Open and hidden charm production in heavy-ion collisions at ultrarelativistic energies

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Abstract
We consider the production of the open charm and J/ψ mesons in heavy-ion collisions at BNL RHIC. We discuss several recently developed pictures for J/ψ production and argue that a measurement at RHIC energies is crucial for disentangling these different descriptions.

1. Introduction

The production of charmonium states J/ψ and ψ' has been measured in nucleus–nucleus (A+A) collisions at CERN SPS over the last 15 years by the NA38 and NA50 Collaborations. This experimental programme was motivated by a suggestion of Matsui and Satz [1] to use J/ψ as a probe of the state of matter created at the early stage of the collision (see also [2] for a recent review).

In the original picture [1], it is assumed that J/ψ particles are created in primary nucleon–nucleon (N+N) collisions when the charm–anticharm quark pairs are produced. The fraction (a few per cent) of initial charm pairs which formed a J/ψ state is subsequently reduced in A+A collisions, because of the J/ψ suppression, by a factor which depends upon the details associated with the impact parameter and energy of the collisions.

The statistical approach, proposed in [3], assumes that J/ψ mesons are created at hadronization according to the available hadronic phase-space similar to other (lighter) hadrons. In this case, the J/ψ multiplicity is the same as in a hadron gas (HG) at complete chemical equilibrium. We call this an HG model.

Recently, a picture of the J/ψ creation via coalescence (recombination) of charmed quarks was developed within several model approaches [4–8]. Similar to the HG model [3], the charmonium states are assumed to be formed at the hadronization stage. However, they are produced as a coalescence of earlier created c and c̄ quarks. Therefore, the J/ψ yield and the total charm yield are connected to each other and differ in general from the HG equilibrium values.

A principal quantity to be studied in our paper is the ratio of the J/ψ multiplicity to the total number of c$\bar{\text{c}}$ pairs, $N_{c\bar{c}}$, created in A+A collisions:

$$R \equiv \frac{\langle J/\psi \rangle}{N_{c\bar{c}}}. \tag{1}$$

In the standard picture of J/ψ suppression, the number of primary J/ψ particles is proportional to the number of directly produced c$\bar{\text{c}}$ pairs with invariant mass below the D$\bar{\text{D}}$ meson threshold [9]. The fraction of the subthreshold pairs in the total number of $N_{c\bar{c}}$ decreases when the collision energy $\sqrt{s}$ grows. The number of initially produced J/ψ particles is further reduced because of their inelastic interactions with the nucleons from the colliding nuclei. This effect becomes stronger when the number of participants increases. In addition, J/ψ particles can be destroyed by the secondary hadrons (co-movers) created in the A+A collision. The density of these hadrons increases with both the number of nucleon participants (N_p) and $\sqrt{s}$. The decrease of the ratio R due to the above-mentioned effects is known as '*normal J/ψ suppression*'. The formation of quark–gluon plasma (QGP) at large values of $\sqrt{s}$ and N_p is supposed to lead to a sudden and strong drop of the ratio (1). This is called '*anomalous J/ψ suppression*'. Therefore, there is an unambiguous consequence of the standard picture [1, 2]: when the collision energy per nucleon pair $\sqrt{s}$ and/or the number of nucleon participants N_p increase, the ratio (1) decreases.

In models where the J/ψ mesons are produced by recombination of charm quarks at the hadronization stage, the picture can be much different. For the large number of c$\bar{\text{c}}$ pairs ($N_{c\bar{c}} \gg 1$) expected at the RHIC energies, the charmonium states can be formed from a c and $\bar{\text{c}}$ which were initially created in different c$\bar{\text{c}}$ pairs [8]. Therefore, the multiplicity of J/ψ due to recombination of charm quarks should be roughly proportional to

$$\langle J/\psi \rangle \sim \frac{(N_{c\bar{c}})^2}{N_{\text{tot}}}, \tag{2}$$

where N_{tot} is the total multiplicity of produced hadrons. In this case ratio (1) may be constant or even increase (we call this '*J/ψ enhancement*')[4] with N_p and/or $\sqrt{s}$ depending on the underlying physics which generates c$\bar{\text{c}}$ pairs and charmonium states.

We consider the canonical ensemble formulation [6] of the statistical coalescence model (SCM) [5] to describe the c and $\bar{\text{c}}$ recombination into the charmonium states at the hadronization stage. A canonical rather than the more familiar grand canonical formalism is required as we must properly account for the small numbers of charm quarks which are present. When a large number of charm quarks are produced in each collision, these two formalisms become equivalent, but the grand canonical formalism fails for small numbers. As in the standard picture, the charm is created at the very early stage of A+A reaction via hard parton collisions. The J/ψ formation is, however, very different in the standard picture of the J/ψ suppression [1] and in the SCM. This leads to very different relations between the open charm and charmonium yields.

In the analysis below we also present two possibilities of charm chemical equilibration. First, we assume that both the open charm and charmonium yields correspond to a chemically equilibrated hadronic gas. This scenario is the most extreme in its assumptions, but provides a benchmark for the dispersion of various theoretical results. Another possibility is that the number of c$\bar{\text{c}}$ pairs, which is an input for the SCM, equals the equilibrium value in the QGP just before its hadronization. These two possibilities are the extremes of scenarios with chemical

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[4] Please note that our definition of J/ψ enhancement does not involve comparison with proton–proton (p+p) collisions. We speak about relative J/ψ enhancement in (semi)central A+A collisions, i.e. in more central collisions with respect to less central ones, or at higher $\sqrt{s}$'s with respect to lower $\sqrt{s}$'s. It should be stressed that the growing value of the ratio R may remain smaller than the corresponding value of R in p+p at the same $\sqrt{s}$.

equilibration of charm. We do not pretend that these extreme cases really represent what happens in heavy-ion collisions. They are studied to complete our analysis. In fact, the hard parton collisions seem to be the dominant source of the $c\bar{c}$ pairs created at the RHIC energies.

The issue we want to address in this paper is how to disentangle the different scenarios of charm and charmonium production. It will be shown that the different pictures have a much different dependence on both N_p and $\sqrt{s}$. Distinctive signatures of each of these pictures will be found. We shall argue that J/ψ measurements at RHIC energies, especially in combination with a measurement of open charm, will go far towards resolving the various pictures. Of course, to really disentangle different scenarios, it is useful to be a little more quantitative. That is the central purpose of this paper.

The outline of this paper is as follows. In section 2 we present the equilibrium HG model for the open charm and charmonium production. The picture of the SCM is described in section 3. The results and conclusions of the paper are presented, respectively, in sections 4 and 5.

2. Hadron gas model

The equilibrium HG model describes the hadron yields measured in A+A collisions in terms of three parameters: volume V, temperature T and baryonic chemical potential μ_B. This model seems to work in the AGS–SPS energy region describing successfully not only the pion and (anti)nucleon yields but (multi)strange hadron abundances as well (see, e.g. [10]). For the RHIC energies the temperature parameter T is expected to be similar to that for the SPS energies: $T = 175 \pm 10$ MeV. The baryonic chemical potential becomes small ($\mu_B < T$) and decreases with the collision energy.

Does the HG model work for the charm and charmonium yields in A+A collisions? The answer to this question is rather uncertain at present. There are no open charm yield data measured in A+A collisions. This may be a reason why there were no attempts to calculate the open charm hadron yield within the HG model.

2.1. Grand canonical ensemble treatment of the hadron gas

The HG model assumes the following formula for the hadron thermal multiplicities in the grand canonical ensemble:

$$N_j = \frac{d_j V}{2\pi^2} \int_0^\infty p^2 \, dp \left[\exp\left(\frac{\sqrt{p^2 + m_j^2} - \mu_j}{T} \right) \pm 1 \right]^{-1}, \tag{3}$$

where V and T correspond to the volume[5] and temperature of the HG system, m_j and d_j denote hadron masses and degeneracy factors. The particle chemical potential μ_j in equation (3) is defined as

$$\mu_j = b_j \mu_B + s_j \mu_S + c_j \mu_C, \tag{4}$$

where b_j, s_j and c_j denote the baryonic number, strangeness and charm of particle j. The baryonic chemical potential μ_B regulates the baryonic density of the HG system whereas strange μ_S and charm μ_C chemical potentials should be found from the requirement of zero value for the total strangeness and charm in the system (in our consideration we neglect small effects of a non-zero electrical chemical potential).

[5] To avoid further complications we use the ideal HG formulae and neglect excluded volume corrections.

Some of the particles which are observed after a nuclear collision result from the decay of resonances which have decoupled after the matter fell out of equilibrium. The total multiplicities N_j^{tot} of such observed particles in the HG model are therefore the sum of

$$N_j^{\text{tot}} = N_j + \sum_R \text{Br}(R \to j) N_R, \tag{5}$$

where $\text{Br}(R \to j)$ are the corresponding resonance decay branching ratios (both strong and electromagnetic decays are taken into account). The temperatures and chemical potentials used in these relationships are those at decoupling. The hadron yield ratios $N_j^{\text{tot}}/N_i^{\text{tot}}$ in the grand canonical ensemble are therefore known functions of T and μ_B variables, and are independent of the volume parameter V.

For the thermal multiplicities of both open charm and charmonium states the Bose and Fermi effects are negligible. This is because the charm quark mass is so large that the system is always dilute for charm quark densities. Therefore, equation (3) is simplified to

$$N_j = \frac{d_j V \, \text{e}^{\mu_j/T}}{2\pi^2} T m_j^2 K_2 \left(\frac{m_j}{T}\right) \simeq d_j V \, \text{e}^{\mu_j/T} \left(\frac{m_j T}{2\pi}\right)^{3/2} \exp\left(-\frac{m_j}{T}\right), \tag{6}$$

where K_2 is the modified Bessel function. The HG model gives the J/ψ yield

$$N_{J/\psi}^{\text{tot}} = N_{J/\psi} + \text{Br}(\psi') N_{\psi'} + \text{Br}(\chi_1) N_{\chi_1} + \text{Br}(\chi_2) N_{\chi_2}, \tag{7}$$

where $N_{J/\psi}$, $N_{\psi'}$, N_{χ_1}, N_{χ_2} are calculated according to equation (6) and $\text{Br}(\psi') \simeq 0.54$, $\text{Br}(\chi_1) \simeq 0.27$, $\text{Br}(\chi_2) \simeq 0.14$ are the decay branching ratios of the excited charmonium states to J/ψ. Note that $T \simeq 170$–180 MeV leads to the HG value of the thermal ratio of $\langle \psi' \rangle / \langle J/\psi \rangle$

$$\frac{\langle \psi' \rangle}{\langle J/\psi \rangle} = \left(\frac{m_{\psi'}}{m_{J/\psi}}\right)^{3/2} \exp\left(-\frac{m_{\psi'} - m_{J/\psi}}{T}\right) = 0.04\text{–}0.05, \tag{8}$$

in agreement with the data [11] in (semi)central ($N_p \gtrsim 100$, which corresponds to the impact parameter range $b \lesssim 8$ fm) Pb+Pb collisions. This fact was first noticed in [12]. At small N_p as well as in p+p and p+A collisions the measured value of the $\langle \psi' \rangle / \langle J/\psi \rangle$ ratio is several times larger than its statistical mechanics estimate (8). Therefore, different versions of the statistical approach considered in this paper will be restricted to A+A collisions with $N_p > 100$. We do not intend here to describe the open and hidden charm production in p+p, p+A and in very peripheral A+A collisions within the statistical models.

Of course, the temperature of decoupling of the hadron gas is quite large, and one should properly be worried about the consistency of such a description. It nevertheless does provide a fair description of the available data on particle production. As such, it provides a standard against which interesting new physics might be calibrated.

2.2. The canonical ensemble for a hadron gas

The numbers of charm quarks produced in high-energy nuclear collisions need not be large. Implicit in the grand canonical ensemble treatment is this large-number assumption. The canonical ensemble treatment is valid for any small number of particles, and we now employ it to properly describe charm abundances.

In the canonical ensemble formulation the thermal charmonium multiplicities are still given by equation (6) for a grand canonical ensemble as the charmonium states have zero net charm. The multiplicities given by this equation (6) for open charm hadrons are modified by an additional multiplicative 'canonical suppression' factor. This has been shown, for example, in [13]. The suppression factor is the same for all individual open charm states. Therefore, if

N_O is the total grand canonical ensemble multiplicity of all open charm and anticharm mesons and (anti)baryons, then the canonical ensemble (c.e.) value of the total open charm is equal to

$$N_O^{c.e.} = N_O \frac{I_1(N_O)}{I_0(N_O)}, \tag{9}$$

where I_0, I_1 are the modified Bessel functions. To find N_O we use equation (6) for thermal multiplicities of the open charm hadrons in the grand canonical ensemble and take the summation over all known particles and resonances with open charm [14]. The factor $I_1(N_O)/I_0(N_O)$ in this equation is due to the exact charm conservation. For $N_O \ll 1$ one has $I_1(N_O)/I_0(N_O) \simeq N_O/2$ and, therefore, the canonical ensemble total open charm multiplicity $N_O^{c.e.} \sim N_O^2 \ll N_O$ is strongly suppressed in comparison to the corresponding grand canonical ensemble result. At SPS energies the canonical ensemble suppression effects, due to small numbers of produced charm, are important for the thermal open charm yield even in the most central Pb+Pb collisions. These suppression effects become crucial when the number of participants N_p decreases. Note that for $N_O \ll 1$ the multiplicities of the open charm hadrons are proportional to V^2 in the canonical ensemble HG formulation (instead of V in the grand canonical ensemble). In the opposite case $N_O \gg 1$, $I_1(N_O)/I_0(N_O) \rightarrow 1$; therefore the grand canonical ensemble and canonical ensemble results coincide: $N_O^{c.e.} \rightarrow N_O$. In a high-energy collision of heavy nuclei, the average numbers of produced (anti)baryons, (anti)strange hadrons and charged particles are much larger than 1. Due to this fact, the baryonic number, strangeness and electric charge of the HG system can be treated according to the grand canonical ensemble. This is not true in general for charm.

The number of $c\bar{c}$ pairs in the HG, $N_{c\bar{c}}^{HG}$, is given by

$$N_{c\bar{c}}^{HG} = \frac{1}{2} N_O \frac{I_1(N_O)}{I_0(N_O)} + N_H, \tag{10}$$

where N_H is the total HG multiplicity of particles with hidden charm.

We can now calculate the J/ψ multiplicity, since this involves only knowing the grand canonical ensemble results for $c\bar{c}$ bound states, and the total open charm multiplicity $N_O^{c.e.}$, since we now have the canonical ensemble results in hand (9). We need the chemical freeze-out parameters V, T, μ_B for A+A collisions. For the RHIC energies the temperature parameter T is expected to be similar to that for the SPS energies: $T = 175 \pm 10$ MeV. To fix the unknown volume parameter V and baryonic chemical potential μ_B we use the parametrization of the total pion multiplicity [15]:

$$\frac{\langle \pi \rangle}{N_p} \simeq C \frac{(\sqrt{s} - 2m_N)^{3/4}}{(\sqrt{s})^{1/4}}, \tag{11}$$

where $C \approx 1.46$ GeV$^{-1/2}$ and m_N is the nucleon mass. Equation (11) is in agreement with both the SPS data and the preliminary RHIC data in Au+Au collisions at $\sqrt{s} = 56$ GeV and $\sqrt{s} = 130$ GeV. The pion multiplicity (11) should be equated to the total HG pion multiplicity N_π^{tot} calculated according to equation (5) which accounts for resonance decays into pionic final states. The HG parameters V and μ_B are found then as the solution of the following coupled equations:

$$\langle \pi \rangle = N_\pi^{tot}(V, T, \mu_B) \equiv V n_\pi^{tot}(T, \mu_B), \tag{12}$$

$$N_p = V n_B(T, \mu_B), \tag{13}$$

where n_B is the HG baryonic density. In these calculations we fix the temperature parameter T.

The baryonic chemical potential for Au+Au collisions at the RHIC energies is small ($\mu_B < T$) and decreases with collision energy. Therefore, most of the thermal HG

multiplicities become close to their limiting values at $\mu_B \to 0$. Consequently, most of the hadron ratios $N_j^{\text{tot}} / N_i^{\text{tot}}$ become independent of the collision energy.

If we assume that the charm quarks reach their chemically equilibrated values, then the chemical potential for charm vanishes. We can then compute the abundances of charmonium. The ratio of J/ψ to negatively charged hadrons is only weakly dependent on the collision energy as a consequence of the considerations of the previous paragraph and is equal approximately to

$$\frac{N_{J/\psi}^{\text{tot}}}{\langle h^- \rangle} \simeq (0.6\text{–}1.5) \times 10^{-6}, \tag{14}$$

for $T = 170$ MeV and $T = 185$ MeV, respectively. The result (14) is close to the estimate [3, 16] of this ratio, 1.0×10^{-6}, extracted from the NA50 data in Pb+Pb collisions at 158 A GeV.

2.3. Summary of results

In this section, we developed the grand canonical ensemble and canonical ensemble formalism for describing abundances of particles in thermal equilibrium. Nucleon and pion abundances were used to constrain the parameters of these treatments: volume and baryon chemical potential. The temperature of the chemical freeze-out was assumed to be equal to its value at SPS energies. Formulae were derived for the abundances of open charm and hidden charm (c$\bar{\text{c}}$ states). In particular, the charm to negative charge particle multiplicity comes out correctly within such a naive treatment.

3. Statistical coalescence model

One has serious reservations about the concept of charm chemical equilibration within the HG approach. The HG system is expected to exist for a much shorter time than is needed for the chemical equilibration of charm. It is more reasonable to assume that the charm quark–antiquark pairs are created at the early stage of A+A reaction and the number of c$\bar{\text{c}}$ pairs remains approximately unchanged during the further evolution. After the hadronization, the system would correspond to the HG picture with a non-equilibrium value of the total charm.

We imagine two possible scenarios. In the first, we imagine the c$\bar{\text{c}}$ pairs reach their equilibrium values in the quark–gluon plasma which precedes the hadronic gas. We call this the QGP+SCM model, where the SCM means statistical coalescence model. In the second, we imagine the c$\bar{\text{c}}$ pairs are produced by hard QCD processes early in the collision, and are unchanged by the presence of the hadronic gas or the quark–gluon plasma. This scenario is referred to as the pQCD+SCM model.

3.1. Charm quarks not in chemical equilibrium

The SCM [5, 6] assumes that the charmonium states are formed at the hadronization stage. This is similar to the HG model [3]. However, the charmonium states are produced via a coalescence of earlier created c$\bar{\text{c}}$ quarks. The number of c$\bar{\text{c}}$ pairs, $N_{\text{c}\bar{\text{c}}}$, differs in general from the result expected in the equilibrium HG. One needs then a new parameter γ_c [5] to adjust the thermal HG results to the required number of $N_{\text{c}\bar{\text{c}}}$ fixed by the early stage evolution. In the grand canonical ensemble, $\gamma_c = e^{\mu/T}$, where μ is the chemical potential for c $+$ c̄ pairs. This is analogous to the introduction of the strangeness suppression factor γ_s [17] in the HG model, when the total strangeness observed is smaller than its chemical equilibrium value.

At high collision energies we find $\gamma_c > 1$ so that the open charm hadron yield is enhanced by a factor γ_c and charmonium yield by a factor γ_c^2 in comparison with the equilibrium HG predictions. The canonical ensemble formulation of the SCM, as shown in [6], gives

$$N_{c\bar{c}} = \frac{1}{2}\gamma_c N_O \frac{I_1(\gamma_c N_O)}{I_0(\gamma_c N_O)} + \gamma_c^2 N_H. \tag{15}$$

Note that the second term in the right-hand side of this equation is proportional to the number of hidden charm states, N_H, and is typically very small compared to the total number of charm quarks. It gives only a tiny correction to the first term. Most of the created $c\bar{c}$ pairs are transformed into the open charm hadrons.

If $N_{c\bar{c}} \gg 1$, equation (15) is simplified to [5]: $N_{c\bar{c}} = \gamma_c N_O/2 + \gamma_c^2 N_H$. This happens for central Au+Au collisions at upper RHIC energy $\sqrt{s} = 200\,\text{GeV}$. For lower RHIC energy $\sqrt{s} = 56\,\text{GeV}$ the value of $N_{c\bar{c}}$ could be close to (or even smaller than) unity so that the canonical ensemble suppression effects for the open charm are still important. Note that for the non-central A+A collisions the total charm essentially decreases so that the canonical ensemble suppression effects become stronger, hence their consideration is necessary to study the N_p dependence of the charmonium production even at the upper RHIC energy.

The above equation for the number of $c\bar{c}$ pairs will be used to find the charm enhancement factor γ_c and calculate the J/ψ multiplicity:

$$\langle J/\psi \rangle = \gamma_c^2 N_{J/\psi}^{\text{tot}}, \tag{16}$$

where $N_{J/\psi}^{\text{tot}}$ is given by equation (7), the equation which accounts for decays of states with hidden charm into the J/ψ.

3.2. Estimating the initial charm production

The number of $c\bar{c}$ pairs, $N_{c\bar{c}}$, in the left-hand side of equation (15) will be estimated in two different ways—in the pQCD approach, $N_{c\bar{c}} = N_{c\bar{c}}^{\text{pQCD}}$, and in the equilibrium QGP just before its hadronization, $N_{c\bar{c}} = N_{c\bar{c}}^{\text{QGP}}$—and then used as the input for the SCM.

The pQCD calculations for $c\bar{c}$ production cross section $\sigma(pp \rightarrow c\bar{c})$ in p+p collisions were first done in [18]. In [19] the cross section $\sigma(pp \rightarrow c\bar{c})$ was calculated in the next-to-leading order pQCD approximation and fit to the existing p+p data for charm production. We have parametrized the $\sqrt{s}$ dependence of the charm production cross section at $10 < \sqrt{s} < 200\,\text{GeV}$ as

$$\sigma(pp \rightarrow c\bar{c}) = \sigma_0 \left(1 - \frac{M_0}{\sqrt{s}}\right)^\alpha \left(\frac{\sqrt{s}}{M_0}\right)^\beta, \tag{17}$$

with $\sigma_0 \simeq 3.392\,\mu\text{b}$, $M_0 \simeq 2.984\,\text{GeV}$, $\alpha \simeq 8.185$ and $\beta \simeq 1.132$. For our fit we used the result of [19] obtained with GRV HO set of proton structure functions [20] for charm quark mass and renormalization scale $m_c = \mu = 1.3\,\text{GeV}$. Formula (17) agrees approximately with the existing data and leads to the value of $\sigma(pp \rightarrow c\bar{c}) \simeq 0.35\,\text{mb}$ at $\sqrt{s} = 200\,\text{GeV}$.

The number of produced $c\bar{c}$ pairs in A+A collisions is proportional to the number of primary nucleon–nucleon collisions:

$$N_{c\bar{c}}^{\text{pQCD}} = N_{\text{coll}}^{\text{AA}}(N_p) \frac{\sigma(pp \rightarrow c\bar{c})}{\sigma_{\text{NN}}^{\text{inel}}}, \tag{18}$$

where $N_{\text{coll}}^{\text{AA}}(N_p)$ is the number of primary nucleon–nucleon collisions, which depends on the number of participants and on the details of nuclear geometry and $\sigma_{\text{NN}}^{\text{inel}} \approx 30\,\text{mb}$ is the inelastic nucleon–nucleon cross sections.

For the most central A+A collision ($N_{\mathrm{p}} \simeq 2A$) the number of primary nucleon collisions can be estimated within the model of homogeneous spherical nucleus [21]:

$$N_{\mathrm{coll}}^{\mathrm{AA}}(N_{\mathrm{p}}) \simeq \frac{9}{8} \frac{A^2}{\pi R_A^2} \sigma_{\mathrm{NN}}^{\mathrm{inel}}, \tag{19}$$

with $R_A \approx 1.12 A^{1/3}$ fm. Therefore N_{p} dependence on the number of produced $c\bar{c}$ pairs can be calculated as[6]

$$N_{c\bar{c}}^{\mathrm{pQCD}} \simeq C \, \sigma(\mathrm{pp} \to c\bar{c}) N_{\mathrm{p}}^{4/3}, \tag{20}$$

where $C \approx 11$ barn^{-1}.

3.3. Initial charm from a chemically equilibrated QGP

We also make the SCM model calculations with $N_{c\bar{c}} = N_{c\bar{c}}^{\mathrm{QGP}}$ on the left-hand side of equation (15). The values of $N_{c\bar{c}}^{\mathrm{QGP}}$ in central Au+Au collisions at different $\sqrt{s}$ are calculated as the equilibrium number of $c\bar{c}$ pairs in the QGP just before its hadronization. The hadronization temperature of the QGP is taken to be equal to the chemical freeze-out temperature in the HG, i.e. we assume that the chemical freeze-out in the HG takes place just after the QGP hadronization. The volume V_{QGP} of the QGP is found then from the requirement of an approximate entropy conservation during the hadronization transition: $S_{\mathrm{QGP}} = S_{\mathrm{HG}}$ (the non-zero but small baryonic chemical potentials in the HG and in the QGP do not affect the QGP volume estimate). The calculation of $N_{c\bar{c}}^{\mathrm{QGP}}$ corresponds to the equilibrium ideal gas of quarks and gluons ($m_{\mathrm{u}} \approx m_{\mathrm{d}} \approx 0$, $m_{\mathrm{s}} \approx 0.12$ GeV, $m_{\mathrm{c}} \approx 1.3$ GeV). In the grand canonical ensemble, one finds for the number of c and $\bar{c}$

$$N_{\mathrm{c}} = N_{\bar{c}} = \frac{d_{\mathrm{c}} V_{\mathrm{QGP}}}{2\pi^2} T m_{\mathrm{c}}^2 K_2\left(\frac{m_{\mathrm{c}}}{T}\right), \tag{21}$$

where $d_{\mathrm{c}} = 6$ is the degeneracy factor for the c quark. Taking into account the exact charm conservation we obtain the number of $c\bar{c}$ pairs in the canonical ensemble:

$$N_{c\bar{c}}^{\mathrm{QGP}} = \frac{1}{2} (N_{\mathrm{c}} + N_{\bar{c}}) \frac{I_1 (N_{\mathrm{c}} + N_{\bar{c}})}{I_0 (N_{\mathrm{c}} + N_{\bar{c}})}. \tag{22}$$

4. Comparison of various mechanisms

In this section we present the results of the SCM and the HG model. Two different inputs for $N_{c\bar{c}}$ in the SCM (15) are studied: $N_{c\bar{c}}^{\mathrm{pQCD}}$ (20) and $N_{c\bar{c}}^{\mathrm{QGP}}$ (22). We call these model formulations as pQCD+SCM and QGP+SCM, respectively. The HG model, discussed in section 2, can be considered as a special case of the SCM when $\gamma_{\mathrm{c}} \equiv 1$. First, we consider central Au+Au collisions ($N_{\mathrm{p}} \simeq 2A$) at different collision energies $\sqrt{s}$. Keeping the temperature parameter T fixed we solve equations (12) and (13) to find the hadronization parameters V and μ_{B}. Equation (11) is used as the parametrization of the $\sqrt{s}$ dependence for the total pion multiplicity. With HG parameters T, V and μ_{B} we calculate the thermal multiplicities of individual open and hidden charm states in the grand canonical ensemble according to equation (6) and then find the grand canonical ensemble total open charm N_{O} and the total hidden charm N_{H} multiplicities taking the summation over all known open and hidden charm states. Using the calculated values of N_{O} and N_{H} we proceed further with three different model formulations along the scheme presented in table 1.

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[6] Strictly speaking, equation (20) does not exactly correspond to the real experimental situation. Au+Au collisions with different centrality rather than central collisions of nuclei with different atomic numbers will be studied at RHIC. Still, formula (20) gives almost the same numerical result as the more realistic calculations [22].
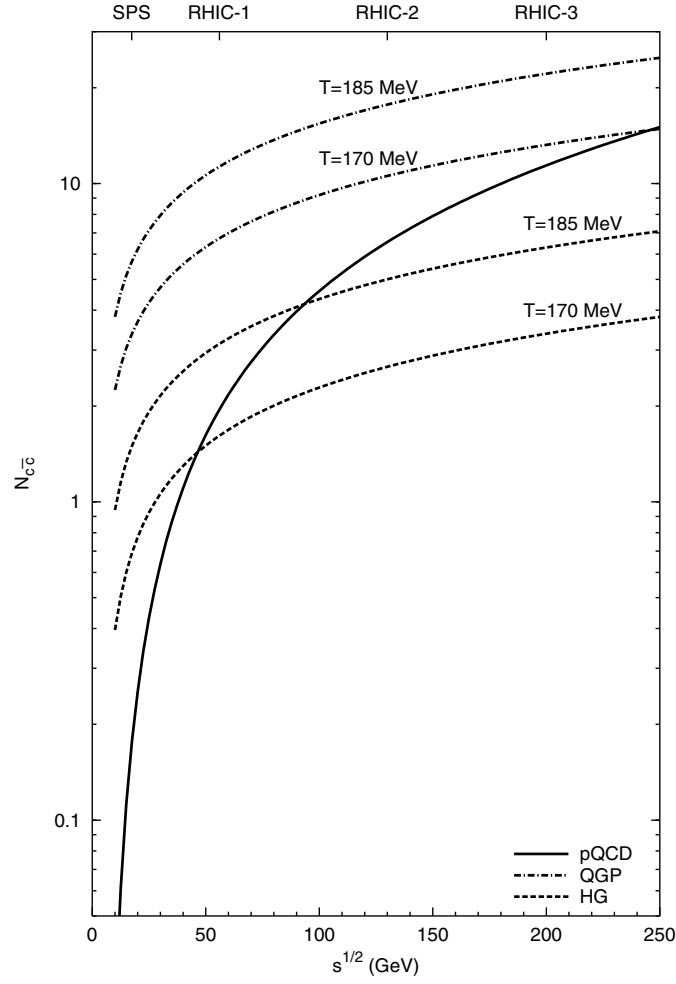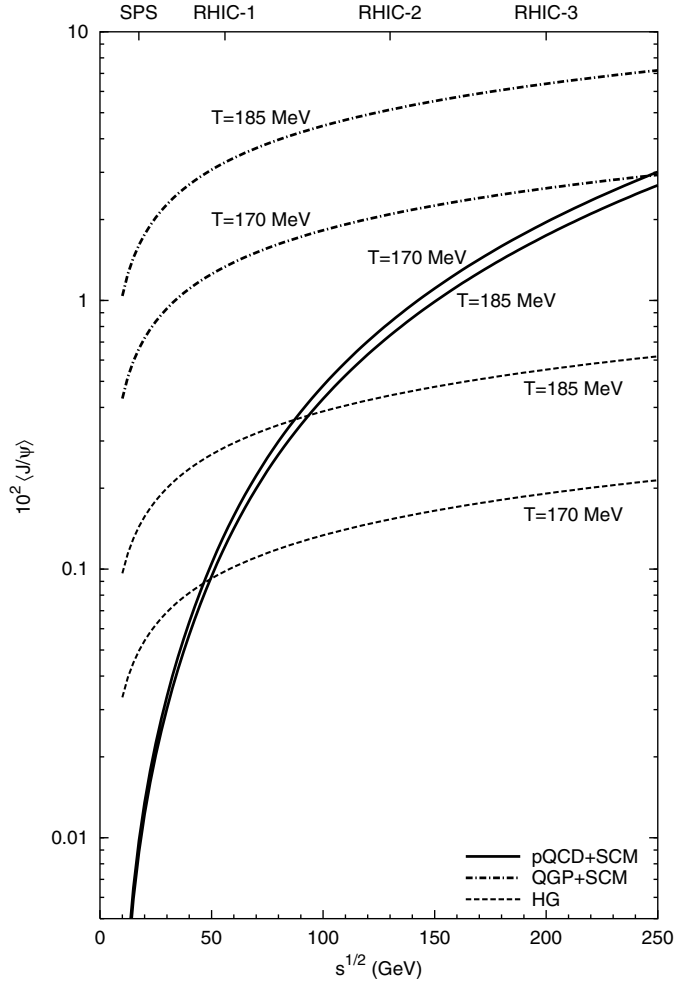
Figure 1. The dependence of the average number of $c\bar{c}$ pairs per Au+Au central collision on the centre-of-mass energy of the nucleon pair. The calculations are done in the framework of three different models: perturbative QCD (pQCD), equilibrium quark–gluon plasma (QGP) and equilibrium hadron gas (HG). The QGP and HG results are shown for two different temperatures: $T = 170 \, \text{MeV}$ and $T = 185 \, \text{MeV}$.

Table 1. Summary of the models.

	pQCD+SCM	QGP+SCM	HG
$N_{c\bar{c}}$	$N_{c\bar{c}}^{\text{pQCD}}$, equation (20)	$N_{c\bar{c}}^{\text{QGP}}$, equation (20)	$N_{c\bar{c}}^{\text{HG}}$, equation (10)
γ_c	Equation (15), $N_{c\bar{c}} = N_{c\bar{c}}^{\text{pQCD}}$	Equation (15), $N_{c\bar{c}} = N_{c\bar{c}}^{\text{QGP}}$	$\gamma_c \equiv 1$
$\langle J/\psi \rangle$	$\gamma_c^2 N_{J/\psi}^{\text{tot}}$, equation (16)	$\gamma_c^2 N_{J/\psi}^{\text{tot}}$, equation (16)	$N_{J/\psi}^{\text{tot}}$, equation (7)

4.1. Number of $c\bar{c}$ pairs as function of energy

The dependence of the number of $c\bar{c}$ pairs, $N_{c\bar{c}}$, on the collision energy is shown in figure 1 for different model formulations. The value of $N_{c\bar{c}}^{\text{pQCD}}$ (20) increases strongly with collision energy and at the highest RHIC energy it becomes several times larger than the thermal equilibrium value $N_{c\bar{c}}^{\text{HG}}$ in the HG model (10). At the SPS energies ($\sqrt{s} \approx 20 \, \text{GEV}$) the
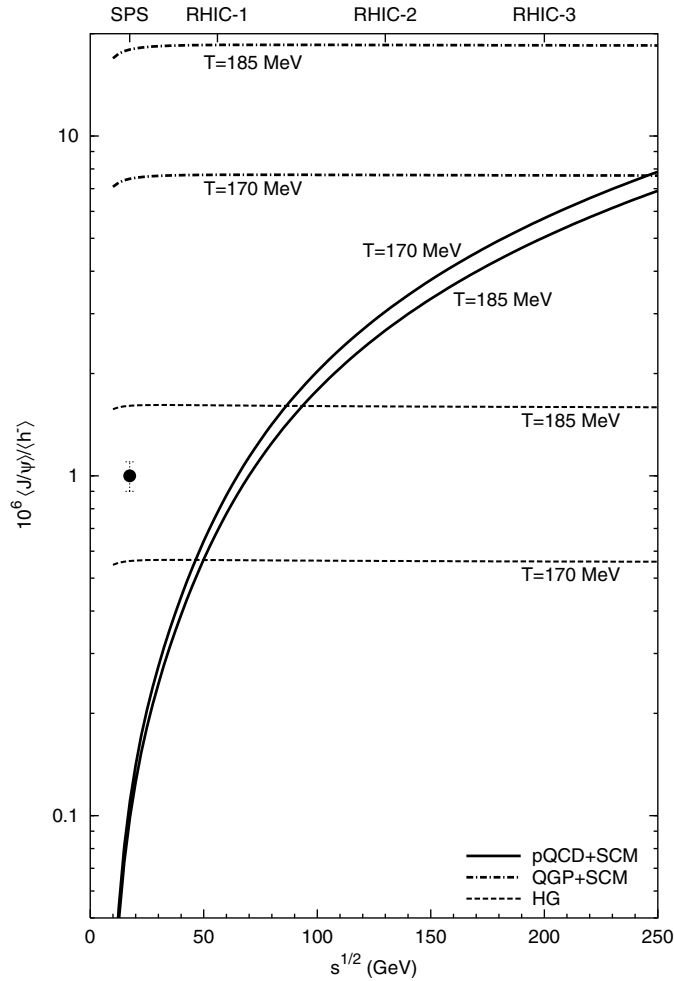
Figure 2. The dependence of the average number of J/ψ particles per Au+Au central collision on the centre-of-mass energy of the nucleon pair. The calculations are done in the framework of three different models: statistical coalescence model with the number of $c\bar{c}$ pairs given by perturbative QCD (pQCD+SCM), statistical coalescence model with the number of $c\bar{c}$ pairs given by equilibrium quark–gluon plasma (QGP+SCM) and equilibrium hadron gas model (HG). All the dependences are shown for two different temperatures: $T = 170\,\text{MeV}$ and $T = 185\,\text{MeV}$.

situation is however different: $N_{c\bar{c}}^{HG} > N_{c\bar{c}}^{pQCD}$. The value of $N_{c\bar{c}}^{QGP}$ defined by equation (22) seems to give an upper limit of the charm production in A+A collisions. The inequality $N_{c\bar{c}}^{QGP} \gg N_{c\bar{c}}^{HG}$ is mainly because of a small mass of charm quark, $m_c \simeq 1.3\,\text{GeV}$, in comparison to the masses of charmed hadrons (D, D*, etc).

4.2. J/ψ multiplicity versus energy

The J/ψ multiplicity is shown in figure 2 and the ratio of J/ψ to negative hadrons in figure 3. The experimental estimate of the J/ψ to h^- ratio extracted from the NA50 data in Pb+Pb collisions at 158 A GeV is also presented in figure 3. It seems to be in agreement with the simple HG results [3] (i.e. $\gamma_c \approx 1$). In the pQCD+SCM model the enhancement of

Figure 3. The dependence of the ratio of the J/ψ-multiplicity to the negative hadron multiplicity in Au+Au central collisions on the centre-of-mass energy of the nucleon pair. The calculations are done in the framework of three different models: statistical coalescence model with the number of $c\bar{c}$ pairs given by perturbative QCD (pQCD+SCM), statistical coalescence model with the number of $c\bar{c}$ pairs given by equilibrium quark–gluon plasma (QGP+SCM) and equilibrium hadron gas model (HG). All the dependences are shown for two different temperatures: $T = 170$ MeV and $T = 185$ MeV. The filled circle shows the value estimated from the NA50 experimental data for Pb+Pb collisions at the SPS energy [2, 14].

the total charm $N_{c\bar{c}}$ by a factor of about 3 is needed to explain the observed J/ψ multiplicity [6]. This prediction of the statistical coalescence model can be tested in the near future (measurements of the open charm are planned at CERN). Such a comparison will also require to specify more accurately the $\langle J/\psi \rangle$ data. A possible source of the open charm enhancement in A+A collisions at the SPS with respect to the direct extrapolation (20) from p+p to A+A may be the broadening of the phase space available for the open charm production due to the presence of the quark–gluon medium [23]. This effect, however, is expected to be small at RHIC energies.
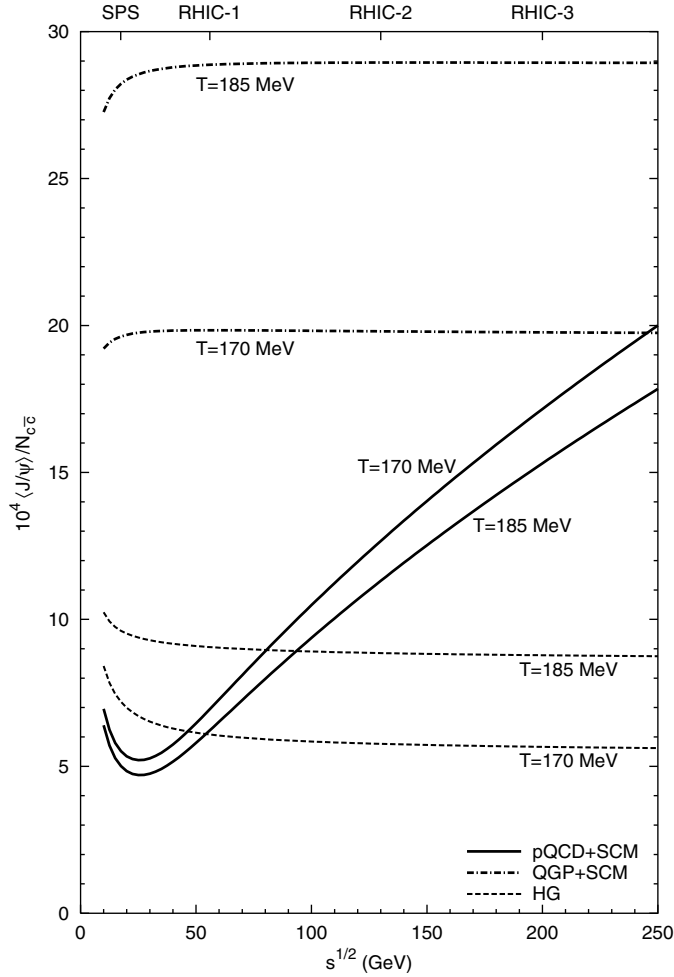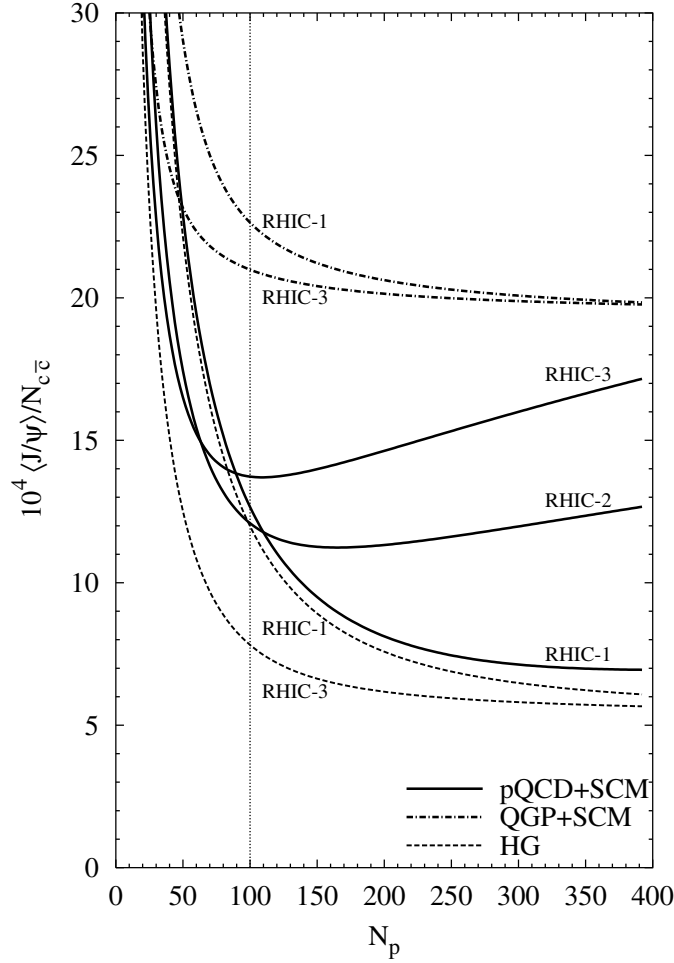
Figure 4. The dependence of the ratio of the J/ψ-multiplicity to the multiplicity of $c\bar{c}$ pairs in Au+Au central collisions on the centre-of-mass energy of the nucleon pair. The calculations are done in the framework of three different models: statistical coalescence model with the number of $c\bar{c}$ pairs given by perturbative QCD (pQCD+SCM), statistical coalescence model with the number of $c\bar{c}$ pairs given by equilibrium quark–gluon plasma (QGP+SCM) and equilibrium hadron gas model (HG). All the dependences are shown for two different temperatures: $T = 170\,\text{MeV}$ and $T = 185\,\text{MeV}$.

### 4.3. J/ψ multiplicity ratio to open charm

The dependence of the J/ψ to $N_{c\bar{c}}$ ratio on the collision energy is shown in figure 4. In the HG model and in QGP+SCM, the ratio of the J/ψ to the number of $c\bar{c}$ pairs is constant at high collision energies. This is because both the J/ψ multiplicity and the number of $c\bar{c}$ pairs are proportional to the system volume V. In contrast, the pQCD+SCM model predicts an essential increase of the ratio by a factor of about 3 from the lowest to the highest RHIC energy.

Results presented in figures 1–4 correspond to the most central Au+Au collisions ($N_p \approx 2A$). The dependence of the ratio R on the number of participating nucleons N_p is shown in figure 5. (Note that statistical models are expected to be valid for $N_p > 100$.)

Figure 5. The dependence of the ratio of the J/ψ-multiplicity to the multiplicity of $c\bar{c}$ pairs on the number of nucleon participants N_p for Au+Au collisions. The calculations are done in the framework of three different models: statistical coalescence model with the number of $c\bar{c}$ pairs given by perturbative QCD (pQCD+SCM), statistical coalescence model with the number of $c\bar{c}$ pairs given by equilibrium quark–gluon plasma (QGP+SCM) and equilibrium hadron gas model (HG). In pQCD+SCM case, the dependences are shown for three collision energies per nucleon pair: RHIC-1 ($\sqrt{s} = 56\,\text{GeV}$), RHIC-2 ($\sqrt{s} = 130\,\text{GeV}$) and RHIC-3 ($\sqrt{s} = 200\,\text{GeV}$). For QGP+SCM and HG, the results at RHIC-1 and RHIC-3 are presented. The freeze-out temperature was fixed at $T = 170\,\text{MeV}$. The models are assumed to be valid for $N_p \geqslant 100$, i.e. to the right from the dotted vertical line.

Within pQCD+SCM, one observes both the J/ψ *suppression* and the J/ψ *enhancement* effects: the ratio (1) decreases with N_p when the number of created $c\bar{c}$ pairs is small, $N_{c\bar{c}}^{pQCD} \lesssim 1$, but it begins to increase with N_p when $N_{c\bar{c}}^{pQCD}$ is essentially larger than unity. In the HG model and in QGP+SCM, the suppression is also observed at small N_p corresponding to $N_{c\bar{c}}^{pQCD} \lesssim 1$. In contrast to pQCD+SCM, the models with statistical charm production do not demonstrate J/ψ enhancement: at large N_p, which correspond to $N_{c\bar{c}}^{pQCD} \gg 1$, the ratio R tends to a constant value.
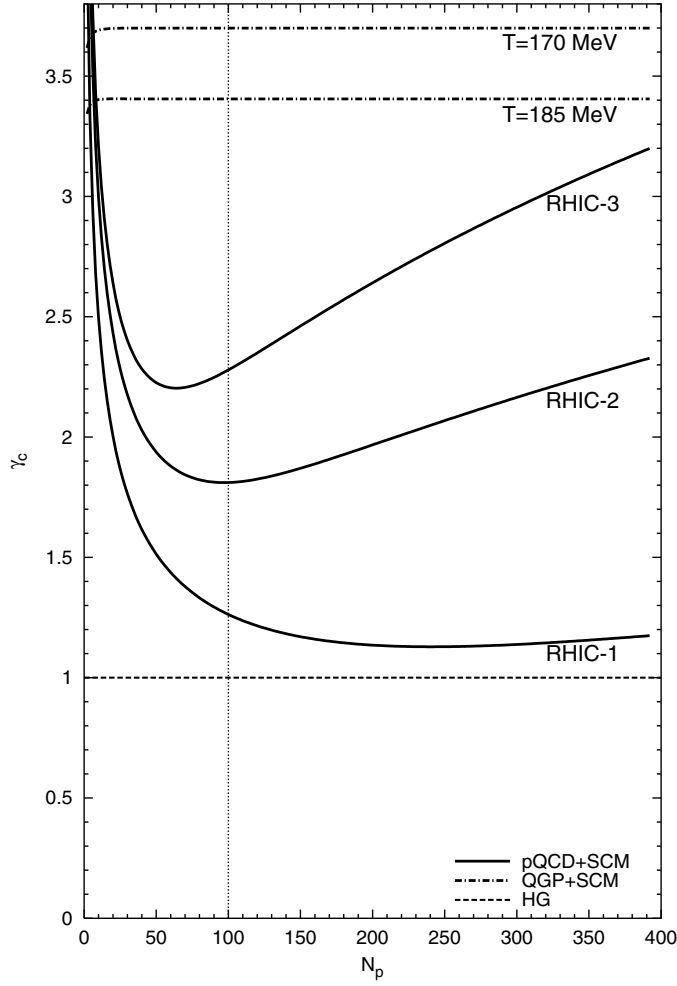
Figure 6. The dependence of the charm enhancement factor γ_c on the number of nucleon participants N_p for Au+Au collisions. The results are presented for three different models: statistical coalescence model with the number of $c\bar{c}$ pairs given by perturbative QCD (pQCD+SCM), statistical coalescence model with the number of $c\bar{c}$ pairs given by equilibrium quark–gluon plasma (QGP+SCM) and equilibrium hadron gas model (HG). In pQCD+SCM case, the dependences are shown for three collision energies per nucleon pair: RHIC-1 ($\sqrt{s} = 56\,\text{GeV}$), RHIC-2 ($\sqrt{s} = 130\,\text{GeV}$) and RHIC-3 ($\sqrt{s} = 200\,\text{GeV}$). The freeze-out temperature was fixed at $T = 170$ MeV. For QGP+SCM, the value of γ_c is shown at two values of freeze-out temperature $T = 170$ MeV and $T = 185$ MeV (the dependence on the collision energy is negligible). For the equilibrium hadron gas model (HG) $\gamma_c \equiv 1$ by the definition. The models are assumed to be valid for $N_p \geqslant 100$, i.e. to the right from the dotted vertical line.

4.4. Centrality dependence of γ_c

The dependence of charm enhancement factor γ_c on the number of participating nucleons N_p is shown in figure 6. In the case of pQCD+SCM model, the behaviour is similar to that of ratio R: γ_c decreases with N_p provided that the number of $c\bar{c}$ pairs is small $N_{c\bar{c}}^{pQCD} \lesssim 1$ and increases at large $N_{c\bar{c}}^{pQCD}$. In contrast, the QGP+SCM model leads to a nearly constant value of γ_c, which can easily be explained comparing equations (15) and (22) and neglecting the

small term $\gamma_c^2 N_H$. From the same reason the dependence of γ_c on $\sqrt{s}$ is also negligible. (Let us remind that in the case of HG model, $\gamma_c \equiv 1$ by the definition).

4.5. Discussion

The results presented in figures 4–6 can be easily studied analytically in the limiting cases. For $N_{c\bar{c}} \gg 1$ one finds from equation (15)

$$R = \frac{\langle J/\psi \rangle}{N_{c\bar{c}}} \sim \gamma_c \approx 2N_{c\bar{c}}/N_O \sim \frac{N_{c\bar{c}}}{V} \sim \frac{N_{c\bar{c}}}{\langle \pi \rangle} \tag{23}$$

where $\langle \pi \rangle$ is the total pion multiplicity. Therefore, in the limiting case $N_{c\bar{c}} \gg 1$, the ratio of the J/ψ to the open charm (or a probability to form the charmonium states) is proportional to the factor γ_c, which in its turn is proportional to the density of $c\bar{c}$ pairs. In the case of pQCD+SCM, this $c\bar{c}$ density increases both with collision energy $\sqrt{s}$ and with the number of nucleon participants N_p:

(1) the increase (20) of $N_{c\bar{c}}^{pQCD}$ with $\sqrt{s}$ is much stronger than the increase of the system volume V or, equivalently, the increase of the total pion multiplicity (11);
(2) the increase of $N_{c\bar{c}}^{pQCD} \sim N_p^{4/3}$ is stronger than $V \sim \langle \pi \rangle$ which is approximately proportional to N_p.

In the case of thermal charm production (QGP+SCM and HG), the number of $c\bar{c}$ pairs is proportional to the system volume. Therefore the ratio R becomes constant when $N_{c\bar{c}} \gg 1$.

The behaviour of the $\langle J/\psi \rangle/N_{c\bar{c}}$ ratio is rather different when $N_{c\bar{c}} \ll 1$ and the canonical suppression factor in equation (15) plays a crucial role. One finds $\gamma_c^2 \approx 4N_{c\bar{c}}/N_O^2$ and then $\gamma_c \sim N_p^{-1/3}$ for pQCD+SCM. The value of R, however, does not depend on γ_c:

$$R = \frac{\langle J/\psi \rangle}{N_{c\bar{c}}} \sim \frac{1}{V} \sim \frac{1}{\langle \pi \rangle}. \tag{24}$$

The ratio (1) decreases with both the collision energy (approximately like $(\sqrt{s})^{1/2}$) and the number of nucleon participants (approximately like N_p^{-1}) and does not depend on the charm production mechanism. This behaviour is similar to the standard picture of the J/ψ suppression.

The J/ψ suppression in pQCD+SCM occurs at the SPS energy. This energy is still 'low' as $N_{c\bar{c}}^{pQCD} < 1$ even in the most central Pb+Pb collisions. The J/ψ suppression (i.e. the decrease of the $J/\psi / N_{c\bar{c}}^{pQCD}$ ratio) with N_p and/or $\sqrt{s}$ continues up to the RHIC energies for the peripheral collisions with small number of nucleon participants. This changes into the J/ψ enhancement when both N_p and $\sqrt{s}$ are 'large'. Therefore, if the $c\bar{c}$ production is described by the pQCD, the statistical coalescence model predicts the decrease of the ratio R (1) with $\sqrt{s}$ and/or N_p (the J/ψ suppression) when $N_{c\bar{c}}^{pQCD} \ll 1$. However, this ratio increases with $\sqrt{s}$ and/or N_p (the J/ψ enhancement) when $N_{c\bar{c}}^{pQCD} \gg 1$. These limiting behaviours are smoothly connected in the intermediate region of $N_{c\bar{c}}^{pQCD} \simeq 1$.

Other models for J/ψ production via coalescence of c and $\bar{c}$ at the hadronization stage [7, 8] give similar *qualitative* result: the J/ψ enhancement for central collisions at high energies. *Quantitative* predictions, however, are rather different. In [8] $\langle J/\psi \rangle = (2.0–3.0) \times 10^{-1}$ per central Au+Au event at $\sqrt{s} = 200$ GeV, whereas our result in pQCD+SCM is $\langle J/\psi \rangle \simeq 1.0 \times 10^{-2}$ (note that the $N_{c\bar{c}}$ estimates are very close: approximately 10–11 $c\bar{c}$ pairs per event). The predictions of the microscopical coalescence model of [7] can be compared with our results at $\sqrt{s} = 130$ GeV. They are rather close for J/ψ to negative hadron ratio: $\langle J/\psi \rangle/\langle h^- \rangle = 1.6 \times 10^{-6}$ [7] and $\langle J/\psi \rangle/\langle h^- \rangle \simeq 2 \times 10^{-6}$ in the pQCD+SCM.

However, the total charm production is rather different: $N_{c\bar{c}} \approx 12$ in [7] and $N_{c\bar{c}} \approx 5$ in our pQCD estimation.

5. Conclusion

The statistical production of J/ψ mesons in Au+Au collisions at RHIC energies has been studied. The obtained results are qualitatively different from the standard picture of the J/ψ suppression in A+A collisions [1, 2] if the average number of produced charmed quark–antiquark pairs per event is large $N_{c\bar{c}} \gg 1$. The standard picture leads to monotonic decrease of the ratio R (the ratio of the J/ψ multiplicity to the total number of charm quark–antiquark pairs) with both the collision energy and the number of participating nucleons. We predict another behaviour. The reason for this difference is the completely distinct physical pictures for charmonia formation. The standard picture supposes the charmonia states to be created *exclusively* in primary nucleon–nucleon collisions at the early stage of the reaction so that all subsequent interactions can only destroy them. We study another possibility: creation of charmed and charmonium particles at the hadronization stage. In our opinion, the most probable scenario of the charmonium production in A+A collisions at the RHIC energies is the following (we call it pQCD+SCM): $c\bar{c}$ pairs are produced exclusively in hard parton collisions at the early stage of A+A reaction (pQCD), and these charmed quarks and antiquarks are redistributed in the open and hidden charm hadrons according to the statistical mechanics prescriptions at the hadronization stage (SCM). This model predicts a strong increase of the ratio R with the collision energy or the number of nucleon participants (J/ψ *enhancement*), provided that the total number of produced $c\bar{c}$ pairs per event is essentially larger than 1. This condition is fulfilled at the highest RHIC energy $\sqrt{s} = 200$ GeV already for $N_p = 100$ and also at lower RHIC energies for the most central Au+Au collisions ($N_p \simeq 2A$).

To get a feeling of possible effects of the charm creation (annihilation) by secondary parton and/or hadron rescatterings we have considered two extreme models of charm chemical equilibration: the quark–gluon plasma (QGP+SCM) and the hadron gas (HG). Both models predict almost constant ratio R at $N_{c\bar{c}} \gg 1$. There is however a quantitative difference: QGP+SCM predicts much larger value of R and much larger value of the total charm than pQCD+SCM and HG models. The predictions of the HG model for the open charm and charmonium production at the upper RHIC energies are below the results of the pQCD+SCM and QGP+SCM.

Our predictions for the behaviour of R at $N_{c\bar{c}} \gg 1$ (J/ψ enhancement) in the pQCD+SCM are in a qualitative agreement with other coalescence models [7, 8], but quantitatively the results are rather different.

We conclude that Au+Au collision experiments at RHIC offer a possibility to disentangle various models of the open charm and charmonia production. Even those models that give very similar results at SPS energies become quantitatively and even qualitatively different at RHIC. Measurements of the J/ψ productions combined with the open charm measurements are needed to give a definite answer about the dominant mechanisms of hidden and open charm production in ultrarelativistic collisions of heavy nuclei.

Acknowledgments

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