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Heterogeneous Multiple Bank Financing, Optimal Business Risk and Information Disclosure

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Abstract

This paper studies optimal risk-taking and information disclosure by firms that obtain financing from both a "relationship" bank and "arm's-length" banks. We find that firm decisions are asymmetrically influenced by the degree of heterogeneity among banks: lowly-collateralized firms vary optimal risk and information precision along with the degree of relationship lending for projects with low expected cash-flows, while highly-collateralized firms do so for projects with high expected cash-flows. Incidences of inefficient project liquidation are minimized if the former firms rely on relationship banking to a low degree, the latter to a large degree.

Keywords: Risk, Relationship Lending, Asymmetric Information, Liquidity Crisis, Efficiency

JEL Classification: G21, L14, D82

1 Introduction

In many countries, firms rely on multiple bank financing. Particularly small- and medium-sized European firms often obtain financing from several banks of which one may be special in the sense of a so-called "relationship bank". Early theoretical work on relationship lending usually saw the relationship bank as the only source of financing for the firm. Relationship banking has been characterized by long-term relations between bank and customer, a large proportion of total firm debt held by the relationship bank, and preferred access to firm-specific information (Fischer, 1990, Elsas, 2004). Potential advantages of relationship lending, such as increased credit availability, intertemporal smoothing of financing conditions, and more efficient credit decisions for borrowers facing financial distress, seem to benefit in particular small, young, and innovative firms that are informationally opaque (Sharpe, 1990, Rajan, 1992, Petersen and Rajan, 1995). These firms typically need to finance projects whose returns are positive only in the long-run and often lack a sufficient track-record to obtain financing from the capital markets. However, the hold-up costs associated with a relationship bank's informational advantage and the ensuing bargaining power may be sufficiently severe to prevent single relationship banking and therefore promote borrowing from multiple "arm's-length" lenders (Von Thadden, 1992, Bolton and Scharfstein, 1996, Detragiache et al., 2000).

The clear-cut results regarding benefits and drawbacks of relationship banking notwithstanding, recent empirical work agreed that firms very often rely on multiple bank financing with a mixture of both relationship and arm's-length lending (Harhoff and Körting, 1998, Ongena and Smith, 2000, Machauer and Weber, 2001). For German data, Brunner and Krahnen (2001) find that the average number of bank relationships is 6 (with a minimum of 1 and a maximum of 30). For a cross-section of European firms, Ongena and Smith (2000) report the average number of bank relationships for instance for Italy as 15.2 and for France as 11.3, with a maximum for the whole data set of 70. Further studies indicate that the number of bank relationships increases in firm size and decreases in the existence of a relationship bank (Ongena and Smith, 2000, Machauer and Weber, 2000, Brunner and Krahnen, 2001).

Even though a wide-spread phenomenon, until recently multiple bank financing has rarely been scrutinized in theoretical work. One of the first papers trying to establish the optimal debt structure in a model of multiple asymmetric bank financing derived the optimal structure from the tradeoff between the bargaining power of a relationship bank and the risk of coordination failure from (symmetric) multiple banking (Elsas et al., 2004). It thereby complemented earlier work on coordination failure in credit markets by Hubert and Schäfer (2002) and Morris and Shin (2004) by a richer structure of bank types. Whereas Morris and Shin (2004) examined coordinating behavior among small, homogeneous lenders only, Hubert and Schäfer (2002) differentiated between small and large creditors, but analyzed the strategies of the different creditor types in separate models. Elsas et al. (2004) were the first to account for the coexistence of a relationship bank lender with various (homogeneous) small banks.

In contrast to the work mentioned so far, this paper investigates the consequences of a heterogeneous multiple banking regime rather than establishing the optimal financing structure. Similarly to the model by Elsas et al. (2004), we emphasize a relationship bank's coordinating role among a multitude of arm's-length banks. In our model, however, coordination effects are due to both the relationship bank's substantial fraction of debt and her superior information about the firm's business prospects, whereas Elsas et al. (2004) put more emphasis on the relationship bank's bargaining power and disregard her informational advantages. Taking the financing structure as given, we are particularly interested in the way heterogeneous multiple bank financing influences firms' risk-taking with regard to the funded business projects and the optimal information disclosure to the relationship bank.

Aspects of optimal risk- and information-policy have also been analyzed by Bannier and Heinemann (2005) for a central bank trying to prevent a coordinated attack on a fixed currency. Similarly to our work, Heinemann and Metz (2002) examined the optimal policy-mix for a firm that aims at minimizing the probability of a liquidity crisis via early withdrawal of credit by a continuum of homogeneous lenders. They find that firms optimally choose maximum risk when the expected project cash-flow is low, but select zero risk for projects with high expected cash-flows. In either case, firms disclose information of maximum precision. The current paper extends this earlier work by assuming a richer structure of creditor types. The model is built around a firm that holds credit relations to several small banks and one relationship bank. Furthermore, whereas the paper by Heinemann and Metz (2002) was limited to firms with large collateral, the current study considers both firms with large and small collateral. As such, we complement the earlier work by focussing additionally on small- and medium-sized firms that typically dispose of only low collateral.

Our results indicate that optimal firm policy is indeed contingent on the level of collateral and on the firm's business prospects. For lowly-collateralized firms, we find that, regarding projects with low expected cash-flows, optimal risk-taking depends on the degree of relationship lending relative to arm's-length lending. If the relationship bank grants a sufficiently large proportion of total firm debt, the firm will choose maximum business risk. For a low proportion of relationship lending, in contrast, the firm optimally decides on minimum project risk. Projects with high expected cash-flows, in contrast, will always be conducted with minimum risk.

Larger firms, that are usually highly collateralized, show a different risk-taking behavior. They vary business risk along with the degree of relationship lending only for projects with high expected cash-flows. Whenever the relationship bank's stake in total firm debt is large, the firm will conduct a project with minimum business risk, but will decide on intermediate riskiness if the relationship bank's proportion of total firm debt is low. For projects with low expected cash-flows, in contrast, the firm will decide on maximum risk. Comparing these results on highly-collateralized firms with the earlier findings by Heinemann and Metz (2002), we see that the financing structure has a decisive influence on optimal firm policy for projects with high expected cash-flows. Whereas homogeneous multiple bank financing will then always induce highly-collateralized firms to choose minimum business risk, they will do so in a heterogeneous financing regime only if the degree of relationship banking is sufficiently high.

With regard to optimal information policy, our model indicates that firms with low collateral will provide their relationship bank with minimally precise information about projects with low expected cash-flow whenever the fraction of relationship lending is sufficiently large, and disclose fully precise information in any other case. Firms with large collateral will deviate from an information disclosure of maximum precision only for projects with high expected cash-flows if the relationship bank's fraction of firm debt is low.

Our results also have implications for the efficiency of firms' businesses. We demonstrate that minimum business risk, combined with fully precise information disclosure to the relationship bank, maximizes ex-ante welfare as it virtually eliminates the incidence of inefficient project liquidation. Choosing maximum risk, in contrast, may merely reduce the ex-ante probability of liquidation, but never eliminates it completely. As such, heterogeneous multiple bank financing may help lowly-collateralized firms to reach maximum efficiency for projects with low expected cash-flows, i.e. eliminate the ex-ante probability of inefficient project liquidation, provided that the degree of relationship banking is not too high. For highly-collateralized firms, in contrast, a heterogeneous financing regime can never be advantageous compared to homogeneous multiple banking. We may summarize our findings by stating that in a system of heterogeneous multiple bank financing ex-ante efficiency is highest if firms with low collateral rely on relationship banking to a relatively low extent, while firms with high collateral employ a large degree of relationship lending.

Aspects of efficient project choice have also been analyzed by Dewatripont and Maskin (1995). They show that multiple bank financing, or "decentralization" in their notation, can lead to efficient project selection when creditors dispose of asymmetric information about project quality. In contrast to our study, however, the credit market structure is derived endogenously as a homogeneous financing regime, whereas we impose a quite restrictive, but to our mind nevertheless reasonable, banking structure with one relationship bank and various arm's-length banks.

Our model essentially analyzes optimal firm policy with regard to both the conduct of business projects and the corresponding financing of projects. A study by von Rheinbaben and Ruckes (2004), in contrast, concentrates mainly on the financing side. They examine a firm's optimal choice of the number of creditors and the extent of information disclosed to them. Their results are based on the trade-off between lower credit costs due to the disclosure of precise information and lower expected operating returns following from information leaks to competitors. They find that highly rated firms disclose only little information, whereas firms with low ratings have to disclose very precise information in order to reduce creditors' uncertainty about their projects. These results may be compared to our findings with regard to highly-collateralized firms that tend to be large, and most often rated, firms. Assuming that high ratings correspond to high expected firm profits, our model states that firms with low ratings provide their relationship bank with completely precise information, whereas firms with high ratings optimally disclose information of only intermediate or even minimal precision, depending on the degree of relationship lending. The similarity of results notwithstanding, information disclosure in our model only affects the relationship bank, whereas in the model by von Rheinbaben and Ruckes (2004) information is disclosed to all creditors.

The remainder of the paper is organized as follows. Section 2 delineates the model of heterogeneous multiple bank financing. Section 3 derives the unique equilibrium, the subsequent section concentrates on basic comparative static results. Section 5 finally analyzes optimal risk-taking and information disclosure for a firm that aims at reducing inefficient project liquidation. Section 6 concludes.

2 The Model

We consider a simple model where the economy consists of three types of agents: a firm, a relationship bank and a continuum of arm's-length banks.[1] The firm plans to run a project with stochastic returns that matures within two time periods. As the firm has no funds available to finance the project, she has to resort to debt financing. In an intermediate stage of the game, lenders may withdraw their loans prematurely, so that the firm is threatened by early liquidation of the project.

The bank financing system is heterogeneous in two respects: first, arm's-length banks dispose of less precise information about the project than the relationship bank. Second, each of the arm's-length lenders grants only a negligible fraction of the full loan to the firm,[2] whereas the relationship lender's proportion of total firm debt is of non-negligible size. In particular, the relationship bank's fraction of total debt amounts to proportion λ, while the small banks provide a combined proportion of $1 - \lambda$ of the full credit. Parameter λ is therefore also taken to characterize the degree of heterogeneity among the involved banks.[3] With regard to banks' information about the project, it is assumed that the relationship bank observes a private signal, x_R, about project quality θ, with $x_R|\theta \sim N(\theta, \frac{1}{c})$, whereas small banks observe individual private signals of $x_S|\theta \sim N(\theta, \frac{1}{b})$. Noise in private signals is supposed to be mutually independent and independent of θ. Moreover, $c \geq b$, so that the relationship bank's private information is at least as precise as any small bank's signal. The distribution of private signals is common knowledge.

The complete structure of the game is as follows:

1. In $t = 0$, the firm approaches the banks in order to request financing for a business project. It offers a repayment of r at maturity ($t = 2$). Based on successful financing decisions,[4] the firm engages in a risky project. It chooses a level of risk that leads to a variance of project cash-flow of $1/a$ and commits to providing the relationship bank with information of precision c. Afterwards, nature selects project quality θ from the commonly known distribution $N(y, 1/a)$. The selected quality θ becomes known to the firm's managers but remains unobservable to bank lenders.

2. In $t = 1$, banks receive private information about θ. Simultaneously, they have to decide whether to extend or withdraw their loans. At the same time, the firm has to decide whether to commit to additional effort V that is necessary for successful completion of the project in $t = 2$, or to terminate the project altogether. The decision to undertake additional effort is tied to refinancing the withdrawn fraction of debt.

3. In $t = 2$, project cash-flow is realized and equals θ if the firm did invest and refinance.

[1] The assumption of a continuum of arm's-length banks is made for simplicity. It can be shown that a finite number of banks does not qualitatively impair the results. See also Morris and Shin (2003)

[2] Arm's-length banks are therefore also referred to as "small" banks.

[3] The higher λ, the larger the proportion of (well-informed) relationship lending relative to (less well-informed) arm's-length lending. For the extreme cases of $\lambda = 1$ and $\lambda = 0$, the model considers single relationship banking and homogeneous multiple banking, respectively.

[4] We abstract from the banks' decision of whether or not to grant a loan to the firm in the first stage of the game. The banks' strategic choice comprises solely the question of whether or not to withdraw the loan prematurely, i.e. in $t = 1$.

Otherwise the project fails and credit cannot be repaid. The final liquidation value of assets is assumed to be zero.

Early withdrawal of capital in $t = 1$ leads to a liquidation value of K $(< r)$ per unit of capital invested. K is also referred to as collateral.[5] Refinancing capital withdrawn by small banks costs the firm W_S, refinancing the relationship bank's fraction of debt leads to costs of W_R per unit of capital. In order to take account of a potential hold-up problem, we assume that $0 \leq r < W_S < W_R \leq 1$. Hence, even though we abstract from a bargaining process between firm and relationship bank, it is more costly to refinance the relationship bank's fraction of debt than the small banks'.

3 Derivation of Equilibrium

Essentially, the depicted model presents a global game in the sense of Carlsson and van Damme (1993), where each player noisily observes the game's payoff structure, which itself is determined by a random draw from a given class of games. Following the solution method of Morris and Shin (2003, 2004), we derive a unique equilibrium in trigger strategies, based on players' indifference conditions, provided that private information is sufficiently precise.[6]

Starting from a process of backwards induction, we find that the firm is indifferent between exerting effort and refinancing the withdrawn parts of credit on the one hand and terminating the project on the other hand, if

$$\begin{aligned} \pi_F(\text{effort and refinance}|\theta) &= \pi_F(\text{terminate}|\theta) \\ \theta - V - \lambda r \text{ prob}(x \geq x_R^*|\theta) - (1-\lambda) r \text{ prob}(x \geq x_S^*|\theta) & \\ -\lambda W_R \text{ prob}(x < x_R^*|\theta) - (1-\lambda) W_S \text{ prob}(x < x_S^*|\theta) &= 0 \,. \end{aligned}$$

Here, it is assumed that the relationship bank follows a trigger strategy around a signal value of x_R^*, so that she withdraws her part of credit whenever $x_R < x_R^*$ and extends credit for $x_R \geq x_R^*$. Likewise, the small banks are supposed to follow trigger strategies around a signal value of x_S^*.[7] The firm will then optimally terminate the project for all project values lower than θ^*, while she will invest effort and refinance the withdrawn part of the credit for higher project values. Trigger value θ^* is given by:

$$\theta^* = V + r + \lambda(W_R - r)\Phi(\sqrt{c}(x_R^* - \theta^*)) + (1-\lambda)(W_S - r)\Phi(\sqrt{b}(x_S^* - \theta^*)) \,. \tag{1}$$

[5] Since the financing structure is exogenous in our model, so that the firm cannot select a different degree of heterogeneity for each project, it is reasonable to think of the project's liquidation value less project-specific simply as the firm's collateral.

[6] For proof of trigger strategies being the uniquely optimal strategies in such global games, see Morris and Shin (2004).

[7] Due to the assumed independence of signals, the proportion of small banks withdrawing their money, defined as the proportion of banks receiving private signals lower than x_S^*, is equivalent to the probability with which any single small bank observes a private signal lower than x_S^*.

The relationship bank is indifferent between foreclosing and extending the loan, if

$$\begin{aligned}\pi_R(\text{foreclose}|x_R) &= \pi_R(\text{extend}|x_R)\\ K &= r\ \text{prob}(\theta \geq \theta^*|x_R)\\ K &= r\left(1-\Phi\left(\sqrt{a+c}\left(\theta^* - \frac{a}{a+c}y - \frac{c}{a+c}x_R\right)\right)\right),\end{aligned}$$

which delivers a trigger value for her private signal of

$$x_R^* = \frac{a+c}{c}\theta^* - \frac{a}{c}y + \frac{\sqrt{a+c}}{c}\Phi^{-1}\left(\frac{K}{r}\right). \tag{2}$$

Hence, whenever the relationship bank observes a signal $x_R < x_R^*$, she forecloses the loan, but extends for $x_R \geq x_R^*$.
Likewise, indifference for the continuum of small banks is given at

$$\begin{aligned}\pi_S(\text{foreclose}|x_S) &= \pi_S(\text{extend}|x_S)\\ K &= r\ \text{prob}(\theta \geq \theta^*|x_S)\\ K &= r\left(1-\Phi\left(\sqrt{a+b}\left(\theta^* - \frac{a}{a+b}y - \frac{b}{a+b}x_S\right)\right)\right).\end{aligned}$$

This, in turn, delivers the trigger signal for small banks as

$$x_S^* = \frac{a+b}{b}\theta^* - \frac{a}{b}y + \frac{\sqrt{a+b}}{b}\Phi^{-1}\left(\frac{K}{r}\right). \tag{3}$$

Plugging the signal values x_R^* and x_S^* in (1) delivers the equilibrium value for the firm's optimal action as

$$\begin{aligned}\theta^* &= V + r + \lambda(W_R - r)\Phi\left(\frac{a}{\sqrt{c}}(\theta^* - y) + \sqrt{\frac{a+c}{c}}\Phi^{-1}\left(\frac{K}{r}\right)\right)\\ &\quad +(1-\lambda)(W_S - r)\Phi\left(\frac{a}{\sqrt{b}}(\theta^* - y) + \sqrt{\frac{a+b}{b}}\Phi^{-1}\left(\frac{K}{r}\right)\right).\end{aligned} \tag{4}$$

The equilibrium given by equations (2), (3) and (4) is unique provided that private information is sufficiently precise relative to public information about θ, i.e. $b, c \geq \frac{a^2}{2\pi}$.

4 Comparative Statics

From the derived equilibrium we know that the firm will terminate the project whenever a project quality lower than θ^* is realized. However, for all $\underline{\theta} \leq \theta < \theta^*$, where $\underline{\theta}$ is given by the firm's indifference condition provided that all lenders extend their loans, i.e. $\underline{\theta} = V + r$, terminating the project is an inefficient action. Only for lower project qualities, termination of the project is warranted due to sufficiently low cash-flows. For values of θ between $\underline{\theta}$ and θ^*, however, the firm will terminate the project only because some fraction of debt has been withdrawn prematurely.

In the following, we assume that the firm aims at preventing inefficient project liquidation. Even though we did not explicitly define a utility function for the firm, it is reasonable to assume that the firm's utility is negatively affected by inefficient withdrawal of credit by its financiers and a thereby implicitly forced termination of an illiquid but essentially still viable project. Hence, as a first step towards finding the optimal policy combination of risk-taking and information disclosure, we have to analyze the different parameters' influence on equilibrium value θ^*. The lower θ^*, the smaller is the range of values θ for which inefficient project termination may be obtained.[8] Before we turn to the impact of riskiness $1/a$[9] and precision c of the relationship bank's information, let us briefly analyze the influence of the a priori expected cash-flow, y, and of the relationship lender's fraction of total firm debt, λ, on trigger value θ^*. Proofs are presented in appendix A.

Proposition 1 *Equilibrium value θ^* decreases in the a priori expected cash-flow of the project, y. It increases in the relationship bank's fraction of debt, λ, whenever refinancing this part of the loan is sufficiently costly. It decreases in λ, however, for low values of W_R only if projects with low (high) expected cash-flows are repaid with low (high) r.*

Whenever the a priori expected project cash-flow is high, banks will prefer to extend their loan in order to reap the credit repayment r instead of confining themselves to the early liquidation value K. In general, it follows from (2) and (3) that banks will extend their loans for a larger range of signals, i.e. x_R^* and x_S^* are reduced, the higher the prior expected cash-flow y and the final repayment r and the lower the early liquidation value K is. Interpreting the impact of λ on θ^* requires considering the strategic behavior of all three types of players: firm, relationship bank and arm's-length banks. The relationship bank may either withdraw her loan or extend it. If she withdraws, the firm's refinancing costs increase in the size of her loan, λ, and in the per-capita costs W_R. Hence, for sufficiently high W_R, the firm will terminate the project for a larger range of project qualities the higher λ, i.e. $\frac{\partial \theta^*}{\partial \lambda} > 0$. If the relationship bank extends her loan, in contrast, a higher λ leads to a lower proportion of (arm's-length) debt that remains to be coordinated on the efficient action "extend". Provided that a sufficiently large proportion of small banks extends, it will be optimal for the firm to proceed with the project, so that θ^* decreases in λ. As already stated above, banks will be more inclined to extend their loans for high values of r. However, once a high repayment has been offered, the firm will only be willing to proceed with the project if its cash-flow is sufficiently high. Anticipating this reasoning by the firm, small banks will extend their loans in that case only if the a priori expected cash-flow, y, is high. For low values of y, in contrast, banks will tend to withdraw their money early. However, they know that they may still reap the final repayment r, which is always higher than the early liquidation value K, if the firm decides not to terminate the project. Even for low project qualities, it will be profitable for the firm to do so if repayment r is relatively low. Consequently, small banks are also willing to extend their loans for projects with low expected cash-flows provided that repayment r is not too high.

[8] For $\theta \leq \underline{\theta}$, terminating the project is the uniquely optimal strategy for the firm, irrespective of the banks' actions. As such, trigger value θ^* as defined in section 3 cannot fall below $\underline{\theta}$.

[9] Note that the riskiness of the firm's project refers to the variance of project cash-flows, $1/a$, while we generally denote a as the "risk parameter". A value of $a = 0$ therefore characterizes maximum risk, while $a \to \infty$ describes a policy of zero risk.

Analyzing the relationship bank's private information, we find that the precision c of her signal has a distinct influence on trigger value θ^*. This effect, however, is contingent on the prior expected cash-flow of the project, y. The same can be shown to be true for the influence of parameter a on θ^*.

Proposition 2 *Whenever the a priori expected cash-flow of the project is sufficiently high, equilibrium value θ^* increases with more precise information held by the relationship bank and with higher riskiness $1/a$ of the project. The opposite holds for sufficiently low expected cash-flows.*

Let us illustrate the implications of proposition 2 for the case of low expected cash-flows. For low values of y, banks are generally reluctant to extend credit since there is a fair chance that the firm will not invest additional effort because of a too low realized value of θ, so that credit may not be repaid. However, if a large business risk leads to a high variance of project cash-flows, the project profit may still turn out to be sufficiently high, despite the low prior expected value y. For decreasing values of a, therefore, the banks will decide to roll over their loans for a larger interval of signal values, so that θ^* decreases. The same holds if the relationship bank obtains more precise information. The more precise her private information becomes, the more she will tend to neglect the informational content of the prior distribution of θ.[10] Since the distribution of her private signal is common knowledge, all other banks know that she will place more weight on her private signal and rely less on y. As the relationship bank decides on a considerable fraction λ of total firm debt, it is reasonable for small banks in this case to neglect y as well. Again, trigger values x_S^* and x_R^* will decrease and hence θ^* will be reduced.

Even though proposition 2 gives a first indication with regard to the influence of a and c on the incidence of inefficient project termination via their impact on θ^*, we still have to overcome two problems in order to find the firm's optimal policy. First, the results delineated in proposition 2 relied on a comparison of expected cash-flow y with threshold functions that are complex functions of a and c (see appendix A). Hence, we did not yet arrive at the absolute effect of risk and information precision on θ^*. Second, the probability of inefficient project termination does not only depend on θ^*, but on the probability that the realized project cash-flow turns out to be lower than θ^* and hence on the whole distribution of θ. These two aspects will be dealt with in the subsequent section.

5 Optimal Information Disclosure and Risk-Taking

In the following, we will analyze the firm's optimal strategy in order to reduce the probability of inefficient project liquidation. The firm hence aims at solving the following optimization problem:

$$\min_{a,c} \{\text{prob}(\theta \leq \theta^*) = \Phi(\sqrt{a}(\theta^* - y))\} \quad \text{s.t.} \quad b, c \geq \frac{a^2}{2\pi} \ ,$$

where θ^* is given by (4). Note that we restrict the firm's decision to assure uniqueness of equilibrium.

[10] Note that the relationship bank's posterior expectation of θ is given as a weighted average of the prior expected value y and her private signal value x_R: $E(\theta|x_R) = \frac{a}{a+c}y + \frac{c}{a+c}x_R$.

5.1 Optimal Information Precision

The impact of the relationship bank's information precision, c, on the probability of inefficient early liquidation depends solely on its effect on θ^*, since

$$\frac{\partial\Phi(\sqrt{a}(\theta^* - y))}{\partial c} = \phi(\sqrt{a}(\theta^* - y))\sqrt{a}\frac{\partial\theta^*}{\partial c} \, .$$

From proposition 2 we know that θ^* increases in the relationship bank's information precision c whenever the a priori expected cash-flow y is sufficiently high. Stated differently, the condition (see appendix A) requires θ^* to be smaller than

$$y - \frac{1}{\sqrt{a+c}}\Phi^{-1}\left(\frac{K}{r}\right) \, . \tag{5}$$

For θ^* larger than the above threshold, equilibrium value θ^* decreases in c. In order to find the optimal precision of information, we follow the analysis of Heinemann and Metz (2002). Yet in contrast to this earlier study, we allow for two different cases: since small- to medium-sized firms typically dispose of only small collateral, whereas larger firms may be provided with a much higher amount of collateral, we consider both the cases of $K < 1/2r$ and $K > 1/2r$.

Generally, it holds that for completely precise information disseminated to the relationship bank ($c \to \infty$), threshold (5) converges to y, while θ^* converges to:

$$\begin{aligned}\theta^*(c \to \infty) \;\; = \;\; & V + r + \lambda(W_R - r)\frac{K}{r} \\ & +(1-\lambda)(W_S - r)\Phi\left(\frac{a}{\sqrt{b}}(\theta^* - y) + \sqrt{\frac{a+b}{b}}\Phi^{-1}\left(\frac{K}{r}\right)\right) \;\; = \;\; \theta_0^c \, .\end{aligned}$$

By committing to a disclosure of fully precise information to the relationship bank, the firm can always achieve an equilibrium value of $\theta^* = \theta_0^c$. In the following analysis, we will therefore differentiate between low expected cash-flows ($y < \theta_0^c$) and high expected cash-flows ($y > \theta_0^c$). Note that a minimum value for c is given by the condition that ensures uniqueness of equilibrium, i.e. $c \geq \frac{a^2}{2\pi} = c^{min}$.

Case 1: $K > 1/2r$

When the firm possesses sufficiently large collateral, it follows that $\Phi^{-1}(K/r) > 0$. Hence, for $c \to \infty$, threshold (5) converges to y from below. Let us first analyze the case of **low expected cash-flow**, i.e. $y < \theta_0^c$. Since θ^* is decreasing whenever $\theta^* > y - 1/\sqrt{a+c}\;\Phi^{-1}(K/r)$, we find that θ^* decreases in c for the whole range of parameter values. Hence, the firm can minimize the probability of inefficient project liquidation by providing its relationship bank with completely precise information.

If, in contrast, **expected cash-flow is high**, so that $y > \theta_0^c$, the following situation is obtained (see Fig. 1): For low precision values c, equilibrium value θ^* will be higher than the threshold function (5), so that θ^* is decreasing in c. Once θ^* equals the threshold (5), a minimum is reached and θ^* starts increasing along with c for higher precision values. The minimum value of θ^* is obtained for a precision value denoted $\tilde{c}$, where the two curves cross.

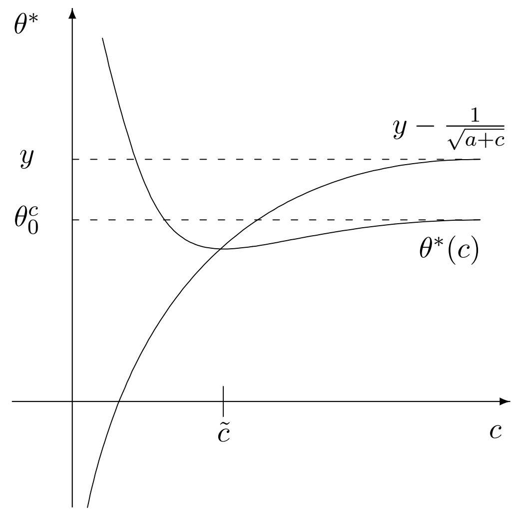

Figure 1: $K > 1/2r$ and $y > \theta_0^c$

However, in order to ensure uniqueness of equilibrium, we require c to be at least as high as $c^{\min} = a^2/(2\pi)$. The optimal precision value c^* in this case is therefore given as

$$c^* = \begin{cases} \frac{a^2}{2\pi} & \text{if} \quad c^{\min} > \tilde{c} \\ \tilde{c} & \text{if} \quad c^{\min} \leq \tilde{c} \end{cases}$$

where $\tilde{c}$ is the precision value for which $\theta^*(c) = y - 1/\sqrt{a+c}\Phi^{-1}(K/r)$.

Case 2: $K < 1/2r$

For $K < 1/2r$, threshold (5) converges to y from above, since $\Phi^{-1}(K/r) < 0$. If the market **expects low cash-flows**, so that $y < \theta_0^c$, the firm's optimal information policy is either to distribute completely precise information to the relationship bank or to decrease information precision to its minimally necessary level, as can be seen from Fig. 2.

If a is sufficiently low, so that $c^{\min}$ takes on very low values, it might be the case, that $\theta^*(c^{\min}) < \theta_0^c$, so that it is advantageous for the firm to distribute as imprecise information as possible. In any other case, however, the firm can minimize the probability of inefficient project liquidation by granting completely precise information to the relationship bank.

From (4) it follows that

$$\begin{aligned} \theta^*(c^{\min}) &= \theta_0^c \\ \sqrt{2\pi}(\theta^* - y) + \sqrt{\frac{2\pi + a}{a}}\Phi^{-1}\Big(\frac{K}{r}\Big) &= \Phi^{-1}\Big(\frac{K}{r}\Big) \\ a &= \frac{2\pi}{\left(\frac{\sqrt{2\pi}(y-\theta^*)}{\Phi^{-1}(\frac{K}{r})}\right)^2 - 1} = \bar{a}\ . \end{aligned}$$

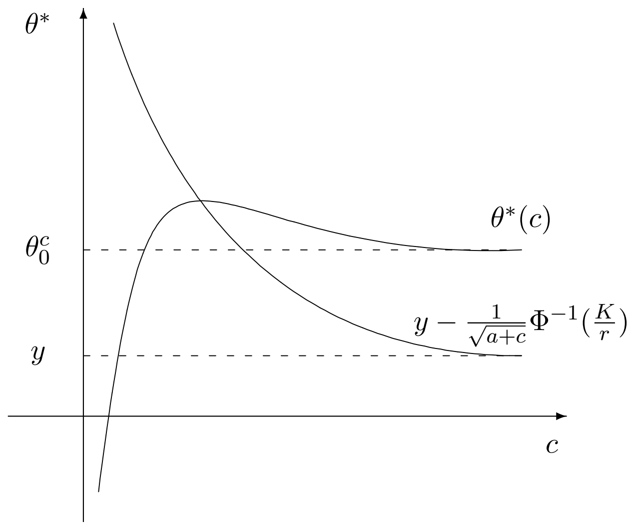

Figure 2: $K < 1/2r$ and $y < \theta_0^c$

Hence, we find that for $a < \bar{a}$ the optimal information precision is given by $c^{\min}$, whereas for $a \geq \bar{a}$, the firm is best off by providing the relationship bank with completely precise information, i.e. $c \to \infty$.

If the market holds very **optimistic expectations** with regard to **cash-flow**, i.e. $y > \theta_0^c$, the optimal information policy is to choose $c^{\min}$, as the condition for θ^* increasing in c is always satisfied.

Summing up the results with regard to optimal information disclosure to the relationship bank, the following proposition holds:

Proposition 3 *For given riskiness $1/a$, optimal information disclosure requires to provide the relationship bank with information of precision as given in Tab. 1.*

Table 1: Results regarding optimal information precision

	$K > 1/2r$	$K < 1/2r$
low expected cash-flow y	$c^* \to \infty$	$c^* = c^{\min}$ for $a < \bar{a}$ $c^* \to \infty$ for $a \geq \bar{a}$
high expected cash-flow y	$c^* = c^{\min}$ for $\tilde{c} < c^{\min}$ $c^* = \tilde{c}$ for $\tilde{c} \geq c^{\min}$	$c^* = c^{\min}$

Here, $c^{min} = a^2/(2\pi)$, $\tilde{c}$ is implicitly defined by $\theta^*(\tilde{c}) = y - 1/\sqrt{a+\tilde{c}}\ \Phi^{-1}(K/r)$and $\bar{a}$ by equality of $\theta^*(c^{min})$ and θ_0^c.

The results derived so far are in line with the intuition behind proposition 2. In general, it holds that for high expected cash-flows, the firm optimally provides the relationship bank with minimally precise information. By doing so, the firm tries to induce the relationship bank to rely less on her private signal - which, due to the assumed normal distributions, may turn out quite low after all -, and more strongly on the "optimistic" prior expected cash-flow. For low expected cash-flows, in contrast, the firm optimally discloses very precise information, i.e. she chooses a high value of c relative to a. If a is sufficiently low, a minimum value of c is adequate to this end.

Note that in the upper left cell (high K and low y), banks experience the highest incentive to foreclose their loans early, while the firm has the highest incentive to terminate the project. In the lower right cell (low K and high y) the opposite holds. This explains the clear-cut results concerning the optimal precision values in these cases. However, we can already see that the optimal information disclosure to the relationship bank is not entirely independent of the chosen business risk. For lowly-collateralized firms this is the case for projects with low expected cash-flows, for highly-collateralized firms for projects with high expected cash-flows. As already indicated, banks have a lower incentive to withdraw their loans prematurely for low values of K. Hence, there is less "persuasion" necessary to avoid early liquidation. For low values of K, therefore, even pessimistic expectations over y (upper right cell) do not necessarily require maximal precision c as long as the variance of cash-flows is sufficiently high (i.e. low a). Since high risk enables the realization of a high cash-flow θ despite low expectation y, banks may still anticipate project continuation and do not have to be distracted from pessimistic prior expectations.

For high K, in contrast, banks experience a high incentive to withdraw their loans prematurely. In case of high expected cash-flows (lower left cell), it may be optimal, however, to induce the relationship bank not to disregard her private information completely, i.e. disclose private information of higher than minimal precision. This is the case for high risk, i.e. low a and hence low c^{min}. Here, the project cash-flow may turn out quite low despite the optimistic prior expectation. Hence, it will be advantageous for the firm if banks do not base their actions too strongly on the prior expectation y.

5.2 Optimal Risk-Taking by the Firm

Given that the firm has already decided on the optimal precision of information to be disclosed to its relationship bank, we now examine the optimal degree of riskiness, $1/a$, that the firm should choose for its project. In particular, we are interested in potential effects of the "degree of heterogeneity", λ, on the optimal value of a.

In contrast to precision parameter c, risk parameter a influences the probability of inefficient project termination in two ways, as can be seen from the term in brackets in the following derivative:

$$\frac{\partial \text{prob}(\theta \leq \theta^*)}{\partial a} = \phi(\sqrt{a}(\theta^* - y)) \left[\frac{1}{2\sqrt{a}}(\theta^* - y) + \sqrt{a}\frac{\partial \theta^*}{\partial a} \right] . \tag{6}$$

Hence, in order to minimize the probability of inefficient project liquidation, the firm not only has to be concerned with the impact of a on θ^*, but also with the difference between θ^* and the expected cash-flow y.

Analyzing the firm's optimal business risk, we again have to consider different cases regarding the value of K relative to r and the a priori expected cash-flow y.

Case 1: $K > 1/2r$

If **expected cash-flow is low**, i.e. $y < \theta_0^c$, we know that the relationship bank should optimally be provided with completely precise information: $c^* \to \infty$. Examining the extreme values of a, i.e. either maximum risk ($a = 0$) or zero risk ($a \to \infty$), while taking into account the optimal information policy, the equilibrium values of θ^* are given by:

$$\theta^*(c \to \infty, a = 0) = V + r + \frac{K}{r}[\lambda(W_H - r) + (1-\lambda)(W_{SB} - r)]$$

and

$$\theta^*(c \to \infty, a \to \infty) = V + r + \lambda(W_H - r)\frac{K}{r} + (1-\lambda)(W_{SB} - r)\ . \tag{7}$$

Equation (7) is derived using the fact that the second term in (4) can also be expressed as $(1-\lambda)(W_{SB} - r)\Phi(\frac{a}{\sqrt{b}}(\theta^* - y) + \sqrt{\frac{a}{b} + 1}\ \Phi^{-1}(\frac{K}{r})) = (1-\lambda)(W_{SB} - r)\Phi(a[\frac{1}{\sqrt{b}}(\theta^* - y) + \sqrt{\frac{1}{ab} + \frac{1}{a^2}}\ \Phi^{-1}(\frac{K}{r})])$. Since $\theta_0^c > y$ holds for all values of a, it has to hold for $a \to \infty$ as well, so that the latter term converges to $(1-\lambda)(W_{SB} - r)\Phi(+\infty) = (1-\lambda)(W_{SB} - r)$.

The partial derivative $\frac{\partial\theta^*(c\to\infty,a)}{\partial a}$ delivers:

$$\frac{\partial\theta^*(c \to \infty, a)}{\partial a} = \frac{(1-\lambda)(W_{SB} - r)\phi(\cdot)[\frac{1}{\sqrt{b}}(\theta^* - y) + \frac{1}{2\sqrt{b(a+b)}}\Phi^{-1}(\frac{K}{r})]}{1 - (1-\lambda)(W_{SB} - r)\phi(\cdot)\frac{a}{\sqrt{b}}}\ ,$$

which is positive whenever $\theta^*(c \to \infty, a) > y - \frac{1}{2\sqrt{a+b}}\Phi^{-1}(\frac{K}{r})$. This condition is satisfied, as $\Phi^{-1}(\frac{K}{r}) > 0$ for $K > 1/2r$ and $y < \theta_0^c$. Hence, we know that θ^* is increasing in a and $\theta^* - y > 0$, so that according to (6) the probability of inefficient project termination increases in a. The optimal riskiness for a firm's business project calls for $a^{**} = 0$, i.e. maximum risk, in this case.[11] The ex-ante probability of inefficient project liquidation is thereby reduced to a level of 1/2.

If, in contrast, **expected cash-flow** y **is high**, i.e. $y > \theta_0^c$, the optimal value of information precision is given as either $c^* = c^{\min}$ or $c^* = \tilde{c}$.

Let us first concentrate on the case of $c^* = c^{\min}$. Here, the equilibrium value θ^* for $a = 0$ is given by

$$\theta^*(c^{\min}, a = 0) = V + r + \frac{K}{r}[\lambda(W_H - r) + (1-\lambda)(W_{SB} - r)] = \theta^*(c, a = 0)\ .$$

We know that $\theta^*(c \to \infty, a) < y$ for all a. Hence it also holds for $a = 0$. $\theta^*(c, a = 0)$, however, is independent of c. Therefore, it must be the case that $\theta^*(c^{\min}, a = 0) < y$ as well.

[11] We use the "double star" (a^{**}) as indication that this value of a minimizes the probability of inefficient project liquidation by taking into account both the effect of a on θ^* and the difference between θ^* and y. In contrast, a^* refers to the value of a that minimizes θ^*.

For the partial derivative of $\theta^*(c^{\min}, a)$ with respect to a, we find:

$$\begin{aligned}
\frac{\partial\theta^*(c^{\min}, a)}{\partial a} = & \frac{1}{1-\lambda(W_H-r)\phi_1(\cdot)\sqrt{2\pi}-(1-\lambda)(W_{SB}-r)\phi_2(\cdot)\frac{a}{\sqrt{b}}} \cdot \\
& \Bigg[-\lambda(W_H-r)\phi_1(\cdot)\frac{\pi}{a^2}\sqrt{\frac{a}{2\pi+a}}\Phi^{-1}\Big(\frac{K}{r}\Big) \\
& +(1-\lambda)(W_{SB}-r)\phi_2(\cdot)\Big[\frac{1}{\sqrt{b}}(\theta^*-y)+\frac{1}{2b}\sqrt{\frac{b}{a+b}}\Phi^{-1}\Big(\frac{K}{r}\Big)\Big]\Bigg] ,
\end{aligned}$$

where $\phi_1(\cdot) = \phi(\sqrt{2\pi}(\theta^* - y) + \Phi^{-1}(\frac{K}{r}))$ and $\phi_2(\cdot) = \phi(\frac{a}{\sqrt{b}}(\theta^* - y) + \sqrt{\frac{a+b}{b}}\Phi^{-1}(\frac{K}{r}))$. This partial derivative is positive, if θ^* is higher than

$$y + \left[\frac{\lambda(W_H-r)\phi_1(\cdot)\pi\sqrt{b}}{(1-\lambda)(W_{SB}-r)\phi_2(\cdot)\sqrt{a^3(2\pi+a)}} - \frac{1}{2\sqrt{a+b}}\right]\Phi^{-1}\Big(\frac{K}{r}\Big) . \tag{8}$$

What happens to threshold (8) for $a \to \infty$? As long as

$$\lambda > \frac{(W_{SB}-r)\phi_2(\cdot)\sqrt{a^3(2\pi a)}}{(W_H-r)\phi_1(\cdot)2\pi\sqrt{b(a+b)}+(W_{SB}-r)\phi_2(\cdot)\sqrt{a^3(2\pi+a)}} = \bar{\lambda} ,$$

threshold (8) converges to y from above, since the term in brackets is positive and $\Phi^{-1}(K/r) > 0$ in the case considered.

Fig. 3 in the appendix exemplifies the behavior of $\theta^*(c^{\min}, a)$ for the case of $\lambda > \bar{\lambda}$.[12] The value of a that reduces θ^* is given by $a^* \to \infty$, as $\theta^*(c^{\min}, a)$ decreases in a. Since $\theta^* < y$, we also know that $\frac{\partial \text{prob}(\theta \leq \theta^*)}{\partial a} < 0$, so that the value of a that minimizes the overall probability of inefficient project liquidation is given by $a^{**} \to \infty$.

For $\lambda \leq \bar{\lambda}$, however, threshold (8)converges to y from below. Here, we have to distinguish two cases: either $\theta^*(c^{\min}, a = 0) < y < \theta^*(c^{\min}, a \to \infty)$ or $\theta^*(c^{\min}, a \to \infty) < \theta^*(c^{\min}, a = 0) < y$. In the first case we find that $a^* = 0$ as given in Fig. 4, where, due to the fact that $\frac{\partial\theta^*(c^{\min},a)}{\partial a} > 0$ and $\theta^* > y$, it holds that $\frac{\partial \text{prob}(\theta \leq \theta^*)}{\partial a} > 0$ and consequently $a^{**} = 0$.

Alternatively, the optimal value of business risk will be given by $a^{**} \to \infty$, if $\theta^*(c^{\min}, a \to \infty) < \theta^*(c^{\min}, a = 0) < y$, as can be seen from Fig. 5. Here, θ^* decreases in a for sufficiently high values of a and $\theta^* < y$, so that $\frac{\partial \text{prob}(\theta \leq \theta^*)}{\partial a} < 0$ and hence projects with zero risk ($a^{**} \to \infty$) will minimize the probability of inefficient liquidation provided that the relationship bank disposes of information with minimal precision.

Whenever optimal information precision is given by $\tilde{c}$, we find that for the extreme values of a the equilibrium value θ^* is given by

$$\theta^*(\tilde{c}, a = 0) = V + r + \frac{K}{r}[\lambda(W_H - r) + (1-\lambda)(W_{SB} - r)]$$

and

$$\theta^*(\tilde{c}, a \to \infty) = y .$$

[12]The following figures will be displayed in appendix B.

Generally, the partial derivative is given as

$$\frac{\partial\theta^*(\tilde{c},a)}{\partial a} = \frac{1}{\sqrt{(a+\tilde{c})^3}}\Phi^{-1}\Big(\frac{K}{r}\Big) > 0 \ .$$

Since $\theta^*(\tilde{c},a) \leq y$, while the partial derivative is positive, the optimal value of a must be an interior solution. Plugging the partial derivative in (6), the impact of a on the overall probability of inefficient project liquidation is given by

$$\frac{\partial\Phi(\sqrt{a}(\theta^*-y))}{\partial a} = \phi(\sqrt{a}(\theta^*-y))\left[\frac{1}{2\sqrt{a}}(\theta^*-y) + \sqrt{\frac{a}{(a+c)^3}}\Phi^{-1}\Big(\frac{K}{r}\Big)\right] \ .$$

The value a^{**} that minimizes this probability, is then found as $a^{**} = \tilde{c}$.

Summarizing the different results for this case of high collateral K, we find the following:

- For $c^* = c^{\min}$:
  - For sufficiently high λ, optimal business risk is characterized by $a^{**} \to \infty$, so that the probability of inefficient project termination amounts to $\Phi(-\infty) = 0$, since $\theta^* < y$.
  - For sufficiently low λ, optimal business risk is achieved with $a^{**} = 0$ and leads to a probability of inefficient project termination of $\Phi(0) = \frac{1}{2}$.
- For $c^* = \tilde{c}$, the optimal value of a is given by $a^{**} = \tilde{c}$, so that $\Phi(\sqrt{a}(\theta^* - y)) = \Phi(\sqrt{\frac{\tilde{c}}{2}}\Phi^{-1}(\frac{K}{r}))$.

Hence, for a sufficiently high degree of relationship banking (i.e. for sufficiently high λ), optimal firm policy is described by $c^* = c^{\min}$ and $a^{**} \to \infty$, since in this case $c^{\min} > \tilde{c}$. For a low degree of relationship banking, in contrast, the firm will prefer a policy combination of $c^* = a^{**} = \tilde{c}$. The policy mix of $c^* = c^{\min}$ and $a^{**} = 0$ is ruled out, since for this value of a it holds that $c^{\min} < \tilde{c}$, so that the optimal precision value is instead given by $\tilde{c}$, as follows from proposition 3.

Case 2: $K < 1/2r$

Let us first analyze the case of **low expected cash-flow**, i.e. $y < \theta_0^c$. For $a < \bar{a}$, optimal information precision for the relationship bank is given by $c^* = c^{\min}$, whereas for $a \geq \bar{a}$, optimal precision is given by $c^* \to \infty$.

If we first concentrate on the case of $c^* = c^{\min}$, we know that due to the assumption of $y < \theta_0^c$ also $\theta^*(c^{\min}, a = 0) > y$. Again, it holds that $\theta^*(c^{\min}, a)$ increases in a whenever θ^* is higher than threshold (8). Since in the current case it is assumed that $K < 1/2r$, however, the threshold will converge to y from below for $a \to \infty$ whenever $\lambda > \bar{\lambda}$. It can therefore be shown that $\theta^*(c^{\min}, a)$ increases in a and, since $\theta^* > y$, also the overall probability of inefficient project termination increases in a, so that the optimal risk parameter is given by $a^{**} = 0$.

For $\lambda < \bar{\lambda}$, however, threshold (8) converges to y from above. Again, two different possibilities arise. Either $\theta^*(c^{\min}, a \to \infty) < y < \theta^*(c^{\min}, a = 0)$, so that θ^* decreases in a. Since here

$\theta^* < y$ for sufficiently low a, the probability of inefficient project liquidation is minimized by selecting a project risk characterized by $a^{**} \to \infty$.

Alternatively, the case of $y < \theta^*(c^{\min}, a = 0) < \theta^*(c^{\min}, a \to \infty)$ could arise as shown in Fig. 6 . Since in this case $\theta^* > y$ and $a^* = \tilde{a_1}$, an intermediate value of a might minimize the overall probability of inefficient project termination.

For $a > \bar{a}$, in contrast, the optimal precision of information is given by $c^* \to \infty$. We know that $\theta^*(c \to \infty, a)$ increases in a whenever $\theta^* > y - 1/(2\sqrt{a+b})\ \Phi^{-1}(K/r)$. For $a \to \infty$, this threshold converges to y from above, since $K < 1/2r$. Again we have to differentiate between two different scenarios. Either $\theta^*(c \to \infty, a \to \infty) < y < \theta^*(c \to \infty, a = 0)$, so that the optimal risk parameter is given by $a^{**} \to \infty$, since $\theta^* < y$ for $a \to \infty$. Alternatively, $y < \theta^*(c \to \infty, a = 0) < \theta^*(c \to \infty, a \to \infty)$, so that Fig. 7 is obtained. Again, an intermediate solution a^{**} might be optimal.

Summing up the results for this case, we find the following:

- For $a < \bar{a}$, the optimal information precision is given by $c^* = c^{\min}$.
  - For $\lambda > \bar{\lambda}$ optimal riskiness is characterized by $a^{**} = 0$. The prior probability of inefficient project termination is thereby reduced to a value of $1/2$.
  - For $\lambda < \bar{\lambda}$, the probability of inefficient project termination can be minimized by choosing a riskiness described by $a^{**} \to \infty$. Since $\theta^* < y$ in this case, $\Phi(\sqrt{a}(\theta^* - y)) = 0$, which is the best result achievable.
- For $a \geq \bar{a}$, optimal information precision is given by $c^* \to \infty$. Choosing $a^{**} \to \infty$ minimizes the probability of project liquidation since $\theta^* < y$ in this case, so that $\Phi(\sqrt{a}(\theta^* - y)) = 0$. Again, since this is the lowest level that can be achieved, intermediate values of a do not have to be considered as alternative solutions.

Let us finally consider the case where **expected cash-flow is high**, i.e. $y > \theta_0^c$. The optimal information precision is given by $c^* = c^{\min}$. $\theta^*(c^{\min}, a)$ increases in a whenever θ^* is higher than threshold (8). For $\lambda > \bar{\lambda}$ and $a \to \infty$, threshold (8) converges to y from below, as can be seen in Fig. 8. Here, we have assumed that $\theta^*(c^{\min}, a = 0) < y < \theta^*(c^{\min}, a \to \infty)$. As follows quite obviously since $\frac{\partial \theta^*}{\partial a} > 0$ and $\theta^* > y$, the optimal risk value is given by $a^{**} = 0$.

If, in contrast, $\theta^*(c^{\min}, a \to \infty) < \theta^*(c^{\min}, a \to \infty) < y$, θ^* is decreasing in a while at the same time $\theta^* < y$, so that the overall optimal value of a is given by $a^{**} \to \infty$.

For $\lambda < \bar{\lambda}$, instead, threshold (8) converges to y from above. Since $\theta^*(c^{\min}, a = 0) < y$ and θ^* decreases in a, the prior probability of inefficient project termination can be minimized by selecting minimum business risk: $a^{**} \to \infty$. Hence for both low and high values of λ, the probability of inefficient project liquidation can be minimized by conducting a policy with parameters $c^* = c^{\min}$ and $a^{**} \to \infty$, so that $\Phi(\sqrt{a}(\theta^* - y)) = 0$.[13]

The following proposition combines the results with respect to optimal risk-taking and information disclosure.

[13] Choosing maximum risk. i.e. $a^{**} = 0$, for $\lambda > \bar{\lambda}$ would reduce the ex-ante probability of project liquidation to a value of $1/2$. A risk policy of $a^{**} \to \infty$ is therefore more efficient and should be preferred.

Proposition 4 *Optimal risk-taking and information disclosure depend on the ratio of the firm's collateral K to repayment r and on the expected cash-flow y. Additionally, the optimal policy mix is influenced by the fraction of relationship lending as compared to arm's-length lending and hence by the degree of heterogeneity in bank financing. The full results are depicted in Tab. 2.*

Table 2: Results regarding optimal information precision and business risk

	$K > 1/2r$	$K < 1/2r$
low expected cash-flow $y < \theta_0^c$	$c^* \to \infty$ $a^{**} = 0$ $\Rightarrow \Phi(0) = 1/2$	$\lambda > \bar{\lambda} : c^* = c^{\min}$ $a^{**} = 0$ $\Rightarrow \Phi(0) = 1/2$ $\lambda < \bar{\lambda} : c^* = a^{**} \to \infty$ $\Rightarrow \Phi(-\infty) = 0$
high expected cash-flow $y > \theta_0^c$	$\lambda > \bar{\lambda} : c^* = c^{\min}$ $a^{**} \to \infty$ $\Rightarrow \Phi(-\infty) = 0$ $\lambda < \bar{\lambda} : c^* = a^{**} = \tilde{c}$ $\Rightarrow \Phi(\sqrt{\frac{\tilde{c}}{2}}\Phi^{-1}(\frac{K}{r}))$	$c^* = c^{\min}$ $a^{**} \to \infty$ $\Rightarrow \Phi(-\infty) = 0$

Both firms with small and those with large collateral are affected by heterogeneous multiple bank financing with regard to their optimal risk-taking and information disclosure decisions. For firms with low collateral, which are likely to be small- to medium-sized firms, the degree of heterogeneity among involved banks is decisive if projects with low expected cash-flows have to be financed. Whenever the proportion of debt obtained from the relationship bank is high, the firm will take on maximum risk and provide the relationship bank with information of only minimal precision. With this policy combination, firms create maximum uncertainty about their business projects, since the relationship bank will tend to neglect her (imprecise) private information and also the remaining small share of arm's-length lending will coordinate more strongly on the prior information about the cash-flow distribution. Due to the maximal variance of θ, the probability of inefficient project liquidation is then reduced to a level of $1/2$. If relationship lending makes up only a small proportion of the full credit amount, however, a large remaining share of small bank lenders has to be coordinated on the efficient action "extend credit". Here, the firm will optimally choose minimum risk for its business project while at the same time disclosing fully precise information to the relationship bank. By doing so, the firm induces all banks to disregard the unfavorable prior information y, so that the ex-ante incidence of project termination can be eliminated. For projects with high expected cash-flows, the firm will not make her optimal policy contingent on the size of relationship lending. In this case, the incentive to withdraw credit early is so low that the firm optimally chooses minimum risk and provides the relationship bank with fully precise information (since for $a \to \infty$, also $c^{\min} \to \infty$).

Large firms, that typically dispose of high collateral, in contrast, vary their optimal risk-taking along with the relationship bank's proportion of total firm debt for projects with high expected cash-flows. Here, we find that for a high degree of relationship lending, the firm will refuse to take on any risk and will keep its relationship bank fully informed, thereby eliminating the ex-ante incidence of early liquidation, while it will raise optimal business risk to an intermediate level and decreases information precision if relationship lending makes up only a small proportion of total firm debt. In the latter case, a relatively large proportion of arm's-length banks has to be coordinated on the efficient action, which is easier to conduct, the more strongly the relationship bank takes into account the prior expected cash-flow value, y. Hence, the precision of information disclosure to the relationship bank has to be reduced as compared to the case of a low proportion of relationship lending. For projects with low expected cash-flows, however, the incentive to withdraw credit early is so large that the firm optimally decides on maximum risk and provides its relationship bank with completely precise information. This policy combination provides a gamble for resurrection and reduces the ex-ante probability of inefficient liquidation to 1/2.

Whereas comparisons to the case of homogeneous multiple bank financing show that large firms cannot increase efficiency, heterogeneous multiple bank financing may put lowly-collateralized firms at an advantage. Interestingly, the latter firms seem to benefit from a heterogeneous system particularly when conducting projects with low expected cash-flows. However, the degree of relationship lending must not become too large. Otherwise efficiency is reduced. Obviously, therefore, small firms benefit from relationship banking in situations of imminent distress but suffer from a potential hold-up problem that is aggravated by the relationship bank's financial power as mirrored by her fraction of total firm debt.

6 Conclusion

Our study underlines the importance of the financing system when analyzing firms' risk-taking and information disclosure. Earlier work on this subject, with limited focus on firms with large collateral, found that in a system of homogeneous multiple bank financing firms maximize risk for projects with low expected cash-flows, but choose minimum risk for projects with high expected cash-flows. In either case, they disclose fully precise information. It has been argued that by doing so firms try to gamble for resurrection in the case of sinister business prospects, but attempt to lock-in good expectations in the opposite case.

In a context of heterogeneous multiple bank financing, optimal firm decisions are more multi-faceted. We can show that firms adjust their optimal risk-taking and information disclosure to the heterogeneity of their bank financing. The adjustment, however, is asymmetric when comparing lowly- and highly-collateralized firms. In this respect, the former vary their optimal decisions along with the degree of relationship banking for projects with low expected cash-flows, while the latter do so for projects with high expected cash-flows.

Comparing the resulting degrees of efficiency from the different firm decisions we find that the highest gains in efficiency can be made if highly-collateralized firms employ a high degree of relationship banking, while lowly-collateralized firms rely on a low degree of relationship banking. Taking into account that highly-collateralized firms may obtain an equivalent degree of efficiency when borrowing from homogeneous multiple lenders as has been shown by Heinemann and Metz (2002), our results may also be interpreted as matching observed financing patterns. Large firms are often found to obtain financing from the capital markets rather than turning to the banking system. Since the capital markets consist of a continuum of homogeneous multiple lenders, this type of financing, according to our results, delivers the highest degree of efficiency to these firms. Small firms, in contrast, are often found to finance mainly via banks. Contrary to the early literature on relationship lending, however, even small firms hold credit relations to more than one bank. Provided that the degree of relationship banking as compared to arm's-length financing is not too large, again, this financing regime supposedly delivers the lowest ex-ante probability of inefficient project liquidation to those lowly-collateralized firms.

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Appendix A

Proof of proposition 1:

$$\frac{\partial\theta^*}{\partial y} = -\frac{\lambda(W_R - r)\phi_c(\cdot)\frac{a}{\sqrt{c}} + (1-\lambda)(W_S - r)\phi_b(\cdot)\frac{a}{\sqrt{b}}}{1 - \lambda(W_R - r)\phi_c(\cdot)\frac{a}{\sqrt{c}} - (1-\lambda)(W_S - r)\phi_b(\cdot)\frac{a}{\sqrt{b}}}$$

where $\phi_b(\cdot) = \phi(\frac{a}{\sqrt{b}}(\theta^* - y) + \sqrt{\frac{a+b}{b}}\Phi^{-1}(\frac{K}{r}))$ and $\phi_c(\cdot) = \phi(\frac{a}{\sqrt{c}}(\theta^* - y) + \sqrt{\frac{a+c}{c}}\Phi^{-1}(\frac{K}{r}))$. Since $b, c > \frac{a^2}{2\pi}$ and $W_R \leq 1$, the denominator of this derivative is always positive, so that y exerts a negative influence on θ^*.

$$\frac{\partial\theta^*}{\partial\lambda} = \frac{(W_R - r)\Phi(\frac{a}{\sqrt{c}}(\theta^* - y) + \sqrt{\frac{a+c}{c}}\Phi^{-1}(\frac{K}{r})) - (W_S - r)\Phi(\frac{a}{\sqrt{b}}(\theta^* - y) + \sqrt{\frac{a+b}{b}}\Phi^{-1}(\frac{K}{r}))}{1 - \lambda(W_R - r)\phi_c(\cdot)\frac{a}{\sqrt{c}} - (1-\lambda)(W_S - r)\phi_b(\cdot)\frac{a}{\sqrt{b}}}$$

Again, the denominator is positive. It is then easy to see that the numerator and hence the partial derivative itself is positive whenever $W_R > r + (W_S - r)\frac{\Phi_b(\cdot)}{\Phi_c(\cdot)}$ and negative if $W_R < r + (W_S - r)\frac{\Phi_b(\cdot)}{\Phi_c(\cdot)}$. In the latter case, we also have to take into account that $0 \leq r < W_S < W_R \leq 1$. In particular, whenever y is sufficiently low, i.e. $y < \theta^* - \Phi^{-1}(K/r)\frac{\sqrt{b(a+c)} - \sqrt{c(a+b)}}{a(\sqrt{c} - \sqrt{b})}$, so that $\frac{\Phi_b(\cdot)}{\Phi_c(\cdot)} > 1$, repayment rate r also has to be low in order to ensure that $W_R < r(1 - \frac{\Phi_b(\cdot)}{\Phi_c(\cdot)}) + W_S\frac{\Phi_b(\cdot)}{\Phi_c(\cdot)}$ for $W_R > W_S$. The opposite holds for sufficiently high y. In this case, $\frac{\Phi_b(\cdot)}{\Phi_c(\cdot)} < 1$ and $W_R < r(1 - \frac{\Phi_b(\cdot)}{\Phi_c(\cdot)}) + W_S\frac{\Phi_b(\cdot)}{\Phi_c(\cdot)}$ requires a sufficiently high repayment r since it has to hold that $W_R > W_S$.
Q.E.D.

Proof of proposition 2:

$$\frac{\partial\theta^*}{\partial c} = -\frac{\lambda(W_R - r)\phi_c(\cdot)[\frac{1}{2}\sqrt{\frac{c}{a+c}}\frac{a}{c^2}\Phi^{-1}(\frac{K}{r}) + \frac{a}{2\sqrt{c^3}}(\theta^* - y)]}{1 - \lambda(W_R - r)\phi_c(\cdot)\frac{a}{\sqrt{c}} - (1-\lambda)(W_S - r)\phi_b(\cdot)\frac{a}{\sqrt{b}}}$$

Since under the stated assumptions the denominator is positive, the partial derivative is positive whenever $y > \theta^* + \frac{1}{\sqrt{a+c}}\Phi^{-1}(\frac{K}{r})$ and negative for $y < \theta^* + \frac{1}{\sqrt{a+c}}\Phi^{-1}(\frac{K}{r})$.

$$\begin{aligned}
\frac{\partial\theta^*}{\partial a} &= \frac{1}{1 - \lambda(W_R - r)\phi_c(\cdot)\frac{a}{\sqrt{c}} - (1-\lambda)(W_S - r)\phi_b(\cdot)\frac{a}{\sqrt{b}}} \cdot \\
&\left[\lambda(W_R - r)\phi_c(\cdot)\left[\frac{1}{\sqrt{c}}(\theta^* - y) + \frac{1}{2c}\sqrt{\frac{c}{a+c}}\Phi^{-1}\left(\frac{K}{r}\right)\right]\right. \\
&\left. + (1-\lambda)(W_S - r)\phi_b(\cdot)\left[\frac{1}{\sqrt{b}}(\theta^* - y) + \frac{1}{2b}\sqrt{\frac{b}{a+b}}\Phi^{-1}\left(\frac{K}{r}\right)\right]\right]
\end{aligned}$$

Since for $c, b > \frac{a^2}{2\pi}$ and $W_R \leq 1$ the denominator is positive, we find that the partial derivative is negative whenever $y > \max\{\theta^* + \frac{1}{2\sqrt{a+b}}\Phi^{-1}(\frac{K}{r}), \theta^* + \frac{1}{2\sqrt{a+c}}\Phi^{-1}(\frac{K}{r})\}$ and positive for $y < \min\{\theta^* + \frac{1}{2\sqrt{a+b}}\Phi^{-1}(\frac{K}{r}), \theta^* + \frac{1}{2\sqrt{a+c}}\Phi^{-1}(\frac{K}{r})\}$. Q.E.D.

Appendix B

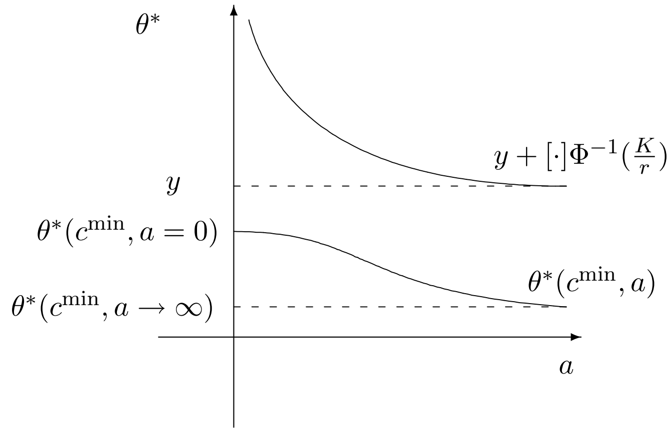

Figure 3: $K > 1/2r$, $y > \theta_0^c$ and $\lambda > \bar{\lambda}$

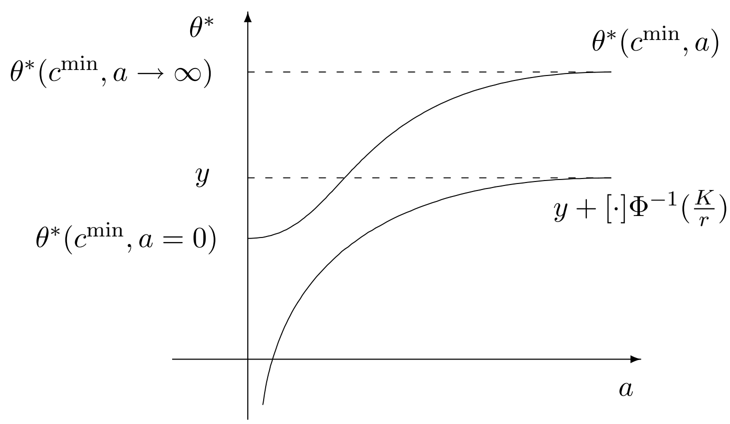

Figure 4: $K > 1/2r$, $y > \theta_0^c$, $\lambda < \bar{\lambda}$ and $\theta^*(c^{\min}, a = 0) < y < \theta^*(c^{\min}, a \to \infty)$

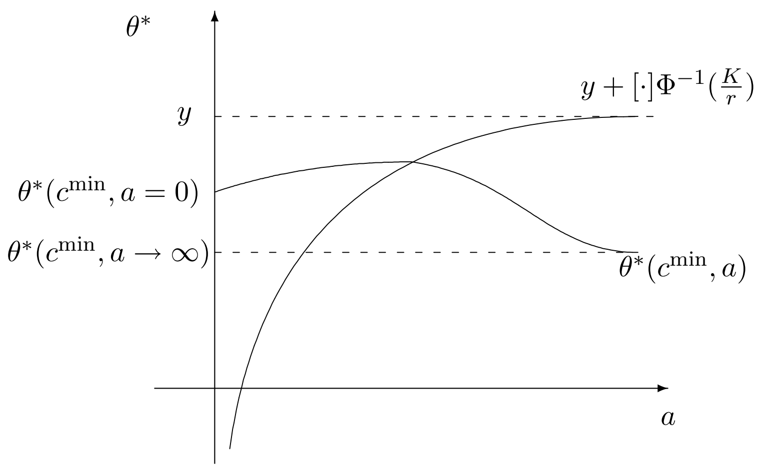

Figure 5: $K > 1/2r$, $y > \theta_0^c$, $\lambda < \bar{\lambda}$ and $\theta^*(c^{\min}, a \to \infty) < \theta^*(c^{\min}, a = 0) < y$

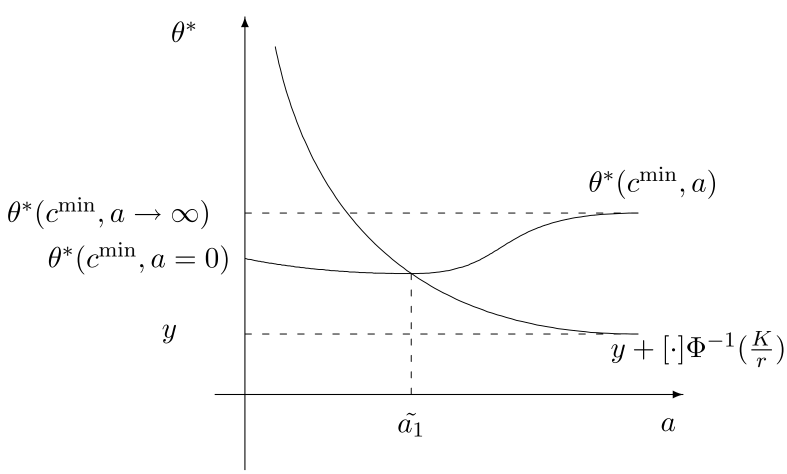

Figure 6: $K < 1/2r$, $y < \theta_0^c$, $\lambda < \bar{\lambda}$ and $y < \theta^*(c^{\min}, a = 0) < \theta^*(c^{\min}, a \to \infty)$

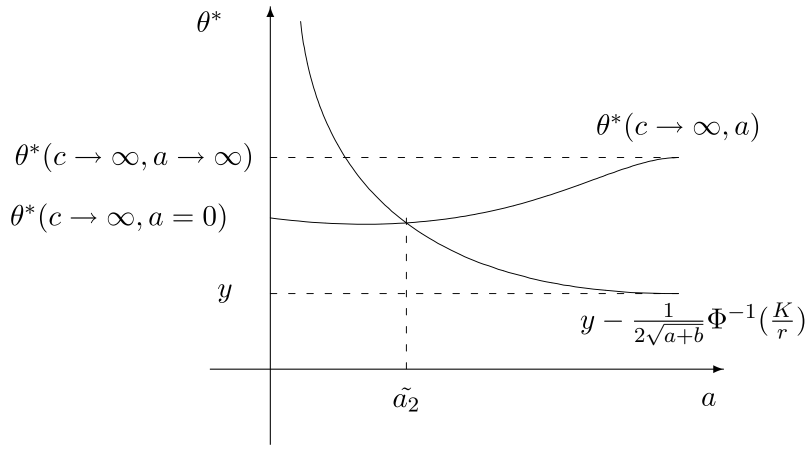

Figure 7: $K < 1/2r$, $y < \theta_0^c$, $a > \bar{a}$ and $y < \theta^*(c \to \infty, a = 0) < \theta^*(c \to \infty, a \to \infty)$

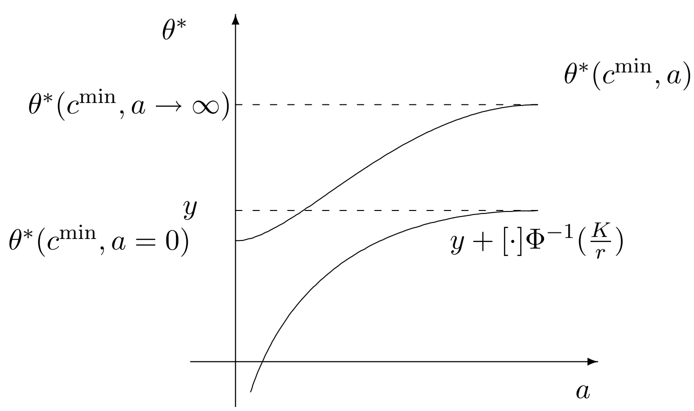

Figure 8: $K < 1/2r$, $y > \theta_0^c$, $\lambda > \bar{\lambda}$ and $\theta^*(c^{\min}, a = 0) < y < \theta^*(c^{\min}, a \to \infty)$

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