Statistical Coalescence Model with Exact Charm Conservation

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Abstract

The statistical coalescence model for the production of open and hidden charm is considered within the canonical ensemble formulation. The data for the J/ψ multiplicity in Pb+Pb collisions at 158 A·GeV are used for the model prediction of the open charm yield which has not yet been measured in these reactions.

The charmonium states J/ψ and ψ' have been measured in nucleus-nucleus (A+A) collisions at CERN SPS over the last 15 years by the NA38 and NA50 Collaborations. This experimental program was motivated by a suggestion [1] to use the J/ψ as a probe of the state of matter created in the early stage of the collision. In this approach a significant suppression of J/ψ production relative to Drell–Yan lepton pairs is predicted when going from peripheral to central Pb+Pb interactions at 158 A·GeV. This is originally attributed to the formation of a quark-gluon plasma, but could be also explained in microscopic hadron models as secondary collision effects (see [2] and references therein).

The statistical approach, formulated in Ref.[3], assumes that J/ψ mesons are created at hadronization according to the available hadronic phase-space. In this model the J/ψ yield is *independent* of the open charm yield. The model offers a natural explanation of the proportionality of the J/ψ and pion yields and the magnitude of the J/ψ multiplicity in hadronic and nuclear collisions.

Recently the statistical coalescence model [4] and the microscopical coalescence model [5] were introduced for the charmonium production. Similar to the statistical model [3], the charmonium states are assumed to be formed at the hadronization stage. However, they are produced as a coalescence of created earlier c-$\bar{c}$ quarks and therefore the multiplicities of open and hidden charm hadrons are *connected* in the coalescence models. In Ref. [4] the charm quark-antiquark pairs are assumed to be created at the early stage of A+A collision and the average number of c-$\bar{c}$ pairs, $N_{c\bar{c}}^{dir}$, is fixed by the model consideration based on the hard scattering approach. The estimated number $N_{c\bar{c}}^{dir}$ seems to be larger than the equilibrium hadron gas (HG) result. This requires the introduction of a new parameter in the HG approach [4] – the charm enhancement factor γ_c (it was denoted as g_c in Ref.[4]). This is analogous to the introduction of strangeness suppression factor

γ_s [6] in the HG model, where the total strangeness observed is smaller than its thermal equilibrium value. Within this approach the open charm hadron yield is enhanced by a factor γ_c and charmonium yield by a factor γ_c^2 in comparison with the equilibrium HG predictions.

The thermal HG calculations in Ref.[4] are done within the grand canonical ensemble (g.c.e.) formulation although the validity of the g.c.e. results for the charm hadron yield was questioned by the authors of [4] themselves. As the total number of charm hadrons is expected to be smaller than unity even for the most central Pb+Pb collisions an exact charm conservation within the canonical ensemble (c.e.) should be imposed[1]. Note also that the c.e. formulation was successfully used in Ref.[7] to calculate the open charm hadron abundances in e^+e^- collisions at $\sqrt{s} = 91.2$ GeV with an experimental input of the total open charm production. In this letter stimulated by the above proposals we consider the c.e. HG formulation for the physical picture suggested in Ref.[4]. The experimental value of the J/ψ multiplicity $\langle J/\psi \rangle$ will be then used to predict the open charm yield within the statistical coalescence model.

The main assumption of Ref.[4] is formulated as

$$N_{c\overline{c}}^{dir} ~=~ \frac{1}{2}\,\gamma_c\,N_O + ~\gamma_c^2~N_H~, \tag{1}$$

where N_O is the total thermal multiplicity of all open charm and anticharm mesons and (anti)baryons and N_H is the total thermal multiplicity of particles with hidden charm. Note that open charm resonance states (not included in [4]) give essential contribution[2] to N_O. The number of directly produced $c\overline{c}$ pairs $N_{c\overline{c}}^{dir}$ in the hard collisions is estimated in Ref.[4] to be equal to $N_{c\overline{c}}^{dir} \cong 0.17$ for Pb+Pb SPS collisions with $N_p = 400$ participants. This number is however not quite confident. The average number of $c\overline{c}$ pairs created in nucleon–nucleon collisions at $\sqrt{s} = 17.3$ GeV was estimated from existing data as $3 \cdot 10^{-4}$ in Ref.[8]. With a linear on N_p extrapolation to central A+A collisions one obtains $N_{c\overline{c}}^{dir} \cong 0.06$ for $N_p = 400$, but assuming that open charm production in central A+A collisions scales as $N_p^{4/3}$ an estimate $N_{c\overline{c}}^{dir} \cong 0.35$ is obtained for $N_p = 400$ [8]. Note also that a recent analysis of the dimuon spectrum measured in central Pb+Pb collisions at 158 A·GeV by NA50 Collaboration [9] suggests a significant enhancement of dilepton production in the intermediate mass region (1.5÷2.5 GeV) over the standard sources. The primary interpretation attributes this observation to the enhanced production of open charm [9]: about 3 times above the pQCD prediction for the open charm yield in Pb+Pb collisions at SPS.

In Ref.[4] N_O and N_H are calculated in the g.c.e.. In the g.c.e. the thermal multiplicities of both open charm and charmonium states are given as (Bose and Fermi effects are negligible):

$$N_j ~=~ \frac{d_j V~e^{\mu_j/T}}{2\pi^2}~T~m_j^2~K_2\left(\frac{m_j}{T}\right) ~\cong~ d_j~V~e^{\mu_j/T}~\left(\frac{m_jT}{2\pi}\right)^{3/2}~\exp\left(-\frac{m_j}{T}\right)~, \tag{2}$$

where V and T correspond to the volume[3] and temperature of HG system, m_j, d_j denote

[1]This was first suggested by K. Redlich and B. Müller (quoted in Ref.[4] and L. McLerran, private communication).

[2]We are thankful to Braun-Munzinger and Stachel for pointing this out.

[3]To avoid further complications we use ideal HG formulae and neglect excluded volume corrections.

particle masses and degeneracy factors and K_2 is the modified Bessel function. The particle chemical potential μ_j in Eq.(2) is defined as

$$\mu_j \ = \ b_j\mu_B \ + \ s_j\mu_S \ + \ c_j\mu_C \ , \tag{3}$$

where b_j, s_j, c_j denote the baryonic number strangeness and charm of particle j. The baryonic chemical potential μ_B regulates the baryonic density of the HG system whereas strange μ_S and charm μ_C chemical potentials should be found from the requirement of zero value for the total strangeness and charm in the system (in our consideration we neglect small effects of a non-zero electrical chemical potential).

In the c.e. formulation (i.e. when the requirement of zero "charm charge" of the HG is used in the exact form) the thermal charmonium multiplicities are still given by Eq.(2) as charmonium states have zero charm charge. The multiplicities (2) of open charm hadrons will however be multiplied by an additional 'canonical suppression' factor (see e.g. [10]). This suppression factor is the same for all individual open charm states. It leads to the total open charm multiplicity N_O^{ce} in the c.e.:

$$N_O^{ce} \ = \ N_O \, \frac{I_1(N_O)}{I_0(N_O)} \ , \tag{4}$$

where N_O is the total g.c.e. multiplicity of all open charm and anticharm mesons and (anti)baryons calculated with Eq.(2) and I_0, I_1 are the modified Bessel functions. For large open charm multiplicity $N_O >> 1$ one finds $I_1(N_O)/I_0(N_O) \rightarrow 1$ and therefore $N_O^{ce} \rightarrow N_O$, i.e. the g.c.e. and c.e. results coincide. For $N_O << 1$ one has $I_1(N_O)/I_0(N_O) \cong N_O/2$ and $N_O^{ce} \cong N_O \cdot N_O/2$, therefore, N_O^{ce} is strongly suppressed in comparison to the g.c.e. result N_O .

Assuming the presence of the charm enhancement factor γ_c the statistical coalescence model within the c.e. should now be formulated as:

$$N_{c\bar{c}}^{dir} \ = \ \frac{1}{2} \, \gamma_c \, N_O \, \frac{I_1(\gamma_c N_O)}{I_0(\gamma_c N_O)} \ + \ \gamma_c^2 \, N_H \ . \tag{5}$$

Therefore, the baryonic number, strangeness and electric charge of the HG system are treated in our approach according to the g.c.e. but charm is considered in the c.e. formulation where the exact charge conservation is imposed.

The logic of Ref.[4] is the following: 1) input $N_{c\bar{c}}^{dir}$ number (it is assumed to be equal to 0.17 for $N_p = 400$) into Eq.(1); 2) calculate the γ_c value; 3) obtain J/ψ multiplicity as $\langle J/\psi \rangle = \gamma_c^2 N_{J/\psi}$, where $N_{J/\psi}$ is given by Eq.(2). Note that the second term in both Eq.(1) and (5) gives only a tiny correction to the first term. Therefore, $\gamma_c \cong 2N_{c\bar{c}}^{dir}/N_O$.

Our consideration differs from that of Ref.[4] in three points. First, we use Eq.(5) instead of (1). Second, in our calculations we take into account all known particles and resonances with open and hidden charm [11]. Third, we will proceed with Eq.(5) in the reverse way. As the $\langle J/\psi \rangle$ multiplicities can be extracted from the NA50 data on Pb+Pb collisions at 158 A·GeV for different values of N_p, we start from the requirement:

$$\langle J/\psi \rangle \ = \ \gamma_c^2 \, N_{J/\psi}^{tot} \ , \tag{6}$$

to fix the γ_c factor. In Eq.(6) the total J/ψ thermal multiplicity is calculated as

$$N_{J/\psi}^{tot} = N_{J/\psi} \;+\; Br(\psi')N_{\psi'} \;+\; Br(\chi_1)N_{\chi_1} \;+\; Br(\chi_2)N_{\chi_2}\;, \tag{7}$$

where $N_{J/\psi}$, $N_{\psi'}$, N_{χ_1}, N_{χ_2} are given by Eq.(2) and $Br(\psi') \cong 0.54$, $Br(\chi_1) \cong 0.27$, $Br(\chi_2) \cong 0.14$ are the decay branching ratios of the excited charmonium states into J/ψ. Eq.(5) will be used then to calculate the value of $N_{c\bar{c}}^{dir}$. This value will be considered as a prediction of the statistical coalescence model: the open charm yield has not yet been measured in Pb+Pb collisions at SPS.

We use two different sets of the chemical freeze-out parameters:

A: $T = 168$ MeV, $\mu_B = 266$ MeV, $\gamma_s = 1$ Ref.[12];

B: $T = 175$ MeV, $\mu_B = 240$ MeV, $\gamma_s = 0.9$ Ref.[13].

They both were fixed by the HG model fit to the hadron yields data in Pb+Pb collisions at 158 A·GeV (the inclusion of open charm and charmonium states does not modify the rest of the hadron yields). For the fixed number of participants N_p the volume V is defined then from $N_p = V n_B$, where $n_B = n_B(T, \mu_B, \gamma_s)$ is the baryonic density calculated in the g.c.e.. With two sets of the chemical freeze-out parameters **A** and **B** we find $N_{J/\psi}^{tot}$ and N_O values using Eq.(2), calculate γ_c factors from Eq.(6) and then calculate $N_{c\bar{c}}^{dir}$ from Eq.(5).

In the above c.e. consideration with exact charm conservation the γ_c parameter regulates the *average* number $N_{c\bar{c}}$ of $c\bar{c}$-pairs in the HG. Therefore, $N_c = N_{\bar{c}}$ is restricted exactly (the c.e.), but the value of $N_c + N_{\bar{c}}$ is restricted on average (the g.c.e). The above c.e. calculations are based therefore on the thermal model distribution for probabilities to observe $0, 1, 2, ...$ of $c\bar{c}$-pairs in the equilibrium HG. One needs then an additional parameter γ_c to adjust these thermal probabilities to the required number of $N_{c\bar{c}}^{dir}$. Another way is to restrict also the $N_c + N_{\bar{c}}$ numbers in the c.e. calculations and use non-thermal probabilities to create $k = 1, 2, ...$ of $c\bar{c}$-pairs in hard collisions. For the fixed number of $c\bar{c}$-pairs equal to k, the average multiplicity of hidden charm can be approximately calculated in the following way. We keep in the c.e. HG partition function the leading terms only with 0 and 1 hidden charm particles and neglect all configurations with 2,3,..,k charmonium particles. This corresponds to the expansion in powers of the small parameter $N_H/(N_O/2)^2 << 1$. It gives:

$$\langle N_H \rangle_k \;\approx\; k^2 \frac{N_H}{(N_O/2)^2}\;, \tag{8}$$

where multiplicities N_O and N_H are calculated in the g.c.e. using Eq.(2).

Because of the assumed hard scattering origin of the $c\bar{c}$ production the Poisson distribution $P(k) = f^k \exp(-f)/k!$ looks quite natural ($k = 0, 1, 2, ...$ is the number of pairs created, $f = N_{c\bar{c}}^{dir}$ is the average number of pairs). The calculations with these 'dynamical' probabilities contain no additional free parameter. All 'dynamical' information needed for the c.e. calculation is now given by the value of f (parameter γ_c does not appear). With Eq.(8) the result for J/ψ yield is:

$$\langle J/\psi \rangle \;\approx\; f(f+1)\, \frac{N_{J/\psi}^{tot}}{(N_O/2)^2}\;, \tag{9}$$

where $N^{tot}_{J/\psi}$ is given by Eq.(7). Using the experimental values for $\langle J/\psi\rangle$ one obtains from Eq.(9) the average number of $c\bar{c}$-pairs $f = N^{dir}_{c\bar{c}}$. The results appear to be rather close to those obtained with thermal probabilities. The reason of this fact is that states with $k = 0$ and $k = 1$ dominate in both thermal and 'dynamical' probability distributions. To illustrate this lets consider an extreme choice: the HG states with $N_c = N_{\bar{c}} = 1$ appear with probability $f = N^{dir}_{c\bar{c}}$, the HG states with $N_c = N_{\bar{c}} = 0$ appear with probability $1 - f$, and states with more than one $c\bar{c}$-pairs are neglected. Under these restrictions the J/ψ multiplicity becomes equal to:

$$\langle J/\psi\rangle \;=\; f\,\frac{N^{tot}_{J/\psi}}{(N_O/2)^2 + N_H}\;. \tag{10}$$

One sees that Eq.(10) is close to Eq.(9) if $f << 1$ and $N_H << (N_O/2)^2$. From Eq.(10) one finds:

$$f \;=\; \frac{\langle J/\psi\rangle}{N^{tot}_{J/\psi}}\;\Big[(N_O/2)^2 + N_H\Big] \;=\; \gamma_c^2\;\Big[(N_O/2)^2 + N_H\Big]\;. \tag{11}$$

This coincides with Eq.(5) at small values of $\gamma_c N_O$.

We present now the model calculations for central Pb+Pb interactions at 158 A·GeV ($N_p = 100 \div 400$). Using the estimates for experimental J/ψ multiplicities and assuming that the system volume V scales linearly with N_p (i.e. $N_p = V n_B(T, \mu_B, \gamma_s)$) we, first, calculate the thermal J/ψ multiplicity $N^{tot}_{J/\psi}$ (7) – including the feeding from the excited charmonium states. Then we use Eq.(6) to find the parameter γ_c. Finally, we calculate the predicted values of $N^{dir}_{c\bar{c}}$ from Eq.(5). The results are presented in Tables 1 and 2 where the sets of the chemical freeze-out parameters **A** and **B** are respectively used.

A reliable extraction of the J/ψ yields from the published data appears to be non-trivial[4], Ref.[14] suggests an approximately linear increase of $\langle J/\psi\rangle$ with N_p. The results for $\langle J/\psi\rangle$ presented in Ref.[3] were evaluated from the data of the NA50 Collaboration [15] using the procedure described in [16]. These results for $\langle J/\psi\rangle$ are used as input for the statistical coalescence model analysis in Tables 1 and 2. Assuming that $N^{dir}_{c\bar{c}}$ scales as N_p^{α} we find from Tables 1 and 2 a value of $\alpha = 1.6 \div 1.7$. This value is larger than $\alpha \cong 4/3$ expected in the hard-collision model. Although the values of $N^{tot}_{J/\psi}$, N_O and γ_c are rather sensitive to the temperature parameter, the model predictions for $N^{dir}_{c\bar{c}}$ remain essentially unchanged.

In conclusion, the statistical coalescence model with an exact charm conservation is formulated. The canonical ensemble suppression effects are important for the thermal open charm yield even at $N_p = 400$. These effects become crucial when the number of participants N_p decreases. From the J/ψ multiplicity data in Pb+Pb collisions at 158 A·GeV the open charm yield is predicted: $N^{dir}_{c\bar{c}} = 0.5 \div 0.6$ in central ($N_p = 360$) collisions. An uncertainty of this prediction is mainly because of uncertainties in different compilations of the $\langle J/\psi\rangle$ data.

[4]We are thankful to K. Redlich and M. Gaździcki for the useful comments.

Table 1

N_p	$\langle J/\psi\rangle \cdot 10^4$ NA50 data Compil. [3]	$N^{tot}_{J/\psi} \cdot 10^4$ Eq.(7) Set **A**	N_O Set **A**	γ_c Eq.(6)	$N^{dir}_{c\bar{c}}$ Thermal Eq.(5)	$N^{dir}_{c\bar{c}}$ Poisson Eq.(9)
100	2.2±02	0.56	0.26	2.0	0.066	0.064
200	3.9±0.2	1.1	0.52	1.9	0.21	0.20
300	6.4±0.6	1.7	0.79	2.0	0.46	0.41
360	6.9±0.7	2.0	0.94	1.9	0.57	0.51

Table 2

N_p	$\langle J/\psi\rangle \cdot 10^4$ NA50 data Compil. [3]	$N^{tot}_{J/\psi} \cdot 10^4$ Eq.(7) Set **B**	N_O Set **B**	γ_c Eq.(6)	$N^{dir}_{c\bar{c}}$ Thermal Eq.(5)	$N^{dir}_{c\bar{c}}$ Poisson Eq.(9)
100	2.2±02	1.1	0.39	1.4	0.072	0.070
200	3.9±0.2	2.2	0.77	1.3	0.23	0.22
300	6.4±0.6	3.3	1.17	1.4	0.50	0.45
360	6.9±0.7	4.0	1.40	1.3	0.62	0.55

It is interesting to compare our estimate $N^{dir}_{c\bar{c}} = 0.5 \div 0.6$ with results predicted in different model approaches. The pQCD inspired models suggest the values of $N^{dir}_{c\bar{c}} = 0.1 \div 0.3$ (the value of $N^{dir}_{c\bar{c}} = 0.17$ is an estimate of Ref.[4]). Much larger value of $N_{c\bar{c}} \cong 3.4$ is obtained in Ref.[5] within the microscopic coalescence model. Even larger value of $N_{c\bar{c}} \approx 8$ is suggested in Ref.[16] assuming the charm equilibration in the quark-gluon plasma at the very early stage of Pb+Pb reaction.

The statistical coalescence model predicts also the N_p dependence of $N^{dir}_{c\bar{c}}$ and the yields of individual open charm states. All these predictions of the statistical coalescence model (the open charm yield has not been measured in Pb+Pb) can be tested in the near future (measurements of the open charm are planned at CERN). This will require to specify more accurately the $\langle J/\psi\rangle$ data.

The charm enhancement factor γ_c found from the $\langle J/\psi\rangle$ data appears to be not much different from unity and its value is rather sensitive to the temperature parameter. Therefore, both the statistical model of Ref.[3] and the statistical coalescence model considered in the present paper lead to similar results for the J/ψ yield. However, the predictions of these two models will differ greatly at RHIC energies: according to [3] the J/ψ to pion ratio is expected to be approximately equal to its value at the SPS, but according to the statistical coalescence model this ratio should increase very strongly. The predictions of the present model for the RHIC energies will be presented elsewhere.

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References

[1] T. Matsui and H. Satz, Phys. Lett. **B178** (1986) 416.

[2] C. Spieles et al, Phys. Lett. **B458** (1999) 137; Phys. Rev. **C60** (1999) 054901.

[3] M. Gaździcki and M.I. Gorenstein, Phys. Rev. Lett. **83** (1999) 4009.

[4] P. Braun-Munzinger and J. Stachel, Phys. Lett. **B490** (2000) 196.

[5] P. Csizmadia and P. Lévai, hep-ph/0008195 (2000);
P. Lévai, T.S. Biró, P. Csizmadia, T. Csögö and J. Zimányi, to be published.

[6] J. Rafelski, Phys. Lett. **B62** (1991) 333.

[7] F. Becattini, Z. Phys. **C69** (1996) 485; J. Phys. **G23** (1997) 1933;
F. Becattini and U. Heinz, Z. Phys. **C76** (1997) 269.

[8] M. Gaździcki and C. Markert, Acta Phys. Polon. B31 (2000) 965.

[9] M.C. Abreu et al. (NA50 Collab.), CERN–EP–2000–012, to appear in Eur. Phys. J. **C**.

[10] K. Redlich and L. Turko, Z. Phys. **C5** (1980) 541;
J. Rafelski and M. Danos, Phys. Lett. **B97** (1980) 279;
J. Cleymans, K. Redlich and E. Suhonen, Z. Phys. **C51** (1991) 137;
M.I. Gorenstein, M. Gaździcki and W. Greiner Phys. Lett. **B483** (2000) 60.

[11] Particle Data Group, Eur. Phys. J. **C15** (2000) 1.

[12] P. Braun-Munzinger, I. Heppe and J. Stachel, Phys. Lett. **B465** (1999) 15.

[13] F. Becattini et al, hep-ph/0002267 (2000).

[14] M. Gaździcki, Phys. Rev. **C60** (1999) 054903.

[15] M.C. Abreu et al., Phys. Lett. **B450** (1999) 456.

[16] M. Gaździcki and M.I. Gorenstein, Acta Phys. Pol. **B30** (1999) 2705.