The Possibility of Introducing Additional Focusing Caused by the Circular Irises in Iris Loaded Accelerator Sections *

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Abstract

To reach high luminosities in future linear colliders short range wakes have to be controlled in the range of X-band frequencies or higher. Rectangular irises can be used to introduce strong focusing quadrupole-like rf-fields. Even circular irises in iris-loaded accelerator structures have the capability of focusing if the particle velocity differs from phase velocity. Theoretical investigations concerning the focusing strength to be expected are presented. Their applicability for linear colliders is discussed.

I. INTRODUCTION

Almost all schemes proposed for future linear colliders rely on travelling-wave iris structures. In order to reach the high luminosities required by experiments wake field effects must be taken into account. Since transversal wakes scale with ω^3 [1] colliders operating at X-band or higher have to care for single bunch beam breakup (SBBU). Additional focusing is required. This can be achieved in several ways. One is to use an external quadrupole system, another is to use microwave quadrupoles (MWQ) [2], [3], and in the special case of an X-band collider it is possible to use short sections of conventional iris structures forming a FOFO-lattice to provide focusing power.

II. THEORY

A. Forces on a Particle

We consider a conventional Iris Structure with circular aperture. If we restrict our considerations on points not far from the beam axis we can write the accelerating E_z-component of a TM_{01}-wave travelling through the structure (see Fig. 1.) as

$$E_z = E_0 \cos(\omega t - kz) \tag{1}$$

where $k = \omega/\beta c$ and $\beta = v_{ph}/c$. From $\mathrm{div}\,\vec{E}$ we get:

$$E_r = -\frac{kr}{2} E_0 \sin(\omega t - kz) \tag{2}$$

Looking for the H_φ-component we find from $\mathrm{curl}\,\vec{H} = \varepsilon \partial_t \vec{E}$:

$$H_\varphi = -\varepsilon \frac{r}{2} E_0 \sin(\omega t - kz) \tag{3}$$

Assuming the velocity of the particle $v_p = c$ and $\beta = 1$ we see an exact cancellation of electric and magnetic forces, $F = F_{el} + F_{mag} = 0$.

We now consider the situation when v_p is different from c and $\beta \neq 1$. The total force on a particle can then be expressed as (see also Fig. 2):

$$F = q \frac{\omega r}{2c} E_0 \left(\frac{\beta_p \beta - 1}{\beta} \right) \sin(k_0 z \delta) \tag{4}$$

where $k_0 = \omega/c$, $\delta = (\beta - \beta_p)/\beta\beta_p$, and q is the charge of the particle. Equation (4) holds for the case that the structure covered by the particle is very short or β and β_p do not differ very much.

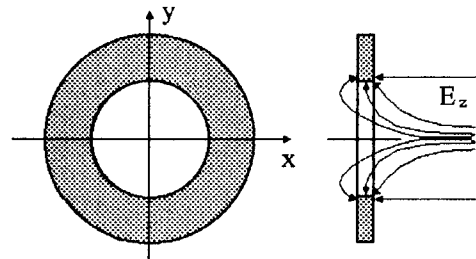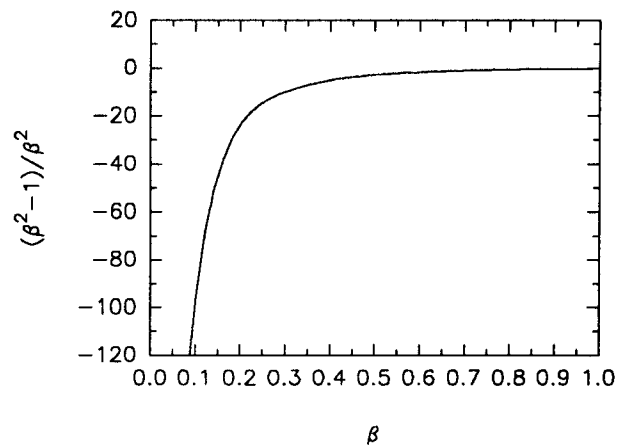Figure 1. Electric field at an iris. The cavity is operated in $\pi/2$-mode.

Figure 2. Total force on a particle plotted versus $\beta = \beta_p$. It can be seen that cancellation of electric and magnetic forces only takes place for velocities near c.

B. Focusing Properties

Since we have to deal with structures of finite length we have to take into account the time a particle needs to traverse the structure. Seeking the transversal momentum gain a charge q experiences while inside the accelerator section this leads to the following expression:

$$\Delta P_\perp = qE_0 \frac{\omega r}{2c^2} \left(\frac{\beta_p \beta - 1}{\beta_p \beta} \right) \frac{\sin(\frac{\omega\delta}{2c}\iota)}{\frac{\omega\delta}{2c}\iota} \sin\Phi \qquad (5)$$

where ι denotes the length of the cavity, and Φ the phase of the particle with respect to the rf. The velocity of the particle must not change while flying through the structure. This condition can be fulfilled by chosing $\Phi = 90°$ or the particle's velocity equal to c. In electron linacs the latter condition is fulfilled for almost every location along the accelerator. In order to achieve a non-zero transversal momentum gain towards the beam axis we look for the optimum velocity difference $\Delta\beta$, where $\beta = \beta_p + \Delta\beta$. Equation (5) can be rewritten:

$$\Delta P_\perp = q\frac{E_0 r}{c} \sin(\frac{\omega\iota}{2c}\tau) \sin(\Phi) \qquad (6)$$

where $\tau = \Delta\beta/(1+\Delta\beta)$. As can be seen the charge experiences a portion of a magnetic field of strength E_0/c. Assuming $E_0 = 100MV/m$ one gets 0.33T magnetic field strength. The maximum of $\Delta P_\perp$ is found for

$$\tau = -\frac{\pi c}{\omega\iota} \qquad (7)$$

where $\Phi = 90°$. It is also possible to choose $\Phi = -90°$ and therefore get a positive τ as long as a phase-slip occurs between particle and wave.

The longitudinal momentum gain ΔP_z is then given by:

$$\Delta P_z = 2q\frac{E_0}{\pi c}\iota \qquad (8)$$

The focusing uses up approximately one third of the maximum accelerating gradient for 0° rf-phase.

Provided that the path of the particle is not changed while traversing the cavity the structure can be considered a thin lens of focal length f.

$$f = -\frac{r P_z}{\Delta P_\perp} \qquad (9)$$

where r denotes the axis offset and P_z the longitudinal momentum of the particle. If it's energy is big compared to it's rest-mass this leads to:

$$f = \frac{U}{E_0 \sin(\frac{\omega\iota}{2c}\tau) \sin(\Phi)} \qquad (10)$$

Here U denotes the voltage seen by the particle. Taking an X-Band cavity of length $\iota = 0.5m$, $E_0 = 100MV/m$, and $U = 3GV$ one gets $\Delta\beta = -0.026$ and a focal length $f = 30m$.

The transversal momentum gain per unit charge is $\Delta P_\perp = 333.56Vs/m^2$.

C. FOFO-Lattice

In principle it is possible to construct a constantly focusing channel by adding up many of these sections. The accelerating gradient is then reduced by one third because this fraction of the rf is used to build up the focusing field. Another way is to arrange the cavities such that a FOFO-lattice is formed (see Fig. 3). The section consists of the focusing cavities F and the drift spaces O of length d.

We now look at the x-component (say) of the motion of a particle through a FOFO. We have to consider the following equation:

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} 1 & d \\ -\frac{1}{f} & 1-\frac{d}{f} \end{pmatrix} \begin{pmatrix} x_0 \\ x_0' \end{pmatrix} \qquad (11)$$

From this one can derive the cosine of the phase advance:
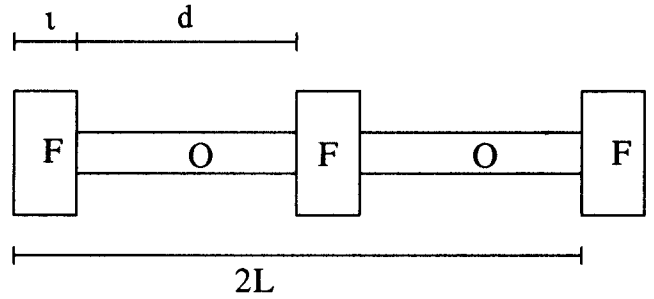
$$\cos(\mu) = 1 - \frac{d}{2f} \qquad (12)$$

Figure 3. Picture of a FOFO-lattice of structure length L. The focusing sections F are separated by drift sections O of length d.
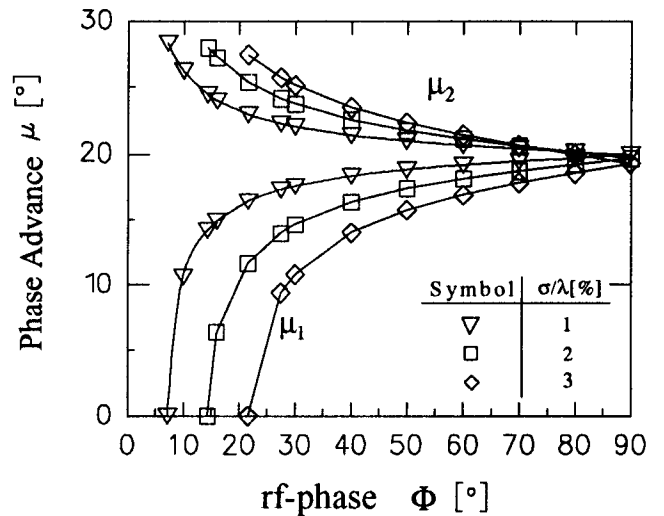

Figure 4. Phase advance for different Φ_0. Plotted for several values σ/λ.

The transversal momentum gain depends on the phase Φ at the entrance. We define the beginning and the end of a bunch of rms-length σ at $\pm 2\sigma$. The corresponding rf-phases are then $\Phi_0 \pm (4\pi\sigma/\lambda)$, where λ denotes the wavelength. Inserting (10) into (12) gives:

$$\frac{\sin^2(\frac{\mu_{1,2}}{2})}{\sin(\Phi_{1,2})} = \frac{\sin^2(\frac{\mu_0}{2})}{\sin(\Phi_0)} \qquad (13)$$

The phase advance of the head (μ_1)and tail particle (μ_2) in a bunch is drawn in Figure 4, where a $\mu_0 = 20°$ is chosen arbitrarily.

D. Additional External Focusing

By adding additional external focusing strength, e.g. by using external quadrupoles the resulting phase advance of the lattice is determined by the superposition of both focusing fields. One gets:

$$\sin^2(\frac{\mu}{2}) = \frac{L E_0}{4U} \left(\sin(\Phi) + \frac{L c \eta_e G_e}{2 E_0} \right) \qquad (15)$$

where η_e is the filling factor of external focusing system. By denoting external focusing strength by $m = L c \eta_e/2E_0$ one comes to an expresion similar to (14).
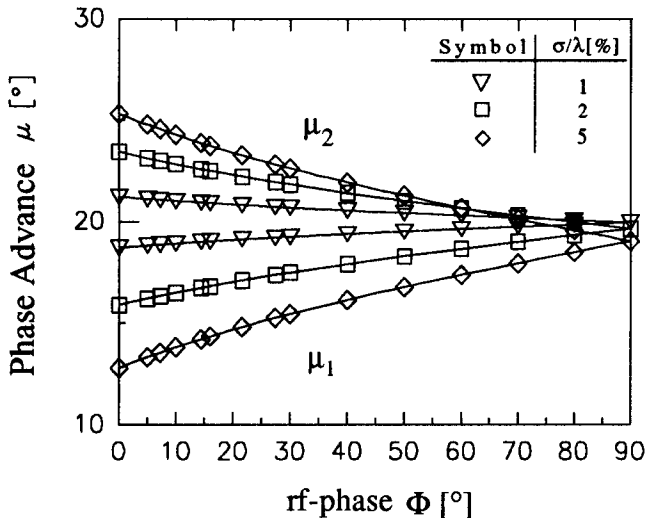

Figure 5. Phase Advance for different Φ_0. Plotted for several values σ/λ. External focusing strength is m = 1.

$$\frac{\sin^2(\frac{\mu_{1,2}}{2})}{m + \sin(\Phi_{1,2})} = \frac{\sin^2(\frac{\mu_0}{2})}{m + \sin(\Phi_0)} \qquad (16)$$

Taking m = 1 a rf-phase of 0° is possible. The Variation in phase advance over the bunch is 12.5° (see Figure 5). Taking $\mu_0 = 60°$ instead of 20° one gets 41° phase width which is only half the value in a quadrupole-FODO.

E. BNS-damping

Since focusing in these structures is phase dependent they can be used for BNS-Damping [4]. Combining (10) (maximum deflection) with (12) and differentiating with respect to the rf-phase (note that $d/d\Phi = (\lambda/2\pi)d/ds$) leads to the BNS-criterion:

$$2U k\beta \frac{dk\beta}{ds} = \frac{\pi\mu^2 E_0}{\lambda L \sin^2(\frac{\mu}{2})} \cos(\Phi) \geq q \frac{dW_\perp}{ds} \qquad (14)$$

It should be mentioned that the rf-wavelength is changed by a factor $1 + \Delta\beta$.

Taking a X-band structure at 11.4GHz of length $\iota = .25$m, $\Phi = 10°$, $L = 2$m, $E_0 = 100$MV/m, and $\mu = 20°$ this leads to $\Delta\beta = -5\%$ and a BNS-damping strength of $1.13 \cdot 10^{10}$Vm^{-3}.

III. DISCUSSION

Taking a X-band structure (e.g. NLC [5]) the rms bunch length is foreseen to be $\sigma = 0.1$mm which corresponds to a $\sigma/\lambda = 0.4\%$. The transversal wake potential can be approximately calculated to $eNdW_\perp/ds = 13.73$GV/m^3 [6] assuming an aperture of 8.6mm and a bunch charge of 1.44nC. The above example shows a BNS-damping strength which is nearly sufficient to compensate for transversal wakes. It is of course possible to change the example-lattice such that BNS-damping is increased (i.e. chose shorter L and ι). Still this type of focusing remains limited to frequencies not higher than X-band for the reason that there is a limitation in the possible accelerating gradient..

IV. REFERENCES

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