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The EPR Paradox

Back from the Future

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Abstract

The Einstein-Podolsky-Rosen (EPR) thought experiment produced a problem regarding the interpretation of quantum mechanics provided for entangled systems. Although the thought experiment was reformulated mathematically in Bell's Theorem, the conclusion regarding entanglement correlations is still debated today. In an attempt to provide an explanation of how entangled systems maintain their correlations, this thesis investigates the theory of post-state teleportation as a possible interpretation of how information moves between entangled systems without resorting to nonlocal action. Post-state teleportation describes a method of communicating to the past via a quantum information channel. The resulting picture of the EPR thought experiment relied on information propagating backward from a final boundary condition to ensure all correlations were maintained. Similarities were found between this resolution of the EPR paradox and the final state solution to the black hole information paradox and the closely related firewall problem. The latter refers to an apparent conflict between unitary evaporation of a black hole and the strong subadditivity condition. The use of observer complementarity allows this solution of the black hole problem to be shown to be the same as a seemingly different solution known as "ER=EPR", where 'ER' refers to an Einstein-Rosen bridge or wormhole.

Declaration

I hereby declare that this thesis has not been submitted, either in the same or different form, to this or any other university for a degree and that it represents my own work. I know the meaning of plagiarism and declare that all of the work in this thesis, save for that which is properly acknowledged, is my own.

KLH Bryan
December 2015

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CHAPTER 1

Introduction

"Einstein said that if quantum mechanics were correct then the world would be crazy. Einstein was right - the world is crazy."
- Daniel M. Greenberger

Quantum mechanics is a field of physics that routinely pushes the boundaries of classical intuition in its attempt to explain the world. Many notions that we draw on to describe the classical world seem to be in direct contradiction with the implications of quantum mechanics. One approach to dealing with the classical-quantum schism is to ignore it completely and simply use the tools provided by quantum mechanics to calculate relevant predictions. However, this leaves us with many questions about how to interpret certain aspects of quantum mechanics mathematics. One of the most notorious issues raised regarding quantum mechanics was presented in the famous Einstein-Podolsky-Rosen (EPR) paper [1].

1.1 The Einstein-Podolsky-Rosen Paradox

The EPR paper sets out to investigate the 'completeness' of quantum mechanics. In order to be considered complete, a theory must provide a description of every element of physical reality. Any part of the physical reality must have an analog within the theory.

The EPR argument has two steps to it. The first step involves setting two alternatives based on the mathematics of quantum mechanics and the 'reality criterion' assumption. The EPR paper presents the reality criterion as an adequate, but not conclusive, description of reality. It requires a component of physical reality to exist if a prediction can be made for that physical value with certainty. In other words, if we can predict a physical value with probability 1, the physical element must exist in reality. The authors then consider the implications of this criterion with regard to the quantum mechanical concept of non-commuting operators. These operators have the peculiar property of not allowing simultaneous measurement [2].

Position and momentum provide such a pair of non-compatible observables with non-commuting operators. By measuring the position we restrict any measurement of momentum and vice versa. By the reality criterion, this seems to imply that the observables associated with non-commuting operators cannot exist simultaneously since we can make no prediction regarding a physical value for one of them if we have a physical value for the other. The EPR paper argues that this leads us to two alternatives. On the one hand, we can accept that non-commuting operators relate to two physical elements which do not exist simultaneously. On the other hand, we can accept that the physical reality of values relating to non-commuting operators do exist simultaneously but quantum mechanics is incomplete in its description of these predictions.

The second step of the EPR argument begins with the assumption that quantum mechanics is complete. As established in the classical realm, completeness requires the specification of all variables in a given system [3]. Consider a setup involving the measurement of non-commuting observables by two spacelike separated experimenters. Although the EPR argument uses position and momentum as its non-compatible observables, we follow Bohm's formulation which produces the same conclusion [4]. In this scenario our non-compatible observables are the spin directions for two particles in the singlet state given by [2]

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\Big[\, |a\uparrow\rangle\,|b\downarrow\rangle \;-\; |a\downarrow\rangle\,|b\uparrow\rangle\,\Big]\,, \tag{1.1}$$

where the particles, α and β, have spins which are anti-correlated along the same direction and $\uparrow$ ($\downarrow$) denotes a spin up (down) along chosen axis given by a and b for particle α and β respectively. Two experimenters, Alice and Bob, are each given one particle from the singlet state and then separated to ensure neither measurement outcome influences the other. This enforces the condition of locality which ensures we do not have nonlocal action between the two experimenters. Both experimenters may choose any direction to measure their particle's spin once they are spacelike separated. Let us assume Alice measures her particle α in the z-direction and gets the result $|a_z\uparrow\rangle$. Due to the correlations of the singlet state, Alice can predict Bob's result in the z-direction with certainty. Therefore, by the reality criterion, Alice's prediction for Bob must correspond to a physical element of reality. This captures the idea of reality as laid out by the EPR authors. By the EPR reality criterion, a prediction with probability 1 of a value associated with an observable of a particle implies that the observable must have a corrresponding element of reality such as the observable would have if the particle was measured. That is to say, an element of reality corresponding to an observable exists if the value assiciated with an observable is discernible by a prediction with certainty as the element of reality associated with the observable is considered to exist independently of any measurement interaction [3]. Alice knows Bob's particle must have a spin given by $|b_z\downarrow\rangle$. Alice's measurement cannot affect Bob's outcome due to their spacelike separation. Therefore the property of spin in the z-direction for both particles must have been determined before separation and not as a result of Alice's measurement. Since Alice could have chosen any direction to measure, this can be applied to all possible measurement directions and we are left with the requirement that the spins for each particle must be determined in every direction before separation.

We now return to the choices available from the first step of the EPR argument. The above setup shows the outcomes of non-compatible observables must have simultaneous reality to ensure the observed correlations hold. Starting from the assumption that quantum mechanics is complete, we are led to the conclusion that non-compatible observables have simultaneous reality. The EPR paper therefore concludes that we must accept quantum mechanics is an incomplete description of reality since it does not describe these non-compatible observables simultaneously.

This argument may appear conclusive but there are a number of concerns and disagreements regarding its approach. Most notable are the concerns raised by Bohr, specifically his worry about discussing experiments that did not happen in a meaningful way alongside experiments which did happen [5]. This can be seen when the EPR authors discuss Alice's measurement outcome in the z-direction alongside what her result could have been had she chosen a different measurement direction. Bohr's conclusion was that reality simply does not allow us to describe such situations. This interpretation is not accepted by everyone and the debate over the EPR paradox continues today. Further discussion on the aspects of the EPR paradox and the issued related to it can be found in [6]. The notion of considering experiments that are actually done alongside hypothetical experiments is referred to as counter-factual definitiveness (CFD) and it is described by Norsen in reference to Bell's theorem [7]. Norsen showed that CFD cannot be removed from Bell's theorem. This theorem provided new insight into the EPR paradox and allowed the EPR argument to be experimentally tested.

1.2 Bell's Theorem

Bell's theorem provided a new formulation of the EPR paradox in which the locality assumption is formalised mathematically [8][9]. This thesis utilises Norsen's review of Bell's work outlined in a series of papers on this topic [10] [11] [12] [13] [7]. The locality condition is identified by Norsen as the only assumption made in Bell's argument. This condition enforces the idea that the probability of a specific outcome must be independent of actions occurring at a spacelike distance. Putting this concept into the context of the EPR's setup involving Alice and Bob, the locality condition requires that Alice's measurement outcome cannot depend on Bob's outcome at a spacelike distance and vice versa. This condition is therefore defined for the EPR setup as

$$P(A|a, b, B, \lambda) \;=\; P(A|a, \lambda)\;, \tag{1.2}$$

where P denotes the conditional probability of Alice measuring result A given the variables on the other side of the vertical divider. The symbols a and b represent the measurement direction chosen for particles α and β respectively, B denotes Bob's measurement outcome and λ represents the complete specification of the initial state provided by the theory in question [11]. Using this locality condition, Bell was able to create an inequality condition that could be tested against the predictions of any theory. When combined with the predictions of quantum mechanics, the inequality seemed to suggest that quantum mechanics required extra variables to account for the correlations of a singlet state. However, Norsen showed the conclusion of

Bell's argument remains the same for any theory whether hidden variables are included or not. It should be pointed out that much of the confusion surrounding 'hidden variables' arises out of a misinterpretation of the λ-term in the locality condition. Bell intended *all* variables, hidden or not, to be included in this term. This term can be included from the start and any hidden variables can be included within it without altering the outcome of Bell's argument. The conclusion is locality must be thrown out. The predictions of quantum mechanics are confirmed by theory and, as stated above, hidden variables cannot resolve the situation. Thus there is only locality to get rid of.

Although the review of Bell's work by Norsen argues for this interpretation, there is still debate surrounding the outcome of Bell's theorem. One concern for this thesis is the use of counter-factual definitiveness in the Bell inequality. This will be explored further and has been shown by Norsen to be built into the Bell locality condition [7]. Another concern for many is the conclusion that Nature operates nonlocally. This implies we will always be stuck with "spooky action at a distance". This is essentially a problem regarding the movement of quantum information. The particles of the singlet state appear to be passing information between themselves over spacelike distances. This nonlocal communication is problematic but there are areas of quantum mechanics which may offer an explanation as to how this feat is achieved.

1.3 Quantum Teleportation

There is a procedure known as quantum teleportation which allows quantum information to be transferred between particles [14]. Although this concept is named 'teleportation', it must be pointed out that no physical object is transported. Rather, quantum teleportation allows the state of a particle to be transferred to another particle. This is accomplished using the features of entangled states, such as the singlet state described in the EPR setup, and the prevailing view is the entangled state provides a 'quantum channel' which facilitates the movement of information [15]. Two experimenters sharing an entangled pair may teleport the unknown state of a third particle using projective measurements and classical communication. Quantum teleportation, which will be explained in greater detail further into this thesis, cannot be used to signal faster than the speed of light [16]. The requirement of classical communication means there is a restriction regarding the time it takes to send a state using this method.

Although this procedure describes sending information between particles, it is not enough to resolve how information is moving in the EPR paradox. In order to explain how the particles in the EPR paradox maintain perfect correlations over spacelike separation, quantum teleportation must be taken one step further.

1.4 Post-state Teleportation

Before embarking on a description of post-state teleportation, the concept of final (or post) state selection must be introduced. In the classical picture, the system dynamics can be determined by evolving forward in time from the initial conditions *or* by evolving backward in time from the final boundary conditions. Using both initial and final boundary conditions is excessive and provides no extra information about the system. In quantum mechanics, however, there is always an element of probability in any prediction of how a system will evolve from initial conditions. One proposal to resolve this suggested including a final boundary condition in conjunction with the initial conditions of the system. This is known as final state selection. Using the final boundary condition, a quantum system can be constrained between the final and initial states. One motivation for this approach is the fact that it enforces time symmetry on the quantum system [17]. It is this concept of final (or post) state selection that is combined with teleportation to produce an interesting result which offers a solution to the EPR paradox.

This thesis explored the work of Lloyd *et al* in which the authors provided a quantum mechanical description of time travel through closed timelike curves (CTCs) [18, 19]. The starting point for Lloyd *et al* was the idea that any form of time travel effectively represents a communication channel from the future to the past. Under this motivation, Lloyd *et al* utilize the quantum teleportation procedure which allows states to be transferred via an entanglement channel. When the quantum teleportation procedure was adjusted to include post selection, the result was a theory with a description of a quantum channel to the past. Lloyd *et al* used this concept to construct a theory of post-state teleportation (PST) [18, 19]. The classical communication needed for standard teleportation becomes irrelevant in PST due to the addition of post selection. The quantum communication channel described in PST is interpreted as an entanglement between the future state and the past state of a system. Within PST, therefore, any influence on the future state (such as post selection) is communicated to the past state via the quantum channel. This will be further explained with a simple example later in this thesis when PST is described as a special case of teleportation. Lloyd *et al* used PST to provide a quantum description of CTCs in their theory of post-selected closed timelike curves (P-CTCs). The theory is intended to be a quantum theory of time travel. Although their results focus on general relativist CTCs, it was pointed out by the authors that PST allows for non-general relativistic time travel [18]. The outcome of Lloyd *et al*'s theory on PST was a scenario in which quantum information could be seen to propagate backwards and forwards in time. It is a description of the movement of quantum information which does not rely on any classical communication. This allowed for a different interpretation for the EPR paradox.

1.5 Applying PST to EPR

This thesis applied the PST procedure to the EPR setup which provided a description of how the information moves during the thought experiment. Alice's measurement of the particle in her possession acts as a final boundary condition for the entangled system. Taking the final boundary condition into account allows the correlations of the entangled particles to be explained without relying on a 'spooky' action occurring over a spacelike distance. Any reliance on nonlocal action is removed. Rather, the information is propagating backwards through time through a quantum channel similar to that described in standard teleportation.

Due to this explanation's dependence on time-traveling information, there were concerns regarding the order of causality and the danger of time-travel paradoxes. Any worries over acausality that arise due to the PST procedure were dealt with within Lloyd *et al*'s work. Lloyd *et al* showed that their theory provided a framework for PST which did not produce time-travel paradoxes [19].

This thesis concluded that quantum mechanics provides a complete description of reality through local actions only. From this point of view, Bell's theorem seems to rely on a locality condition that is too strong to apply to Nature. When the assumptions regarding Bell locality were explored by Wiseman, it became apparent that much of the confusion regarding Bell's theorem arises from misunderstanding Bell locality as a single assumption [20]. Rather, it is the combination of the requirements of relativity along with the CFD assumption which Wiseman identified as a form of predetermination. This is also shown to be assumed by the authors of the EPR paradox. This thesis suggests that the reliance on CFD in these arguments leads to the problematic outcome of a nonlocal description of Nature and this can be avoided if principles of the theory of quantum gravity are taken into account; namely, observer complementarity.

As the use of time travel to resolve the paradox may be troubling, another quantum information paradox is investigated and its possible solutions compared to the PST solution of the EPR paradox.

1.6 The Black Hole Information Paradox

One of the strangest puzzles regarding black holes concerns the fate of matter falling past the event horizon. Hawking showed that black holes evaporate and release radiation over time [21]. This evaporation process is complicated but it can be understood heuristically by the pair-creation process [22]. Two particles are created, a particle-antiparticle pair, along the black hole horizon. These entangled pairs characterise the Unruh vacuum state on the black hole horizon [23]. One, the negative energy particle, falls in to the black hole while the other radiates towards infinity. The negative particle annihilates with matter inside the black hole, reducing the black hole mass, while the other is part of the radiation emmitted by black holes. The information paradox arises when the state of the outgoing radiation is considered. Quantum mechanics dictates that a pure state must evolve into a pure state in order to preserve unitarity [2]. Therefore a pure state which collapses to form a black hole must evaporate into radiation that is also a pure state. Attempting to fit this requirement with the strange place that is a black hole produces a paradox. Consider a spacetime slice constructed to intersect the black hole horizon and avoid the singularity as long as possible, known as a 'nice slice'. This 'nice slice' remains strictly spacelike outside of the black hole and only curves within the black hole in order to avoid the singularity. As an observer falls into the black hole, the nice slice would see not only the infallen observer but also the radiation emitted by the black hole as a result of the observer falling in. This implies that the information exists twice on the same spacelike slice; a direct violation of quantum mechanics. The cloning can be avoided by assuming that any emitted radiation is maximally mixed. However this means any information falling into the black hole is lost forever and this violates the unitarity requirement of quantum mechanics.

This paradoxical choice between cloned information or destroyed information is the crux of the black hole information problem. The paradox was originally resolved by the horizon complementarity solution which allowed for cloned states on the horizon provided that no observer viewed both copies [24]. This solution, however, was challenged by the recent discussion regarding the possible violation of quantum monogamy on the horizon. Now commonly referred to as the 'firewall' argument, the problem has stimulated the debate surrounding black hole horizons and the question of whether or not these areas of space appear approximately flat. The problem was raised by Mathur [25, 26, 27] and similar discussions were presented by Braunstein [28] and Itzhaki [29] as well as others but it was the recent 'AMPS' paper that brought the debate to boiling point where AMPS refers to the authors Almheiri, Marolf, Polchinski and Sully [30]. The 'firewall' problem regards the possible entanglements between systems inside, just outside and far away from the black hole with regard to whether there is a possibility of violating the rules of quantum mechanics. The AMPS conclusion was that there must be a 'energized curtain' or firewall on the horizon which thermalizes any infalling matter. The two solutions to the black hole information paradox that this thesis examines are reviewed with reference to the firewall problem.

The first solution investigated is the final state solution which was proposed by Horowitz and Maldacena which uses a procedure similar to post-state teleportation to ensure information escapes the black hole without producing any paradoxes [31]. To achieve this, a specific final boundary condition was enforced at the singularity. This boundary condition, along with the entangled particle pairs of the Unruh vacuum, allowed infalling information to be 'teleported' out of the black hole. The procedure, however, is not as straight forward as standard quantum teleportation. The information appeared to follow a path which propagated backwards in time in order to escape the black hole. This bears a remarkable resemblance to Lloyd *et al*'s description of the movement of information in PST. Indeed, Lloyd *et al* reference Horowitz and Maldacena's final state solution in the the introduction to their paper on PST [18] and although the final state solution was published years before the firewall discussion, Lloyd *et al* propose that the solution could escape the problems raised by the AMPS paper [32].

The second solution to the black hole information paradox that is considered is Maldacena and Susskinds's "EPR=ER" conjecture [33, 34]. This conjecture built on work of the AMPS firewall proposal. The AMPS argument will be explored in greater detail further in this thesis but one assumption to note is the expectation that there can be no way for a distant system to communicate instantaneously with the interior of the black hole. Although an action on a distant system would eventually affect systems near the horizon, this effect cannot be immediate due to how far away the distant system is. This assumption is the starting point of the "EPR=ER" argument. Maldacena and Susskind's position is the AMPS argument proved there must be a way for the distant system to communicate with matter near the black hole. There must be a "causal shortcut" [33]. Any action on the distant system sends a signal through the 'shortcut' and disturbs the system inside the black hole. The suggestion is that the 'shortcut' between the two systems is an Einstein-Rosen (ER) bridge which is an example of a type of wormhole. The connection to the EPR paradox is the requirement of entanglement. In the black hole picture the distant system is entangled with the interior of the black hole. Maldacena and Susskind argue that in order for two systems to be connected by an ER bridge, they must be entangled with each other [33]. The EPR paradox, meanwhile, requires two particles to be prepared in an entangled state. This connection allows for the transmission of signals between the systems and hence avoids the paradoxes concerning the movement of information in both the black hole and the EPR scenario. This resolution through the use of ER bridges leads Maldacena and Susskind to their argument for ER=EPR.

The ability of both the final state solution and the ER=EPR argument to resolve the black hole information paradox leaves the problem open of which solution to choose as only one can be the correct resolution of the paradox. To this end, this thesis argued for a perspective which views the two solutions as two sides of the same argument. Although they appear to present different answers, it may be possible to consider them as the same solution.

1.7 Observer Complementarity

Observer complementarity was a concept that arose out of 'horizon complementarity' which described the strange behavior of Nature around black hole horizons [24, 35]. Both versions enforce the same principle: separated observers may disagree on their descriptions of certain events but Nature will never allow either observer to witness a paradoxical situation. This will be explored more thoroughly further on but what must be pointed out is that observer complementarity directly contradicts CFD. Observer complementarity requires that each observer only account for their local situation and that they cannot meaningfully compare local results with spacelike separated events which have not affected them yet. CFD, on the other hand, requires discussing events which are not only spacelike separated but also which have not actually occurred but only could have occurred. These two concepts, therefore, cannot work together within the same theory.

This thesis argued that, since observer complementarity appears to arise as a consequence of a theory of quantum gravity, it is natural to apply it to an ordinary quantum system if it can be used to resolve a paradoxical situation. This suggested that it is CFD that should be discarded. The implication is that CFD enforced a description of reality that was too strong and this turns the EPR paradox into a non-starter from the beginning. The use of observer complementarity allowed for the PST interpretation of the EPR paradox and can, in a sense, be seen to imply the use of time traveled information when the observer complementarity assumption is combined with the principles of relativity.

The use of observer complementarity also allowed the choice between the final state solution and the ER=EPR argument to be resolved. By assuming observer complementarity from the start of the black hole information paradox scenario, the argument can be made that both the final state solution and the ER=EPR argument are the same solution but they appear different due to the relative perspectives of their starting points.

1.8 Conclusion

This thesis' argument draws the conclusion that the EPR paradox is resolved by post-state teleportation. The movement of information in EPR is explainable through the mechanisms of PST. Through Lloyd *et al*'s work it is apparent that PST is equivalent to a quantum theory of wormholes, in other words a theory of ER bridges. This allowed a connection to be made between the use of PST-type procedure in the final state solution and the ER=EPR argument.

The use of time traveling information allows for the resolution of the EPR paradox as well as the black hole information paradox.

CHAPTER 2

The Possibility of Paradox

"I like to think that the moon is there even if I am not looking at it"
- Albert Einstein

In 1935 a paper was published which posited a problem with the predictions of quantum mechanics: the EPR paper [1]. The issues raised in the EPR paper continue to be discussed today and, as of yet, no one has provided a solution which satisfies all interested parties. The debate rages on. In this section, the structure of the EPR thought experiment will be fleshed out and discussed with reference to later analysis of the resulting paradox.

2.1 The EPR paradox

As mentioned in the introduction, the EPR thought experiment has two parts to it. The first is the setup of a choice as a result of the assumptions made by the EPR authors. The reality criterion, by the authors own explanation, is not intended to be a conclusive description of reality but rather an adequate one for the purposes of the EPR paradox [1]. The requirement of the reality criterion is that a physical element must exist in reality if a prediction can be made for that element with certainty. This is simply the requirement that a prediction for a physical system must correspond to an existing element of reality if the prediction has a probability of 1. Something must exist to correspond to a prediction made with certainty. The implication of this assumption with regard to the formulation of quantum mechanics led to the choice presented by the EPR authors. The presence of non-commuting operators resulted in a restriction on the simultaneous measurement of the observables relating to these operators [2]. In the formulation of quantum mechanics, therefore, there exist groups of observables that cannot be simultaneously measured or predicted with certainty. This is most easily illustrated with regard to the Heisenberg uncertainty principle [2]. The more certain a measurement or prediction about a variable such as momentum, the more uncertain a measurement or prediction regarding its conjugate, the position of the same system. If a particle's momentum is confidently predicted then the position cannot be predicted with any certainty. When the reality criterion was applied, the result was a strange statement about the reality of any non-commuting observables. Observables represent what variables are measurable in a given system. The reality criterion dictates, as discussed earlier, that providing a prediction with certainty for a value associated with an observable means that there must be an element of reality to correspond to that observable as is the case when an observable is measured. However, if the momentum is measured and its value is known with certainty then by the same criterion the position observable has no corresponding element of reality. The implication was that the presence of non-commuting operators in quantum mechanics results in observables and corresponding elements of reality being unable to exist simultaneously. The EPR paper discussed this outcome and posited two choices. Either the elements of reality corresponding to non-commuting observables could not exist simultaneously *or* quantum mechanics was incomplete in its description of reality and there exist 'hidden variables' which would allow accurate predictions for non-commuting observables. In order to resolve this choice, the EPR argument considered a measurement scenario involving non-commuting observables.

As mentioned in the introduction, this thesis will be using Bohm's interpretation of the EPR setup [4]. Any group of non-commuting observables would produce the same result and for the purposes of this thesis those observables will be the spin directions of a pair of entangled particles given by equation (1.1). The EPR argument first assumed that quantum mechanics is complete in its description of non-commuting observables and proceeded from there.

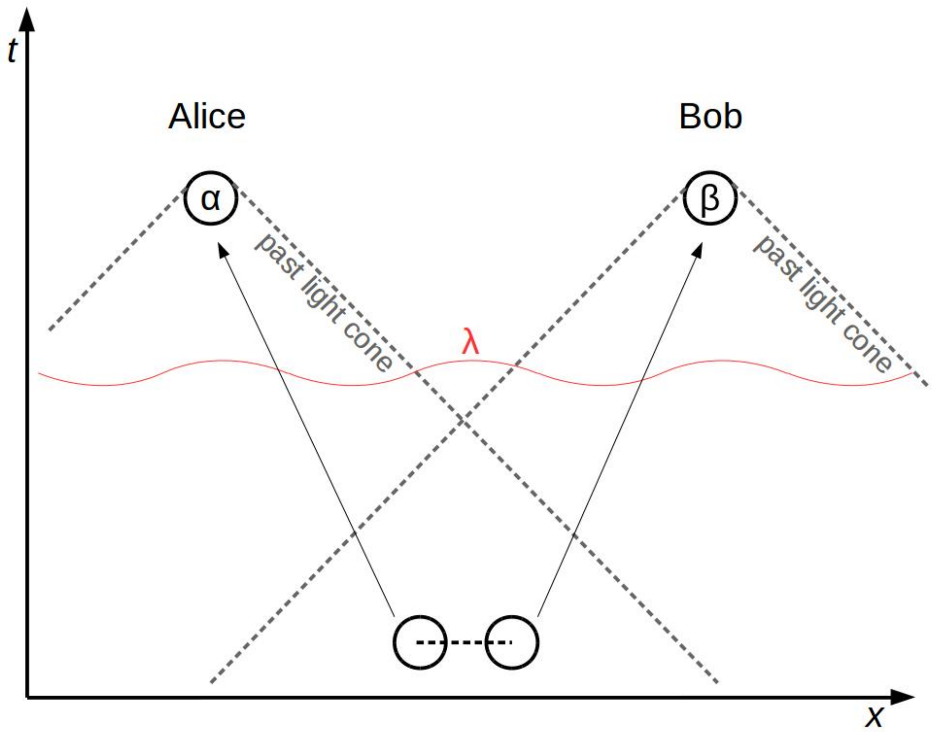

Figure 2.1: The EPR Setup. *The symbols α and β represent the two entangled particles shared between Alice and Bob respectively. The term λ incorporates all possible information contained in the entangled particles' shared past.*

The setup, shown in figure 2.1, continues as outlined in the introduction. After being given their respective particles, Alice and Bob separate to a spacelike distance to ensure they have no effect locally on each other's experiments. In this way the experiment is set up to detect any possible violation of locality. This is the enforcement of the locality assumption which is an application of the principle of special relativity [36]. This principle restricts how quickly an action may affect a distant system. Specifically, no action at location A can affect a system at location B faster than light takes to travel from A to B. The locality assumption enforced the principle of special relativity so that Alice's actions could not instantaneously affect Bob at his distant location. The locality concept was mathematically defined by Wiseman *et al* [20] as

$$\forall\, a, B, b, \lambda,\;\; P_\theta(B|a, b, c, \lambda) = P_\theta(B|b, c, \lambda), \tag{2.1}$$

where P_θ is the probability of outcome B given the assignment of variables following the divider, θ refers to the model or theory used to calculate the probability of the event under question, A and B are Alice and Bob's respective outcomes while a and b represent their choice of measurement direction. This condition dictated that the probability of Bob getting result B must be independent of Alice's actions at her spacelike separated location. The symbol λ represents all possible information regarding causes in the past of both particles before they can no longer affect each other by local actions. Any cause and effect between the particles must occur before separation to avoid violating this application of the principle of relativity. Although the EPR authors did not explicitly describe their locality condition as a mathematical statement, this description by Wiseman *et al* allowed for a clearer discussion regarding the EPR paradox and the following work of Bell. The assumptions enforced in this model can be summarised as follows: a locality assumption based on the causality enforced by relativity, a completeness assumption invoked by allowing λ to contain all available variables of the system, the correctness of the predictions of quantum mechanics, and the assumption that each experimenter can exercise free will in their choices. For further discussion of these assumptions see Norsen's work [10, 11, 13]. As was pointed out in the introduction, considering the outcomes of the two experimenters measurements led to the conclusion that the spin values for every direction for each particle must be determined before they are separated. Alice's measurement will always reveal information about Bob's particle with probability 1. The correlations between spin in entangled particles means Alice will be able to make a prediction regarding Bob's particle no matter which direction either she or Bob chooses. By the reality criterion, as per the previous discussion, this prediction must have a corresponding element of reality. Bob's particle must have the correlated spin in the direction Alice measured regardless of which direction Bob measured. The implication is that non-commuting observables must have simultaneous reality. This outcome allowed the EPR argument to decide between the two choices presented in the first section of the argument. Since the Alice/Bob setup started from the assumption that quantum mechanics was complete and resulted in a scenario requiring the simultaneous existence of non-commuting observables, the EPR authors asserted that quantum mechanics is incomplete. There must be 'hidden variables' that are not accounted for in quantum mechanics which would allow for predictions regarding non-commuting variables. By the EPR definition of reality, non-commuting observables must exist simultaneously and quantum mechanics simply does not provide an accurate description of them.

Concerns regarding this outcome have been present since the EPR paper was published. Bohr, as previously mentioned, had a problem with the use of hypothetical measurements alongside measurements that were actually performed[5]. In the EPR paradox all possible outcomes for Alice are compared regardless of which direction was actually chosen for the measurement. Bohr's suggestion was that this was simply an indescribable situation. One and only one measurement may be performed on the particle in Alice's possession. This measurement changes the state of Alice's particle through the interaction and so no further measurement of the original particle is possible. Considering hypothetical measurements for all the possible spin directions requires comparing results that could not possibly all exist together due to the nature of the quantum measurement process. Bohr therefore concluded that the EPR argument was flawed in its approach. This counter-argument brings a crucial point to the fore: the EPR paradox has a built in predetermination assumption, also known as counter-factual definitiveness (CFD) [20]. Wiseman *et al* defined this assumption mathematically as

$$\forall\, A, a, B, b, \lambda,\ P_\theta(A, B|a, b, c, \lambda) \in \{0, 1\}, \tag{2.2}$$

where A, a, B, b, λ and P_θ represent the same variables as before and c represents the preparation of the initial state before the particles are separated. The implication is that for any measurement scenario, including possible hidden variables, the prediction for any outcome is determined with probability 0 or 1. The outcome is certain and can be predicted for any possible setup given the initial conditions and any hidden variables. No matter which measurement direction is chosen, the probability of the outcomes is known with certainty and, by the reality criterion, must relate to an element of reality. P_θ has meaning in the EPR paradox [20]. If the outcomes are set in this manner the result of a measurement that was not performed can be discussed meaningfully regardless of which measurement was actually performed. This is clearly taken advantage of in the EPR paradox when Alice's result is compared to hypothetical results, had Alice chosen a different measurement direction. This notion of discussing hypothetical measurements alongside actual results is the crux of CFD and is shown by Norsen to be integral to Bell's theorem [11].

This thesis will explore the consequences of and alternatives to CFD further in order to suggest that assumptions made in the EPR argument (and later in Bell's theorem) are overly strong. But first Bell's theorem will be explained in greater detail.

2.2 Bell's Theorem

Norsen's review of Bell's work was used to investigate the implications of Bell's theorem. This provided a view of Bell's theorem as a second step to the EPR paradox. Bell's mathematical formulation of the requirements of locality allowed the EPR paradox to be put to the test experimentally. There is still much debate over what Bell's theorem proved and much of the disagreement rests on which assumption must be discarded [20].

Norsen identified a single assumption: Bell locality [11]. As mentioned in the introduction, this was given by equation (1.2) and enforced the condition that only local events may influence a system. Before equation (1.2) is applied to predictions of quantum mechanics, the requirement for separability must be taken into account [10]. Given the EPR setup, Alice and Bob cannot affect each other locally with their experiments. Therefore the joint probability for Alice and Bob's outcomes must be separable so that each experimenter can calculate an individual probability based on local variables only. This was intended as a consequence of the Bell locality condition rather than a separate assumption [13]. The joint probability must still be preserved for the singlet state given by equation (1.1). The separability condition was given by

$$P(A, B|a, b, \lambda) \;=\; P(A|a, \lambda) \cdot P(B|b, \lambda), \tag{2.3}$$

where P, A, a, B, b and λ represent the same variables as before. The locality condition was also applied to the individual probabilities to ensure that only local actions are taken into account. The locality condition given by equation (1.2) is then compared to the predictions of quantum mechanics. The predictions of quantum mechanics along with experimental results gave the following description of the singlet state given by equation (1.1):

- The probability of measuring a spin up, $\uparrow$, or a spin down, $\downarrow$, particle is 50% in any direction. This gave

  $$P(A =\uparrow |a, \lambda) \;=\; \frac{1}{2}, \tag{2.4}$$

  where λ contains the predictions of quantum mechanics in the form of a wave function [13]. In terms of the EPR setup, Alice has a 50% chance of measuring her particle to be spin up.

- Experiments also show that Alice's result must maintain an overall anti-correlation. The requirements of the singlet state are that

  $$P(A =\uparrow |a, \lambda, b = a, B =\downarrow) \;=\; 1. \tag{2.5}$$

  When Bob's measurement direction is the same as Alice's, the spins of the two particles must be anti-correlated.

The locality condition presented no problem for this description when Alice and Bob chose different directions to measure, $a \neq b$. However, when the two directions coincide there is a violation of Bell locality. Substituting equation (2.4) and equation (2.5) into equation (1.2) yields

$$P(A = \uparrow | a, \lambda) \; = \; \frac{1}{2} \; \neq \; 1 \; = \; P(A = \uparrow | a, \lambda, b = a, B = \downarrow). \tag{2.6}$$

The predictions of quantum mechanics violated the locality condition. With or without hidden variables, Norsen showed that Bell locality will always be violated by quantum mechanics without qualifying the exact type of hidden variable or quantifying the degree of violation [11]. For earlier discussions see the work done in [6]. The predictions of quantum mechanics have been tested and verified by experiment and so Bell's only conclusion was to discard Bell locality. Therefore the implication of Bell's theorem is that Nature behaves nonlocally.

The use of CFD is present in equation (2.6) where two possible scenarios are compared. In one, Alice has no variables for Bob's system which is equivalent to Bob having not performed his measurement; there is no dependence on Bob's outcome. In the second scenario Bob has a measured result and so it is implied he has done an experiment. Either side of equation (2.6) presume a different scenario but compare the results on equal footing. This concept is analogous to a man who takes his umbrella to work and claims this is the reason it has not rained since it always rains when the man forgets it. CFD discusses an event that *might* have happened alongside an event that did happen. That this condition is built into Bell's theorem becomes clearer when the EPR locality condition, equation (2.1), is compared to Bell locality, equation (1.2). In equation (2.1), there is no dependence on the distant observer's result while equation (1.2) does rely on the distant result. Wiseman *et al* showed that Bell locality can be viewed as the combination of two assumptions [20]. The first is the locality assumption utilised in the EPR paradox and the second is the use of CFD or predeterminism to justify comparing hypothetical results with actual ones. The implications of separating Bell locality into two assumptions will be considered in the later discussion of an alternative to CFD.

The conclusion that Nature is nonlocal can be troubling to some. Within the EPR setup it is the information regarding the spin directions that appears to travel nonlocally between the entangled particles. This is therefore a problem regarding the movement of information between entangled particles and there are areas of quantum mechanics that provide an interesting option for such scenarios.

CHAPTER 3

Teleportation to the Past

"The 'paradox' is only a conflict between reality and your feeling of what reality 'ought to be'."
- Richard Feynman

3.1 Quantum Teleportation

The word 'teleportation' may bring to mind scenes from science fiction stories but this is the term being used to describe a remarkable procedure that is made possible by entanglement [14]. In order to explain the teleportation procedure and its properties this thesis used the outline provided by Timpson [16]. This process can be broken down into 4 distinct steps as shown in figure 3.1.

Step 1 Two particles are prepared in one of the four Bell states given by

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle|\uparrow\rangle + |\downarrow\rangle|\downarrow\rangle), \quad (3.1a)$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle|\uparrow\rangle - |\downarrow\rangle|\downarrow\rangle), \quad (3.1b)$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle|\downarrow\rangle + |\downarrow\rangle|\uparrow\rangle), \quad (3.1c)$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle). \quad (3.1d)$$

These states form the maximally entangled Bell basis [16]. After the pair of particles are prepared, Alice and Bob each receive one. Alice also receives a spin-$\frac{1}{2}$ particle in an unknown state given by

$$|\psi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle. \quad (3.2)$$

Alice and Bob then depart to their separate locations for the next step. If the unknown state is system 1 and the two entangled particles are systems 2 and 3 given to Alice and

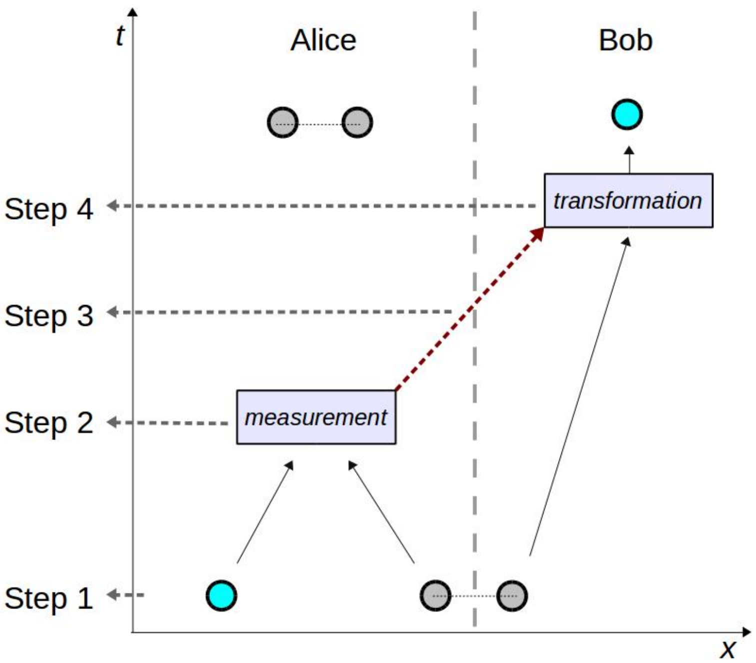

Figure 3.1: The Teleportation Procedure. *Step 1: The Bell state is prepared and the entangled particles sent to their respective experimenters along with the unknown state. Step 2: Alice performs a projective measurement on her two particles which results in one of the 4 Bell states. Step 3: Alice sends her result to Bob. Step 4: Bob performs the related transformation on his particle. Alice is left with an entangled pair while Bob has a particle in the original unknown state.*

Bob respectively, then the initial joint state of all 3 particles is given by

$$|\psi\rangle_1 |\Psi^-\rangle_{23} = \frac{1}{\sqrt{2}}(\alpha |{\uparrow}\rangle_1 + \beta |{\downarrow}\rangle_1)(|{\uparrow}\rangle_2 |{\downarrow}\rangle_3 - |{\downarrow}\rangle_2 |{\uparrow}\rangle_3), \quad (3.3)$$

where the entangled particles are in Bell state $|\Psi^-\rangle$.

Step 2 Alice now performs a joint measurement on the two particles in her possession. When this projective measurement is in the Bell basis, an interesting result occurs. To illustrate this, equation (3.3) is rewritten as

$$|\psi\rangle_1 |\Psi^-\rangle_{23} = \frac{1}{2}\Big[|\Phi^+\rangle_{12} (\alpha |{\downarrow}\rangle_3 - \beta |{\uparrow}\rangle_3) + |\Phi^-\rangle_{12} (\alpha |{\downarrow}\rangle_3 + \beta |{\uparrow}\rangle_3) + |\Psi^+\rangle_{12} (-\alpha |{\uparrow}\rangle_3 + \beta |{\downarrow}\rangle_3) + |\Psi^-\rangle_{12} (-\alpha |{\uparrow}\rangle_3 - \beta |{\downarrow}\rangle_3)\Big]. \quad (3.4)$$

A similar description of the reworking of equation (3.3) is given in Appendix A. What is important to notice is that while Bob's particle had no dependence on the unknown state $|\psi\rangle$, the states for his particle are related to the unknown state by simple unitary transformations such that

$$|\psi\rangle |\Psi^-\rangle_{23} = \frac{1}{2}\Big[|\Psi^+\rangle_{12} (-i\sigma_y |\psi\rangle_3) + |\Phi^-\rangle_{12} (\sigma_3 |\psi\rangle_3) + |\Psi^+\rangle_{12} (\sigma_z |\psi\rangle_3) + |\Psi^-\rangle_{12} (\mathbb{1} |\psi\rangle_3)\Big], \quad (3.5)$$

where $\sigma_{x,y,z}$ and $\mathbb{1}$ represent the Pauli spin matrices and the identity respectively [16]. When Alice now measures her two particles (1 and 2) in the Bell basis, she has four possible outcomes and each outcome is correlated to a specific state for Bob's particle. Once Alice has her result, the wave function given by equation (3.5) collapses to one outcome and Alice now knows how Bob's particle differs from the particle in the unknown state. More importantly, Alice knows what unitary operation Bob needs to perform on the particle in his possession in order to transform it into a particle in unknown state $|\psi\rangle$. It must be noted that Alice has no information regarding the particulars of the state $|\psi\rangle$ and the variables α and β remain unknown throughout.

Step 3 This step restricts the teleportation procedure from signaling faster than light. In order for Bob to transform his particle into the unknown state, Alice first has to communicate her results. This classical communication is vital to the teleportation procedure and it ensures that no faster-than-light communication is allowed.

Step 4 The final step is Bob's transformation. Once he receives news of Alice's result Bob can perform the corresponding unitary operation and be sure he has in his possession a particle in unknown state $|\psi\rangle$. Alice, meanwhile, is left with 2 particles entangled in some Bell state.

The above procedure depends only on local operations at Alice and Bob's respective locations. The specifications of the unknown state $|\psi\rangle$ are also irrelevant to the procedure as neither experimenter measures $|\psi\rangle$ to determine α or β at any point. The state of particle 1 is 'destroyed' by the procedure as is the entanglement between particle 2 and 3. Therefore, there is no cloning occurring during the teleportation procedure. The entanglement is conserved as Alice is left with an entangled pair and the unknown state is conserved in Bob's particle. Since it is the entanglement between particle 2 and 3 that allows the particulars of the unknown state to be transferred to particle 3, the interpretation was that the entanglement provided a quantum channel which transmitted the information about particle 1 to particle 3 [15]. Alice's measurement collapses equation (3.5) and breaks the entanglement and this action causes the transmission of information to Bob's particle. It is important to note that no body of matter was transported during the procedure. Rather, the information capturing the state of a system was transmitted between particles. This brings to the fore the idea that information can be seen as separate form the body of matter containing it.

The setup used in quantum teleportation is similar to that seen in the EPR paradox. However, although teleportation utilised the features of entangled particles, it does not explain how entangled particles maintain their correlations over spacelike distances. In order to provide an explanation for entanglement the teleportation procedure must be taken further, into the realm of post-state selection.

3.2 Final State Selection

Lloyd *et al*'s theory of post-state teleportation relied on selecting the final state of a system [18]. This feature of final state selection is utilised in the two-state vector formulation of quantum mechanics developed by Aharonov *et al* [17]. The basic principle requires the specification of both the initial and the final states of the system in question. This formulation led to the weak value theory [37]. A weak value is a value for a state conditioned on the final and initial states. It is essentially a property ascribed to a state between an initial and final measurement. Conceptually this can be seen as two states evolving in time, one forward and one backward, to converge in the middle and constrain the state at some intermediary time. In order for a measurement to yield this weak value at the intermediary time, the final and initial states must remain undisturbed. Therefore the measurement interaction must minimise any disturbance on the system which would alter the boundary states and so is known as a 'weak measurement'. This measurement technique is associated with a large uncertainty in the result and so the state is only 'weakly' defined at the intermediate time [38]. Although Lloyd *et al* point out that their theory of post-state teleportation does not require weak value theory [18], a discussion of features shared by both theories provides motivation for the procedure.

The nature of quantum mechanics is probabilistic: evolving an initial state forward in time does not lead to one determined outcome but rather to one out of a group of possible outcomes, each with their own probability of occurring. The measurement process essentially changes (collapses) the system into a new wave function and taking a final result does not generally allow a prediction of past values for a quantum system [37]. The result is a time asymmetry within quantum mechanics. By selecting a final boundary condition, however, the weak value theory and the theory of post-state teleportation enforced time-symmetry in their formulations [18]. This use of a boundary condition acting backwards through time contradicted the idea of an 'arrow of time' which is a notion given by the second law of thermodynamics. This 'arrow' establishes time as something that moves from the past, which exists, to the future, which has not yet happened. As pointed out by Aharonov *et al*, however, the idea that quantum mechanics *must* contain a similar arrow of time is an assumption which may be challenged [38]. There exists the possibility that the experience of a preferred direction of time is a result of macroscopic phenomena and is not essential to a formulation of quantum mechanics which occurs on the microscopic scale. The use of final boundary conditions in conjunction with initial conditions allowed for a more complete description of quantum systems [38] and this, along with the result of time symmetry, provides a solid motivation for exploring the results of including post-state selection within quantum procedures.

3.3 Post-State Teleportation

Lloyd *et al* use post-state teleportation to provide a quantum mechanical description of time travel through closed timelike curves. These curves can be represented as two connected wormholes such that the system loops back on itself and so closed timelike curves are commonly referred to as wormholes. The relativistic environment of wormholes, however, is not essential to accomplish time travel with Lloyd *et al*'s theory. In order to illustrate the post-state teleportation described by Lloyd *et al* in [19, 18], consider the teleportation setup where Alice and Bob begin by sharing an entangled state. For the sake of this example the entangled state is taken to be the Bell state $|\Psi^-\rangle$ although any of the four Bell states would produce the same result. To be as clear as possible, the three systems involved in this procedure, which is illustrated in figure 3.2, are defined as follows:

System 1: The unknown state which must be transferred

System 2: Half of the entangled pair which receives the unknown state

System 3: Half of the entangled pair which is sent to Alice

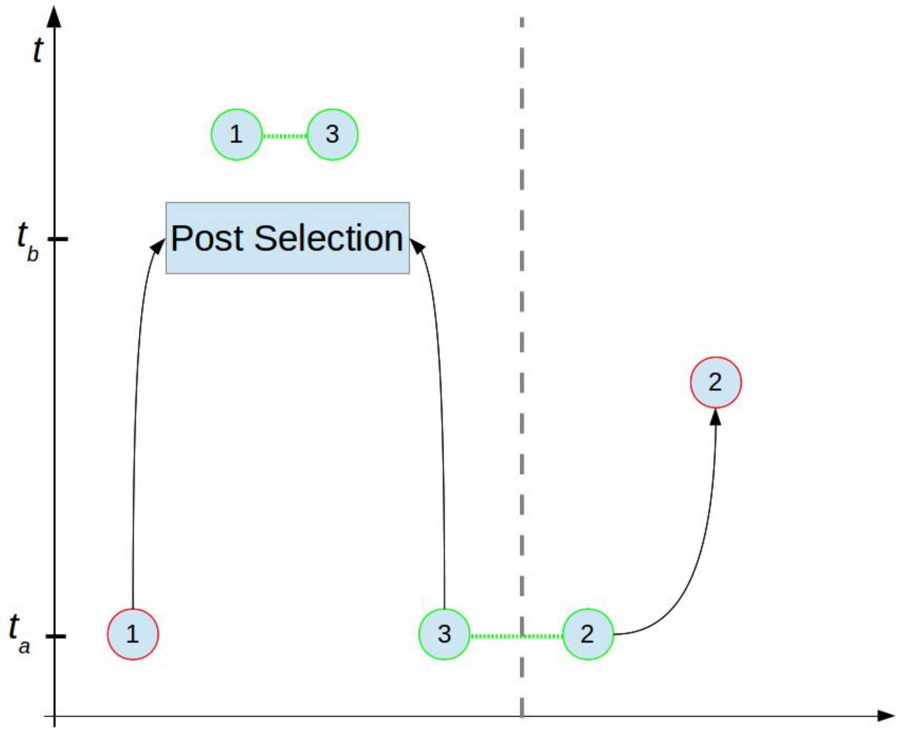

Figure 3.2: The Post-State Teleportation Procedure. *Alice recieves particles* 1 *and* 3 *while Bob receives particle* 2. *If Alice post-selects for the correct state, Bob's particle must be in the unknown state from* t_a.

To begin with, at time t_a, system 2 and system 3 are in the state $|\Psi^-\rangle$ while system 1 is in the unknown state $|\psi\rangle$. The joint state of all three systems is therefore given by

$$|\psi\rangle_1 |\Psi^-\rangle_{23} . \tag{3.6}$$

After the entangled pair is separated in the teleportation procedure, Alice must perform a projective measurement at time t_b on system 1 and system 3 to produce a result in the Bell basis. To see the implications of Alice's result, Lloyd *et al* expanded the joint state given by equation (3.6) to

$$(-|\Psi^-\rangle_{13}|\psi\rangle_2 - |\Psi^+\rangle_{13}\sigma_z|\psi\rangle_2 + |\Phi^-\rangle_{13}\sigma_x|\psi\rangle_2 + i|\Phi^+\rangle_{13}\sigma_y|\psi\rangle_2)/2, \qquad (3.7)$$

where $\{\Psi^\pm, \Phi^\pm\}$ are the entangled Bell states and $\sigma_{x,y,z}$ are the Pauli spin matrices [18]. An explicit description of how equation (3.6) is arrived at is given in Appendix (A).

The teleportation procedure requires Alice to send her projective measurement's result to Bob. Bob needs this information in order to know what transformation to perform on his system to complete the transfer of the unknown state from system 1 to system 2. It must be emphasised that the matter forming the particle is not transported, only the state characterising the system. This is a common feature of all teleportation protocols. Lloyd *et al*'s focus, however, was on the scenario when Bob's transformation turns out to be the identity $\mathbb{1}$. This case is simplistically shown in figure (3.2). In this scenario, performing the identity transformation is equivalent to performing no transformation at all. Lloyd *et al* argued that this can be interpreted as Bob being in possession of a system in the unknown state from t_a [18]. As long as Alice measurement results in $|\Psi^-\rangle$ Bob does not need to perform any transformation on his particle. This scenario does not allow for any signaling faster then light since Bob must wait for the classical communication of Alice's result to know what transformation to perform. Until Bob hears from Alice, he cannot be sure what her outcome is and therefore Bob cannot know how the state of his particle differs from the unknown state. However, even if Bob does not know it until he hears from Alice, he is in possession of the unknown state $|\psi\rangle$ before the particle originally in the unknown state is available. Any measurement Bob makes on his particle between t_a and t_b will provide a result consistent with the unknown state without any communication from Alice. Lloyd *et al* pointed out that this would be true for Bob even if Alice was delayed in receiving system 1. In other words, Bob could be in possession of the unknown state *before* it is made available. This reasoning led Lloyd *et al* to conclude that a valid interpretation for this procedure is that the information regarding the unknown state is propagating backwards from Alice's measurement at t_b to the earlier time t_a. Essentially, the measurement of system 1 and system 3 at t_b influences the state of system 2 at t_a.

When Bob's transformation is the identity $\mathbb{1}$, the classical communication from Alice is not required to transfer the state. This is only the case when Alice's measurement produces the specific result of $|\Psi^-\rangle$ or, more generally, produces the same Bell state at t_b that system 2 and system 3 are in at t_a. This enforces the post-selection requirement in Lloyd *et al*'s theory of time travel. Indeed, Lloyd *et al* began their discussion by asserting that it was the combination of quantum teleportation along with post-selection which allowed for a quantum theory of time travel [19]. The post-selection of Alice's measurement result at t_b allows the teleportation procedure to be modified to one which does not rely on classical communication to transfer states and so the concept of post-selection is crucial to Lloyd *et al*'s quantum description of time travel. In the scenario presented in figure (3.2), it is apparent that the information contained in the unknown state is transferred to system 2 before the unknown state is necessarily available. Alice might receive system 1 only moments before her measurement whereas a measurement of system 2 by Bob would result in the unknown state at any point after t_a. The only connection between Alice and Bob is the shared entangled pair and so it must be the measurement at t_b, which affects system 3, that transmits the information regarding the unknown state to system 2. However, as Lloyd *et al* argued, the fact that Bob can measure system 2 to be in the unknown state at any time after t_a suggests that the information made available at t_b is propagating backwards to the entangled pair at t_a [19]. In other words, the effect of Alice measurement at t_b is influencing the conditions at t_a. The unknown state is transfered not only through space to system 2 but also through time to t_a. This interpretation was labeled by Lloyd *et al* as post-state teleportation (PST) and this special case of teleportation, where Bob's transformation is $\mathbb{1}$, opened the door for a Lloyd *et al*'s quantum mechanical description of time travel through wormholes.

Lloyd *et al*'s focus was not limited to describing time travel in the special case of teleportation. Rather, using post-state selection along with quantum teleportation, allowed Lloyd *et al* to present a quantum mechanical description of time travel through closed timelike curves (CTCs). CTCs are more commonly know to the world as wormholes. They describe areas of spacetime which curve around to such a degree that to travel through them would put oneself in a former time; essentially a device which allows time travel to the past, the possibility of which was proved by Gödel [39]. It should be noted that, while the possibility may exist, work has been done to show that there may be no way to use wormholes. This idea is captured by the chronology protection conjecture developed by Hawking [40]. This conjecture suggests that quantum effects prevent matter from using wormholes as a form of time travel [41]. However, this concept is not necessarily at odds with the form of time travel this thesis will employ later. First, the theory of time travel developed by Lloyd *et al* is explained. Lloyd *et al* set out to reconcile the possibility of CTCs with quantum mechanics using teleportation and post-selection, both procedures that are already used in quantum mechanics. Any form of time travel represents a channel of some type which leads from the future to the past. This motivated Lloyd *et al* to employ the mathematics of quantum teleportation which already describes a "quantum communication channel" that allows for the transmission of information [18].

The example given by Lloyd *et al* in [18] that concerns a CTC is conceptually illustrated in figure 3.3 and, again, concerns three systems:

System 1: Enters the CTC after being post-selected at t_b. Given by state $|\psi\rangle$

System 2: Exits the CTC at t_a. Entangled with system 3.

System 3: The reference or purification system, it remains outside the CTC.

Scrutinising equation (3.6) led Lloyd *et al* to the conclusion that post-selecting system 1 and 3 to be in the state $|\Psi^-\rangle$ ensured that system 2 would be in the state $|\psi\rangle$. The teleportation procedure prescribed a projective measurement at t_b while the post state selection dictated that the state $|\Psi^-\rangle$ is selected for. The result of combining these procedures leaves only one possible state available in equation (3.7). If the post-selection effectively kills all the possible states of system 2 except the one associated with $|\Psi^-\rangle_{13}$, then system 2 must be in the same

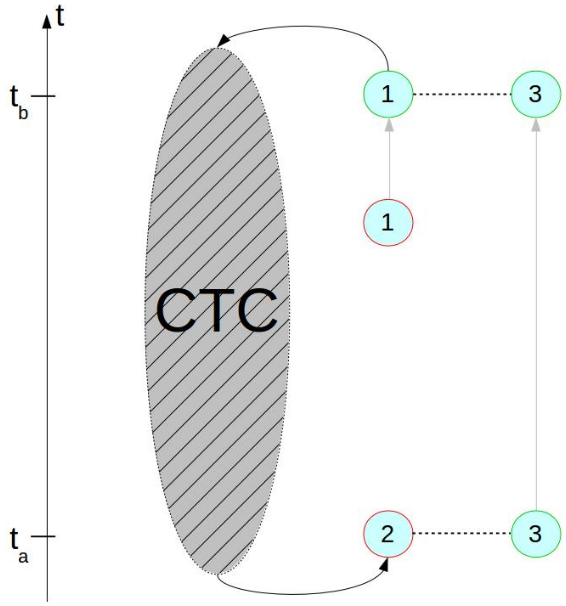

Figure 3.3: The P-CTC Procedure. *System* 1*, originally unknown, is sent into the CTC after undergoing post-selection along with system* 3*. System* 2 *emerges from the CTC in the original unknown state of system* 1*.*

state as system 1 was in before post-state selection, in a manner similar to that of the special teleportation case. Unlike the teleportation example, however, this example deals specifically with systems entering into and emerging from a CTC. The expansion of equation (3.6) by Lloyd *et al* showed that utilising post-selection ensured the system exiting the CTC at t_a (in the past) would be in the same state as the system entering the CTC at t_b (in the future). From equation (3.7) it is clear that, if the post-selection is successful, the state of system 2 emerging from the CTC is $|\psi\rangle$. The PST procedure allowed Lloyd *et al* to compile a quantum mechanical description of time travel through CTCs which was dubbed post-state CTCs (P-CTCs) [18]. Lloyd *et al* were not the first to try reconcile CTCs with quantum mechanics. In [18], Lloyd *et al* compare their results with that of Deutsch who also attempted a quantum theory of CTCs [42]. Deutsch's theory of CTCs depends on a self-consistency condition enforced on the states traveling through the CTCs as well as a 'maximum entropy rule' which determined which state must be chosen should more than one state be consistent with the self-consistency condition [18]. The P-CTC theory was shown to be consistent with the path-integral approach formulated to describe CTCs while Deutsch's CTC theory fails to do this [18]. The calculation explicitly showing this consistency is given in Appendix B. Conceptually the path-integral formulation of CTCs works by letting system 3 evolve normally through time while system 1 is evolved backwards through time to system 2. The result of this protocol is that the time-traveled system's state remains unchanged. Another important difference that arose between the two theories was the preservation of correlations. Lloyd *et al* pointed out that Deutsch's theory failed to maintain all possible correlations the system entering the CTC may have had. The P-CTC procedure based on PST, however, ensured that all correlation going in were present when the system emerged. To summarise, Deutsch's consistency condition is replaced by the post-selection. Considering the correlations in the PST procedure and a closer look at the entanglements in figure 3.3 illustrates an interesting development for PST.

Consider the entanglement experienced by system 3. At t_a system 3 is entangled with system 2 and at t_b system 3 is entangled with system 1. At first glance, the post-selection to entangle system 1 and system 3 may appear to break the entanglement of system 3 with system 2. However, this is not the case in the P-CTC procedure. System 3 is entangled with *both* system 1 and system 2. This presents a problem when quantum monogamy is considered. This refers to the condition that a quantum system may not be strongly entangled with multiple systems. This condition is a corollary of the strong subadditivity statement proven in [43]. Strong subadditivity refers to an inequality equation governing how the entropy of a system must be constrained with regard to the entropy of the subsystems which make up the whole. By tracing over individual subsystems, the entropies of specific sections of the system may be measured against one another. The corollary to the strong subadditivity statement which related to quantum monogamy was proved in [44] and is given as

$$S(\rho^A) + S(\rho^B) \ \leq \ S(\rho^{AC}) + S(\rho^{CB}), \tag{3.8}$$

where S denotes the entropy of the density matrix given by ρ and the superscript A, B and C refers to three subsystems within a larger system. The entropy is compared between the subsystems. This is accomplished by tracing out either one or two subsystems from the total density matrix of the complete system. This allows equation (3.8) to limit possible entanglements within a group of three subsystems. Consider a situation in which subsystem C is strongly entangled with both subsystem A and subsystem B. A strongly entangled system has low entropy and each subsystem within the entangled system will have an individual entropy that is higher than the entropy of the entire entangled system. This results in $S(\rho^A)$ being greater than $S(\rho^{AC})$ and similarly $S(\rho^B)$ would be greater than $S(\rho^{CB})$. This combination violates equation (3.8) as the left hand side, comprised of single subsystem entropies, outweighs the right hand side which consists of the entropies of entangled pairs. This outcome led to the conclusion that system C can only be strongly entangled with either system A *or* system B but not both. Hence a quantum system must respect monogamy and may entangle strongly with only one other system at a time.

In identifying similar states, one interpretation is that two states are considered to be identical if they have the same purifier. This is the stance taken in the following discussion. By asserting that system 3 is entangled with two other systems in the PST procedure, the monogamy appears to be violated. However, closer examination of the situation shows this is not the case. System 3 represents the purifying system for this procedure. For any given system, there is always a state (even if it is only hypothetical) which can act as its purifier. Entangling system 1 and system 3 ensures a pure state for the combined systems. System 1 is now sent back in time to emerge as system 2. However, system 2 is shown to be in the same state as system 1. Therefore system 2 which emerges from the CTC will have the same purifier as system 1 that entered; namely system 3. Therefore system 3 is entangled with two systems but these two systems are the *same* system existing in the past and the present. Lloyd *et al*'s interpretation was that system 1 and system 2 are in fact one and the same. System 2 simply represents system 1 after it has experienced time travel. It should be noted that system 1 and 2 do not necessarily have to be the same state due to the implications of the monogamy discussion alone. It could be argued that system 3 is simply entangled with two different systems at two different times, t_a and t_b. However, it is the consideration of system 3's perspective which leads to the conclusion that system 2 is system 1 at a different time. As far as system 3 is concerned, it does not interact and its state does not change throughout the procedure. System 3 simply evolves according to the normal rules of quantum mechanics and it does not differentiate between system 2 and system 1. Therefore it is system 3's perspective which leads to the conclusion that system 1 and system 2 can be considered to be the same system. This interpretation holds for both the P-CTC and PST procedures. While the PST example may not deal with a system actually entering a CTC, the state of system 2 at t_a is shown to be the same as the unknown state of system 1 at t_b. The interpretation by Lloyd *et al* that this unknown state travels backwards in time from t_b to t_a allows PST to avoid any problems regarding quantum monogamy. The backwards propagation of the unknown state through time opens the door for the interpretation that system 2 is a time traveled version of system 1, provided the post-selection was successful.

This special case of the quantum teleportation procedure highlights an important feature of Lloyd *et al*'s theory of post-state teleportation (PST): sending information back in time with this procedure does not require jumping into wormholes [18]. Even without a CTC present in the example, the information regarding the unknown state appears to be experiencing time travel. The state (not the particle) is sent back in time using only post-selection and the teleportation procedure. Therefore PST can be applied independently of CTCs. This technique is similar to that used by Maldacena and Horowitz to ensure information escapes from black holes [31] which will be explored later in this thesis.

Any discussion of time travel inevitably leads to realm of paradox. When something is sent to the past, that object's past now lies in front of it with regard to the classical arrow of time and contradictions begin to arise. A commonly used example of this is the grandfather paradox in which a time traveler goes back in time and shoots their grandfather before the time traveler's parents are born. With the grandfather dead, the parents are never born and so neither is the time traveler. However, if the time traveler is never born, no one shoots the grandfather and all the required people are born which allows the time traveler to be born and go back in time to shoot grandad. If the grandfather is killed, the time traveler is never born and there is no-one to shoot the grandfather. Headache ensues.

This type of logical contradiction occurs when effect no longer follows cause along the classical arrow of time. Paradoxes illustrate the flaws associated with most theories regarding time travel and they need to be resolved in order for a theory of time travel to be considered consistent. Lloyd *et al* achieve this by ensuring the probability of a scenario leading to paradox is always null [19]. The outcome of the PST, as always in quantum mechanics, is reliant on probability. There are multiple possible results only one of which is associated with successful post-state teleportation: only one outcome allows for time travel. What Lloyd *et al* showed was that the post-selected state which resulted in paradox was orthogonal to the post-selected state allowing time travel. The post-selection can either result in the time travel state or the paradox-inducing state, but never both. Therefore, when the time travel is successful, the state being transported to the past does not produce a paradox. When the post-selection results in a state that would produce a paradox if it were sent to the past, the PST procedure does not allow time travel to occur. Lloyd *et al* prepared an experimental test of a version of the grandfather paradox employing a photon attempting to change its polarisation in the past [19]. This experiment illustrated the use of PST in describing non-relativistic time travel. Even without a handy wormhole, entangled particles can be used to implement a P-CTC scenario as is seen in the special teleportation case. By utilising this feature of PST, Lloyd *et al* managed to construct an experimental situation to test their theory. This experiment consisted of sending a photon through a quantum teleportation circuit and post-selecting for the state allowing for time travel thereby implementing the PST procedure. The circuitry was designed to 'fire' a 'quantum gun'; essentially this allowed the photon to attempt to 'kill' its past self by changing its state in the past. The results showed that the 'gun' simply did not 'fire' when the post-selection was successful. Essentially, in the case where a paradox would result, Lloyd *et al* showed that the probability of the post-selection resulting in time travel is null [19].

Another time travel paradox is discussed by Lloyd *et al* in [19] and is referred to as the 'unproved theorem paradox'. Unlike the grandfather paradox, the unproved theorem paradox deals with the movement of information through time rather than physical matter. The scenario begins with an experimenter (Alice) learning a proof for a theorem from a book. Alice then attempts to send the proof back to a location in the past where the author of said book may find it. This paradox is also more relevant in the case were the chronology protecting conjecture turns out to be correct as it involves information moving through wormholes rather than systems of matter. Rather than go through any wormhole herself, Alice simply ensures that the information regarding the proof makes it to the past. The author of the book then happens upon the proof in the past. Blown away by the mathematical elegance, the author includes the proof in the book where Alice will have learnt it from. The question is, where did the proof originate? Alice learns it from the book and so the author must have worked out the proof but the scenario dictates that the author learns the proof from the time traveled copy courtesy of Alice. Both parties appear to have learnt the proof from the other and the origin of the proof is vague and undefined.

Similarly to the grandfather paradox resolution, there are multiple possible outcomes for the PST procedure. For the unproved theorem scenario, this means that the proof traveling through time will be in a mixed state comprised of all possible proofs. Therefore there is a possibility of any one of those proofs emerging in the past and Alice cannot be sure that the proof which the author finds in the past is the version that she intended to send. The author 'collapses' the possibilities when he reads the proof. PST ensures that, in the case when the proof would transmit correctly and produce a paradox, the 'collapse' will never be able to be verified by Alice. Alice can only say that a version arrived in the past, not that the original proof was actually transmitted. Alice simply cannot definitively send the proof back to the author. The PST procedure automatically corrects for paradoxical results by ensuring that the probability of paradox is always eventually overwhelmed by the probability of consistency.

Lloyd *et al* also showed their path-integral formulation for CTCs could be given as a "generic quantum evolution" [18]. This allowed the evolution of a state traversing a CTC to be calculated in a manner similar to any conventional evolution. The result they calculated was the same as one arrived at in Hartle's previous work on the subject [45]. A description of the general case is given in Appendix C.

This thesis' motivation for examining PST was to explore the movement of information in quantum systems. With this in mind, the PST procedure described in figure 3.2 is revisited. In this example, the information seems to experience time travel as it appears the state is teleported before it is made available. How is this achieved? Lloyd *et al*'s claim is that the post-selection combined with the teleportation procedure results in a quantum channel to the past [18]. The effects on the systems in the future (the post-selection measurement) are felt in the past via a quantum channel. Lloyd *et al* suggested that entanglement channel occurs between the future and past systems. The reference system connects the future measurement with the system in the past and so the interpretation was that this reference system is entangled through time. This allows any effect from an interaction in the future to propagate backwards in time and affect any correlations the reference system had in the past. In this way the information regarding the unknown state is teleported back in time via the reference system's entanglement. This interpretation of PST opens an interesting avenue for understanding the movement of information in the EPR paradox.

3.4 Applying PST to the EPR Paradox

The EPR scenario and Bell's theorem led to serious questions regarding the movement of information in entangled systems. The conclusion that Nature must be acting nonlocally to enforce the correlations associated with entangled particles has troubled physicists and fueled the debate surrounding the EPR paradox. In this chapter, the PST procedure is applied to the EPR setup in an attempt to explain this movement of information without resorting to nonlocal action. It may be argued that this approach of "spooky action in time" is no better than "spooky action at a distance". Though it may simply be a matter of taste, this thesis suggests that the PST approach is preferable as it is consistent with the two-state vector formalism discussed earlier.

Consider the EPR setup with regard to the PST procedure. Alice and Bob share an entangled state between them given by equation (1.1) which represents the joint state of all relevant systems, namely Alice and Bob's respective particles α and β. The systems involved in the

EPR setup can be mapped onto those in the PST procedure in the following way. System 1, the unknown state, represents Alice's system in the past (at t_a). System 2 represents Alice's system in the future (at t_b). Finally, system 3 represents the system given to Bob which is entangled with Alice's at t_a and t_b. Equation (1.1) gives the joint state of the two systems and represents the initial conditions of the EPR scenario. The standard approach is to leave this as the only boundary condition which affects the outcomes of either experimenter's measurement. In the PST procedure, however, it was seen that a final boundary condition imposed by post-selection proved useful in describing quantum systems. Consider the measurement made by Alice on her particle at time t_b as given in figure 3.4. In order to apply PST to the EPR setup, this measurement is interpreted as the post-selected final boundary condition. Similar to the teleportation procedure, Alice has a set of possible outcomes for each measurement direction and her measurement selects one of these outcomes. Consider a measurement for Alice in the z-direction which results in the state $|a_z \uparrow\rangle$. In the original EPR scenario this measurement result appears to instantaneously affect Bob's particle to ensure his result would be $|b_z \downarrow\rangle$ if the same direction is measured by both experimenters. But PST does not transmit the effects of the final conditions in a nonlocal manner. Rather, the disturbance of the post-selected measurement propagates through time along the entanglement channel between the system in the future and in the past. According to PST, the system at t_b is causally linked with its past self at t_a thanks to its continual entanglement with its purifier: system 3. This provides a quantum communication channel for the disturbance to reach the past. Once Alice measures her system she disturbs it and this effect is transmitted to the system in the past. In other words, Alice measures her particle to be in a specific state and the disturbance produced by the measurement propagates back to Alice's system at t_a and sets her system in that state. In effect, therefore, the direction of spin in the z-direction is set from t_a, before the systems are separated. This is enforced from a future boundary condition, however, rather than any action in the past of the singlet state.

Figure 3.4 gives a conceptual picture of the movement of information in this interpretation of the EPR scenario. The dashed red lines represent the entanglement between the future and past of each system. Quantum mechanics dictates that once Alice's measurement is performed and her system's spin is set for the z-direction, Bob's system's spin is also set for the z-direction due to the correlations of the entangled systems. However, instead of Alice's outcome affecting Bob's system over a spacelike separation, PST allows for an explanation involving only local interactions. If Alice's measurement creates a disturbance which propagates backwards in time to set her system's spin at t_a, then Bob's system's spin is also set at t_a for that measurement direction. The entanglement correlations ensure that once Alice's spin direction is set, Bob's spin direction is also determined. PST allows this spin direction to be set at t_a by employing a quantum channel between the future and past of Alice's system. So if Alice's system is determined for the z-direction at t_a then Bob's system must *also* be set for the z-direction at t_a. The interaction between the two systems which determines the spin directions is therefore a local action as it happens when the systems are near each other at t_a and before they are separated to spacelike distances. The information regarding the outcome propagates backwards

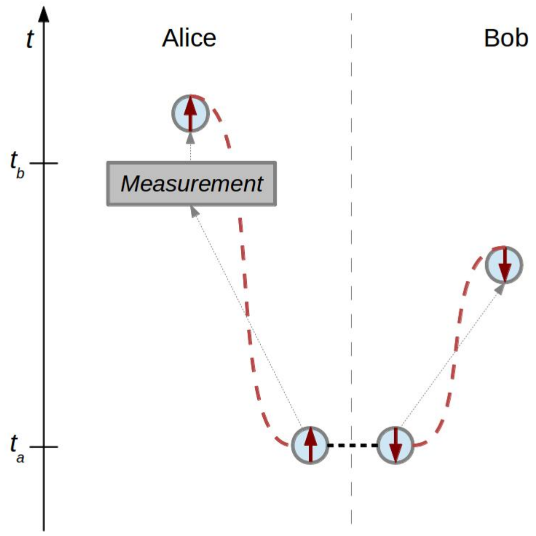

Figure 3.4: The PST-EPR Setup. *The red lines and arrows show the features added to the EPR setup due to the PST interpretation while the rest of the scenario is similar to that described previously.*

to when the systems are initially entangled and the spins are set before the particles separate. There is no need to resort to nonlocal action under the PST interpretation as the systems can interact locally at t_a. In a sense, this is a 'hidden variable' solution to the EPR paradox where the variables necessary to determine the spins are 'hidden' in the future and both initial and final boundary conditions are required to fully determine the entangled systems. By applying the PST procedure to the EPR scenario, Alice's measurement becomes a final boundary condition. Alice's outcome selects for one of the possible terms in the singlet state and kills off the remaining terms. Bob's system can only be in the state associated with Alice's outcome. Using the PST interpretation of entangled systems means Alice's outcome can propagate along a quantum channel to the past and set the systems in their correlated states before they are separated. The PST procedure accounts for the correlations of the entangled systems while employing only local interactions in the EPR scenario.

This solution to the EPR paradox runs into possible paradoxes of its own due to the time travel aspect. Asserting that information can travel to the past to influence systems opens the door for the usual suspects in time travel problems. Could this mean that future Alice can communicate with past Alice and Bob? Unfortunately for those who hope to beat lotteries with knowledge from the future, this possibility is ruled out by Lloyd *et al*'s resolutions for time travel paradoxes within the PST procedure. As was seen in the discussion on PST, this procedure cannot be used to communicate to the past. To see how this is enforced in the EPR scenario, consider Alice's measurement. Alice can only have full knowledge of the entangled systems' spin in the z-direction once her system has been measured. Even though the PST procedure determines the spin at t_a, Alice can only learn this when she performs her measurement at t_b. Past Alice has no way to learn what future Alice knows except by evolving through time to t_b to perform the measurement. Bob can also not receive any signals from the future through the PST procedure. Once the experimenters separate, Bob can have no knowledge of Alice's outcome except through classical means. If Bob measures a different direction to Alice, his result will be uncorrelated to Alice's. If Bob measures the same direction as Alice, he will not know it is correlated until he hears from Alice. His measurement outcome will be one of the possible results allowed by the system and Bob's result cannot tell him whether Alice has measured the same direction or not. Even if the experimenters conspire to measure the same direction they cannot signal each other without Bob receiving a classical communiqué from Alice. Alice's measurement outcome will always be one of two possibilities. Which possibility is finally seen to be the result cannot be known until the measurement is performed and so Alice and Bob cannot set up in advance an outcome to signal each other. This is similar to the discussion regarding the PST case. The outcome of Alice's measurement will always be a result of probabilities and so any signaling to Bob will always have to rely on classical communication from future Alice. This restricts Alice from using PST to create paradoxes by sending Bob the winning lottery numbers.

The PST procedure allows a complete description of the EPR scenario. The spins of entangled systems can be described as determined without discarding the probabilistic nature of quantum mechanics. All outcomes still rely on probabilities but the properties of the systems can be determined by including a final boundary condition and allowing information to propagate backwards through time. At no point does the PST procedure rely on nonlocal action. This puts it at odds with the conclusions reached by Bell's interpretation of the EPR paradox. Before comparing these two approaches for resolving the EPR paradox further, another very well known information paradox is discussed along with a possible solution which has much in common with the PST procedure. The EPR paradox is also not the only problem regarding quantum information that can be resolved using time traveled information.

CHAPTER 4

Quantum Information and Black Holes

"Black holes are where God divided by zero."
- Albert Einstein

4.1 The Black Hole Information Paradox

As the name suggests, the black hole information paradox concerns the movement of information after it has interacted with a black hole. The popular view of black holes is as cosmic drains which pull all matter, even light, into themselves with an inescapable force. This suggested that information regarding matter that has fallen into a black hole is never retrievable once the system has passed the point of no return characterized by the event horizon. This horizon represents the distance around the black hole where the gravitational force is so strong that light cannot move fast enough to escape the pull towards the singularity at the black hole's center. Hawking altered this view of black holes when he showed that they evaporate by particle emission, given enough time [21]. The evaporation process is a complicated one and beyond the scope of this thesis. However, at a glance, the production of particles arises from dynamically evolving spacetime and can be described by applying a quantum field theory framework to curved space [46]. This complex evaporation procedure can be understood more simply through the pair production mechanism [22]. Although this provides only a heuristic understanding of black hole evaporation, it is sufficient for the analysis in this thesis. The discussion so far has focused on entangled pairs providing quantum channels and the pair production procedure results in just such an entangled pair. The pair of particles in question are the particle-antiparticle duo which characterise the Unruh vacuum state [23] which is that experienced by those freely falling past the horizon. Usually the two particles involved in the pair production procedure would have very short lifetimes as virtual particles and effectively annihilate one another due to their anti-correlated energy levels. The particles avoid this fate around the horizon due to their interaction with a field strong enough to separate them. One particle has negative energy (the antiparticle) while the other is positive (the particle). The evaporation process relies on the antiparticle (or negative energy particle) falling past the event horizon and into the black hole. The produced pair separates and the positive energy particle is able to radiate away from the black hole [22]. This accomplishes the task of having the black hole radiate matter away but in order to be considered evaporation, the black hole must lose mass via this procedure. Back to the infalling half of the Unruh pair. The negative energy particle falls into the black hole where it meets the matter that has been pulled into the black hole or the matter which collapsed to form the black hole in the first place. One part of this infallen matter collides with one part of the infallen antimatter and the result is mutual annihilation. This means there is now less mass inside the black hole thanks to the infalling antimatter particle. The pair created near the horizon allow matter to radiate away from the black hole while ensuring that matter is annihilated within the black hole. This is a very simplistic picture of black hole evaporation and, in honour of its discoverer, the emitted particles are known as Hawking radiation.

Black holes were originally in the domain of the massive; a result of general relativity. However, their horizon provides a location in Nature where the typical scales of action are comparable to the Plank scale. This makes black holes a unique system as they produce a 'laboratory' where the strength of gravitational forces can be considered comparable to the strength of other forces affecting particles. As far as a 'lab' scale goes, black holes provide an environment where the predictions of quantum mechanics can be combined with the laws of relativity. This combination is what led to the black hole information paradox. As with the evaporation process, there are multiple ways to understand the information paradox. This thesis has used the 'nice slice' interpretation to explain the paradox. Here, and in the following, 'nice slice' refers to a single spacetime slice (or hypersurface) rather than a family of slices. The term 'nice slice' refers to a slice of spacetime of nearly constant time; essentially a surface of nearly constant time through spacetime. The time of the slice is kept constant outside the black hole and, after passing the horizon, it is kept 'nice' by keeping the slice as smooth as possible while avoiding the singularity for as long as possible. Consider now what this spacetime slice would see as matter falls in and the black hole evaporates. An observer on the nice slice would see the in-falling matter crossing into the black hole. It would *also* see the radiation emitted by the black hole as a result of this in-falling matter. If Alice is imagined to jump into a black hole, the nice slice outside the black hole would see Alice hit the horizon and be emitted as radiation. However, the nice slice continues inside the black hole and so it would see Alice heading for the singularity. This implies that there are two versions of Alice on the same spacetime slice: one fallen inside the black hole and one emitted as radiation. This amounts to cloning and it is a violation of quantum mechanics. However, there is no paradox yet as the emitted radiation can be constrained to be in a maximally mixed state. If there is no information in the emitted radiation concerning the matter that fell in, then Alice only exists once on the nice slice. The emitted radiation can be said to hold no information about Alice if it is maximally mixed. Now enters the paradox. Quantum mechanics dictates that if the initial state was pure then the final state must also be pure. In other words, if a pure system enters the black hole, then a pure system must exist when the evaporation process is complete. This enforces the principle that information cannot be lost in a quantum system. Therefore, in order to adhere to the regulations of quantum mechanics, the emitted radiation cannot be allowed to be maximally mixed. But quantum mechanics will also not allow cloning on the same spacetime slice. So the situation is left in paradox, with two options available that both contradict the established rules. Black holes radiate, but *what* do they radiate?

4.2 Horizon Complementarity

Before investigating the final state solution proposed by Horowitz and Maldacena, it is worth exploring the first widely accepted solution to the black hole information paradox: horizon (or black hole) complementarity [24]. The aim of this solution is to ensure that information can escape the black hole and, from the paradox discussion, this seems to entail the cloning of in-falling matter. Horizon complementarity, therefore, sets out to present a scenario in which the cloning of these in-falling state does not amount to a violation of quantum mechanics. Once again, Alice and Bob are called upon to be fearless experimenters in order to illustrate this procedure. Alice bravely falls into the black hole while Bob observes the process as he orbits far from the horizon at a distance which is several times the radius of the black hole. Assuming the black hole is not too young, the radiation of an infalling body is released relatively quickly [47]. This means that from Bob's perspective, although he never sees Alice cross the horizon, he can assume from his knowledge of Physics that Alice is 'thermalised' at the horizon. Bob then sees Alice emitted as pieces of Hawking radiation. This is *not* what Alice experiences. From the perspective of the experimenter crossing the horizon, the space is flat and uninteresting [48]. This means Alice must cross the horizon and experience no ill-effects until she gets closer to the singularity. This illustrates the necessity for cloning states in order for information to escape the black hole. Bob, as a stationary observer positioned some distance away from the horizon, sees Alice emitted as radiation. But Alice exists inside the black hole. Therefore there are two versions of Alice, pre- and post-thermalisation, that could exist on the same spacetime slice. If Bob can retrieve no information about Alice from the Hawking radiation, then information is lost in the black hole. So either Alice will not experience flat space or Bob will not retrieve any information. To resolve this, horizon complementarity asserts that cloning can occur on the horizon without producing a paradox. If Alice is cloned at the horizon, one copy falls in to experience the singularity and one copy is thermalised on the horizon and collected by Bob as radiation. Horizon complementarity maintains that this does not lead to a paradox as Alice and Bob can no longer communicate their experiences to each other once Alice has crossed the horizon. Alice certainly cannot communicate with Bob from inside the black hole and so there is little to worry about there but Bob could collect the information about Alice, in the form of radiation, and jump into the black hole after Alice. If Bob could receive a message from Alice after entering the black hole, he would experience two copies of the same system and this experience would violate quantum mechanics. It seems like a problem for horizon complementarity but there is an argument in which it is made clear that Alice simply could not have enough energy to send such a message to Bob while still fitting inside the black hole.

In other words, Alice would have to have more energy than the black hole in order to send a message to Bob before she hits the singularity [49]. This ensured neither experimenter could communicate with the other after Alice falls into the black hole and, therefore, no paradox can be observed by either experimenter even though Alice is cloned on the horizon. This encapsulates the idea of horizon complementarity. Since the paradox is only applicable when a global view of the situation is taken, the requirements of each experimenters' description of events is restricted to only what they observe individually and Nature prevents them from observing a paradoxical situation. This means Alice may experience flat space and Bob sees Alice thermalised and, even though these are contradictory events, there is no paradox if Alice and Bob cannot share their experiences with each other. In this way horizon complementarity allows information to escape the black hole while preventing any experience of paradox for the systems involved. As compelling as this argument is, it has not been without complications.

4.3 The Firewall Problem

One such complication is the worry that the principle of quantum monogamy may be violated during the process of retrieving information from black holes. This relates to 'old' black holes; aged older than the Page time which refers to the half-life of the black hole in terms of entropy units [50]. The concern over quantum monogamy was raised by Mathur in [25, 26, 27] and similar concerns were discussed by Braustein *et al* [28, 51, 52] and Itzhaki [29] among others. However, it was the arrival of a recent paper, commonly referred to as the AMPS paper after the authors' initials, which sparked a large debate regarding the issue of quantum monogamy violation in black hole evaporation [30]. The AMPS's argument began by identifying the relevant postulates of horizon complementarity:

1 Information is not destroyed by a black hole but is available from the Hawking radiation which is eventually in a pure state after evaporation is complete.

2 An observer outside the black hole would not witness any violations of effective quantum field theory.

3 The horizon of the black hole obeys the equivalence principle such that an observer falling past the horizon would experience nothing unusual to flat space.

The AMPS paper then pointed out that attempting to maintain all three of these assumptions leads to a violation of quantum theory; specifically the requirement of quantum monogamy. In order to understand the AMPS conclusions, consider three subsystems involved in the process of black hole evaporation:

A: Subsystem within the black hole.

B: Hawking radiation emitted late in the lifetime of a black hole.

C: Subsystem of the Hawking radiation emitted early in the black hole's lifetime.

These three systems are each required to have specific relations with one another in order to preserve the postulates that horizon complementarity attempts to reconcile. Subsystem **C** represents the radiation emitted early from the black hole. In order for the Hawking radiation to be found in a pure state, subsystem **C** must be highly entangled with subsystem **B**. These two systems together will eventually form the pure state of the Hawking radiation necessary to preserve the information radiating from the black hole. This correlation between subsystem **B** and **C** is also necessary to ensure quantum field theory is respected by an observer outside the black hole. However, if an observer is falling through the horizon and experiences only flat space then subsystem **B** exterior to the horizon be highly entangled with subsystem **A** which lies inside the horizon. Essentially, the horizon complementarity postulates 1 and 2 require the subsystems **B** and **C** to be highly entangled while postulate 3 requires subsystems **A** and **B** to be highly entangled [32]. This, AMPS pointed out, is a violation of quantum monogamy which demands subsystem **B** be highly entangled with only *one* subsystem at a time. AMPS pointed out that this approach to the black hole information paradox cannot be resolved by horizon complementarity as an observer could access the exterior entangled subsystems as well as the subsystems entangled across the horizon. This access implied that an observer, under the horizon complementarity interpretation, would witness a paradox of quantum monogamy.

The AMPS authors offered a possible solution to this violation of quantum monogamy: an energetic area along the horizon which thermalises any in-falling matter [30]. This solution essentially disregards postulate 3 and suggests that an in-falling observer would *not* experience flat space but would rather be met with a highly energised horizon which they would doubtlessly not survive passing. This notion of a high energy zone on the horizon is commonly referred to as a 'firewall' and, in this case, it is a solution in response to the violation of quantum monogamy. The concept is also referred to as an 'energetic curtain' along the horizon in work done on the concept by Braunstein [51] [52]. But this is not the only possible response to the firewall paradox. In the discussion regarding PST, the possibility of a violation of quantum monogamy also arose. However, the PST procedure proved to be capable of avoiding any violations of quantum monogamy in a scenario also focused on the movement of quantum information. With that in mind, the final state solution to the black hole information paradox is now examined.

4.4 The Final State Solution

The final state solution was introduced by Horowitz and Maldacena [31]. The main concept behind this approach was to impose a final boundary condition within the black hole interior; specifically at the singularity. The role of this boundary condition is to enforce the demand that no information be destroyed by the black hole. Rather than dismissing unitarity as a requirement in the black hole environment and assuming information is lost, the final state solution ensured unitarity was preserved in the black hole evaporation process by implementing their final boundary condition on the system within the black hole horizon. To see how this boundary condition preserves unitarity, the black hole evaporation process is re-examined, again using the heuristic picture of particle creation.

The final state solution considered the particles created by pair production in the vicinity of the black hole's horizon. This pair production of particles is characterised by the Unruh vacuum state, which describes entangled pairs of particles whose correlations are maintained over spacelike distances [23]. As discussed in the explanation of black hole evaporation, these particles follow very different paths once they are separated by the forces at the horizon. One falls into the black hole while the other radiates away from the horizon and each represents a separate system in the final state solution. As well as these two systems given by the vacuum state, Horowitz and Maldacena also introduced the system representing the infalling matter. This system crosses the black hole horizon and falls through towards the singularity.

The next step of the final state solution's argument was considering what befell the systems which crossed into the black hole interior. For clarification in the following discussion, the Unruh vacuum state pair and the infalling matter are labeled as such:

System 1: A part of the infallen matter which possesses the information that must be retrieved in order to ensure unitarity.

System 2: The positive energy half of the Unruh vacuum state pair which eventually radiates away from the black hole as Hawking radiation.

System 3: The negative energy half of the Unruh vacuum state pair which falls into the black hole.

Horowitz and Maldacena suggested that the final boundary condition can be seen as a 'measurement' upon the two infalling systems. This boundary condition therefore acts as a post-selection measurement. The two systems which are affected by the boundary condition are those which cross the horizon: systems 1 and 3. The final boundary condition essentially restricts what state the two infalling systems can be found in when they are 'measured' at the singularity. Horowitz and Maldacena argued that enforcing a specific state at the singularity allows the information contained in system 1 to be sent to system 2 in a procedure similar to quantum teleportation. System 3 undergoes the 'measurement' at the singularity along with system 1 *but* system 3 also remains entangled with system 2 while the latter is close to the horizon. Any measurement on system 3 will therefore disturb system 2. As was seen in the quantum teleportation procedure, a projective measurement on a system in an unknown state along with half of an entangled pair allows the unknown state to be transmitted to the other half of the entangled pair [15]. Utilising this feature of entanglement, Horowitz and Maldacena enforced their final boundary condition on the joint state of system 1 and system 3 at the singularity. This condition not only constrains the joint state of systems 1 and 3 but also affects system 2 at the horizon due to the entanglement correlations of systems 2 and 3. Specifying the relevant final boundary condition for the infalling particles at the singularity forces system 2 into the state of the infalling system 1. In this way the information contained in system 1 is transmitted out of the black hole to system 2 on the horizon which ultimately carries the information away as Hawking radiation.

The similarities between the final state solution and quantum teleportation are striking as Lloyd *et al* pointed out [18]. Both procedures employed an entangled pair which provided a quantum entanglement channel along which information can propagate. In the final state solution, the information held by the infalling matter (system 1) is transferred out via the entanglement channel connecting the two particles created on the horizon (systems 2 and 3). Unlike the quantum teleportation procedure, however, the final state solution does not rely on classically communicating the result of the 'measurement' of systems 1 and 3 to the distant system 2 [31]. In the quantum teleportation procedure Bob had to wait for news of Alice's result to know which unitary transformation to apply to his system to ensure it was in the correct state the experimenters had hoped to teleport. In the final state solution there is no chance of classically communicating the result of a measurement done inside the black hole to a system outside the horizon. But the final state avoided the need of this classical communication and so bears more of a similarity to the PST procedure than standard quantum teleportation. The result of a measurement on system 1 and system 3 is constrained by the final boundary condition and this restriction is designed to ensure that the state of system 1 is transferred to system 2 without any further transformations being necessary. As long as system 1 and system 3 are 'measured' to be in the correct final state, then system 2 must be in the state given by system 1 as it fell into the black hole. There is no need for any communication of the result of system 1 and system 3's interaction.

This assurance that the correct state is transmitted to the boundary and radiated away in system 2 provides a picture in which the firewall dilemma may be circumvented. To start, consider the entanglement across the horizon. The firewall argument points out that entanglement across the horizon and entanglement between early and late radiation is one entanglement too much. However, the final state solution relies on the trans-horizon entanglement in order for the teleportation procedure to function. Without the entangled pairs produced at the horizon the final state solution procedure cannot transmit states from inside the black hole to the outgoing radiation. This entanglement must be there to ensure the state is teleported out. Taking this entanglement as given, the entanglement of the early and late Hawking radiation is considered. If a pure state collapsed to form a black hole, then the Hawking radiation which that black hole evaporated as must also be found to be in a pure state after the black hole completely vanishes. This is accomplished by the final state solution as the infalling states are all sent to the horizon to radiate away via the teleportation and final state selection combination. Any one bit of information falling into the black hole is guaranteed by the post-selection teleportation to be transmitted to the Hawking radiation. Thus the combination of all the information bits making up the Hawking radiation after the evaporation is complete will relate to the combination of all the bits of information which collapsed into the black hole at the beginning of the procedure. If both the black hole and Hawking radiation incorporate the same information then defining one as pure requires that the other must be as well since they encompass the same information. In this way, the final state solution ensures the Hawking radiation is ultimately entangled with itself as each bit of infalling information is successfully transmitted to the Hawking radiation. Lloyd and Preskill offered a description of the final state solution which suggested that the reasoning leading to the firewall assumption may not apply in this scenario as possible violations are considered to be correctable by a quantum gravity theory [32]. As was seen in the PST discussion, combining teleportation and post-selection allows system 3 to be entangled with multiple systems. This does not violate quantum monogamy as the two systems involved are seen to be one and the same system but at different 'times'. That is to say, system 3 is entangled with system 1 and system 2 but system 2 represents system 1 after it has experienced time travel. The final state solution approach therefore circumvents the reasoning which led to the firewall paradox by presenting a case which allows for multiple entanglements and cloned systems without allowing for the problems these concepts usually produce.

As with PST, the final state solution raises concerns regarding the arrow of time for the flow of information out of the black hole. By removing the requirement of classical communication of the result of the 'measurement' of systems 1 and 3, system 2 can be said to be in the transferred state from the moment it propagates away from the horizon. No further interaction occurs regarding system 2 and so it must be in the correct state once it leaves the horizon and the vicinity of its entangled partner. However, this state is only made available to the entangled pair when system 3 interacts with system 1 at the singularity. This suggests that, in order to adhere to local action only, the information held in system 1 must propagate along a quantum channel connecting system 3 in the future (at the singularity) with system 3 in the past (at the horizon). System 3 is 'measured' with system 1 at the singularity and the information propagates to the past where system 3 was in contact with system 2.

The final state solution is very similar to the procedure described by PST and Lloyd *et al* refer to Horowitz and Maldacena's work on the final state solution in their paper on PST [18]. The core of the similarities between these scenarios lies in the combination of post-selection with the quantum teleportation procedure. Even though one approach is applied to the environment surrounding a black hole and the other is not, their respective outcomes are the same in that they allow information to propagate to locations where it was previously thought to be impossible to send any information. PST allows the transmission of information backwards through time while the final state solution ensures information does not just travel back in time but out of black holes as well.

4.5 The ER=EPR Argument

The AMPS paper shook up the standard interpretation of the black hole information paradox. Before the firewall version was published, the commonly accepted view was that horizon complementarity allowed cloning of states at the horizon of black holes without violating quantum mechanics. Once the AMPS paper was examined, however, it was clear that more than 'simple' horizon complementarity was needed in order to avoid the black hole information paradox. AMPS own suggestion, that the horizon was characterised by a firewall, a solution which caused much consternation in the physics community as it left the expectation that a black hole horizon appears as nearly flat space on shaky ground [30].

In an attempt to restore the assumption of approximately flat space at the horizon, Maldacena and Susskind proposed a new interpretation of the firewall argument [34]. As discussed, the AMPS paper presented a scenario involving an evaporating black hole in which quantum monogamy was violated. The systems next to the horizon (but not inside the black hole) would have to be entangled with both the early radiation and the systems inside the horizon in order to satisfy all three assumptions made in the black hole information paradox scenario which led to horizon complementarity. This violation of quantum mechanics led AMPS to discard one of these assumptions; namely, the expectation of flat space experienced by an observer falling past the horizon.

Ensuring that the states inside the horizon, given by A, are entangled with the states outside (but near) the horizon, given by B, requires that the system A must be 'identified' with the distant Hawking radiation (R) which evaporated from the black hole earlier. This identification refers to an entanglement between the systems A and R_B where R_B is a subsystem of R [24]. Without this connection between A and R, the system B can only be entangled with the distant Hawking radiation R and not the system A inside the horizon. What AMPS showed was that assuming a connection between A and R led to a troubling outcome. If A is identified with R then any action on R must affect A. In other words, disturbing R can result in a disturbance in A which would be observed by an infalling observer. An experimenter Alice could affect R and create particles behind the horizon which would be seen by observer Bob as he fell past the horizon [33]. The AMPS argument can then be summarised as follows. The distance between A and R is too great to allow a disturbance at R to affect A. The system R will have radiated away to a point very distant from the black hole and any disturbance at this point does not have time to affect A before Bob could fall past the horizon. If Alice disturbs R, there should be particles for Bob to encounter as he falls past the horizon. AMPS argue that any particles behind the horizon should be there regardless of any disturbance at R since there is no time for the effect of the disturbance to reach A. Whether or not Alice disturbs R, Bob must encounter particles behind the horizon for any black hole. This led AMPS to suggest that a firewall on the black hole horizon is inevitable [33].

Maldacena and Susskind took a different stance on this argument and suggested that what the AMPS paper had in fact proved was that there must be a way for a disturbance at R to affect A [34]. Maldacena and Susskind start from the point that the AMPS's conclusion of a firewall relied on the assumption that there is no possible way for a disturbance at R to propagate to A. This is not necessarily the case however. Maldacena and Susskind argued that the AMPS paper could be seen as a proof that there must be a "short-cut" connecting A and R [33]. If such a short-cut existed, a disturbance at R could propagate through the connection and affect A in time for Bob to observe particles behind the horizon. Maldacena and Susskind's suggestion for a mechanism which would allow for such a shortcut was an Einstein-Rosen bridge, commonly referred to as a wormhole [34]. This wormhole would connect the interior of the black hole (A) with the system it is entangled with, the radiation R. This would allow Alice to disturb R and affect what Bob experienced as he fell past the horizon. Maldacena and Susskind called their idea ER=EPR to emphasise that such a connection via a wormhole relied on the two connected systems being in an entangled state. The interior A must be entangled with the distant radiation R in order for there to be an Einstein-Rosen bridge connecting the two systems [34]. Using this mechanism, the firewall paradox is avoided in the sense that the firewall at the horizon is not required automatically but is rather the result of an interference at R. This interference can influence the states at the horizon despite being so far away due to the wormhole connecting the entangled systems.

The concept that entangled objects can be connected via a wormhole is relevant to the EPR argument in that it allows for actions at Alice's location to disturb Bob's system even though the experimenters are separated by spacelike distances. If the entangled pairs involved in the EPR setup are linked by a wormhole, then an action on one of the pair can be felt by the other. Any disturbance caused by Alice's measurement on her system could be transmitted through such a wormhole to influence the system at Bob's location. By assuming that entangled pairs are linked in this way, the ER=EPR argument provides a mechanism through which the entangled pairs maintain their entangled correlations without requiring the spin directions to be determined when the particles are prepared. Alice's measurement will still result in a probabilistic outcome consistent with quantum mechanics. The wormhole allows the measurement at Alice's location to influence Bob's system over spacelike distances which provides a mechanics for Alice's result to influence Bob's result instantaneously regardless of the distance between them. Bob's system would therefore be influenced by the actions at Alice's location. This essentially describes a mechanism which allows for 'spooky action at a distance' between entangled particles.

This solution to the EPR paradox is very similar to that obtained by applying the PST procedure. In both approaches information regarding Alice's result is propagated along a quantum channel to Bob's location. In the case of ER=EPR, this channel is described as a wormhole. In the PST procedure, Alice's result is transmitted to the particles at the preparation stage. This allows the directions of spin to be set in the past as a result of future boundary conditions. Both approaches maintain the probabilistic nature of quantum mechanics. Alice's result will never be determined before her measurement and the directions of the respective spins are only set as a result of Alice's measurement. Both approaches also provide a mechanism to maintain the correlations seen in quantum entanglement actions. The ER=EPR solutions propagates the disturbance of Alice's measurement along a 'shortcut' through spacetime. The PST procedure propagates the disturbance of Alice's measurement backwards through time to the moment when the systems were initially entangled. So both procedures allow Alice's measurement to influence Bob's result in a manner that appears to violate the restrictions of relativity.

However, this violation of relativity is only apparent. In the case of ER=EPR, an action at one location influences the state of a system far away from the source of influence. This appears to be a form of spacelike communication but is explained through the use of a channel connecting the two relevant systems. The ER bridge, or wormhole, allows for a situation in which it looks like relativity is violated but only when the information is not considered to travel through the 'shortcut' provided. The PST procedure, on the other hand, relies on a quantum channel connecting the future and past systems of one particle. The state of Alice's system in the future propagates backwards to set the state of Alice's system in the past. Bob is free to measure his system from this preparation time onwards and his result will always correlate to Alice's due to the influence of the future boundary condition. This also appears to violate relativity as the cause seems to follow effect. This acausal communication is only apparent from the classical perspective. From the perspective of quantum mechanics, the system should be invariant under time reversal and so acausal influence does not present a problem within the quantum mechanics framework. As was seen in Lloyd *et al*'s work, the PST theory is used to describe travel through wormholes without changing the structure of the procedure. PST, therefore, can be seen to be equivalent to any description of communication through wormholes. This suggests that ER=EPR and the PST procedure are two sides of the same coin with regard to the EPR paradox in that they both provide mechanisms for Alice's actions to influence events at Bob's location regardless of the distances between the observers. The ER=EPR argument begins from the idea that there is a connection between the systems A and R_B. In terms of the EPR paradox, this relates to a 'shortcut' between Alice's system and Bob's system which allows the information regarding correlated properties to be transmitted. The PST procedure, however, began with teleportation and showed that the information may be propagated between the future and past systems, thereby ensuring correlations through local actions only. The interpretation of PST is that the system found in the past is a 'time traveled' version of the system in the future. This connection ultimately has the same result as the connection suggested by ER=EPR. The final state solution uses a technique essentially equivalent to the PST procedure and it provides a mechanism for the interior of a black hole to transmit information to the exterior. This is also achieved by the ER=EPR argument. So, although the ER=EPR approach begins with the 'shortcut' connecting states and works towards the transmission of information, it can be seen to be the same result as the final state solution (and ultimately PST) which begins with a mechanism for the transmission of information and works towards a connection between states.

Both resolutions result in an apparent violation of the rules of relativity for an observer experiencing the classical arrow of time. The use of communication channels in both ER=EPR and the final state solution present a situation in which the appearance of this violation is the result of information propagating through these communication channels. The ER=EPR solution is consistent with the final state solution and so also consistent with the PST resolution to the EPR paradox. The use of quantum channels to transmit information in both procedures and the appearance of a violation of relativity in both cases strongly suggests that, further than simply being consistent, these two approaches are really the same procedure. The difference lies in the starting point for each argument. ER=EPR begins with the notion of a wormhole and later shows this to be responsible for transmission of information while PST begins with a quantum procedure to transmit information and later equates this to the use of wormholes. This suggests that the conclusion should be ER=EPR=PST.

CHAPTER 5

An Observation of Complementarity

"[The black hole] teaches us that space can be crumpled like a piece of paper into an infinitesimal dot, that time can be extinguished like a blown-out flame, and that the laws of physics that we regard as sacred, as immutable, are anything but."
- John Wheeler

The EPR argument's conclusion relies heavily on the assumption that hypothetical results can be compared with actual results. By assuming the outcome is set regardless of whether a measurement is preformed or not the EPR authors assumed predetermination or counterfactual definitiveness (CFD) [20]. This assumption is also present in Bell's reformulation of EPR. Norsen showed that the concept of Bell locality necessarily includes CFD [11]. For earlier discussions on Bell locality, see [6]. This was shown to be the case for stochastic formulations of Bell's inequality in the Clauser-Horne-Shimony-Holt reformulation [53]. Even when extended to probabilistic theories, CFD is maintained although in a weaker form known as counterfactual meaningfulness [7]. Rather than being a single assumption of locality based on only the principle of relativity, the Bell locality assumption included the assumption of CFD within it. This is pointed out by Wiseman in [20]. Not only must Alice's measurement not effect Bob at his distant location but Bob's actual result must be compared to a hypothetical result had he chosen another direction to measure. This is the application of the idea that any possible measurement has a set outcome which may be discussed regardless of whether the measurement was actually performed. Although this may seem to be a reasonable assumption, it is the basis for concern for some; most notably Bohr who suggested that considering hypothetical outcomes against actual results was an indescribable situation in quantum mechanics [5]. For a good summary of this discussion, see [3]. The development of observer complementarity lies in favor of Bohr's viewpoint.

Observer complementarity was the result of expanding the concept of horizon complementarity to apply to all theoretical approaches. Horizon complementarity restricts what an observer on either side of a black hole horizon must account for [24]. Essentially, horizon complementarity allowed the cloning of states across the horizon without allowing for a paradox. This is achieved by ensuring that observers only account for systems in their local vicinity. Observer complementarity extends this view to all theories by asserting that *any* observer should only account for local events in their vicinity. This can be tied to Bohr's objection to the EPR argument as a hypothetical result occurs in no observer's region since it does not occur at all. By observer complementarity, therefore, a hypothetical result cannot be meaningfully discussed by any one. To illustrate this alternative to CFD, the EPR experiment is considered from the perspective of observer complementarity.

5.1 Observer Complementarity and EPR

Consider the measurement made by Alice in the EPR setup. The EPR authors argue that, since Alice's result allows her to predict a value for Bob's result with certainty, the system at Bob's location must have an element of reality to correspond with the prediction. This leads them to the conclusion that all spin directions must be determined from the start of the experiment. However, this is not the conclusion when observer complementarity is applied. Alice's result allows her to predict Bob's result but her prediction has no bearing on reality until there is a local interaction between Alice and the system at Bob's location. Although Alice can predict Bob's result it remains a prediction only and not a reality until an actual experiment is performed and verified by Alice. Should Bob perform a measurement which verifies Alice's prediction it would still require some form of classical communication for Alice to verify the result. Since Alice can only account for her local system, her prediction does not allow her to verify any element of reality, as previously defined, at Bob's location. Observer complementarity simply does not allow Alice to draw any conclusions regarding Bob's reality from her spacelike separated location. From this viewpoint the next step of the EPR argument cannot take place. Considering a hypothetical result for Bob's measurement is what led the EPR authors to conclude that quantum mechanics is incomplete and Bell to conclude that Nature is nonlocal according to the Bell locality definition as explained by Wiseman [20]. However, without the hypothetical result this is a moot point. Therefore, from the perspective of observer complementarity, quantum mechanics is complete in its description. This supports Bohr's position that imagined results can not be described alongside actual results in quantum mechanics. The quantum mechanical description of the EPR experiment is complete as it accounts for each observers' local system. At both Alice's and Bob's location quantum mechanics provides a complete description of the respective systems. This approach also preserves the correlations of the entangled state. Although Alice and Bob only account for local actions, it can be shown that the overall correlations are maintained by considering a third observer who checks both Alice and Bob's results. This was verified in previous work [54].

Observer complementarity provides a way of removing the concerns raised in the EPR paradox. However, it is at odds with the use of CFD. Arguing for the use of observer complementarity is essentially an argument against predetermination. Which should be given preference? Observer complementarity can fall back on its origin for support in this case. The original form of complementarity arose out of a theory of quantum gravity which should be considered as fundamental. Seeing aspects of a fundamental theory materialise in other theories that are not considered fundamental is expected [55]. Although this thesis ignores gravity throughout, the claim is that the results of a fundamental theory could show up in all aspects of the theory. The use of observer complementarity in areas other than those dealt with by a theory of quantum gravity is justified if it can be used to resolve a paradoxical situation. Therefore observer complementarity can be considered preferable to the classical notion of predetermination or CFD since observer complementarity arises from a fundamental theory and resolves a paradox rather than producing one. Observer complementarity is the preferable perspective to take in the EPR experiment. By itself, however, observer complementarity does not account for how the correlations are maintained. At this point the PST theory can be used to save the day. As was seen, applying PST to the EPR setup allowed the information regarding the spin directions to be propagated between the entangled particles without resorting to nonlocal action. The results of Alice's measurement influenced the entangled pair before they were separated. PST allowed the future to influence the past and so preserves the correlations between the entangled particles no matter how far they are taken from each other.

It could also be argued that using observer complementarity as a starting point leads to the PST interpretation of the EPR paradox. Consider the relevant theories which influence the EPR setup. Firstly, the principle of relativity is considered. This principle enforces the idea that Alice can only report on local interactions until she has classically communicated with Bob. From this viewpoint Alice has access to two interactions in the EPR setup. One: the preparation of the entangled state to be shared between Alice and Bob, and two: the measurement Alice performs at her spacelike separated location. Any action at Bob's location once the experimenters are separated is excluded. This is consistent with the prescription given by observer complementarity. Second, the predictions of quantum mechanics are taken into account. The EPR authors focused on the correlations enforced by quantum mechanics. However, this requires discussing Bob's result from Alice's locations. Since observer complementarity states that Alice can only account for actions at her location, the focus should be on her result. As quantum mechanics dictates, Alice's actual result is only one out of a possibility of outcomes. That is to say, Alice's outcome is a probabilistic result rather than a predetermined one; she will have an equal chance of measuring a spin up as she does of measuring a spin down. Quantum mechanics, therefore, seems to require that the result of Alice's measurement not be set in stone in any sense but only guaranteed once the measurement is performed. This, again, is consistent with observer complementarity. Alice must have a probabilistic result since she cannot yet know Bob's result. With these two expectations, the entire setup is now considered. Observer complementarity leads to the conclusion that Bob cannot affect Alice and that Alice's result must obey the probabilistic nature of quantum mechanics. What then of the correlations also dictated by quantum mechanics? How could a probabilistic result possibly correspond to a result predicted with certainty due to the correlations of entanglement?

The only systems Alice has access to are her measurement and the initial preparation. Therefore, similarly to the EPR conclusion, the correlations must be enforced at the time when the entangled system is prepared. In order to respect relativity, the two systems can only interact at this time when they are entangled together. The EPR took this to mean the spins are set before the measurement as this is the last time the systems interact. However, this is not the only interpretation. In order to maintain the quantum mechanical situation of probabilistic results, Alice's measurement must be responsible for the spin results in the direction she chooses to measure. Alice's result must only be determined when she actually measures her system. However, this result occurs in the future of the preparation event. Alice and Bob can only measure their systems *after* they have been entangled and separated. It is at this point that the PST interpretation enters. If Alice's action at her measurement position must set the systems' spin and the system can only interact in the past of this measurement, then the effects of this measurement must be propagated backwards through time to allow a measurement in the future to determine a state in the past. PST accomplishes this and maintains not only the requirement of relativity and the spin correlations but also the probabilistic nature of quantum mechanics. This conclusion was reached using the observer complementarity perspective as a starting point and so, in some sense, observer complementarity implies the need for PST in the EPR paradox.

5.2 Observer Complementarity and Black Holes

This thesis has argued that the ER=EPR argument and the final state solution produced the same resolution to the black hole information paradox. Even though they appear to be two separate solutions to the paradox, their use of quantum channels to allow the transmission of information results in the same conclusion but based on different starting points. The question remains how the two interpretations are arrived when both attempt to solve the same problem and why it looks like there are two solutions when only one solution can be the correct solution to the paradox. This is where observer complementarity comes to the rescue.

To answer this, the argument which led to horizon complementarity is reconsidered. As stated before, observer complementarity is simply the expansion of horizon complementarity to situations other than black hole horizons. Therefore, assuming horizon complementarity is equivalent to assuming observer complementarity and this is the starting point for the following discussion. The relevant postulates resulting from assuming complementarity, as described in the discussion of the AMPS paper, produce certain restrictions on the observers in and around the black hole. Notably, they must experience approximately flat space as they cross the horizon. The assumption of observer complementarity also states that they need not account for any system beyond their own horizon of experience. With this assumption in hand, Alice is considered first. As in the original discussion, Alice falls into the black hole. From her perspective, the space around the horizon appears flat and she does not become thermalized as she crosses the horizon boundary. This means she will see the entanglement of positive and negative energy particle pairs on her way in. This slightly concerns Alice, as the emitted (positive) particles appear not to carry any information regarding the infalling state out of the black hole and so Alice thinks the information regarding the infalling state will be lost to Bob outside the black hole. For the sake of this argument, Alice is assumed to be a hypothetical being capable of surviving the tidal forces inside the black hole. This allows Alice to remain whole as she nears the singularity where she sees the negative energy particles annihilating with particles of matter that has fallen into the black hole. Her knowledge of quantum teleportation allows her to conclude that this annihilation event could act as a measurement which teleports the infallen state to the entangled partner outside the black hole. Alice performs some calculations as she falls and concludes that the 'measurement' of the negative energy particle and the infallen matter particle must be in a specific final state to allow the information to be teleported to the horizon where it will be carried away by the Hawking radiation. This method causes Alice brief consternation as she recognises this is sending information backwards in time. However, observer complementarity comes to the rescue and Alice realises that she has no access to the outgoing particle in the past and so need not account for its state after she had fallen in. Alice then, from inside the black hole, happily concludes that the final state solution saves the information while preserving all relevant theories and she heads towards the singularity.

But Bob is still outside the black hole. He has seen Alice thermalized and is now concerned that the expectation of approximately flat space at the horizon is incorrect. He collects the early and late Hawking radiation and verifies that it is indeed entangled. This means, in order not to violate monogamy of entanglement, there should be no entangled pairs across the horizon and so no experience of flat space. How does Bob resolve this problem? Bob decides that there must be a way for his action on the early radiation to affect the states inside the black hole. If the effect of his action of this radiation is felt in the interior of the black hole, he can expect to be thermalized when he follows Alice. So long as there is no effect on the radiation, the horizon will appear flat but once Bob interferes and measures the entanglement then the horizon must act like a firewall and thermalize him as he enters. To this end, Bob concludes that a wormhole must allow the effect of his action to propagate to the interior and produce the required result on the horizon. He need not account for Alices experience of flat space as he cannot ever communicate his results to her and vice versa. Bob cheerfully assumes ER=EPR and heads towards the firewall he caused with his disturbance.

This illustrates how each observer arrives at a different interpretation of the solution based on their vantage point in the proceedings. If Alice and Bob could compare results they would each argue that what the other had experienced was wrong or that they had been misled. Alice would tell Bob that the horizon is flat; no monogamy is violated thanks to the teleportation of states. Bob, on the other hand, would tell Alice that the violation of monogamy is only avoided thanks to his handy wormhole connecting the interior and exterior of the black hole. This discussion, however, would never occur. Alice and Bob cannot ever compare results and so are they left to accept their own interpretations of the solution. The arguments of ER=EPR and the final state solution produce the same result but it is the perspective taken, inside or outside the black hole, that leads to one interpretation being preferred over the other. Both allow the interior and exterior of the black hole to transmit information between them but in the final state solution the ‘measurement’ sends information outwards from the interior to the horizon of the black hole and in the ER=EPR argument the information travels into the black hole from a disturbance of the Hawking radiation. This further suggests that ER=EPR and the final state solution can be considered to be the same solution to the black hole information paradox and the application of observer complementarity provides an explanation as to why the solutions look different. As the final state solution relies on the same procedure as PST, this can be encapsulated by ER=EPR=PST.

It should be noted that the idea of information propagating backwards in time may be difficult to swallow. The world of classical experience only allows for movement through time in a single direction: forward. However, the quantum world continually forces a reassessment of many classical notions that have previously been taken for granted. It is not enough, therefore, to say an idea is not what is classically expected and so should be discarded. Although the classical experience involves an arrow of time, the quantum experience may have no such restriction. As this interpretation of time-traveling information within the observer complementarity perspective allows the EPR paradox to be resolved, it should not be discarded on the grounds that it jars with classical notions. As long as this solution to the EPR paradox does not produce paradoxes of its own, there is no reason to discard it offhand in favor of classical preconceptions.

CHAPTER 6

Conclusion

"How wonderful that we have met this paradox. Now we have some hope of making progress."
- Niels Bohr

The EPR paradox began a debate which is still under way. The focus of the paradox was the correlations of entangled particles and the apparent ability for such entangled pairs to communicate over vast distances in an instantaneous manner. The EPR paradox involved a setup in which Alice and Bob share an entangled pair of particles in order to investigate the properties of entanglement. The EPR authors used the result of this setup to claim that quantum mechanics is incomplete since relativity dictates that systems cannot influence each other instantaneously over great distances. Although the EPR authors argued that the only way to resolve the paradox is to accept that there are hidden variables not accounted for in quantum mechanics, the disagreement over this conclusion led Bell to reformulate the EPR setup in mathematical terms. Bell provided an inequality which was to settle the matter of the paradox. The consideration of the EPR paradox using Bell's inequality contradicted the EPR authors' conclusion and suggested a nonlocal form of Nature. This fueled the debate surrounding the topic of entanglement in quantum mechanics and the manner in which entangled systems maintained their correlations. Bell's conclusion was that quantum mechanics accounted for all possible variables and the solution to the paradox was to accept the nonlocal character of the Nature.

In assessing the EPR paradox, the problem was identified to be how to explain the movement of information between entangled systems when they are too far apart to communicate instantaneously. With this in mind, the quantum teleportation procedure was reviewed as a method of transferring information using entangled pairs of particles. However, while the quantum teleportation procedure employed entangled systems to transmit information, the only explanation for this movement of information was that the procedure utilised a quantum channel connecting the entangled pair. In order provide an explanation of how entangled systems transmit information, a version of quantum teleportation was investigated which combined the standard quantum teleportation procedure with the concept of final state selection.

Final state selection consisted of enforcing a final boundary condition which affects the system in the past. This raises concerns regarding causality in the classical realm where the arrow of time dictates that effect must follow cause. The quantum realm, however, does not have the restriction of a prescribed arrow of time and so considering the selection of final states is allowable in quantum procedures. Lloyd *et al* used this feature of quantum mechanics to combine final state selection with the quantum teleportation procedure and create their theory of post-state teleportation (PST). This provided them with a framework to describe a quantum state before and after it has journeyed through a relativistic wormhole. Although explaining this form of quantum time travel was the aim of Lloyd *et al* theory, their result was also applicable to non-relativistic circumstances. Specifically, the combination of quantum teleportation with final state selection led to a form of time travel for quantum states without employing a wormhole. This led Lloyd *et al* to conclude that the quantum channel transmitting information must be connecting a future system with its past self. Although PST can be used to describe time travel through a wormhole, the wormhole is not necessary to achieve time travel. The PST procedure of simply combining final state selection and quantum teleportation produced a time traveling state. The quantum channel allowed a future boundary condition to influence a system in the past and, in the teleportation procedure, transmit an unknown state to Bob before Alice receives said state to send to Bob. The suggested interpretation was that entanglement allowed the measurement act in the future to affect the initial state of the entangled pairs in the past.

This interpretation allowed for an explanation of how the entangled systems in the EPR paradox could be transmitting information in an apparently instantaneous manner regardless of the distance between the systems. If Alice's measurement in the EPR setup is regarded as a final boundary condition, the PST interpretation can be used to explain how Bob's particle is aware of Alice's result as soon as Alice measures her system. The ability for Alice's state in the future to influence her system in the past means the information regarding her outcome can be transmitted to a point where her particle is in close contact with Bob's particle. The result of applying PST reasoning to the EPR paradox is a description which maintains the completeness of quantum mechanics while only using local actions to enforce correlations between the entangled pairs. Alice's measurement sets her particle's properties from the time of measurement all the way back to when the particle's initial state was prepared in the past. By setting properties for Alice's particle in the past through PST, Bob's particle can be made aware of Alice's outcome by local action only. When (in the past) the particles are still close enough for local action, the effect of Alice's measurement propagates backwards in time to set both particles' states. The explanation of how information propagates in entangled systems could therefore be that the systems untilise time travel to allow future condition to influence past states. From the classical perspective this would look like instantaneous action over spacelike distances and appear to violate the rules enforced by relativity. However, the quantum perspective which allows for time travel is aware that there is only local action involved in the communication of entangled systems. The possibility of a time travel paradox occurring is taken care of within the PST theory. The procedure ensures that consistency is always maintained thanks to the feature of probability within quantum mechanics. The possibility of communicating faster than the speed of light is also eliminated as each experimenter must corroborate the other's outcome before their result could be used to transmit a message.

In recognition of the difficulty in accepting a interpretation involving time travel, another paradox involving quantum information was explored to illustrate the use of time travel-like solutions in other situations. The paradox in question regards the apparent destruction of information after it has fallen into black holes: the black hole information paradox. Quantum mechanics dictates that the information must be recoverable while the black hole evaporation procedure seems to suggest the infallen information is lost forever. This problem was once resolved by horizon complementarity which allowed states to be cloned at the horizon of the black hole provided neither side of the black hole was witness to both copies. This solution was put under fire with the recent claims that it breaks the requirement of monogamy of entanglement. Two solutions to this problem were considered in this thesis.

The first, the final state solution, appeared to be an application of the PST procedure as it involved final state selection as well as a quantum teleportation-like channel. The final state solution recovered the information apparently lost in the black hole by providing a mechanism which transmitted the information from inside the black hole to the particles in the horizon. The information appeared to travel a path which propagated backwards in time from the singularity in the future to the horizon in the past in a manner similar to that described in the PST procedure. The problem of violating monogamy could also be avoided by the final state solution and so the recent 'firewall' problem is not an issue. Although the final state solution predates the PST procedure, they both employ the same components to achieve a similar result and could be viewed as applications of the same theory. The resolution of the EPR paradox through the PST procedure is therefore similar to the resolution of the black hole information paradox through the final state solution.

A second solution to the black hole information paradox which was considered was the ER=EPR argument. This arose as solution to the problem of monogamy violation in the black hole environment. Where the 'firewall' arguments suggested the horizon be inhospitable in order to maintain the correct amount of entanglement, the ER=EPR argument suggested that the problem of violating monogamy actually suggested that there be a communication channel connecting the interior of the black hole to distant radiation. This meant that should an observer witness the exterior entanglement, his action on it would be transmitted to the interior to ensure a break in entanglement across the horizon and so produce the 'firewall'. However, for those not witness to the exterior entanglement, there would be no effect to be transmitted and the entanglement across the horizon would remain. The mechanism allowing for this transmission was identified as an Einstein-Rosen (ER) bridge or wormhole and it was suggested that entanglement is a requirement for such a channel; hence ER=EPR. This allows for an explanation for the correlations of entangled systems such as those in the EPR paradox. If entangled systems are connected by a wormhole, a disturbance on one could be felt by the other in what would appear to be an instantaneous manner. The disturbance would, however, be using the wormhole as a shortcut to maintain the correlations through 'local' action.

The fact that the ER=EPR argument and the final state solution both resolve the monogamy violation in the black hole information paradox is a problem as only one solution can be correct. The discussion of observer complementarity proved to be the key in finding the correct solution.

Observer complementarity refers to the application of horizon complementarity to situations other than black hole horizons. By observer complementarity, every observer is only responsible for describing systems within their own experience and not those of a distant observer. In terms if the EPR paradox and Bell's theorem, the use of observer complementarity reduces both arguments to none starters. In both EPR and Bell's theorem, the outcome of local experiments are compared to the results of spacelike separated experiments as well as the results of hypothetical measurements. This comparison is not allowable under observer complementarity as neither distant nor hypothetical experiments occur within the experience of the local observer. The feature which allowed this comparison in the EPR paradox and Bell's theorem was the counter-factual definitiveness (CFD) or predetermination assumption which is built into both arguments. Observer complementarity, however, is in direct contradiction with the principle of CFD and so a choice must be made between them. The origin of observer complementarity is relied on in order to motivate a choice between observer complementarity and CFD. Since observer complementarity arose from a fundamental theory of gravity, it could be expected to apply to other areas. It is preferable due to its ability to resolve paradoxical situations without producing more paradoxes. CFD, meanwhile, produces the result of nonlocal action which is of concern to those keeping under the speed limit of light. Adopting observer complementarity in the EPR paradox resolves the problems of correlations between the observers and PST provides an explanation of how the correlations are maintained without nonlocal action. An argument was made that assuming observer complementarity and the principles of relativity leads one to the PST resolution of the EPR paradox.

The question remained open over which argument was the correct solution to the black hole information paradox. The answer lay in assuming observer complementarity and reassessing the situation leading to the information paradox and the problem of violating quantum monogamy. The conclusion was that observer complementarity provided a perspective in which both the ER=EPR argument and the final state solution were describing the same solution only from different viewpoints. Both ER=EPR and the final state solution (and so PST) present a communication channel which is disguised behind apparently paradoxical behavior that appears to contradict relativity. The contradictions are only apparent, however, and the channel provides a means to maintain entanglement correlations without violating the principles of relativity. The use of observer complementarity, therefore, allows for a resolution of the black hole information paradox as well as the EPR paradox.

This thesis has investigated a perspective which allows for a description of how entangled systems may transmit information between themselves. Although the use of time travel in the solution may be difficult to accept, the paradoxes of unlikely lottery winners or prematurely dead grandfathers are avoided within this description. It is only the classical notions of how Nature should behave which suggest it should be dismissed and quantum mechanics has worked hard to prove how wrong preconceived classical notions can be in this Universe.

APPENDIX A

Joint State in Teleportation Procedure

The initial joint state is given by $|\psi\rangle_1 |\Psi^-\rangle_{23}$ [18]. When working through this calculation, an error was discovered regarding a missing minus sign. In order to arrive at the rewritten equation, the starting point was taken to be

$$- |\psi\rangle_1 |\Psi^-\rangle_{23} . \tag{A.1}$$

For an explicit calculation, the unknown state $|\psi\rangle$ is represented by

$$|\psi\rangle \;=\; \frac{1}{\sqrt{2}}(a\,|0\rangle + b\,|1\rangle), \tag{A.2}$$

where the notation is consistent with that defined previously in this thesis. The states $\{\Psi^{\pm}, \Phi^{\pm}\}$ giving the Bell basis are defined earlier in the thesis. Using the Bell basis and (A.2), the expression given by (A.1) are expanded to

$$\begin{aligned} - |\psi\rangle_1 |\Psi^-\rangle_{23} &= -\frac{1}{\sqrt{2}}\Big(a\,|0\rangle + b\,|1\rangle\Big)_1 \Big(\,|01\rangle - |10\rangle\,\Big)_{23} \\ &= \frac{1}{\sqrt{2}}\Big(-a\,|0\rangle - b\,|1\rangle\Big)_1 \Big(\,|01\rangle - |10\rangle\,\Big)_{23}. \end{aligned}$$

Multiplying the brackets out on the left hand side results in

$$\begin{aligned} - |\psi\rangle_1 |\Psi^-\rangle_{23} &= \frac{1}{\sqrt{2}} \cdot \frac{2}{2}\Big(a\,|0\rangle_1 |10\rangle_{23} - a\,|0\rangle_1 |01\rangle_{23} + b\,|1\rangle_1 |10\rangle_{23} - b\,|1\rangle_1 |01\rangle_{23}\Big) \\ &= \frac{1}{\sqrt{2}} \cdot \frac{2}{2}\Big(a\,|010\rangle - a\,|001\rangle + b\,|110\rangle - b\,|101\rangle\Big)_{123} \\ &= \frac{1}{2\cdot\sqrt{2}}\Big(-2a\,|001\rangle + 2b\,|110\rangle - 2b\,|101\rangle + 2a\,|010\rangle\Big)_{123}. \end{aligned}$$

The terms within the brackets is then expanded to

$$
\begin{aligned}
-\left|\psi\right\rangle_1 \left|\Psi^-\right\rangle_{23} = \frac{1}{2\cdot\sqrt{2}}\Big(& -a\left|001\right\rangle + a\left|100\right\rangle - b\left|011\right\rangle + b\left|110\right\rangle \\
& -a\left|001\right\rangle - a\left|100\right\rangle + b\left|011\right\rangle + b\left|110\right\rangle \\
& +b\left|000\right\rangle - b\left|101\right\rangle + a\left|010\right\rangle - a\left|111\right\rangle \\
& -b\left|000\right\rangle - b\left|101\right\rangle + a\left|010\right\rangle + a\left|111\right\rangle \Big)_{123}
\end{aligned}
$$

$$
\begin{aligned}
-\left|\psi\right\rangle_1 \left|\Psi^-\right\rangle_{23} = \frac{1}{2\cdot\sqrt{2}}\Bigg[& -\Big(a\left|001\right\rangle - a\left|100\right\rangle + b\left|011\right\rangle - b\left|110\right\rangle\Big) \\
& -\Big(a\left|001\right\rangle + a\left|100\right\rangle - b\left|011\right\rangle - b\left|110\right\rangle\Big) \\
& +\Big(b\left|000\right\rangle - b\left|101\right\rangle + a\left|010\right\rangle - a\left|111\right\rangle\Big) \\
& +\Big(-b\left|000\right\rangle - b\left|101\right\rangle + a\left|010\right\rangle + a\left|111\right\rangle\Big)\Bigg]_{123}.
\end{aligned}
$$

The states for the three systems are then separated out as follows

$$
\begin{aligned}
-\left|\psi\right\rangle_1 \left|\Psi^-\right\rangle_{23} = \frac{1}{2\cdot\sqrt{2}}\Bigg[& -\Big(\left|01\right\rangle - \left|10\right\rangle\Big)_{13}\Big(a\left|0\right\rangle + b\left|1\right\rangle\Big)_2 \\
& -\Big(\left|01\right\rangle + \left|10\right\rangle\Big)_{13}\Big(a\left|0\right\rangle - b\left|1\right\rangle\Big)_2 \\
& +\Big(\left|00\right\rangle - \left|11\right\rangle\Big)_{13}\Big(b\left|0\right\rangle + a\left|1\right\rangle\Big)_2 \\
& +\Big(\left|00\right\rangle + \left|11\right\rangle\Big)_{13}\Big(-b\left|0\right\rangle + a\left|1\right\rangle\Big)_2\Bigg].
\end{aligned}
$$

Multiplying through by the $\frac{1}{\sqrt{2}}$ term allows the terms associated with systems 1 and 3 to be rewritten using the Bell basis, which results in

$$
\begin{aligned}
-\left|\psi\right\rangle_1 \left|\Psi^-\right\rangle_{23} = \frac{1}{2}\Bigg[& -\left|\Psi^-\right\rangle_{13}\Big(a\left|0\right\rangle + b\left|1\right\rangle\Big)_2 - \left|\Psi^+\right\rangle_{13}\Big(a\left|0\right\rangle - b\left|1\right\rangle\Big)_2 \\
& +\left|\Phi^-\right\rangle_{13}\Big(b\left|0\right\rangle + a\left|1\right\rangle\Big)_2 + \left|\Phi^+\right\rangle_{13}\Big(-b\left|0\right\rangle + a\left|1\right\rangle\Big)_2\Bigg].
\end{aligned}
$$

Using the Pauli spin matrices given by

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{A.3}$$
$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

the terms associated with system 2 are rewritten as

$$-\left|\psi\right\rangle_1 \left|\Psi^-\right\rangle_{23} = \frac{1}{2}\Bigg[-\left|\Psi^-\right\rangle_{13} \Big(a\left|0\right\rangle + b\left|1\right\rangle\Big)_2 - \left|\Psi^+\right\rangle_{13} \sigma_z \Big(a\left|0\right\rangle + b\left|1\right\rangle\Big)_2 + \left|\Phi^-\right\rangle_{13} \sigma_x \Big(a\left|0\right\rangle + b\left|1\right\rangle\Big)_2 - \left|\Phi^+\right\rangle_{13} i\sigma_y \Big(a\left|0\right\rangle + b\left|1\right\rangle\Big)_2 \Bigg].$$

This is the desired expression as given in Chapter 3.

APPENDIX B

PST: Path Integral Approach

As discussed in the PST chapter, Lloyd *et al*'s theory can be shown to be consistent with the path integral approach to CTCs. The following discussion closely follows the work done in [18]. The transitional amplitude for a CTC is calculated using the 'conventional' path integral method first which remains as conventional as possible while allowing for time travel to the past. The transitional amplitude is then recalculated within the PST framework in order to show that the two are consistent with one another. As in Lloyd *et al*'s calculation, the following overlooks many of the subtleties in defining Hamiltonians and constructing path integrals. The general case, which avoids such ambiguities, is given in Appendix C.

The Path Integral Transitional Amplitude

The transition of a system between an initial state $|I\rangle$ and a final state $|F\rangle$ can be described by a transitional amplitude represented by

$$\langle F| e^{-\frac{i}{\hbar}H\tau} |I\rangle \, . \tag{B.1}$$

By completeness, $\int d\alpha \, |\alpha\rangle \langle\alpha| = 1$ and so (B.1) is rewritten as

$$\begin{aligned} \langle F| e^{-\frac{i}{\hbar}H\tau} |I\rangle &= \int_{-\infty}^{+\infty} dx \; dy \, \langle F| y\rangle \langle y| e^{-\frac{i}{\hbar}H\tau} |x\rangle \langle x| I\rangle \\ &= \int_{-\infty}^{+\infty} dx \; dy \; F^*(y) \langle y| e^{-\frac{i}{\hbar}H\tau} |x\rangle \, I(x), \end{aligned} \tag{B.3}$$

where $F(y)$ and $I(x)$ are written using the wave function notation in which

$$\langle\beta|\,\alpha\rangle = \psi_\alpha(\beta)$$
$$\langle\alpha|\,\beta\rangle = \psi^*_\alpha(\beta).$$

Equation (B.3) is then rewritten as

$$\langle F|\, e^{-\frac{i}{\hbar}H\tau}\,|I\rangle = \int_{-\infty}^{+\infty} dx\; dy\; I(x)F^*(y)\int_x^y \mathcal{D}x(t)e^{\frac{i}{\hbar}S}, \tag{B.4}$$

where $\int_x^y \mathcal{D}x(t)e^{\frac{i}{\hbar}S}$ represents the Feyman path integral and S is the action.

The 'Conventional' Addition of a CTC

In order to allow for a CTC, the spacetime is separated into two parts such that

$$\begin{aligned}\langle F|_C\,\langle F'|\,e^{-\frac{i}{\hbar}H\tau}\,|I\rangle\,|I'\rangle_C &= \langle F|\left[\int dy'dx\,\langle F'|\,y'\rangle\,\langle y'|\,e^{-\frac{i}{\hbar}H\tau}\,|x\rangle\,\langle x|\,I\rangle\right]|I'\rangle\\
&= \langle F|\left[\int dy'dx\;F'^*(y')I(x)\,\langle y'|\,e^{-\frac{i}{\hbar}H\tau}\,|x\rangle\right]|I'\rangle\\
&= \int dx'dy\,\langle F|\,y\rangle\,\langle y|\left[\int dy'dx\;F'^*(y')I(x)\,\langle y|\,e^{-\frac{i}{\hbar}H\tau}\,|x\rangle\right]|x'\rangle\,\langle x'|\,I'\rangle\\
&= \int dx'dy\;F^*(y)\,\langle y|\left[\int dy'dx\;F'^*(y')I(x)\,\langle y'|\,e^{-\frac{i}{\hbar}H\tau}\,|x\rangle\right]|x'\rangle\,I'(x')\\
&= \int_{-\infty}^{+\infty} dxdx'dydy'\;F^*(y)F'^*(y')I'(x')I(x)\,\langle y|\,\langle y'|\,e^{-\frac{i}{\hbar}H\tau}\,|x\rangle\,|x'\rangle\\
&= \int_{-\infty}^{+\infty} dxdx'dydy'\;F^*(y)F'^*(y')I(x)I'(x')\int_{x,x'}^{y,y'}\mathcal{D}x(t)e^{\frac{i}{\hbar}S}.\end{aligned} \tag{B.6}$$

The 'conventional' approach allows the system C to be sent to a previous time while remaining unchanged to accomplish the notion of a system traveling through a CTC. The remaining system represents the chronology-respecting section and this evolves according to standard rules. Achieving this result requires enforcing periodic boundary conditions consistent with the boundaries of the CTC. This results in transitional amplitude for the chronology-respecting section being given by

$$\langle F|\,e^{-\frac{i}{\hbar}H\tau}\,|I\rangle = \int_{-\infty}^{+\infty} dxdx'dydy' I(x)F^*(y)\delta(x'-y')\int_{x,x'}^{y,y'}\mathcal{D}x(t)e^{\frac{i}{\hbar}S}, \tag{B.7}$$

where the δ-function is responsible for fixing the initial and final boundary conditions in the CTC system.

The Path Integal in the PST Framework

Equation (B.7) represents the 'conventional' approach to calculating the the transitional amplitudes for CTC systems. To be consistent with this approach, the PST method must be shown to also produce (B.7). To this end,

$$\langle F|_C \langle \Psi| e^{-\frac{i}{\hbar}H\tau} \otimes \mathbb{1} |I\rangle |\Psi\rangle_C \tag{B.8}$$

is calculated, where $|\Psi\rangle \propto \int dx\, |xx\rangle$ represents a maximally entangled state as given by EPR [1]. As per the PST method, the Hamiltonian only acts on the time traveling system and the first Hilbert space given by $|\Psi\rangle$ [18].

Equation (B.6) is used for the time traveling system and the first Hilbert space(denoted by x and y respectively) such that

$$\langle F|_C \langle \Psi| e^{-\frac{i}{\hbar}H\tau} \otimes \mathbb{1} |I\rangle |\Psi\rangle_C = \left[\int dy\, \langle F|\, y\rangle \langle y|\right] \langle \Psi| e^{-\frac{i}{\hbar}H\tau} \otimes \mathbb{1} \left[\int dx\, |x\rangle \langle x|\, I\rangle\right] |\Psi\rangle_C \,. \tag{B.9}$$

Using the position representation given by $|\Psi\rangle = \int dy\; dz\; \delta(y-z)\, |y\rangle\, |z\rangle$, equation (B.9) is rewritten as

$$\begin{aligned}
&\langle F|_C \langle \Psi| e^{-\frac{i}{\hbar}H\tau} \otimes \mathbb{1} |I\rangle |\Psi\rangle_C \\
&= \int dy\, \langle F|\, y\rangle \langle y| \int dy' dz\; \delta(y'-z)\, \langle y'| \langle z| e^{-\frac{i}{\hbar}H\tau} \otimes \mathbb{1} \int dx\, |x\rangle \langle x|\, I\rangle \int dx' dz'\; \delta(x'-z')\, |x'\rangle\, |z'\rangle \\
&= \int dx dx' dy dy' dz dz'\; F^*(y) I(x) \delta(y'-z) \delta(x'-z')\, \langle y| \langle y'| \langle z| e^{-\frac{i}{\hbar}H\tau} \otimes \mathbb{1} |x\rangle |x'\rangle |z'\rangle \,.
\end{aligned} \tag{B.11}$$

Since the Hamiltonian only acts on the time traveling system and the first Hilbert space, the substitution

$$\langle z| \mathbb{1} |z\rangle \times \langle y| \langle y'| e^{-\frac{i}{\hbar}H\tau} |x\rangle |x'\rangle = \langle z| \mathbb{1} |z\rangle \times \int \mathcal{D}x(t) e^{\frac{i}{\hbar}S},$$

can be made into (B.11) such that

$$\begin{aligned}
&\langle F|_C \langle \Psi| e^{-\frac{i}{\hbar}H\tau} \otimes \mathbb{1} |I\rangle |\Psi\rangle_C \\
&= \int dx dx' dy dy' dz dz'\; F^*(y) I(x) \delta(y'-z) \delta(x'-z')\, \langle z| \mathbb{1} |z\rangle \int \mathcal{D}x(t) e^{\frac{i}{\hbar}S} \\
&= \int dx dx' dy dy' dz dz'\; F^*(y) I(x) \delta(y'-z) \delta(x'-z') \delta(z-z') \int \mathcal{D}x(t) e^{\frac{i}{\hbar}S}.
\end{aligned} \tag{B.13}$$

The expression $\delta(x'-z) = \int dz'\; \delta(x'-z') \delta(z-z')$ allows (B.13) to be simplified to

$$\langle F|_C \langle \Psi| e^{-\frac{i}{\hbar}H\tau} \otimes \mathbb{1} |I\rangle |\Psi\rangle_C = \int dx dx' dy dy' dz \; F^*(y) I(x) \delta(x'-z)\delta(y'-z) \int \mathcal{D}x(t) e^{\frac{i}{\hbar}S}. \quad \text{(B.14)}$$

Similarly, the expression $\delta(x' - y') = \int dz \; \delta(x' - z)\delta(y' - z)$ allows (B.14) to be simplified to

$$\langle F|_C \langle \Psi| e^{-\frac{i}{\hbar}H\tau} \otimes \mathbb{1} |I\rangle |\Psi\rangle_C = \int dx dx' dy dy' \; F^*(y) I(x) \delta(x' - y') \int \mathcal{D}x(t) e^{\frac{i}{\hbar}S}. \quad \text{(B.15)}$$

The equation (B.15) now matches the expression given by equation (B.7) which proves the PST approach is consistent with the ‘conventional’ path integral approach.

APPENDIX C

PST: The General Case

The path integral formulation was extended by Lloyd *et al* to encompass the general quantum evolution case. The description given here of this derivation closely follows Lloyd *et al*'s work in [18].

First, the generic case of quantum evolution is described using the Kraus decomposition [56] such that

$$\begin{aligned} \mathcal{L}[\rho] &= Tr_E[U(\rho \otimes |e\rangle \langle e|)U^\dagger] \\ &= \sum_i \langle i| U |e\rangle \rho \langle e| U^\dagger |i\rangle , \end{aligned} \tag{C.2}$$

where $|e\rangle$ represents the purification or environment's initial state and U represents the unitary operator describing the interaction of the system in the initial state ρ and the environment. The use of the Kraus operator, $B_i \equiv \langle i| U |e\rangle$, allows (C.2) to be simplified to

$$\mathcal{L}[\rho] = \sum_i B_i \rho B i^\dagger \tag{C.3}$$

The nonlinear nature of the PST method results in a different decomposition equation. The scenario of Alice and Bob transmitting states to one another was employed and so the subscript A and B refers states in Alice and Bob's possession respectively while R refers to the reference or purification state. The initial state is take to be $\rho = |\Psi_A^0\rangle \langle \Psi_A^0|$. The initial entangled state is given by $|\Psi\rangle = \sum_{i,j} |\psi_B\rangle |\psi_R\rangle$ while the final entangled state is represented by $|\Psi\rangle = \sum_{k,l} |\psi_A\rangle |\psi_R\rangle$. This results in

$$\mathcal{N}[\rho] = Tr_{A,R}\Bigg(U\left|\Psi_A^0\right\rangle\left\langle\Psi_A^0\right| \times \left[\sum_{i,j}\left|\psi_B^i\right\rangle\left|\psi_R^i\right\rangle\left\langle\psi_B^j\right|\left\langle\psi_R^j\right|\right]U^\dagger \\ \otimes\left[\mathbb{1}_B \otimes \sum_{k,l}\left|\psi_A^k\right\rangle\left|\psi_R^k\right\rangle\left\langle\psi_A^j\right|\left\langle\psi_R^j\right|\right]\Bigg). \tag{C.5}$$

The reference system is then traced out of (C.5) resulting in

$$\mathcal{N}[\rho] = Tr_A\Bigg(\sum_{r'}\left\langle\psi_R^{r'}\right|U\left|\psi_A^0\right\rangle\left\langle\psi_A^0\right| \times \left[\sum_{i,j}\left|\psi_B^i\right\rangle\left|\psi_R^i\right\rangle\left\langle\psi_B^j\right|\left\langle\psi_R^j\right|\right]U^\dagger \otimes \left[\sum_{k,l}\left|\psi_A^k\right\rangle\left|\psi_R^k\right\rangle\left\langle\psi_A^j\right|\left\langle\psi_R^j\right|\right]\left|\psi_R^{r'}\right\rangle\Bigg) \\ = Tr_A\Bigg(\sum_{r'}U\left|\psi_A^0\right\rangle\left\langle\psi_A^0\right| \times \left[\sum_{i,j}\left|\psi_B^i\right\rangle(\delta_{r',i})\left\langle\psi_B^j\right|\left\langle\psi_R^j\right|\right]U^\dagger \otimes \left[\sum_{k,l}\left|\psi_A^k\right\rangle\left|\psi_R^k\right\rangle\left\langle\psi_A^j\right|(\delta_{l,r'})\right]\Bigg). \tag{C.7}$$

Using the fact that $\delta_{r',i}$ and $\delta_{l,r'}$ result in $\delta_{i,l}$, the substitution $r' = i = l$ is made which results in

$$\mathcal{N}[\rho] = Tr_A\Bigg(U\left|\psi_A^0\right\rangle\left\langle\psi_A^0\right| \times \left[\sum_{i,j}\left|\psi_B^i\right\rangle\left\langle\psi_B^j\right|\left\langle\psi_R^j\right|\right]U^\dagger \otimes \left[\sum_{k}\left|\psi_A^k\right\rangle\left|\psi_R^k\right\rangle\left\langle\psi_A^j\right|\right]\Bigg). \tag{C.8}$$

The next step involves tracing out Alice's system to determine what state Bob was left with after the procedure. This gives

$$\mathcal{N}[\rho] = \sum_{a'}\left\langle\psi_A^{a'}\right|\Bigg(U\left|\psi_A^0\right\rangle\left\langle\psi_A^0\right| \times \left[\sum_{i,j}\left|\psi_B^i\right\rangle\left\langle\psi_B^j\right|\left\langle\psi_R^j\right|\right]U^\dagger \otimes \left[\sum_{k}\left|\psi_A^k\right\rangle\left|\psi_R^k\right\rangle\left\langle\psi_A^j\right|\right]\Bigg)\left|\psi_A^{a'}\right\rangle \\ = \sum_{a'}\left\langle\psi_A^{a'}\right|U\left|\psi_A^0\right\rangle\left\langle\psi_A^0\right| \times \left[\sum_{i,j}\left|\psi_B^i\right\rangle\left\langle\psi_B^j\right|\left\langle\psi_R^j\right|\right]U^\dagger \otimes \left[\sum_{k}\left|\psi_A^k\right\rangle\left|\psi_R^k\right\rangle\delta_{i,a'}\right]. \tag{C.10}$$

Since $\delta_{i,a'}$ only gives values for $a' = i$, the substitution $\sum_{a'}\delta_{a',i} = \sum_i$ is used to simplify (C.10) to

$$\mathcal{N}[\rho] = \sum_{i,j}\left\langle\psi_A^i\right|U\left|\psi_A^0\right\rangle\left\langle\psi_A^0\right| \times \left[\sum_{i,j}\left|\psi_B^i\right\rangle\left\langle\psi_B^j\right|\left\langle\psi_R^j\right|\right]U^\dagger \otimes \left[\sum_{k}\left|\psi_A^k\right\rangle\left|\psi_R^k\right\rangle\right]. \tag{C.11}$$

The operator $U^\dagger$ does not act on the system given by R which allows the remaining reference states to be reduced such that

$$\mathcal{N}[\rho] = \sum_{i,j} \langle\psi_A^i| U |\psi_A^0\rangle \langle\psi_A^0| \times \left[\sum_{i,j} |\psi_B^i\rangle \langle\psi_B^j|\right] U^\dagger \otimes \left[\sum_k |\psi_A^k\rangle \delta_{j,k}\right]. \tag{C.12}$$

Similarly to $\delta_{i,a'}$, $\delta_{j,k}$ only gives values for $j = k$ and so the substitution $\sum_k \delta_{j,k} = \sum_j$ is used to simplify (C.12) to

$$\mathcal{N}[\rho] = \sum_{i,j} \langle\psi_A^i| U |\psi_A^0\rangle \langle\psi_A^0| \Big[|\psi_B^i\rangle \langle\psi_B^j| \Big] U^\dagger |\psi_A^j\rangle . \tag{C.13}$$

The terms in (C.13) are then reshuffled in order to group the i and j-terms together such that

$$\begin{aligned} \mathcal{N}[\rho] &= \sum_{i,j} \Big[\langle\psi_A^i| U |\psi_B^i\rangle \Big] |\psi_A^0\rangle \langle\psi_A^0| \Big[\langle\psi_B^j| U^\dagger |\psi_A^j\rangle \Big] \\ &= \Big[\sum_i \langle\psi_A^i| U |\psi_B^i\rangle \Big] |\psi_A^0\rangle \langle\psi_A^0| \Big[\sum_j \langle\psi_B^j| U^\dagger |\psi_A^j\rangle \Big]. \end{aligned} \tag{C.15}$$

The result given in (C.15) is simplified using Kraus operators which results in the expression

$$\begin{aligned} \mathcal{N}[\rho] &= C |\psi_A^0\rangle \langle\psi_A^0| C^\dagger \\ &= C\rho C^\dagger, \end{aligned} \tag{C.17}$$

where $C, C^\dagger$ is the trace with respect to the CTC of U and the initial state $|\psi_A^0\rangle \langle\psi_A^0|$ is given by ρ. From Bob's perspective, without access to systems A and R as they have been traced out, (C.17) implies

$$\begin{aligned} \rho_B &= Tr_{A,R}[\rho_{A,R,B}] \\ &= \sum_{i,j} \langle\psi_A^i| U |\psi_A^i\rangle \Big[|\psi_B^0\rangle \langle\psi_B^0| \Big] \langle\psi_A^j| U^\dagger |\psi_A^j\rangle , \end{aligned} \tag{C.19}$$

which puts Bob in possession of a system in the state ρ.

As the evolution given by (C.19) is nonlinear, a renormalisation of the final state is necessary such that a state ρ interacting with a CTC would transform via

$$\mathcal{N}[\rho] = \frac{C\rho C^\dagger}{Tr[C\rho C^\dagger]}, \tag{C.20}$$

and, in the circumstance where $C = 0$, it is assumed the evolution does not occur.

The result given in (C.20) is equivalent to that described by Hartle in [45] using the decoherent histories framework as well as being of the same form as the final state solution developed by Horowitz and Maldacena in [31].

Bibliography

[1] A. Einstein, B. Podolsky, and N. Rosen, *Can quantum-mechanical description of physical reality be considered complete?*, *Physical review* **47** (1935), no. 10 777.

[2] J. J. Sakurai and J. Napolitano, *Modern quantum mechanics.* Addison-Wesley, 2011.

[3] A. Peres, *Quantum mechanics: concepts and methods*, 1993.

[4] D. Bohm, *Quantum theory.* Courier Corporation, 2012.

[5] N. Bohr, *Can quantum-mechanical description of physical reality be considered complete?*, *Physical review* **48** (1935), no. 8 696.

[6] A. Brandenburger and N. Yanofsky, *A classification of hidden-variable properties*, *Journal of Physics A: Mathematical and Theoretical* **41** (2008), no. 42 425302.

[7] T. Norsen, *Counter-factual meaningfulness and the Bell and inequalities*, *arXiv preprint quant-ph/0606084* (2006).

[8] J. S. Bell *et. al.*, *On the Einstein-Podolsky-Rosen paradox*, *Physics* **1** (1964), no. 3 195–200.

[9] J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics: Collected papers on quantum philosophy.* Cambridge university press, 2004.

[10] T. Norsen, *EPR and Bell locality*, *arXiv preprint quant-ph/0408105* (2004).

[11] T. Norsen, *Bell locality and the nonlocal character of nature*, *arXiv preprint quant-ph/0601205* (2006).

[12] T. Norsen, *Local causality and completeness: Bell vs. Jarrett*, *Foundations of Physics* **39** (2009), no. 3 273–294.

[13] T. Norsen, *John S. Bells concept of local causality*, *American Journal of Physics* **79** (2011), no. 12 1261–1275.

[14] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, *Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels*, *Physical review letters* **70** (1993), no. 13 1895.

[15] M. B. Plenio and V. Vedral, *Teleportation, entanglement and thermodynamics in the quantum world*, *Contemporary physics* **39** (1998), no. 6 431–446.

[16] C. G. Timpson, *The grammar of teleportation*, *The British journal for the philosophy of science* **57** (2006), no. 3 587–621.

[17] Y. Aharonov, P. G. Bergmann, and J. L. Lebowitz, *Time symmetry in the quantum process of measurement*, *Physical Review* **134** (1964), no. 6B B1410.

[18] S. Lloyd, L. Maccone, R. Garcia-Patron, V. Giovannetti, and Y. Shikano, *Quantum mechanics of time travel through post-selected teleportation*, *Physical Review D* **84** (2011), no. 2 025007.

[19] S. Lloyd, L. Maccone, R. Garcia-Patron, V. Giovannetti, Y. Shikano, S. Pirandola, L. A. Rozema, A. Darabi, Y. Soudagar, L. K. Shalm, *et. al.*, *Closed timelike curves via postselection: theory and experimental demonstration*, *arXiv:1005.2219v1 quant-ph* (2010).

[20] H. M. Wiseman and E. G. Cavalcanti, *Causarum investigatio and the two Bell's theorems of John Bell*, *arXiv preprint arXiv:1503.06413* (2015).

[21] S. W. Hawking, *Black hole explosions*, *Nature* **248** (1974), no. 5443 30–31.

[22] S. W. Hawking, *Particle creation by black holes*, *Communications in mathematical physics* **43** (1975), no. 3 199–220.

[23] W. G. Unruh, *Notes on black-hole evaporation*, *Physical Review D* **14** (1976), no. 4 870.

[24] L. Susskind, L. Thorlacius, and J. Uglum, *The stretched horizon and black hole complementarity*, *Physical Review D* **48** (1993), no. 8 3743.

[25] S. D. Mathur, *What exactly is the information paradox?*, in *Physics of Black Holes*, pp. 3–48. Springer, 2009.

[26] S. D. Mathur, *The information paradox: a pedagogical introduction*, *Classical and Quantum Gravity* **26** (2009), no. 22 224001.

[27] S. D. Mathur, *What does strong subadditivity tell us about black holes?*, *Nuclear Physics B-Proceedings Supplements* **251** (2014) 16–21.

[28] S. L. Braunstein, H.-J. Sommers, and K. Życzkowski, *Entangled black holes as ciphers of hidden information*, *arXiv preprint arXiv:0907.0739* (2009).

[29] N. Itzhaki, *Is the black hole complementarity principle really necessary?*, *arXiv preprint hep-th/9607028* (1996).

[30] A. Almheiri, D. Marolf, J. Polchinski, and J. Sully, *Black holes: complementarity or firewalls?*, *Journal of High Energy Physics* **2013** (2013), no. 2 1–20.

[31] G. T. Horowitz and J. Maldacena, *The black hole final state*, *Journal of High Energy Physics* **2004** (2004), no. 02 008.

[32] S. Lloyd and J. Preskill, *Unitarity of black hole evaporation in final-state projection models*, *Journal of High Energy Physics* **2014** (2014), no. 8 1–30.

[33] L. Susskind, *Entanglement is not enough*, *arXiv preprint arXiv:1411.0690* (2014).

[34] J. Maldacena and L. Susskind, *Cool horizons for entangled black holes*, *Fortschritte der Physik* **61** (2013), no. 9 781–811.

[35] C. R. Stephens, B. Whiting, *et. al.*, *Black hole evaporation without information loss*, *Classical and Quantum Gravity* **11** (1994), no. 3 621.

[36] A. Einstein, *Relativity: The special and the general theory.* Princeton University Press, 2015.

[37] Y. Aharonov and L. Vaidman, *Properties of a quantum system during the time interval between two measurements*, *Physical Review A* **41** (1990), no. 1 11.

[38] Y. Aharonov and D. Rohrlich, *Quantum paradoxes: quantum theory for the perplexed.* John Wiley & Sons, 2008.

[39] K. Godel, *An example of a new type of cosmological solutions of Einstein's field equations of graviation*, *Reviews of Modern Physics* **21** (1949), no. 3 447–50.

[40] S. W. Hawking, *Chronology protection conjecture*, *Physical Review D* **46** (1992), no. 2 603.

[41] M. Visser, *The quantum physics of chronology protection*, *The future of theoretical physics and cosmology: celebrating Stephen Hawkings 60th birthday* (2003) 161–175.

[42] D. Deutsch, *Quantum mechanics near closed timelike lines*, *Physical Review D* **44** (1991), no. 10 3197.

[43] E. H. Lieb and M. B. Ruskai, *Proof of the strong subadditivity of quantum-mechanical entropy*, in *Inequalities*, pp. 63–66. Springer, 2002.

[44] H. Araki and E. H. Lieb, *Entropy inequalities*, in *Inequalities*, pp. 47–57. Springer, 2002.

[45] J. B. Hartle, *Unitarity and causality in generalized quantum mechanics for nonchronal spacetimes*, *Physical Review D* **49** (1994), no. 12 6543.

[46] N. Birrell and P. Davies, *Quantum field theory in curved space*, 1982.

[47] P. Hayden and J. Preskill, *Black holes as mirrors: quantum information in random subsystems*, *Journal of High Energy Physics* **2007** (2007), no. 09 120.

[48] L. Susskind, *Twenty years of debate with Stephen*, *The Future of Theoretical Physics* (2003) 330–47.

[49] J. Bekenstein, *Universal bound on the entropy to energy ratio for bounded systems*, *Physical Review D* **23** (2000) 287.

[50] D. N. Page, *Average entropy of a subsystem*, *Physical review letters* **71** (1993), no. 9 1291.

[51] S. L. Braunstein, S. Pirandola, and K. Życzkowski, *Better late than never: information retrieval from black holes*, *Physical review letters* **110** (2013), no. 10 101301.

[52] S. L. Braunstein, *Black hole entropy as entropy of entanglement, or its curtains for the equivalence principle*, *arXiv preprint arXiv:0907.1190* **1** (2009).

[53] J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, *Proposed experiment to test local hidden-variable theories*, *Physical review letters* **23** (1969), no. 15 880.

[54] K. Bryan and A. Medved, *Quelling the concerns of EPR and Bell*, *arXiv preprint arXiv:1401.5988* (2014).

[55] C. Isham and J. Butterfield, *Spacetime and the philosophical challenge of quantum gravity in physics meets philosophy at the Planck scale eds callender c and huggett n*, 2000.

[56] K. Kraus, A. Bohm, J. D. Dollard, and W. Wooters, *States, effects, and operations fundamental motions of quantum theory*, 1983.