THE EFFECT OF A SINGLE HOM-DAMPER CELL WITHIN A CHANNEL OF UNDAMPED CELLS *

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Abstract

The effect of a single HOM-damper cell within a channel of undamped cells is described theoretically using an equivalent circuit model. From this a simple equation can be derived which relates the Q-value of the single damping-cell, the bandwidth of the passband under consideration, and the additional phase shift which is introduced by the damper cell to provide energy flow into the damper cell. This equation immediately shows the limitations of such single cell damping systems. Comparisons with experimental results are shown.

Introduction

The wake field effects in accelerator sections for future linear colliders will be reduced either by damping, by detuning or a combination of both [1, 2]. In the latter case it is foreseen to employ heavily HOM-damped cells within a stack of undamped ones. This leads to several problems concerning the propagation of energy of the higher order modes under consideration from the undamped cells into the damping system. For example it was not known which is the Q of the single damper cell to be provided in order to achieve maximum damping in a channel of given bandwidth. If the chosen Q-value of the damper cell is too low for a given bandwidth of a certain passband no energy flow will take place from the undamped cells into the damper cell. In this case the damper cell behaves like an obstacle and separates a structure into two nearly undamped parts oscillating independently. If the Q-value of the damper cell and the bandwidth of a given passband are well matched the presence of the damper cell introduces an additional phase shift from cell to cell and energy flow can take place. We will derive a simple equation which shows the correlation between Q-value of the single damper cell, the bandwidth of the passband, and the additional phase shift using an equivalent circuit model. We will apply this formula to three-, six- and twelve cell constant impedance structures loaded by a single wall slotted damping cell and will compare these calculations to experiments.

Theory

We consider a chain (see Fig. 1) of capacitively coupled circuits terminated by full end cells. We assume electric

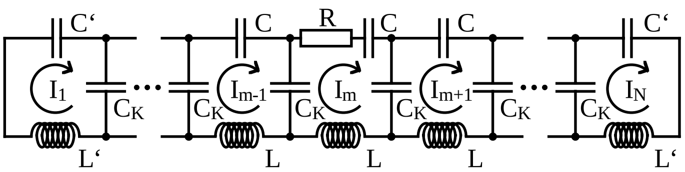

Fig. 1 Capacively coupled resonator model loaded by a single damping cell and with full end cells under electric boundary conditions.

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$$\left\{i\omega L + \frac{1}{i\omega C}\left(1 + \frac{C}{C_K}\right)\right\} I_1 - \frac{1}{i\omega C_K} I_2 = V_1 \qquad (1)$$

. . . .

$$\left\{i\omega L + \frac{1}{i\omega C}\left(1 + 2\frac{C}{C_K}\right)\right\} I_{m-1} - \frac{1}{i\omega C_K}\left(I_{m-2} + I_m\right) = V_{m-1} \qquad (2)$$

$$\left\{i\omega L + \frac{1}{i\omega C}\left(1 + 2\frac{C}{C_K}\right) + R\right\} I_m - \frac{1}{i\omega C_K}\left(I_{m-1}+I_{m+1}\right) = V_m \qquad (3)$$

$$\left\{i\omega L + \frac{1}{i\omega C}\left(1 + 2\frac{C}{C_K}\right)\right\} I_{m+1} - \frac{1}{i\omega C_K}\left(I_m+I_{m+2}\right) = V_{m+1} \qquad (4)$$

. . . .

$$\left\{i\omega L + \frac{1}{i\omega C}\left(1 + \frac{C}{C_K}\right)\right\} I_N - \frac{1}{i\omega C_K} I_{N-1} = V_N \qquad (5)$$

boundary conditions [3, 4] (the remaining iris of the left- and the right end cell is closed by a metallic surface). All circuits are lossfree except for the m^{th} cell which is loaded by a resistance R. L and C being the inductance and the capacitance of the circuits, C_K is the coupling capacitance. V_m corresponds to a driving voltage in the m^{th} circuit due to a antenna or the mirror charges of a passing particle and I_m is the current in the m^{th} circuit. In the special case, where the resistance of the damper cell is small there are N solutions of the homogeneous equations ($V_m = 0$) of the form [4]

$$I_m^n = A_n \cos\left(\Phi_n \left(m - \frac{1}{2}\right)\right) e^{i\omega_n t} \qquad \begin{matrix}(m = 1, 2, \dots, N)\\ (n = 0, 1, \dots, N-1)\end{matrix} \qquad (6)$$

where n is the mode and m the cell number. Φ_n is the phase shift per cell which is given by

$$\Phi_n = n\frac{\pi}{N} \qquad (n=0, 1, \dots, N-1) \qquad (7)$$

and

$$\omega_n = \omega_o \sqrt{1 + 4\frac{C}{C_K}\sin^2\left(\frac{\Phi_n}{2}\right)} \quad , \qquad \omega_o = \frac{1}{\sqrt{LC}} \qquad (8)$$

which is the dispersion relation for the chain of N capacitively coupled resonators (full end cells, elctric boundary conditions).

For further steps we need only equation (3) for the damper cell. Multiplying (3) by the conjugate complex current and comparing the real parts yields:

$$R\, I_m I_m^* - \mathrm{Re}\left(\frac{1}{i\omega C_K}\left(I_{m-1} I_m^* + I_{m+1} I_m^*\right)\right) = \mathrm{Re}\left(V_m I_m^*\right) \qquad (9)$$

This equation is a statement regarding energy flow. The first term on left hand side is the power dissipated in the damper cell, the second term on the left is the power transmitted from the (m-1)th cell and the (m+1)th cell to the mth cell by the coupling mechanism. The right hand side resembles the power fed into the damper cell due to an antenna or a particle passing. In the situation where there is no drive in the damper cell, we simply have:

$$R\, I_m I_m^* = \mathrm{Re}\left(\frac{1}{i\omega C_K}\left(I_{m-1} I_m^* + I_{m+1} I_m^*\right)\right) \qquad (10)$$

For all modes we assume, that the dominant change in the field due to the presence of the damper turns out to be a phase shift α_m. If we assume that the structure is excited only from the right end (antenna in the right end cell) we may then write for the current term on the right hand side:

$$I_{m-1} I_m^* + I_{m+1} I_m^* = a_{m-1} e^{i\alpha_{m-1}} a_m e^{-i\alpha_m} - a_{m+1} e^{i\alpha_{m+1}} a_m e^{-i\alpha_m} \qquad (11)$$

The negative sign on the right hand side indicates, that power is leaving the damper cell by the coupling mechanism. Introducing

$$\alpha_{m-1} - \alpha_m = \alpha_m - \alpha_{m+1} = \Delta\alpha \qquad (12)$$

leads to

$$I_{m-1} I_m^* + I_{m+1} I_m^* = a_{m-1}\, a_m e^{i\Delta\alpha} - a_{m+1}\, a_m e^{-i\Delta\alpha} \qquad (13)$$

a_m , a_{m+1}, and a_{m-1} are the unperturbed Amplitudes which are given by (6). Instead of (10) we then have:

$$R\, a_m^2 = \mathrm{Re}\left(\frac{1}{i\omega C_K}\left(a_{m-1}\, a_m e^{i\Delta\alpha} - a_{m+1}\, a_m e^{-i\Delta\alpha}\right)\right) \qquad (14)$$

In the case of a beam driving the channel there would be an equal amplitude in the (m-1)th and (m+1)th cell thus leading to a power flow into the damper from both sides.

$$R\, a_m^2 = \mathrm{Re}\left(\frac{1}{i\omega C_K}\left(a_{m-1}\, a_m e^{i\Delta\alpha} + a_{m+1}\, a_m e^{i\Delta\alpha}\right)\right) \qquad (15)$$

Using relationship (14) or (15) one can easily calculate the additional phase shift from cell to cell which is introduced by the damper cell to provide energy flow from the undamped cells into the damper cell. The resultant quality factor for the whole structure can be calculated using

$$Q_s = \omega \frac{\frac{1}{2} L \sum_{\nu=1}^{N} I_\nu I_\nu^*}{\frac{1}{2} R\, I_m I_m^*} = \frac{\omega}{\omega_o} Q \frac{\sum_{\nu=1}^{N} I_\nu I_\nu^*}{I_m I_m^*} \qquad (16)$$

For a specific example, consider 0-mode operation. According to equation (6) we have

$$a_m = a_{m-1} = a_{m+1} \qquad (17)$$

and we get

$$R = \frac{2}{\omega\, C_K} \sin(\Delta\alpha) \qquad (18)$$

or if we divide by ωL this relation becomes

$$\frac{1}{Q} = 2\left(\frac{\omega_o}{\omega}\right)^2 \frac{C}{C_K} \sin(\Delta\alpha) = K \sin(\Delta\alpha) \qquad (19)$$

where the bandwidth K is defined by

$$K = 2\,\frac{|\omega_\pi - \omega_o|}{\omega_\pi + \omega_o} \approx 2\,\frac{C}{C_K} \qquad (20)$$

and

$$\omega = \omega_o \qquad (21)$$

Depending on the chosen Q-value of the single cell damper and the given bandwidth of the passband under consideration we get to different cases. If the product

$$Q\,K \geq 1 \qquad (22)$$

becomes a number larger or equal to one equation (18) provides a proper phase shift between 0- and 90 degrees from cell to cell.

In this case an energy transport from the undamped cells into the damper cell is possible and the damping system works. If the product

$$Q\,K < 1 \qquad (23)$$

becomes a number lower than one, equation (19) does no longer hold, i.e.there exist no phase shift to provide energy transport into the damper cell. In this case it is behaving like an obstacle. This case corresponds to the situation in which the quality factor of the damping cell is chosen too low for a given bandwidth and the possible group velocities in the passband which are connected to the phase shift are always too low to provide energy flow.

Furthermore from equation (19) we recognize that the introduced phase shift for a 0-mode in a forward coupling passband is always positive and any number between 0- and 90 degrees.

Let us now consider a more complex example, the $2\pi/3$-mode. Then we have

$$a_{m-1}\, a_m = C^2 \cos\left(\frac{2\pi}{3}\left(m - \frac{3}{2}\right)\right) \cos\left(\frac{2\pi}{3}\left(m - \frac{1}{2}\right)\right)$$

$$a_{m+1}\, a_m = C^2 \cos\left(\frac{2\pi}{3}\left(m + \frac{1}{2}\right)\right) \cos\left(\frac{2\pi}{3}\left(m - \frac{1}{2}\right)\right)$$

Thus we get

$$R = \mathrm{Re}\left(\frac{1}{i\omega C_K}\left(\frac{\cos\left(\frac{2\pi}{3}\left(m - \frac{3}{2}\right)\right)}{\cos\left(\frac{2\pi}{3}\left(m - \frac{1}{2}\right)\right)} e^{i\Delta\alpha} - \frac{\cos\left(\frac{2\pi}{3}\left(m + \frac{1}{2}\right)\right)}{\cos\left(\frac{2\pi}{3}\left(m - \frac{1}{2}\right)\right)} e^{-i\Delta\alpha}\right)\right)$$

Here the phase shift depends on the position of the damper.

If the damper is located in a position with the maximum field strength we have to use

$$\frac{1}{Q} = -2 \left(\frac{\omega_o}{\omega_{2\pi/3}}\right)^2 K \sin(\Delta\alpha) \tag{24}$$

and in the position of half maximum field strength of the unperturbed mode we have to use

$$\frac{1}{Q} = -\frac{1}{2} \left(\frac{\omega_o}{\omega_{2\pi/3}}\right)^2 K \sin(\Delta\alpha) \tag{25}$$

where

$$\omega_{2\pi/3} = \omega_o \sqrt{1 + 3 \frac{C}{C_K}} \tag{26}$$

The negative sign in equations (24) and (25) leads to a negative phase shift which lowers the phase shift of the $2\pi/3$-mode to an appropriate value, i. e. the phase shift has always to be moved in the direction of larger group velocities, i.e. $\pi/2$-mode, that means for the $2\pi/3$-mode a lower phase shift from cell to cell. The phase shift of the $2\pi/3$-mode can become any number between 90- and 120 degrees.

Comparison with experimental results

For experimental proof of the theory three-, six- and twelve cell constant impedance structures [5], loaded by a heavily HOM-damped cell, where used. Cell geometry is identical to the DESY/THD-collider prototype. The damping system is a wall slotted iris. The slots lead into rectangular waveguides whose cut off frequency was chosen well above the fundamental passband.

We start our comparison with the 0-mode. In this case one can use the measured Q-value of the TM_{110}-pill box mode of the single damper cell directly, since the TM_{110}-pill box mode geometry remains nearly the same in a longer structure.

The Q-value of the TM_{110}-pill box mode of the strongest wall slotted damper cell which was applied to a three- and a six cell structure was in the region of 6. Using equation (19) and taking into an account that the bandwidth K of the second dipole passband is 0.1 leads to a phase shift of about

$$\Delta\alpha = 65^o \tag{27}$$

The resulting quality factor of the whole structure can be calculated applying equation (16) taking the new phase shift (27) into an account. The agreement between the measured- and the calculated Q_s-values can be obtained from Table 1. Furthermore we have applied a weaker damper cell with a Q-value of about 86 to a twelve cell structure. In this case we found a phase shift of about

$$\Delta\alpha = 3{,}3^o \tag{28}$$

for the 0-mode. Again applying formula (16) leads to a calculated quality factor Q_s = 980. A quality factor Q_s = 950 was measured.

Now we take a look on the $2\pi/3$-mode in the damped six cell structure. The $2\pi/3$-mode is a member of the first dipole

TABLE 1
Comparison of Measured- and Calculated Q_s-Values

structure	Q_s-measured	Q_s-calculated
3-cell structure	50	54
6-cell structure	510	480

passband which turns out to be of very low bandwidth. K is of the order of $1*10^{-2}$ which is of course a very small number. Furthermore the damper cell was located in a position with half of the maximum field strength. Thus we have to apply equation (25). If we take Q=6 equation (25) does no longer hold. No energy transport is possible. The structure separates into two nearly independently oscillating parts which could be confirmed by experiment.

Conclusion

Using an equivalent circuit model a formula giving the Q_s of a chain of coupled cells, loaded by a single damper cell, was derived. The method is easy to use and provides reasonable accurate results. It appears that the idea of introducing a phase shift between cells is successful in describing the behaviour of a stack of iris cells loaded by a single damper cell. The relation obtained shows immediately the properties and limitations of a single cell damping system.

Deviations between experiment and theory are due to the fact, that the simple equivalent circuit model takes only one space harmonic of the field geometry into account and thus changes in field geometry are not considered, i.e. the measured quality factor Q of the single damper cell which has to be inserted into equations (18), (24) and (25) is different for the 0-mode and for the $2\pi/3$-mode because of the changed mode geometry. For example the single cell Q-value of the damper cell has to be somewhat higher for the $2\pi/3$-mode than for the 0-mode since the strongest field strength for the $2\pi/3$-mode is mainly located within the iris whereas the field of the 0-mode is very similar to the TM_{110}-pillbox mode and thus the coupling to the waveguides is much stronger for the 0-mode.

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