Strange quark matter within the Nambu–Jona-Lasinio model

I.N. Mishustin[1,2,3]**, L.M. Satarov**[1,2]**, H. Stöcker**[2]**, and W. Greiner**[2]

[1] *The Kurchatov Institute, Russian Research Centre, 123182 Moscow, Russia*

[2] *Institut für Theoretische Physik, J.W. Goethe Universität,*

  D–60054 Frankfurt am Main, Germany

[3] *The Niels Bohr Institute, DK–2100 Copenhagen Ø, Denmark*

Equation of state of baryon rich quark matter is studied within the SU(3) Nambu–Jona-Lasinio model with flavour mixing interaction. Possible bound states (strangelets) and chiral phase transitions in this matter are investigated at various values of strangeness fraction r_s. The model predictions are very sensitive to the ratio of vector and scalar coupling constants, $\xi = G_V/G_S$. At $\xi = 0.5$ and zero temperature the maximum binding energy (about 15 MeV per baryon) takes place at $r_s \simeq 0.4$. Such strangelets are negatively charged and have typical life times $\sim 10^{-7}$ s. The calculations are carried out also at finite temperatures. They show that bound states exist up to temperatures of about 15 MeV. The model predicts a first order chiral phase transition at finite baryon densities. The parameters of this phase transition are calculated as a function of r_s.

I. INTRODUCTION

Almost 30 year ago A.B. Migdal has put forward a brilliant idea of pion condensation in nuclear matter [1]. Soon it was realized that this phenomenon may lead to the existence of density isomers [2]. At about this time Lee and Wick proposed another mechanism

leading to the appearance of an abnormal nuclear state [3]. It is related to the restoration of chiral symmetry at high baryon density. These ideas have motivated nuclear community to initiate new experimental programs aimed at producing hot and dense nuclear matter in energetic collisions of heavy nuclei. These experiments have started in Dubna and Berkeley and then continued in Brookhaven and CERN. Nowadays exciting expectations are associated with new ultrarelativistic heavy ion colliders, RHIC and LHC.

The general goal of present and future experiments with ultrarelativistic heavy ions is to study the equation of state and dynamical properties of strongly interacting matter. Now the main interest lies in investigating the chiral and deconfinement phase transitions, predicted by QCD. The ultimate goal is to produce and study in the laboratory a new state of matter, the Quark–Gluon Plasma (QGP). This state of matter can be reached only at high temperatures or particle densities when elementary constituents, quarks and gluons are liberated from hadrons.

Since the direct application of QCD at moderate temperatures and nonzero chemical potentials is not possible at present, more simple effective models respecting some basic symmetry properties of QCD are commonly used. One of the most popular models of this kind, which is dealing with constituent quarks and respects chiral symmetry, is the Nambu–Jona-Lasinio (NJL) model [4,5]. In recent years this model has been widely used for describing hadron properties (see reviews [6,7]), phase transitions in dense matter [8–13] and multiparticle bound states [14–17].

In the previous papers [18,19] we have used the NJL model to study properties of the quark–antiquark plasma out of chemical equilibrium. In fact, we considered a system with independent densities of quarks and antiquarks. We have found not only first order transitions but also deep bound states even in the baryon–free matter with equal densities of quarks and antiquarks. In the present paper the emphasis is put on investigating the possibility of bound states and phase transitions in equilibrated matter at various flavour compositions. In particular, we consider the possibility of bound states in quark matter

with a significant admixture of strange quarks, i.e. strangelets. Thermal properties of strange quark matter are also studied.

The paper is organized as follows: in Sect. II a generalized NJL model including flavour–mixing terms is formulated in the mean–field approximation. The model predictions for strange matter and characteristics of its bound states at zero temperature are discussed in Sect. III. Effects of finite temperatures are considered in Sect. IV. Possible decay modes of new bound states are discussed in Sect. V. Main results of the present paper are summarized in Sect. VI.

II. FORMULATION OF THE MODEL

Below we use the SU(3)–flavour version of the NJL model suggested in Ref. [20]. The corresponding Lagrangian is written as ($\hbar = c = 1$)

$$\mathcal{L} = \overline{\psi} \left(i \not{\partial} - \hat{m}_0\right) \psi + G_S \sum_{j=0}^{8} \left[\left(\overline{\psi}\, \frac{\lambda_j}{2}\, \psi\right)^2 + \left(\overline{\psi}\, \frac{i\gamma_5 \lambda_j}{2}\, \psi\right)^2\right]$$
$$- G_V \sum_{j=0}^{8} \left[\left(\overline{\psi}\, \gamma_\mu \frac{\lambda_j}{2}\, \psi\right)^2 + \left(\overline{\psi}\, \gamma_\mu \frac{\gamma_5 \lambda_j}{2}\, \psi\right)^2\right]$$
$$- K \left[\det_f \left(\overline{\psi}\, (1 - \gamma_5)\, \psi\right) + \det_f \left(\overline{\psi}\, (1 + \gamma_5)\, \psi\right)\right]. \tag{1}$$

Here ψ is the column vector consisting of three single–flavour spinors ψ_f, $f = u, d, s$, $\lambda_1, \ldots, \lambda_8$ are the SU(3) Gell-Mann matrices in flavour space, $\lambda_0 \equiv \sqrt{2/3}\, \boldsymbol{I}$, and $\hat{m}_0 = \mathrm{diag}(m_{0u}, m_{0d}, m_{0s})$ is the matrix of bare (current) quark masses. At $\hat{m}_0 = 0$ this Lagrangian is invariant with respect to $\mathrm{SU}_\mathrm{L}(3) \otimes \mathrm{SU}_\mathrm{R}(3)$ chiral transformations. The second and third terms in Eq. (1) correspond, respectively, to the scalar–pseudoscalar and vector–axial-vector 4–fermion interactions. The last 6–fermion interaction term breaks the $U_A(1)$ symmetry and gives rise to the flavour mixing effects.

In the mean–field approximation the Lagrangian (1) is reduced to

$$\mathcal{L}_{\mathrm{mfa}} = \sum_f \overline{\psi}_f \left(i \not{D} - m_f\right) \psi_f - \frac{G_S}{2} \sum_f \rho_{Sf}^2$$

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$$+ \frac{G_V}{2} \sum_f \rho_{Vf}^2 + 4K \prod_f \rho_{Sf} \,, \tag{2}$$

where $\slashed{D} = \slashed{\partial} + i\,\gamma_0 G_V \rho_{Vf}$ and

$$\rho_{Sf} = <\overline{\psi}_f \psi_f>\,, \tag{3}$$

$$\rho_{Vf} = <\overline{\psi}_f \gamma_0 \psi_f> \tag{4}$$

are scalar and vector densities of quarks with flavour f. Angular brackets correspond to the quantum–statistical averaging. The constituent quark masses, m_f, are determined by the coupled set of gap equations

$$m_f = m_{0f} - G_S\,\rho_{Sf} + 2K \prod_{f' \neq f} \rho_{Sf'}\,. \tag{5}$$

The NJL model is an effective, non–renormalizable model. To regularize the divergent contribution of negative energy states of the Dirac sea, one must introduce an ultraviolet cut–off. Following common practice, we use the 3–momentum cut–off $\Theta(\Lambda - p)$ in divergent integrals [1]. The model parameters m_{0f}, G_S, K, Λ can be fixed by reproducing the observed masses of π, K, and η' mesons as well as the pion decay constant f_π. As shown in Ref. [20], a reasonable fit is achieved with the following values:

$$m_{0u} = m_{0d} = 5.5 \text{ MeV}, \quad m_{0s} = 140.7 \text{ MeV}, \tag{6}$$

$$G_S = 20.23 \text{ GeV}^{-2}, \quad K = 155.9 \text{ GeV}^{-5}, \quad \Lambda = 0.6023 \text{ GeV}. \tag{7}$$

Motivated by the discussions in in Refs. [6,21], we choose the following value of the vector coupling constant[2]

$$G_V = 0.5\,G_S = 10.12 \,\text{GeV}^{-2}\,. \tag{8}$$

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[1]Here $\theta(x) \equiv \frac{1}{2}(1 + \text{sgn}\,x)$.

[2]See discussion of this question in Ref. [19]

Let us consider homogeneous, thermally (but not, in general, chemically) equilibrated quark–antiquark matter at temperature T. Let $a_{\boldsymbol{p},\lambda}\,(b_{\boldsymbol{p},\lambda})$ and $a_{\boldsymbol{p},\lambda}^{+}\,(b_{\boldsymbol{p},\lambda}^{+})$ be the destruction and creation operators of a quark (an antiquark) in the state $\boldsymbol{p}, \lambda$, where $\boldsymbol{p}$ is the 3-momentum and λ is the discrete quantum number denoting spin and flavour (color indices are suppressed). It can be shown [18] that quark and antiquark phase–space occupation numbers coincide with the Fermi–Dirac distribution functions:

$$< a_{\boldsymbol{p},\lambda}^{+}\, a_{\boldsymbol{p},\lambda} >\equiv n_{\boldsymbol{p}f} = \left[\exp\left(\frac{E_{\boldsymbol{p}f} - \mu_{Rf}}{T} \right) + 1 \right]^{-1}, \tag{9}$$

$$< b_{\boldsymbol{p},\lambda}^{+}\, b_{\boldsymbol{p},\lambda,f} >\equiv \overline{n}_{\boldsymbol{p}f} = \left[\exp\left(\frac{E_{\boldsymbol{p}f} - \overline{\mu}_{Rf}}{T} \right) + 1 \right]^{-1}, \tag{10}$$

where $E_{\boldsymbol{p}f} = \sqrt{m_f^2 + \boldsymbol{p}^2}$ and $\mu_{Rf}, \overline{\mu}_{Rf}$ denote the reduced chemical potentials of quarks and antiquarks:

$$\mu_{Rf} = \mu_f - G_V \rho_{Vf}, \tag{11}$$

$$\overline{\mu}_{Rf} = \overline{\mu}_f + G_V \rho_{Vf}. \tag{12}$$

The explicit expression for the vector density can be written as

$$\rho_{Vf} = \rho_f - \overline{\rho}_f, \tag{13}$$

where

$$\rho_f = \nu \int \frac{\mathrm{d}^3 p}{(2\pi)^3}\, n_{\boldsymbol{p}f}, \quad \overline{\rho}_f = \nu \int \frac{\mathrm{d}^3 p}{(2\pi)^3}\, \overline{n}_{\boldsymbol{p}f} \tag{14}$$

are, respectively, the number densities of quarks and antiquarks of flavour f and $\nu = 2N_c = 6$ is the spin–color degeneracy factor. The net baryon density is obviously defined as

$$\rho_B = \frac{1}{3} \sum_f \rho_{Vf}. \tag{15}$$

The physical vacuum ($\rho_f = \overline{\rho}_f = 0$) corresponds to the limit $n_{\boldsymbol{p}f} = \overline{n}_{\boldsymbol{p}f} = 0$.

In general the chemical potentials μ_f and $\overline{\mu}_f$ are independent variables. The assumption of chemical equilibrium with respect to creation and annihilation of $q\overline{q}$ pairs leads to the conditions

$$\overline{\mu}_f = -\mu_f, \quad f = u, d, s \, . \tag{16}$$

These conditions automatically follow from the relations valid for any $q\overline{q}$ system chemically equilibrated with respect to strong interactions:

$$\mu_i = B_i \, \mu_B + S_i \, \mu_S + Q_i \, \mu_Q \ (i = u, d, s, \overline{u}, \overline{d}, \overline{s}) \, . \tag{17}$$

Here three independent chemical potentials, μ_B, μ_S, μ_Q are fixed by the net baryon number B, strangeness S and electric charge Q of the system.

In heavy-ion collisions at high bombarding energies the partonic matter created is characterized by $B \simeq 0$ and $S \simeq 0$, therefore $\overline{\mu}_f \simeq \mu_f \simeq 0$. On the other hand, strangelets are finite droplets with nonzero B and S. Using Eq. (17) one can see that in this case μ_B, $\mu_S \neq 0$ and, therefore, the inequality $\mu_s > \mu_{u,d}$ should hold. This conclusion shows that strong fluctuations are needed for strangelet formation in high energy nuclear collisions.

If strangelet life times are long enough, the equilibrium with respect to weak processes:

$$s \to u + e^- + \overline{\nu}_e, \ u + e^- \to s + \nu_e, \ s + u \leftrightarrow u + d, \tag{18}$$

may be also achieved. Assuming that $\mu_e = \mu_{\overline{\nu}} = 0$ one arrives at the following conditions

$$\mu_u = \mu_d = \mu_s \, . \tag{19}$$

As will be shown below, constituent masses m_s of strange quarks in (mechanically) stable strangelets exceed chemical potentials of u, d quarks. At $\mu_s > m_s$ and $T \to 0$ the β equilibrium conditions (19) can be realized only after all s quarks decay.

Using conditions (16) in the limit of zero temperatures one can see that the density of antiquarks vanishes in baryon–rich chemically equilibrated matter, $\overline{\rho} \to 0$. In the mean–field approximation the reduced chemical potential of quarks with flavour f coincides with their Fermi energy:

$$\mu_{Rf} = \sqrt{m_f^2 + p_{Ff}^2} \, , \tag{20}$$

where

$$p_{Ff} = \left(\frac{6\pi^2 \rho_f}{\nu} \right)^{1/3} \tag{21}$$

is the corresponding Fermi momentum.

Within the NJL model the energy density and pressure of matter as well as the quark condensates ρ_{Sf} contain divergent terms originating from the negative energy levels of the Dirac sea. As noted above, these terms are regularized by introducing the 3–momentum cutoff $\theta(\Lambda - |\boldsymbol{p}|)$. Then the scalar density is expressed as

$$\rho_{Sf} = \nu \int \frac{d^3 p}{(2\pi)^3} \frac{m_f}{E_{\boldsymbol{p}f}} \left[n_{\boldsymbol{p}f} + \overline{n}_{\boldsymbol{p}f} - \theta(\Lambda - p) \right] . \tag{22}$$

The energy density corresponding to the Lagrangian (2) can be written [19] as

$$e = e_K + e_D + e_S + e_V + e_{FM} + e_0 . \tag{23}$$

This expression includes:

the "kinetic" term

$$e_K = \nu \sum_f \int \frac{d^3 p}{(2\pi)^3} E_{\boldsymbol{p}f} \left(n_{\boldsymbol{p}f} + \overline{n}_{\boldsymbol{p}f} \right) , \tag{24}$$

the "Dirac sea" term

$$e_D = -\nu \sum_f \int \frac{d^3 p}{(2\pi)^3} E_{\boldsymbol{p}f} \, \theta(\Lambda - p) , \tag{25}$$

the scalar interaction term

$$e_S = \frac{G_S}{2} \sum_f \rho_{Sf}^2 , \tag{26}$$

the vector interaction term

$$e_V = \frac{G_V}{2} \sum_f \rho_{Vf}^2 \tag{27}$$

and the flavour mixing term

$$e_{FM} = -4K \prod_f \rho_{Sf} . \tag{28}$$

A constant e_0 is introduced in Eqs. (23) in order to set the energy density of the physical vacuum equal to zero. This constant can be expressed through the vacuum values of constituent masses, m_f^{vac}, and quark condensates, ρ_{Sf}^{vac}. These values are obtained by selfconsistently solving the gap equations (5) in vacuum, i.e. at $n_{\boldsymbol{p}f} = \overline{n}_{\boldsymbol{p}f} = 0$.

Explicit analytic formulae for the energy density and the gap equations may be obtained in the case of zero temperature. At $T \to 0$ one has [18]

$$e_K + e_D = \frac{\nu}{8\,\pi^2} \sum_f \left[p_{Ff}^4 \Psi\left(\frac{m_f}{p_{Ff}}\right) - \Lambda^4 \Psi\left(\frac{m_f}{\Lambda}\right) \right] , \tag{29}$$

$$\rho_{Sf} = \frac{\nu}{8\pi^2} \sum_f \left[p_{Ff}^3 \Psi'\left(\frac{m_f}{p_{Ff}}\right) - \Lambda^3 \Psi'\left(\frac{m_f}{\Lambda}\right) \right] , \tag{30}$$

where

$$\Psi(x) \equiv 4 \int_0^1 dt\, t^2 \sqrt{t^2 + x^2} = \left(1 + \frac{x^2}{2}\right)\sqrt{1 + x^2} - \frac{x^4}{2} \ln \frac{1 + \sqrt{1 + x^2}}{x} . \tag{31}$$

For a system with independent chemical potentials for quarks (μ_f) and antiquarks ($\overline{\mu}_f$) one can use the thermodynamic identity for the pressure of $q\overline{q}$ matter

$$P = \sum_f (\mu_f \rho_f + \overline{\mu}_f \overline{\rho}_f) - e + sT , \tag{32}$$

where s is the entropy density

$$s = -\nu \sum_f \int \frac{d^3 p}{(2\pi)^3} \left[n_{\boldsymbol{p}f} \ln n_{\boldsymbol{p}f} + (1 - n_{\boldsymbol{p}f}) \ln(1 - n_{\boldsymbol{p}f}) + n_{\boldsymbol{p}f} \to \overline{n}_{\boldsymbol{p}f} \right] . \tag{33}$$

As discussed in Ref. [18], bound states of $q\overline{q}$ matter and first order phase transitions in $q\overline{q}$ matter are possible if its equation of state $P = P(\mu_f, \overline{\mu}_f, T)$ contains regions with negative pressure or isothermal compressibility, respectively. At $T = 0$ the state of mechanical equilibrium with vacuum ($P = 0$) corresponds to minimum of the energy per particle, $\epsilon = E / \sum_f (N_f + N_{\overline{f}}) = e / \sum_f (\rho_f + \overline{\rho}_f)$, where $N_{f(\overline{f})}$ is the number of quarks (antiquarks) of flavour f. In the case of pure quark matter $N_{\overline{f}} = 0$, its baryon number $B = \sum_f N_f / 3$ and its energy per baryon $E/B = 3\epsilon$.

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To characterize the flavour composition we introduce the strangeness fraction parameter

$$r_s \equiv \frac{S}{3B} = \frac{\overline{\rho}_s - \rho_s}{3\rho_B}. \tag{34}$$

Below only the isospin–symmetric mixtures where $N_u = N_d$ and $N_{\bar{u}} = N_{\bar{d}}$ will be considered. It can be shown [19] that in the dilute limit, when all single flavour densities $\rho_f, \overline{\rho}_f$ are small, ϵ tends to the sum of the constituent quark and antiquark masses in vacuum, weighted according to r_s :

$$m_q^{\text{vac}}(r_s) = (1 - r_s)\, m_u^{\text{vac}} + r_s\, m_s^{\text{vac}}. \tag{35}$$

In the case of chemically equilibrated matter at $T = 0$ and fixed r_s one has

$$\epsilon(\rho_B \to 0, r_s) = m_q^{\text{vac}}(r_s) \tag{36}$$

A bound multiparticle state exists if there is a nontrivial minimum of ϵ as function of ρ_B and the binding energy (BE) per baryon is positive:

$$\text{BE} = 3\left[m_q^{\text{vac}}(r_s) - \epsilon_{\min}(r_s) \right] > 0. \tag{37}$$

III. STRANGE QUARK MATTER AT ZERO TEMPERATURE

Let us consider first quark matter with nonzero net baryon density at $T = 0$. In the chemically equilibrated system the density of valence antiquarks will be zero for each flavour ($\overline{\rho}_f = 0$). Fig. 1 shows the energy per quark as a function of baryon density, $\rho_B = \frac{1}{3}\sum_f \rho_f$. Different curves correspond to different r_s, which in this case is the relative concentration of strange quarks. In accordance with Eq. (36), at $\rho_B \to 0$ the energy per quark tends to the corresponding vacuum mass. With growing density both the attractive scalar and repulsive vector interactions contribute to ϵ (see the discussion of this question in Ref. [19]).

It is interesting that at $r_s \leq 0.7$ the attractive interaction is strong enough to produce a nontrivial local minimum at a finite ρ_B. In the pure u, d matter $(r_s = 0)$ this minimum is unbound by about 20 MeV as compared to the vacuum masses of u and d quarks. On the other hand, it is located at a baryon density of about $1.8\,\rho_0$, which is surprisingly close to the saturation density of normal nuclear matter. Of course, the location of this minimum depends on the model parameters. Nevertheless, one can speculate that nucleon–like 3–quark correlations, not considered in the mean–field approach, will turn this state into the correct nuclear ground state.

When r_s grows from 0 to about 0.4, the local minimum is getting more pronounced and the corresponding baryon density increases to about $3.2\,\rho_0$. At larger r_s, the minimum again becomes more shallow and disappears completely at $r_s \simeq 0.7$. At $0.2 < r_s < 0.6$ the minima correspond to the true bound states, i.e. the energy per quark is lower than the respective vacuum mass. But in all cases these bound states are rather shallow: even the most strongly bound state at $r_s \simeq 0.4$ is bound only by about 15 MeV per baryon. Nevertheless, the appearance of local minima signifies the possibility for finite droplets to be in mechanical equilibrium with the vacuum at $P = 0$. It is natural to identify such droplets with strangelets, which are hypothetical objects made of light and strange quarks [22–27].

It should be emphasized here that β–equilibrium is not required in the present approach (see the discussion below). That is why our most bound strangelets are predicted to be more rich in strange quarks $(r_s > 1/3)$ than in the approaches assuming β–equilibrium [22,23,25], which give $r_s < 1/3$. As a result, these strangelets will be negatively charged[3]. Indeed, the ratio of the charge Q to the baryon number B is expressed through r_s as

$$\frac{Q}{B} = \frac{2}{3}\frac{\rho_u}{\rho_B} - \frac{1}{3}\frac{\rho_d}{\rho_B} - \frac{1}{3}\frac{\rho_s}{\rho_B} = \frac{1}{2}(1 - 3r_s). \tag{38}$$

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[3]Negatively–charged strangelets have been also considered in Refs. [24,26].

For $r_s \simeq 0.4$ this gives $Q/B \simeq -0.1$. In light of recent discussions (see e.g. Ref. [27]) concerning possible dangerous scenarios of the negatively–charged strangelet production at RHIC, we should emphasize that the strangelets predicted here are not absolutely bound[4], i.e. their energy per baryon is higher than that for the normal nuclear matter. Hence, the spontaneous conversion of normal nuclear matter to strange quark matter is energetically not possible.

Fig. 2 shows the constituent masses of u and s quarks as functions of baryon density. The dropping masses manifest a clear tendency to the restoration of chiral symmetry at high densities. The dots indicate the masses at the local minima in the respective energies per baryon shown in Fig. 1. Note that the stronger is reduction of constituent masses the deeper are the corresponding bound states. For the metastable state at $r_s = 0$, which is a candidate for the nuclear ground state, the masses of u and s quarks are equal to 0.3 and 0.9 of their vacuum values respectively. For the most bound state at $r_s \simeq 0.4$ the respective mass ratios are reduced to 0.15 and 0.6.

The behaviour of the u and s chemical potentials is shown in Fig. 3. At $r_s \neq 0, 1$, due to the contribution of the vector interaction (see Eqs. (11), (20)), μ_u and μ_s grow practically linearly at large baryon densities. In accordance with a previous discussion, one can see that the conditions $\mu_s > \mu_{u,d}$ hold at baryon densities corresponding to bound states of strange matter.

The properties of the multiparticle bound states are summarized in Figs. 4–5. Fig. 4 shows the binding energy per baryon, Eq. (37). The maximum binding, about 15 MeV, is realized at $r_s \simeq 0.4$. One should bear in mind, that in the case of baryon–rich matter local minima of ϵ result from a strong cancellation between the attractive scalar and repulsive vector interactions. Therefore, it is very sensitive to their relative strengths. The results presented above are obtained for $G_V = 0.5\,G_S$. For comparison in Figs. 4 and 5 we also

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[4]The analogous conclusion has been made in Ref. [17].

present the model predictions for $G_V = 0$. In this case the maximum binding energy increases to about 90 MeV per baryon and the corresponding r_s value shifts to about 0.6. It is interesting to note that for $G_V = 0$ the bound state appears even in the pure u, d matter. The corresponding binding energy is about 20 MeV per baryon.

The dots in Fig. 5 indicate the positions of some conventional baryons. By inspecting the figure, one can make a few interesting observations. First, conventional baryons are more bound than strangelets even at $G_V = 0$. This is an indication that baryon–like 3–quark correlations might be indeed very important in the baryon–rich quark matter. Second, the bound state energies grow monotonously with r_s.

IV. QUARK MATTER AT FINITE TEMPERATURES

In this section we study properties of deconfined matter at finite temperatures. The calculations for this case can be done by using general formulas of Sect. II with the quark and antiquark occupation numbers given by Eqs. (9)–(10). Unless stated otherwise, the results given below correspond to the ratio $G_V/G_S = 0.5$.

Figs. 6(a) represents the pressure isotherms for the case of zero net strangeness $(r_s = 0)$, which is appropriate for fast processes where net strangeness is conserved, e.g. in relativistic nuclear collisions. One can see a region of spinodal instability, $\partial_\rho P < 0$, which is characteristic for a first order phase transition. The corresponding critical temperature is about 35 MeV. The dashed line (binodal) shows the boundary of mixed phase states. Bound (zero–pressure) states exist only at temperatures below 15 MeV.

Generally, the equation of state of the chemically equilibrated quark matter is characterized by two quantities: net baryon charge, B, and net strangeness, S. Therefore, it is interesting to study thermal properties of this matter at $S \neq 0$. Such states can be reached in neutron stars. They can also be realized via the distillation mechanism accompanying a QCD phase transition in heavy–ion collisions [28]. Fig. 6(b) shows the pressure isotherms for $r_s = 0.4$. As discussed above, this case corresponds to most bound strange

matter at $T = 0$. At this value of r_s bound states exist at $T < 30$ MeV. The dashed line in Fig. 6(b) again shows the boundary of the two–phase region. It is obtained by the solution of the Gibbs conditions $P^{(1)} = P^{(2)}$, $\mu_u^{(1)} = \mu_u^{(2)}$, $\mu_s^{(1)} = \mu_s^{(2)}$ where indices 1 and 2 denote two coexisting phases. As compared to the case $r_s = 0$ the chiral phase transition takes place in a wider region of T and ρ_B. Applying the Gibbs conditions, one can see that for $r_s = 0$ the equilibrium pressure isotherms are constant in the mixed phase region (the Maxwell construction). But this is not the case for $r_s = 0.4$. In accordance with general conclusions of Ref. [29], in the case of two conserving charges (baryon number and strangeness) the Maxwell construction is modified in such a way that the equilibrium pressure at fixed T increases with ρ_B in the mixed–phase domain. However, this increase is small (several percents) and hardly visible in Fig. 6(b). For example, for $T = 30$ MeV the equilibrium pressure changes from 5.36 to 5.44 MeV/fm^3. It is interesting, that local values of r_s in the coexisting phases are slightly different $(r_s^{(1)} < 0.4 < r_s^{(2)})$ and only the global strangeness ratio is fixed $(S/3B = 0.4)$.

Fig. 7 shows the critical temperatures for the existence of phase transitions and bound states in the equilibrated strange matter as functions of r_s. One can see that both temperatures first grow with r_s and then drop to zero at $r_s \sim 0.8$. The maximal values of respectively 50 MeV and 30 MeV are realized at some intermediate $r_s \sim 0.4$. As demonstrated earlier in Fig. 4, this value of r_s corresponds to the most bound state of strange quark matter at $T = 0$. So we see an obvious correlation: the deeper is a bound state at $T = 0$ the stronger is a phase transition at finite temperatures.

It should be emphasized here again that the thermal properties of asymmetric baryon–rich quark matter are very sensitive to the relative strength of scalar and vector interactions [19]. If we take $G_V = 0$, as in most calculations in the literature, the corresponding critical temperature at $r_s = 0$ increases to about 70 MeV. On the other hand, if one takes $G_V = 0.65\,G_S$, the zero–pressure states disappear completely. The calculation shows that there is no phase transition at $G_V > 0.71\,G_S$. It is interesting to note that in all cases

this first order phase transition occupies the region of densities around the normal nuclear density ρ_0.

A more detailed information about the first order (chiral) phase transition predicted by our model is given in Figs. 8 where the critical temperature T_c and baryon chemical potential $\mu_B = \mu_u + 2\mu_d = 3\mu_u$ are shown for different values of r_s. Again one can see that maximal $T_c \simeq 50\,\text{MeV}$ corresponds to $r_s \simeq 0.4$.

Two phases coexisting in the mixed-phase domain of the chiral phase transition are characterized by different values of constituent quark masses. If $q\bar{q}$ matter evolves from a high to a low density state, crossing the two–phase domain, the transition from chirally restored states (with low quark masses $m_f \sim m_{0f}$) to the chirally broken phase (where $m_f \sim m_f^{\text{vac}}$) takes place. Therefore, a part of the internal energy should be transformed into the rest mass. If the total entropy [5] S and the effective number of degrees of freedom are conserved, the temperature will decrease during this transition. This is clearly seen in Fig. 9, where isentropes with entropy per baryon $S/B = 2$ and $S/B = 5$ are shown for the case $r_s = 0$. The analogous effect has been predicted within the linear σ model in Ref. [30]. The drop of temperature as a result of this cooling mechanism may serve as an observable signature of the chiral phase transition in nuclear collisions. The above behaviour is qualitatively different from the one expected in the deconfinement transition where the number of degrees of freedom is smaller in the dilute (hadronic) phase and the temperature may increase during hadronization [31].

These results demonstrate that the chiral phase transition is rather similar to a liquid–gas phase transition in normal nuclear matter. The critical temperature and baryon density in the present case ($T_c \sim 30$ MeV, $\rho_{Bc} \sim \rho_0$) are not so far from the values predicted by the conventional nuclear models [32] ($T_c \sim 20$ MeV, $\rho_{Bc} \sim 0.5\,\rho_0$). One may expect that in a more realistic approach, taking into account nucleonic correlations, the

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[5]This quantity should not be confused with the net srangeness introduced in Sec. II.

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chiral transition may turn into the ordinary "liquid–gas" phase transition. If this would be the case, one should be doubtful about the possibility of any other QCD phase transition of the liquid–gas type at a higher baryon density. At least only one phase transition of this type is predicted within the NJL model.

V. DISCUSSION OF DECAY MODES

Let us discuss briefly possible decay channels of bound states of quark matter described above. In strange quark matter (without antiquarks), flavour conversion is only possible through weak decays. As follows from Fig. 3, at densities corresponding to zero pressure, the condition $\mu_s > \mu_u$ holds. This means that weak processes of the types $s \to u + e^- + \overline{\nu}_e$, $s + u \to u + d$ are allowed. Since there is no local barrier at any r_s (see Fig. 5), in a system produced initially at some $r_s \neq 0$, all strange quarks will eventually be converted into light u, d quarks. Schematicly this conversion process is shown in Figs. 9(a) where the initial $(r_s \neq 0)$ and final $(r_s = 0)$ states corresponds to the left and right diagram, respectively.

This picture is very different as compared to the one based on the MIT bag model. Because the quark masses are kept constant $(m_f = m_{0f})$, the condition $\mu_s > m_s$ will be first satisfied at a relatively low baryon density $\propto m_{0s}^3$. At higher densities a certain fraction of s quarks will always be present in a β–equilibrated matter (see Fig. 9(b)).

In contrast, in the NJL model the s quark mass is a function of both baryon density and strangeness content. As one can see from Fig. 9(a), at any given r_s the condition $\mu_{u,d} = \mu_s$ can be satisfied only at sufficiently high baryon densities which correspond to positive pressure. On the other hand, at the points of zero pressure we always have $\mu_{u,d} < \mu_s$. Therefore, weak decays will proceed until a system reaches $r_s = 0$ (see right picture in Fig. 9(a)).

The life times of strangelets at $T = 0$ can be roughly estimated by analogy to the neutron decay. The matrix element of s-quark β decay is proportional to $(\Delta E)^{5/2} \sin \theta_c$

where ΔE is the energy gain in the reaction and θ_c is the Cabibbo angle ($\sin\theta_c \simeq 0.22$ [33]). As follows from our calculations (see Fig. 3), the energy gain $\Delta E = \mu_s - \mu_u$ depends on r_s, ranging from about 200 MeV at $r_s = 0.4$ to 100 MeV at $r_s = 0$. By scaling with corresponding quantities for the neutron decay, one can express the life time of a strangelet as

$$\tau \sim \tau_n \left(\frac{\Delta m}{\Delta E}\right)^5 \sin^{-2}\theta_c, \tag{39}$$

where $\Delta m = m_n - m_p \simeq 1.2$ MeV and $\tau_n \simeq 880\, s$ is the life time of neutron [33]. This estimate gives $\tau \sim 10^{-9} \div 10^{-7}$ depending on r_s. It is not surprising that this is close to the life times of charged pions and conventional hyperons.

VI. CONCLUSIONS

By using the NJL model, we have investigated the equation of state of chemically equilibrated deconfined quark matter at various temperatures, baryon densities and strangeness contents. The model predicts the existence of loosely bound, negatively–charged strangelets with maximal binding energies of about 15 MeV per baryon at $r_s \sim 0.4$. Similarly to Ref. [17], no absolutely stable strange quark matter has been found. The estimated life–times of these states may be as long as 10^{-7} s. It is shown that properties of baryon–rich quark matter are very sensitive to the relative magnitude of the vector and scalar interactions. At the standard values of vector and scalar couplings, $G_V/G_S = 0.5$, the metastable bound states of chemically equilibrated matter exist at $T < 15$ MeV, while at $G_V = 0$ this temperature increases to 40 MeV.

The calculations reveal the first order chiral phase transition at finite baryon densities and moderate temperatures. In the case of zero net strangeness ($r_s = 0$) the critical temperature is in the range of 30 MeV and the critical baryon density is around ρ_0. We believe that this phase transition is reminiscent of the ordinary liquid–gas phase transition in nuclear matter. We have found that the maximal critical temperature $T_c \simeq 50$ MeV is

reached for the ratio of net strangeness to baryon charge $S/B \simeq 1.2$. The model predicts a strong cooling of matter during the phase transition due to generation of the constituent mass.

We hope that these results will generate a certain optimism in searching for unusual states of matter in relativistic heavy-ion collisions.

ACKNOWLEDGMENTS

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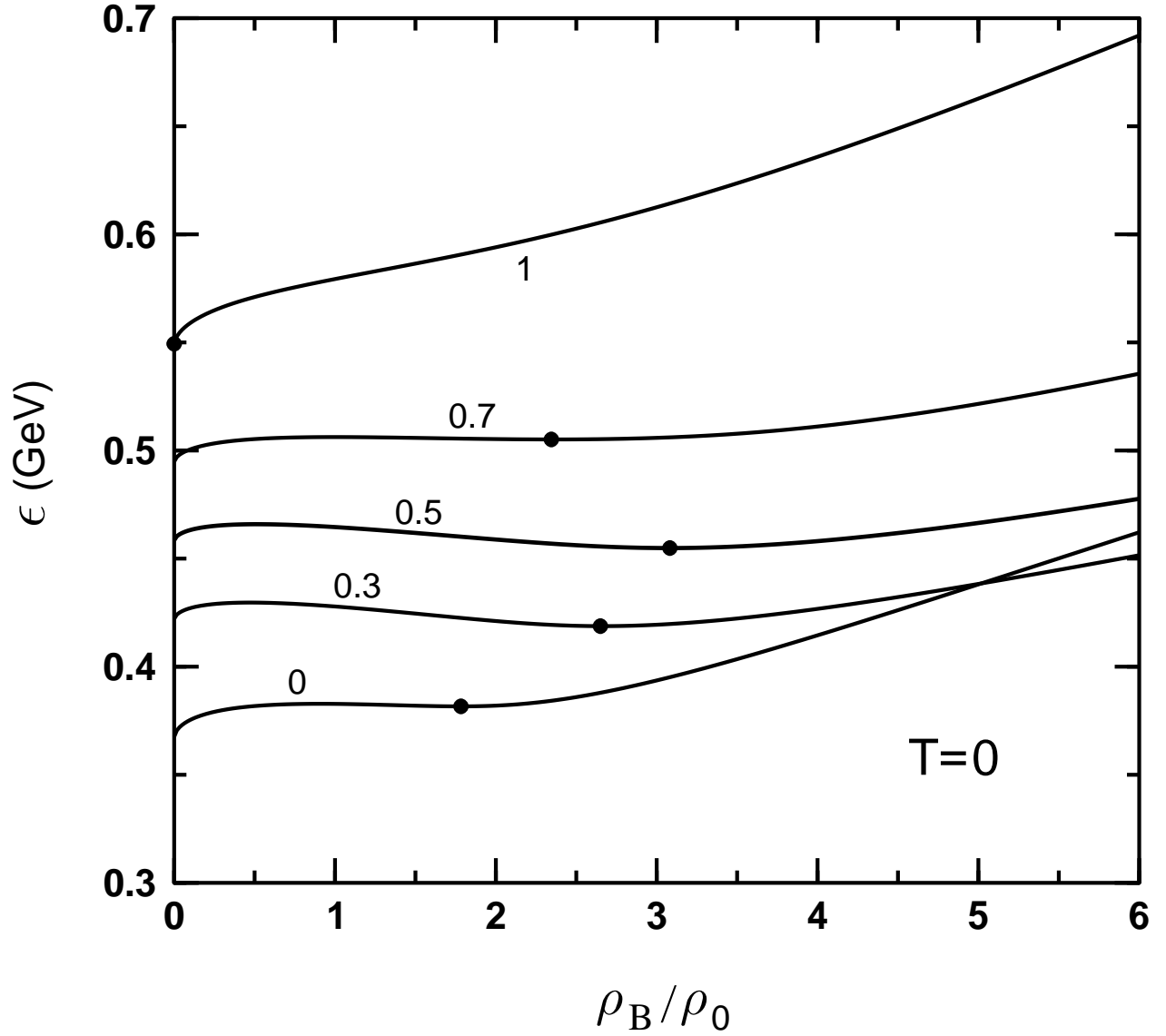
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FIG. 1. Energy per particle ϵ in pure quark matter at zero temperature as function of baryon density at different values of strangeness fraction r_s. Points indicate local minima of ϵ. $\rho_0 = 0.17\,\mathrm{fm}^{-3}$ is normal nuclear density.
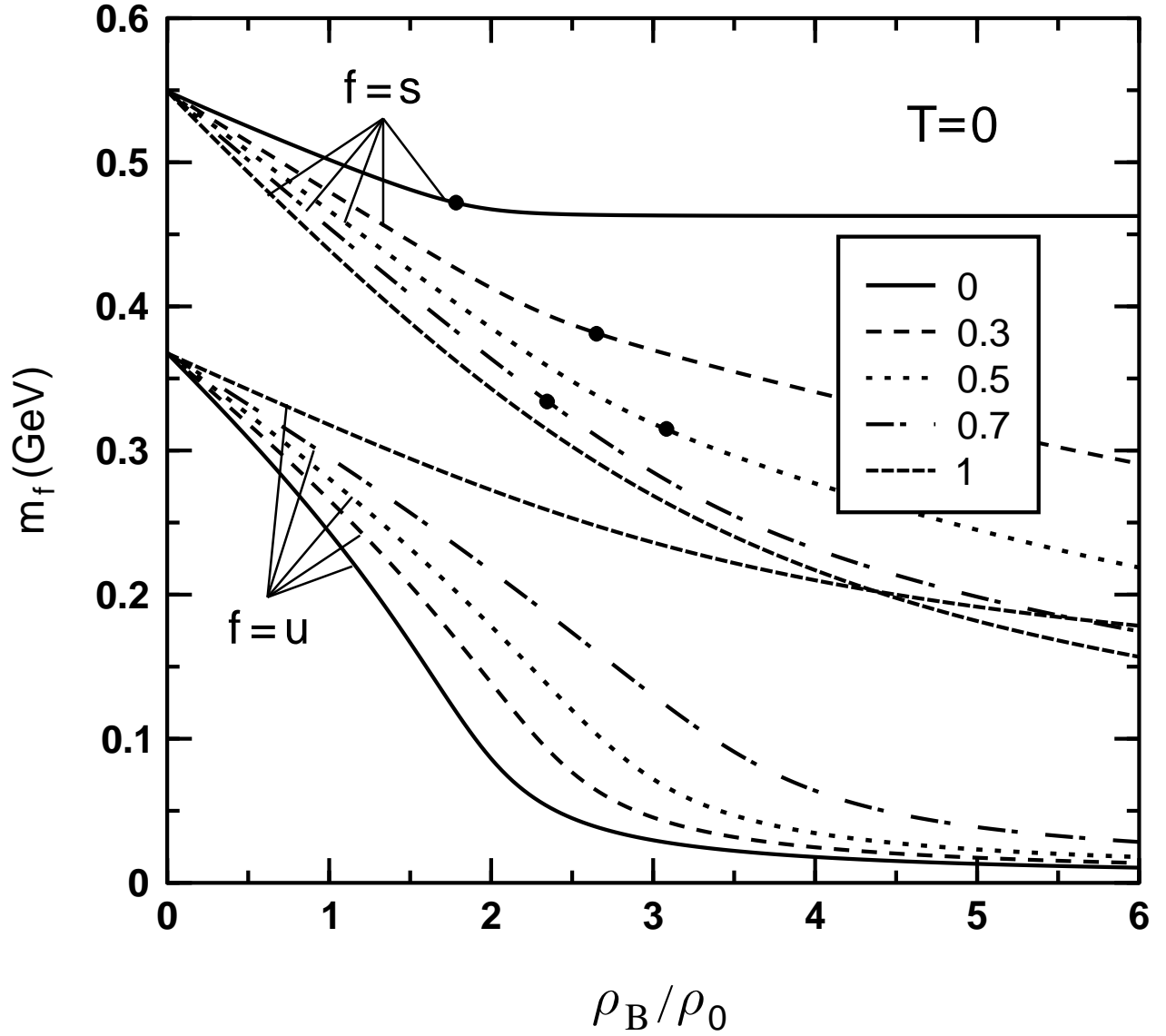
FIG. 2. Constituent masses of u and s quarks vs baryon density. Figures in the box show values of strangeness fraction r_s. Dots correspond to minima of energy per particle at given r_s.
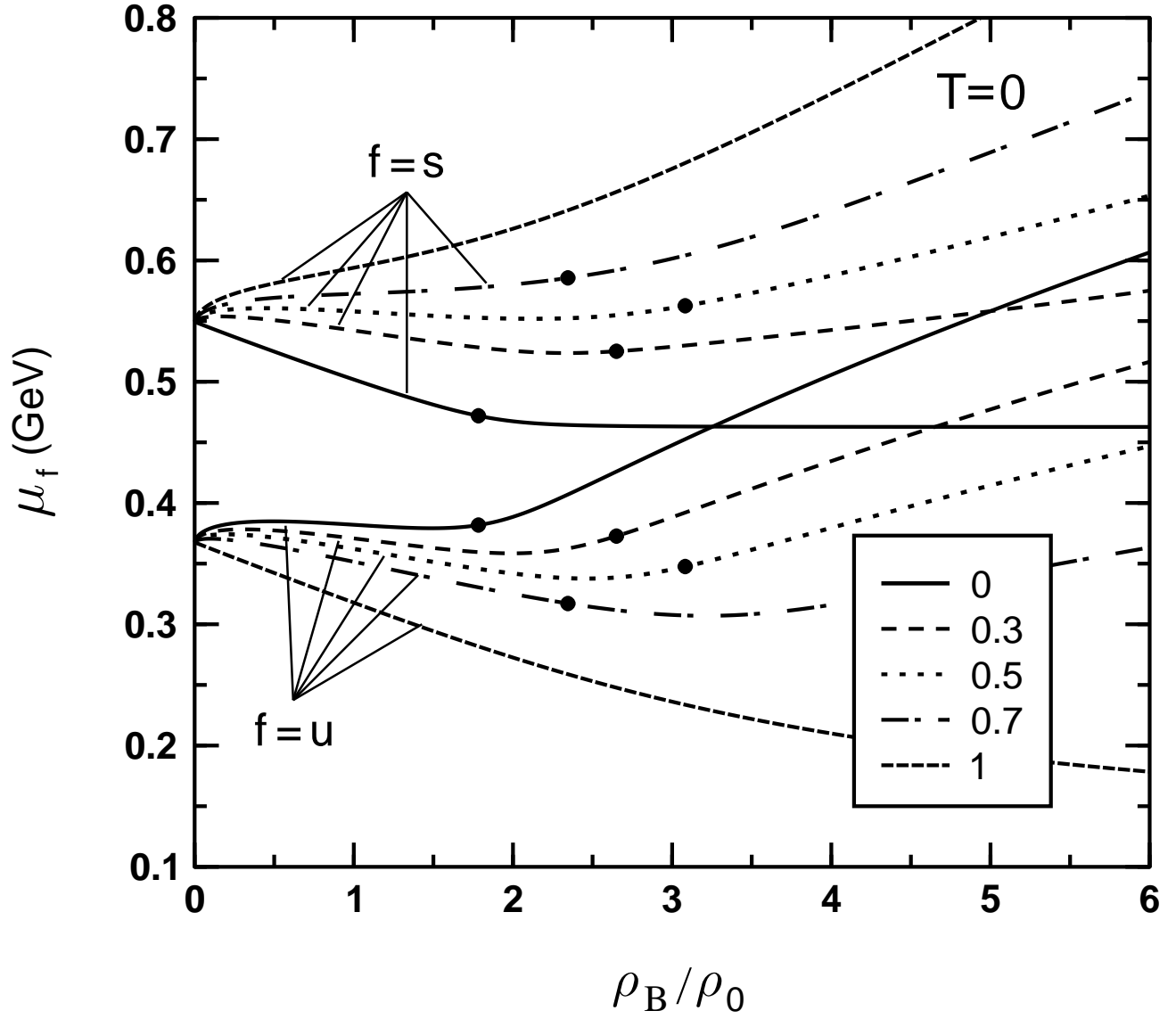
FIG. 3. The same as in Fig. 3, but for chemical potentials of u and s quarks.
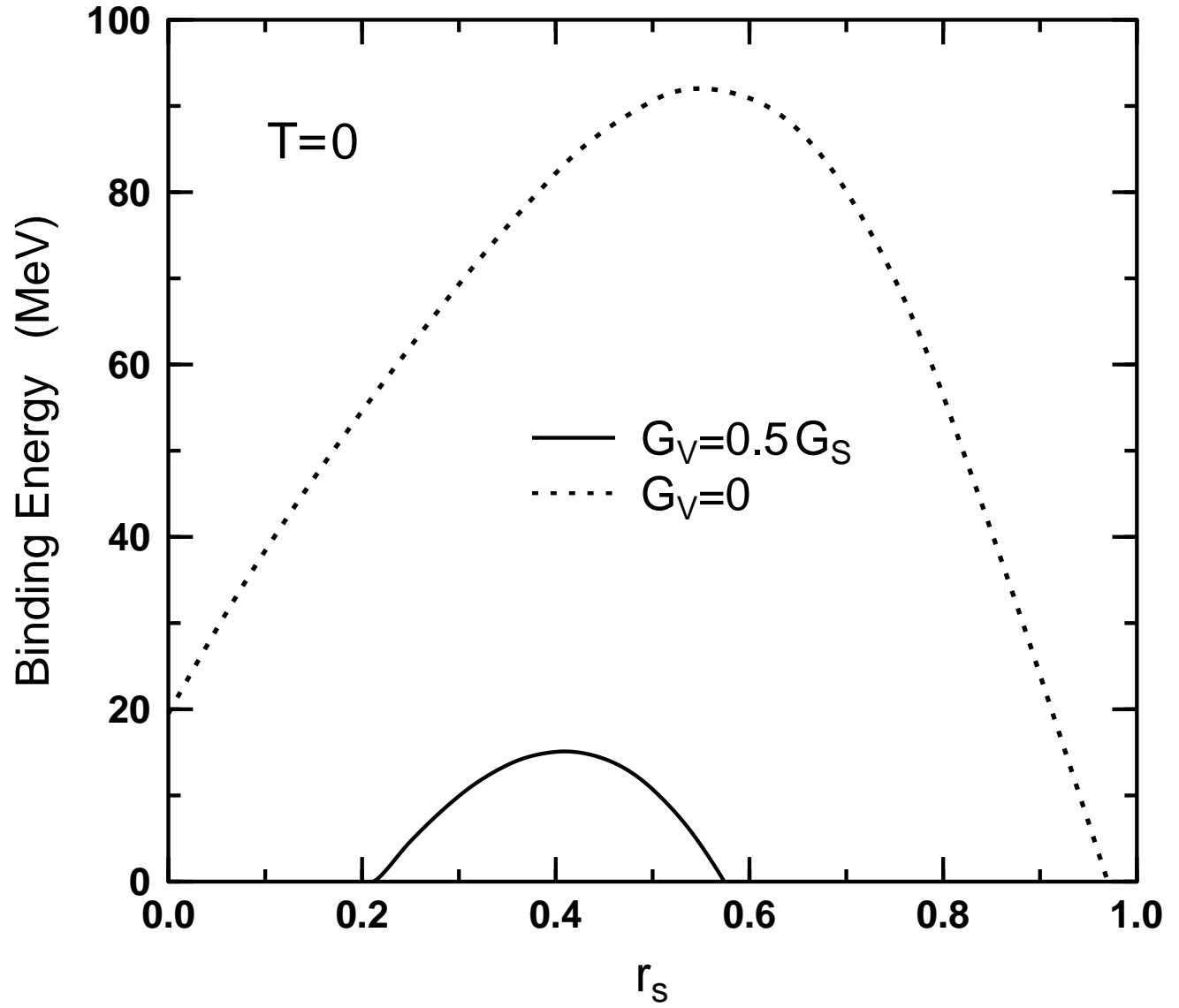
FIG. 4. Binding energies per baryon in pure quark matter as functions of strangeness fraction r_s. Dotted line shows the results of calculations when the vector interaction is switched off ($G_V = 0$).
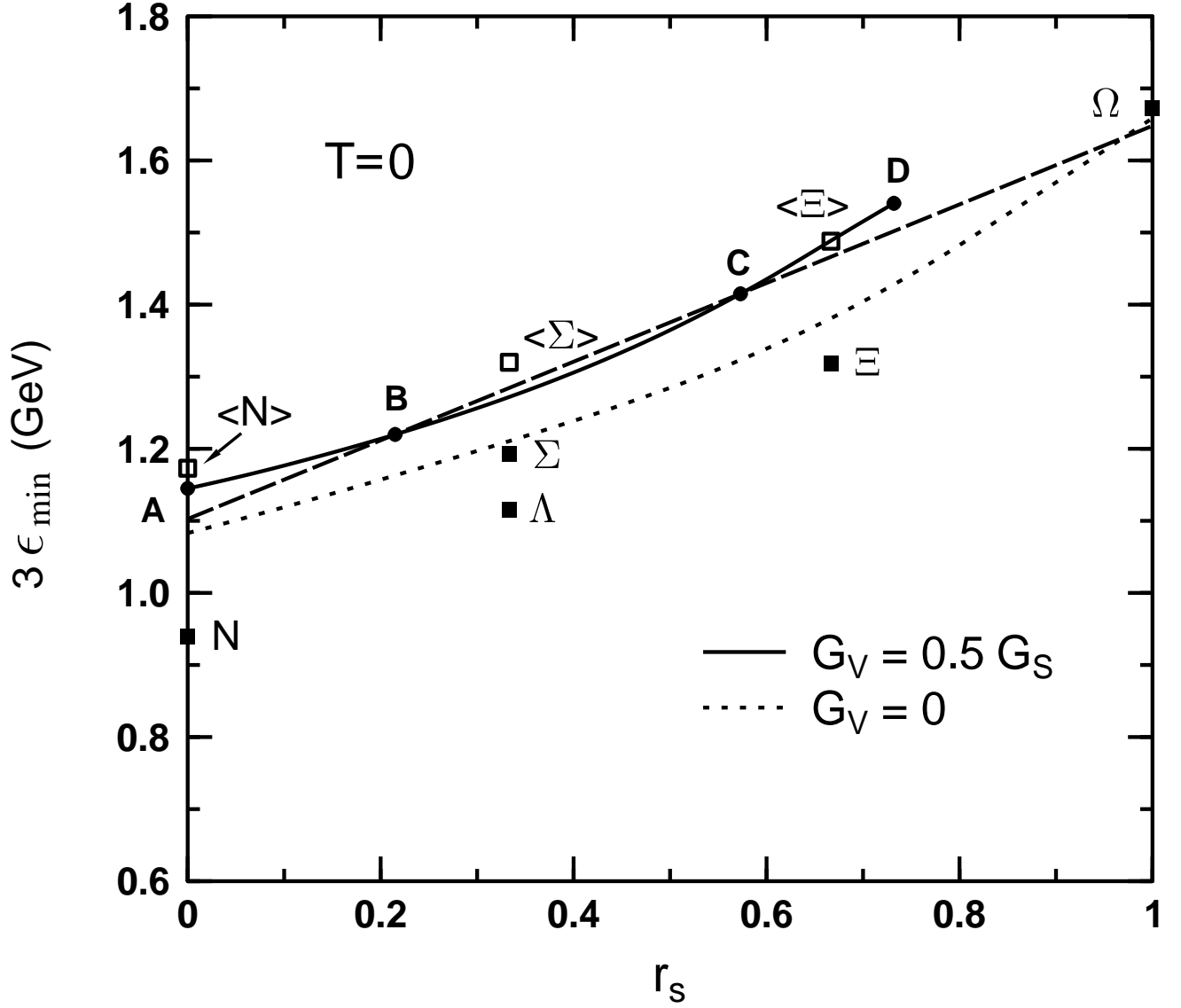
FIG. 5. Minimal energies per baryon in pure quark matter as functions of strangeness fraction r_s. Dashed line shows the same energy in the limit of zero particle densities, Eq. (36). Different parts of the solid line correspond to metastable (AB and CD) or bound (BC) states. Filled squares show masses of lightest baryons. Open squares represent the spin–isospin averaged masses of these baryons, e.g. $< N > = [m_N + 4 m_{\Delta(1232)}]/5$ (for details see Ref. [19]). Dotted line in the lower plot shows the results in the limit $G_V \to 0$.
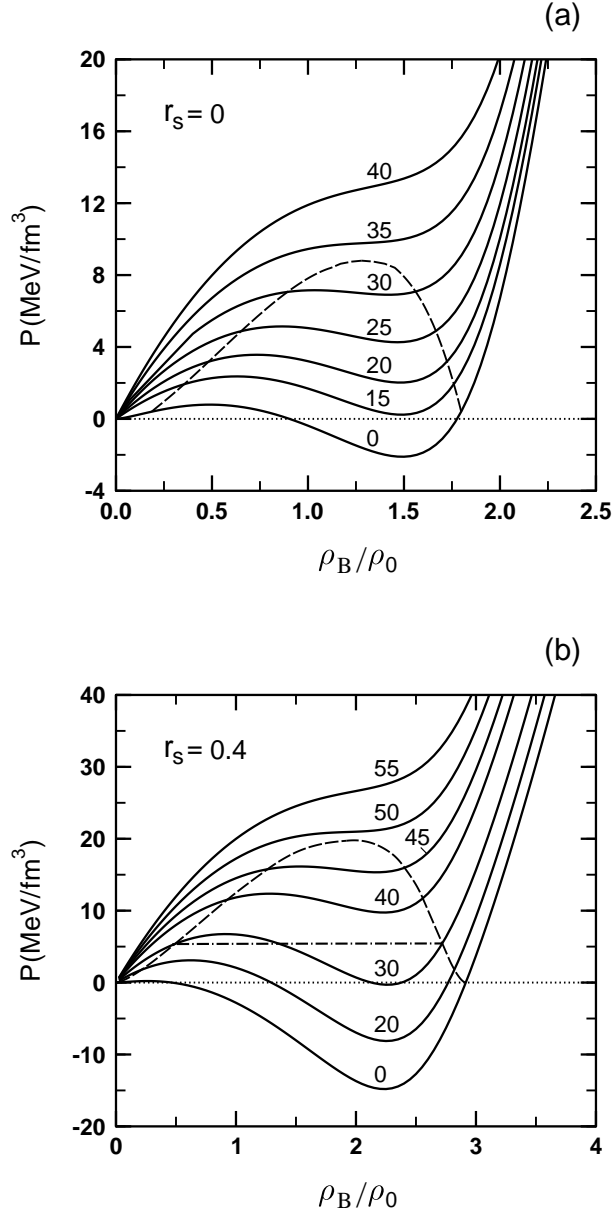
FIG. 6. Pressure isotherms for chemically equilibrated quark matter with $r_s = 0$ (upper part) and $r_s = 0.4$ (lower part). Temperatures are given in MeV near the corresponding curves. Boundaries of spinodal regions are shown by the dashed lines. The dashed–dotted line in lower plot shows the equilibrium pressure in the mixed phase–domain at $T = 30$ MeV.
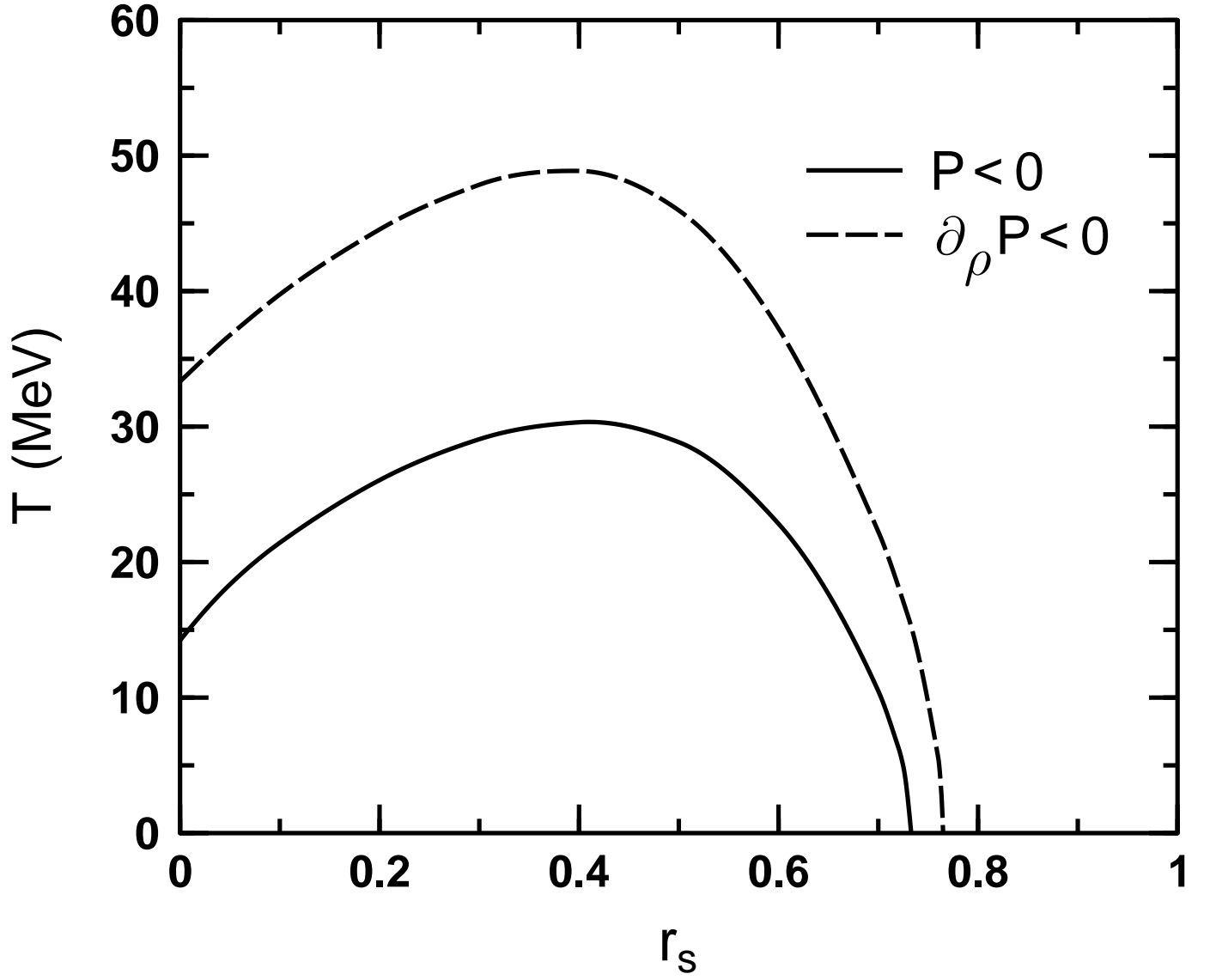
FIG. 7. Critical temperatures for existence of bound states ($P < 0$) and phase transition ($\partial_\rho P < 0$) in equilibrium quark matter as functions of strangeness fraction.
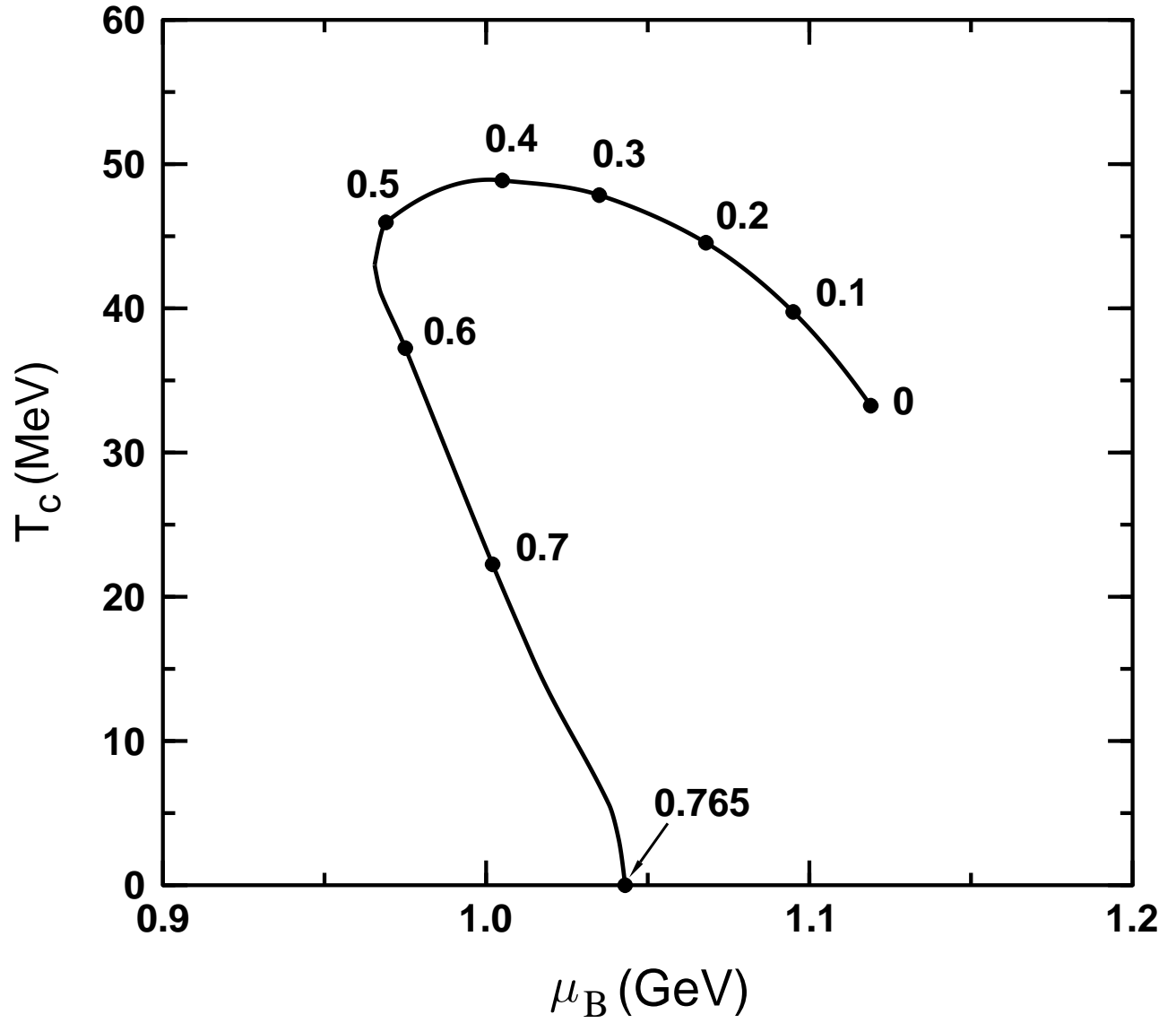
FIG. 8. Critical temperatures and baryon chemical potentials in equilibrated quark matter at fixed strangeness fractions r_s (indicated by figures near corresponding points).

(a)

$r_s = 0$

T (MeV)

— S/B=2
—·— S/B=5

C

PT

μ_B (GeV)

(b)

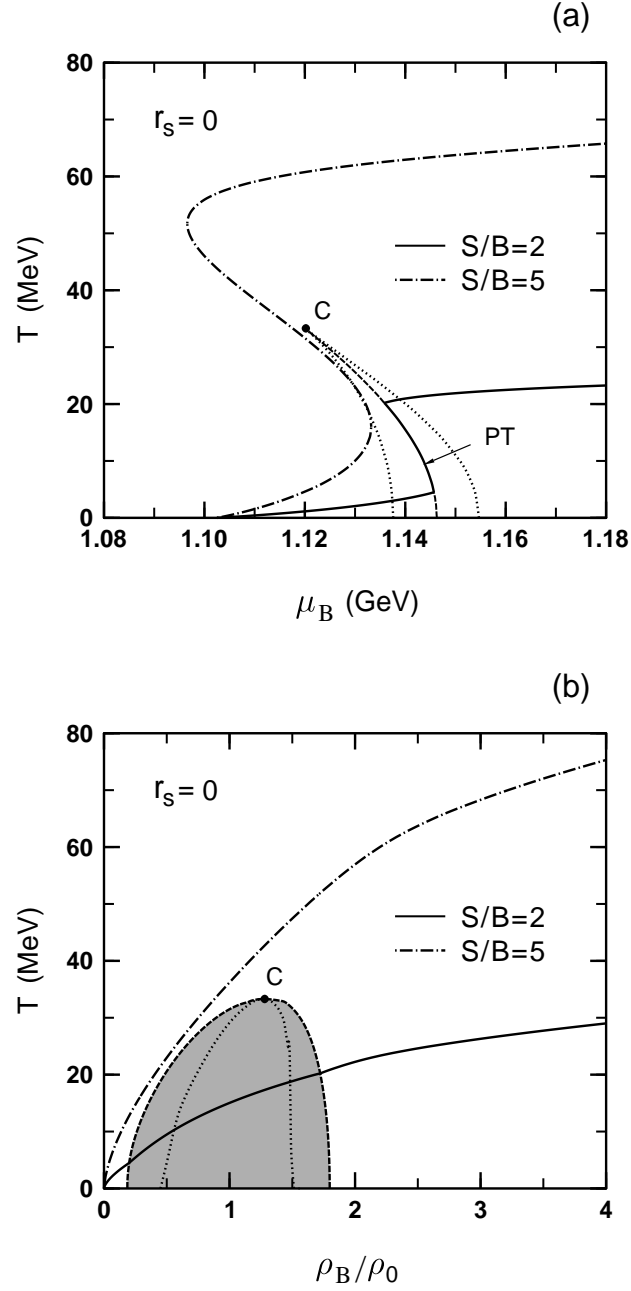$r_s = 0$

T (MeV)

— S/B=2
—·— S/B=5

C

ρ_B/ρ_0

FIG. 9. Isentropes $S/B = 2$ (solid line) and $S/B = 5$ (dashed–dotted line) of quark matter with zero net strangeness. Lower (upper) part corresponds to the $\mu_B - T$ ($\rho_B - T$) plane. Dotted and dashed lines represent, respectively, spinodal and binodal boundaries of the two–phase region (shown by shading in the lower plot). Point C marks the critical point.
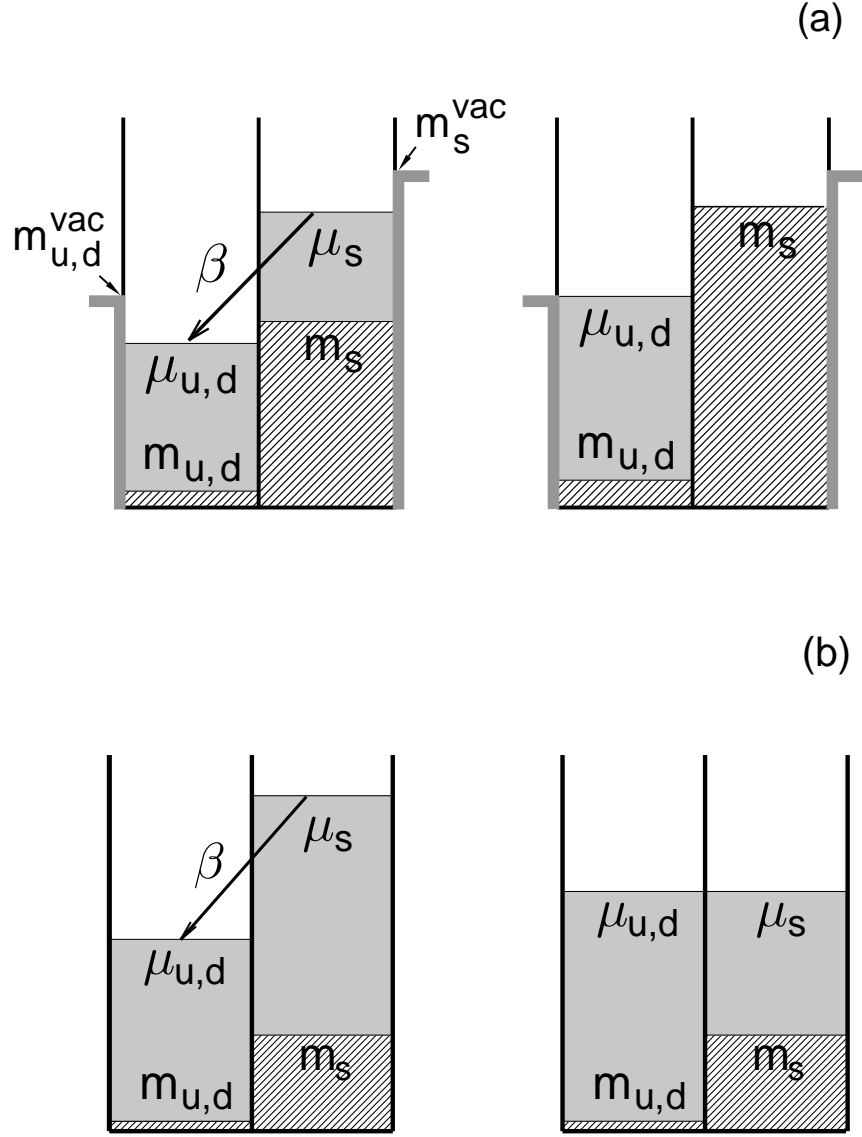
FIG. 10. Schematic pictures of energy levels (shown by shading) occupied by light and strange quarks in cold quark matter at different strangeness fractions r_s. Left and right upper diagrams (a) shows the results for strange ($r_s = 0.4$) and nonstrange ($r_s = 0$) matter predicted within the NJL model. Low panel (b) represents the same results within the MIT bag model [19]. Hatched boxes in upper and low panels shows, respectively, the constituent and bare quark masses. Arrows show weak decay processes $s \rightarrow u + e + \overline{\nu}_e$ and $s + u \rightarrow u + d$.