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Heterogeneous Multiple Bank Financing Under Uncertainty: Does it Reduce Inefficient Credit Decisions?

No. 149
March 2005

WORKING PAPER SERIES: FINANCE & ACCOUNTING

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No. 149
March 2005

ISSN 1434-3401

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Abstract

Small and medium-sized firms typically obtain capital via bank financing. They often rely on a mixture of relationship and arm's-length banking. This paper explores the reasons for the dominance of heterogeneous multiple banking systems. We show that the incidence of inefficient credit termination and subsequent firm liquidation is contingent on the borrower's quality and on the relationship bank's information precision. Generally, heterogeneous multiple banking leads to fewer inefficient credit decisions than monopoly relationship lending or homogeneous multiple banking, provided that the relationship bank's fraction of total firm debt is not too large.

Keywords: Relationship lending, Uncertainty, Asymmetric information, Credit market competition, Financial distress

JEL Classification: G21, L14, D82

1 Introduction

In many, particularly European, countries the business sector is characterized by a large number of small to medium-sized, mostly owner-run firms. Typically, these firms use bank lending as the only source of obtaining capital for their business projects.[1] Over time, the business relations to a bank may develop into what is called a "housebank" relationship. The advantages of such relationship lending are well-known. Based on a sustained monitoring process, the relationship bank may efficiently renegotiate contracts, as such facilitating intertemporal transfers. Apparently, the softening of the budget constraint is initially advantageous for small and informationally opaque firms that run a high risk of financial distress. However, once a borrower is informationally tied to a bank, this exclusive relationship gives rise to a certain degree of monopoly power so that the relationship bank may extract quasi-rents from the borrower (Rajan, 1992; Detragiache et al., 2000).

Considering unlisted, small and medium-sized firms that are not as easily able to disclose reliable information about their business prospects as larger, listed firms, one might conjecture that the advantages of a relationship lending system may outweigh the disadvantage represented by the hold-up problem. In reality, however, we observe that particularly in the sector of small and medium-sized businesses heterogeneous multiple bank financing prevails: firms typically hold credit relations to many banking institutions of which one may be special in the sense of a relationship lender (Ongena and Smith, 2000; Machauer and Weber, 2001; Brunner and Krahnen, 2001).

This paper explores the reasons for the dominance of heterogeneous multiple banking in a theoretical model. We can show that under certain conditions, heterogeneous multiple bank financing reduces the incidence of inefficient credit termination and hence of inefficient firm liquidation. The theoretical framework entails elements from the class of global games.[2] The concept of global games has first been applied to credit markets by Hubert and Schäfer (2002) and Morris and Shin (2004). While Hubert and Schäfer (2002) compare the isolated effects of monopoly lending by a relationship lender to those of a multitude of small lending institutions, Morris and Shin (2004) examine coordination failures among homogeneous multiple lenders that force solvent but illiquid borrowers into a liquidity crisis. Both models, however, ignore that firms often hold multiple credit relations to different banking institutions that may be classified as heterogeneous according to the degree of informativeness about the borrower and the bargaining power that they dispose of. Elsas et al. (2004) were among the first to consider the coexistence of a relationship bank with a multitude of "arm's-length" banks in a theoretical framework. They derive the optimal debt structure in an asymmetric banking system from the trade-off between the bargaining power of the relationship bank and the threat of coordination failure from multiple banking. However, Elsas et

[1] Only recently did German Mittelstand firms, for instance, open up to other forms of lending via the capital markets. However, as the Economist (November 2003) remarks: "Though starved of bank loans, German companies are reluctant to tap alternative sources of capital".

[2] In a global game each player noisily observes the game's payoff structure, which itself is determined by a random draw from a given class of games (Carlsson and van Damme, 1993)

al (2004) assume that all banks are fully informed about their borrowers. Our model in contrast, even though closely related to the former work, puts the different informational positions of bank lenders in the main focus. We assume that very precise information about a firm's business prospects can be obtained only through long-lasting relations and hence is available only for the relationship bank. The resulting lack of as precise information for arm's-length banks matches particularly well the financing of non-listed, small to medium-sized firms that are in the main focus of this paper. Whereas Elsas et al. (2004) stress the coordinating role of a relationship bank based on her substantial fraction of total firm debt, we additionally analyze the effects of her informational advantage.

Generally, our model also lends from work on debt structures and the effects of multiple lenders, for instance by Bolton and Scharfstein (1996), Dewatripont and Maskin (1995) or Carletti (2004). The latter, in particular, studies the benefits and drawbacks of endogenous monitoring activities in multiple relationship banking. Similarly, our model entails the basic trade-off between hold-up problem and soft-budget constraint arising from informed lending via a relationship bank. However, this trade-off is embedded in a heterogeneous multiple banking system, with both relationship and arm's-length lending, rather than a homogeneous multiple financing regime as in Carletti (2004). This adds a third dimension to the effects of relationship lending: apart from holding-up the firm on the one hand and smoothing intertemporal transfers on the other, the relationship bank may also coordinate arm's-length banks behavior and hence influence the efficiency of credit decisions.

In contrast to the aforementioned work, we do not aim at establishing the optimal debt structure. Rather, we analyze the effect of heterogeneous multiple bank financing on the incidence of inefficient credit termination, and hence on inefficient project liquidation, as compared to single relationship banking on the one hand and homogeneous multiple banking on the other. As such, we complement the earlier work by Hubert and Schäfer (2002) by a richer structure of lender types. Additionally, we examine the effects that different degrees of information precision among relationship and arm's-length banks have on the efficiency of credit decisions. In contrast to Elsas et al. (2004) we can show that the outcome is not only influenced by the relationship bank's "size", i.e. her fraction of total firm debt, but also by her informational advantage over arm's-length banks.

Our results are multi-faceted and differentiate between a system where the relationship bank disposes of very but not infinitely precise information and the limit where she possesses completely precise information about her borrowers. In the former case, we find that for firms running projects with low expected cash-flows, i.e. for firms perceived to be illiquid but solvent, the incidence of inefficient credit termination and hence of firm liquidation decreases in the relationship bank's information precision, whenever a system of heterogeneous multiple bank financing has been employed. Based on the perceived firm (respectively project) quality, the relationship bank lowers the credit repayment in a renegotiation process. She thereby influences the credit decisions of arm's-length banks and increases credit access for the firm. Hence, the relationship bank coordinates arm's-length banks' credit decisions towards the efficient equilibrium. The stabilization of financially distressed firms notwithstanding, a drawback of the heterogeneous multiple banking regime is that firms running projects with high expected cash-flows face a higher likelihood of inefficient credit termination and project liquidation the more precise their relationship bank's information becomes. Here, the relationship bank's actions coordinate the small banks towards the inefficient equilibrium. Similarly to our results, Elsas et al. (2004) find that if banks posses fully precise information, firms with projects yielding low expected cash-flows prefer financing within a heterogeneous multiple banking system, while projects with high expected cash-flows tend to be financed without a relationship bank.

Comparing heterogeneous multiple bank financing to monopolistic relationship banking, we find that in the case of finitely precise information, inefficient project liquidation is always less likely under the first regime. Obviously, the interplay of coordination-, hold-up and intertemporal-smoothing effect eases the firm's credit constraints as compared to single relationship lending, which lacks the coordination mechanism, so that inefficient credit decisions are less probable in a heterogeneous system. Yet, compared to homogeneous multiple bank financing, a heterogeneous regime should only be preferred if the firm's cost of capital caused by the relationship bank are relatively small, i.e. if her fraction of total firm debt and the charged repayment rate are sufficiently low. Hence, contingent on the bargaining power of the relationship bank, the hold-up problem may become severe enough relative to the coordination- and intertemporal-smoothing effect to render a homogeneous system more beneficial than a heterogeneous banking regime.

Additionally, we consider the limiting case where the relationship bank disposes of completely precise private information about her borrowers. Here, our model shows that she always charges the maximum feasible repayment, irrespective of the quality of projects to be financed, so that the intertemporal-smoothing effect vanishes. Yet, coexistence of relationship and arm's-length banking still reduces the likelihood of inefficient credit termination compared to monopoly relationship banking, since the coordination-effect beneficially influences the firm's credit constraints. When compared to a homogeneous multiple banking context, however, a heterogeneous system again fares better only if the relationship bank's fraction of debt is relatively small.

Our results match a number of stylized facts established from empirical studies on relationship lending in a heterogeneous banking system. Elsas and Krahnen (1998) find that housebanks develop a truly distinct lending behavior despite competition from other banking institutions. Based on data from the German credit market, they show that relationship banks increase financing to borrowers of deteriorating, but still sustainable, quality. Similarly, Foglia et al. (1998), based on Italian data, conclude that the existence of relationship lending from one specific banking institution increases firms' general access to credit. Obviously, the relationship bank plays a coordinating role on the credit market for all other financiers, that, being deprived of as precise information, act as arm's-length lenders. Even some of our more detailed results are matched by empirical findings. In particular, D'Auria et al. (1999) concur that with regard to the pricing of loans, a privileged relationship to one particular lender may be preferable, i.e. leading to lower cost of capital, as long as the relationship bank's share of total borrowing does not get so large as to represent a virtual monopoly.

The remainder of the paper is organized as follows. Section 2 describes the general model of heterogeneous multiple bank financing in which a firm obtains credit from a relationship bank and a multitude of arm's-length banks. Section 3 examines the characteristics of this financing system provided that the relationship bank disposes of finitely precise private information. The results are compared to a market where the relationship bank acts as a monopolist and to a homogeneous multiple banking regime. Section 4 studies the limiting case where the relationship bank obtains private information of infinitely high precision. Again, a comparative statics analysis is followed by a comparison of different banking regimes. Section 5 concludes.

2 The Model

The model considers a simple economy with three types of agents: a firm, a relationship bank and a continuum of arm's-length banks.[3] Both types of lenders are approached by the firm in order to obtain financing for a risky business project. The project matures within three time periods. At an intermediate stage, lenders may withdraw their money prematurely so that the project is threatened by early liquidation.

The bank financing system is heterogeneous in two respects. First, it is assumed that the housebank has been having long-term relations with the firm and therefore disposes of more precise information about the firm than arm's-length banks. Second, the relationship bank is supposed to grant a loan of non-negligible size. In particular, her loan provides a fraction λ of total firm debt. Arm's-length banks, in contrast, grant loans that are negligibly small individually but amount to a combined proportion of $1-\lambda$ of the full credit. Arm's-length banks are therefore distinguished from the relationship bank by a smaller financial "size" and a lower precision of information about the borrower's quality.

The model has the following time structure:

- In $t = 0$, the firm approaches the banks in order to raise capital for a risky business project. It offers a repayment of r per unit of capital in $t = 3$. Provided that financing decisions have been successful, the firm invests in the project.

- In $t = 1$, project quality θ is realized from a normal distribution $N(\mu_\theta, \sigma_\theta)$. θ is unobservable to the banks. Instead, they receive noisy private information about it: the relationship bank observes a private signal $x_R = \theta + \varepsilon$ with $\varepsilon \sim N(0, \frac{1}{\alpha})$, while the small banks observe individual private signals $x_{AL_i} = \theta + \varepsilon_i$ with $\varepsilon_i \sim N(0, \frac{1}{\beta})$. It is assumed that ε and ε_i are independent of each other and of θ. Based on her private information, the relationship bank decides whether to renegotiate the repayment and extend credit or to foreclose the loan. The arm's-length banks

[3] Empirical studies found that for medium-sized European firms, the number of banking relationships varies between 1 and 70 (Ongena and Smith, 2000). In the model, the assumption of a continuum of arm's length lenders is made for simplicity and does not qualitatively influence the results.

observe her actions and decide whether to extend or withdraw their loans as well.[4] Early foreclosures deliver a liquidation value of K per unit of capital for both types of lenders.

- In $t = 2$, project quality θ becomes uncovered. The firm makes her choice whether to invest additional effort V into the project and refinance the foreclosed fraction of loans or to terminate the project altogether. In the latter case, the final liquidation value of the project is normalized to zero.

- In $t = 3$, if the project has not been terminated prematurely, the project's cash-flow is realized and equals θ. Credit is repaid with the firm holding the residual claim.

With regard to banks' information it is assumed that $\alpha > \beta$, so that the relationship bank disposes of at least as precise posterior information about θ as arm's-length banks. Furthermore, all bargaining power is supposed to rest with the relationship bank. The repayment rate renegotiated by the relationship bank is referred to as r_R. Since small to medium-sized firms, that are in the main focus of this paper, typically do not dispose of large collateral, it is assumed that $K < r, r_R$. Whereas the early liquidation value K is the same for all banks, we assume that the relationship bank causes higher refinancing costs per unit of capital, W_R, than the small banks, W_{AL}, i.e. $W_R \geq W_{AL} \geq r$, which gives rise to a hold-up problem. Note that W_R is the maximum feasible amount of repayment that the relationship bank can demand of the firm. Furthermore, in order to simplify the analysis, effort V is normalized to zero. Finally, we assume $\sigma_\theta \to \infty$, i.e. we examine the limiting case where the variance of θ becomes very large, so that banks dispose of hardly any prior information about the project quality. Again, we believe this assumption to be justified for small and medium-sized firms that are informationally opaque. Furthermore, since the focus of our study is on the banks' conditional beliefs after observing their noisy private signals, imposing an improper prior distribution on θ considerably simplifies the analysis without altering the core results.

Note that we do not analyze the initial financing decisions by the banks. In essence, we assume that the firm already holds credit relations to the banks, which have to be either confirmed by extending the loans in the intermediate period or terminated by withdrawing the money prematurely. Our model is therefore focussed solely on the credit continuation decision.

Solving the sequential model backwards, the firm has to decide whether to refinance the withdrawn fraction of loans or to terminate the project altogether. If the project is terminated, the firm receives no payoff at all. If the withdrawn money is refinanced, the firm receives payoff $\pi_F(\text{refinance}) = \theta - \lambda r_R - (1-\lambda)(1-l)r - (1-\lambda)lW_{AL}$ whenever the relationship bank decides to renegotiate the repayment and extends credit. Here,

[4] The relationship bank may therefore use her actions as a signal to the arm's-length banks. A similar approach has been chosen by Pagratis (2004) in an analysis of financially distressed firms and the role that banks play in the restructuring of debtors' contractual obligations. In contrast to Pagratis, we study both firms perceived to be in distress, i.e. running projects with low expected cash-flows, and those with projects yielding high expected cash-flows.

l denotes the proportion of small banks that foreclose their loans. If, instead, the relationship bank chooses to withdraw her loan prematurely, the firm's payoff from refinancing is given by $\pi_F(\text{refinance}) = \theta - \lambda W_R - (1-\lambda)(1-l)r - (1-\lambda)lW_{AL}$.

Assuming that it is optimal for the firm to employ a trigger strategy, we find the following. If the relationship bank extends her part of the loan, the firm will terminate the project rather than refinance the withdrawn money whenever the realized project value θ is lower than

$$\theta_1^* = \lambda r_R + (1-\lambda)r + (1-\lambda)(W_{AL} - r)l.$$

If, instead, the housebank chooses to foreclose the loan, the firm will terminate the project for values of θ lower than

$$\theta_2^* = \lambda W_R + (1-\lambda)r + (1-\lambda)(W_{AL} - r)l.$$

In order to interpret the equilibrium values θ_1^* and θ_2^*, it is useful to introduce two boundary values for θ. Let $\underline{\theta_1}$ be the threshold value of the project cash-flow that renders the firm indifferent between terminating the project and refinancing the withdrawn part of the loan, if all financiers (including the relationship bank) extend their loan,

$$\underline{\theta_1} = \lambda r_R + (1-\lambda)r. \tag{1}$$

Similarly, let $\overline{\theta_2}$ be the threshold value that makes the firm indifferent between terminating and refinancing if all financiers withdraw their loan,

$$\overline{\theta_2} = \lambda W_R + (1-\lambda)W_{AL}. \tag{2}$$

Hence, for $l \to 0$, θ_1^* converges to $\underline{\theta_1}$ from above, for $l \to 1$, θ_2^* converges to $\overline{\theta_2}$ from below.

In the following, we will assume that all players in this game optimally employ trigger strategies.[5] In particular, suppose that small banks switch their decision from foreclosing to extending the loan exactly after observing a private signal of x_{AL}^*. The proportion l of arm's-length banks withdrawing their money early is then given by those who receive private information lower than x_{AL}^*, i.e. $l = \text{prob}(x_{AL} \leq x_{AL}^* | \theta)$.[6] The equilibrium values for the project cash-flow, θ_1^* and θ_2^*, are then resolved as

$$\theta_1^* = \lambda r_R + (1-\lambda)r + (1-\lambda)(W_{AL} - r)\Phi(\sqrt{\beta}(x_{AL}^* - \theta_1^*)) \tag{3}$$

and

$$\theta_2^* = \lambda W_R + (1-\lambda)r + (1-\lambda)(W_{AL} - r)\Phi(\sqrt{\beta}(x_{AL}^* - \theta_2^*)) \ , \tag{4}$$

where $\Phi(\cdot)$ denotes the cumulative normal distribution.[7]

Fig. 1 shows the structure of project cash-flows and the corresponding equilibrium outcomes. Whenever the realized cash-flow of the firm's project is below $\underline{\theta_1}$, the

[5]For a proof of trigger strategies being optimal in global games, see Morris and Shin (2004).

[6]Note that, due to the assumed independence of signal noise, the proportion of banks receiving private information lower than x_{AL}^* is equal to the probability with which an individual bank observes a signal lower than x_{AL}^*.

[7]Likewise, $\varphi(\cdot)$ denotes the normal density.

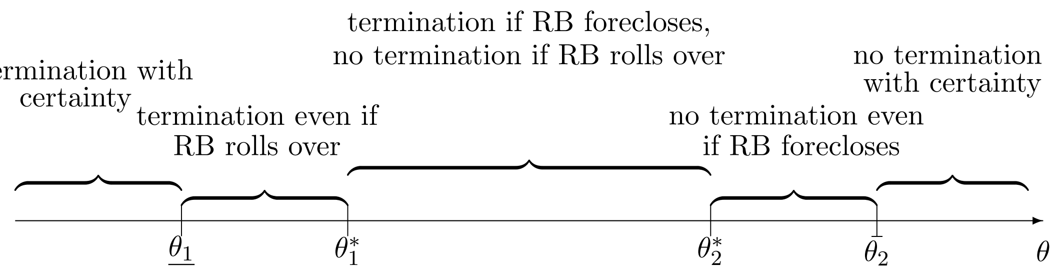

Figure 1: Range of project values

project will be terminated with certainty, even if all small banks and the relationship bank (RB) extend their loans. Terminating the project is the efficient choice for this range of very low project qualities. If $\underline{\theta_1} < \theta < \theta_1^*$, the proportion of arm's-length banks withdrawing their loans is still so high that the firm will decide not to continue the project even if the relationship bank renegotiates the repayment and rolls over the loan. However, terminating the project is not efficient any more, since it is not warranted by the project quality: the firm would have proceeded with the project had only the proportion of withdrawn credit been smaller. For $\theta_1^* < \theta < \theta_2^*$, the outcome depends entirely on the behavior of the relationship bank. If she decides to renegotiate and extend credit, the firm will choose to refinance the money withdrawn from the small banks. If, however, the relationship bank forecloses the loan, the firm will choose to terminate the project. Whenever the project quality is higher than θ_2^* but lower than $\overline{\theta_2}$, the firm will decide to proceed with the project even if the relationship bank chooses to foreclose the loan. For $\theta > \overline{\theta_2}$, the project cash-flow is so high that the project succeeds with certainty, no matter what the small banks and the relationship bank decide to do.[8]

For x_{AL}^* to be a trigger for the arm's-length banks' decision of whether to withdraw or extend credit, they have to be indifferent between the two actions when observing a private signal of $x_{AL_i} = x_{AL}^*$:

$$\begin{aligned}\pi_{AL}(\text{foreclose}|x_{AL_i} = x_{AL}^*) &= \pi_{AL}(\text{extend}|x_{AL_i} = x_{AL}^*)\\ K &= r \cdot \text{prob}(\theta \geq \theta_2^*|x_{AL_i} = x_{AL}^*)\\ &\quad + r \cdot \text{prob}(\theta_1^* \leq \theta \leq \theta_2^*|x_{AL_i} = x_{AL}^*, x_R \geq x_R^*)\ .\end{aligned}$$

Whenever a project value higher than θ_2^* is realized, the firm will continue the project irrespective of the relationship bank's decision. For project values between θ_1^* and θ_2^*, however, continuation of the project critically depends on the relationship bank's action. Even though arm's-length banks can observe her decision, they still have to calculate

[8]The interval $[\underline{\theta_1}, \overline{\theta_2}]$ is the typical range where multiple equilibria occur in a homogeneous banking regime whenever the project value θ is common knowledge. Hence, in this interval both credit withdrawal and credit extension are self-sustaining equilibria that lead to (inefficient) project termination or (efficient) project continuation, respectively.

the probability that the project value lies in the interval $[\theta_1^*, \theta_2^*]$, conditional on their observation and private information x_{AL}.

For the relationship bank a trigger value of x_R^* is determined by a similar condition of indifference:

$$\begin{aligned} \pi_R(\text{foreclose}|x_R = x_R^*) &= \pi_R(\text{renegotiate and extend}|x_R = x_R^*) \\ K &= r_R \cdot \text{prob}(\theta \geq \theta_1^* | x_R = x_R^*), \end{aligned} \tag{5}$$

where

$$r_R = \arg\max_{r \leq W_R} r \cdot \text{prob}(\theta \geq \theta_1^* | x_R) \, ,$$

which leads to[9]

$$r_R = \frac{1 - \Phi(\sqrt{\alpha}(\theta_1^* - x_R))}{\sqrt{\alpha}\frac{\partial \theta_1^*}{\partial r_R}\varphi(\sqrt{\alpha}(\theta_1^* - x_R))} \quad \text{for all} \quad x_R \geq x_R^*. \tag{6}$$

The optimal repayment for the relationship bank is then the r_R that balances two different effects on her expected payoff. First, there is a positive effect of an increase in r_R due to higher repayment per unit of capital whenever credit is repaid. Second, however, a higher repayment to the relationship bank affects the threshold value θ_1^* that has to be exceeded by cash-flow θ for the project to be continued and hence for credit to be repaid. Interestingly, the effect of r_R on θ_1^* may be both positive and negative. On the one hand, a higher repayment to the relationship bank leads to higher cost of capital to the firm. This, in turn, requires a higher project cash-flow θ for the project to be continued. Hence, it reduces the incidence of credit repayment via an increase in θ_1^*, which may be denoted as the "cost"-effect of r_R on θ_1^*. On the other hand, small banks, knowing about the relationship bank's informational advantage, may interpret an increase in r_R as a sign that the relationship bank observed more optimistic information about the project value. This, in turn, makes it more reasonable for small banks to extend their loans, so that θ_1^* decreases. This second impact may be referred to as the "information"-effect of r_R on θ_1^*. Both effects are strongly influenced by the relationship bank's fraction of total firm debt, λ, since it determines both the fraction of capital that succumbs to the renegotiation process but also the fraction $1 - \lambda$ of debt that remains to be coordinated on either extending or withdrawing.

The relationship bank's indifference condition (5) leads to the following equation defining her trigger signal x_R^*:

$$x_R^* = \theta_1^* + \frac{1}{\sqrt{\alpha}}\Phi^{-1}\left(\frac{K}{r_R}\right) . \tag{7}$$

Whenever the relationship bank observes private information lower than x_R^* she will foreclose the loan. For private information higher than x_R^*, however, she will renegotiate the repayment rate and roll over the loan.

[9] See also appendix A.

Based on the strategy of the firm and the observed behaviour of the relationship bank, arm's-length banks are indifferent between foreclosing and extending their loans, whenever they observe a private signal of[10]

$$x^*_{AL} = x^*_R + \sqrt{\frac{\alpha+\beta}{\alpha\beta}}\Phi^{-1}\left[\frac{r\int_{\theta^*_1}^{\theta^*_2}\Phi(\sqrt{\alpha}(\theta - x^*_R))\varphi(\sqrt{\beta}(\theta - x^*_{AL}))d\theta}{K - \Phi(\sqrt{\beta}(x^*_{AL} - \theta^*_2))}\right]. \quad (8)$$

For lower signals they will withdraw, for higher signals they will extend credit. The equilibrium in trigger strategies is hence described by equations (3), (4), (7) and (8).

Note that both the hold-up and intertemporal-smoothing effect of the relationship bank's actions are manifested in r_R. Whenever the relationship bank requests a higher repayment rate, i.e. $r_R > r$, this may be attributed to her ability to hold-up the firm, whereas a decrease in repayment, i.e. $r_R < r$, clearly eases the firm's budget constraints. The relationship bank's coordination-effect, in contrast, shows up in the development of threshold value θ^*_1 (and also of θ^*_2) and may be decomposed into the cost-effect and the information-effect as described above.

The following section analyzes the equilibrium in more detail, with emphasis on the relationship bank's action serving as a (coordinating) signal. We furthermore compare the model of heterogeneous multiple credit relations to a setting with a relationship bank acting as monopolist and also to a completely homogeneous credit market consisting solely of arm's-length banks.

3 Analysis of Financial Systems - Finitely Precise Information

3.1 Heterogeneous Multiple Bank Financing

One of the key features of relationship lending is that the housebank may renegotiate the rate that has to be repaid on her fraction of debt.[11] Since the bargaining result is observable, the relationship bank may try to signal her information to the rest of the market. Before we investigate the impact of the bargaining process on the equilibrium, we will analyze in which way the relationship bank's information influences the renegotiated repayment rate r_R. Based on these results, we will then examine how her decision to foreclose or to renegotiate the repayment rate and extend the loan impacts the threshold values θ^*_1 and θ^*_2 of the project. Since for all $\theta \geq \underline{\theta_1}$ credit termination is inefficient, we will interpret a decrease in θ^*_1 and θ^*_2 as a reduction in the probability of inefficient credit termination, respectively of project liquidation, as it reduces the state space of inefficient credit decisions.

[10]For the derivation of the equilibrium equation for arm's-length banks, see appendix B.

[11]Note that due to the assumption that all the bargaining power rests with the relationship bank, the renegotiated repayment, in a take-it-or-leave-it offer, extracts all the ex-ante expected surplus from the firm.

For the effect of the relationship bank's private information x_R on the repayment rate r_R, we find the following:

Lemma 1 *For $\partial\theta_1^*/\partial r_R$ sufficiently high (low), the relationship bank will request a higher (lower) repayment rate with improving information, i.e. higher signal x_R.*

Proof:

The effect of the relationship bank's private information on the renegotiated repayment rate is given by:

$$\frac{\partial r_R}{\partial x_R} = \frac{\sqrt{\alpha}\frac{\partial\theta_1^*}{\partial r_R}[\varphi(\cdot) + 1 - \Phi(\cdot)] - [1 - \Phi(\cdot)]\frac{\partial^2\theta_1^*}{\partial r_R \partial x_R}}{\sqrt{\alpha}\varphi(\cdot)[\frac{\partial\theta_1^*}{\partial r_R}]^2} ,$$

where $(\cdot) = (\sqrt{\alpha}(\theta_1^* - x_R))$. The sign of this derivative depends on the ratio of the second-order derivative of θ_1^* to its first-order derivatives. In particular, $\frac{\partial r_R}{\partial x_R}$ is positive if $\frac{\partial^2\theta_1^*}{\partial x_R \partial r_R} < \frac{1-\Phi(\cdot)+\varphi(\cdot)}{1-\Phi(\cdot)}\sqrt{\alpha}\frac{\partial\theta_1^*}{\partial r_R}(1 - \frac{\partial\theta_1^*}{\partial x_R})$, which is the case for sufficiently high $\partial\theta_1^*/\partial r_R$. In contrast, r_R decreases in x_R if $\frac{\partial^2\theta_1^*}{\partial x_R \partial r_R} > \frac{1-\Phi(\cdot)+\varphi(\cdot)}{1-\Phi(\cdot)}\sqrt{\alpha}\frac{\partial\theta_1^*}{\partial r_R}(1 - \frac{\partial\theta_1^*}{\partial x_R})$, which holds for sufficiently low $\partial\theta_1^*/\partial r_R$. Q.E.D.

A higher signal x_R increases the relationship bank's posterior expectation with respect to project cash-flow. Hence, she may try to charge a higher repayment in order to extract the higher expected net profit from the firm. However, she knows that threshold θ_1^* is influenced by her choice of r_R. As a consequence, $\partial r_R/\partial x_R$ is positive only if the repayment rate r_R has a strong positive impact on the threshold value θ_1^*. In this case, the cost-effect of the relationship bank's action dominates, so that with improving information an increase in r_R is necessary to offset the (increasing) effect on θ_1^* and the resulting negative impact on her expected profit. If instead threshold θ_1^* decreases in r_R, a lower private signal induces the relationship bank to request a higher repayment rate. In this case, the information-effect of her action is sufficiently strong to reduce the proportion of credit withdrawn by arm's-length banks along with r_R, so that with deteriorating expectations about the project's cash-flow, a higher repayment rate is necessary to compensate the relationship bank for the low probability of credit repayment.

Likewise, the impact of the relationship bank's information precision α on the renegotiated repayment rate r_R depends on the repayment rate's effect on threshold θ_1^*:

Lemma 2 *For $\partial\theta_1^*/\partial r_R > 0$, more precise private information induces the relationship bank to bargain for a lower (higher) repayment rate whenever her private information is sufficiently low (high). For $\partial\theta_1^*/\partial r_R < 0$, the opposite holds.*

Proof:

The partial derivative

$$\frac{\partial r_R}{\partial \alpha} = \frac{-[\frac{1}{2}(\theta_1^* - x_R)\frac{\partial \theta_1^*}{\partial r_R} + \sqrt{\alpha}\frac{\partial \theta_1^*}{\partial r_R}\frac{\partial \theta_1^*}{\partial \alpha}][\varphi(\sqrt{\alpha}(\theta_1^* - x_R)) + \Phi(\sqrt{\alpha}(x_R - \theta_1^*))]}{\alpha(\frac{\partial \theta_1^*}{\partial r_R})^2\varphi(\sqrt{\alpha}(\theta_1^* - x_R))}$$
$$- \frac{\Phi(\sqrt{\alpha}(x_R - \theta_1^*))[\frac{1}{2\sqrt{\alpha}}\frac{\partial \theta_1^*}{\partial r_R} + \sqrt{\alpha}\frac{\partial \theta_1^{*2}}{\partial r_R \partial \alpha}]}{\alpha(\frac{\partial \theta_1^*}{\partial r_R})^2\varphi(\sqrt{\alpha}(\theta_1^* - x_R))}$$

is negative for $\frac{\partial \theta_1^*}{\partial r_R} > 0$, whenever

$$x_R < \theta_1^* + 2\sqrt{\alpha}\frac{\partial \theta_1^*}{\partial \alpha} + \frac{\Phi(\sqrt{\alpha}(x_R - \theta_1^*))\left[\frac{1}{\sqrt{\alpha}} + 2\sqrt{\alpha}\frac{\frac{\partial^2 \theta_1^*}{\partial r_R \partial \alpha}}{\frac{\partial \theta_1^*}{\partial r_R}}\right]}{\varphi(\sqrt{\alpha}(\theta_1^* - x_R)) + \Phi(\sqrt{\alpha}(x_R - \theta_1^*))}$$

and positive otherwise. The opposite holds if $\frac{\partial \theta_1^*}{\partial r_R} < 0$. Q.E.D.

Let us illustrate this result for the case where the relationship bank's cost-effect outweighs the information-effect, so that θ_1^* increases in r_R. If the relationship bank receives optimistic information about the firm's project (i.e. a high signal x_R), she will charge a higher repayment rate the more precise her private information is. Precise optimistic posterior information will ascertain the relationship bank that the project will be continued, so that demanding a high repayment maximizes her profit even though θ_1^* increases in r_R. If, in contrast, the relationship bank is pessimistic about the project's prospects due to low posterior information x_R, she will grant a lower repayment rate the more precise her information is. In that case, higher information precision convinces her that the firm may not be able to continue the project, since its cash-flow is very likely to be too low. Reducing the repayment rate eases the firm's financial constraints and increases the probability of credit repayment, thereby maximizing the relationship bank's payoff despite the lowering of the repayment rate r_R.

Should, however, the information-effect of the relationship bank's action dominate, so that θ_1^* decreases in r_R, she will charge a lower repayment rate with more precise optimistic information and a higher repayment rate with more precise pessimistic information. Since small banks will interpret an increase in r_R as more optimistic information observed by the relationship bank, they will extend their loans for a larger interval of signals and hence reduce the firm's financial constraints exactly when it is needed most: in the case of (perceived) financial distress. This, in turn, allows the relationship bank to increase the repayment rate (in order to maximize her payoff) even though she has precise pessimistic information about the project value. In contrast, since the information-effect leads a lower repayment rate r_R to increase θ_1^* and hence raises the incidence of project failure, the relationship bank can lower r_R only if she is sure that the project's payoff will be sufficiently high, i.e. if she has precise and optimistic information about θ.

Apart from the relationship bank's information parameters x_R and α, also her fraction of total firm debt, λ, has an impact on the bargained repayment rate r_R.

Lemma 3 *Whenever $\partial\theta_1^*/\partial r_R$ is sufficiently high (low), the bargained repayment rate r_R increases (decreases) along with λ.*

Proof:

The impact of λ on the bargained repayment rate is given by

$$\frac{\partial r_R}{\partial \lambda} = -\frac{\alpha\frac{\partial\theta_1^*}{\partial r_R}\frac{\partial\theta_1^*}{\partial\lambda}[\varphi(\sqrt{\alpha}(\theta_1^* - x_R)) + \Phi(\sqrt{\alpha}(x_R - \theta_1^*))] + \sqrt{\alpha}\frac{\partial^2\theta_1^*}{\partial r_R \partial\lambda}\Phi(\sqrt{\alpha}(x_R - \theta_1^*))}{\alpha\varphi(\sqrt{\alpha}(\theta_1^* - x_R))\left(\frac{\partial\theta_1^*}{\partial r_R}\right)^2}$$

This partial derivative is positive as long as $\frac{\partial^2\theta_1^*}{\partial\lambda\partial r_R} < \frac{\Phi(\cdot)-\varphi(\cdot)-1}{1-\Phi(\cdot)}\sqrt{\alpha}\frac{\partial\theta_1^*}{\partial\lambda}\frac{\partial\theta_1^*}{\partial r_R}$, which tends to be the case if $\frac{\partial\theta_1^*}{\partial r_R}$ is sufficiently high. Otherwise, r_R will decrease in the housebank's proportion of credit, λ. Q.E.D.

If the cost-effect of the relationship bank's repayment renegotiation is sufficiently strong, a higher value of λ must induce an increase in the repayment rate to make up for the reduction in expected payoff following from a low probability of credit repayment and vice versa.

In the following, we will analyze the influence of the renegotiated repayment rate r_R on the equilibrium values θ_1^* and θ_2^* that the cash-flow has to exceed for the project to be continued.

Lemma 4 *Equilibrium values θ_1^* and θ_2^* increase (decrease) along with r_R if x_{AL}^* rises (falls) in r_R.*

Proof:

$$\frac{\partial\theta_1^*}{\partial r_R} = \frac{\lambda + (1-\lambda)(W_{AL} - r)\sqrt{\beta}\varphi(\sqrt{\beta}(x_{AL}^* - \theta_1^*))}{1 + (1-\lambda)(W_{AL} - r)\sqrt{\beta}\varphi(\sqrt{\beta}(x_{AL}^* - \theta_1^*))} \cdot \frac{\partial x_{AL}^*}{\partial r_R}$$

$$\frac{\partial\theta_2^*}{\partial r_R} = \frac{(1-\lambda)(W_{AL} - r)\sqrt{\beta}\varphi(\sqrt{\beta}(x_{AL}^* - \theta_2^*))}{1 + (1-\lambda)(W_{AL} - r)\sqrt{\beta}\varphi(\sqrt{\beta}(x_{AL}^* - \theta_2^*))} \cdot \frac{\partial x_{AL}^*}{\partial r_R}\ .$$

Q.E.D.

The influence of r_R on the strategy of arm's-length banks plays a pivotal role for the model. In particular, it determines whether the information-effect or the cost-effect of the renegotiation process dominates, so that the threshold values θ_1^* and θ_2^* either decrease or increase in r_R. Even though we can show that small banks' trigger signal x_{AL}^* decreases in r_R whenever r takes on extreme values and K lies in an intermediate region,[12] the sign of this derivative is not fully determined (see appendix C). However, a combination of the results derived so far allows interesting interpretations with regard to the efficiency of credit decisions and is given in proposition 1. It sums up the effects of the relationship bank's private information precision on credit extension and project continuation decisions. The results differentiate between a firm (respectively its project)

[12] This combination of parameter values describes a situation where premature withdrawal of credit is not very attractive to banks while extending the credit is either very desirable (high value of r) or not desirable at all (low value of r).

perceived as sound by the relationship bank, which is the case if the housebank observes high posterior information x_R, and a firm perceived as distressed, i.e. for low posterior information x_R.

Proposition 1 *Increasingly (decreasingly) precise private information held by the relationship bank reduces (raises) the incidence of inefficient credit termination decisions for firms perceived as distressed. The opposite holds for firms perceived as sound.*

Table 1 delineates the mechanisms for the case of an increase in the relationship bank's information precision.

Table 1: Effects of an increase in α

	firm perceived as sound	firm perceived as distressed
$\frac{\partial x^*_{AL}}{\partial r_R} > 0$	$r_R \uparrow \Rightarrow x^*_{AL} \uparrow, \theta^*_1 \uparrow, \theta^*_2 \uparrow$	$r_R \downarrow \Rightarrow x^*_{AL} \downarrow, \theta^*_1 \downarrow, \theta^*_2 \downarrow$
$\frac{\partial x^*_{AL}}{\partial r_R} < 0$	$r_R \downarrow \Rightarrow x^*_{AL} \uparrow, \theta^*_1 \uparrow, \theta^*_2 \uparrow$	$r_R \uparrow \Rightarrow x^*_{AL} \downarrow, \theta^*_1 \downarrow, \theta^*_2 \downarrow$

No matter whether cost-effect ($\partial x^*_{AL}/\partial r_R > 0$) or information-effect ($\partial x^*_{AL}/\partial r_R < 0$) dominate, whenever the relationship bank obtains more precise information, the risk of inefficient credit termination is reduced for projects perceived as distressed by the relationship bank, but is increased for projects with high expected cash-flows. The opposite holds for a decreasing precision of information held by the relationship bank.

For firms perceived to be sound, the coordination-effect of the relationship bank's decisions always leads to an increase of inefficient credit decisions the higher the relationship bank's information precision becomes. Whenever the cost-effect of the renegotiation process dominates, so that x^*_{AL} rises in r_R, the relationship bank has to ask for higher repayment r_R with increasing precision of her (optimistic) information in order to maximize her profit (lemma 2). The higher value of r_R and the resulting hold-up costs make small banks more reluctant to extend credit, so that x^*_{AL} increases. Hence, the project cash-flow has to exceed higher threshold values in order to let the project be continued by the firm. As a consequence, the prior probability of project liquidation increases (lemma 4). If, in contrast, the information-effect of the renegotiation process dominates, more precise information leads the relationship bank to ask for a lower repayment rate. Small banks will interpret this intertemporal smoothing as a sign of a lower project quality and again will foreclose their loans for a larger range of signals. Hence, θ^*_1 and θ^*_2 are increased and the incidence of inefficient credit termination is raised.

For firms with low expected project cash-flows, the coordination-effect, in combination with either hold-up or intertemporal smoothing, always decreases the likelihood of inefficient credit decisions. If the cost-effect of r_R dominates, increasing precision of her (pessimistic) information induces the relationship bank to lower the repayment rate in order to soften the firm's budget constraints. Due to the dominance of the cost-effect, small banks acknowledge the reduced pressure on the firm's repayment amount and consequently extend their loans even for lower values of their private signals. Thus, the probability of inefficient credit termination is reduced. Whenever the information-effect of the renegotiation process dominates, however, after observing more precise information the relationship bank will demand a higher repayment rate in order to make up for the lower perceived probability of credit repayment. This increase in the firm's hold-up costs will, however, be interpreted by small banks as a sign of an improved firm quality, so that they extend their loans even for lower private information, i.e. θ_1^* and θ_2^* are decreased.

As can be seen, the various cells in Table 1 present different combinations of cost-effect (in the upper row) and information-effect (lower row) with either hold-up or intertemporal-smoothing mechanism. If we assume that the firm is best-off with a low repayment and low values of θ_1^* and θ_2^*, while the relationship bank aims at low values of θ_1^* and θ_2^* as well but prefers a high repayment rate r_R, we can see that with increasing information precision perceived by the relationship bank the optimal position for the firm is the upper right cell, where the cost-effect is combined with intertemporal-smoothing, whereas the relationship bank prefers the lower right cell, which combines the information-effect with a hold-up. Firms running projects with high expected cash-flows, in contrast, benefit from a decrease in the relationship bank's information precision.

3.2 Comparing Financial Systems

In the following, we will first compare the system of heterogeneous multiple bank financing of section 3.1 to single relationship banking, i.e. a monopoly of the relationship bank ($\lambda = 1$). Secondly, we will analyze the case where only arm's-length debt is available ($\lambda = 0$), so that the firm holds multiple credit relations with homogeneous small banks.

3.2.1 Single Relationship Lending

Let us assume that in the first stage of the game ($t = 0$) the only source of financing is the relationship bank. In the later stages, further financiers are supposed to exist, so that refinancing in $t = 2$ is still a viable option to the firm.

From the firm's indifference condition it follows that the critical value of the project up to which the firm always chooses to terminate the project rather than refinance is $\hat{\theta} = W_R$. By renegotiating the repayment rate, the relationship bank tries to maximize her profit from extending the loan. Feasibility requires $\hat{r}_R \leq W_R$, so that

$$\hat{r}_R = \arg\max_{r \leq W_R} r \cdot \text{prob}(\theta \leq \hat{\theta} | x_R = \hat{x}_R) = W_R$$

Hence, in a monopoly situation, the relationship bank always demands the maximum repayment, so that the intertemporal-smoothing effect vanishes and the firm succumbs to maximal hold-up. The relationship bank is then indifferent between withdrawing and extending the loan if:

$$\begin{aligned} \pi_R(\text{withdraw}) &= \pi_R(\text{extend}) \\ K &= W_R \cdot \text{prob}(\theta \geq \hat{\theta} | x_R = \hat{x}_R) \\ \hat{x}_R &= W_R + \frac{1}{\sqrt{\alpha}} \Phi^{-1}(\frac{K}{W_R}) \ . \end{aligned} \tag{9}$$

Based on the equilibrium values $\hat{r}_R$, $\hat{\theta}$ and $\hat{x}_R$, the following lemma holds:

Lemma 5 *The incidence of inefficient project termination in a monopoly relationship lending regime is at least as high as with heterogeneous multiple bank financing.*

Proof:

$$\theta_1^* = \lambda r_R + (1-\lambda)[(1-l)r + lW_{AL}] \leq W_R = \hat{\theta},$$

since due to feasibility of the bargaining process $r_R \leq W_R$. Q.E.D.

3.2.2 Homogeneous Multiple Bank Financing

If the only source of financing is a continuum of small banks, the firm's indifference condition delivers a trigger value of $\tilde{\theta} = r + (W_{AL} - r)l$, up to which she terminates the project but proceeds for higher cash-flows. The small banks' indifference condition yields the equilibrium value for the private signal

$$\tilde{x}_{AL} = \tilde{\theta} + \frac{1}{\sqrt{\beta}} \Phi^{-1}(\frac{K}{r}) \ .$$

Hence, since all banks with private information lower than $\tilde{x}_{AL}$ will foreclose, equilibrium value $\tilde{\theta}$ can be expressed as $\tilde{\theta} = r + (W_{AL} - r)\Phi(\sqrt{\beta}(\tilde{x}_{AL} - \tilde{\theta}))$. Substituting for $\tilde{x}_{AL}$ then delivers

$$\tilde{\theta} = r - K + \frac{W_{AL}K}{r} \ .$$

When comparing homogeneous multiple bank financing to the heterogeneous system of section 3.1, we have to distinguish two different scenarios: either the relationship bank renegotiated the repayment rate and rolled over the loan or she foreclosed the loan. Let us first analyze the case of "efficient coordination" of lenders, in which the relationship bank extends the loan. It might be expected that in a heterogeneous system the incidence of inefficient credit termination is lower than in a homogeneous setting, i.e. $\theta_1^* < \tilde{\theta}$, since due to the relationship bank's non-negligible fraction of debt a lower amount of coordination among the individual small banks is necessary to yield the efficient outcome. We find that this is indeed the case if:[13]

$$r_R < r + (1 - \frac{1}{\lambda})(W_{AL} - r)\Phi(\sqrt{\beta}(x_{AL}^* - \theta_1^*)) - \frac{K}{\lambda} + \frac{W_{AL}K}{r\lambda} \ .$$

[13]Respectively if $K > r\lambda\frac{r_R - r}{W_{AL} - r} + r(1-\lambda)\Phi(\sqrt{\beta}(x_{AL}^* - \theta_1^*))$.

If, in contrast, the relationship bank withdraws the loan, we find that the event of inefficient termination of projects is lower with heterogeneous than homogeneous multiple bank financing, i.e. $\theta_2^* < \tilde{\theta}$, if:[14]

$$\lambda < \frac{\frac{W_{AL}K}{r} - K - (W_{AL} - r)\Phi(\sqrt{\beta}(x_{AL}^* - \theta_2^*))}{W_R - r - (W_{AL} - r)\Phi(\sqrt{\beta}(x_{AL}^* - \theta_2^*))} \ .$$

Lemma 6 *Whenever the relationship bank causes relatively small costs for the firm, i.e. if either the repayment rate r_R or her fraction of total debt λ are sufficiently small, the incidence of inefficient project liquidation is lower in a system with heterogeneous rather than homogeneous multiple credit relations.*

In order to interpret this result, consider that a low fraction of total firm debt held by the relationship bank and a low repayment rate charged by her reduce the severity of the hold-up problem that is at play in the heterogeneous multiple banking regime but does not play a role an a homogeneous system.

4 Analysis of Financial Systems - Infinitely Precise Information

4.1 Heterogeneous Multiple Bank Financing

Apart from being able to renegotiate the repayment and providing a non-negligible fraction of total firm debt, the relationship bank disposes of relatively more precise private information about the firm and its project. In the extreme, we may analyze the case in which the relationship bank's information is completely precise, i.e. $\alpha \to \infty$. It is easy to see that the equilibrium values x_R^{**}, x_{AL}^{**}, θ_1^{**} and θ_2^{**} then converge in the following way:

$$\begin{aligned} x_R^{**} &\to \theta_1^{**} \\ x_{AL}^{**} &\to \theta_1^{**} \\ \theta_1^{**} &\to \lambda r_R + \frac{1}{2}(1-\lambda)(W_{AL} + r) \\ \theta_2^{**} &\to \lambda W_R + (1-\lambda)r + (1-\lambda)(W_{AL} - r)\Phi(\sqrt{\beta}(\theta_1^{**} - \theta_2^{**})) \ . \end{aligned}$$

With completely precise information, the relationship bank will demand the maximum feasible repayment, i.e. $r_R = W_R$, so that hold-up costs are maximized and intertemporal smoothing no longer exists. Since arm's-length banks know that the relationship bank disposes of fully precise information, their optimal strategy is to do the same as she does. The formerly individual threshold values θ_1^{**}, x_R^{**} and x_{AL}^{**} therefore converge to one identical value. It is easy to see that threshold values θ_1^{**} and θ_2^{**} increase in the relationship bank's "size" λ, the repayment rates r and r_R and in refinancing costs W_{AL} and W_R, so that the information-effect of the renegotiation process is completely dominated by the cost-effect, irrespective of the project quality.

[14] Respectively, if $K > r\lambda\frac{W_R - r}{W_{AL} - r} + r(1-\lambda)\Phi(\sqrt{\beta}(x_{AL}^* - \theta_2^*))$.

4.2 Comparing Financial Industries

A completely informed relationship bank acting as monopolist is characterized by the following equilibrium values:

$$\hat{\hat{\theta}} = \hat{\hat{x}}_R = W_R \; .$$

As before, we find that the event of inefficient project liquidation is never higher with heterogeneous multiple bank financing than when the relationship bank acts as monopolist, i.e. $\theta_1^{**} \leq \hat{\hat{\theta}}$.

When comparing threshold values $\theta_1{}^*{*}$ and θ_2^{**} from a heterogeneous system, given that the relationship bank disposes of completely precise private information, with the threshold level for the project's value in a homogeneous system, we find that $\theta_1^{**} < \tilde{\tilde{\theta}}$ whenever[15]

$$\begin{aligned} W_R &< r + (1 - \frac{1}{\lambda})(W_{AL} - r)\frac{1}{2} - \frac{K}{\lambda} + \frac{W_{AL}K}{r\lambda} \\ \Leftrightarrow \lambda &< \frac{W_{AL}(\frac{K}{r} - \frac{1}{2}) - K + \frac{1}{2}r}{W_R - \frac{1}{2}(W_{AL} + r)} \; , \end{aligned}$$

i.e. for sufficiently low refinancing costs respectively for sufficiently low fraction of firm debt held by the relationship bank. For $\theta_2^{**} < \tilde{\tilde{\theta}}$ it has to hold that:[16]

$$\lambda < \frac{W_{AL}\frac{K}{r} - K - (W_{AL} - r)\Phi(\sqrt{\beta}(\theta_1^* - \theta_2^*))}{W_R - r - (W_{AL} - r)\Phi(\sqrt{\beta}(\theta_1^* - \theta_2^*))} \; .$$

Since with completely precise information, the relationship bank will charge the maximum feasible repayment rate irrespective of the project value, heterogeneous multiple bank financing can only reduce the event of inefficient project liquidation compared to a homogeneous banking regime, if the relationship bank's fraction of total firm debt is sufficiently small, so that the severity of the hold-up problem for the firm's cost of capital does not become too strong.

Lemma 7 *Even with completely precise private information held by the relationship bank, the incidence of inefficient project liquidation in a system of heterogeneous multiple bank financing is reduced in comparison to a monopoly situation. Compared to a homogeneous system, however, a heterogeneous regime can only mitigate the event of inefficient project liquidation if the relationship bank's fraction of total firm debt is sufficiently low.*

5 Conclusion

Heterogeneous multiple bank financing, where firms obtain credit from both a well-informed relationship bank and a multitude of small, less well-informed arm's-length

[15]Respectively, if $K > r\lambda\frac{W_R - r}{W_{AL} - r} + r(1-\lambda)\frac{1}{2}$. Note that this condition is more restrictive than the one under finitely precise information whenever $x_{AL}^* > \theta_1^*$ and vice versa.

[16]Respectively, it has to hold that $K > r\lambda\frac{W_R - r}{W_{AL} - r} + r(1-\lambda)\Phi(\sqrt{\beta}(\theta_1^{**} - \theta_2^{**}))$.

banks, predominates in many countries. In particular small and medium-sized businesses seem to prefer such heterogeneous banking regimes over other ways to obtain capital. Our study points to the asymmetric informational positions of the involved banking institutions as an explaining factor for the prevalence of such heterogeneous financing systems.

Assuming that with increasing duration of the credit relation the relationship bank's information about her borrowers becomes increasingly precise, we find that heterogeneous multiple bank financing reduces the incidence of inefficient credit termination for firms perceived to be in financial distress. For firms conducting sound projects, the opposite holds. With increasing length of the credit relation, they face a higher probability of inefficient project liquidation. If we accept that small and medium-sized business typically run a higher risk of financial distress than larger firms, we might conjecture that particularly those smaller firms are interested in keeping up multiple credit relations with asymmetric banking institutions.

Contrasting earlier work on the question of the optimal financing regime (Elsas et al, 2004), we can show that even with infinitely precise information by the relationship bank, a system of heterogeneous multiple bank financing may still reduce the incidence of inefficient termination of credit for all firms as compared to homogeneous credit relations or a monopoly lending regime. As a sufficient condition, the relationship bank's fraction of total debt has to remain sufficiently small as to not represent a virtual monopoly. This result is in contrast to Elsas et al. (2004), who found that with completely precise information held by all banks, firms with high expected cash-flows will tend to finance without a relationship bank.

Given the analysis of different financial systems, it might be interesting to establish the optimal debt structure, taking into account the relationship bank's informational advantage. Obviously, the optimal financing structure should depend mainly on the borrower quality. As our model suggests, however, the transparency of this variable should play an additional role. Tentatively, one might conjecture that more intransparent firms should opt for a larger fraction of relationship lending. A detailed examination of this question is, however, outside the scope of this paper and is left for future research.

Appendix A

For r_R to yield a maximum payoff to the relationship bank, the second-order condition demands that:

$$\frac{\partial^2 R_R}{\partial r_R^2} = \sqrt{\alpha}\varphi(\sqrt{\alpha}(\theta_1^* - x_R))\left[-2\frac{\partial\theta_1^*}{\partial r_R} - r_R(\frac{\partial\theta_1^*}{\partial r_R})^2 - r_R\frac{\partial^2\theta_1^*}{\partial r_R^2}\right] < 0 ,$$

where R_R denotes the relationship bank's expected payoff from extending the loan. This condition is satisfied, if either $\frac{\partial\theta_1^*}{\partial r_R} > -\frac{1}{r_R\sqrt{\alpha}} + \sqrt{\frac{1-r_R^2\sqrt{\alpha}}{r_R^2\alpha}\frac{\partial^2\theta_1^*}{\partial r_R^2}}$ or $\frac{\partial\theta_1^*}{\partial r_R} < -\frac{1}{r_R\sqrt{\alpha}} - \sqrt{\frac{1-r_R^2\sqrt{\alpha}}{r_R^2\alpha}\frac{\partial^2\theta_1^*}{\partial r_R^2}}$, i.e. feasibility requires $\frac{\partial\theta_1^*}{\partial r_R}$ to be either sufficiently large or sufficiently small. Note that we do not rule out the case where the renegotiation leads the housebank to grant a subsidy to the firm.

Appendix B

The information structure in the model is as follows. Given the realized project value θ, private signals are distributed as $x_R|\theta \sim N(\theta, \frac{1}{\alpha})$ and $x_{AL}|\theta \sim N(\theta, \frac{1}{\beta})$. For the financiers, after observing their private information, the unknown project value θ is distributed according to $\theta|x_R \sim N(x_R, \frac{1}{\alpha})$ and $\theta|x_{AL} \sim N(x_{AL}, \frac{1}{\beta})$. Moreover, each arm's-length bank believes the relationship bank's information to be given by $x_R|x_{AL} \sim N(x_{AL}, \frac{\alpha+\beta}{\alpha\beta})$. Additionally, $\text{Cov}(x_R, \theta|x_{AL}) = \frac{1}{\beta}$, so that $\text{Corr}(x_R, \theta|x_{AL}) = \frac{\frac{1}{\beta}}{\sqrt{\frac{\alpha+\beta}{\alpha\beta}}\sqrt{\frac{1}{\beta}}} = \sqrt{\frac{\alpha}{\alpha+\beta}}$.

Derivation of the arm's-length banks' indifference condition and threshold x_{AL}^*:

$$\frac{K}{r} = 1 - \text{prob}(\theta \leq \theta_2^*|x_{AL} = x_{AL}^*) + \frac{\text{prob}(\theta_1^* \leq \theta \leq \theta_2^*, x_R > x_R^*|x_{AL} = x_{AL}^*)}{\text{prob}(x_R > x_R^*|x_{AL} = x_{AL}^*)} \quad (10)$$

Denote $\frac{\text{prob}(\theta_1^* \leq \theta \leq \theta_2^*, x_R > x_R^*|x_{AL} = x_{AL}^*)}{\text{prob}(x_R > x_R^*|x_{AL} = x_{AL}^*)}$ by m. We then have:

$$m = \frac{\int_{\theta_1^*}^{\theta_2^*}\int_{x_R^*}^{\infty} \frac{1}{2\pi\frac{1}{\sqrt{\beta}}\sqrt{\frac{\alpha+\beta}{\alpha\beta}}\sqrt{1-\frac{\alpha}{\alpha+\beta}}} e^{-\frac{1}{2(1-\frac{\alpha}{\alpha+\beta})}\cdot\left[\left(\frac{\theta-x_{AL}^*}{\frac{1}{\sqrt{\beta}}}\right)^2 - 2\sqrt{\frac{\alpha}{\alpha+\beta}}\frac{\theta-x_{AL}^*}{\frac{1}{\sqrt{\beta}}}\frac{x_R-x_{AL}^*}{\sqrt{\frac{\alpha+\beta}{\alpha\beta}}} + \left(\frac{x_R-x_{AL}^*}{\sqrt{\frac{\alpha+\beta}{\alpha\beta}}}\right)^2\right]} dx_R d\theta}{1-\Phi(\sqrt{\frac{\alpha\beta}{\alpha+\beta}}(x_R^* - x_{AL}^*))}$$

$$m = \frac{\int_{\theta_1^*}^{\theta_2^*}\int_{x_R^*}^{\infty} \frac{\sqrt{\alpha\beta}}{2\pi} e^{-\frac{1}{2}\alpha\left[(\theta-x_R)^2 + \frac{\beta}{\alpha}(\theta-x_{AL}^*)^2\right]} dx_R d\theta}{1-\Phi(\sqrt{\frac{\alpha\beta}{\alpha+\beta}}(x_R^* - x_{AL}^*))}$$

$$m = \frac{\int_{\theta_1^*}^{\theta_2^*}\left[1-\int_{-\infty}^{x_R^*}\frac{1}{\sqrt{2\pi}\sqrt{\frac{1}{\alpha}}}e^{-\frac{1}{2}\left(\frac{x_R-\theta}{\sqrt{\frac{1}{\alpha}}}\right)^2}dx_R\right]\frac{1}{\sqrt{2\pi}\sqrt{\frac{1}{\beta}}}e^{-\frac{1}{2}\left(\frac{\theta-x_{AL}^*}{\sqrt{\frac{1}{\beta}}}\right)^2}d\theta}{1-\Phi(\sqrt{\frac{\alpha\beta}{\alpha+\beta}}(x_R^*-x_{AL}^*))}$$

$$m = \frac{\int_{\theta_1^*}^{\theta_2^*}\left[1-\Phi(\sqrt{\alpha}(x_R^*-\theta))\right]\varphi(\sqrt{\beta}(\theta-x_{AL}^*))d\theta}{1-\Phi(\sqrt{\frac{\alpha\beta}{\alpha+\beta}}(x_R^*-x_{AL}^*))}$$

$$m = \frac{\int_{\theta_1^*}^{\theta_2^*}\Phi(\sqrt{\alpha}(\theta-x_R^*))\varphi(\sqrt{\beta}(\theta-x_{AL}^*))d\theta}{\Phi(\sqrt{\frac{\alpha\beta}{\alpha+\beta}}(x_{AL}^*-x_R^*))} \quad (11)$$

Plugging this in (10), we find:

$$\frac{K}{r} = 1-\Phi(\sqrt{\beta}(\theta_2^*-x_{AL}^*)) + \frac{\int_{\theta_1^*}^{\theta_2^*}\Phi(\sqrt{\alpha}(\theta-x_R^*))\varphi(\sqrt{\beta}(\theta-x_{AL}^*))d\theta}{\Phi(\sqrt{\frac{\alpha\beta}{\alpha+\beta}}(x_{AL}^*-x_R^*))} \quad (12)$$

In order to solve for the optimal strategy of arm's-length banks, it follows from equation (4) that

$$\theta_2^*-x_{AL}^* = -\frac{1}{\sqrt{\beta}}\Phi^{-1}\left(\frac{\theta_2^*-\lambda W_R-(1-\lambda)r}{(1-\lambda)(W_{AL}-r)}\right)$$

Plugging this into (12) yields

$$\frac{K}{r}-\frac{\theta_2^*-\lambda W_R-(1-\lambda)r}{(1-\lambda)(W_AL-r)} = \frac{\int_{\theta_1^*}^{\theta_2^*}\Phi(\sqrt{\alpha}(\theta-x_R^*))\varphi(\sqrt{\beta}(\theta-x_{AL}^*))d\theta}{\Phi(\sqrt{\frac{\alpha\beta}{\alpha+\beta}}(x_{AL}^*-x_R^*))}$$

which finally delivers equation (8).

Appendix C

From (8) it follows that the derivative of x_{AL}^* with respect to r_R can be expressed as

$$\frac{\partial x_{AL}^*}{\partial r_R} = \frac{1+(1-\lambda)(W_{AL}-r)\sqrt{\beta}\phi(\sqrt{\beta}(x_{AL}^*-\theta_1^*))}{1-\lambda}\left[\frac{1}{\sqrt{\alpha}}\frac{\partial\Phi^{-1}(\frac{K}{r_R})}{\partial r_R}+\sqrt{\frac{\alpha+\beta}{\alpha\beta}}\frac{\partial\Phi^{-1}\left(\frac{r\int_{\theta_1^*}^{\theta_2^*}\Phi(\sqrt{\alpha}(\theta-x_R^*))\phi(\sqrt{\beta}(\theta-x_{AL}^*))d\theta}{K-\Phi(\sqrt{\beta}(x_{AL}^*-\theta_2^*))}\right)}{\partial r_R}\right]$$

For intermediate values of K and $r\to 0$ or $r\to\infty$, the second term in brackets on the right-hand-side will converge to $-\infty$ respectively $+\infty$, so that overall $\frac{\partial x_{AL}^*}{\partial r_R}<0$, since $\frac{\partial\Phi^{-1}(\frac{K}{r_R})}{\partial r_R}<0$.

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