# Bremsstrahlung from a Microscopic Model of Relativistic Heavy Ion Collisions

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## Abstract

We compute bremsstrahlung arising from the acceleration of individual charged baryons and mesons during the time evolution of high-energy Au+Au collisions at the Relativistic Heavy Ion Collider using a microscopic transport model. We elucidate the connection between bremsstrahlung and charge stopping by colliding artificial pure proton on pure neutron nuclei. From the intensity of low energy bremsstrahlung, the time scale and the degree of stopping could be accurately extracted without measuring any hadronic observables.

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# I. INTRODUCTION

The long awaited Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory has now begun operation. It is widely expected that heavy ion collisions at RHIC will result in the creation of hot and dense deconfined QCD matter, termed the Quark-Gluon-Plasma (for a review on the properties and signatures of the plasma, we refer to [1]). Apart from searching for this plasma, a question of great interest is how bulk matter behaves in a very energetic collision. This is an old question, going back to the 1950's and therefore pre-dating QCD by 20 years. An early hypothesis, due to Landau [2], is that matter would come to a complete halt before it is driven to explode by the extremely high pressure. Another hypothesis is due to Bjorken [3] where projectile and target pass through each other but leave behind a zone of hot and dense matter.

To find out whether the initial nuclei are left with none of their original velocity or whether they will be traveling with almost the original velocity at the end of the collision, the most obvious method is to measure the final hadron rapidity distributions as is done by the various experimental collaborations; for example, see [4,5]. This method is certainly direct and straightforward, but it is not the simplest. Indeed, to do this properly it is necessary to measure various baryon species in a wide range of acceptance, not an easy task in a collider environment such as RHIC. Furthermore, the measurement of the final-state baryons cannot really distinguish between primordial baryons (initially contained in the two colliding nuclei), which have undergone massive stopping and newly created baryons, which are already produced at their final rapidity values.

Although strong stopping has been observed in collisions of heavy nuclei at the GSI/SIS, BNL/AGS and CERN/SPS accelerators [4,5], it is by no means clear whether this trend will continue up to the far higher available collision energies at RHIC (the maximum achievable energy at the SPS is $\sqrt{s} = 8.6$ GeV/nucleon for Pb+Pb, whereas RHIC will collide Au+Au at up to $\sqrt{s} = 100$ GeV/nucleon). The pertinent question is what particular measurements will provide information on the degree of energy, momentum, baryon, and electric charge

stopping. The degree of stopping correlates with the maximum energy density achieved in the collision, albeit in complicated ways.

Already more than 20 years ago bremsstrahlung was suggested as a simple means to determine the degree of stopping in nucleus-nucleus collisions [6]. That first paper concerned nuclear collisions at laboratory energies of order 1 GeV per nucleon. This was later followed by theoretical studies at laboratory energies of order 100 GeV per nucleon in [7,8]. More recently, very soft photons were studied in [9] along with a detector design to measure them at RHIC. This was followed by further theoretical studies in [10,11] where it was shown that the coarse grained space-time evolution of central nucleus-nucleus collisions at RHIC could be discerned by measuring photons with energies up to 200 MeV. Bremsstrahlung exploits the fact that initially the ions are moving relativistically and are highly charged (Z=79 for Au). Therefore they copiously emit bremsstrahlung when they experience any deceleration due to strong interactions. Because of the high energies involved, these relativistically moving, highly charged target and projectile will radiate bremsstrahlung in the extreme forward or backward directions. The intensity is determined mainly by the amount of slowing down experienced during the collisions. So by strategically placing photon detectors around the beam pipe in a narrow cone, the amount of stopping can be determined [10]. In addition, the space-time evolution of the colliding matter can be probed by this much simpler method [11].

In [11] and [12], various models for the time evolution of heavy ion collisions at RHIC were used. There are also some similar earlier works on the subject using numerical dynamical models but at lower energies [13–15]. In this paper a microscopic hadronic model, referred to as the Ultra-relativistic Quantum Molecular Dynamics model (UrQMD) [16,17], will be used to calculate bremsstrahlung. Since this is a dynamical model that follows the space-time evolution of the initial target and projectile as well as all the produced hadrons, it should allow us to discover what can be expected for bremsstrahlung from realistic collisions. Being able to follow the detailed collision history will give us a better insight into what bremsstrahlung can tell us about the collisions. The method of calculation will be detailed

in Sect. II and the results presented in Sect. III.

In [18] a quantity $\mathcal{S}$, obtainable both from hadron rapidity distributions and from bremsstrahlung measurements, was defined so that it is equal to unity when there is full charge stopping and to zero when there is total transparency. We will discuss the extraction of $\mathcal{S}$ from the microscopic evolution of heavy ion collisions as treated by UrQMD. This together with results from UrQMD, will be presented in Sect. III and IV.

## II. ULTRA-RELATIVISTIC QUANTUM MOLECULAR DYNAMICS MODEL

### A. The Model

The UrQMD model is a dynamical transport model that follows the time evolution of a heavy-ion collision in the entire many-body phase space. UrQMD has been applied to heavy-ion collisions in the energy range from a few hundreds of A·MeV to several hundreds of A·GeV using the same basic concepts and physics inputs at all energies. A detailed description of the underlying concepts and comparisons to experimental data are available in Refs. [16,17].

The model includes explicitly 55 different baryon species (nucleons, deltas, hyperons, and their known resonances [19] up to masses of 2.25 GeV) and 32 different meson species (including the known meson resonances [19] up to masses of 2 GeV), as well as their respective anti-particles and all isospin-projected states. At RHIC energies, the treatment of subhadronic degrees of freedom is of great importance. These degrees of freedom enter in UrQMD via the introduction of a formation time for hadrons produced in string fragmentation. Hadrons produced through string decays have a non-zero formation time, $\tau_f$, which depends on the energy-momentum of the particle. Newly formed particles cannot interact during their formation time. The leading hadrons interact within their formation time with a reduced cross-section, which is taken to be proportional to the number of their original constituent quarks.

All hadrons are propagated (in a relativistic cascade sense) according to Hamilton's equations of motion, supplemented by a relativistic Boltzmann-Uehling-Uhlenbeck collision term involving all hadron states. The collision term is based on tabulated or parameterized experimental cross-section (when available). Resonance absorption and scattering is handled via the principle of detailed balance. If no experimental information is available, the cross-section is either calculated via a One-Boson-Exchange model or via a modified additive quark model.

## B. The Method

The formula for computing classical bremsstrahlung from a space and time-dependent current is well-known [20]. For our purposes, it is convenient to express the current as a sum of separate currents denoting the flow of individual moving charges. For a charge $q_i$ with position $\boldsymbol{r}_i(t)$ and velocity $\boldsymbol{v}_i(t)$ the current is

$$\boldsymbol{J}_i(\boldsymbol{r}_i, t) = q_i \boldsymbol{v}_i(t) \, \delta \left( \boldsymbol{r}_i - \boldsymbol{r}_i(0) - \int_0^t dt' \, \boldsymbol{v}_i(t') \right) \, . \tag{1}$$

The intensity and number of photons emitted with frequency $\omega$ in the direction $\boldsymbol{n}$ is

$$\frac{d^2 I}{d\omega d\Omega} = \omega \frac{d^2 N}{d\omega d\Omega} = \omega^2 \, |\mathbf{A}|^2 \tag{2}$$

where the amplitude is

$$\mathbf{A} = \sum_i \int \frac{dt \, d^3 \boldsymbol{r}_i}{4\pi^{3/2}} \, \left( \boldsymbol{n} \times [\boldsymbol{n} \times \boldsymbol{J}_i(\boldsymbol{r}_i, t)] \right) \, e^{i\omega(t - \boldsymbol{n} \cdot \boldsymbol{r}_i(t))} \, . \tag{3}$$

The sum is over all charged particles present in the system at any given time $t$. Unlike in [9,11], we are using units with $\alpha = e^2/4\pi = 1/137$ which accounts for the odd factor of $1/2\sqrt{\pi}$ in Eq. (3).

For low energy photons the details of the individual hadron collisions, such as whether they are elastic or inelastic, are unimportant. For most of the hadron collisions and decays of relevance in this study the typical proper, physically estimated spatial and temporal extent is on the order of 1 fm. Photons with energy and momentum less than about 200 MeV will

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not resolve these details. Consider the following examples. Suppose that the center-of-mass of a set of particles involved in one of the microscopic collisions or decays moves with speed $v$ along the beam axis with a corresponding Lorentz factor $\gamma$ relative to the frame in which the bremsstrahlung is computed, which is the nucleus-nucleus center-of-velocity frame. If the proper time for this collision or decay is $\tau \approx 1$ fm/c then in the computational frame it takes a time $\gamma\tau$. The ability of a photon to resolve this is determined by the phase in Eq. (3) which is $\omega\gamma\tau(1 - v\cos\theta)$. A photon with $\omega < 1/\left(\tau\gamma(1 - v\cos\theta)\right)$ will not be sensitive to the fine details of this process, only to the gross features: namely, the difference between the incoming and outgoing currents. If $v = 0$ then $\omega$ would have to be comparable to $1/\tau \approx 200$ MeV. In the other limit, $\gamma \gg 1$, the radiation is strongly peaked at $\theta \approx 1/(2\gamma)$ [20], and so $\omega$ would have to be as large as $\gamma/\tau$ to resolve the details. These sorts of details are not desired; a proper description of them would require knowledge of the dynamics on the level of quarks and gluons, not hadrons. Since there is very little radiation away from the forward direction, and since one does not expect transverse motion of the afore-mentioned frames of reference with large $\gamma$, the limitation to photons with energy less than 200 MeV should be safe.

As a consequence of the above discussion we treat the accelerations of the participating hadrons as instantaneous. In fact, that is exactly how UrQMD treats collisions and decays, even when string formation and decay is involved. For example, a string may decay by emitting a hadron at $(\mathbf{x}_1, t_1)$, then a second hadron at $(\mathbf{x}_2, t_2)$, and so on. Electric charge is conserved by each of these instantaneous events. In UrQMD particles created with high energy have an associated formation time. Within this time they are not allowed to interact via the strong interactions. If that hadron has an electric charge it does contribute to the electromagnetic radiation amplitude immediately, otherwise charge conservation would be violated! We do not justify or criticize the UrQMD approach; rather, our goal is to point out that bremsstrahlung can be used to test it and other detailed dynamical models of high energy nuclear collisions.

For each hadron involved in a microscopic event $e$ with space-time coordinate $(\mathbf{x}_e, t_e)$,

whether it be a collision or a decay, we write the acceleration as a delta function at the time of the event in terms of its velocity $v_e^{\text{in}}$ before and $v_e^{\text{out}}$ after the event

$$\boldsymbol{a}_e(t) = (\boldsymbol{v}_e^{\text{out}} - \boldsymbol{v}_e^{\text{in}}) \, \delta(t - t_e) \,. \tag{4}$$

Equivalently, the velocity can be written as a stepwise function

$$\boldsymbol{v}_e(t) = \boldsymbol{v}_e^{\text{out}} \theta(t - t_e) + \boldsymbol{v}_e^{\text{in}} \theta(t_e - t) \,. \tag{5}$$

After some manipulation of Eq. (3) one gets the contribution to the full amplitude $\mathbf{A}$ contributed by event $e$ as

$$
\begin{aligned}
\mathbf{A}_e &= \frac{i}{4\pi^{3/2}\omega} \, e^{i\omega(t_e - \boldsymbol{n}\cdot\boldsymbol{r}_e)} \sum_j q_j \left( \frac{(\boldsymbol{n}\cdot\boldsymbol{v}_{je}^{\text{out}})\boldsymbol{n} - \boldsymbol{v}_{je}^{\text{out}}}{1 - \boldsymbol{n}\cdot\boldsymbol{v}_{je}^{\text{out}}} - \frac{(\boldsymbol{n}\cdot\boldsymbol{v}_{je}^{\text{in}})\boldsymbol{n} - \boldsymbol{v}_{je}^{\text{in}}}{1 - \boldsymbol{n}\cdot\boldsymbol{v}_{je}^{\text{in}}} \right) \\
&= \frac{i}{4\pi^{3/2}\omega} \, e^{i\omega(t_e - \boldsymbol{n}\cdot\boldsymbol{r}_e)} \sum_j q_j \left( \frac{\boldsymbol{n} - \boldsymbol{v}_{je}^{\text{out}}}{1 - \boldsymbol{n}\cdot\boldsymbol{v}_{je}^{\text{out}}} - \frac{\boldsymbol{n} - \boldsymbol{v}_{je}^{\text{in}}}{1 - \boldsymbol{n}\cdot\boldsymbol{v}_{je}^{\text{in}}} \right) \,.
\end{aligned}
\tag{6}
$$

This expression is valid for both decays and for elastic and inelastic collisions. The sum over $j$ is a sum over all hadrons participating in the event. If a hadron is created in the event its $v_e^{\text{in}}$ is set to zero, since that is equivalent to removing it from the incoming current. Similarly, a hadron which is annihilated in the event has its $v_e^{\text{out}}$ set to zero. Note also that any hadron whose velocity does not change at $t_e$ gives no net contribution to the amplitude.

We use UrQMD to generate an ensemble of nucleus-nucleus collisions with impact parameter $b \leq 3$ fm at RHIC energy of $\sqrt{s} = 100$ GeV/nucleon. For each nucleus-nucleus collision we compute the amplitude in the center-of-mometum frame by summing over its space-time evolution of microscopic scatterings and decays. We have computed 160 events for the results in the sections to come. The moduli squared of the individual nucleus-nucleus collisions are then averaged over this ensemble of events. Coherence is included within a given nucleus-nucleus collision, not between different collisions. These data enable one to calculate from Eq. (6) the soft radiation amplitude and therefore the photon intensity distribution that can reasonably be expected at RHIC.

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## III. RESULTS

### A. $\theta$ and $\omega$ dependence

Using the method described in the previous sections we compute bremsstrahlung arising from central Au-Au collisions at RHIC. The angular dependence of the intensity distribution at $\omega = 10$ MeV is plotted in Fig. 1. As expected at such high beam energies, relativistic effects tend to focus the radiation in the extreme forward and backward directions. Such focussed emission should facilitate practical measurements of photons. The $\omega$ dependence for several different angles are plotted in Fig. 2. In [11] the $\omega$ dependence was studied in both Landau-like and in Bjorken-like scenarios. The Bjorken-like scenario results in an essentially flat $\omega$-distribution, while a Landau-like scenario results in an oscillatory behavior. The oscillatory behavior in the latter scenario follows from interference between the stopping phase and the re-acceleration phase of the nuclear collision. Fig. 2 shows Bjorken-like behavior at small angles but Landau-like behavior at larger angles. For $\theta = 10^0$ and $20^0$ there are definitely hints of oscillation.
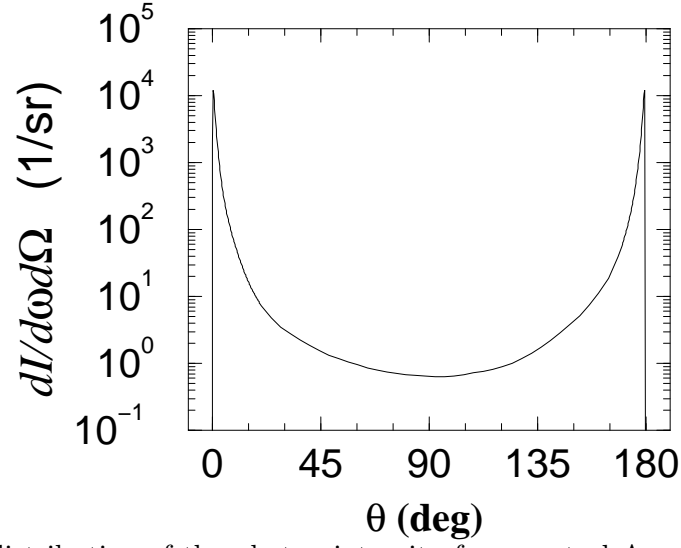
FIG. 1. Angular distribution of the photon intensity from central Au+Au collisions at RHIC at a photon energy of 10 MeV. This shows the expected extreme forward-backward focussing.
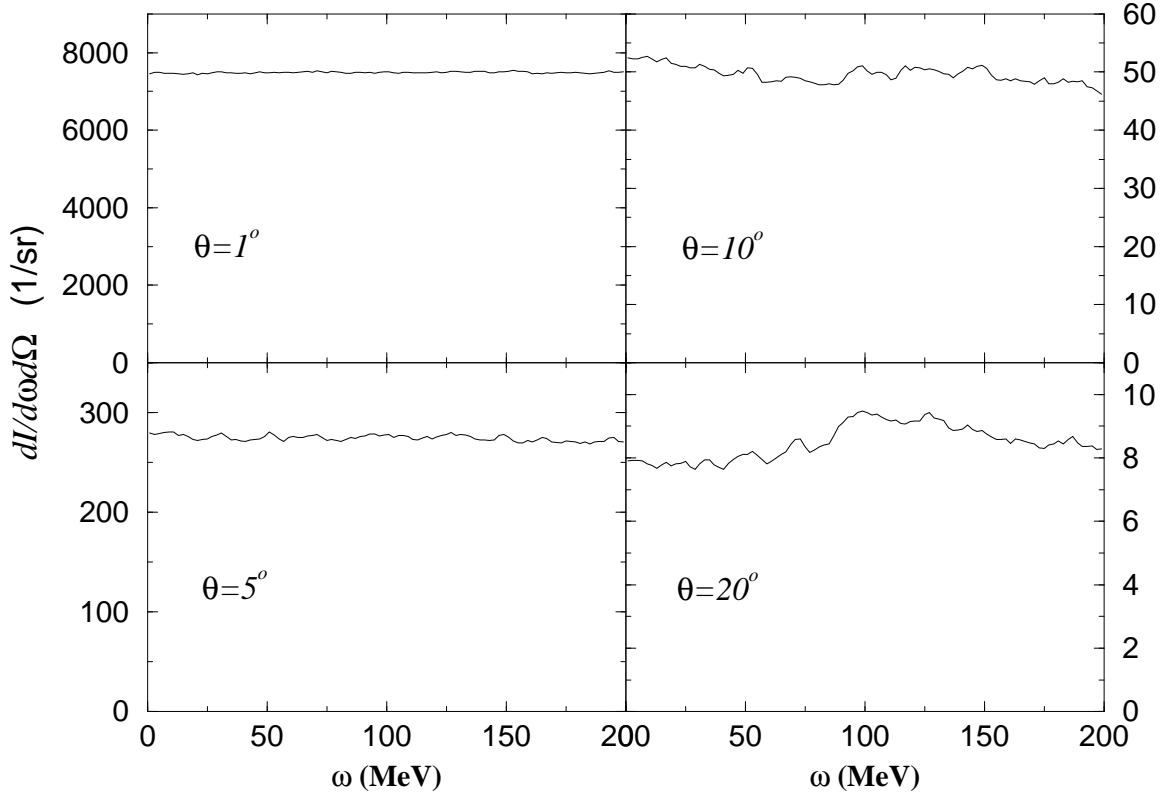


FIG. 2. Photon intensity versus the energy $\omega$ of the photons at fixed angles. The intensity is almost independent of photon energy. At larger angle $\theta$, though, there is a trace of oscillation.
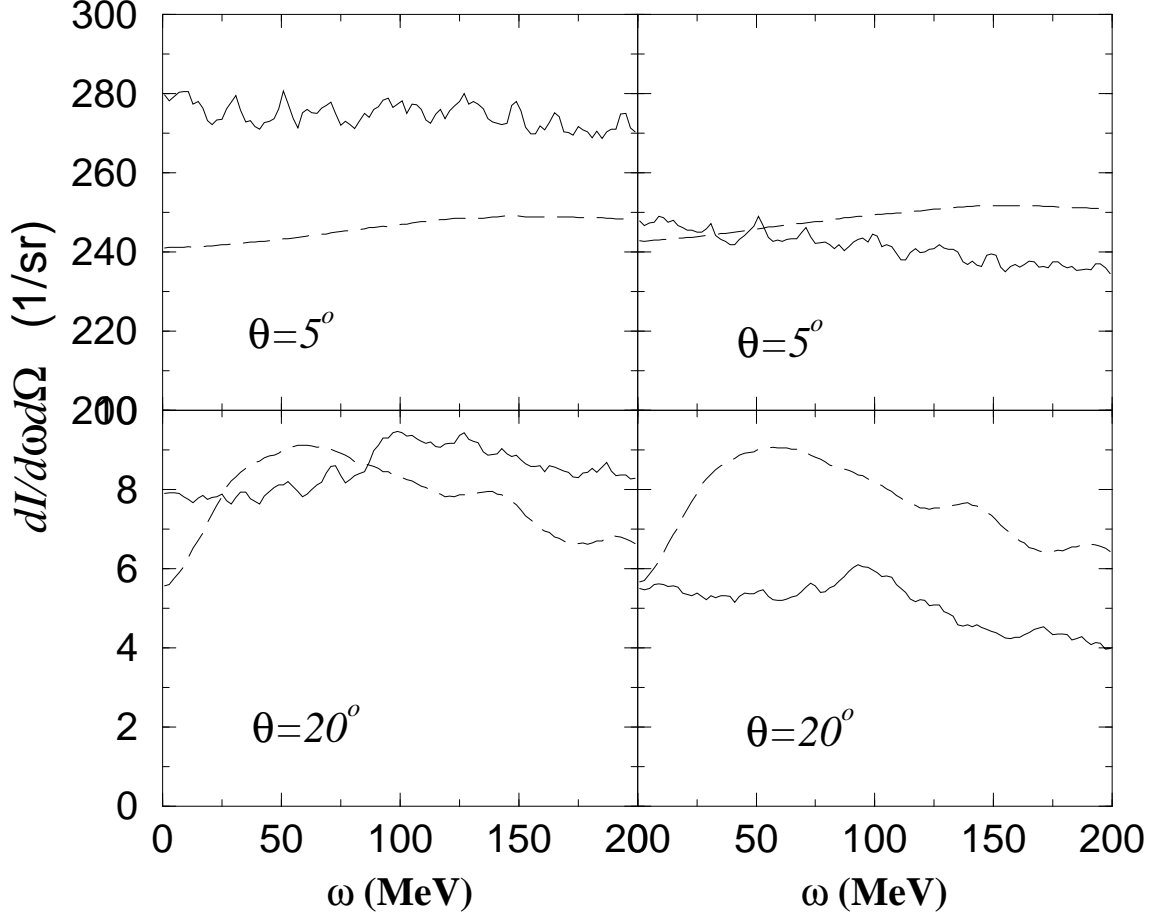
FIG. 3. $\omega$-dependence of the bremsstrahlung intensity at two angles are shown. The solid lines are the total bremsstrahlung and the dashed lines are photons from baryons only. The right panel shows what happens when the transverse velocities are arbitrarily set to zero. The baryonic contributions show little dependence whereas the dependence of the total contributions are quite visible. Oscillations in the baryonic component are clearly apparent.

The oscillation can be better seen in Fig. 3 where only $\theta = 5^o$ and $20^o$ are plotted. The left panels show the total bremsstrahlung (solid lines) and the bremsstrahlung originating from baryons only (dashed lines). In the right panels, we show the result of arbitrarily setting the transverse velocities to zero. It may be seen that bremsstrahlung from the baryons is essentially independent of their transverse motion. We will make use of this fact in a later section on stopping. On the contrary, the charged meson contribution and, therefore, the total receives a noticeable contribution, especially at larger angles. Without the mesons it is clearly apparent that there are oscillations from the baryons, as pointed out

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in [11]. The relative sensitivity of the mesons has two origins. First, most of the mesonic component comes from pions which are very light compared to protons and therefore suffer proportionately greater acceleration for the same momentum kick. Second, entropy drives a net increase of protons relative to neutrons, with a consequent excess of $\pi^-$ over $\pi^+$ to conserve charge.

## B. A=Z=197 on A=N=197

To elucidate the nature of stopping we have collided an artificial nucleus consisting entirely of 197 protons with another artificial one consisting entirely of 197 neutrons. Initially, the proton nucleus travels in the positive z-direction and the neutron nucleus in the negative z-direction. If the final protons all move with positive rapidity and the neutrons all with negative rapidity then we clearly have a Bjorken-like scenario. With such an asymmetric charge distribution the bremsstrahlung would have a highly asymmetric $\theta$-distribution too.

The angular distribution of 10 MeV photons radiated during central collisions of these artificial nuclei, as computed by UrQMD, is plotted in Fig. 4. Instead of two strong peaks in the extreme forward and backward directions, as in Fig. 1, there is now a strong peak only in the forward direction. In the backward direction there is only a weak peak. The reason is that the original protons suffer a massive deceleration to rapidities much less than their original beam rapidity $y_0$. This is most clearly seen in the net nucleon rapidity distributions, $p - \overline{p}$ and $n - \overline{n}$, shown in Fig. 5. Apart from the remnants of the initial (artificial) nuclei appearing as peaks at $y \sim y_0$ for protons and $y \sim -y_0$ for neutrons, the nucleons spread out rather evenly throughout the entire rapidity range. Since the elementary nucleon-nucleon collisions are known to be strongly forward-backward peaked, this flatness of the nucleon distributions in nucleus-nucleus collisions is as likely a result of repeated charge exchange as it is from single hard nucleon-nucleon scattering or sequential baryon-baryon collisions. From the point of view of electric charge there is no way or means to distinguish. Our conclusion from UrQMD is that electric charge suffers severe deceleration during the collision.
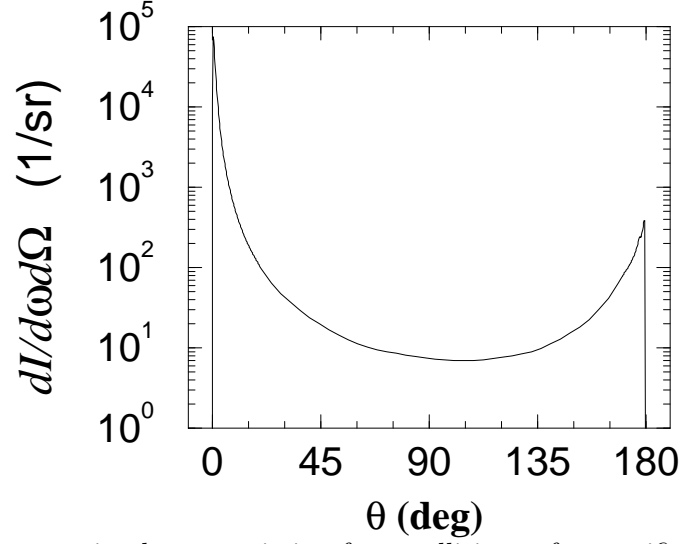
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FIG. 4. Highly asymmetric photon emission from collisions of an artificial 197 proton nucleus with an artificial 197 neutron nucleus.
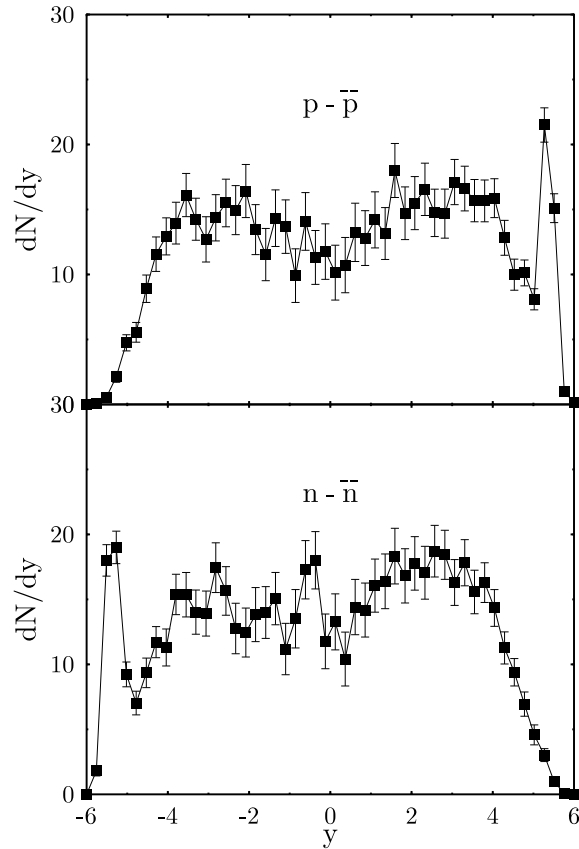


FIG. 5. The final net rapidity distribution of protons and neutrons from the collisions of artificial nuclei. Except for the peaks from the remnants of the initial nuclei, the nucleons spread out quite evenly over the entire rapidity range.

## C. Time dependence

Another question of interest is the time scale of nucleus-nucleus collisions. There is a paucity of direct means of getting any estimates and a paucity of meaningful observables. One might think of Hanbury-Brown and Twiss hadron interferometry, but this only relates to the time interval over which the final state hadrons emerge. In [11] the possibility of using bremsstrahlung for this purpose was discussed but that, in part, relied on the collisions being Landau-like. Theoretically we can examine the time-dependence of the emitted bremsstrahlung. In Fig. 6, the ratio of the instantaneous to final intensity has been plotted at two extreme values of $\theta$ as a function of time. It shows that at small angle, for example at $\theta = 1^o$, most of the soft bremsstrahlung were emitted within the first 5 fm/c. This results from the fact that in the longitudinal direction the bulk of the charges have finished their acceleration within a very brief duration upon first contact. In the transverse direction, on the other hand, it takes much longer, something like 100 fm/c, for 90% of the soft photons to be emitted. This is logical since we expect the transverse expansion to become significant only at the later stages. These facts here are essentially energy independent or weakly dependent in the range $\omega < 200$ MeV as seen in Fig. 2.
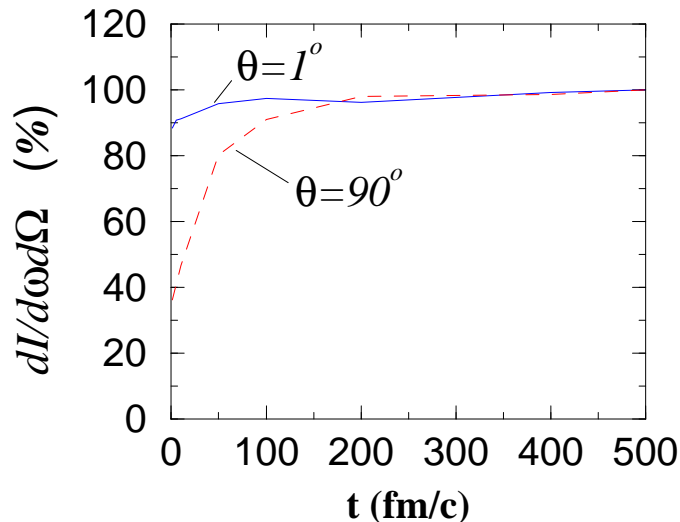
FIG. 6. The percentage of the bremsstrahlung final intensity at two values of $\theta$ as a function of time. Solid line is result at $\theta = 1^o$ and dashed line for $\theta = 90^o$. Here $\omega = 10$ MeV.

## D. Global Charge Stopping Time

In Sect. III A, we have seen that there are signs of oscillation in the intensity versus energy plots in Fig. 2 and Fig. 3 which revealed a partial Landau-like collision picture. In [11] the Landau-like scenario was found to have one advantage over the Bjorken-like scenario, which was that the amplitude and frequency of the oscillation was sensitive to the global charge stopping time $t_f$. We now attempt to extract $t_f$ from the UrQMD data using the simple Landau-like model in [11]. In that paper a flat rapidity distribution, or using the defined quantity from [18] $S = 1/2$, was assumed throughout. We will see below that UrQMD data yields somewhat more than one-half stopping, or $S > 1/2$. Therefore we have to adjust the intensity distribution from the simple model by a constant upward shift in order to fit the data. At smaller angles the oscillation is either unclear or its amplitude too small to be of practical use. So it is at $\theta = 20^o$ that the adjusted intensity from the model will be fitted to that of UrQMD. The result is shown in Fig. 7. It is seen that the range $1.2 < t_f < 1.4$ fm/c essentially covers the data. It can therefore be deduced that the bulk of the charges settled extremely quickly.
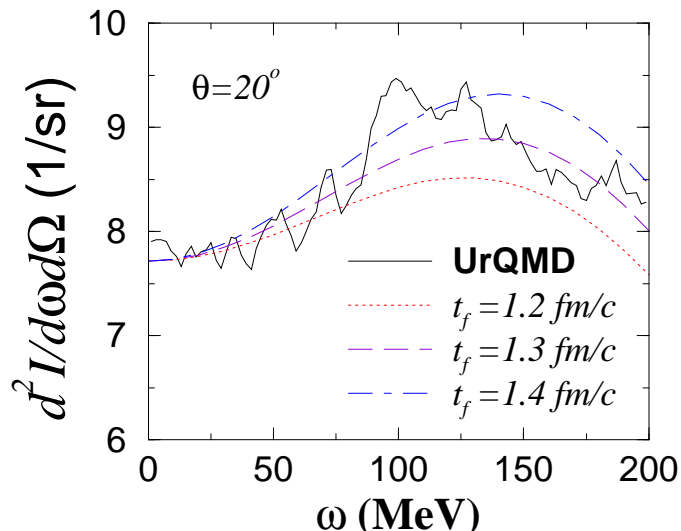


FIG. 7. The simple Landau-like model from [11] is fitted to the UrQMD data with three values of $t_f$.

To see if this is indeed the case, snapshots of the net-baryon and net-charge rapidity

distributions have been taken from UrQMD at several times. These are shown in Fig. 8. We see that these rapidity distributions settled already at times around 1 fm/c and do not change much after that[1]. The global stopping time determined above, although very short, is therefore sensible. The point is that the value of $t_f$ obtained from bremsstrahlung agrees with the time it takes for the baryon and electric charge rapidity distributions to acquire their final form. We have now shown that $t_f$ can be determined from bremsstrahlung, a piece of information that no hadronic measurement could provide.
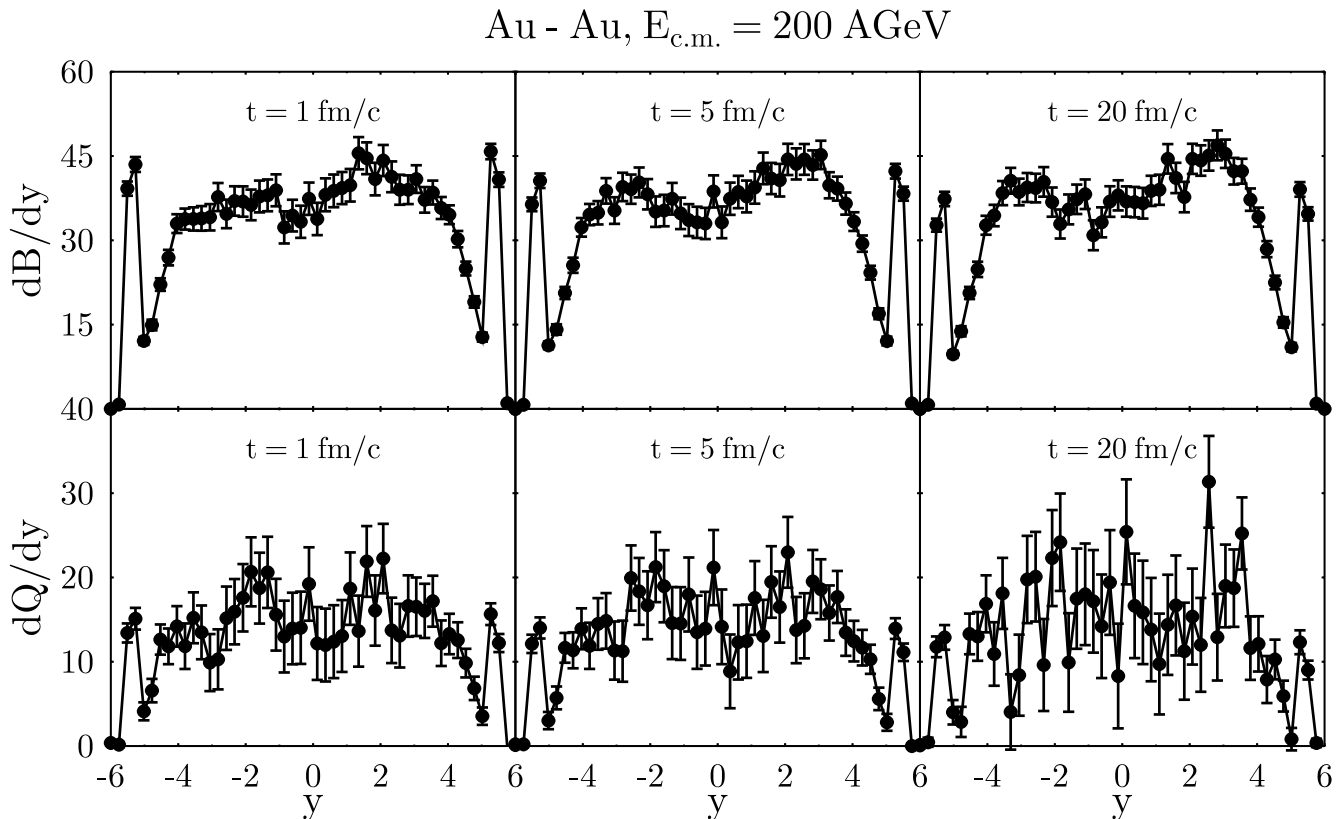


FIG. 8. Net-baryon and net-charge rapidity distribution from UrQMD at various times.

[1]The net-charge distribution shows larger fluctuations at $t = 20$ fm/c than at previous times. This may be due to resonance decays into mesons.

## IV. INFERENCE OF CHARGE VS. BARYON STOPPING

In an ideal world for measuring stopping, bremsstrahlung would be emitted only from charged baryons and anti-baryons and the contribution from the many produced mesons, which can be both positively as well as negatively charged, would cancel each other. For Au+Au collisions, the largest contributions do indeed come from baryons and anti-baryons but there remains a non-negligible contribution from mesons, as discussed in Sect. III A. The mesonic contribution has to be subtracted somehow if bremsstrahlung is to be used to probe baryon stopping, as opposed to charge stopping. To do this, we have to understand from the soft photons' point of view the essential dynamics of the sources. We find it illuminating and instructive to use another, much simpler, model that can reproduce, as far as bremsstrahlung is concerned, the generated data from UrQMD. In this section we will pretend that the bremsstrahlung from UrQMD is experimental data and show that the procedure described below is capable of extracting the degree of stopping in UrQMD.

Several facts from UrQMD prove to be useful in this regard. We notice that for soft photon emission there is a fundamental difference between the contributions from baryons and from mesons. For the emission from baryons and anti-baryons the precise values of their transverse velocities (relative to the beam direction) are unimportant. One could set these to zero without affecting bremsstrahlung very much at all; recall Fig. 3. On the contrary, transverse acceleration of the mesons is very important. We could therefore model the radiation from UrQMD by assuming that baryons and anti-baryons move only longitudinally with no transverse velocity. Information on the charged baryons and anti-baryons for soft photon emission is therefore all encoded in the final rapidity distributions. This is, of course, not sufficient as we have already mentioned that mesons contribute a finite amount to the radiation. In some sense the mesons are "noise" in this context and their transverse velocities are an important part of it. We have found that the "noise" can be reasonably subtracted by modeling the motion of the mesons in the manner to be described below.

In [18] it was suggested to measure low energy photons at the special angle $\theta_{1/2} \simeq 7.93^o$ at RHIC to determine the amount of stopping via the stopping variable $\mathcal{S}$.

$$\mathcal{S} = 1 - \frac{(1 - v_0^2 \cos^2 \theta_{1/2})}{2 v_0^2 \cos \theta_{1/2}} \int_{-\infty}^{+\infty} dy \, \frac{v(y) \rho(y)}{1 - v(y) \cos \theta_{1/2}} \tag{7}$$

Here $v(y) = \tanh(y)$ is the velocity along the beam axis and $\rho(y)$ is the baryon rapidity distribution normalized to 2 for collisions between symmetric nuclei. $\mathcal{S}$ ranges from 0 for no stopping to 1 for full stopping. The angle $\theta_{1/2}$ depends on the beam velocity $v_0$ and is chosen so that $\mathcal{S} = 1/2$ for any distribution that belongs intuitively and obviously to one-half stopping, such as $\rho(y) \sim \delta(y - y_0) + \delta(y + y_0) + 2\delta(y)$ or $\rho(y) = $ constant. Defining baryon stopping in terms of the above variable permits a direct connection between soft bremsstrahlung and baryon stopping, as discussed in [18].

In the soft photon limit, due to destructive interference, only the initial and final charges contribute to the bremsstrahlung. This permits a well-known simplification. That is, the detailed collision history can be ignored and only the initial and final particles need to be taken into account. In the presence of the above mentioned "noise" the relation between $\mathcal{S}$ given above and the soft photon intensity distribution needs to be modified. Without loss of generality, we choose the directional vector $\boldsymbol{n}$ to be $\boldsymbol{n} = (\sin\theta, 0, \cos\theta)$. We define

$$\boldsymbol{M} = \sum_{i \in \text{mesons}} \frac{q_i \, \boldsymbol{v}_i}{1 - \boldsymbol{n} \cdot \boldsymbol{v}_i} \tag{8}$$

for the sum over products of the charge and velocity factor over the final mesons. With the aforementioned mesonic "noise" included we have to modify the relation between $\mathcal{S}$ and the bremsstrahlung intensity distribution $dI/d\omega d\Omega$ at the angle $\theta_{1/2}$. The required modification amounts to simply replacing $\mathcal{S}$ as appeared in Eq. (13) of ref. [18], which is repeated here for convenience

$$\mathcal{S} = \frac{1 - v_0^2 \cos^2 \theta_{1/2}}{v_0^2 \, \sin 2\theta_{1/2}} \left( \frac{4\pi^2}{\alpha Z^2} \frac{d^2 I}{d\omega \, d\Omega} \Big|_{\substack{\omega \to 0 \\ \theta = \theta_{1/2}}} \right)^{1/2}, \tag{9}$$

by

$$\mathcal{S} \longrightarrow \left\{ \left( \mathcal{S} - \frac{(1 - v_0^2 \cos^2 \theta_{1/2})(M_z - M_x \cot \theta_{1/2})}{2 Z v_0^2 \cos \theta_{1/2}} \right)^2 + \frac{(1 - v_0^2 \cos^2 \theta_{1/2})^2}{4 Z^2 v_0^4 \cos^2 \theta_{1/2}} M_y^2 \right\}^{1/2}. \tag{10}$$

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Here $v_0 \simeq 0.999956$ at RHIC.

With the above replacement, the procedure to get $\mathcal{S}$ is less straightforward but neverthe-less possible. We find that the final meson contribution to soft photon emission is consistent with that of $N_+$ positively and $N_-$ negatively charged particles emerging from the collisions, apart from the expected forward-backward bias, in random directions. With this observation we arrive at the following procedure to obtain an estimate of $\mathcal{S}$.

(1) $N_+$ and $N_-$, which represent the approximate numbers of the corresponding charged mesons, are generated randomly within a given range. Models such as UrQMD can provide reasonable values; for example, on average $\langle N_+ \rangle \simeq \langle N_- \rangle \simeq 3000$ at RHIC.

(2) The difference $\Delta Q = N_+ - N_-$, where $\Delta Q \ll N_+, N_-$, need not be equal to zero but is bounded within the range $0 \leqslant \Delta Q < 2Z$. In fact, as seen in experiment at SPS and from the UrQMD model, the average value of $\Delta Q$ is in the lower end of the range below 40. The upper limit is only a statistically improbable theoretical possibility.

(3) Equal numbers of these charges will have positive and negative longitudinal velocities. Apart from that rapidities are randomly assigned according to the triangular distri-bution shown in Fig. 9 (this is the simplest yet reasonable distribution which agrees fairly well with realistic distributions). This gives a sensible forward-backward bias.
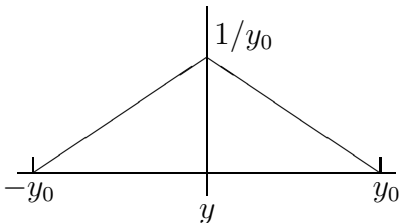


FIG. 9.

(4) The velocities of the majority of the charges are close to the speed of light. Slower longitudinally moving charges contribute negligibly to the bremsstrahlung because of the denominator in Eq. (8). For these charges, the transverse component of $M_x$ and

$M_y$ are not negligible. For the purpose of giving transverse velocities to our simulated mesonic charges, a maximum transverse velocity $v_{\text{max}}$ close to the initial speed $v_0$ is necessary to give reasonable results. Note that this limit on the velocities is for $v_x$ and $v_y$. It does not affect $v_z$, which is distributed according to the discussion in point (3) above.

(5) The direction of the transverse motion of each individual charged particle is picked randomly subject to the constraint that the total sum of the transverse velocities is approximately zero.

(6) With the above steps in setting the velocities, directions and numbers of the mesonic charges, the 3-vector $M$ and therefore the intensity distribution are calculated over a number of events. Using the relation between Eq. (10) and the intensity distribution given in [18], the mean $dI/d\omega d\Omega$ in the soft photon limit and the standard deviation $\sigma$ of the intensity generated from UrQMD can be recovered by suitable choice of the value of $\mathcal{S}$.

(7) When experimental information becomes available, one simply replaces the intensity from the UrQMD model at the angle $\theta_{1/2}$ with experimental measurements and repeat the above procedure to obtain $\mathcal{S}$.

For Au+Au at RHIC and at the angle $\theta_{1/2}$ from the beam direction the intensity distribution and its standard deviation of soft photons from the UrQMD model are, respectively

$$\left.\frac{dI}{d\omega d\Omega}\right|_{\substack{\omega \to 0 \\ \theta=\theta_{1/2}}} = 94.1 \quad \text{and} \quad \sigma = 71.8 \ . \tag{11}$$

(Note the large intrinsic fluctuation in the intensity; this is characteristic of soft photons as computed in UrQMD.) The other relevant quantities from the model are

$$\langle \Delta Q \rangle \simeq 19 \quad \text{and} \quad \mathcal{S} = 0.547 \ . \tag{12}$$

The latter means that there is just over half stopping [18] in the collisions predicted by UrQMD at RHIC. In Fig. 10, the net-proton and net-neutron rapidity distribution from

Au+Au at RHIC are plotted. They show exactly what the quantity $\mathcal{S}$ is designed to indicate, which is there is just over half stopping. The standard deviation of the intensity, $\sigma$, is quite large, indicating very large fluctuations in the charge rapidity distribution on an event by event basis. There are no such fluctuations in hydrodynamic treatments of heavy ion collisions, of course.

To the parameters already introduced, we now add $\mathcal{I}$

$$\mathcal{I}(\theta) = \int_{-\infty}^{+\infty} dy \frac{v(y)\,\rho(y)}{1 - v(y)\cos\theta} \ . \tag{13}$$

In terms of $\mathcal{I}$ the baryon stopping parameter $\mathcal{S}$ is

$$\mathcal{S} = 1 - \frac{(1 - v_0^2\cos^2\theta_{1/2})}{2v_0^2\cos\theta_{1/2}}\,\mathcal{I}(\theta_{1/2}) \ . \tag{14}$$

We can now vary them to see if it is possible to recover the results from UrQMD.
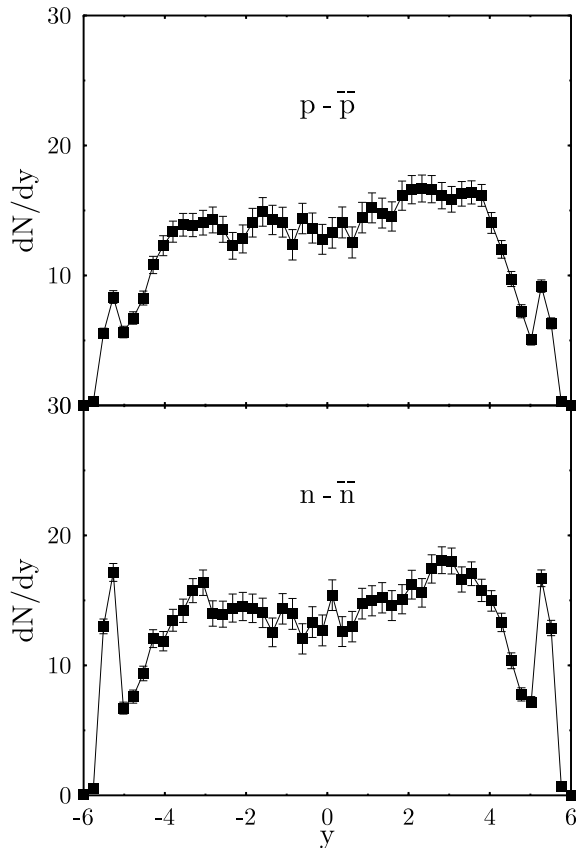


FIG. 10. Final net-proton and net-neutron rapidity distribution from Au+Au collisions at RHIC.

After some trial and error, certain facts can be established. Of all the parameters the intensity is most sensitive to $\mathcal{S}$, which is good. The next most important parameter is $\langle \Delta Q \rangle$. The rest are for fine tuning the results. These are tabulated in Table I with various input parameters. Initially setting both $\langle \Delta Q \rangle$ and $\mathcal{I}$ to zero gives a result that is too large (see the first row of the table). $v_{\max}$ is temporarily set to 0.997 here for exploratory purpose. Next, increasing $\langle \Delta Q \rangle$ to the largest but very unlikely value of $2Z$ is still not sufficient to reduce the intensity to the actual value. This is shown in the second row of the table. On the other hand, by setting $\langle \Delta Q \rangle = 0$ and increasing $\mathcal{I}$ to about 48.5 reasonable agreement can be obtained. From experiments at SPS, from models, and from very general considerations, we expect that some of the initial positive charge will be transferred to the mesons. Reasonable values for $\langle \Delta Q \rangle$ are within the range 10–30. This range can help us get a bound on the value of $\mathcal{S}$ because once a fit has been established for, say $\langle \Delta Q \rangle = 10$, increasing this to 30 will require a corresponding decrease in $\mathcal{I}$ to compensate.

The next question is what value should $v_{\max}$ be used. Fortunately, it turns out that $\mathcal{S}$ is not very sensitive to $v_{\max}$. To see this, we picked the angle $\theta = 5^o$. From UrQMD we know the following results.

$$\left. \frac{dI}{d\omega d\Omega} \right|_{\substack{\omega \to 0 \\ \theta = 5^o}} = 280 \quad \text{and} \quad \sigma = 171 \tag{15}$$

Also the value $\mathcal{I}$ at this angle is $\mathcal{I}(5^o) = 95.0$. In the next four rows in the table, it is shown that only with $v_{\max} = 0.999$ or higher is this value of $\mathcal{I}$ able to yield intensities that can agree with that from the model. Below this value, the resulting intensities are just short. $\mathcal{S}$ is only defined and meaningful at $\theta_{1/2}$ so the $\mathcal{S}$ column was left blank. Although in real experiments one does not know *a priori* the value of $\mathcal{I}$, the remaining entries in Table I should be enough to show that a precise knowledge of the value of $v_{\max}$ is unnecessary. The range of values of $v_{\max}$ from 0.997 to 0.999 and higher produce a range of $\mathcal{I}$ that covers the correct value. Here the covered range based on the $\langle \Delta Q \rangle =10$ and 30 limit for $v_{\max} =0.997$ and 0.999 are shown. They are sufficiently narrow to convey the fact that baryon stopping is somewhat over halfway, exactly what has been plotted in Fig. 10.

## TABLES

TABLE I. Various parameters used for fitting $\mathcal{S}$ to the intensity distribution from UrQMD.

| $\langle N_+ \rangle$ | $\langle \Delta Q \rangle$ | $v_{\max}$ | $\theta$ | $\mathcal{I}$ | $\mathcal{S}$ | $\frac{dI}{d\omega d\Omega}$ | $\sigma$ |
|---|---|---|---|---|---|---|---|
| 2929 | 0 | 0.997 | $\theta_{1/2}$ | 0.0 | 1.0 | 269.2 | 120.2 |
| 2929 | 158 | 0.997 | $\theta_{1/2}$ | 0.0 | 1.0 | 146.1 | 94.3 |
| 2987 | 0 | 0.997 | $\theta_{1/2}$ | 48.5 | 0.532 | 95.6 | 76.7 |
| 3032 | 10 | 0.997 | $5^o$ | 95.0 | NA | 273.9 | 165.5 |
| 2969 | 10 | 0.998 | $5^o$ | 95.0 | NA | 274.0 | 178.0 |
| 3009 | 10 | 0.999 | $5^o$ | 96.5 | NA | 280.7 | 184.5 |
| 2979 | 30 | 0.999 | $5^o$ | 91.0 | NA | 280.5 | 160.7 |
| 2978 | 10 | 0.999 | $\theta_{1/2}$ | 48.0 | 0.537 | 94.1 | 76.1 |
| 3002 | 30 | 0.999 | $\theta_{1/2}$ | 45.9 | 0.557 | 94.3 | 78.3 |
| 2978 | 10 | 0.997 | $\theta_{1/2}$ | 47.5 | 0.542 | 94.4 | 76.2 |
| 2994 | 30 | 0.997 | $\theta_{1/2}$ | 43.0 | 0.585 | 95.8 | 71.7 |

Our procedure yields the value of stopping around $\mathcal{S} = 0.537 - 0.585$, which is close enough to the value 0.547 from UrQMD in Eq. (12). So, with our extraction procedure to separate the charged baryon and charged meson contribution, soft bremsstrahlung from nuclear collisions can definitely be used to probe the degree of stopping. The procedure described here is relatively simple for subtracting the charged meson contributions. When experimental data are available eventually, one could of course use other more complex models such as UrQMD itself for the subtraction but the advantages of our procedure are that it is much simpler, more efficient therefore can be done by almost anyone using a very small program under fifty lines of codes.

## V. CONCLUSION

In this paper we have used the microscopic, dynamical model UrQMD to compute bremsstrahlung emitted by baryons and mesons as they are created, annihilated, and scattered during a central collision between Au nuclei at RHIC. Fine grained details on time and length scales shorter than about 1 fm do not matter from the point of view of emission of photons of energy less than about 200 MeV. Therefore, as discussed in earlier papers and demonstrated here with a specific microscopic model, bremsstrahlung measurements can probe the space-time evolution of high energy heavy ion collisions. UrQMD suggests that electric charge easily diffuses in rapidity space, giving rise to an approximately flat charged rapidity distribution. This was concretely demonstrated by colliding pure proton nuclei on pure neutron nuclei in the computer. We also examined the connection between charge stopping and baryon stopping, and showed that the noise caused by net charge transfer from protons to mesons can be simulated sufficiently accurately so that the two measures of stopping can still be related. In particular, the bremsstrahlung intensity at small angles approximately scales with the degree of stopping as quantified in the variable $\mathcal{S}$.

Bremsstrahlung is an interesting and useful way to study the dynamics of high Energy heavy ion collisions. Unfortunately, it is unlikely than the initial round of experiments at RHIC will be able to measure photons of sufficiently low energy as to be useful in this context. Fortunately, there are still two collision halls vacant at RHIC which could accommodate a dedicated soft photon detector. Future experiments should take advantage of these unique possibilities.

# REFERENCES

[1] See the proceedings of the Quark Matter conferences, the most recent in print being: Nucl. Phys. **A661**, (1999).

[2] L. D. Landau, Izv. Akad. Nauk SSSR (Physics Series) **17**, 51 (1953); S. Z. Belenkij and L. D. Landau, Uspekhi Fiz. Nauk, **56**, 309 (1955); Nuovo Cimento Suppl. **3**, 15 (1956).

[3] J.D. Bjorken, Phys. Rev. D **27**, 140 (1983).

[4] T. Wienold et al. Nucl. Phys. A **610**, 76 (1996).

[5] NA49 collaboration, Phys. Rev. Lett. **82**, 2471 (1999).

[6] J. Kapusta, Phys. Rev. C **15**, 1580 (1977).

[7] J. D. Bjorken and L. McLerran, Phys. Rev. D **31**, 63 (1985).

[8] A. Dumitru, L. McLerran, H. Stöcker and W. Greiner, Phys. Lett. B **318**, 583 (1993).

[9] S. Jeon, J. Kapusta, A. Chikanian and J. Sandweiss, Phys. Rev. C **58**, 1666 (1998).

[10] J.I. Kapusta and S.M.H. Wong, *Proceedings of the XXIX International Symposium on Multiparticle Dynamics*, August 1999, edited by I. Sarcevic and C.-I. Tan (World Scientific Singapore, 2000), p. 346 [hep-ph/9909573].

[11] J.I. Kapusta and S.M.H. Wong, Phys. Rev. C **59**, 3317 (1999).

[12] U. Eichmann, C. Ernst, L.M. Saratov, and W. Greiner, Phys. Rev. C **62**, 044902 (2000).

[13] D. Vasak, B. Müller and W. Greiner, J. Phys. G **11**, 1309 (1985).

[14] T. Stahl, M. Uhlig, B. Müller, W. Greiner, D. Vasak, Z. Phys. A **327**, 311 (1987).

[15] R. Heuer, B. Müller, H. Stöcker, and W. Greiner, Z. Phys. A **330**, 315 (1988).

[16] S. A. Bass, M. Belkacem, M. Bleicher, M. Brandstetter, L. Bravina, C. Ernst, L. Gerland, M. Hofmann, S. Hofmann, J. Konopka, G. Mao, L. Neise, S. Soff, C. Spieles,

H. Weber, L. A. Winckelmann, H. Stöcker, W. Greiner, Ch. Hartnack, J. Aichelin and N. Amelin, Progr. Part. Nucl. Phys. **41**, 255 (1998).

[17] M. Bleicher, E. Zabrodin, C. Spieles, S. A. Bass, C. Ernst, S. Soff, L. Bravina, M. Belkacem, H. Weber, H. Stöcker and W. Greiner, J. Phys. G: Nucl. Part. Phys. **25**, 1859 (1999).

[18] S.M.H. Wong, Phys. Lett. B **480**, 65 (2000).

[19] Particle Data Group, Eur. Phys. J. C **3**, 1 (1998).

[20] J. D. Jackson, *Classical Electrodynamics*, 2nd ed. (Wiley, N.Y. 1975).