# Unitarity constraints on the ratio of shear viscosity to entropy density in higher derivative gravity

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ABSTRACT:

We discuss corrections to the ratio of shear viscosity to entropy density $\eta/s$ in higher-derivative gravity theories. Generically, these theories contain ghost modes with Planck-scale masses. Motivated by general considerations about unitarity, we propose new boundary conditions for the equations of motion of the graviton perturbations that force the amplitude of the ghosts modes to vanish. We analyze explicitly four-derivative perturbative corrections to Einstein gravity which generically lead to four-derivative equations of motion, compare our choice of boundary conditions to previous proposals and show that, with our new prescription, the ratio $\eta/s$ remains at the Einstein-gravity value of $1/4\pi$ to leading order in the corrections. It is argued that, when the new boundary conditions are imposed on six and higher-derivative equations of motion, $\eta/s$ can only increase from the Einstein-gravity value. We also recall some general arguments that support the validity of our results to all orders in the strength of the corrections to Einstein gravity. We then discuss the particular case of Gauss-Bonnet gravity, for which the equations of motion are only of two-derivative order and the value of $\eta/s$ can decrease below $1/4\pi$ when treated in a nonperturbative way. Our findings provide further evidence for the validity of the KSS bound for theories that can be viewed as perturbative corrections to Einstein Gravity.

# Contents

## 1. Introduction

The gauge–gravity duality [1, 2] allows one to holographically map black brane thermodynamics and hydrodynamics in the Anti-deSitter (AdS) bulk to their gauge-theory correspondents at the AdS boundary [3, 4].

For a large class of strongly coupled fluids (essentially, any with a two-derivative gravity dual), the ratio of the shear viscosity $\eta$ to the entropy density $s$ is, in appropriately chosen units, remarkably low: $\frac{\eta}{s} = \frac{1}{4\pi}$ [5, 6]. The uncertainty principle can be used to argue that $\eta/s$ should have a lower bound of order unity [6]; leading Kovtun, Son and Starinets (KSS) to propose that $1/4\pi$ is a universal lower bound on this ratio. However, investigations of higher-derivative gravity theories have revealed that the so-called KSS bound can apparently be violated [7, 8]. Many subsequent inquiries have appealed to gauge-theory causality as the physical principle that bounds $\eta/s$ (*e.g.*) [9]-[14].

We have recently asserted [15] that a unitary ghost-free extension of Einstein gravity can, at best, saturate the KSS bound. We have observed that $\eta$ is a gravitational coupling and any such coupling can only increase from its Einstein value in unitary theories [16, 17, 18]. The source of tension between our claim and the apparently bound-violating gravity theories is that, even though higher-derivative gravity theories contain ghosts, these are typically at the Planck scale and therefore expected to be irrelevant to the calculation of hydrodynamic transport coefficients like $\eta$.

Let us briefly recall why ghosts are problematic for any classical or quantum field theory (also see, *e.g.*, [19]-[23]). If ghosts are present, then any state, including the vacuum, would be catastrophically unstable due to the spontaneous creation of positive and negative energy particles having zero total energy. For a classical theory of gravity, in particular, ghost gravitons lead to instabilities at arbitrarily short time scales or superluminal propagation in both the gravitational and matter sectors. These issues have to do with the wrong sign of the kinetic energy and not with the relative sign of the kinetic and mass terms. Although the latter can, if "incorrectly" chosen, have further effects that undermine both stability and causality.

The necessity to remove ghosts becomes even more acute when the gauge–gravity duality is considered. From the field-theory side, only a finite number of states is permissible, which corresponds to a truncated spectrum of bulk excitations. This so-called "stringy exclusion principle" [24] has been argued to prohibit ghosts [25].

From an effective-theory perspective, the situation is not so clear cut when all the ghosts have a mass on the order of the ultraviolet cutoff. One might then reason that it is now safe to disregard their presence. This could be true in many instances, but this reasoning will be shown to be incorrect for a calculation of $\eta$. So, to proceed in a sensible manner, one should ensure that the ghost has decoupled before the computation is carried out. This could be accomplished through a choice of boundary conditions (BC's).

Later on, we use four-derivative theories of gravity to explicitly show that the Planck-scale ghosts do indeed infiltrate the existing schemes of calculating $\eta$. Then, motivated by considerations of unitarity, we propose a new set of BC's for the higher-order equations of motion that forces the amplitude of the ghost modes to vanish. Applying the new prescription to four-derivative gravity theories, we find that $\eta/s = 1/4\pi$ to leading order in the strength of the corrections to Einstein gravity. We then argue that, after imposing the new BC's on six and higher-derivative equations of motion, $\eta/s$ can only increase above $1/4\pi$. These results support our previous claim that $1/4\pi$ is indeed a lower bound on $\eta/s$ for any unitary weakly coupled extension of Einstein gravity.

Let us now suppose that a gauge field theory is given and its AdS gravity dual is known. So, just how does one go about determining $\eta/s$ from a black brane theory on the AdS side? For any two-derivative theory of gravity, [1] this is a well understood matter, and the resulting answer of any method must agree with the well-known Einstein result of $\eta/s = 1/4\pi$ [5, 6]. But, for higher-derivative gravity theories, the situation is still quite ambiguous.

The Wald Noether-charge formalism [26, 27] is appropriate and accepted for the calculation of the entropy or its density $s$. For the shear viscosity $\eta$, the situation is less clear. The earliest calculations of $\eta$ for Einstein gravity used holographic techniques to express the field-theory viscosity — which follows from the Kubo formula — in terms of horizon-valued parameters of the AdS brane theory. (For a review and

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[1] The two-derivative class is meant to include theories that are effectively two-derivative; *e.g.*, any $f(\mathcal{R})$ theory can be expressed as Einstein gravity plus a scalar.

references, see [4].) Later, the physical connection between the boundary and black brane theories has been made more explicit via the membrane-fluid interpretation of horizon hydrodynamics [3].

For any two-derivative gravity theory, one can also obtain $\eta$ from the horizon value of the coefficient of the kinetic term of the transversely polarized gravitons [28]-[31]. This is the same as reading off the coefficient of the graviton propagator $\langle h_{xy} h_{xy} \rangle$, where $h_{\mu\nu}$ is the first-order correction to the background metric [2].

For higher-derivative theories, it is reasonable to apply a similar procedure. Except that, in this case, one first requires a well-defined prescription for dealing with the higher (than second) order derivatives which inevitably turn up in the action. A popular approach has appeared in different yet equivalent guises in the literature [32]-[37]. For instance, according to [33], one is instructed to iteratively apply the field equation to the action, consistently trading off the perturbative-order higher derivatives for lower ones (while ignoring terms of second or higher perturbative order), until a two-derivative action has been obtained. Then, the viscosity is extracted from the kinetic term of this effective action. Identical results have been obtained from a canonical-momentum formulation of $\eta$ [36], which can be viewed as a higher-derivative generalization of the membrane paradigm [3] as interpreted in [30].

An alternative approach [29] prescribes ignoring the higher-order derivatives and extracting $\eta$ straight from the kinetic term. In this approach, $\eta$ is presumed to maintain its identity as the coefficient of the propagator. For theories limited to four derivatives, the two approaches are in agreement. This agreement does not, however, persist for theories with six or more derivatives, nor could it be expected to. For the sake of completeness, we will elaborate further on these methods in an appendix. Our goal in this paper is not to argue for the merit of one prescription versus the other. Rather, it is our contention that both of these approaches should be corrected.

The rest of the paper proceeds as follows: In the next section, we introduce

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[2]$x$ and $y$ represent transverse brane directions that are mutually orthogonal to one another, as well as to the propagating direction of gravitons moving along the brane.

the explicit models which are four-derivative extensions of Einstein gravity. Here, our conventions are set and much of the necessary formalism is presented, with particular emphasis on deriving the linearized field equations. In Section 3, we present a detailed discussion on the calculations of the shear viscosity for such higher-derivative theories. A thorough inspection of the field equations confirms that these existing methods are equivalent to reading off the shear viscosity from the coefficient of the kinetic term. Using this observation, we are then able to demonstrate explicitly that ghosts have infiltrated the earlier attempts at evaluating $\eta$. Section 4 begins with a proposal for new BC's, as part of a revised prescription for evaluating the shear viscosity. This choice forces the decoupling of the ghosts that are inherent to higher-derivative theories. Subsequently, we explain how our new prescription protects the KSS bound and elaborate on the physical basis for the bound's validity. In Section 5, we resolve an apparent contradiction between our findings and the special case of Gauss–Bonnet gravity. Section 6 contains a brief summary of the results and their significance. The paper concludes with an appendix, where we provide a detailed description of previous methods for calculating $\eta$.

## 2. Equations of motion

### 2.1 Specific 4-derivative models

For clarity and concreteness, we consider the case of a 5-dimensional AdS black brane and focus on the class of four-derivative theories of pure gravity. Let us now set the notation and conventions. The background metric can be expressed as

$$ds^2 \;=\; -F(r)dt^2 + \frac{dr^2}{F(r)} + \frac{r^2}{L^2}\left[dx^2 + dy^2 + dz^2\right]\;, \qquad (2.1)$$

where $F(r) = \frac{r^2}{L^2}\left[1 - \frac{r_h^4}{r^4}\right]$, while $L$ and $r_h$ respectively denote the radius of curvature and black brane horizon. The function $F$ vanishes on the horizon, $F(r_h) = 0$.

The theories of interest are completely described by the Lagrangian of Einstein gravity in AdS space,

$$\mathcal{L}_E = \mathcal{R} + \frac{12}{L^2}\;, \qquad (2.2)$$

and three perturbative corrections as follows: [3]

$$\mathcal{L}_A = \alpha L^2 \mathcal{R}^2 \; , \tag{2.3}$$

$$\mathcal{L}_B = \beta L^2 \mathcal{R}_{ab} \mathcal{R}^{ab} \; , \tag{2.4}$$

$$\mathcal{L}_C = \gamma L^2 \mathcal{R}_{abcd} \mathcal{R}^{abcd} \; . \tag{2.5}$$

The validity of the gauge–gravity duality mandates the hierarchy $r_h \gg L \gg l_p$ (where $l_p$ is the Planck length), thus making the dimensionless coefficients $\alpha$, $\beta$, $\gamma \sim l_p^2/L^2$ much smaller than unity. We also consider, with particular emphasis in Section 5, the special Gauss–Bonnet combination of the three perturbations or $\mathcal{L}_{GB} \equiv \lambda \left[ \mathcal{L}_A - 4\mathcal{L}_B + \mathcal{L}_C \right]$ .

Our interest is in the case of weak gravity, and so the linear expansion of the metric about its background $g_{ab} = \overline{g}_{ab} + h_{ab} + \mathcal{O}[h^2]$ is valid. Similarly, we need linearized expressions for the various curvature tensors; for instance,

$$\mathcal{R}_{abcd}(h) \; = \; \frac{1}{2} \left[ \overline{\nabla}_c \overline{\nabla}_b h_{ad} \; + \; \overline{\nabla}_d \overline{\nabla}_a h_{bc} \; - \; \overline{\nabla}_d \overline{\nabla}_b h_{ac} \; - \; \overline{\nabla}_c \overline{\nabla}_a h_{bd} \right] \; , \tag{2.6}$$

from which the contracted forms follow. An overline indicates the background geometry.

To obtain the linearized graviton field equations, we use the following identity [27, 38]:

$$\frac{\delta \mathcal{L}}{\delta g^{pq}} \; - \; 2\nabla_a \nabla_b \left[ \frac{\delta \mathcal{L}}{\delta \mathcal{R}_a{}^{pq}{}_b} \right] \; + \; \mathcal{R}_{abcp} \left[ \frac{\delta \mathcal{L}}{\delta \mathcal{R}_{abc}{}^q} \right] \; - \; \frac{1}{2} g_{pq} \mathcal{L} \; = \; 0 \; . \tag{2.7}$$

One can obtain the desired equations by expanding out each term to linear order in the $h$'s. (In our case, the first term is trivially vanishing.) For the current analysis, this treatment is preferred over the more familiar procedure of varying with respect to the metric, as the absence of explicit derivatives in the Lagrangian means that boundary terms need not be stipulated. To determine $\eta$, we compute the $xy$ component of Eq. (2.7), knowing that the $\{x,y\}$ sector does not mix with other polarizations. The calculations are carried out in the transverse, traceless gauge and,

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[3]For a four-derivative theory, any derivative of the Riemann tensor reduces to a surface term and can therefore be ignored.

often, at the horizon. The brane horizon is the most appropriate surface for an analysis of $\eta$ from the gravity side; however, none of our conclusions are sensitive to this choice. Deriving the Einstein equation for $\mathcal{L}_E$ and multiplying through by a factor of $-2$, one then obtains

$$\Box_E \, h_{xy} \;\equiv\; \left[ \Box_L + \overline{\mathcal{R}} + \frac{12}{L^2} \right] h_{xy} \;=\; 0 \,, \tag{2.8}$$

where $\Box_L \equiv \Box - \overline{\mathcal{R}}^x{}_x - \overline{\mathcal{R}}^y{}_y - 2\overline{\mathcal{R}}^{xy}{}_{xy}$ and $\Box = \overline{g}^{ab}\overline{\nabla}_a\overline{\nabla}_b$ . The operator $\Box_L$ is the spin-2 Lichnerowicz d'Alembertian [39] (*i.e.*, the analogue of $\Box$ for a graviton in curved space) for the background metric. When the field equation is re-expressed in terms of $\phi = \overline{g}^{xx}h_{xy}$, it is formally equivalent to the Klein–Gordon equation for a massless scalar throughout the spacetime. On the horizon, in particular, Eq. (2.8) becomes

$$\Box_E \, h_{xy} \;=\; \Box \, h_{xy} \;=\; 0 \,. \tag{2.9}$$

This observation proves to be important later on.

Let us next consider the leading-order contributions from the corrections. After tedious but straightforward calculations, we obtain

$$\mathcal{L}_A \;\rightarrow\; -40\alpha\Box \, h_{xy} + \cdots \,, \tag{2.10}$$

$$\mathcal{L}_B \;\rightarrow\; \beta L^2 \left[ \Box^2 \;-\; 2\frac{\partial_r F}{r}\Box \right] h_{xy} + \cdots \,, \tag{2.11}$$

$$\mathcal{L}_C \;\rightarrow\; 4\gamma L^2 \left[ \Box^2 \;+\; \left( -\frac{1}{2}\partial_r^2 F - \frac{1}{2}\frac{\partial_r F}{r} + 7\frac{F}{r^2} \right)\Box \right] h_{xy} + \cdots \,. \tag{2.12}$$

The ellipsis represent perturbative-order mass terms, which will be omitted for the remainder of the paper, as these are inconsequential to the current discussion. In any event, one can eliminate these by expressing the on-shell form of the full field equation in terms of $\phi = h^x{}_y$ .

The $\Box^2$ terms in the equation of motion are focal to the ensuing discussion, so let us be more explicit how they arise. After linearizing Eq. (2.7), one finds that a $\Box^2$ term can only come from $\overline{\nabla}_a\overline{\nabla}_b\mathcal{X}(h)^a{}_{xy}{}^b$ , where $\mathcal{X}^{abcd} \equiv \delta\mathcal{L}/\delta\mathcal{R}_{abcd}$ . Considering, for instance, the 4-index Riemann-squared case, one then has $\mathcal{X}^a{}_{xy}{}^b = 2\gamma L^2 R^a{}_{xy}{}^b$ , which can be linearized via Eq. (2.6). In this case, only the second

term in Eq. (2.6) is relevant, leading to $2\gamma L^2 \overline{\nabla}^a \Box \overline{\nabla}_a h_{xy}$. Next, commutator relations such as $[\overline{\nabla}_a, \overline{\nabla}_b] V_c = \overline{\mathcal{R}}_{abc}{}^d V_d$ can be utilized to attain (up to mass terms) $2\gamma L^2 \left[ \Box^2 + \overline{R}_a{}^{bac} \overline{\nabla}_b \overline{\nabla}_c \right] h_{xy}$ or $2\gamma L^2 \left[ \Box^2 - \frac{4}{L^2} \Box \right] h_{xy}$, with the latter form following from symmetries of the background as explained below. Finally, restoring all numerical factors, one recovers the $\Box^2$ term in Eq. (2.12).

An important caveat about $\mathcal{L}_C$ is that, for this case only, one finds terms that cannot be expressed directly in terms of $\Box$; with these always being of the form $\overline{\mathcal{R}}^{axb}{}_x \overline{\nabla}_a \overline{\nabla}_b h_{xy}$ (as well as the obvious $x \leftrightarrow y$ analogue). However, we are interested in the $t$ and $r$ sectors because time derivatives are responsible for the ghosts (if any) and only radial derivatives will survive once the hydrodynamic limit is imposed to calculate $\eta$. Then, since only the terms with $a, b = \{r, t\}$ are relevant and $\overline{\mathcal{R}}^{tx}{}_{tx} = \overline{\mathcal{R}}^{rx}{}_{rx} = -\frac{1}{2} \frac{\partial_r F}{r}$, we can make the substitution $\overline{\mathcal{R}}^{axb}{}_x \overline{\nabla}_a \overline{\nabla}_b h_{xy} \rightarrow -\frac{1}{2} \frac{\partial_r F}{r} \Box h_{xy}$. The remaining discrepancy $\left[ \overline{\mathcal{R}}^{zxz}{}_x - \left( -\frac{1}{2} \frac{\partial_r F}{r} \right) \right] \overline{\nabla}^z \overline{\nabla}_z h_{xy} = \frac{1}{2} \frac{L^2}{r^4} \left[ 2F - r \partial_r F \right] k^2 h_{xy}$ (with $k \equiv -i \overline{\nabla}_z$) may be regarded as a perturbative-order mass term.

The Gauss–Bonnet combination leads to the correction

$$\mathcal{L}_{GB} \rightarrow -2\lambda L^2 \frac{\partial_r F}{r} \Box \, h_{xy}, \tag{2.13}$$

where the background relations $\frac{\partial_r F}{r} + 2\frac{F}{r^2} = \frac{4}{L^2}$ and $\partial_r^2 F = 4\frac{F}{r^2} - \frac{\partial_r F}{r}$ have been used to re-express the contributions from $\mathcal{L}_A$ and $\mathcal{L}_C$ respectively. As a useful check, we find that this Gauss–Bonnet correction reduces in the AdS vacuum ($r_h = 0$) to $-4\lambda \Box h_{xy}$, in agreement with the known result [40].

For future reference, the horizon forms of Eqs. (2.10-2.13) are as follows:

$$\mathcal{L}_A \rightarrow -40\alpha \Box \, h_{xy}, \tag{2.14}$$

$$\mathcal{L}_B \rightarrow \beta L^2 \left[ \Box^2 - \frac{8}{L^2} \Box \right] h_{xy}, \tag{2.15}$$

$$\mathcal{L}_C \rightarrow 4\gamma L^2 \Box^2 \, h_{xy}, \tag{2.16}$$

$$\mathcal{L}_{GB} \rightarrow -8\lambda \Box \, h_{xy}. \tag{2.17}$$

The background metric has been used in the above and any subsequent computations. Since our main interest is in corrections to $\eta$ up to leading order in $l_p^2/L^2$,

this choice is justified for contributions that come directly from the perturbative part of the Lagrangian. But what about those from the Einstein part? To understand why the background metric still suffices, let us consider the following: Given the Einstein Lagrangian and some perturbative correction, the leading-order effect on the near-horizon geometry can only be to shift $L$ and $r_h$ from their background values. We can also, to leading order, re-express any contribution from the perturbative part in terms of the same shifted values of $L$ and $r_h$. Now, if our primary interest is to compute $\eta/s$ for a given theory, then all dependence on $L$ and $r_h$ will cancel out of this ratio. Meanwhile, if our main interest is to compare $\eta$ for two different theories, then it is natural to do so at a fixed value of brane temperature $T = r_h/\pi L^2$ and, hence, at common values of $L$ and $r_h$. So that, for current purposes, any correction to the background geometry never does come into play.

We can restate the above argument in another way. As already discussed, one can fully determine $\eta$ for two-derivative gravity from the coefficient of the kinetic term in the Lagrangian or, equivalently, from the coefficient of the $\square$ terms in the field equation. For the Einstein part of the Lagrangian, the latter coefficient is universally $-1/2$. This is true anywhere in the spacetime and irrespective of the solution. So that, to understand how $\eta$ is corrected for an extended theory, we need only to determine how the additional ("non-Einstein") parts of the Lagrangian *explicitly* contribute. And, with these additions already being of order $l_p^2/L^2$, the Einstein background metric (2.1) suffices.

## 2.2 Generic 4-derivative gravity

Let us emphasize an important property of *any* four-derivative theory of pure gravity and, in fact, any higher-derivative extension that depends only on the Riemann tensor (and its derivatives): After the common factor of $L^2$ (*cf*, Eqs. (2.3-2.5)) has been factored out of the corrected part of the field equation, the coefficient of the $\square^2$ term is a geometry-independent number (possibly zero) and, hence, a spacetime invariant. By geometry independent, we mean that it can depend on the form of the Lagrangian but not on the solution to the field equations. This outcome follows from

the observation that the prefactor for the $\Box^2$ term is dimensionless; meaning that it can not depend on the Riemann tensor nor its derivatives, as any of these have a strictly positive mass dimension (nor can it depend explicitly on the metric, as the Lagrangian is only a function of the curvature). On the other hand, the prefactor for the linear-$\Box$ term has a dimension of mass squared, and so it can depend on the Riemann tensor.

Consequently, for a generic four-derivative theory at any radius $r$, the corrected field equation can be written as

$$\Box_E h_{xy} \; + \; \epsilon L^2 \left[ a\Box^2 + \frac{b(r)}{L^2}\Box \right] h_{xy} \; = \; 0 \;, \tag{2.18}$$

where $\epsilon \sim l_p^2/L^2$ is a perturbative coefficient, $a$ is a fixed number of order unity and $b(r)$ is a radial function of order unity; all of which are dimensionless. On a constant-radius surface, $b$ can also be regarded as a number. We are interested in corrections to $\eta/s$ to leading order in $\epsilon$ and therefore need to consider a finite (small) $\epsilon$.

We need not limit considerations to the $\{x,y\}$-polarization channel for the gravitons, as $\eta$ can also be extracted from the so-called shear and sound channels (see, e.g., Appendix B of [7]). For the shear channel (the sound channel follows along similar lines), the relevant gravitons are $h_{ra}$, $h_{ta}$, $h_{za}$ with $a = \{x,y\}$. The field equations for this class are more complicated but can be simplified by choosing the radial gauge $h_{r\mu} = 0$ ( $\mu = \{r,t,z,a\}$ ) and then considering the gauge-invariant combination $\mathcal{Z} = g^{xx}k h_{tx} + g^{xx}\omega h_{zx}$ ( $\omega \equiv +i\overline{\nabla}_t$ , $k \equiv -i\overline{\nabla}_z$ ). In this way, the coupled set of equations is turned into a single equation for a single graviton mode $\mathcal{Z}$. This equation is necessarily of the same generic form as Eq. (2.18), as none of the above arguments are specific to the $\{xy\}$-polarization class. Then, just as we go on to show how the shear viscosity can be extracted from the kinetic coefficient of $h_{xy}$, the same can be deduced for the kinetic coefficient of $\mathcal{Z}$ [41] (and analogously for the sound channel [42]).

## 3. Calculating $\eta$

### 3.1 Contributions to $\eta$

Next, we consider the shear viscosity $\eta_X$ for the extended theories $\mathcal{L}_E + \mathcal{L}_X$ , with $X = \{A,B,C,GB\}$. But, first, let us recall what is known about two-derivative theories: The kinetic coefficient of the transverse gravitons at the horizon fully determines the shear viscosity for both the "graviton fluid" on the black brane and its field-theory dual at the AdS boundary. This follows from a rigorous calculation in [31] (also, [43]) that applies to any two-derivative (but otherwise arbitrary) $5D$ gravity theory. There it was shown that the Kubo form of the field-theory viscosity $\eta_{FT}$ can be expressed purely in terms of the bulk geometry. They then imposed the standard choice [3] of incoming plane wave BC at the horizon and the Dirichlet BC at the outer boundary to arrive at

$$\eta_{FT} \;=\; \eta_{grav} \;=\; \frac{1}{16\pi l_p^3} \frac{r_h^3}{L^3} K(r = r_h) \,, \tag{3.1}$$

where $K$ is the kinetic coefficient of the transverse gravitons (here, normalized so that its Einstein value is unity).

Supposedly, we can determine how each extension modifies the Einstein viscosity $\eta_E$ by inspecting the horizon forms of the corrected field equations (2.14-2.17). The kinetic coefficients, in particular, should tell us how the viscosity is modified. The scalar-squared and Gauss–Bonnet cases ($A$ and $GB$) are effectively two-derivative theories, and so the associated corrections can be read off directly from Eqs. (2.14) and (2.17). We then have $\eta_A = \eta_E\,[1 - 40\alpha]$ and $\eta_{GB} = \eta_E\,[1 - 8\lambda]$ ; both in agreement with the already-known results. [4]

However, for the Ricci and Riemann-tensor squared cases ($B$ and $C$), there are now $\Box^2$ terms present in the field equations. Following the standard prescription, one would conclude from Eqs. (2.15) and (2.16) that $\eta_B = \eta_E\,[1 - 8\beta]$ and $\eta_C = \eta_E$ ,

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[4] For $\eta_{GB}$, see (e.g.) [7]. That the scalar-squared case ($A$) is a match, even if not documented explicitly, follows from the confirmation of $\eta_A/s_A = \eta_E/s_E$ , as must be true for any $f(\mathcal{R})$ theory. Note that $s_A = s_E\,[1 - 40\alpha]$ follows from Wald's formula [26, 27].

which are indeed in perfect agreement with those obtained from any of the previously described methods. [5]

On the horizon, $\Box h_{xy}$ and the background field equation $\Box_E h_{xy}$ are interchangeable up to order $l_p^2/L^2$. Then, as any $\Box$ must be of order $l_p^2/L^2$, $\Box^2 \ll \Box$ certainly follows. Off the horizon the situation changes, since it is $\Box_E h_{xy} = [\Box + 2F(r)]h_{xy}$ that vanishes on shell, and so $\Box^2 \ll \Box$ is no longer valid. A single $\Box^2$ term now makes a generic contribution of

$$\Box^2 = \Box_E^2 - 4F\Box_E + \cdots \, , \tag{3.2}$$

where the dots signify that we are disregarding mass terms as usual. In terms of our generic theory of Eq. (2.18), $b \to b - 4aF$. The $\Box^2$ term has, through its prefactor $a$, entered into the calculation of the kinetic coefficient. [6]

It would seem that the horizon value of $b$ and, hence, $\eta$ are insensitive to the $\Box^2$ terms. We will, contrary to this expectation, show that the $\Box^2$ terms are always implicated with ghosts; with these directly influencing the value of $b$ throughout the spacetime — including at the horizon!

## 3.2 Ghosts in the machinery

It is well known that the presence of a $\Box^2$ in the equation of motion signals that a ghost lurks in the theory. The usual argument for ignoring this is that the Planck-scale ghost becomes infinitely massive once the ultraviolet cutoff has been sent to infinity [44]. However, for the case at hand, $\epsilon \sim l_p^2/L^2$ needs to be kept finite. If one takes the strict decoupling limit $\epsilon = 0$ and all corrections originating from the higher-derivative terms vanish, as expected.

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[5]Similar to case $A$, the Ricci-square result follows from knowledge of the entropy density via Wald's formula and that $\eta_B/s_B = \eta_E/s_E$ [7]. Also, to avoid any confusion, the ratio $\eta_C/s_C$ changes by an overall factor of $1 - 8\gamma + \mathcal{O}[\gamma^2]$, same as for Gauss-Bonnet. Only that, for the Riemann-squared case, this correction comes entirely from the entropy [29].

[6]This distinction between the horizon and other surfaces seems contradictory to the claim that $\eta_{FT}$ can be calculated at any radius in the spacetime [30]. However, this claim has only been established for two-derivative theories.

Let us begin here by exposing the ghosts. We will do so using a generic four-derivative theory, as described by Eq. (2.18), to emphasize that the ghosts are neither model nor radius specific. First, Eq. (2.18) can be recast, using Eq. (3.2), into

$$\left[ \Box_E \,+\, \epsilon a L^2 \Box_E^2 \,+\, \epsilon \tilde{b} \Box_E \right] h_{xy} \;=\; 0 \,, \tag{3.3}$$

where $\tilde{b} \equiv b - 4aF$ is a radially dependent parameter that generally differs from $b$ but $a$ is the exact same number as before. Or, up to leading order in $\epsilon$,

$$(1 + \epsilon b)\Box_E \left[ 1 \,+\, \epsilon a L^2 \Box_E \right] h_{xy} \;=\; 0 \,, \tag{3.4}$$

with the tilde on $b$ henceforth implied.

The factorization in Eq. (3.4) implies that, at order $\epsilon$, there are two separable modes in this problem. Let us then write the graviton as $H = H_0 + H_1$ (with tensor indices implied), where these are meant (and shortly will be shown) to satisfy

$$\Box_E H_0 \;=\; 0 \,, \tag{3.5}$$

$$\left[ \epsilon a \Box + \frac{1}{L^2} \right] H_1 \;=\; 0 \,. \tag{3.6}$$

To explicitly expose the two distinct modes, let us invoke the standard practice in flat spacetime of inverting the linearized operator and then factorizing. That the inversion process extends in a straightforward manner from flat space to AdS space follows from the fact that $\Box_E$ is a scalar. Then $\Box = -p^2$ of flat space is replaced with an appropriately generalized momentum; say $\Box_E = -\tilde{p}^2$. On the horizon of a black brane, $\tilde{p}^2 = p^2$. Then,

$$\frac{1}{\Box_E \left[ 1 \,+\, \epsilon a L^2 \Box_E \right]} \;=\; \frac{1}{\Box_E} \;-\; \frac{1}{\Box_E \,+\, (\epsilon a L^2)^{-1}} \tag{3.7}$$

The first term on the right can be identified as the propagator of the massless Einstein graviton. Meanwhile, the second term is the propagator of a massive ghost graviton, as evident from the "wrong" sign relative to the former. This outcome does not depend on the sign nor magnitude of the perturbative coefficient $\epsilon$, which only determines the mass of the ghost. Let us recall that $a, b \sim \mathcal{O}[1]$ and $\epsilon \sim l_p^2/L^2$, and so the ghosts have a squared mass $M^2 \sim (\epsilon L^2)^{-1} \sim 1/l_p^2$. As already stressed at the

end of Section 2, we are looking at corrections to leading order in $\epsilon$. Consequently, the very massive ghosts may indeed affect the result.

If a four-derivative theory does have ghosts, these persist throughout the entire spacetime because of the invariance of the dimensionless constant $a$ associated with the $\Box^2$ term. [7] Hence, the ghosts are not confined to the AdS bulk interior and can contribute to the field-theory viscosity to leading order in $\epsilon$.

Now suppose that we want to restore the normalization factor of $(1 + b)$ to the mode equations (3.5,3.6). According to any of the standard methods, the factor $(1 + \epsilon b)$ is strictly attributed to the massless graviton. This follows from $\eta$ always going as the value of $(1 + \epsilon b)$ at the horizon (see Subsection 3.1) and the implicit assumption that any ghost mode would have ultimately decoupled. On this basis, the mode equations should be reformulated as

$$(1 + \epsilon b)\Box_E H_0 \;=\; 0 \;, \tag{3.8}$$

$$\left[\epsilon a \Box + \frac{1}{L^2}\right] H_1 \;=\; 0 \;. \tag{3.9}$$

But, as $b = b(r)$ does not commute with $\Box_E$, this formulation is problematic in that it could not be obtained through a process of factorization (*cf*, Eq. (3.4)). Hence, whatever might be the actual coupling for the massless graviton, it can be generically different than the standard result of $(1 + \epsilon b)$. This difference can be attributed to the fact that the gravitational coupling $(1 + \epsilon b)$ can only be attributed to the sum of the modes and not to any single one of them. This argument shows that the ghost mode can indeed impact the calculation of $\eta$.

In summary, we have seen that the ghosts have, through their association with the coupling correction $b$, managed to infiltrate into the coefficient of the kinetic terms and, thus, into the calculation of $\eta$. This is true insofar as the squared mass in units of the AdS radius $L^2 M^2 \sim \epsilon^{-1} \sim L^2/l_p^2$ is kept large but finite. In the next section, we will propose a procedure that guarantees the complete decoupling of the ghost mode and allows for an unambiguous calculation of $\eta/s$.

[7]The same claim can, in fact, be made for any higher-derivative theory that depends only the Riemann tensor. The inclusion of other types of fields could jeopardize the invariance of $a$, but this parameter would still be typically non-vanishing at all radii.

# 4. New (and improved) prescription for calculating $\eta$

We conclude that, to proceed, one should ensure that the influence of the ghosts is completely eliminated before the calculation of $\eta$ is carried out. This could be accomplished through a carefully imposed choice of BC's, as we explain next.

## 4.1 An exorcizing boundary condition

Let us reconsider the generic field equation (3.4). For a generic extension of Einstein's theory, one can expect two BC's per each order of $\epsilon$. This enables one to individually fix each order of the solution to fulfill the stated conditions [45]. Here, we have rearranged the solution so that it is separated according to the different degrees of freedom rather than perturbative order. Nevertheless, as is evident from Eqs. (3.5) and (3.6), both of the modes (massless graviton and ghost) still satisfy a simple quadratic equation and, thus, both have a corresponding incoming and outgoing solution. And so, with four BC's at our disposal, we are using one of these to stipulate that the ghost is an incoming plane wave at the horizon and another to kill off the ghost by, for instance, imposing the Dirichlet BC at the horizon. [8] The remaining two BC's are then used to ensure that the massless graviton satisfies the usual pair of BC's: Dirichlet (or vanishing wavefunction) at the AdS boundary and incoming plane wave at the black brane horizon.

Our new BC's are compatible with the standard set of incoming plane wave at the horizon and Dirichlet on a radial shell [9] for the *total* wavefunction. Let us recall that the total solution can be expressed as $H = H_0 + H_1$ , with $H_{0/1}$ respectively labeling the massless graviton and massive ghost. We also recall the relevant pair of quadratic equations,

$$\Box_E H_0 \; = \; 0 \, , \tag{4.1}$$

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[8] Generically, these hydrodynamic modes are regular everywhere, except at the horizon where regularity must be imposed by hand (see, *e.g.*, [46]). Hence, fixing the wavefunction to vanish at the horizon is a sufficient condition for the mode to be vanquished throughout the spacetime.

[9] As stressed in [41], there is nothing special about the outer *AdS* boundary in this regard, even if the standard choice. The Dirichlet condition can readily be imposed on any radial shell exterior to the horizon. This includes at the horizon when taken as a suitable limit of the stretched horizon.

$$\left[ \epsilon a \Box_E + \frac{1}{L^2} \right] H_1 \; = 0 \; , \tag{4.2}$$

with both admitting plane-wave solutions.

Adopting the standard incoming plane-wave solution and using the dimensionless (inverted) radial coordinate $u = r_h^2/r^2$ , we can express the total wavefunction of the mixed-index gravitons $\Phi_{0/1} \equiv g^{xx} H_{0/1}$ as the sum $\Phi(u, \omega, q) = \mathcal{C}_0 \Phi_0 + \mathcal{C}_1 \Phi_1$ such that

$$\Phi_0 \; = \; [f(u)]^{-i\omega/2} \, \Upsilon_0(u, \omega, q) \; , \tag{4.3}$$

$$\Phi_1 \; = \; [f(u)]^{-i\omega/2} \, \Upsilon_1(u, \omega, q) \; . \tag{4.4}$$

Here, $f(u) = 1 - u^2$ (essentially, $|g_{tt}|$), $\omega$ and $q$ represent a dimensionless frequency and wavenumber (as defined in, *e.g.*, [41]), $\mathcal{C}_{0/1}$ are normalization constants and $\Upsilon_{0/1}$ are model-dependent functions of $u, \omega, q$ . The latter functions can be uniquely fixed, up to normalization, by imposing regularity at the horizon (with regularity assured elsewhere).

The hydrodynamic limit implies $\omega, q \ll 1$ , so that — in practice — one expands $\Phi_{0/1}$ out in increasing powers of $\omega$ and $q$ and considers only the first few terms in the series. For instance, Einstein gravity ( $\epsilon = 0$ ) leads to $\Upsilon_0 = 1 + \mathcal{O}[\omega^2, q^2]$ [46]. [10] Technically, however, the hydrodynamic limit should only be applied at the very end of the calculation.

For $\Phi_0$ , the Dirichlet BC is imposed at the outer boundary ( $u \to 0$ ) and then the dispersion relation $\omega = \omega(q)$ is fixed to attain

$$[f(u)]^{-i\omega/2} \, \Upsilon_0(u, \omega, q) \big|_{u \to 0} = 0 \; . \tag{4.5}$$

Meanwhile, the normalization $\mathcal{C}_0$ is fixed in the standard way

$$\mathcal{C}_0^{-1} \; = \; [f(\overline{u})]^{-i\omega/2} \, \Upsilon_0(\overline{u}, \omega, q) \big|_{\overline{u} \to 0} \; , \tag{4.6}$$

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[10] At the horizon, either of our modes has precisely this same form. For the massless graviton, this is trivially so at any radius. For the massive ghost, this follows from the red shift at the horizon making this mode effectively massless.

where $\bar{u}$ is a constant "cutoff" scale. Consequently, the renormalized wavefunction $\widehat{\Phi}_0 \equiv \mathcal{C}_0 \Phi_0$ satisfies

$$\left. \widehat{\Phi}_0 \right|_{u \to 0} \;=\; 1 \;. \tag{4.7}$$

For the ghost mode $\Phi_1$, we now deviate from the usual convention and rather fix its normalization with the constraint $\mathcal{C}_1 = 0$. Since regularity has already been imposed at the horizon through the correct choice of the functional form $\Upsilon_1$, the renormalized wavefunction $\widehat{\Phi}_1 \equiv \mathcal{C}_1 \Phi_1$ is compliant with the horizon Dirichlet BC

$$\left. \widehat{H}_1 \right|_{u \to 1} \;=\; 0 \;. \tag{4.8}$$

As a result, the total normalized wavefuntion $\widehat{\Phi} = \widehat{\Phi}_0 + \widehat{\Phi}_1$ reduces to $\widehat{\Phi}_0$,

$$\widehat{\Phi}(u,\omega,q) \;=\; \left[ \frac{f(u)}{f(\bar{u})} \right]^{-i\omega/2} \left. \frac{\Upsilon_0(u,\omega,q)}{\Upsilon_0(\bar{u},\omega,q)} \right|_{\bar{u} \to 0} \;. \tag{4.9}$$

In this way, we have ensured that the amplitude of the heavy ghost mode is made to vanish without insisting on the vanishing of $\epsilon$.

The effect of imposing the BC's that "exorcise" the ghost is to set the right-most term in Eq. (3.7) to vanish. We can then re-invert the remaining operator to obtain a "unitarized" version of the field equation (3.4) in which the rightmost factor has been set to unity:

$$(1 + \epsilon c) \, \Box_E \, h_{xy} \;=\; 0 \;, \tag{4.10}$$

where the order-unity radial function $c(r)$ is related to but not necessarily equal to the original radial function $b(r)$. As already discussed, this normalization factor may well have changed in the described process.

Because the value of the function $c$ could have been "infected" by the ghost mode, it is desirable to bypass the individual evaluation of $\eta$ and $s$, which would require an exact calculation of $c$. Rather, we will calculate the ratio $\eta/s$ from which the exact value of the normalization function $c$ actually drops.

The field equation Eq. (4.10) can be derived from an effective action with the following Lagrangian:

$$\mathcal{L}_{eff} \;=\; -\frac{1}{2} \left( 1 + \epsilon c \right) \overline{g}^{pq} \overline{\nabla}_p h^{xy} \, \overline{\nabla}_q h_{xy} + \cdots \;. \tag{4.11}$$

Since this is now a two-derivative theory of gravity, we can directly invoke Eq. (3.1) to determine the shear viscosity; hence, given $c$, $\eta$ goes as $(1 + \epsilon c)\eta_E$.

Now, what about the entropy density? Once the ghosts have been exorcized, the gravitational theory is Einstein's with a renormalized gravitational coupling. This follows from [47, 48, 49], where it has been made clear that a consistent two-derivative theory for a massless spin-two field is uniquely described by Einstein gravity. In general, differently polarized gravitons would "perceive" different values for the couplings. However, for a consistent two-derivative theory, there can be no such distinction between the polarizations [47, 48, 49]. So that, by eliminating the ghosts, we have also rendered gravity to be polarization indifferent.

We can then call upon the analysis from Section III of [28], where the Wald entropy has been reinterpreted as an effective coupling for the $r, t$-polarized gravitons, to deduce that $s$ goes as $(1 + \epsilon c)s_E$. Meaning that the ratio $\eta/s$ must necessarily be equal to the Einstein value of $1/4\pi$. That is, the "ghost-busted" theory may have a shifted value of $\eta$, but it comes with the assurance that $s$ will be corrected from its Einstein value by the very same amount.

## 4.2 Comparison to another choice of boundary conditions

We would like to be precise on how our newly proposed prescription differs from those previously used; in particular, the effective-action method [33] and those similar to it (*e.g.*) [36]. Given a four-derivative theory, a prescription for calculating $\eta$ is a three-step process: (i) find the equations of motion, (ii) solve the equations of motion, (iii) decide on how to extract the viscosity. There obviously can be no dispute on the first step and, given that the action has been consistently reduced to a two-derivative form, there should be no contention about the third. Where our new prescription then differs is only in the second step, where a clear distinction has been made between the appropriate choice of BC's and, hence, on the resulting solution. It is interesting to compare these two sets of solutions in detail, as we do next.

Let us begin here by recalling Eqs. (4.10) and (4.2); these being the field equations for the (post-normalized) massless graviton $H_0$ and the ghost graviton $H_1$,

respectively:

$$(1 + c)\Box_E H_0 \;=\; 0 \;, \tag{4.12}$$

$$\left[\epsilon a \Box_E + \frac{1}{L^2}\right] H_1 \;=\; 0 \;. \tag{4.13}$$

We want to compare these equations with those obtained in the "usual" manner by which one expands the solution according to perturbative order (rather than isolating the degrees of freedom). Let us label the zeroth-order and first-order modes respectively as $h_0$ and $\epsilon h_1$. Then, the previous field equations translate into

$$\Box_E h_0 \;=\; 0 \;, \tag{4.14}$$

$$\left[\epsilon a L^2 \Box_E + \epsilon b\right] \Box_E h_0 \;+\; \epsilon \Box_E h_1 \;=\; 0 \;. \tag{4.15}$$

The standard choice of BC's [45] is incoming at the horizon and Dirichlet at the outer boundary for both $h_0$ and $h_1$.

To avoid clutter, let us consider the simple but instructive case of $b = c$. Comparing the two sets of field equations, one finds that compatibility (to first-perturbative order) requires the following dictionary:

$$h_0 \;=\; H_0 + \frac{L^{-2}}{\Box_E} H_1 \;, \tag{4.16}$$

$$h_1 \;=\; -a L^2 \Box_E H_0 - \frac{L^{-2}}{\Box_E} b H_1 \;, \tag{4.17}$$

so that the two sets of modes share a non-local relationship. Now, it follows from the above equations and $\Box_E \sim \epsilon$ (so $\frac{1}{\Box_E} \sim \frac{1}{\epsilon}$) that $H_0$ is $h_0$ with a perturbative correction whereas $H_1$ is strictly is of order $\epsilon$, as it must be since the ghost is required to vanish in the Einstein or $\epsilon \to 0$ limit.

Comparing our exorcizing BC's to the standard set, we see that the difference between them is quite subtle, as it only appears at order $\epsilon$ for both $h_0$ and $h_1$. From the discussion in the preceding subsections, it is clear that the values of $\eta$ and of $\eta/s$ are sensitive to the choice of BC's. In particular, an order-$\epsilon$ difference in the BC's for the massless graviton results in an equally small but still significant change in the value of $\eta/s$.

## 4.3 General arguments

For theories leading to equations of motion with more than four derivatives, it is quite possible that not all of the additional degrees of freedom are ghosts (see the discussion on six-derivative equations at the end of this section); meaning that the ghost-reduced action need not be limited to two derivatives nor even be local. The latter because our prescription would, in general, yield a non-local field equation due to summing over two or more inverted operators.

For such cases, we propose proceeding as follows: First, after re-inverting the sum of the non-ghost propagators, one should convert this ghost-free field equation into a local form by expanding the propagators of massive modes (with masses $M_i$) in terms of $\Box/M_i^2 \ll 1$. Then, following the effective-action method [33], one should reduce the order of this field equation by treating the higher-derivative terms perturbatively while iteratively applying a lower-order field equation and the BC's. However, the initial field equation for starting the iterative purposes is, in general, no longer that of the background. It is, rather, the equation one obtains by perturbatively expanding and then suitably truncating the ghost-free field equation. Finally, one should, as before, translate the reduced field equation into a two-derivative effective action and then apply Eq. (3.1).

One might then wonder as to how $\eta/s$ would turn out for such higher-derivative theories. We will argue below that, on general grounds, this ratio can now change but only in such a way that it increases relative to its Einstein value

We have advocated elsewhere [15] that any gravitational coupling, such as $\eta$, can only increase from its Einstein value for a unitary extended theory. This is because any consistent extension will necessarily introduce new degrees of freedom to supplement the Einstein graviton. Given that these are unitary, the extended theory can only introduce additional channels that act to increase the couplings.

Let us recall the relevant discussion from [15]. Given our gauge choice, the $h_{xy}$ gravitons can only couple linearly to other particles of spin 2 *and* of the same polarization. Near the horizon, the associated 1PI graviton propagator [11] takes on a

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[11]See (*e.g.*) [18] for the fully decomposed form of the graviton propagator. Note that this flat-

particularly simple form:

$$\langle h_x{}^y(q) h_y{}^x(-q) \rangle = \frac{\rho_E(q^2)}{q^2} + \sum_i \frac{\rho_i(q^2)}{q^2 + m_i^2} , \qquad (4.18)$$

where $q^2 = -q^\mu q_\mu$ is the spacelike momentum, $\rho_E$ is the gravitational coupling for the Einstein graviton, and the $\rho_i$'s are the couplings for the additional spin-2 particles of an extended theory — any of which could be massive ( $m_i^2 \neq 0$ ) or massless ( $m_i^2 = 0$ ). [12] The couplings can, as indicated, depend on the energy scale $q$ ; and we work in units such that $\rho_E(0) = 1$ correctly fixes the Newtonian force at large distances.

The essential point here is that the couplings are really spectral densities of the schematic form $\rho = \sum_n \langle 0|h|n\rangle \langle n|h|0\rangle$ [51] and, as such, assured to be positive as long as all the inserted states have a positive norm. Meanwhile, the Einstein coupling can itself be modified in either direction (*cf*, Eq. (4.10)), but this is an illusionary effect, as one would always recalibrate her instruments to maintain the $\rho_E(0) = 1$ normalization. In any event, any such modification to the Einstein coupling would immediately cancel out of the ratio $\eta/s$, as Einstein gravity must be insensitive to the polarization of gravitons (which is just a restating of the equivalence principle). Conversely, the "non-Einstein" gravitons are generally sensitive to the polarization but can still only act to increase $\eta/s$ from the Einstein value of $1/4\pi$.

To sum up, the microscopic theory is, on the basis of unitarity, telling us that $\eta/s$ can only increase from the Einstein value. This point allows us to clarify what was previously meant by "considerations of unitarity". For addressing such matters of principle, a Planck-scale ghost is obviously unreliable, as would be the case for any mode near or above the effective-theory cutoff. On the other hand, including

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space form of the propagator is appropriate near any horizon — including that of a brane in AdS space — as any horizon has an effectively flat geometry. This is so because $\Box_E \propto -\partial_t^2 + \partial_{r_*}^2$ as $r \to r_h$ (where $r_* \equiv \int dr/F$ is a generalized "Tortoise" coordinate).

[12]By the same reasoning as in the prior footnote, any particle near the horizon is effectively massless and "perceives" an effective $2D$ geometry. Hence, one can set all the $m_i^2$ to zero and need only consider the $t$ and $r_*$ components of $q^\mu$. Note, though, that the full $5D$ tensorial structure of the gravitons is still maintained. See [50] for further explanation.

the ghost in the calculation enables $\eta/s$ to change freely in any direction. Hence, we conclude that, to obtain results from the effective theory that are consistent with microscopic unitarity, it is necessary to impose BC's that eliminate these unreliable modes. Consistency then requires that a physically meaningful quantity such as $\eta/s$ should no longer be sensitive to cutoff-scale physics.

Applying the preceding ideas to our generic four-derivative model, we expect that the ratio $\eta/s$ must remain independent of the perturbative coefficient $\epsilon$ at linear order. If this were not so, one could always reverse the direction of the $\eta/s$ correction by simply changing the sign of $\epsilon$. Reassuringly, this is exactly what our revised prescription ensures! After all, the non-Einstein degree of freedom is inevitably a ghost (*cf*, Eq. (3.7)); so that, with our choice of BC's, the physically relevant theory is guaranteed to reduce to Einstein's.

### 4.4 Six and higher-derivative equations of motion

Let us now discuss how the situation that we have described in such detail for the case of four-derivative theories is modified for theories leading to six-derivative equations of motion. We will only provide a sketch, relegating a detailed discussion to a subsequent paper. Six or higher-derivative equations can be quite different from four-derivative theories. With six derivatives, for example, one would obtain two additional degrees of freedom, only one of which need be a ghost. To illustrate this, let us consider "toy-model" versions of a six-derivative linearized operator. For instance, $\mathcal{G}_1 \equiv \Box_E + \zeta\Box_E^2\left(c - \frac{1}{c}\right) - \zeta^2\Box_E^3$ (with $\zeta$ a perturbative coefficient, $c \neq 1$ parameterizing mass and units of $L = 1$), which factorizes to give

$$\frac{1}{\mathcal{G}_1} = \frac{1}{\Box_E} - \frac{c^2}{c^2 + 1}\frac{1}{\left[\Box_E + \frac{1}{c\zeta}\right]} - \frac{1}{c^2 + 1}\frac{1}{\left[\Box_E - \frac{c}{\zeta}\right]}. \tag{4.19}$$

In this case, both of the massive gravitons are ghosts and should be eradicated from the calculation. On the other hand, $\mathcal{G}_2 \equiv \Box_E - \zeta\Box_E^2\left(c + \frac{1}{c}\right) + \zeta^2\Box_E^3$ leads to

$$\frac{1}{\mathcal{G}_2} = \frac{1}{\Box_E} - \frac{c^2}{c^2 - 1}\frac{1}{\left[\Box_E - \frac{1}{c\zeta}\right]} + \frac{1}{c^2 - 1}\frac{1}{\left[\Box_E - \frac{c}{\zeta}\right]}. \tag{4.20}$$

This second case is more interesting because only one of the massive gravitons is a ghost, with the choice depending on whether $c$ is greater or less than unity. For such a model, it can then be expected that $\eta/s$ does change, as a new unitary degree of freedom would generally be sensitive to graviton polarization. Nonetheless, seeing that $\mathcal{G}_2$ only differs subtly from $\mathcal{G}_1$, one is now able to envision a scenario whereby the emergence of a new degree of freedom is correlated with a strictly positive correction to $\eta/s$. Provided that the ghosts have been properly handled, it is our contention that this is, indeed, what must happen. [13]

## 5. Gauss–Bonnet redux

There is still an important example to be dealt with; the Gauss–Bonnet gravity model $\mathcal{L}_E + \mathcal{L}_{GB}$ . This theory leads to a second, rather than fourth, order equation for the graviton as can be seen from Eq. (2.17). Hence, the issue of the existence of ghosts or choice of boundary conditions becomes irrelevant. Obviously, unitarity considerations cannot be applied in a useful way in this case.

Yet, we have found (as did past studies such as [7]) that

$$\frac{\eta_{GB}}{s_{GB}} = \frac{1}{4\pi}\left[1 - 8\lambda\right] \; , \tag{5.1}$$

as $s_{GB}$ is known to be functionally equivalent to $s_E$. Evidently, the ratio will decrease below $1/4\pi$ whenever $\lambda$ is positive. So, this theory seems to provide a counter-example to the main theme of our paper, as it apparently demonstrates a ghost-free theory for which the viscosity–entropy ratio can be less than its Einstein value. However, this conflict is only an apparent one, as we now explain.

There are two distinct ways of viewing the Lagrangian $\mathcal{L}_E + \mathcal{L}_{GB}$ ; as representing a theory unto itself (*i.e.*, distinct from Einstein's) or as a perturbatively corrected version of the Einstein Lagrangian. If one chooses the former point of view, then the

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[13]One might be concerned that $s$ — also being a type of gravitational coupling — should likewise increase, possibly faster than $\eta$. Nonetheless, we have shown elsewhere [15] that it is always possible to choose fields and coordinates such that $s$ is calibrated to its Einstein value. Then, since $\eta \geq \eta_E$ is a "gauge"-invariant statement, the previous claim follows.

microscopic theory is Gauss–Bonnet gravity and there can be no contradiction with what we have already said. All methods of calculations agree as they must, and the issue of imposing boundary conditions in a higher-derivative theory never arises.

On the other hand, from the effective field-theory perspective, Einstein's theory is viewed as the infrared limit of the quantum gravity theory and the various corrections ($\mathcal{L}_X$) are regarded as a consequence of integrating out heavy degrees of freedom. Then, even if the Lagrangian has the finely tuned Gauss–Bonnet form ( $\beta = -4\alpha = -4\gamma$ ) at some scale, it will be of the generic form at other scales and lead to quartic equations of motion for the graviton.

So the issue now becomes an order of limits: Should one first impose the Gauss–Bonnet fine-tuning condition and then calculate $\eta/s$, leading to Eq. (5.1) and possibly to values of $\eta/s$ lower than $1/4\pi$? Or should one first eliminate the ghosts from $\mathcal{L}_B$, $\mathcal{L}_C$, then impose the Gauss–Bonnet fine-tuning condition and only afterwards evaluate the ratio? If one has a reason to regard the Gauss–Bonnet theory as fundamental, then the former choice is correct. With this interpretation, Gauss–Bonnet gravity can no longer be regarded as a perturbative correction of Einstein's theory and, so, is outside of the scope of our previous claims. On the other hand, if the Gauss–Bonnet model is to be regarded as the leading-order modification of the Einstein Lagrangian, the latter choice should be made and the correct calculation leads to a renormalized Einstein theory with the outcome $\eta_{GB}/s_{GB} = 1/4\pi$ .


# 6. Concluding discussion

Let us summarize: We have shown that, for higher-derivative theories that extend Einstein gravity, earlier choices of BC's for the purpose of calculating the shear viscosity have resulted in contributions from ghosts to the value of $\eta$. We have proposed a different choice of BC's that decouples the ghosts. An immediate consequence of our proposal is that, when the extensions are limited to four derivatives, the ratio of shear viscosity to entropy density saturates its Einstein value to leading order in the strength of the corrections; that is, saturates the KSS bound. We have gone on to

argue, on general grounds related to unitarity, that our prescription can be expected to enforce the KSS bound for theories leading to six and higher-derivative equations of motion and to all orders in the strength of the corrections. In such general cases, the saturation of the bound should no longer persist, as any gravitational coupling would naturally increase (from Einstein's) after additional unitary degrees of freedom have been introduced into the gravitational sector. The validity of our claims under these more generic circumstances remains an interesting, open question, which we hope to address with explicit calculations.

## 7. Acknowledgments

## A. Previous proposals

Let us be more specific on how the shear viscosity has previously been evaluated for a higher-derivative theory. We begin with the "effective-action" method [33] and remind the reader that other approaches — in particular, the generalized canonical-momentum treatment of [36] — are know to yield identical results.

Let us consider the Lagrangian $\mathcal{L} = \mathcal{L}_E + \epsilon \mathcal{L}_X$, where $\mathcal{L}_X$ is an arbitrary four-derivative extension and $\epsilon$ controls the perturbation. To quadratic order in the $xy$ gravitons of the metric, $\mathcal{L}$ can be written as

$$\sqrt{-g}\mathcal{L} \;=\; \widetilde{A}(u)\phi''\phi \;+\; \widetilde{B}(u)\phi'\phi' \;+\; \widetilde{E}(u)\phi''\phi'' \;+\; \widetilde{F}(u)\phi''\phi' \;+\; \cdots, \qquad (A.1)$$

where terms of lower-derivative order, as well as terms vanishing in the hydrodynamic limit of vanishing frequency and transverse momentum, are denoted by "$\cdots$". In this appendix, we use the radial coordinate $u = 1/r^2$ and set $L = r_h = 1$ so that the horizon is at $u_h = 1$. Additionally, $\phi = h^y_x$, a prime denotes a differentiation with respect to $u$ and $\widetilde{A}, \widetilde{B}, \widetilde{E}, \widetilde{F}$ indicate model-dependent radial functions. (For Einstein gravity, $\widetilde{E} = \widetilde{F} = 0$.) The background metric (2.1) becomes

$$ds^2 = -\frac{f(u)}{u} dt^2 + \frac{du^2}{4u^2 f(u)} + \frac{1}{u} \left[ dx^2 + dy^2 + dz^2 \right] , \qquad (A.2)$$

with $f(u) = 1 - u^2$ and $f(1) = 0$.

As a first step, one integrates by parts (with the relevant surface terms presumed to exist [32]) to obtain

$$\sqrt{-g}\mathcal{L} = \widetilde{E}\phi''\phi'' + \left[ \widetilde{B} - \widetilde{A} - \frac{\widetilde{F'}}{2} \right] \phi'\phi' + \cdots . \qquad (A.3)$$

The associated field equation is then

$$\left[ \widetilde{E}\phi'' \right]'' + \left[ \widetilde{A} - \widetilde{B} + \frac{\widetilde{F'}}{2} \right] \phi'' + \left[ \widetilde{A} - \widetilde{B} + \frac{\widetilde{F'}}{2} \right]' \phi' + \cdots = 0 . \qquad (A.4)$$

Next, one is instructed to use the zeroth-order or Einstein field equation, $\phi'' = -Z'\phi' + \cdots$ with $Z \equiv [\ln(\sqrt{-g}g^{uu})]'$, to reduce the higher-derivative $\widetilde{E}$ term, as $\widetilde{E}$ is already of order $\epsilon$. Twice applying the zeroth-order equation in the prescribed manner, one ends up with

$$\left( \left[ (Z\widetilde{E})' - Z^2\widetilde{E} + \widetilde{B} - \widetilde{A} - \frac{\widetilde{F'}}{2} \right] \phi' \right)' + \cdots = 0 , \qquad (A.5)$$

and the omitted terms now also include those of order $\epsilon^2$.

Eq. (A.5) implies an effective Lagrangian of the following form:

$$\sqrt{-g}\mathcal{L}_{eff} = -\frac{1}{\overline{g}^{uu}} \left[ (Z\widetilde{E})' - Z^2\widetilde{E} + \widetilde{B} - \widetilde{A} - \frac{\widetilde{F'}}{2} \right] \overline{g}^{uu}\partial_u\phi\partial_u\phi + \cdots . \qquad (A.6)$$

The shear viscosity can be identified with the coefficient of the kinetic term for the $\phi$'s at the horizon [29, 30, 31], and so one would deduce that

$$\eta \propto -\frac{1}{\overline{g}^{uu}} \left[ (Z\widetilde{E})' - Z^2\widetilde{E} + \widetilde{B} - \widetilde{A} - \frac{\widetilde{F'}}{2} \right]_{|u\to 1} , \qquad (A.7)$$

where the horizon limit is imposed only at the end.

Let us confirm that this result is indeed equivalent to that obtained from the canonical-momentum formalism of Myers *et. al.* [36]. Generalizing the methodology of [30], these authors have essentially identified $\eta$ with the (inverse) ratio of $\phi$ to its canonical conjugate, as computed at the AdS boundary and in the hydrodynamic limit. A nice consequence of the latter limit is that the same identification can be made on any radial surface in the spacetime, including at the black brane horizon. In this manner, for a generic four-derivative theory, the shear viscosity has been formulated strictly in terms of the horizon values of $\widetilde{A},\widetilde{B},\widetilde{E},\widetilde{F}$ and the metric. Retrieving their formula from Eq. (3.22) of [36], we obtain (after some minor manipulations)

$$\eta \;\propto\; -\frac{1}{\overline{g}^{uu}}\left[Y\left(\frac{\widetilde{E}}{Y^2}Y'\right)' + \widetilde{B} - \widetilde{A} - \frac{\widetilde{F}'}{2}\right]_{\big|_{u\to1}} , \tag{A.8}$$

where $Y \equiv \sqrt{-\overline{g}_{tt}\overline{g}^{uu}}$ . Comparing the last two equations, one can see that it is enough to establish the equivalence of $\mathcal{E}_1 \equiv (Z\widetilde{E})' - Z^2\widetilde{E}$ and $\mathcal{E}_2 \equiv Y\left[\frac{\widetilde{E}}{Y^2}Y'\right]'$ on the horizon. For this purpose, it is useful to recall that $\widetilde{E}$ is the coefficient of the four-derivative term (*cf*, Eq. (A.1)); meaning that it generically contains two factors of $\overline{g}^{uu} \sim Y$ and is quadratically vanishing on the horizon. Hence, we can rather work with $W \equiv \widetilde{E}/f^2$ ( $f \sim Y$ ), with $W$ assured to be regular as $u \to 1$ .

Let us now specialize to the background metric of Eq. (A.2), although, as one can check, this is not a necessary requirement. Making the substitutions $\widetilde{E} = f^2W$ , $Z = \frac{f'}{f} - \frac{1}{u}$ , $Y = 2\sqrt{u}f$ into $\mathcal{E}_1$ and $\mathcal{E}_2$, one finds after simplifying that

$$\frac{1}{f}\mathcal{E}_1 \;=\; f''W \;+\; \left(f' - \frac{f}{u}\right)W' , \tag{A.9}$$

$$\frac{1}{f}\mathcal{E}_2 \;=\; \left(f'' - \frac{3}{4}\frac{f}{u^2}\right)W \;+\; \left(f' + \frac{1}{2}\frac{f}{u}\right)W' . \tag{A.10}$$

Finally, after imposing the horizon limit, one ends up with

$$\frac{1}{f}\mathcal{E}_{1,2} \;\to\; f''(1)W(1) \;+\; f'(1)W'(1) \tag{A.11}$$

in either case.

It is instructive to focus on a specific gravity theory. Let us consider, for instance, the Riemann (tensor) squared model or $\mathcal{L} = \mathcal{L}_E + \mathcal{L}_C \;=\; \mathcal{R} + 12 + \gamma\mathcal{R}_{abcd}\mathcal{R}^{abcd}$ , for

which the background metric (A.2) is appropriate and the explicit forms of $\widetilde{A},\widetilde{B},\widetilde{E},\widetilde{F}$ are already given in Eq. (3.22) of [36]. [14] Substituting these into Eq. (A.7) or Eq. (A.8) and then simplifying, we have

$$\eta \; \propto \; 1 \; + \; 8\gamma\left[f'(1) - f''(1)\right] \; , \tag{A.12}$$

where the Einstein term has been "normalized" to unity and note that the correction term is, for this particular background, a vanishing quantity.

Let us next consider the approach where one continues to identify $\eta$ with the horizon coefficient of the kinetic term for the $xy$-polarized gravitons in the original action. The premise being that the gravitational coupling for a given class of gravitons can be extracted from their kinetic terms [28] and that the shear viscosity is a measure of the coupling for the $xy$ gravitons. One can, in principle, make this identification by expanding out the Lagrangian to quadratic order in $h_{xy}$. It is, however, simpler to extract the kinetic term by a method proposed in [29], which can be viewed as a generalization of Wald's entropy formula [26, 27, 28] and is tantamount to varying the Lagrangian by $\mathcal{R}^{xy}{}_{xy}$ . For the just-discussed Riemann-squared model, this implies

$$\eta \; \propto \; 1 \; + \; 4\gamma\left[\overline{\mathcal{R}}^{xy}{}_{xy}\right]_{|u\to 1} \; = \; 1 \; - \; 4\gamma f(1) \; . \tag{A.13}$$

Here, the correction term is also vanishing but the functional form apparently differs from that of the previous computations. Nevertheless, we can help to establish the claimed equivalence (for four-derivative theories) as follows:

First, let us expand the curvature component out as $\overline{\mathcal{R}}^{xy}{}_{xy} = \overline{\mathcal{R}}^{x}{}_{x} - \overline{\mathcal{R}}^{xu}{}_{xu} - \overline{\mathcal{R}}^{xt}{}_{xt} - \overline{\mathcal{R}}^{xz}{}_{xz}$ . Next, we use the equality $\overline{\mathcal{R}}^{x}{}_{x} = \overline{\mathcal{R}}^{t}{}_{t}$ , which follows from the spacetime being static and Poincare invariance on the brane. Making this substitution and then expanding out $\overline{\mathcal{R}}^{t}{}_{t}$ in terms of its four-index constituents, we have $\overline{\mathcal{R}}^{xy}{}_{xy} = \overline{\mathcal{R}}^{tu}{}_{tu} + \overline{\mathcal{R}}^{ty}{}_{ty} + \overline{\mathcal{R}}^{tz}{}_{tz} - \overline{\mathcal{R}}^{xu}{}_{xu} - \overline{\mathcal{R}}^{xz}{}_{xz}$ . Straightforward evaluation at the horizon then leads to $\overline{\mathcal{R}}^{xy}{}_{xy} = 2\left[f'(1) - f''(1)\right]$ . Inserting this into Eq. (A.13), we verify a functional form for $\eta$ that agrees perfectly with Eq. (A.12).

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[14]Their $c_1$ is the same as our $\gamma$ and they denote $\widetilde{A},\widetilde{B},\widetilde{E},\widetilde{F}$ without tildes. Otherwise, the conventions are basically in agreement.

Such agreement does not generally persist for theories with six or more derivatives. For instance, let us consider the six-derivative corrected theory $\mathcal{L} = \mathcal{L}_E + \zeta \mathcal{R}_{abcd} \mathcal{R}^{cdbe} \mathcal{R}^a{}_e$, for which the effective-action method has already been used in [52] to obtain $\eta = \eta_E [1 - 32\zeta]$. On the other hand, extracting the shear viscosity from the real action as prescribed in [29], one rather finds that $\eta = \eta_E [1 - 16\zeta]$. Note that the equations of motion for this model are limited to terms with no more than four derivatives, so that the two methods can already be in disagreement for theories leading to four-derivative equations of motion.

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