power–law function indicated by a solid line yields $P \cong 8$ (for details see Table 1), the value similar to one obtained for quarkonium spectra at 1800 GeV. The power–law fits to 30 and 1800 GeV data, indicated in Fig. 2 for a comparison, are both below the spectrum at 540 GeV. This observation may suggest a possible violation of the scaling for π^0 mesons at 1800 GeV. This is because one may expect a monotonic increase of $\rho(m_T)$ at fixed m_T with increasing collisions energy and this expectation is not obeyed by the 540 GeV data. We note, however, that the measurements of π^0 spectrum at high p_T region were indirect due to difficulty to resolve single photons from π^0 decay. Therefore, a direct measurements of π^0 spectrum at 1800 GeV are necessary in order to verify a validity of the scaling in this domain.

What is the origin of the power–law m_T–scaling observed in p($\overline{\text{p}}$)+p interactions and not (yet) seen in the collisions of heavy nuclei [14]? The production of heavy hadrons (like J/ψ and Υ mesons) as well as the power–law behaviour of pion spectra at high p_T are fitted by QCD inspired models [4,15]. However different assumptions and parameters enter in these models for π^0 and quarkonia description. Thus the observed power–law m_T–scaling seems to be an unexpected and interesting feature of the data which requires explanation. In this context we list known to us developments which can be helpful in attempts to understand the power–law m_T–scaling. It was recently pointed out [16] that quantum fluctuations of the string tension may account for the thermal features of the hadron spectra. A possible appearance of the power–law instead of Boltzmann spectrum was suggested within thermal field theory [17] in the low temperature and high mass limit. Generalization of standard thermodynamics to non–extensive systems [18] leads to distribution of energy in the canonical ensemble with the power–law tail for high energies.

Finally we repeat the basic results presented in this letter. Properly normalized multiplicities and m_T spectra of neutral mesons from η to Υ ($m \cong 0.5 \div 10$ GeV/c^2) and m_T spectra of π^0 meson ($m_T >$ 1 GeV/c^2) obey the m_T–scaling. The scaling function has approximately the power–law form: $m_{(T)}^{-P}$. The values of parameter P and normalisation constant C resulting from the fits to data on production of different mesons at fixed collision energy are similar. This scaling behaviour resembles that expected in statistical mechanics: the parameter P plays the role of temperature and the normalisation constant C is analogous to the system volume. Thus the basic modification of the statistical approach needed to reproduce the experimental results on hadron production in p($\overline{\text{p}}$)+p interactions in the large $m_{(T)}$ domain is the change of the shape of the distribution function. The Boltzmann function $e^{-E^*/T}$ appearing in the standard statistical mechanics has to be substituted by a power–law function $(E^*/\Lambda)^{-P}$.

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Table 1 The values of the parameters (P and C) resulting from the power–law fits ($\rho(m_{(T)}) = C \cdot m_{(T)}^{-P}$) to various data sets discussed in this letter (for details see text). The values of χ^2/ndf are also given.

data set	P	$C \cdot 10^2$ (GeV$^(P-3)$)	χ^2/ndf
neutral meson multiplicity at ≅30 GeV	10.1 ± 0.3	3.6 ± 1.0	1.02
m_T–spectra of π^0 at ≅30 GeV	9.8 ± 0.1	6.1 ± 0.9	1.4
m_T–spectra of quarkonia at 1800 GeV	7.7 ± 0.4	3.4 (+4.4,-2.0)	1.13
m_T–spectra of π^0 at 540 GeV	8.1 ± 0.1	129 ± 15	1.3

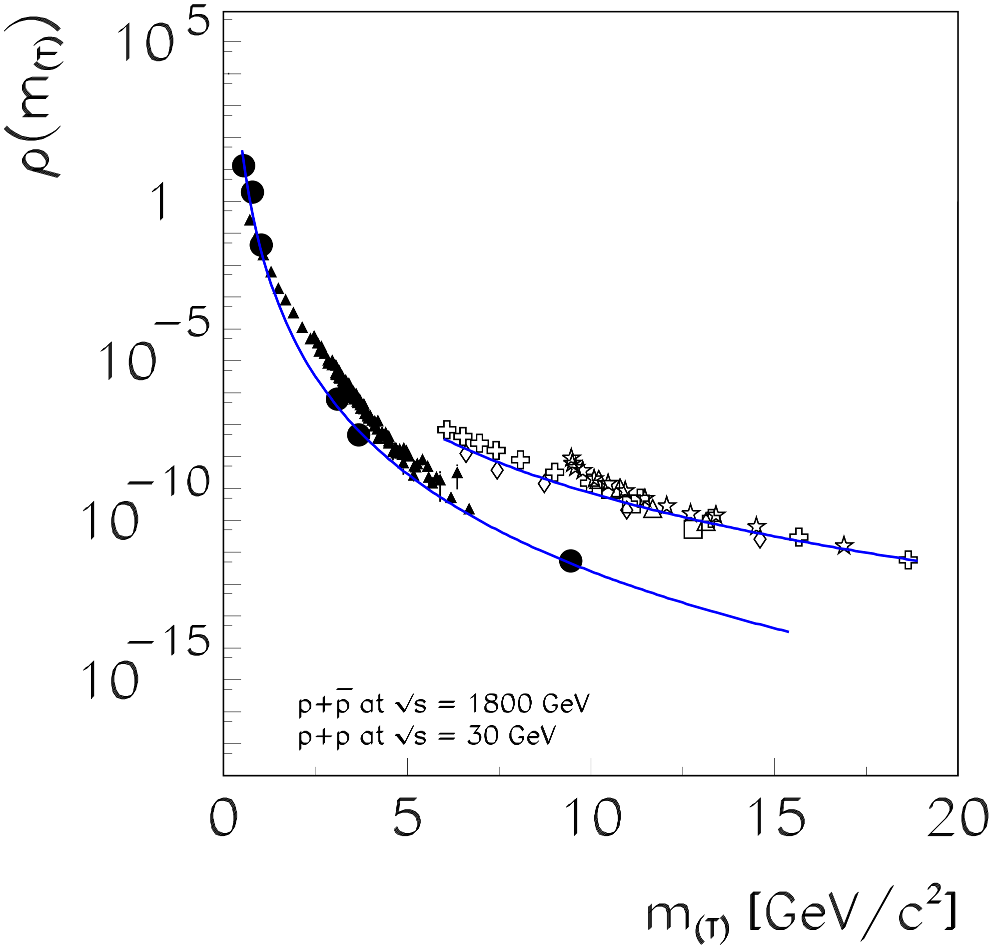

FIG. 1. Mean multiplicity of neutral mesons (full dots) scaled to $\rho(m)$ (P.3) and transverse mass spectra of π^0 mesons (full triangles) scaled to $\rho(m_T)$ (P.2) produced in p+p interactions at $\sqrt{s} \approx 30$ GeV as well as the quarkonium $\rho(m_T)$ spectra (J/ψ – crosses, ψ' – diamonds, $\Upsilon(1s)$ – stars, $\Upsilon(2s)$ – triangles, $\Upsilon(3s)$ – squares) for p+$\overline{\text{p}}$ at $\sqrt{s} = 1800$ GeV. The solid lines indicate power–law fits to the data.

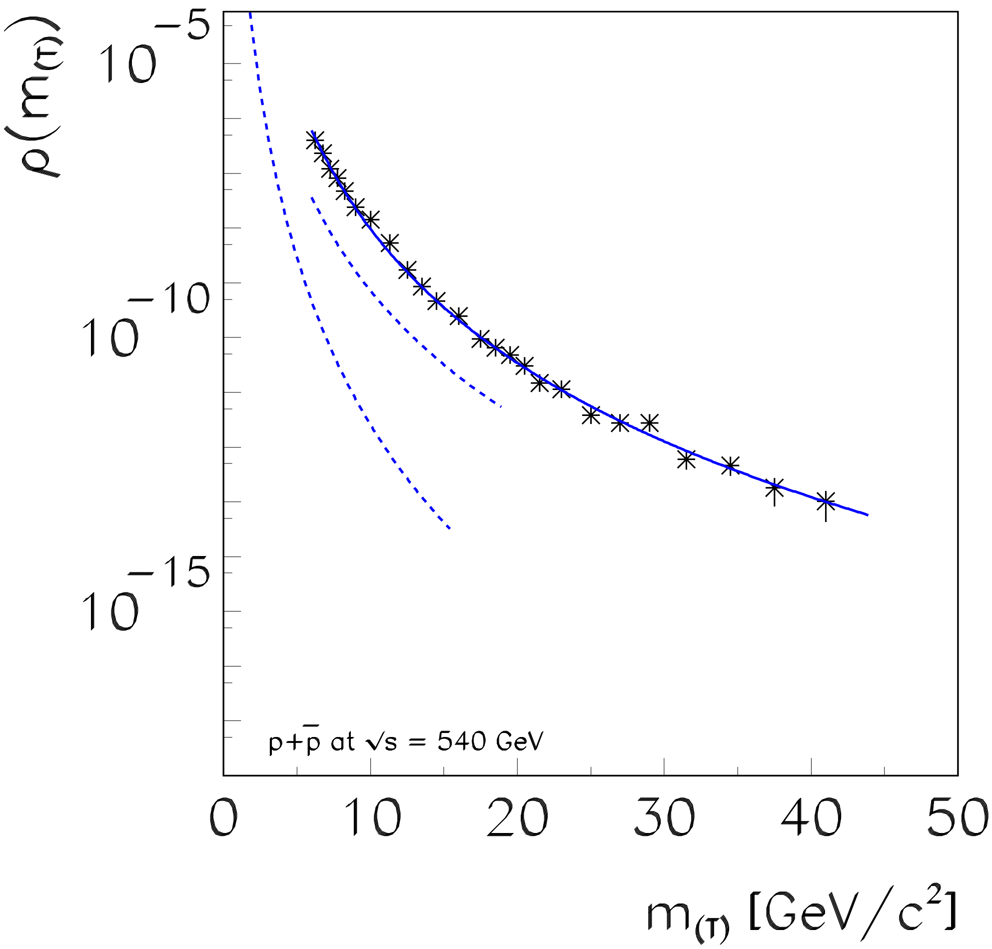

FIG. 2. The $\rho(m_T)$ spectra of π^0 meson for p+$\overline{\mathrm{p}}$ interactions at $\sqrt{s}$ = 540 GeV. The solid line indicates a power–law fit to the 540 GeV data, whereas the two dashed lines show the corresponding fits to the results at 30 and 1800 GeV (see Fig. 1).