Dynamic canonical suppression of strangeness in transport models

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It is investigated whether canonical suppression associated with the exact conservation of an U(1)-charge can be reproduced correctly by current transport models. Therefore a pion-gas having a volume-limited production and annihilation cross section for $\pi\pi \leftrightarrow K\bar{K}$ is simulated within two different transport prescriptions for realizing the inelastic collisions. It is found that both models can indeed dynamically account for the canonical suppression in the yields of rare strange particles.

I. INTRODUCTION

As the properties of hadronic gases provide important theoretical means of describing heavy ion collisions, there exist several models to address this problem. Among the most prominent are statistical thermal models for equilibrated hadronic gases.

One result is a suppression in the yields for rare particles when treating the conservation of a corresponding U(1)-charge exactly [1, 2, 3]. The importance of such a treatment is fortified by recent experimental results [4]. The suppression effect can eventually be formulated in terms of a volume-dependence [1].

A dynamical interpretation has recently been offered by the formulation and solution of kinetic master-equations, which are capable of covering the time evolution of the system as well [2].

In the following we will demonstrate that standard hadronic transport models dynamically show canonical suppression. More explicitly the U(1)-charge considered is the strangeness whose net value is taken to be zero throughout our calculations. Due to simplicity we consider only inelastic reactions of the type $\pi\pi \leftrightarrow K\bar{K}$, where the kaons and anti-kaons bear strangeness +1 and −1 respectively.

An abundant number of kaons justifies a grand canonical description of the system, in which the strangeness is conserved only on the average $\langle N_K\rangle - \langle N_{\bar{K}}\rangle = 0$. The density of particles having strangeness +1, the kaons, can then be computed

$$n_K^{\mathrm{gc}} = \frac{Z_K^1}{V}, \tag{1}$$

where Z_K^1 denotes the relativistic one-particle partion function for non-interacting particles (kaons in this case)

$$Z_K^1(V,T) = g_K \frac{VT}{2\pi^2} m_K^2 K_2\left(\frac{m_K}{T}\right). \tag{2}$$

A rare number of kaons however demands the conservation of strangeness to be treated exactly, i.e. $N_K - N_{\bar{K}} = 0$. A consistent realization of this constraint eventually leads to a canonical partition function describing the system [1, 3] and thus to the following kaon density

$$n_K^{\mathrm{c}} = \eta\; n_K^{gc}, \tag{3}$$

with the canonical suppression factor $0 \le \eta \le 1$ being

$$\eta = \frac{n_K^{\mathrm{c}}}{n_K^{\mathrm{gc}}} = \frac{I_1(x)}{I_0(x)} \tag{4}$$

and $x = 2Z_K^1$. Pursuing the kinetic master equation approach yields the same results for the equilibrated kaon densities [2].

Investigating the canonical suppression factor (4) more closely, using the asymptotic behaviours

$$\lim_{x\to\infty} \frac{I_1(x)}{I_0(x)} \to 1 \qquad\qquad \lim_{x\to 0} \frac{I_1(x)}{I_0(x)} \to \frac{x}{2}, \tag{5}$$

one finds that in the grand canonical limit the kaon density n_K is independent of the reaction volume, whereas in the canonical regime, with the number of kaons $\langle N_K\rangle \ll 1$, it scales linearly with the volume as $x \propto V$. This volume dependent behaviour of the kaon density (3) provides a convenient reference for comparison between simulation and theory.

II. DYNAMICAL SIMULATIONS

The simulation setup consists of a large box of 20 fm side length holding a relativistic gas of pions. This pion-gas provides a heat bath for a much smaller reaction volume of variable size centered within the large box. Inside this likewise box-shaped reaction volume processes $\pi\pi \leftrightarrow K\bar{K}$ are allowed, covering all possible isospin states of pions and kaons. The kaons are reflected by the walls of the reaction volume, whereas these walls are permeable for the pions. After equilibration, the kaon density within the reaction volume should be governed by (3) which holds as a reference for the transport model results. Therefore different sizes of the inner reaction volume are simulated and the number of kaons $\langle N_K\rangle$ is extracted by averaging over many timesteps. The timesteps are sufficiently large such that no correlations are to be expected.

We implemented this scenario using two different types of transport descriptions - the microscopic transport model UrQMD [5] and a realization of a stochastic transport model borrowed from a recent and elaborated parton cascade [6]. The former model makes use of a geometrical interpretation of cross sections in order to solve transport equations, whereas the latter relies on the calculation of transition probabilities.

The UrQMD model provides full space time dynamics for hadrons and strings. It is a non-equilibrium model based on the covariant propagation of hadrons and strings and the geometric interpretation of cross sections. For our studies it was modified such that only reactions $\pi\pi \leftrightarrow K\bar{K}$ together with all possible elastic $2 \leftrightarrow 2$ processes remained possible. We had to choose sufficiently small cross sections such that the mean free path remained large compared to the interaction length $\lambda_{\mathrm{m.f.p.}} = (n\sigma_{22})^{-1} \gg \sqrt{\sigma_{22}/\pi}$. Otherwise the difference in the collision times of the involved particles viewed from the lab frame causes a decrease in the collision rates as pointed out in [6]. As different densities are involved, forward and reverse reactions are not affected in the same way and the change in reaction rates leads to a shifted equilibrium value of the kaon density. For details see [7].

The simulation of $2 \leftrightarrow 2$ processes within the stochastic method is based on the calculation of a collision probability for each possible particle pair per unit volume Δx^3 and unit time Δt via

$$P_{22} = v_{rel}\sigma_{22}\frac{\Delta t}{\Delta x^3}. \qquad (6)$$

v_{rel} denotes the relative velocity and σ_{22} is the cross section for the considered $2 \leftrightarrow 2$ process. Any so obtained probability is then compared with a random number between 0 and 1 to decide whether the collision should take place or not.

The initial conditions in any model are chosen such that the pion gas acquires a temperature of $T = 170$ MeV. The appropriate number of pions and the total energy of the system are calculated via the use of a grand canonical partition function for pions alone. Our heat bath volume of 8000 fm^3 corresponds to a population of 1348 pions bearing a total energy of 747.5 GeV. Initially each pion is assigned the same fraction of the total energy, giving one half of the particles momenta in the positive x-direction, while the remaining particles start out bearing momenta in the negative x-direction. The spatial distribution is random.

A fundamental and necessary verification of the simulations' reliability is of course a check for thermalization. Figure 1 nicely demonstrates that the energy distribution of the pions becomes thermal in both models.

Figure 2 then displays our results in terms of the extracted kaon density plotted versus the reaction volume along with the theoretical predictions for the grand canonical (1) and canonical (3) behaviour. The minor fluctuations in the results provide a visual indication for the irrelevance of statistical errors. It is manifest that canonical suppression is reproduced by both models, i.e., as expected the kaon yield is suppressed with respect to the grand canonical limit for small reaction volumes and thus small numbers of produced kaons.

The canonical suppression for the number of kaons $\langle N_K\rangle$ being considerably smaller than 1, as pointed out in [2], originates dynamically from an enhancement of the annihilation process by $1/\langle N_K\rangle$ as compared to standard,

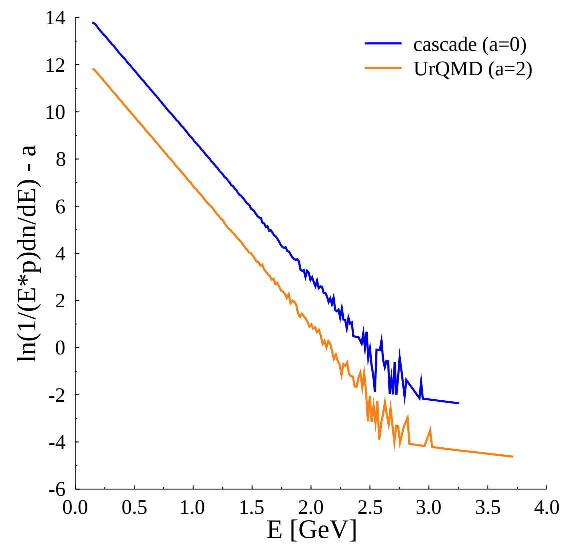

FIG. 1: Logarithmic energy spectra for the simulated pion-gases.

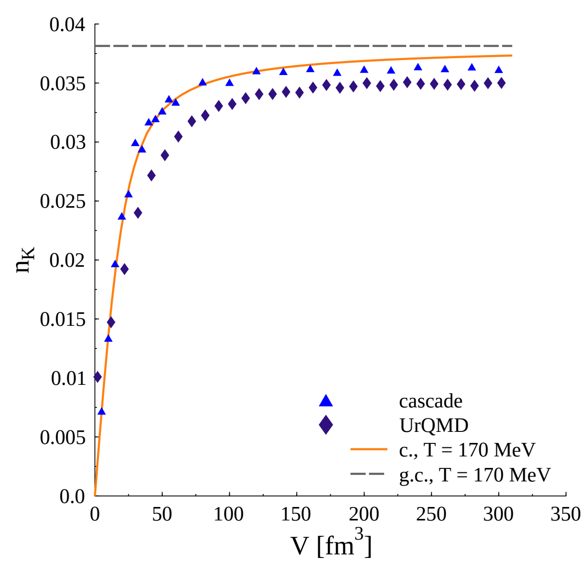

FIG. 2: Kaons density versus reaction volume as extracted from simulations with UrQMD (diamonds) and the stochastic cascade (triangles). For comparison the dashed line indicates the grand canonical behaviour. The solid line shows the canonical volume dependence of the kaon density as expected from (3) for a temperature of $T = 170$ MeV.

grand canonical formulation of the Boltzmann equation. The reason is that any kaon in the system requires the existence of a corresponding anti-kaon due to strangeness conservation. Thus the probability of finding a particle anti-particle pair turns into a highly correlated conditional probability. The so enhanced annihilation probability then leads to the canonical suppression in the kaon yields. As shown by our results, this behaviour is automatically included in the considered transport models.

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