An Investigation of the Link Between the Typical Geometry Errors and the Van Hiele Levels of Geometric Thought of Grade 9 Learners

By

Catherina Steyn

Submitted in fulfilment of the requirements for the

Degree of Masters in Education

in

The Faculty of Education

at the

Nelson Mandela Metropolitan University

Supervisor: Dr Tulsi Morar

2016

DEPARTMENT OF ACADEMIC ADMINISTRATION
EXAMINATION SECTION
SUMMERSTARND NORTH CAMPUS
PO Box 77000
Nelson Mandela Metropolitan University
Port Elizabeth
6013

Enquiries: Postgraduate Examination Officer

Nelson Mandela Metropolitan University

for tomorrow

DECLARATION BY CANDIDATE

NAME: **Catherina Steyn**

STUDENT NUMBER: s186042410

QUALIFICATION: Masters in Education

TITLE: An Investigation of the Link Between the Typical Geometry Errors and the Van Hiele Levels of Geometric Thought of Grade 9 Learners

DECLARATION:

In accordance with Rule G5.6.3, I hereby declare that the above-mentioned treatise/ dissertation/ thesis is my own work and that it has not previously been submitted for assessment to another University or for another qualification.

SIGNATURE: ____________________

DATE: **16/08/2016**

ACKNOWLEDGEMENTS

I would like to thank all those who made this study possible:

In the first place I would like to thank my Heavenly Father, for, through His strength and love, I was able to embark on this journey.

The National Research Foundation (NRF) is gratefully acknowledged for their financial support of this study (Reference number: SFH 150716127072). Opinions expressed in this research and conclusions arrived at, are those of the author and are not necessarily to be attributed to the NRF.

Ernst and Ethel Eriksen Trust for their financial support of this study.

Dr Tulsi Morar, my supervisor for encouragement, positive comments and guidance in the process of becoming a little more academic. Thank you for believing in me.

Dr Lyn Webb for her input as a critical reader and for her encouragement.

Prof Zalman Usiskin and Prof Michael de Villiers for their time and willingness to part with their expertise in the setting of the Van Hiele test.

The two schools for their willingness to co-operate and allowing me to work with their students.

The ladies who helped with the proofreading, editing and setting up of the final document: Gail Hawkins and Redène Steenberg.

My family who have always been there for me and who have supported me no matter what.

Finally and most importantly, I would like to thank my two daughters, Elizma and Johanette, who patiently encouraged and helped me throughout the entire process.

ABSTRACT

South African learners perform poorly in the geometry sections of both national and international assessments. Numerous assessment reports mention multiple errors that keep re-occurring and play a big role in the learners' poor performance. For this research, the link between the grade 9 learners Van Hiele levels of thought and the typical errors that they made were investigated.

In this mixed method study, 194 grade 9 learners in two schools in Port Elizabeth, South Africa were tested using a Van Hiele based test. A test was set up containing multiple-choice and open-ended questions and was used to determine firstly, the predominant level of geometric reasoning of the learners and secondly, to determine their typical errors. Semi-structured interviews were held with six learners to gain more insight into some of the typical errors uncovered in the tests.

The quantitative data revealed that the learners' predominant levels of geometric thought were low. Furthermore, the qualitative data revealed typical error patterns concerning angles and sides, parallel lines, hierarchy of quadrilaterals and incorrect reasons in the proofs. The quantitative and qualitative data was merged to determine if the errors could be linked to the Van Hiele levels.

From the findings, it was concluded that most of their typical errors could be linked to the Van Hiele levels of the learners.

Keywords:

Van Hiele, geometry, multiple-choice tests, open-ended tests, error analysis, mixed method study

ACRONYMS

ANA	-	Annual National Assessment
CAQDAS	-	Computer Aided Qualitative Data Analysis Software
CAPS	-	Curriculum and Assessment Policy Statements
FET	-	Further Education and Training
HSRC	-	Human Sciences Research Council
NCS	-	National Curriculum Statement
NDP	-	National Development Plan
NMMU	-	Nelson Mandela Metropolitan University
NWU	-	North West University
NSC	-	National Senior Certificate
OBE	-	Outcomes Based Education
SACMEQ	-	Southern and Eastern Africa Consortium for Monitoring Educational Quality
TIMSS	-	Trends in Mathematics and Science Study
UNICEF	-	United Nations Children’s Fund

TABLE OF CONTENTS

CHAPTER 3

LITERATURE REVIEW ON PERFORMANCE AND ERRORS

CHAPTER 4

METHODOLOGY

CHAPTER 5

PERFORMANCE ON VAN HIELE LEVELS: RESULTS AND DISCUSSION

CHAPTER 6

TYPICAL ERRORS IN GEOMETRY: RESULTS AND DISCUSSION

CHAPTER 7

LINK BETWEEN THE VAN HIELE LEVELS AND TYPICAL ERRORS

CHAPTER 8

CONCLUSION

LIST OF ANNEXURES

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1

OVERVIEW OF THE STUDY

1.1 INTRODUCTION AND BACKGROUND TO THE STUDY

"*I do not hesitate to confess that to a certain extent a similar pleasure may be found by absorbing ourselves in questions of pure geometry."* (Albert Einstein)

Contrary to what Albert Einstein thought of geometry, it is often not a source of pleasure for many learners and teachers and over the ages, many curriculum developers have re-evaluated the justification for geometry in the curriculum (Gonzalez and Herbst, 2006).

The inclusion of Euclidean geometry into the curriculum has been a point of debate in many countries (De Villiers, 1996; Mariotti, 2004). The debate was also held in South Africa when geometry was removed from the grade 10 – 12 mathematics curriculum in 2006 and reintroduced in 2012. Including geometry in the curriculum has many advantages for mathematics, as geometric representations can help learners in their understanding of fractions and multiplication in arithmetic, the graphs of functions (Jones, 2002), solving of triangles in trigonometry and graphical representations of data in statistics. According to Jones (2002), spatial reasoning is important in other curriculum areas as well including science, geography, art, design and technology. Improving the learners' knowledge of geometry could also help to "develop their skills of visualisation, critical thinking, intuition, perspective, problem-solving, conjecturing, deductive reasoning, logical argument and proof" (Jones, 2002:125).

In the diagnostic reports of the grade 12 National Senior Certificate (NSC) examinations in 2014 and 2015, the performance of learners in geometry was poor (Department of Basic Education, 2014b, 2015a). The poor geometry performance does not begin in the secondary school, as the reports of the Annual National Assessment (ANA) on the grade 1 to 7 and grade 9's indicate that geometry is seen as a weakness in all the grades that were tested (Department of Basic Education, 2014d). The poor performance is mirrored in international assessments such as the TIMSS research project of mathematics and science education worldwide where the mathematics results from South Africa are far below the rest of the world (Mullis, Martin, Foy & Arora, 2012).

Questions in the Geometry and Trigonometry sections scored the lowest marks in the 2014 and 2015 grade 12 NSC examinations (Department of Basic Education, 2014b, 2015b). One-third of the second paper in mathematics in the secondary school consists of Euclidean geometry (Department of Basic Education, 2011a, 2011b) and an improvement in geometry will, therefore, have a positive impact on the mathematics scores.

In addition, the National Development Plan (NDP) has set a goal of 350 000 learners passing mathematics in South Africa in the year 2024 (National Planning Commission, 2012). Currently less than half of that number passes grade 12 Mathematics. In the years from 2011 to 2013, the national pass rate for mathematics in the NSC grade 12 examination increased from 46.3% to 59,1%. However, it dropped to 53,5% in 2014 with the first examination on the new CAPS curriculum which included a section on geometry for the first time since its exclusion from grade 12 in 2008 (Department of Basic Education, 2014a). The effective teaching and learning of geometry should, therefore, be of great concern to mathematics educators and education policy makers in South Africa.

1.2 STATEMENT OF THE PROBLEM

This research stems from problems the researcher encountered in the 25 years of teaching geometry at secondary schools and in the 10 years of facilitating mathematics to part-time teaching students. The schools were fee-paying (quintile 5) schools in suburban areas but the part-time teaching students who were facilitated to improve their teaching qualifications were from non-fee-paying (quintile 1 to 3) schools in the township and rural areas in previously disadvantaged communities in Mpumalanga.

The researcher and most of her colleagues at the school expended a lot of effort teaching geometry to learners. However, it was found that many of the learners did not perform as expected. In the 1950's, this problem led two Dutch educators, Pierre van Hiele and Dina van Hiele-Geldof to investigate the way learners' reason and think about geometry (Van Hiele, 1986). This research led to the development of their theory of different levels of geometric reasoning (Van De Walle, 2004).

One factor that has a huge impact on the geometry performance is the number of errors that learners make (Department of Basic Education, 2015a). The researcher found that errors made by many learners in the junior phase seem to resurface again when they are confronted with the geometry riders in the senior grades. In addition, many of the part-time teaching students, some of whom had been teachers for many years, made the same errors as the learners.

Ball and Friel (1991) state that delving into learners' answers and especially incorrect answers could give important clues on how they reason. The possibility that the analysis of the errors could be linked to the level of reasoning in a learner caught the attention of the researcher.

Grade 9 learners were chosen for this study because grade 9 is the final year in the GET or junior secondary phase and it is important to identify and address the typical errors in the junior grades before the learners progress to the senior grades.

1.3 PURPOSE OF THE STUDY

The purpose of this mixed method study was to provide more insight into the geometric reasoning of grade 9 learners according to the Van Hiele levels of geometric thought in order to uncover the typical errors that influence their performance and finally to determine whether there is a link between the level of geometric reasoning and the errors.

The first phase of the study was quantitative and data concerning the learners' Van Hiele level of geometric performance was collected using a test combining multiple-choice and open-ended questions. Two Port Elizabeth schools, a township and a suburban school, were purposively selected and 194 Grade 9 learners were recruited.

In the second phase of the study, qualitative methods were used to explore the typical errors that those learners made. All their multiple-choice answers were entered into a spreadsheet and analysed for error patterns while the 60 open-ended answer sheets were coded for errors. The results on the level of reasoning determined in the first phase were used to purposively select 60 learners' open-ended answer sheets. The answer sheets were coded to uncover error patterns. Interviews with 6 learners were used to follow up on the typical errors uncovered in the tests.

In the third phase, the data from the quantitative and qualitative phases were merged to find links between the Van Hiele levels and the typical errors that the learners made.

The research design is summarised in the diagram below.

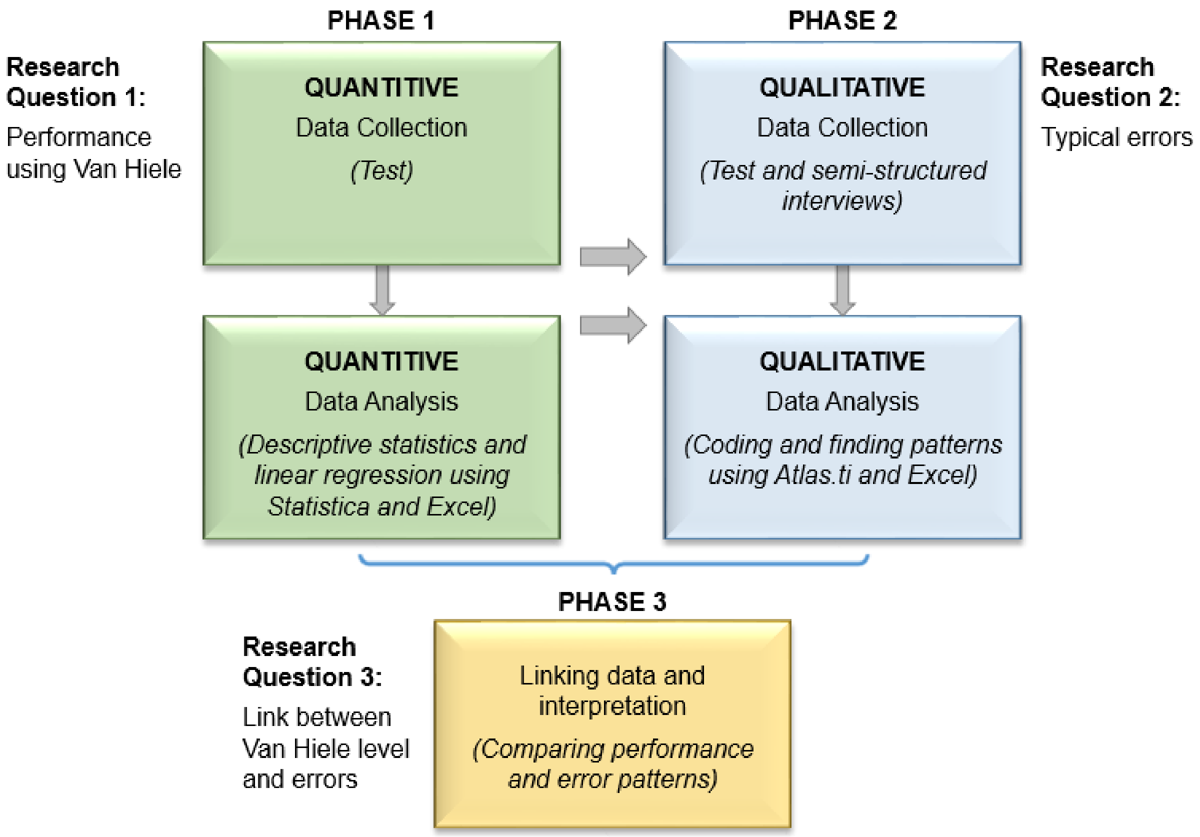


Figure 1.1: Illustration of the phases in the research and how the research questions link to the research design (Researcher's own design).

1.4 RESEARCH QUESTIONS

In order to guide the study on the possible link between typical errors of the grade 9 learners and their levels of geometric thought, the researcher set out to answer the following research questions:

- What is the level of geometric thought of grade 9 learners according to the Van Hiele theory?

- What are the typical errors that the grade 9 learners make in geometry?
- Is there a link between the Van Hiele level of geometric thought and the typical errors that the grade 9 learners make?

1.5 THEORETICAL FRAMEWORK

The Van Hiele theory of the level of geometric thought was used to guide this study. The research of the Dutch couple, Pierre van Hiele and Dina van Hiele-Geldof has provided us with a framework for geometric thinking (Van Hiele, 1986). The following table (Table 1.1) provides a short outline of the Van Hiele levels:

Table 1.1: Van Hiele levels of geometric thought and the thinking process involved

	Name of level	Thinking process
1	**Visualisation (Recognition)**	Shapes and what they look like
2	**Analysis (Descriptive)**	Properties of shapes
3	**Informal Deduction (Ordering or relational)**	Classes of shapes rather than individual shapes
4	**Deduction**	Relationships among properties of geometric objects.
5	**Rigour**	Deductive axiomatic systems for geometry

According to Van Hiele (1986), the five levels of geometric thought are sequential and students must move through all the lower levels before reaching the higher levels. The advancement through the levels is not age-dependant as in the theory of Piaget (Van Hiele, 1986) although age does determine the number of geometrical experiences that a learner has come across (Fuys, Geddes, & Tischler, 1988). Geometric experience is the greatest single factor influencing advancement through the levels. Instruction or language at a higher level than that of the learners will lead to a lack of communication which, in turn, could negatively impact on the advancement through the levels (Van

De Walle, 2004). To ensure instruction at the correct level, the assessment of the levels of geometric thought is a crucial task that is complicated by the fact that learners may not be at the same level for different concepts (Battista, 2007; Pusey, 2003).

Many different forms of assessment have been used in various studies to assess the Van Hiele levels of geometric thought. In one study in the United States, Usiskin first conducted an investigation in the early 1980's to further the description of the levels. These descriptions were then used to set up a multiple-choice test to determine the learners' understanding in terms of the Van Hiele levels of thought (Usiskin, 1982). Other tests and assessments have been set up and used by various other studies (Burger and Shaughnessy, 1986; Fuys et al., 1988; Mayberry, 1983; Senk, 1989; Wu and Ma, 2006). Some of these tests and other newer tests have been used to assess learners, teaching students and in-service teachers in South Africa (Atebe and Schäfer, 2010; De Villiers and Njisane, 1987; Siyepu, 2005; Smith and De Villiers, 1989; Van der Sandt and Nieuwoudt, 2003; Van Putten, 2008).

1.6 SIGNIFICANCE OF THE STUDY

The Van Hiele theory provides a framework for evaluating the level of geometric thought of the learners (Van De Walle, 2004). Furthermore, Mayberry (1983) suggested that teaching could be much more efficient if appropriate experiences are designed to help learners progress through the Van Hiele levels thus improving their performance.

If the Van Hiele levels and errors are linked, the errors may give educators a clue as to the level of geometric thought of the learners. Analysing the errors may be a starting point to detecting the Van Hiele levels and where to focus the efforts of teaching in order to ultimately improve our geometry results.

Through this study, the researcher hopes to make a small contribution to the knowledge of the levels of geometric reasoning and the potential for the use of errors.

1.7 DELIMITATION OF THE STUDY

The following points were not focussed on in this study.

The **factors** that have an impact on the teaching and learning were not studied in the schools. For example, teaching methods, language levels and socio-economic status of parents or the influence of technology on the teaching of geometry was not investigated.

The tests were done after the geometry section of the grade 9 curriculum was completed. The extent of the **content that was covered** by the teachers was not investigated. Therefore the lack of knowledge or experience in a certain section of work due to teaching was not taken into account when the Van Hiele levels were assessed. The content of the **curriculum** itself was also not investigated.

Whilst certain misconceptions were detected, only the typical errors were investigated. Other possible **reasons** for the errors were not investigated.

The influence of the **exclusion of geometry** from 2008 to 2013 was not investigated. Furthermore, for this study, only Euclidean and not analytical geometry was investigated.

1.8 EXPLANATION OF TERMS

The terminology that was used was related to the South African grade 9 curriculum e.g. alternate interior angles, represent the angles obtained when two lines are intersected by a transversal line and congruency refers to two triangles being equal in terms of size and shape.

The two sections of geometry covered by the NCS are Euclidean and analytical geometry. Below is an explanation of what is understood by the two terms:

- **Euclidean geometry** is a mathematical system attributed to the mathematician Euclid, which he described in his textbook on geometry: The Elements.
- **Analytic geometry**, also known as coordinate geometry or Cartesian geometry, is the study of geometry using a coordinate system.

In this study, only Euclidean geometry was studied and the word geometry is used to refer to Euclidean geometry.

All South African public ordinary schools are categorised into five groups, called **quintiles** that contain 20% of all learners. The quintiles refer to the financial resources of the community around the school, as well as certain infrastructural factors. Quintile one is the 'poorest' quintile, while quintile five is the 'least poor'. Schools in quintiles 1, 2 and 3 have been declared non-fee paying schools, while schools in quintiles 4 and 5 are fee-paying schools. A schools' quintile ranking is important as it determines the amount of government funding that a school receives each year and whether or not the school can charge fees (Grant, 2013).

CAPS is the abbreviation for Curriculum and Assessment Policy Statement which refers to the new curriculum that was implemented in 2012 and currently used in South Africa.

Typical errors represent the errors that were commonly found in the answer sheets and during the interviews of the group of grade 9 learners in this investigation. In other words, typical errors are the errors that occurred frequently in the group and were deemed to be representative of the group.

1.9 STRUCTURE OF THE DISSERTATION

Chapter 2 discusses the theoretical background of the dissertation. In order to place this study within the existing literature, an in-depth literature study was done and a summary of the literature is provided in chapter 3. This is followed in chapter 4 by a discussion of the research design which guided the methodology in this study. The results and findings that are linked to the research questions are discussed in chapter 5, 6 and 7. Chapter 8 presents the conclusions and recommendations.

CHAPTER 2

THEORETICAL FRAMEWORK

2.1 INTRODUCTION

> *'Theory informs our understanding of issues, which in turn assists us in making research decisions and sense of the world around us'* (May, 2011:27).

In this study, the Van Hiele theory was chosen as a theoretical framework to guide the research on investigating the performance and the typical errors of learners in grade 9 geometry. A brief description of the relevance of this theory to research on geometry education and the current study is explained, followed by a description of the theory.

Pegg (1992:19) pointed out that a theory is to the advantage of the research if it satisfies the criteria of "*explanation", "unification"* and "*prediction*":

2.1.1 Explanation

> *"Explanation offers reasons for what is observed;"*(Pegg, 1992:19)

The Van Hiele theory has been used to explain the poor performance of learners in geometry worldwide (Atebe & Schäfer, 2010b; Fuys et al., 1988; Usiskin, 1982). According to the theory when learners function at a different level than the level of teaching, learning does not take place effectively. Therefore when the questions are at a higher level than the learners' levels, they will not be able to perform as expected. The Van Hiele theory also explains the progression from recognising shapes and using intuitive reasoning to being able to construct geometrical proofs and produce scientific reasoning (Atebe & Schäfer, 2010a; Battista, 2007).

The Van Hiele theory helped to explain the original problem that led to this study – the poor performance of learners in geometry.

2.1.2 Unification

"Unification, that is, synthesise or link together previous work;"(Pegg, 1992:19)

Van Hiele acknowledges the influence of work done by theorists such as Selz, Piaget and Van Parreren as well as the Gestalt theory in the setting up of his theory (Van Hiele, 1986). Since the theory became more widely known it has been used by many researchers and has influenced geometry teaching practice in countries such as Russia, Japan and the United States (Pusey, 2003). Through its widespread use, numerous studies are linked with each other (Pusey, 2003) and this study gained from the insights developed by others.

2.1.3 Prediction

"Prediction, that is, offers insights into new areas that have not been explored."(Pegg, 1992:19)

Various levels of thought help to predict why the teaching and learning of geometry is not effective and therefore why performance is not as expected. With the Van Hiele theory in mind, researchers have also investigated the influence of language (Feza & Webb, 2005; Howie & Plomp, 2003; Roux, 2005), curriculum (Pusey, 2003) and other challenges linked to the teaching and learning of geometry.

In this chapter seminal studies are used to describe the Van Hiele theory with respect to the levels, properties, learning phases and some of the critique aimed at the theory. The setting of a valid test to assess the Van Hiele levels was central to the outcome of many studies and the literature concerning the assessment of the Van Hiele levels is reviewed. Finally, spatial ability and a few other theories that also impact on the teaching and learning of geometry are reviewed. The theory's implication for teaching and learning in South Africa is discussed in the next chapter using some of the more recent studies.

2.2 THE VAN HIELE THEORY

The Van Hiele theory originated in the late 1950's in the doctoral research of two Dutch educators, Pierre van Hiele and Dina van Hiele-Geldof and provides us with a framework for geometric thinking (Van Hiele, 1986). Pierre van Hiele found that even after a lot of effort and hard work in teaching geometry to students, many still did not succeed and this led him to question the geometrical thought process (Van Hiele, 1986). Pierre van Hiele focussed on building a theory to explain the levels of geometric thought, whereas his wife, Dina van Hiele, focussed on a teaching method that would result in their learners progressing to a higher level (Pusey, 2003). Dina van Hiele died shortly after they completed their doctoral dissertations.

The theory was used outside the Netherlands in the early 1960’s by the Russians. The Russians also experienced challenges in geometry education and therefore a theory on the levels of geometric thought attracted their attention. The theory was introduced in the United States by Wirszup in the 1970’s and then it started to gain popularity (Fuys et al., 1988). Since then many studies on geometric thought have used the Van Hiele theory and support the Van Hiele levels as suitable to characterise geometric reasoning (Battista, 2007; Pusey, 2003).

A short explanation of the Van Hiele levels, the properties of the levels and the learning phases is given below.

2.2.1 The Van Hiele Levels of Geometric Thought

Van Hiele (1986) distinguished between five sequential levels of geometric thought and acknowledged two major factors that determine a learner's levels namely: ability and previous geometry experiences. These experiences are not just gained in a classroom but include all the experiences that a child has been exposed to since birth (Van Hiele, 1986).

Studies by Burger & Shaughnessy (1986); Fuys et al. (1988); Mayberry (1983) and Usiskin (1982) supported the presence of levels of reasoning. In 1982, after the Chicago study group research project in the USA, Usiskin concluded that the Van Hiele levels are a good predictor of the geometric performance of learners (Usiskin, 1982). However, Frykholm (1994) also indicated the converse, that general mathematical performance is a predictor of Van Hiele levels.

The levels were originally numbered by the Van Hieles from level 0 to level 4. This numbering system was still used by some of the seminal authors including Fuys et al. (1988), Hoffer (1981), Burger & Shaughnessy (1986) and Mayberry (1983). However, Usiskin (1982) and Pegg (1992) started numbering the levels from 1 to 5 instead, in order to allow for the pre-recognition level to be called level 0. In this study, the numbering system of level 1 to 5 is used.

Although researchers agree that there should be levels, not all agree on how many levels there should be. Van Hiele himself later reduced the number of levels to three (Teppo, 1991) whereas others for example Pegg (1997) and Battista (2007) suggested expanding the levels by including sub-levels. In the table below the levels from level 1 to 5 as used in this study is explained.

Table 2.1: The Van Hiele Levels of geometric thought

Level	Name of level	Thinking process	Explanation of process
1	**Visualisation (Recognition)**	Shapes and what they look like	Sort and classify shapes e.g. squares based on their appearance. Properties are not recognised.
2	**Analysis (Descriptive)**	Properties of shapes	Recognise the properties of the shapes but the properties are independent of one another.
3	**Informal Deduction (Ordering or relational)**	Classes of shapes rather than individual shapes	Develop relationships between properties. Can see the hierarchy of properties and shapes. Shapes go together because of the properties.
4	**Deduction**	Relationships among properties of geometric objects.	Can work with abstract statements about geometric properties and make conclusions based more on logic than intuition. Proof can be done.
5	**Rigour**	Deductive axiomatic systems for geometry	An interest in the axiomatic systems themselves and not just the deductions within a system. Non-Euclidean geometry can also be applied.

Source:(Pegg, 1992; Van De Walle, 2004)

The properties of the levels were added to the theory to further describe the levels of geometric thought and are discussed in the section below.

2.2.2 Properties of the levels

The Van Hieles added the properties: sequential, advancement, intrinsic and extrinsic and linguistic to clarify certain presumptions that they had about the levels of thought. These properties and a short discussion using comments of the seminal authors are given below.

2.2.2.1 Sequential (Fixed sequency)

According to Van Hiele, the levels are sequential and learners must pass through and acquire the lower levels before proceeding to next level (Van De Walle, 2004). This sequential nature of the theory has been confirmed in studies done by Fuys et al. (1988) and Mayberry (1983). However, Clements & Battista (1992) and Mayberry (1983) questioned the discrete nature of the levels.

Due to the sequential nature, learners cannot skip a level. Although, if the learners receive instruction it may allow them to progress more quickly. Some learners may mimic being at a higher level after memorising the rules and definitions used in a higher level. Crowley (1987) attributes this to the reduction of the subject matter to a lower level so that it can be learnt without any understanding taking place.

2.2.2.2 Advancement (Attainment)

In order to advance from one level to the next requires "direct instruction, exploration and reflection" by the learner (Pegg, 1992:21). This is one of the differences between the theories of Van Hiele and Piaget. In Piaget's theory, development is age dependent whereas in Van Hiele progress to the next level depends more on the content and method of instruction than on the age of the learner (Mason, 2003). However, one cannot completely disregard age because age usually determines the number of geometrical experiences to which the learner has been introduced.

Learners must engage in more sophisticated thinking to move to the next level (Pegg, 1992). The teachers cannot force the learners to a higher level of thought but should provide problems and guide learners in the process (Pegg, 1992). Fuys, Geddes, & Tischler (1988) found that progression through the levels takes time. This agreed with Dina Van Hiele findings that it took 20 to 50 lessons before a class could progress from level 2 to 3 (Pegg, 1992). In planning their instruction teachers should allow enough time for this process to take place.

2.2.2.3 Intrinsic and extrinsic (Adjacent)

Concepts that are implicitly understood at one level become explicitly understood when learners reach the next level e.g. at level 1 a shape is perceived but the properties are not distinguished while at level 2 the properties are now discovered (Crowley, 1987). The properties of an object may be discovered whilst the learners are on a lower level but they do not realise that these are the properties unless they have reached the higher level of thought (Fuys et al., 1988).

2.2.2.4 Linguistics (Distinction)

Each level has its own language or linguistic symbols and way of thinking (Van Hiele, 1986). If the language that the teacher uses is at a higher level than the level of the learner, the learner will not be able to follow the thought processes and there will be a lack of communication (Mayberry, 1983; Van De Walle, 2004). Learners must be confident with the terminology needed to function on a level before they can move to the next level. The learning of new terminology is especially important for progression from level 2 to 3 (De Villiers, 2012). Van Hiele (1986) stated that when a learner reaches level 4 where they are required to build proofs, there must be no uncertainty of basic terminology and concepts such as lines, points, surfaces, etc.

Pegg (1992) considers the Van Hiele levels to be a broad structure upon which teaching and learning can be based. In the next section the 5 learning phases identified by Dina van Hiele to help in the progression of learners to a higher level, will be described.

2.2.3 Learning phases

Pierre van Hiele asserted that the true worth of knowing the levels is being able to use them, to know where to start teaching and how to help learners progress (Van Hiele, 1986). Pierre van Hiele was more concerned about the different levels of reasoning that the learners pass through whereas Dina van Hiele was concerned about how to help the learners progress through the levels. According to Dina van Hiele, learners must progress through 5 learning phases in order to progress from one level of reasoning to the next. Advancement through the learning phases and to the following level of reasoning is supported and accelerated due to the direct instruction from the teachers (Van Hiele, 1985). Often the learners must pass through the learning phases more than once before they can move on to the next level (Mason, 2002). The learning phases are summarised in table 2.2 below:

Table 2.2: Phases of learning for a learner to progress from one level to the next, together with the actions of the teacher and learners in each phase.

Learning Phases of the Van Hiele Levels of Reasoning			
Phase	**Event**	**Student Actions**	**Teacher Actions**
1	**Information (Inquiry)**	Receives examples and non-examples of the material. Observations are made, questions are raised and level specific vocabulary is learnt.	Provides and discusses the material.
2	**Guided (Directed) Orientation**	Starts examining the material by drawing, folding and measuring. Gradually recognises the structures.	Guides the learning by carefully sequencing the material used.
3	**Explicitation**	Discovers the properties of the material. Tries to express them whilst learning and using the terminology involved.	Role is less visible in this phase. Introduces the appropriate terminology.
4	**Free Orientation**	Starts doing more difficult tasks, learns more about the properties and starts forming connections between e.g. different shapes. Works more independently in completing the tasks.	Gives tasks that can be completed in more than one way. Encourages reflection on the tasks and gives prompts to guide learners in forming connections if necessary.
5	**Integration**	Summarises all that is learnt and reflects back on all the properties of the new material. Starts to interpret how this new knowledge can be applied.	Assists learners in this phase by showing them how their new knowledge fits in with the rest.

(Adapted from Crowley, 1987; Fuys et al., 1988; Martin, 2007; Van Hiele, 1985, 1986).

2.2.4 Critique of the model

Although many agree with Usiskin (1982:6) in saying that the Van Hiele theory is *"elegant, comprehensive and has a wide applicability"* critique has also been levelled at the theory. The major points of critique and some of the solutions offered by the seminal authors are discussed below:

First critique is that the theory does not describe the process within the levels adequately (Pegg, 1997; Clements, 2003). One of the aims of the study done by Fuys et al. (1988) was to set up more detailed descriptions of each level.

A second critique is levelled against the discrete nature of the levels. Learners do not progress from one level to the next in jumps but rather in small steps resulting in a more continuous progress (Battista, 2007). The discontinuity between the levels is emphasised by the number of learners who seem to fit in between levels (Pusey, 2003). Learners seem to be at different levels for different concepts or tend to oscillate between the levels thus raising the question of whether learners can be placed on a certain level (Clements & Battista, 1992; Mayberry, 1983). Burger & Shaughnessy (1986), Crowley (1987) and Battista (2007) have therefore argued that a specific level cannot be ascribed to a learner.

To deal with this concern Pegg (1997) expanded levels 2 and 3 by merging the Van Hiele Theory with the SOLO taxonomy of Biggs and Collins (SOLO Taxonomy is explained in the next section). Battista's (2007) answer to the second critique was to elaborate on the levels by adding sub-levels. Gutiérrez, Jaime, & Fortuny (1991) addressed the critique by assigning degrees of acquisition in a specific level. Gutiérrez et al. (1991) reasoned that initially, students were not conscious of the thinking methods specific to a new level and hence had no degree of acquisition on this level of reasoning. The learners progressed through the degrees of acquisition as they became more experienced, thus passing through low, intermediate, high and complete degrees of acquisition in each level. Students attained complete acquisition of the level when they had complete mastery of this way of thinking and used it without difficulty. It was found that learners with a lower acquisition at a certain level would fall back to a lower level when they encountered more difficult questions (Gutiérrez et al., 1991). However, according to Pegg (1992) once a student has reached a certain level of understanding of one concept it will take less time for the student to reach the same level of acquisition with other concepts.

A third critique is that the theory does not make provision for the learners who do not meet the criteria for the first level. Usiskin (1982), Senk (1989) and Pegg and Davey (1989) identified students that did not meet the criteria of the first level in their studies. Through interviews with pre-service teachers, Mayberry (1983) also questioned whether there should be a level before visualisation. In addition, Clements (2003) found that the theory was not accurate in describing the thinking of very young learners. The original numbering of the level from 0 to 4 was changed to level 1 to 5 and a pre-recognition level or level 0 was added before level 1 (Clements, 2003).

A fourth critique is aimed at the methods of assessment of the levels. Battista (2007) recognised that although we have gained a lot of knowledge about learners thinking in geometry, it is still very difficult to assess the cognitive processes. The critique against each of the methods will be discussed further in the section on the methods of assessment.

A fifth critique is against level 5 or the rigour level. Usiskin (1982) regarded this level as non-existent or that the level could not be tested. This fact was also later acknowledged by Van Hiele himself in his later work (Van Hiele, 1986). Therefore many of the researchers do not include this level in their assessments (Van Putten, 2008).

Despite the fact that the Van Hiele theory has been critiqued it has been used extensively in many studies worldwide providing much knowledge about geometric thinking (Battista, 2007). The assessment of the Van Hiele levels forms a major part of many studies and is reviewed in the next section.

2.3 SETTING UP A VALID TEST TO ASSESS THE VAN HIELE LEVELS

The assessment of the levels of geometric thought is a difficult task that is further complicated by the fact that learners may not be at the same level for different concepts (Battista, 2007; Pusey, 2003).

A variety of instruments have been designed to assess the learners' levels, for example multiple-choice tests (Usiskin, 1982), open-ended tasks (Smith & De Villiers, 1989), interviews (Atebe & Schäfer, 2008; Burger and Shaunessy, 1986; Fuys, Geddes, & Tischler, 1988; Mayberry, 1983) and proof tests (Senk, 1989). Most of the assessments only focus on the performance of the learners at a certain stage but in some studies, it also focuses on the progress that a learner makes in response to the teaching (Fuys et al., 1988).

2.3.1 Multiple-choice tests

Multiple-choice tests based on geometry have been developed in a variety of studies (Hendricks, 2012; Mogari, 2003; Siyepu, 2005; Van Putten, 2008; Watson, 2012). One test that has been used frequently is a test developed by Usiskin (1982) and the Cognitive Development and Achievement in Secondary Schools Group (CDASSG). The necessity to be able to determine the level of geometric thought of learners led them to develop a 25 question multiple-choice test. Crowley (1990), Wilson (1990) and Smith & De Villiers (1989) raised doubts as to whether reasoning could be tested with the items used in the test. Smith and De Villiers based their concern on their findings after comparing the multiple-choice test with open-ended question tests. However, this test has been used in a great many studies and has also been used as the foundation for setting up similar tests.

Rodriquez & Haladyna (2013) found that reasoning could indeed be assessed by carefully wording the questions in the multiple-choice tests. The setting of high-quality multiple-choice tests can, therefore, be very difficult and time-consuming and it is always a good practice to pilot the test before using it to determine the validity of the test (Brown, 2002). The results of multiple-choice tests could be skewed due to learners guessing the answers, which could be countered by adding more test items (De Villiers & Njisane, 1987) and carefully selecting the answers so that the correct choice is not so obvious (Brown, 2002).

In spite of the critique, the multiple-choice test method of the Van Hiele levels has been used extensively because it is easy to administer, far less time consuming than any of the other methods and is more practical to use when large numbers of learners must be assessed (Battista, 2007).

2.3.2 Interviews

Fuys et al. (1988) motivated their preference to use interviews over multiple-choice question papers by saying that the learners' responses to questions on a certain topic in a multiple-choice test will give a teacher information about what the learner knows but it may not reveal accurate information about how the learner thinks or whether the learner has the potential to think at a certain level.

A wealth of information about the levels of geometric thought and reasoning has been obtained through interviews. Burger & Shaughnessy (1986), Clements & Battista (1992), Fuys et al. (1988), Gutiérrez et al. (1991) and Mayberry (1983) used interviews to assess the learners and, more recently, interviews were also used by Abu & Abidin (2013); Atebe & Schäfer (2008); Bleeker, Stols, & Van Putten (2013); Dindyal (2007); Khembo (2011) and Kim (2011). Teachers were empowered to conduct interviews with very young learners as an alternative method of assessment in a study by Moss, Hawes, Naqvi, & Caswell (2015).

Another general advantage of the interview method is that the interviewer can also see the gestures of the interviewee. However "seeing" could also become a disadvantage as the interviewer's gestures could guide the interviewee in a specific direction. This disadvantage could be lessened by following an interview protocol (Opdenakker, 2006).

A disadvantage of the interview method is that it is a time-consuming method of assessment. Individual interviews generally take between 40 and 90 minutes (Fuys et al., 1988) and therefore, a much smaller sample can be used in the studies.

Another disadvantage is that the data could also be subject to bias from the interpreter of the interview data. In the re-analysis of the same data by various people in the study done by Fuys et al. (1988) the responses of the learners were classified differently by different interpreters. However, in their study, they regarded this as proof of the levels not being discrete and that some learners were between levels.

2.3.3 Proof or open-ended tests

Although interviews are believed to give a better picture of learners' reasoning than multiple-choice tests, many other pen-and-paper tests were developed and used (Battista, 2007). Smith & De Villiers (1989) found that their open-ended test outperformed a multiple-choice test in all aspects. Their test consisted of one worded answers, explanations for the answers and short proofs. Gutiérrez & Jaime (1998) devised a test with more than one question related to the same problem and they also supplied clues when the learners could not answer the questions. The number of clues used was also an indication of the level of thought.

Another type of test is a proof test where learners are required to complete and do proofs. Proof tests were developed and used by Senk (1982) as part of the CDASSG project in America. More than 1 500 learners were tested using proof tests and Senk (1982) found a strong relation between the Van Hiele levels and writing proofs. This was also the conclusion of a study by Ndlovu & Mji (2012).

The disadvantages of the open-ended or proof tests are that they are generally more time-consuming than multiple-choice tests. Often fewer items are asked in the test due to time constraints, which decreases the reliability of the test (Briggs, Alonzo, Schwab, & Wilson, 2006). There is also the potential problem of bias when the test is scored (Briggs et al., 2006). McBride & Carifio (1995) found that the scoring methods had a significant influence on the results. Their findings also suggested that a more structured scoring scheme supplied more consistent results when the tests were scored by different people.

However, these types of tests still seem to be more reliable than the multiple-choice type tests (Smith & De Villiers, 1989) and are often used.

2.4 OTHER THEORIES OR METHODS FOR DESCRIBING LEARNERS' GEOMETRICAL THOUGHT

In reviewing the literature for this study, a few theories and models were found that have been used when describing geometric thought. Although the main focus of this study is on the Van Hiele theory, Piaget's developmental theory and the SOLO taxonomy are briefly discussed here. Spatial ability seems to play very important role in geometric understanding (Martin, 2007) and is also included in this discussion. Although Bloom's taxonomy and other cognitive categorisations are not used to describe geometric reasoning as such, they have played an important role in the hierarchy of questions in the schooling system's formal tests and are therefore also included in the discussion.

2.4.1 Spatial Ability

> *"Spatial reasoning (or spatial ability, spatial intelligence or spatiality) refers to the ability to recognize and (mentally) manipulate the spatial properties of objects and the spatial relations among objects."* (Mulligan, 2015:513).

People with good spatial sense can more easily relate to geometric descriptions of objects and positions (Van De Walle, 2004). Thus there seems to be a strong link between spatial ability and geometric ability (Bishop, 1980). According to Battista (2007), spatial reasoning contributes to formal geometric reasoning and provides cognitive tools for formal geometric analysis.

The importance of spatial ability was known but not recognised for its role in geometric ability before research in the 1940s and 1950s (Unal, Jakubowski, & Corey, 2009). There now seems to be renewed interest in the development of spatial ability of learners as the importance of spatial ability is seen not only in the development of geometrical reasoning but also for mathematical reasoning in general (Mulligan, 2015). Also, in a world where the advancement of technology is important, it seems even more important to improve the spatial abilities of our learners (Mulligan, 2015).

Another problem that Blanco found was that in teaching the greatest emphasis was placed on the definitions of shapes but the analysis of the properties was avoided and visualisation was not important at all (Blanco, 2001). This resulted in students finding it difficult to change this picture and find solutions using non-standard figures.

2.4.1.1 Relationship to Van Hiele Levels

French (2004) linked the first Van Hiele level of visualisation to the spatial exploring of geometry. When learners examine shapes and learn the correct terminology they increase their spatial sense and lay the foundations for the higher levels. If the learners have a poor spatial ability and cannot correctly identify shapes, angles and parallel lines it will become apparent in the typical errors that they make (French, 2004).

Van de Walle (2004) states that goal of the curriculum in the primary school in terms of geometric reasoning and spatial ability should be to help learners to progress past the second Van Hiele level of analysis. This progression is necessary for the learners to be prepared for the deductive geometry curriculum of the secondary school (Van De Walle, 2004).

2.4.1.2 How to improve spatial ability

Piaget stated that physically manipulating concrete objects was necessary for the discovery of shapes. This was also the conclusion of Bishop (1980), Clements & Battista (1992) and French (2004). The initial discovery of geometry starts with experimental activities that involve drawing, cutting, folding, tracing and fitting of shapes. Bishop (1980) found that when primary school learners used physical manipulation of objects they performed better in spatial ability tests when compared to learners who did not.

Piaget commented that there is a danger that mathematics teachers often focus on the abstract and language without using more concrete examples (Martin, 2007). Battista (2007) confirmed this by saying that too much emphasis is placed on the cognitive processes involved in learning. He also felt that the social-cultural processes should receive much more attention because of the huge influence it has on spatial reasoning and motivation of learning. The social-cultural background of the learners influences how they perceive the world around them and also how they explore shapes.

2.4.2 Piaget's Developmental Theory

Jean Piaget (1896 – 1980) was a Swiss developmental psychologist. He believed that we are different to animals because of abstract symbolic reasoning (Jogymol Kalariparampil Alex, 2012). In his theory, Piaget described the developmental nature of children's thinking in various domains linked to the child's age (Labinowicz, 1980). His theory focussed on the child's perception, adaptation and manipulation of the environment around them and he proposed four stages of cognitive development:

sensorimotor, preoperational, concrete operations and formal operations. According to Piaget the stages of cognitive development are based on a person's mental structures resulting from biological ageing and environmental experiences (Labinowicz, 1980). The development is not linked with or necessarily influenced by instruction (Pusey, 2003). The stages are summarised in table 2.3 below.

Table 2.3: Piaget's Stages of Cognitive Development

	Stages	Age (years)	Description
Preparatory prelogical stages	Sensorimotor	0 to 2	Coordination of sensory and motor input
	Preoperational	2 to 7	Ability to represent actions through thought and language
Advanced logical thinking stage	Concrete operations	7 to 11	Logical thinking limited to concrete events
	Formal operations	11 +	Logical thinking applied to abstract events and was unlimited.

Source: (Labinowicz, 1980)

Van Hiele acknowledges and mentions Piaget in his work (Van Hiele, 1986) According to Van Hiele, there are important terminological differences between his theory and that of Piaget. Van Hiele had two main objections against Piaget's theory. Firstly, Piaget was not aware of more than two stages of thought and therefore concluded that all development takes place in one period. Secondly, Piaget studied the "psychology of development" of children whereas Van Hiele studied the "progress of the learning process of intelligence" (Van Hiele, 1986:100). Also, according to Van Hiele, development takes place as a result of particular instruction and is not just the result of the growth of general mental structures (Lehrer, Jenkins, & Osana, 1998). However, Van Hiele acknowledges that it was Piaget who first introduced levels and the idea that a person on a lower level could not understand a person at a higher level (Van Hiele, 1986).

2.4.3 SOLO taxonomy (Structure of the Observed Learning Outcomes)

A structure was devised by Biggs and Collis in 1982 by which the mode of functioning and quality of the responses of children can be evaluated in all subjects in the curriculum (Pusey, 2003). They developed this structure to help teachers improve their instruction (Pusey, 2003). This structure was called the SOLO Taxonomy.

Pegg (1997) regarded the taxonomy as particularly useful in evaluating learning in tasks that follow after teaching a certain topic. Pegg argued that the Van Hiele levels did not adequately describe the process of progression through the levels. Therefore he used the SOLO Taxonomy developed by Biggs and Collis to extend the second and third Van Hiele levels.

The SOLO Taxonomy was influenced by the Piagetian tradition and describes five modes of functioning that is also linked to age (Jones, Collis, & Watson, 1993). It further distinguishes five levels of responses within a mode of functioning: pre-structural, uni-structural, multi-structural, relational and extended abstract. This taxonomy views learning as a process that moves from a recall of facts to abstract thinking and reasoning (Jones et al., 1993).

The SOLO taxonomy measures the mode of functioning and the quality of the response at a certain time. It is not regarded as being a fixed measure as the learners' responses change as circumstances change (Pusey, 2003).

2.4.4 Taxonomies for cognitive categorisation

National school examinations boards need to have a set of criteria that can be used to measure the level of difficulty of the examination items to ensure equality of the examinations over the years and alignment with the curriculums (Berger, Bowie, & Nyaumwe, 2010). A number of different taxonomies have been developed over the years to evaluate cognitive levels, classroom tasks and alignment with the curriculum (Berger et al., 2010).

In education, Bloom's taxonomy is most probably one of the most widely used measures to evaluate questions (Anderson, 1999). Therefore, although Bloom's taxonomy is not well aligned with the Van Hiele theory (McBride & Carifio, 1995), it will be discussed here.

Bloom's taxonomy originated in 1956 and was updated in the late 1990's (Anderson, 1999). The original theory was developed before the cognitive processes involved in teaching and learning were understood thus necessitating the update (Anderson, 1999). The framework was originally designed to classify questions for item bank purposes (Long, Dunne, & De Kock, 2014).

The original terms used in the taxonomy were the nouns: Knowledge, Comprehension, Application, Analysis, Synthesis and Evaluation. Knowledge was the most basic level and evaluation the most complex level. The revised terms are the verbs: Remembering, Understanding, Applying, Analysing, Evaluating and Creating. In the report on the progress of the group revising the taxonomy, Anderson explained that the nouns were changed to verbs to represent thinking as an active process. The nouns in the subcategories were also changed to verbs and the subcategories were reorganised (Anderson, 1999).

In South Africa, the Curriculum and Assessment Policy Statement (CAPS) requires that questions in the question papers are set on various cognitive levels as identified in TIMMS 1999: Knowledge, routine procedure, complex procedure and problem solving (Department of Basic Education, 2011a). The categories are very similar to Bloom's Taxonomy, both the original and revised versions (Long et al., 2014).

Long et al. (2014) stated that the use of Bloom and similar taxonomies is not adequate to provide feedback about teaching and learning. However, although they offer critique against the use of Bloom's Taxonomy, they also acknowledge that it is necessary to have a framework to use in aligning assessment practices with classroom practices (Long et al., 2014).

2.5 CHAPTER SUMMARY

The Van Hiele theory has been used extensively internationally and locally. This theory provides a structure by which learners' progression through the development of geometric reasoning can be assessed. Despite critique against the theory, it is widely accepted and acknowledged because it can assist researchers in understanding the issues involved in the teaching and learning of geometry.

In the next chapter literature regarding the research questions on the performance and typical errors of learners in geometry, will be reviewed.

CHAPTER 3

LITERATURE REVIEW ON PERFORMANCE AND ERRORS

3.1 INTRODUCTION

The inclusion of geometry and especially mathematical proof at school level has been a point of debate in many countries. In South Africa, it was excluded from the FET (Further Education and Training) mathematics curriculum for six years with the phasing out starting from grade 10 in 2006. However the skills learnt in geometry do not only apply to mathematics but also to other areas of the school curriculum (Gonzalez & Herbst, 2006) and therefore geometry has seen a “rebirth” in many countries (De Villiers, 1996; Mariotti, 2004). It was reintroduced in South Africa for grade 10 learners in 2012. The first group of grade 12s to write geometry after its re-introduction, completed the NSC in 2014.

The diagnostic reports on the National Senior Certificate examination for 2014 and 2015 paint a bleak picture of their geometry performance (Department of Basic Education, 2014b, 2015a). They emphasise that many errors resulted from learners not knowing the basic terminology and geometry concepts that should have been mastered in the junior secondary (grade 7 – 9) phase (Department of Basic Education, 2014b, 2015a).

The theoretical framework of this study was discussed in the previous chapter. This chapter discusses the literature related to the research questions. The first section of this chapter discusses the performance of South African learners in international and national tests that were not based on the Van Hiele levels in order to gain an overview of the mathematics performance. Secondly, the performances of the learners in terms of the Van Hiele levels are reviewed. Thirdly, literature on errors and the typical errors relevant to learners in grade 9 geometry is reviewed. The last section discusses the literature on studies that found a connection between errors and the Van Hiele levels.

3.2 PERFORMANCE OF LEARNERS IN SOUTH AFRICA (NOT BASED ON VAN HIELE)

A number of studies and tests have been used to assess the performance of learners with regards to mathematics in various grades in South Africa. Two international studies: the **SACMEQ** (Southern and Eastern Africa Consortium for Monitoring Educational Quality) **Project** and **TIMSS** (Trends in International Mathematics and Science Study) as well as two standardised national tests: the **ANA** (Annual National Assessment) and **NSC** (National Senior Certificate) are discussed to give an overview of the general performance in mathematics in South Africa. A comparison of the reports on the assessments indicates that geometry seems to be an area of concern in all the grades.

3.2.1 International assessments

If the performance of South African learners in mathematics is compared to other countries internationally, one soon realises the seriousness of the problem. The international learner performance is far better than that of the South African learners. Two well-known international assessments of learners' mathematics performance that include sections on geometry are mentioned below in order to highlight the problem:

The **SACMEQ II and III Projects,** which were conducted in 2000 and 2007, assessed the reading and mathematics skills of Grade 6 learners in 14 countries in east and southern Africa. The mathematics assessments covered the following domains: Number, measurement and space/data (Saito, 2011). South Africa's overall mathematics achievements in these assessments were amongst the lowest (Department of Education, 2008; Saito, 2011) and the measurement questions scored the lowest average percentages of the three domains (Saito, 2011). The most recent **SACMEQ III,** conducted in 2007, showed that there was no improvement in the South African grade 6 mathematics performance over the seven-year period from the year 2000. **SACMEQ III** also tested grade 6 teachers and the data shows that many South-African mathematics teachers lacked basic content knowledge and could not answer questions that their learners were required to answer (Spaull, 2013).

The **TIMSS** (Trends in International Mathematics and Science Study) assesses Grade 8 learning achievement in mathematics and science in various countries internationally every four years (Mullis et al., 2012). South Africa took part in this assessment in 1995, 1999, 2002 and 2011. The study was conducted with a relatively large sample e.g. for TIMSS 2011 the Human Sciences Research Council (HSRC) conducted the study in 285 schools testing 11 969 grade 9 learners in South Africa.

Table 3.1: Average mathematics scores of South African learners in the TIMSS study from 1995 to 2011

Grade	Year	Average score
9	2011	352
9	2002	285
8	2002	264
8	1999	275
8	1995	276

Source: Human Sciences Research Council, 2011

South Africa's performance has been disappointing. South Africa scored amongst the bottom six countries, far below the international lowest benchmark of 400 and South Africa compared grade 9 learners with the grade 8 learners in the other countries. Although geometry was also a weakness in many other countries, South African learner's mean scores in geometry were significantly lower than the international average scores (Mullis et al., 2012).

3.2.2 National assessments

The bleak picture painted by the international studies is confirmed by South African studies and national assessments.

The **Annual National Assessment** (ANA) uses nationally standardised tests for learners from grade 1 to 6 and 9. This assessment was successfully implemented in September 2012 for the first time in all South African schools (Department of Basic Education, 2014c).

The average percentage for the Mathematics scores in ANA for 2012 to 2014 are presented in the table below (Department of Basic Education, 2014c). The decline in percentages from grade 1 to grade 9 is especially concerning. The reports further indicate that geometry is a weakness in all the grades (Department of Basic Education, 2014c).

Table 3.2: National mathematics averages for ANA in the different grades from 2012 to 2014.

GRADE	NATIONAL AVERAGE MATHEMATICS PERCENTAGE		
	2012	**2013**	**2014**
1	68	60	68
2	57	59	62
3	41	53	56
4	37	37	37
5	30	33	37
6	27	39	43
9	13	14	11

Source: Adapted from Department of Basic Education, 2014b

The **National Senior Certificate** (NSC) is a nationally standardised examination written by all the grade 12 learners in South Africa. In the senior or FET phase, the learners must select between mathematics and mathematical literacy. The total number of mathematics and mathematical literacy gives an indication of the total number of learners who wrote the grade 12 examinations from 2011 to 2015.

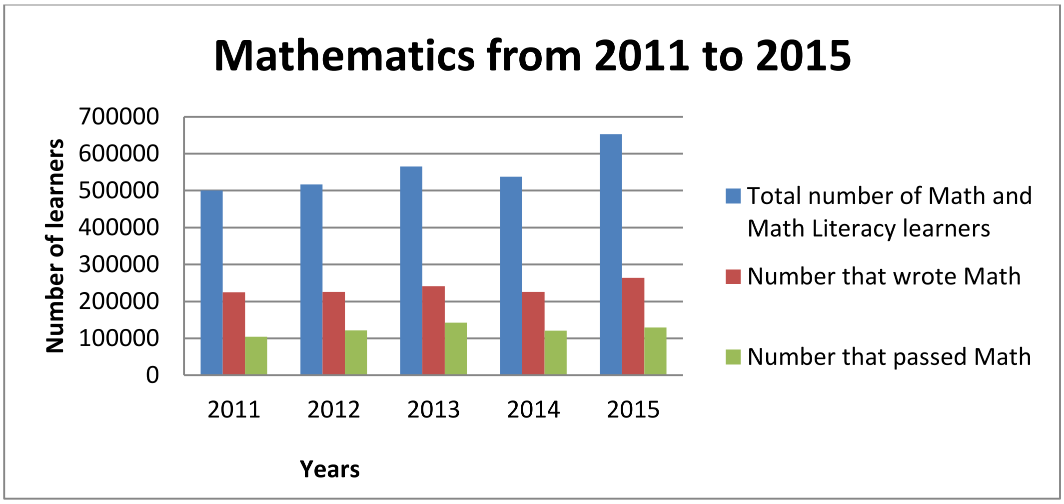


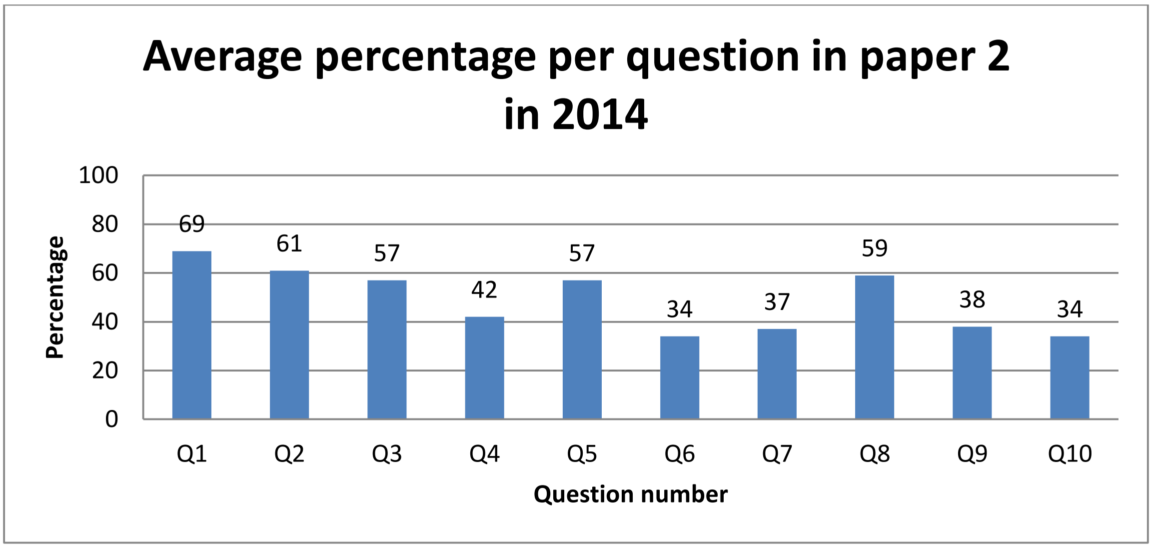
Figure 3.1: Graph showing the number of learners who wrote and passed the NSC mathematics examinations from 2011 to 2015. (Source: Adapted from Department of Basic Education, 2014a, 2015)

From the graph, it can be seen that only about a quarter of the countries' learners are passing mathematics and although there is an upwards trend in the number of learners who are writing mathematics, the percentage of learners passing the subject is still low.

Geometry plays a big role in the overall performance of learners in mathematics as space, shape and measurement contributes between 35 – 40% of the assessment in grades 7 – 9 (Department of Basic Education, 2011c) and Euclidean geometry contributes 33% of the second paper in mathematics in the grade 10 – 12 phase (Department of Basic Education, 2011a).

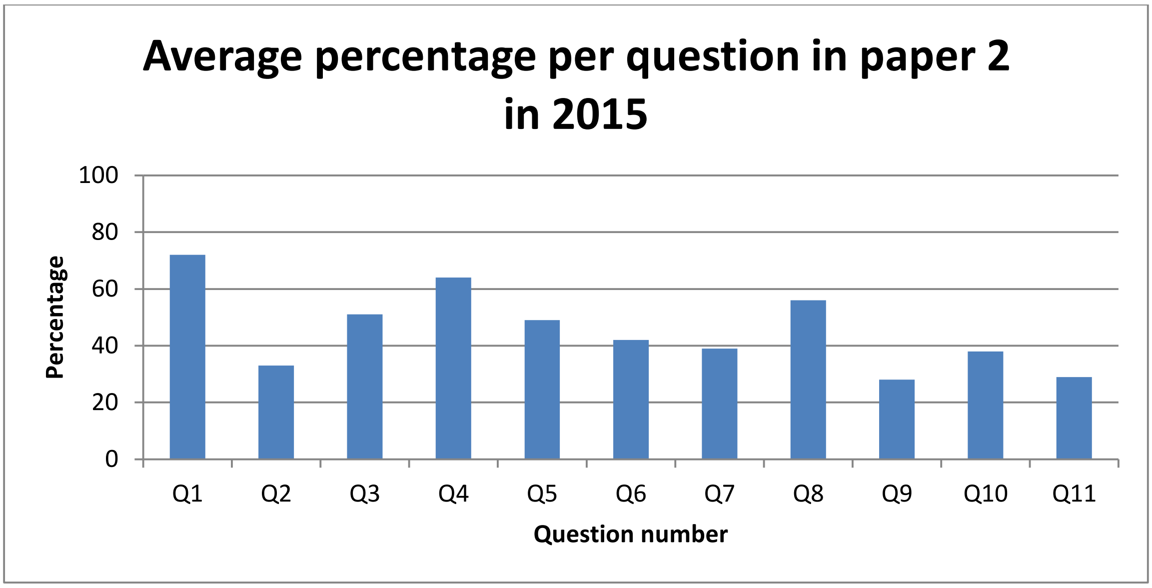
In 2006, Euclidean geometry had been moved out of the mathematics NSC paper 2 to be part of a non-compulsory paper 3 in grade 10. That group was the first group not to write about Euclidean geometry in grade 12 in 2008 (Van Putten, Howie and Stols, 2010). Learners writing the paper 3 mathematics were credited with an additional subject. However, in 2012, the topics covered by the paper 3 mathematics were placed back into the mathematics papers and phased in from grade 10. Paper 3 mathematics was then removed and Euclidean geometry has once again been included as a part of the grade 12 NSC paper 2. These changes could have contributed to the drop in numbers of learners writing and passing mathematics in 2014 (see figure 3.1).

The following graph and table were taken from the diagnostic reports of the 2014 and 2015 grade 12 final examinations in Mathematics of the Department of Basic Education to illustrate the performance in geometry compared to the other sections of mathematics. The graphs were based on a random sample of papers from all sections representing South Africa and therefore provide a good indication of the challenging sections in the second papers.



Question	Section covered
Q1	Data Handling
Q2	Data Handling
Q3	Analytical Geometry
Q4	Analytical Geometry
Q5	Trigonometry
Q6	Trigonometry
Q7	Trigonometry
Q8	Euclidean Geometry
Q9	Euclidean Geometry
Q10	Euclidean Geometry

Figure 3.2: The average percentages per question in paper 2 of the 2014 grade 12 NSC paper (Source: Department of Basic Education, 2014a:122)



Question	Section covered
Q1	Data Handling
Q2	Data Handling
Q3	Analytical Geometry
Q4	Analytical Geometry
Q5	Trigonometry
Q6	Trigonometry
Q7	Trigonometry
Q8	Euclidean Geometry
Q9	Euclidean Geometry
Q10	Euclidean Geometry
Q11	Euclidean Geometry

Figure 3.3: The average percentages per question in paper 2 of the 2015 grade 12 NSC paper (Source: Department of Basic Education, 2015:152)

The graphs clearly show that learners performed poorly in the Euclidean geometry rider questions namely, questions 9 and 10 in 2014 and questions 9 to 11 in 2015.

In the diagnostic reports, it was reported that many of the learners did not even attempt the questions with the more complex riders (Q10 in 2014 and Q11 in 2015)

(Department of Basic Education, 2015a). There could be many reasons for this but one explanation could be similar to the conclusion of Daymunde (2010) after a study in the USA. Daymunde concluded that learners who did not complete certain sections of the paper were not able, either because they had not prepared themselves efficiently or because of the inefficient teaching that took place. They tended to use inefficient strategies or spent too much time in answering the questions and therefore did not complete all the sections.

In both the 2014 and 2015 NSC diagnostic reports it was reported that many geometry errors were due to incorrect use of terminology and poor knowledge of the basic concepts taught in the junior secondary curriculum (Department of Basic Education, 2014a; 2015). This seems to correspond with the reports of poor geometry knowledge in the ANA assessments.

The poor geometry performance of the learners in South Africa has concerned many researchers, many of whom have used the Van Hiele theory to gain more insight into the problem. The next section provides an overview of South African studies using Van Hiele levels.

3.3 THE VAN HIELE LEVELS OF GEOMETRIC THOUGHT IN SOUTH AFRICA

South African scholars, similarly to the rest of the world, experiences challenges with geometry (Alex & Mammen, 2012; Marchis, 2012; Mayberry, 1983; Vighi & Marchini, 2007). In order to gain a better understanding of the challenges experienced, a number of South African researchers have used the Van Hiele theory to evaluate learners, teachers and pre-service teaching students.

South African researchers (De Villiers & Njisane, 1987; Bennie, 1998; Cranfield, 2001; Siyepu, 2005; Atebe & Schäfer, 2010b; Alex, 2012) found that many of the learners were still at level 1 and 2 in the senior mathematics phase. Luneta (2015) graded one thousand grade 12 final examination papers and found that most of those learners were below level 2. The low level of geometric thought in South Africa is a matter of concern as after the junior secondary phase (Grades 7 – 9) the learners should have reached level 3 of geometric thought before they choose mathematics in the senior secondary (Grades 10 – 12) phase (De Villiers, 2012). The skill to solve geometric riders in the senior secondary phase requires that learners be ready for deductive geometry (Department of Basic Education, 2011a; Van De Walle, 2004). Therefore if they reason below level 3 they can only do geometry proofs by rote memorization (Davey & Holliday, 1992).

In addition, most of the pre-service teaching students who were tested in studies functioned at the lower Van Hiele levels. Van der Sandt & Nieuwoudt (2003) found that the average achievement of pre-service teachers on level 1 was 62% and Van Putten (2008) found that the average of the students on level 1 in her study was 42,5%. The average percentages on levels 2 and 3 in both studies were much lower.

Even more concerning is that studies imply that teachers also seem to be functioning at lower Van Hiele levels. Van der Sandt & Nieuwoudt (2003) found that the grade 7 teachers in their study only achieved an average of 71% on Van Hiele level 1 and a 45% average on level 2. If complete acquisition of a level requires 85%, then these results indicate that the teachers had also not yet acquired level 1. These poor results were confirmed in the SACMEQ III study (Venkat & Spaull, 2014) and by Khembo (2011) using grade 6 teachers. Mji & Makgato (2006) found that teachers teaching Euclidean geometry in the senior secondary phase, have problems with the solution of the geometric riders that the learners are expected to solve.

This low level of thought of the teachers is concerning because many international studies show that the learners' proficiency in mathematics is directly related to their teachers' content knowledge (Dicky, 2011; Fennema & Franke, 1992; Ma, 1999). In South Africa, many learners, especially in under-resourced schools (quintile 1 – 3), are assigned to teachers who do not have the necessary qualifications and knowledge to teach mathematics (Taylor, Fleisch, & Shindler, 2008). The Department of Education expressed concern that the teaching of mathematics in schools is rarely the first choice of talented maths graduates. In many cases, the teachers of mathematics had been taught by teachers with similar problems (Van Putten, 2008). Consequently, there is a vicious cycle of poor teaching, poor learner achievement and a constant shortage of competent teachers (Department of Education, 2008; Spaull, 2013).

Competent teachers are necessary because learning mathematics is complex, it takes time and is often not straight forward (Even & Tirosh, 2008). Good content knowledge is essential for teaching but does not necessarily lead to effective learning. The teachers also need pedagogical knowledge to know how to guide learners through the complex process of mathematical learning (Fennema & Franke, 2001). Learners on different levels of geometric thought need to receive teaching that relates to their levels of thinking. Learners demonstrate a better understanding of the geometrical concepts if teachers keep their Van Hiele levels in mind when preparing their lessons and use the Van Hiele phases of learning to help learners progress through the levels (Atebe & Schäfer, 2011; Howse & Howse, 2014; Van Hiele, 1986).

Teachers must be able to select relevant examples and exercises when preparing lessons and must be able to sequence the content of the lesson and select a method for teaching the relevant procedures (Bansilal, Brijlall & Mkhwanazi, 2014). Teachers are also challenged by developing lessons with activities on more than one level in a class in order to reach all the learners on the different levels (Bleeker, Stols & Van Putten, 2013).

In addition, the transition between the levels of geometric thought in the high school geometry curriculum occurs rapidly and this, in combination with a full syllabus, contributes to the failure of students (De Villiers, 1996; Siyepu, 2005).

The low level of geometric thought of learners is the product of many factors in the teaching and learning environment and is reflected in the low performance of learners. The numerous errors that learners make can be seen as a measure of the misconceptions that result from poor teaching and learning. The next section reviews error analysis in mathematics and other typical geometry errors.

3.4 ERROR ANALYSIS

Errors are viewed negatively as something that has gone wrong and that remediation is needed (Borasi, 1987). However, the analysis of errors can be very positive and can provide a wealth of information that can be used to ensure effective teaching and learning (Radatz, 1979). The section below discusses various viewpoints on errors and misconceptions as well as how errors can be used to improve teaching and learning. Lastly, the typical geometry errors found in other studies, applicable to both the junior secondary phase and the current study, are summarised.

3.4.1 What are errors and misconceptions?

The focus of this study is on the typical geometry errors made by learners and how they are related to the Van Hiele levels. However in the study of literature related to errors the researcher found the use of the terms "errors" and "misconceptions" confusing. Some authors such as Kim (2011) use the terms errors and misconceptions interchangeably and other authors such as Ashlock (2006), Hansen (2011) and Rach, Ufer, & Heinze (2013) distinguish between errors and misconceptions. For the purpose of this study, the researcher distinguishes between errors and misconceptions. Therefore, in this section, the researcher attempts to clarify and show the relationship between the two terms.

Errors, as defined by Radatz (1979:179), are not simply "the absence of correct answers or the result of unfortunate accidents" but "the consequence of definite processes whose nature must be discovered". The Oxford Dictionary defines an error as: "*The state, quality, or condition of being wrong; a mistake, an accidental wrong action or a false statement not made deliberately.*" while a misconception is defined as: "*A view or opinion that is incorrect because it is based on faulty thinking or understanding*" (Oxford University Press, 2016). The difference between errors and misconceptions can be further explained as: Error patterns are the symptoms of misconceptions (Ashlock, 2006; Olivier, 1992) and misconceptions are just one of the factors that lead to errors (Hansen, 2011). Therefore it can be concluded that errors are wrong actions and misconceptions are the wrong ideas that lead to errors.

To summarise and explain how the researcher views the link between misconceptions and errors a visual representation is given below.

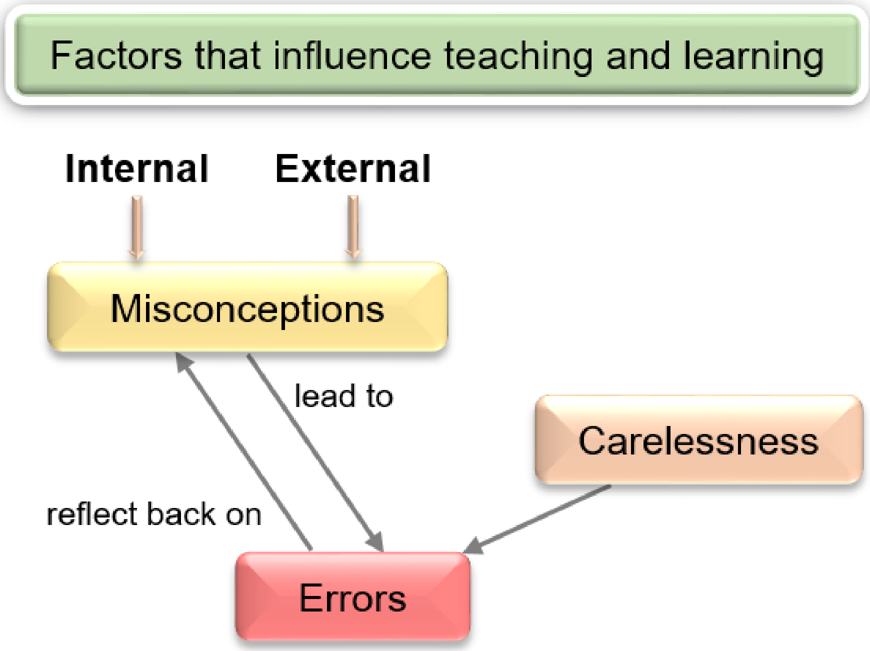


Figure 3.4: Diagram that shows the relationship between the causes of misconceptions and errors as used in this study.

3.4.2 Use of errors

Errors made in mathematics can be a very important instructional tool if handled correctly (Mcnamara & Shaughnessy, 2011; Venkat & Adler, 2012). Teachers and learners could use errors to detect misconceptions and make teaching and learning more efficient.

3.4.2.1 By teachers

Correct answers are rewarded and errors are either punished or dismissed in the traditional behaviourist classroom. Errors were not seen as opportunities to learn about the type of reasoning used by a learner but were avoided so as not to reinforce the incorrect answer in the learners mind (Melis, 2004). Although much has changed, teachers still often avoid errors in the learning process and tend to focus on the correct method (Spychiger, Kuster, & Oser, 2006). It must also be kept in mind that although good teaching may reduce the number of learner errors, the errors cannot be avoided altogether (Hansen, 2011). However, when teachers were trained to use errors as a teaching tool, Heinze & Reiss (2007) noted a marked improvement in geometry results in their study.

a) More efficient teaching

Errors are often avoided as limited time is available in the full curriculum (Hansen, 2011). However, time constraints could be avoided if specific errors were selected by the teacher for class discussions or projects (Borasi, 1987). Time would also be saved if re-teaching a section of work could be avoided by simply addressing the errors and misconceptions and directing future teaching to the areas of need, therefore utilising the class time more efficiently (Autrey, Burroughs, & Fertig, 2013; Gardee & Brodie, 2015). Finding error patterns during assessment gives the teacher an indication of the learners' strengths and weaknesses and also which activities to use so that the learners do not continue practising the incorrect concepts and procedures (Ashlock, 2006).

b) Detection of errors

The diagnosis of errors should be done continually in order for effective teaching and learning to take place. Teachers need to look for error patterns and hypothesise on possible causes all the time (Ashlock, 2006), because if the causes are not addressed they will just lead to more errors (Hansen, 2011).

The problem is that many of the misconceptions and certain errors are only recognised by the teacher during formal assessment rather than during teaching (Kembitzky, 2009). Radatz (1979) noted, as a further challenge, that although errors can be detected during tests, which characterise most of the formal assessment situations, they often do not supply enough information to analyse the misconceptions. Adding to this, Ellerton & Olson (2005) warned that some misconceptions went through undetected in assessments because the answers were correct (that is, there are no visible errors). It is also interesting that incorrect procedures often produce correct answers which reinforce the learners' belief that they have learned the correct procedure (Ashlock, 2006).

From the above, it can be concluded that the detection of misconceptions is not always simple and that the use of standard formal assessments is not always effective.Other methods should also be used. One method of detecting a misconception would be to use an analysis tool e.g. a rubric that learners can use to analyse themselves after each exercise (Spychiger et al., 2006). Kembitzky (2009) encouraged learners to write about their errors to help them uncover their misconceptions. Kendall, as described by Kembitzky (2009), used a similar method by encouraging learners to keep a log of their errors. However, the success of this and other methods depends on the accuracy with which the students can analyse their own misconceptions (Spychiger et al., 2006). Another more secure method would be for the teacher to use an interview to question the learners about concepts and procedures to identify any error patterns and relate them to misconceptions (Ashlock, 2006), but this method could be very time-consuming.

c) Remediating errors

Although the detection of errors is important the process should not stop after the errors are detected, they need to be remedied. However, the remediating of errors is not a simple task and the following points should be kept in mind. Firstly, to correct the misconceptions, one needs to change the conceptual framework of the learner (Olivier, 1992). This conceptual framework of the learners is not simple to correct, so to simply point out the misconception to the learner is usually not effective (Daymunde, 2010). According to Borasi (1987) simply re-explaining the topic again or giving additional exercises is also not effective.

In order to be more effective, it is essential that the correct diagnosis of the type of error is made and that the learners' individual differences, as well as their difficulties in learning mathematics, be taken into account (Borasi, 1987). Olivier (1992) suggests that teachers should try to link the old concepts to the new ones and expose the misconceptions in a positive manner.

Secondly, some of the misconceptions and errors in geometry may be the result of a language problem or a problem with reading and interpreting the questions. The problem might not be that the learner does not know the answer, but simply cannot read or does not understand the question. Newman's Error Analysis was designed as a "simple diagnostic procedure" (White, 2010:133) to help identify problems concerning word problems but, as Ellerton & Clements (1996) pointed out, many of the problems in all aspects of mathematics originate from the learners' inability to read and interpret all the types of questions. Therefore problems concerning the reading and interpreting of geometry questions should also be taken into account. Many of the remedial mathematics programmes fail due to an over-emphasis on the mathematical procedures, with little attention given to reading and comprehension of the questions (Ellerton & Clements, 1996).

Thirdly, the learners themselves may not be ready to acknowledge their mistakes (Hansen, 2011). Teachers could help them by creating an environment where learners feel comfortable to make errors and to correct them (Schleppenbach, Flevares, Sims, & Perry, 2007).

The use of errors can have a very positive impact on teaching but may not be easy for all teachers due to various factors. One factor could be a lack of pedagogical knowledge which could be improved if teachers work collaboratively with each other and discuss learners' errors (Ashlock, 2006).

The attitudes of teachers is another factor that plays a huge role in teaching (Salifu & Agbenyega, 2013). Cranfield (2001) noted that the teachers in his study disliked geometry since they were at school. They only taught geometry because they were forced to. Consequently, they avoided geometry and did not make sure that the learners grasped the concepts. This resulted in a large number of errors and poor performance of the learners due to misconceptions. Teachers should also have more empathy with learners' errors and misconceptions because, in general, they do not make mistakes intentionally (Olivier, 1992). Ryan & McCrae (2005) believe that for teachers to respect learners' errors they must first learn to recognise and value their own errors.

Finally, errors should not only be used for remedial purposes but can also be used to explore mathematics. If other circumstances in which the errors could actually be true are explored, the learners could gain a deeper understanding of the concepts (Borasi, 1987). New discoveries in mathematics are often made by analysing the errors or exceptions to the rule e.g. Saccheri's non-Euclidean geometry (Borasi, 1987). In the same way, errors can be used in the classroom to help learners discover new sections of the work or to clarify concepts (Borasi, 1987; Heinze & Reiss, 2007).

Error management can also form an important part of the problem-solving process (Heinze & Reiss, 2007). In a study in by Schleppenbach, Flevares, Sims, & Perry, (2007) the teachers planned classes so that the learners would make errors in order to promote a discussion of the errors.

Lastly, teachers should not just focus on detecting the errors, immediately correcting them and then moving on to the next concept, but should also spend time on error prevention measures (Rach et al., 2013).

3.4.2.2 By learners

The exploring of errors is hampered by the negative way in which learners often view errors (Hansen, 2011). Heinze & Reiss (2007) found a significant improvement in geometric reasoning and proof in classrooms where errors were treated in a positive manner. Learners should use their errors positively to improve their faulty knowledge structures and help them from repeating these errors. Learners must be given the opportunity to "explain" and "fix-up" their own errors as this could help them to become more motivated (Borasi, 1987).

Heinze & Reiss (2007), Daymunde (2010) and Rach, Ufer, & Heinze (2013) found that the learners' achievements improved when they analysed their own errors. However, Daymunde (2010) also noted that the improvement seemed to be more prominent in the learners whose grades were in the middle of the class. The learners who were struggling or those who were excelling seemed to benefit less from the analysis of their errors. She also noted that parental involvement was very beneficial.

Melis (2004) used computer-based learning to assist learners in correcting their own errors. She also suggested that it helped learners when they identified and corrected somebody else's errors. By looking at another learner's errors they can learn about their own thinking on a subject. This promotes critical thinking but she also warns that the learner should not correct an incorrect answer with yet another incorrect answer. The process should, therefore, be monitored closely (Melis, 2004).

According to Allen (2007), some of the factors that lead to misconceptions are informal thinking patterns and poor memory and therefore learners should concentrate on improving this. Daymunde (2010) found that careless errors had a serious impact on performance. Learners should be aware of how to reduce their careless errors if they want to improve their geometry performance.

3.4.3 Factors that could lead to errors

Radatz identified five factors that lead to errors. These factors are: language difficulties, difficulties obtaining spatial information, deficient mastery of prerequisite skills, facts and concepts, incorrect associations or rigidity of thinking and faulty application of rules or strategies (Radatz, 1979). Hansen added that errors could also result from carelessness, misinterpretation of symbols or text, lack of relevant experience or knowledge and the inability to check answers (Hansen, 2011). The classification of errors by Radatz has been applied to the analysis of misconceptions in geometry studies (Kim, 2011).

Errors due to language difficulties, prerequisite skills and concepts, rigidity of thinking and misinterpretations due to insufficient examples is expanded on below.

3.4.3.1 Language difficulties

In a culturally diverse country and with 11 official languages it is inevitable that language and culture will play a role in the classroom. Feza & Webb (2005) and Roux (2005) ascribed the low Van Hiele levels of learners in South Africa to language backgrounds and learners being taught in a second language. Another matter that complicates the problem is that technical mathematical terminology when translated, does not necessarily have the same meaning as the English word, making it difficult for a second language learner to understand mathematics (Mji & Makgato, 2006). These findings were also supported in the studies by Atebe & Schäfer (2010a) and Henning & Dampier (2012).

There is much literature on how learners understand mathematical language and the difficulties that they experience (Johnstone-Wilder et al., 2007). A misunderstanding of the mathematical language is often a source of learner misconceptions and results in errors (Radatz, 1979). Misconceptions often result if the instruction or language is at a higher level than that of the learners because of a lack of communication (Van De Walle, 2004).

Each level of geometric thought has its own language or linguistic symbols but language plays a bigger role in the progression of learners from Van Hiele levels 1 (Visualisation) to level 2 (Analysis) where they move from the visual holistic perception of figures to the discerning and naming of properties of the figures (De Villiers, 1998). The transition from Van Hiele level 1 to level 2 poses a bigger problem for second language learners since it requires the acquisition of terminology by which figures can be described and explored (De Villiers, 1998). Reasoning, as is required for solving of FET geometry riders, is also much more difficult for a second language learner (Atebe & Schäfer, 2010a).

The literacy level of the learners was stated as an area of concern in the diagnostic reports of the grade 12 NSC papers and in the TIMSS report of Howie as it led to many errors in answering the questions (Department of Basic Education, 2014b, 2015a; Howie, 2002).

In studies using the Newman's error analysis attention has been drawn to the impact of language factors on mathematics learning. The evidence generated in these studies point to learners experiencing more problems with the language than with the standard algorithms in mathematics (Ellerton & Clements, 1996; White, 2010).

In contrast to all the studies on the impact of language, Atebe & Schäfer found that conceptual misunderstandings played a bigger role than language problems in the incorrect use of terminology (Atebe & Schäfer, 2008). In a study by Wijaya, Van den Heuvel-Panhuizen, Doorman & Robitzsch (2014) they found that the errors were also linked to the types of tasks that were given to the learners.

3.4.3.2 Prerequisite skills and concepts

Rationalists such as John Locke believed that children were born as “blank slates” (Pistorius, 1982). However, learners do not come to a classroom as blank slates as they already have a whole set of concepts and perspectives that were formed prior to them entering a classroom (Kembitzky, 2009). Some of these concepts are actually misconceptions which need to be modified before learners build new connections during instruction (Ashlock, 2006). Misconceptions should, therefore, be detected as soon as possible before too many incorrect connections are made which become increasingly difficult to correct (Olivier, 1992).

It is important that the misconceptions with which learners enter classrooms should be discovered before the learning of new knowledge leads to more misconceptions (Biber, Tuna, & Korkmaz, 2013). The challenge is to deal with the misconceptions that were formed before the learner entered the classroom (Kembitzky, 2009) and which is just further reinforced if the current teacher lacks the content knowledge to correct the misconceptions (Olivier, 1992).

Olivier (1992) has a slightly different perspective to misconceptions. He emphasised that although many people think that misconceptions are the result of previous incorrect teaching and learning that misconceptions are mostly the result of learning concepts that could be seen as correct in earlier grades but not correct in later grades. In order to help learners build a conceptual framework for understanding teachers and learners themselves often over-simplify certain concepts. When learners try to fit new knowledge to this over-simplified framework a mismatch occurs (Olivier, 1992).

3.4.3.3 Rigidity of thinking

Sometimes the learners are not at a level that opens them to learning the new concepts (Ashlock, 2006). When teaching takes place at a higher level of geometric thought the learner may try to re-interpret the new concept in terms of insufficient background knowledge or reasoning level, thus forming incorrect concepts (Ellerton & Clements, 1996). This is where the Van Hiele theory and the importance of investigating the levels of geometric thought of the learners become important.

A Thai study by Vaiyavutjamai & Clements (2006) found that some individuals tended to resist change although they were corrected a few times. This seems to imply that some individuals form an emotional and intellectual bond with the misconceptions that they have constructed (Allen, 2007). Allen also found that learners with “unstable” conceptions gave different answers when they were confronted with the questions at different times. Learners who were not confident about concepts reverted to guessing which also led to different answers to the same question.

3.4.3.4 Misinterpretations due to insufficient examples

Many researchers acknowledged that the type of examples used during teaching were found to impact on errors (French, 2004; Hansen, 2011; Johnstone-Wilder et al., 2007). In a study in Spain with 3rd year student teachers, Blanco (2001) recognised that the students misconceptions were the result of the type of examples used by the teachers and the interaction of the learners with the content. Standard examples were given in the textbooks and used during teaching. He found that very few other resources were used and that examples of figures in, for example, different orientations were very limited (Blanco, 2001). Teachers often confine themselves to the use of a single textbook and therefore limit the variety of problems (Ma, 1999).

Often not only the examples but also the exercises used during teaching are often also based on the same standard examples thus minimising the experimentation with other possible situations and deeper comprehension of the concepts. No transfer or connection of the knowledge with other problems is formed (Blanco, 2001).

The teacher should take into account that in the initial process of learning the concept teachers should start off with what they named “best examples”. These “best examples” should be linked to the world of the learner and what they already know (Fuys & Liebov, 1997), for example, a rectangle in the upright position so that it resembles a door. Thereafter learners should also be exposed to “non-examples” of the figure, for example, rectangles in other positions and also non-rectangular shapes.

3.4.3.5 Errors due to carelessness

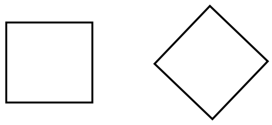
Some of the errors could not be placed on one of these levels and were placed under “careless” (Ellerton & Clements, 1996). Finally, Daymunde (2010) found that careless errors also made a great impact on performance. She classified the careless error types that led to the greatest reduction of marks into the following four categories: Accidentally skipped the question, ran out of time, didn’t follow directions and misread the question.

3.4.4 Typical errors of the Junior Secondary Phase

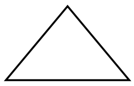
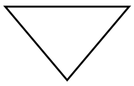
Some errors are described repeatedly in literature and seem to be commonly made by learners. In the section below I will describe a few of the common geometry errors made by learners as applicable to the junior secondary phase (grades 7 – 9).

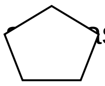
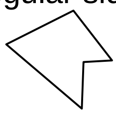
3.4.4.1 *Recognising closed figures*

The orientation or rotation of a figure confuses learners into not recognising the following figures.



Young learners often recognise the first figure as a square but in the second position, it is seen as a diamond and not as a square (French, 2004). French writes that even if the shape is turned in front of the learner some still do not recognise it as a square and would respond that it has “become a diamond”. The learners do not recognise the figure by its properties but by a mental representation (prototype) of what the figure looks like.

Johnstone-Wilder et al. (2007) similarly found that learners recognised a triangle when it was placed on its base in the horizontal position  but not in this  inverted position.

It is not only the orientation that seems to confuse the learners but also the lengths of the sides of the shapes. Learners fail to recognise shapes with irregular side lengths, for example is  as a pentagon but not 

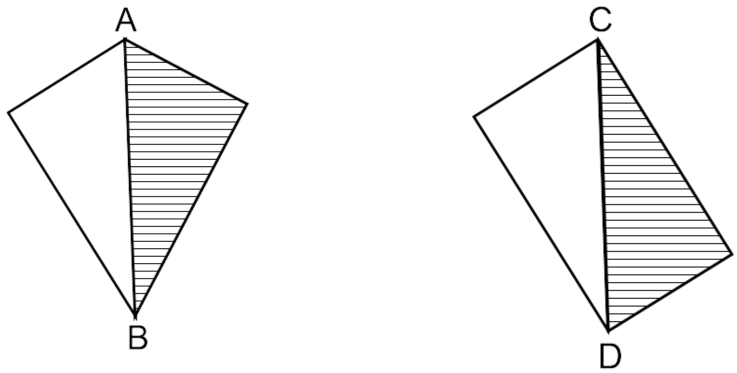
(Hansen, 2011; Luneta, 2015).

Kembitzky (2009) also noted that learners would stick to certain properties and fail to use or recognise others that could be crucial to the correct answer. They were also misled by the visual appearance of the figures. Therefore they would not see the angle as being perpendicular because it did not look perpendicular although the evidence pointed towards it being perpendicular, or they would think that triangles were congruent because they looked congruent although there was not enough evidence to support it (Kembitzky, 2009).

Misconceptions and errors could also be the result of learners memorising specific properties of shapes without understanding the concepts (Oberdorf & Taylor-Cox, 1999). Sometimes learners cannot distinguish between the shapes because they have put in too little effort to learn the properties (Ozerem, 2012).

3.4.4.2 Lines of symmetry

Learners often err when asked to determine whether a line drawn through a figure is



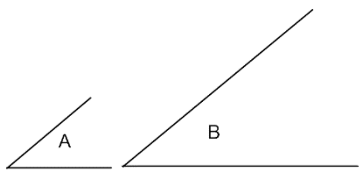
an axis of symmetry or whether a figure has an axis of symmetry e.g.

Learners tend to see e.g. both AB and CD in the sketch above as axes of symmetry. It is not obvious to them that they are not symmetrical and the teacher should be prepared to use folding, cutting or computer simulations to illustrate this to them (French, 2004).

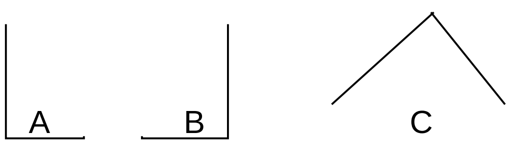
3.4.4.3 Angles

According to French (2004), the concept of angles seems to more difficult for the learners to grasp than the concept of length. Learners should, therefore, be introduced to the concept of angles by relating it from an early age.

A common error that learners make is to see angle B as bigger than angle A simply because the length of the "arms" are longer (French, 2004).



Right angles are also often confused if the orientation changes e.g. A and B may be seen as right angles and not C (Hansen, 2011) or A as a left angle and B as a right angle (French, 2004).

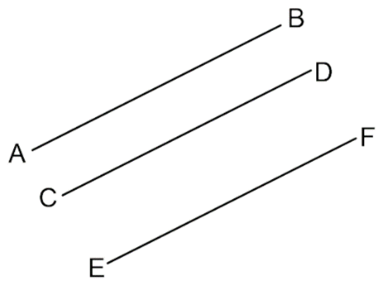


Similarly, perpendicular lines are not recognised if not in a standard position (Hansen, 2011).

Learners also tend to confuse terminology and symbols of angles such as supplementary and complimentary angles (Bhaskar, 2014; Padmavathy, 2015).

3.4.4.4 Parallel lines

Fielker (as cited in French (2004)) found that if three parallel lines were given, the learners knew that AB//CD and CD//EF but not AB//EF because CD was seen as being in the way.

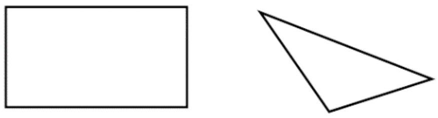


Another error encountered is that learners do not recognise lines of unequal length as being parallel (French, 2004; Hansen, 2011). Similarly to the problems experienced when the orientation of closed figures is changed, learners also seem to errantly only perceive lines as parallel when they are horizontally or vertically orientated but not when they are diagonal. This problem is found when they are not exposed to diagonal parallel lines during teaching (Johnstone-Wilder et al., 2007).

The angles that are formed when parallel lines are present seem to pose a problem as seen in the following studies. Kembitzky (2009) found that learners could not make deductions about e.g. supplementary angles when parallel lines were given. The learners could not relate angles to each other to prove that lines were parallel unless they were given angles that directly related to parallel lines, for example, alternate-, corresponding or co-interior angles. In the study by Ozerem (2012) learners simply could not detect the alternate and corresponding angles. Biber, Tuna, & Korkmaz (2013) found that misconceptions about parallel lines in a study of grade 8's also lead to errors in calculating the angles in questions with parallel lines.

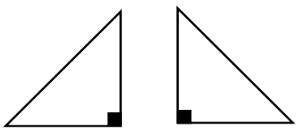
3.4.4.5 Terminology errors

French (2004) describes the use of incorrect terminology as a result of a confusion of mathematical terms with everyday language. For example the confusion of the word diagonal with the everyday meaning (line at an incline) of the word.



No diagonals Three diagonals

Another confusion with everyday language is right – and left angle (French, 2004; Johnstone-Wilder et al., 2007).



Right angle Left angle

Terminology errors seem to be a problem in many countries for various reasons. One reason may be that terms are not repeated enough times. Autrey et al. (2013) in the USA developed lessons to help students with the solution of geometry problems. At the end of a lesson, many students had forgotten the terminology. They found that more emphasis had to be placed on the terminology and the terms had to be repeated using various strategies in the lessons.

Another reason could be the poor knowledge of the teachers, as was found in a study with pre-service teachers by Bhaskar (2014) and Padmavathy (2015) in India and teachers (teaching in their second language English) by Ndlovu and Mji (2012) in South Africa. The teachers in Ndlovu and Mji's study lacked basic geometrical knowledge and vocabulary.

Sometimes teachers use the incorrect terminology in an effort to simplify things for learners (Oberdorf & Taylor-Cox, 1999; Olivier, 1992). It often has the opposite effect by causing confusion later on. Oberdorf and Taylor-Cox give an example of this by noting that teachers often refer to rectangles using words such as the long and tall shape and squares as the short and fat shape. They also teach that they are completely different shapes. Teachers should focus on the correct use of mathematics terminology from the start (Oberdorf & Taylor-Cox, 1999).

3.4.4.6 Hypotenuse and Pythagoras

The report of the grade 12 NSC examination of 2015 stated that one of the common errors was that learners could not identify the hypotenuse in a right-angled triangle. This then leads to an error in using Pythagoras to calculate the length of the hypotenuse (Department of Basic Education, 2015a). This incorrect identification of sides in a right-angled triangle was also highlighted as one of the problems in an Indian study by Bhaskar (2014) and in a USA study by Kembitzky (2009).

3.4.4.7 Calculation of ratios

Another common error found in the grade 12 NSC examination 2015 was that learners could not calculate the ratio of sides in two triangles. This resulted in them not being able to prove triangles were similar (Department of Basic Education, 2015a). Problems concerning ratios of geometric figures seem to stem from problems with the calculation of ratios in general (Hansen, 2011).

3.4.4.8 Hierarchy of quadrilaterals

Learners fail to see that a rectangle is a special type of parallelogram. This according to Van Hiele indicates that learners are not on level 3 (informal deduction) (Van Hiele, 1986). According to De Villiers (1994), learners often use partition classification to make sense of the quadrilaterals. This classification is seen as uneconomical as a parallelogram is a quadrilateral with opposite sides equal and parallel, opposite angles equal, diagonals of different length halving each other, but not perpendicularly. The hierarchical classification that a parallelogram is a quadrilateral with opposite sides parallel is seen as more economic and therefore preferable. De Villiers also cautions that the use of partition classification does not necessarily imply that the learner cannot reason on a higher level.

Teachers often teach learners initially that squares and rectangles are two completely different shapes. This causes confusion later on when squares are classified as a special case of a rectangle (Oberdorf & Taylor-Cox, 1999).

3.4.4.9 Carelessness

In a study by Ozerem (2012) using grade 7 learners, one-quarter of the learners ascribed their errors to hastiness and negligence. Careless errors whilst doing the calculations included forgetting to divide or multiply with certain values or not substituting correctly into formulas (Daymunde, 2010; Ozerem, 2012).

In a study using 9^{th} grade learners, Daymunde (2010) found that she could classify the careless error types into the following four categories: accidentally skipping the question, ran out of time, didn’t follow directions and misread the question.

3.4.4.10 Proof questions

Senk (1985) found a low level of achievement in the answering of proof type questions (Van Hiele, levels 3 or 4) and the conclusion drawn by Gutiérrez & Jaime (1998) is that this is due to the learners not functioning on a high enough level (Level 3) of geometric thought.

For many learners who battle with proof questions, the problems may start with the teachers. In a study of teachers who were improving their qualifications, Ndlovu and Mji (2012) realised that many teachers did not understand how to answer a geometry proof question. The common error that the teachers made in answering the proof questions were to merely give a list of properties of the figure (Ndlovu & Mji, 2012). Some of the teachers in the study supplied their own reasons when they could not find enough evidence in the sketch and others could not organise their thoughts in a logical manner. The problems experienced by the teachers become evident when one looks at the reports of the grade 12 NSC examination. Many of the grade 12 learners either did very poorly in the proof tests or did not even attempt to answer them. One of the most common mistakes was that they could not follow an argument. Their reasons for the statements were also very poorly answered especially when the converse of a theorem was used (Department of Basic Education, 2014b, 2015a). Omitting reasons or giving irrelevant reasons for answers is a common problem (Department of Basic Education, 2015a; Ndlovu & Mji, 2012; Ozerem, 2012).

An explanation for the teachers' challenges with proofs is that they are still at a low Van Hiele level as discussed in section 3.3. above. The link between the Van Hiele levels and the occurrence of common errors is discussed in the next section.

3.5 RELATIONSHIP BETWEEN ERRORS AND VAN HIELE LEVELS OF GEOMETRIC THOUGHT

Each Van Hiele level represents a characteristic thought pattern. If the learner has not mastered a certain level they will exhibit a certain way of thinking that can still be connected to the previous level of thought (Fuys, Geddes & Tischler, 1988). During a formal assessment, learners are expected to correctly answer questions which are deemed to be appropriate for the age and cognitive level of the learner (Department of Basic Education, 2011b). A learner who has not mastered the relevant cognitive level will make typical errors linked to their level of reasoning.

Teachers should know at which Van Hiele level the learners are functioning in order to effectively address the errors but also to avoid them. When teachers ask questions they expect different answers in the different grades as they expect the learners to be at a certain level (Van De Walle, 2004). If the learners do not answer according to the level on which the teacher regards them to be, the answer is seen as an error (Olivier, 1992). Therefore the errors might be an indication that the learners are not on the specific level of thought. Learners who are at a lower level will not understand an explanation at a higher level. They will then try and form connections between the new knowledge and their own level of thinking (Van Hiele, 1985) which could lead to misconceptions. It is, therefore, crucial that teachers should correctly sequence problems in a classroom so that the learners can progress through the Van Hiele levels (Van Hiele, 1986) and reach a stage where they can perform in the assessments. It is futile to address the sequencing of statements (level 3 and 4) in geometric riders if the learners still lack the basic concepts or terminology because they are still at the visualisation (level 1) or analysis (level 2) stage (Van Hiele, 1986).

If we consider the feedback and analysis of errors in the Department of Education Reports (Department of Basic Education, 2014b, 2015a) we will find that they indicate that progression is not achieved in the schooling system. One recurrent error is the incorrect reasoning that learners use in the proof questions which indicates that they have not passed through the second van Hiele level where they form connections between the terminology and the visual picture. Many of the learners still make errors because they recognise figures based on their appearance and not on their properties, because they are still at the first Van Hiele level (Visualisation) (Luneta, 2015).

Biber, Tuna, & Korkmaz (2013) did a study with grade 8's in Turkey. Their results indicated that one of the main causes for errors were that learners focussed on the visual appearance of figures and not on the properties thus exhibiting Van Hiele level 1 reasoning. They also noted that the learners preferred to learn by rote when they failed to understand the logic but unfortunately forgot the knowledge very quickly (Biber et al., 2013).

Although errors seem to be linked to the Van Hiele levels there also seems to be a link between achievement and errors. High achieving learners seemed to have better mental organisational skills than the lower achievers (Autrey et al., 2013). They, therefore, recognised relationships in geometrical figures more quickly and progressed more quickly.

Van Hiele regarded the ability to recognise the hierarchy of quadrilaterals, namely. that a rectangle is a special parallelogram, to be ascribed to level 3 (informal deductive) thinking (Van De Walle, 2004). However, De Villiers asked whether knowing that a learner can classify quadrilaterals according to the hierarchy implies that they are at a higher level than a learner who uses a partition definition of a quadrilateral (De Villiers, 1994).

Ndlovu & Mji (2012) conducted a study with in-service teachers who were taking a course to improve their qualifications. They identified various errors in answers to proof questions and linked them to specific Van Hiele levels:

Firstly, the incorrect use of terminology and poor basic geometry knowledge was linked to Van Hiele Level 1 (Visualisation). The types of errors also included false reasons, citing the theorem to be proven as a reason, referring to, for example, opposite sides of a triangle as being parallel.

Secondly, supplying reasons that may be true but in a completely inappropriate order in the proof was linked to Van Hiele level 2 (Analysis). Also, they regarded the error of simply listing the properties of the figure to belong to level 2.

Thirdly, teachers that could follow an argument in a proof but supplied unnecessary information were placed at Van Hiele level 3. They stated that these teachers incorrectly thought that more information would lead to higher scores. Thus their answers were not "efficient" and the arguments were difficult to follow. Also included in this group were teachers who knew the correct reasons but used the incorrect sequencing in the proof.

3.6 CHAPTER SUMMARY

The poor performance of learners of mathematics in geometry detected in the international and national studies over a number of years is an area of concern. There are many factors that influence this and that may influence the performance and level of reasoning of learners in geometry but the factors are not discussed in detail because they are not the focus of the study.

Errors play a significant role in the poor performance and can be used as a diagnostic tool for uncovering misconceptions in geometry. If errors are analysed effectively they can be used to make the teaching and learning of geometry more efficient. A positive attitude towards errors can help learners use the errors to uncover and correct the misconceptions.

Finally, the errors could also be linked to the Van Hiele levels of geometric thought of the learners.

CHAPTER 4

METHODOLOGY

4.1 INTRODUCTION

This research stems from problems encountered during 25 years of teaching geometry at secondary school level. The researcher also facilitated mathematics for 10 years to part-time teaching students. The schools where the researcher was a teacher were suburban schools but the part-time teaching students were mostly from township and rural schools in previously disadvantaged communities in Mpumalanga. In these schools, the researcher found that learners still battled with geometry when they reached the FET (grade10-12) phase. Errors that the learners made in the junior phase seemed to resurface again when learners were confronted with the more difficult geometry riders in the senior grades. Many of the part-time teaching students, some of whom had been teachers for many years, still made the same errors as the learners.

The challenges that the learners and students experienced with geometry encouraged the researcher to start this study.

A mixed method research design was used to answer the following questions:

- What is the level of geometric thought of grade 9 learners according to the Van Hiele theory?
- What are the typical errors that the grade 9 learners make in geometry?
- Is there a link between the Van Hiele level of geometric thought and the typical errors that the grade 9 learners make?

The methods used to determine the performance of the grade 9 learners, to uncover some of the typical errors and the analysis of the different types of data generated will now be discussed.

4.2 RESEARCH DESIGN

The paragraphs below explain and motivate the choice of the mixed method design and methodology.

4.2.1 Rationale for using a mixed method design

Mixed method designs combine quantitative and qualitative methods and approaches into one study. This design has gained popularity over the last few decades (Creswell & Plano Clark, 2007; Driscoll, Salib, & Rupert, 2007; Greene, Caracelli, & Graham, 1989; Johnson & Onwuegbuzie, 2004). A quantitative approach is deductive in nature whereas a qualitative approach is inductive (De Vos, Strydom, Fouché, & Delport, 2007). Both quantitative and qualitative methods of inquiry were necessary to answer the different research questions above and therefore a mixed method design was chosen.

A further motivation was that a mixed method design has been used in similar educational studies for example: Daymunde (2010) examined the effect of test error analysis; Van Putten (2008) examined the Van Hiele levels of pre-service teachers; Stols, Mji, & Wessels (2008) examined the influence of computer programmes on the teachers of geometry, Luneta (2015) analysed question papers and Kembitzky (2009) addressed misconceptions in geometry.

4.2.2 Quantitative approach

A quantitative approach using the Van Hiele theory of geometric thought was used to direct the collection and analysis of data on the performance of the learners (Creswell and Plano Clark, 2007). The learners were tested using multiple-choice questions and open-ended questions in a test that was designed using the Van Hiele theory. Using the results of the test it was possible to determine the predominant level of geometric thought in the sections assessed in the test. The number of correct responses was recorded and analysed using statistical methods.

4.2.3 Qualitative approach

In a qualitative approach, the data is collected and analysed to find a pattern and then conclusions are drawn from the emerging patterns formed (Creswell and Plano Clark, 2007). This approach was used to answer the question on the typical errors that learners make. The qualitative data was collected using both the test and semi-structured interviews. The error patterns in the test were coded. The semi-structured interviews were used for triangulation to strengthen the validity of the results (Greene et al., 1989) and to gather further information on selected typical errors.

The qualitative data was interpreted and linked to the quantitative data during the final analysis to answer the research question on finding links between the Van Hiele levels and typical errors.

The diagram below summarises the procedures used in this study. The diagram is an adapted version of the diagram by Creswell and Plano Clark (2007:46). In the first phase, the quantitative data was collected. In the second phase, qualitative data was collected and the final phase of the study was the linking of the two data sets.

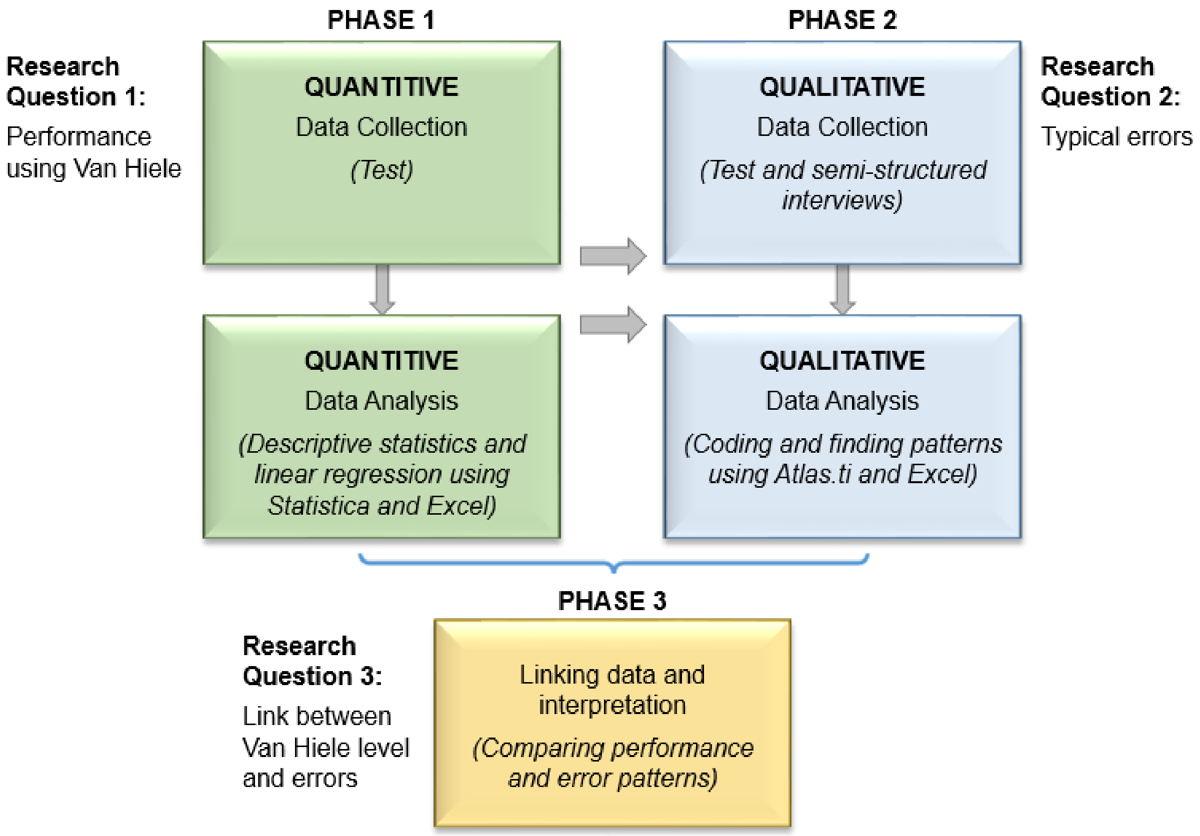


Figure 4.1: Illustration of the phases in the research and how the research questions link to the research design (Researchers own design).

4.3 RESEARCH METHODS

4.3.1 Sampling in this study

This section discusses the choice of schools, choice of grades and sampling size for the quantitative and qualitative phases.

4.3.1.1 Choice of schools

Schools in different areas and school categories (quintiles) have different challenges and also different levels of performance. Therefore, in order to get a comprehensive picture of the typical performance and errors associated with the Van Hiele levels of geometric thought, two schools from different quintiles were purposively selected.

The quintiles are awarded by the Department of Education according to the socio-economic status of the school community. The lowest, quintile 1, also known as the non-fee paying schools are situated in the lowest socio- economic category and the highest, quintile 5, also known as the fee-paying schools are situated in a higher socio-economic community.

Two secondary South African schools were selected: One was a quintile 3 township school and the other a quintile 5 suburban school. The township school was in Missionvale, Port Elizabeth and the suburban school was in Walmer, Port Elizabeth.

The two schools are different with regards to quintiles as well as the home language of the learners. The home language of most of the learners in the quintile 3 school was not the same as their language of teaching and learning. In the quintile 5 school , most of the learners were taught in their home language. The school in quintile 3 and 5 were selected to represent typical schools in those quintiles.

4.3.1.2 Choice of grade

Many of the errors found in the senior phase relate to basic terminology and concepts that should have been taught in the junior secondary phase. It is, therefore, important to identify the errors the learners make at the end of grade 9, which is the final year in the GET (General Education and Training) or junior secondary phase.

In addition, at the end of the grade 9 year the learners select their subjects for grades 10 – 12. The grade 9 mathematics scores are often the determining factor as to whether mathematics is taken in the senior phase.

4.3.1.3 Sample size for quantitative phase

A total number of 194 learners wrote the tests. The township school contributed 123 and the suburban school 71 grade 9 learners who were purposively selected for testing.

4.3.1.4 Sample size for qualitative phase

All of the learners' multiple-choice sections of the tests (123 + 71 = 194 scripts) were analysed for error patterns.

Stratified sampling (Cohen, Manion & Morrison, 2007) was used to select 60 open-ended question sections of the tests to code. The answer sheets were grouped into level 0 to 3 according to the highest level in which they acquired >60% for the average of the two sections of the test. Learners who did not acquire >60% at any level were placed on level 0. No learners in the township school could be selected on level 3 because none acquired >60% in that level and only 3 acquired an average of more than 60% on level 2. In the suburban school, only 4 learners acquired >60% on level 3. The numbers of learners selected per level are summarised in the table below.

Table 4.1: The number of learners' open-ended question papers that were selected per school and per level

School	Number of learners selected per level				
	Level 0	Level 1	Level 2	Level 3	Total
Township	13	13	3	0	28
Suburban	9	9	9	4	32
Total	22	22	12	4	60

For the semi-structured interviews, stratified sampling (Cohen et al., 2007) was used to select 6 learners – three from each school. The number of learners per level is given in the table below.

Table 4.2: The number of learners that were interviewed per school and per level

School	Number of learners interviewed per level				
	Level 0	Level 1	Level 2	Level 3	Total
Township	0	2	1	0	3
Suburban	1	1	0	1	3
Total	1	3	1	1	6

4.3.2 Data collection approach and instruments

A test consisting of two sections: multiple-choice and open-ended questions, was used after which semi-structured interviews were conducted. Originally the test was set up as two separate tests but was combined to complement each other and for greater validity. The test served a dual purpose: firstly to determine the Van Hiele level of reasoning of the learners and secondly, to identify typical errors. The content of the test was aligned with the Van Hiele levels of thought as well as with the contents of the grade 9 curriculum. The purpose of the semi-structured interviews was to gain more insight into the errors.

4.3.2.1 Data collection action plan

The tests were scheduled for the end of September after the schools had completed the geometry section of the mathematics curriculum.

At the township school, the researcher met with the headmaster twice to explain the nature of the study and to arrange for the grade 9s to take part in the study. The grade 9 classes wrote the test in the school hall. The date was set just one week before their quarterly test cycle started. Arrangements were made with an invigilator who lectured at a college and had experience with invigilation to help on the day of the test. She was briefed beforehand. The school also provided two teachers to help with the invigilation. The additional support of the invigilators enabled the researcher to ensure that the process ran smoothly.

In the suburban school, the researcher also met with the headmaster to gain permission and explain the nature of the study. Thereafter she was referred to the deputy headmaster and head of the mathematics department to arrange a date. The learners wrote the test during the quarterly test cycle. The learners wrote in two classrooms and two teachers were asked by the deputy headmaster to help with the invigilation so that the researcher was free to go back and forth between the classes to make sure that the process ran smoothly.

4.3.2.2 Test design

The setting of a test in alignment with the Van Hiele levels is not a simple task (Battista, 2007; Usiskin, 1982). Fuys, Geddes and Tischler (1988) broadened the descriptions of each Van Hiele level in their research project to make it simpler to place the questions within a Van Hiele level. Their descriptions, as well as information gained from the descriptions in Burger and Shaughnessy (1986), Mayberry (1983), Usiskin, (1982) and Van Hiele (1986), were used to set up the test. Even after carefully setting up questions to align with the Van Hiele levels, there are still other factors such as the level of difficulty of each question (Wilson, 1990), time allocation, number of questions and content that should be taken into account.

The language and content of the tests that were set up for use in other countries are somewhat different to the language and content used in South Africa (Van Putten, 2008) e.g. internationally congruent is used to indicate equal angles, sides and figures whereas in South Africa it is only used to indicate equal figures. Therefore it was necessary for the process of setting up a test, to contextualise the questions so that the grade 9 learners in South Africa would understand.

Briggs et al., (2006) suggest that a combination of tests should be used to obtain more reliable results and still be time effective. Therefore two tests were set up initially: a multiple-choice and an open-ended question test.

In order to ensure that the test set met the requirements of a Van Hiele test design, the researcher e-mailed Professor Michael de Villiers and Professor Zalman Usiskin who both have vast experience in the field of Van Hiele and geometry. Professor Michael de Villiers advised me to combine the two tests into one test with two sections.

The appropriateness of the test for grade 9s was evaluated by a teacher who had experience in teaching the grade 9s.

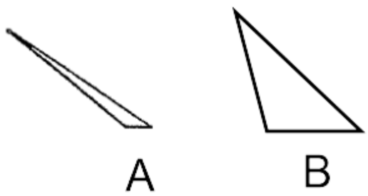
The setting of the multiple-choice and open-ended sections is discussed below.

Multiple-choice section

The multiple-choice test format for this study was designed using the format of the test used by Usiskin in 1980 – 1981 (Usiskin, 1982).

After piloting the test with two learners, a draft of the multiple-choice test section was sent to Professor Zalman Usiskin. Usiskin commented on the type of questions and that the some of the questions may not test what was intended. His comments made the researcher realise how a change to a geometrical sketch can change the difficulty of the question considerably.

For example, B is more easily discernable as a triangle than A



Usiskin also noted that Van Hiele stated that algorithms and formulas should not be used in the test. Further, he advised that no fixed levels can be assigned to learners. His advice and comments were used to improve the questions.

The research supervisor had concerns with the time allotted for this section of the test. He thought that 40 minutes might not be enough and this issue was carefully monitored during the pilot test. The multiple-choice section was piloted with 57 learners. The learners in the pilot test completed the test well within the time limit and it was decided to leave the time unchanged. After the pilot study, the final changes were made to the test and it was printed at the university printers.

The initial multiple-choice section of the test had 30 questions on levels 1 to 3 because the learners in grade 9 are expected to be at that level. However, two questions at level 4 were included to make provision for learners who may be further ahead than expected. Thus the total number of multiple-choice questions was 32.

The multiple-choice section had questions on the following content: quadrilaterals, angles on straight lines, parallel lines, triangles and congruency (see table 3 below).

Each question had 5 choices (a-e). The answers were selected as follows: one correct answer for the expected level, one "almost" correct answer or correct on a lower level, three incorrect answers (see Appendix A for the test). Table 4.3 below provides an analysis of the questions based on Van Hiele levels.

Table 4.3: Analysis of Test based on van Hiele levels

Content of question	Number of questions on each level				
	Level 1	Level 2	Level 3	Level 4	Total
Quadrilaterals	3	5	5	1	14
Triangles and congruency	1	3	1	0	8
	0	1	2	0	
Parallel lines and Angles	1	0	2	0	8
	3	2	0	0	
Total	8	11	10+1(22)	1+1(30)	32

Open-ended question section

The open-ended question section of the test consisted of 22 questions. This section of the test was an adapted version of the test used by Smith and De Villiers in 1987 as part of the RUMEUS (Research Unit of Mathematics Education University of Stellenbosch) study group. The open-ended questions consisted of the following:

- one word answers and then supplying reasons for their answers
- yes/no questions and supplying reasons, and
- short proof questions on parallel lines, similarity and congruency.

After the open-ended questions were checked by the same teacher who checked the multiple-choice section for suitability for grade 9, it was sent to Professor Michael de Villiers, who was part of the original group that set up the test at the University of Stellenbosch, for his comments. He gave very good advice on the levels of the answers which was then used in combination with the guidelines compiled by Fuys et al. (1988), Burger & Shaughnessy (1986) and Usiskin (1982) to compile a memorandum.

The open-ended questions were piloted with two learners to ensure language and time allotted was suitable for grade 9. The content of the questions is indicated below in table 4.4.

Table 4.4: Analysis of the content and score in the open-ended question section

Content of questions	Score	Total number of questions
Quadrilaterals	22	6 + 1(combination with congruency)
Triangles and congruency	24	9
	22	
Parallel lines and Angles	7	7
	6	
Total	60	22

4.3.2.3 Interviews

The semi-structured interviews were used to probe the errors picked up during the marking of the tests. Merriam (1998) explains that semi-structured interviews are guided by a list of questions but the order or wording of the questions is not exact. A few of the common errors were selected after the coding of the question papers and a list of questions was set up that were adapted as the interviews progressed (see Appendix B). The interviews took 25–35 minutes to complete. The interview times were set up to cause the least disruption to the school programme.

Cut-outs of shapes of closed figures (3-6 sided figures) were used together with sticks and a question where the learner had to provide proof, from the open-ended section of the test. The sorting activities of the figures used in the interviews were influenced by the methods used by Fuys, Geddes and Tischler (Fuys et al., 1988).

The interview method was piloted and the video camera tested using two learners one week before the interviews started. The video recording did not focus on the learners' faces but focused on the table and the shapes they manipulated and recorded their verbal responses to the questions.

The learners' explanations during the interviews were used to evaluate the results of selected questions in the tests and to confirm the validity of the errors that have an influence on the level of geometric knowledge of the learners.

4.4 DATA ANALYSIS AND PROCEDURES

Data analysis is the process by which data is disassembled, sorted and labelled in order to discover useful information on which conclusions and decisions can be based (Friese, 2012).

In this study, two types of data, quantitative and qualitative, were generated and they were analysed differently. The analysis of the data is described below under the headings of the instruments used to collect the data.

4.4.1 Analysis of the multiple-choice question data

The multiple-choice sections were marked and results analysed according to the percentage of the questions answered correctly on each Van Hiele level. The results were entered into an Excel spreadsheet and then entered into Statistica by a statistician at Nelson Mandela Metropolitan University.

The multiple-choice questions were analysed twice. Firstly, the multiple-choice questions were entered on a spreadsheet using a 0 (incorrect) and 1 (correct). The percentage of attainment on each Van Hiele level was calculated.

The percentage of learners who attained more than 60% on a Van Hiele level was calculated and compared with the open-ended section of the test after which a comparison between the two schools was made. After the open-ended section was marked an average percentage for the two sections on each level was calculated.

Secondly, all the questions in the multiple-choice section were used in the analysis of errors. All the answers given by the learners were re-entered into another Excel datasheet but this time, the specific response given by the learners e.g. a, b, c, d or e was entered. The learners were grouped according to the highest level in which the learners had more than 60% (level 0 to 3). Level 0 was used for the learners who did not get >60% in any level. A percentage for each choice in the questions was calculated. The occurrence of certain incorrect choices helped in finding the typical errors made by learners.

The error patterns that emerged from the multiple-choice papers were then compared with the patterns from the open–ended question tests. Finally, these errors were linked to the Van Hiele levels of the learners.

4.4.2 Analysis of the open-ended question data

The open-ended question section of the answer sheets was used twice in the analysis process. Firstly, the questions were marked and graded according to the performance on the Van Hiele levels of geometric thought. The results were entered into Excel and Statistica for comparison with the results from the multiple-choice section and to calculate an average of the two sections. Secondly, using stratified sampling a number of answer sheets were selected to be scanned and coded for error patterns using Atlas ti.

4.4.2.1 Quantitative analysis of the open-ended questions

The tests were marked using a memorandum. After the first papers were marked the memorandum was checked and some comments were added to ensure consistent marking.

It was difficult to decide on how to grade the yes/no questions because many learners just marked the yes or no and did not supply a reason. Therefore, after collaborating with the research supervisor it was decided to disregard the yes/ no answers unless the learners supplied a reason. The tests were then remarked and a percentage allocated to each level on each answer sheet.

The average percentages for the tests and the scores for each question were entered into an Excel spreadsheet and analysed. The results of the open-ended questions were compared with the results from the multiple-choice questions. The results of the two schools were also compared. Descriptive statistics were used to describe and analyse the data.

4.4.2.2 Qualitative analysis of the open-ended questions

The analysis of qualitative data can be a challenge (Carcary, 2011; Fuys et al., 1988) and therefore the Computer Aided Qualitative Data Analysis Software (CAQDAS), Atlas.ti, was used to manage and document the process of analysis in this study more effectively.

After the marking and grading of the papers for the quantitative process, the open-ended question sections of 60 answer sheets were selected and scanned and inserted as primary documents into Atlas.ti. The scanned tests were arranged in primary document families according to the highest level in which they had more than 60%.

The errors in the content of the answers were coded deductively and the codes arranged into code families. The word "families" is a term used in Atlas.ti to describe groups of codes or primary documents (Friese, 2012).

Krefting (1991) suggests using coding and then recoding after a time lapse to increase the dependability of the study. Therefore after a time lapse of three weeks, the answer sheets were studied again and errors were re-coded. The results were compared and thereafter some codes were combined, other codes were changed and a few were added. A complete list of codes is given in Appendix E.

The frequency of the errors and the different types of errors were used to describe the typical errors made by the grade 9 learners.

4.4.3 Analysis of the interview data

The analysis of the interview data was the final stage before the data from the quantitative and qualitative analysis were linked.

The videotapes of the interviews were added to the primary documents in Atlas Ti. The use of CAQDAS makes it less tedious to code the interview data (Carcary, 2009). Atlas.ti was used to create a code book. Codes were added similar to the codes used to categorise the errors in the open-ended questions.

The central themes were compared to the results of the test analysis to answer the question on the link between the errors and the Van Hiele levels.

4.4.4 Final analysis of data to answer the research question

The quantitative data on Van Hiele levels and the qualitative data on the error analysis were merged to answer the research question on how the Van Hiele levels link to the errors made by the learners.

The merging consisted of combining the qualitative data in the form of texts with the quantitative data in the form of numeric information. The qualitative data was transformed by counting the number of occurrences of each error type using Atlas.ti. The frequency of error patterns was then matched to the Van Hiele level of geometric thought of the learners.

Creswell and Plano Clark (2007) mention the issue of overinflated counts. Overinflated counts are explained as a higher number of occurrences of a certain idea due to respondents repeating themselves. In this study, the same error often occurred more than once in a question. Overinflating of counts was reduced by counting a certain error in a specific question only once.

4.5 METHODOLOGICAL ACCOUNTING OF THE STUDY AS A WHOLE

The terms validity, reliability and objectivity are normally used to describe the worth of quantitative research whereas trustworthiness and authenticity are used for qualitative research (Krefting, 1991). The purpose of this section is to describe the measures that were taken to ensure the validity, reliability and trustworthiness of this study.

4.5.1 Validity

According to Carcary (2009), validity is a measure of how well the research measures what it sets out to investigate.

The validity of a study hinges on the validity of the data generation and the interpretation of the data. The measures taken to ensure greater validity in this study are described below.

Validity of the data generation

This measures how suited the instrument is for collecting data to answer the research questions (Carcary, 2009), as well as the quality of the data collection process (Creswell and Plano Clark, 2007).

The validity of the research instruments in this study was sought by using other experts to evaluate the tests. The test was also based on tests that had been used previously in other studies, as advised by Creswell and Plano Clark (2007).

The experimenter effect (Thomas, 2013) was minimal during the test as the tests were written under clinical circumstances (Test environment – no chatting, each at own desk, no copying, strict invigilation). Although care was taken to ensure the correct interview procedures, the experimenter effect might have had an influence on the learners during the interviews.

Validity of the interpretation

The guidelines set out by Carcary (2009), to ensure greater validity of the interpretation, was used to set up the following guidelines for this research:

- The test answer sheets were all scored using a memorandum. Thereafter a number of test answer sheets were moderated in order to ensure the quality of the scoring.
- The errors in the open-ended questions were coded and recoded to ensure greater validity.
- The data was recorded on a spreadsheet and spot checks were conducted to ensure that the mistakes were minimised during data capturing.

- The results were analysed systematically to make sure that nothing was left out. The quantitative analysis of the data was done by the statistician.
- The interpretations of the results were regularly compared to interpretations found in similar studies in literature to validate the interpretations. The findings were shared with teachers from the two schools involved, ensuring that the interpretations were accurate.

4.5.2 Reliability

Reliability gives an indication of how well the method would perform if the study was repeated (Carcary, 2009) and how consistent the results would be when the instrument is repeated on different occasions (Thomas, 2013).

The setting of the test was important but was not the focus of this study. The test was therefore not retested to determine the reliability but the test was piloted and very similar results were obtained which could indicate a relative sense of reliability. The results of the multiple-choice section were also compared to the results of the open-ended section of the test.

4.5.3 Trustworthiness of the data

Although it is necessary to talk about the validity and reliability of the study the two terms are more suited towards a quantitative study. In a qualitative study, the terms trustworthiness and authenticity are more often used.

It is very difficult to get the same results in qualitative studies even when studies are repeated using very similar circumstances (Carcary, 2009) simply because “human behaviour is never static” (Merriam, 1991). Therefore the following is recommended in order to improve a study’s trustworthiness:

Investigator’s and study’s position

Merriam (1998) suggests that the researcher should explain the assumptions and theory behind the investigation, the position of the researcher, the sampling method and a description of the sample and the context of the study.

Van Hiele's theory was used in this study as described in the theoretical framework in chapter 2. As a mathematics teacher, the researcher did not teach at the two schools in this study and was, therefore able to retain the position of a researcher. A description of the sample, as well as the data collection methods, are described in section 4.3 of this chapter.

Triangulation

A variety of strategies should be used during data collection to ensure greater trustworthiness of the data (Krefting, 1991) because the use of several methods minimises or cancels out the biases of one method (Seale, 1999).

Multiple methods were used in this study. A test combining two methods namely, multiple-choice questions and open-ended questions were used together with the semi-structured interviews.

Audit trail

A researcher should give a detailed description of how the research was carried out in order for another investigator to be able to carry out a similar study (Carcary, 2009). In other words, the researcher must clearly indicate the way decisions were made in all the phases of the study so that another researcher will be able to follow the process (Carcary, 2009).

4.6 DELIMITATION OF THE STUDY AREA

The learners' Van Hiele levels and the typical errors that they made were the phenomena studied in the setting of two schools.

Although the background factors have a significant impact on the performance and errors, these were not investigated for the scope of this study. For example, the socio-economic status of parents and the influence of technology on the teaching of geometry were not investigated and tested. The gender and age of the learners were also not included in the investigation.

The influence of the exclusion of geometry on the grade 12 learners' from 2008 to 2013 was also not investigated.

The setting of the test was important but was not the focus of this study. The study was set up as an instrument to assess the level of the learners and was therefore not retested at a later stage or in another setting to determine the reliability.

The focus of the study was on euclidean and not analytical geometry.

4.7 ETHICS STATEMENT

Ethics is a set of moral principles which offers rules and behavioural expectations about the correct conduct towards experimental subjects and respondents (De Vos et al., 2007).

Consent for the study was obtained at three levels:

1. The university. Ethics clearance for the study was gained from the NMMU ethics committee (see Appendix D).
2. The authorities in charge of the schools. Written consent for the study was obtained from the Department of Education (see Appendix C) and the school principals before commencing with data collection.
3. The learners and their parents. The purpose and aims of the research were explained in writing to the parents and learners in the selected schools. The aims and procedures were explained again orally to the learners before they wrote the test. They were assured of the confidentiality of the tests and interviews. The respondents were informed that they had the right to withdraw from the study at any time.

None of the information gained would be used to harm or discredit any of the learners, teachers or schools. The final results of the study will be made known to the schools so that they can use the findings to improve the situation at the school.

4.8 CHAPTER SUMMARY

This chapter described the mixed method research design in order to explain and motivate the choice of the mixed method design and methodology. Both qualitative and quantitative methodologies were needed to answer the research questions on performance and errors. The research was conducted in 3 phases. Firstly a test containing multiple-choice and open-ended questions was set using the advice of professors Zalman Usiskin and Michael de Villiers. Here I want to acknowledge the input of these leading experts in the field of Van Hiele testing. The test was also evaluated for suitability for grade 9s by an experienced teacher. The test was piloted and then written at two schools involving a total of 194 learners. During this phase, the tests were analysed and marked to determine the Van Hiele level of thought of the learners. Secondly, the tests were also analysed and coded to uncover the typical errors that the grade 9 learners make. Interviews were conducted with 6 learners in this phase. In the final phase, the two sets of data were merged to find links between the levels of geometric thought and the typical geometric errors of the grade 9 learners.

The results and discussion of the data collected in each phase to answer the research questions are given in the next three chapters.

CHAPTER 5

PERFORMANCE ON VAN HIELE LEVELS: RESULTS AND DISCUSSION

5.1 INTRODUCTION

This mixed method study was done in three phases. The collection of the data to answer the first two research questions was done during the first two phases. The third phase combined the data collected in the first two phases to answer the third question.

The research questions were:

- What is the level of geometric thought of grade 9 learners according to the Van Hiele theory?
- What are the typical errors that the grade 9 learners make in geometry?
- Is there a link between the Van Hiele level of geometric thought and the typical errors that the grade 9 learners make?

This chapter will discuss the results of the data collected during the first phase to answer the first research question. The diagram below (*cf*. chapter 4) summarises the link between the phases of data collection and the research questions.

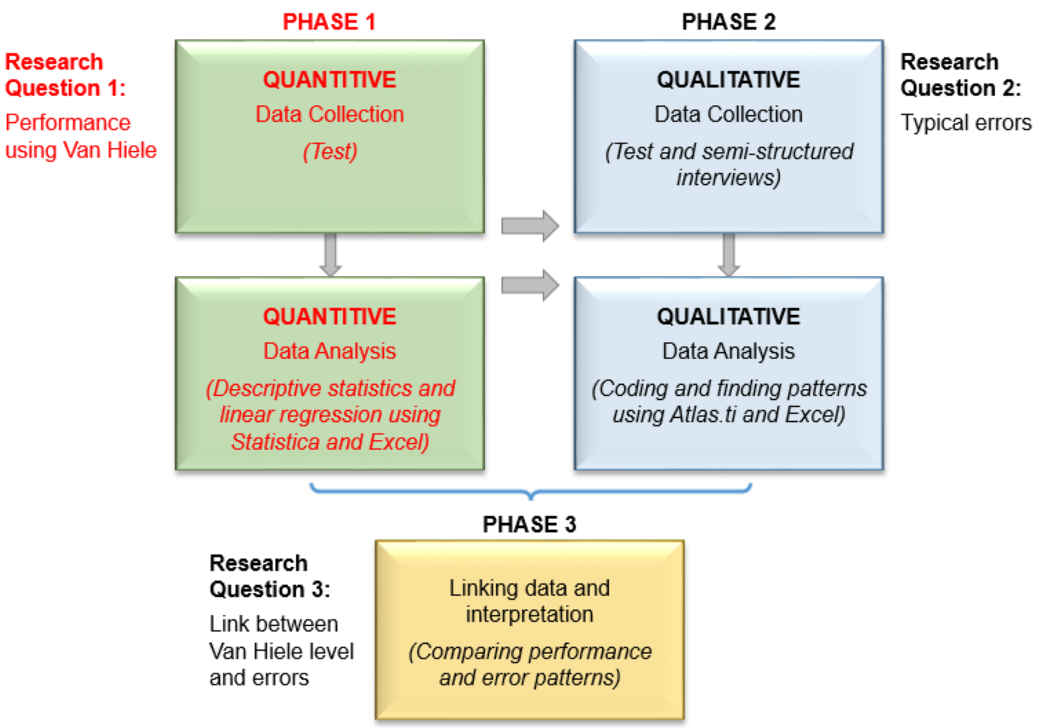


Figure 5.1: Illustration of where phase 1 fits into the research design.

Throughout the next chapters the following abbreviations will be used:

- L 0 represents the level that the learners are at if they achieved < 60% on L1
- L 1, L 2 and L 3 represent the first to third Van Hiele levels of geometric thought

5.2 PERFORMANCE ACCORDING TO VAN HIELE

In the first phase, the quantitative data was collected and analysed. The learners' Van Hiele level of geometric thought was assessed with the aid of a test consisting of two sections: a multiple-choice and an open-ended question section. The tests were marked and a percentage was calculated for each section.

The learners' results were entered into a spreadsheet and then analysed in the following way:

- the average percentages per level for the whole group to gain an indication of the overall performance of the group (see Table 5.1) and per Van Hiele level per school to gain an indication of each school's performance (see Table 5.2).
- the percentage of learners achieving more than 60% on each Van Hiele level (see Figure 5.2)

5.2.1 Performance of the group of grade 9 learners

The table below displays a definite descending trend in the averages from level 1 to level 3 as was suggested by the literature that was studied.

Table 5.1: The descriptive statistics of the grade 9 learners

		Avg L1	Avg L 2	Avg L 3
N	Valid	194	194	194
	Missing	0	0	0
Mean		51.64	36.47	14.34
median		49.75	33.75	13.50
Std Deviation		18.49	17.84	11.17
Minimum		17.0	2.0	0.0
Maximum		100.0	93.0	67.5
Percentiles	25	38.13	23.5	8.13
	50	49.75	33.75	13.5
	75	62.88	45.88	18

The very low average of 14.34% on the third and the low average of 36.47% on the second level imply that most of the learners are not proficient at those levels. This is a concern because learners are expected to perform much better in grade 9.

However, more concerning are the results of level 1. The table shows that at the 75th percentile on level 1, the average percentage is 62.88%, therefore only little more than one quarter of the learners have more than 60% and can be regarded as proficient on this level. One would expect all learners to obtain proficiency in this level as this level is already introduced to the learners in the foundation phase (grade R to 3) at school and the learners should be able to visually identify, name, compare and operate comfortably with geometric figures.

To conclude, the performance of the grade 9 learners in this study indicates that most of them are still at a very low level of geometric reasoning and may not be ready for the geometric challenges of the FET phase.

5.2.2 Comparison of the schools

The schools (suburban and township) in the two quintiles face different challenges and to get an indication of the performances of the two schools the average percentages were compared. The results are given below.

Table 5.2: Difference in the performance of the township and suburban schools in the multiple-choice and open-ended questions per level of geometric thought

	Mean	Mean	t-value	df	p	Valid N	Valid N	Std.Dev.	Std.Dev.	Cohen's d	Practical significance
	Q3	**Q5**				**Q3**	**Q5**	**Q3**	**Q5**		
MC L1	44.97	57.93	-4.82	192	0.0000	123	71	17.21	19.45	0.72	Medium
MC L2	35.49	53.75	-7.34	192	0.0000	123	71	14.84	19.48	1.09	Large
MC L3	19.23	27.92	-4.07	192	0.0001	123	71	12.06	17.54	0.61	Medium
OE L1	45.17	68.06	-8.28	192	0.0000	123	71	17.99	19.47	1.23	Large
OE L2	24.12	42.25	-6.48	192	0.0000	123	71	16.81	21.79	0.97	Large
OE L3	4.24	9.80	-3.27	192	0.0013	123	71	7.35	16.23	0.49	Small

MC indicates the multiple-choice questions and OE indicates the open-ended questions. Q3 represents the township school and Q5 the suburban school

Cohen's d is an effect size measure that indicates the practical significance of the difference in two sets of results with a reading greater than 0.8 indicating a significantly large difference. From the table above it is evident that there is a large difference in the results of the two schools with respect to the level 2 questions in the section and in both the level 1 and 2 questions in the open-ended section. From this, one might conclude that the performance in the township school is significantly lower than the suburban school on the first and second level.

The learners had to write down reasons for their answers in the open-ended question section. The significantly larger differences are in the open-ended question paper could be the result of a language problem. This aspect is discussed in chapter 8.

5.2.3 Percentage of learners achieving more than 60%

Literature indicates that is not possible to assign a fixed Van Hiele level to a learner (Clements and Battista, 1992; Mayberry, 1983) as learners may not be on the same level for each section of the work and the levels may change when a new section or topic is introduced. However, since an indication of the predominant level of geometric thought was necessary for this study, an average percentage of more than 60% was regarded as indicative of proficiency at that level.

It was therefore decided to use the criteria of the highest level in which the learners had more than 60% as an indication of the learners' predominant level at the time when the tests were conducted. An average of more than 60% on a level implied that the learner should be able to reason comfortably on that level although they might still be prone to mistakes (Gutiérrez, Jaime & Fortuny, 1991). Learners who do not meet the requirements of the lowest level, level 1, may be placed at level 0 (Clements, 2003). Therefore all learners who did not achieve more than 60% on any level were ascribed to level 0.

This was done with due consideration that the percentages could differ if the content of the test was changed. The researcher calculated the percentages by using the average score of the multiple-choice and open-ended section of the test.

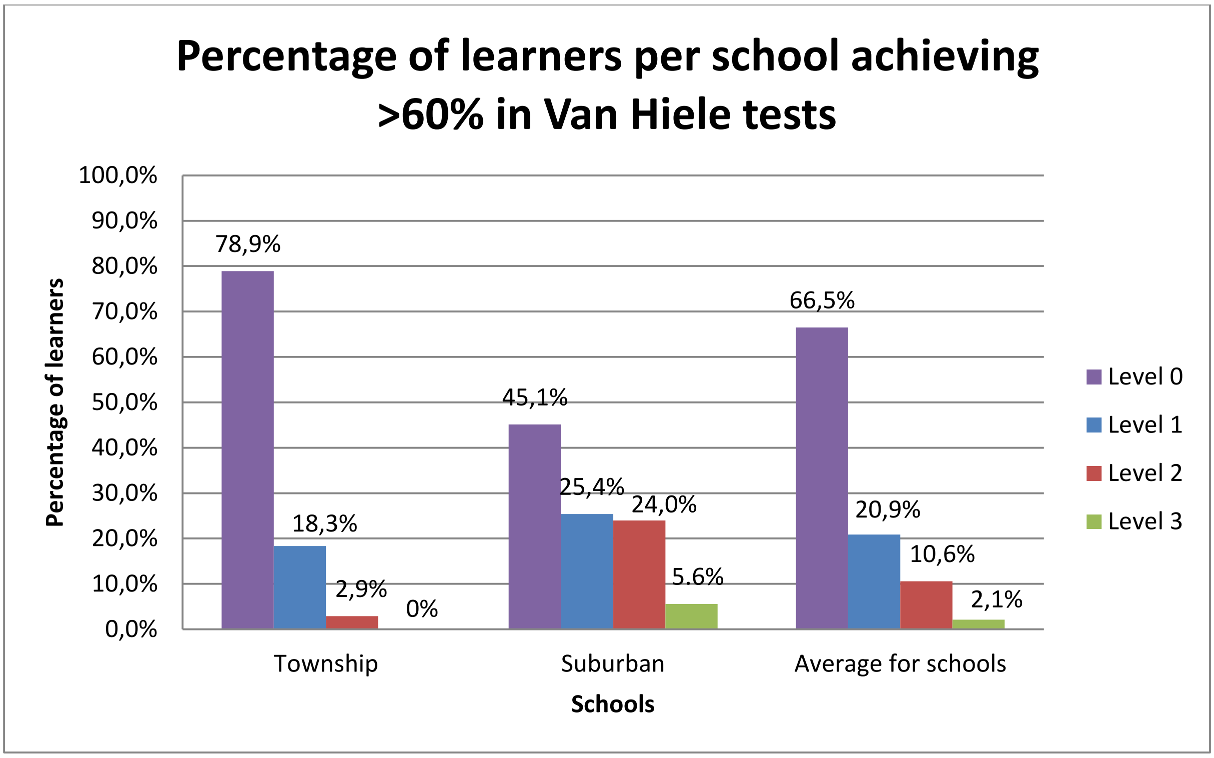


Figure 5.2: The percentage of learners per school per level according to the criteria set out above.

None of the learners in the township school had more than 60% on the third level. Only 5.6 % of the learners in the suburban school had more than 60% on level 3 and only 10.6 % of the 194 learners had more than 60% on level 2. If we consider that learners should at least be proficient on level 2 before entering the FET phase these learners may not be ready for geometry in the senior phase (Van De Walle, 2004).

The predominant level of the learners calculated in this phase of the study was used in the sampling of the learners for phase 2 and in finding a link between the errors and the Van Hiele levels in phase 3.

5.3 LEVEL 4 QUESTIONS IN THE MULTIPLE-CHOICE SECTION

The questions in the multiple-choice section were set to assess the learners geometric thought on Van Hiele levels 1 (Visualisation), level 2 (Analysis), level 3 (Informal deduction) and level 4 (Deduction).

Only the results of the questions on levels 1 to 3 were taken into account when analysing the multiple-choice section. The results of the two level 4 questions were not considered because none of the learners who had more than 60% for level 3 in the multiple-choice section had one of the questions on level 4 correct. Most of the other learners who had a level 4 question correct did not have more than 60% for any level. It was therefore concluded that it was most likely that they had guessed the correct answer.

5.4 SUMMARY OF THE CHAPTER

The purpose of the first phase of the research was to answer the first research question, namely: *What is the level of geometric thought of grade 9 learners according to the Van Hiele theory?*

From the data, it may be concluded that the Van Hiele level of geometric thought of the grade 9 learners in this study was low. Two-thirds of the learners failed to achieve more than 60% in level 1 and only one-fifth acquired more than 60% in level 1. Only 4 of the 194 learners who were tested managed more than 60% on level 3.

The results of other South African studies (Alex, 2012; Atebe and Schäfer, 2010; Siyepu, 2005) also indicated that the levels of geometric thought of learners are lower than one would expect. This could be one of the reasons why the performance in geometry in the grade 12 final examination is so poor.

The next chapter discusses the analysis of errors.

CHAPTER 6

TYPICAL ERRORS IN GEOMETRY: RESULTS AND DISCUSSION

6.1 INTRODUCTION

In this phase, qualitative methods were used to collect the data to answer the second research question "*What are the typical errors that the grade 9 learners make in geometry?"* The tests were systematically analysed to uncover the errors. The multiple-choice and open-ended question sections of the test are discussed separately below and thereafter the interview results. Although the content of most of the questions in the two sections overlapped, some topics were dealt with in more detail in the multiple-choice section (e.g. differences in orientation of figures) and others in the open-ended section (e.g. proof questions). Therefore a decision was made to combine the results in order to obtain a more complete picture.

The highest level on which the learners had >60% in the test during the first phase was used to group the learners for the second phase of the study. During the second stage, the errors were analysed to determine the errors made by the learners. Finally, the data was analysed and interpreted to find a link between the typical errors and the Van Hiele levels.

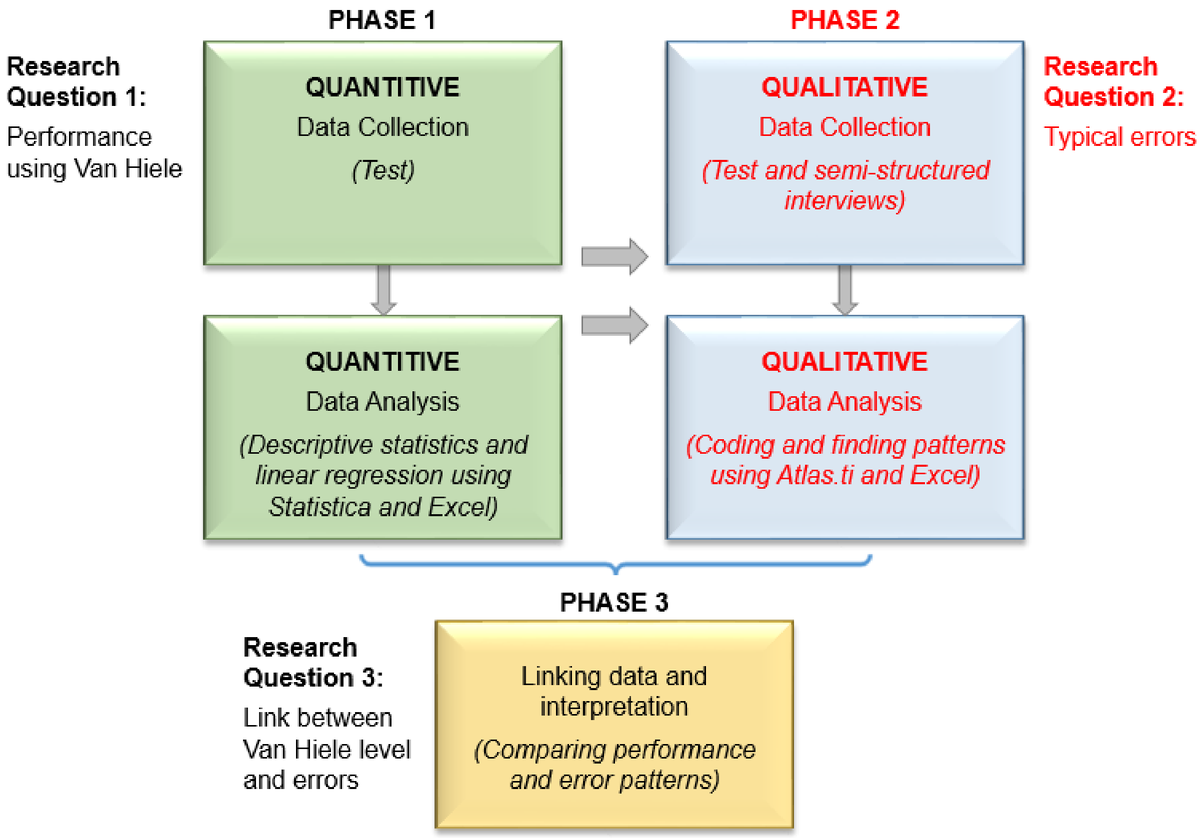


Figure 6.1: Illustration of where phase 2 fits into the research design.

6.2 TYPICAL ERRORS FOUND IN THE MULTIPLE-CHOICE SECTION OF THE TEST

The responses of all the learners in the multiple-choice section of the test were entered into a spreadsheet. The percentage of each response (a) to (e) for the entire group on that level was calculated and then compared to the questions and options given in the test. The incorrect choices were analysed to identify certain error patterns.

The full results of the analysis of each question can be found in Appendix F. This section summarises the typical errors that emerged.

6.2.1 Terminology and basic concepts

Many of the incorrect choices that the learners made were the result of poor knowledge of terminology or basic concepts. The learners' poor knowledge was particularly evident in questions 3 (parallel lines), 5 (alternate interior- and corresponding angles), 7 and 14 (supplementary angles), 9 (vertically opposite angles), 16 (acute angles), 17 (exterior angle) and 21 (similarity).

Furthermore, learners confused the terms similarity and congruency, alternate interior and corresponding angles, vertically opposite and supplementary angles. Two examples of questions where this confusion was evident and a summary of learners' responses are given below:

Example 1:

Q7. Angle A_1 and A_2 are ?

a) complementary
b) supplementary
c) corresponding
d) vertically opposite
e) none of the above

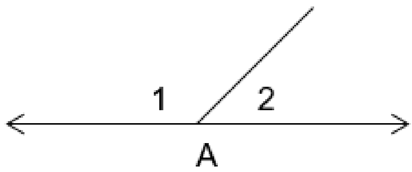


Table 6.1: Analysis of Question 7

Q7		Frequency	%
Valid	a	14	7.2
	b	38	19.6
	c	51	26.3
	d	72	37.1
	e	19	9.8
	Total	194	100.0

Table 6.1 shows that 37.1% of the learners confused the terms vertically opposite angles and supplementary angles. This error was also found in questions 6, 7, 14 and 16 where the learners did not recognise that angles on a straight line are supplementary.

Example 2:

Q 9. The two angles $\hat{1}$ and $\hat{4}$ in the sketch are called

a) supplementary
b) complementary
c) alternate interior
d) vertically opposite
e) all of the above

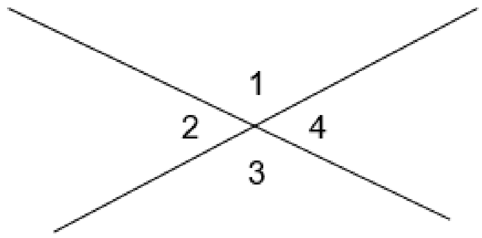


Table 6.2: Analysis of question 9

Q 9		Frequency	%
Valid	a	10	5.2
	b	8	4.1
	c	48	24.7
	d	123	63.4
	e	5	2.6
	Total	194	100.0

Table 6.2 demonstrates that 24.7% of the learners confused the terms alternate interior angles and vertically opposite angles.

In question 17 learners also confused the alternate interior angles with the exterior angle of a triangle.

The poor knowledge of terminology often has further repercussions in answering other questions. In question 16 many learners could not make deductions about the number of acute angles in a right-angled triangle, because they did not know the definition of an acute angle.

6.2.2 Orientation of figures

Learners tended to make errors in identifying figures that were not in an upright or standard position e.g. in questions 2 (triangles), 4 (rectangles) and 5 (alternate interior angles) where the figures were given in various orientations. An example of a question where orientation played a role and the summary of learners' responses are given below:

Example:

Q4. Which of these are rectangles?

a) P and SR
b) Q
c) Q and R
d) R
e) All of them

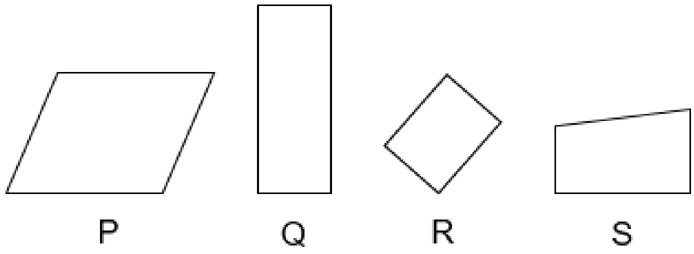


Table 6.3: Analysis of question 4

Answers to Q 4		Frequency	%
Valid	a	16	8.2
	b	105	54.1
	c	62	32.0
	d	4	2.1
	e	7	3.6
	Total	194	100

Table 6.3 indicates that 54.1 % of the learners did not recognise R in the slanted position as a rectangle.

6.2.3 Quadrilaterals

The other typical errors included the learners' knowledge of the properties of quadrilaterals as seen in questions 4 (rectangles), 8 (rectangles), 10 (rectangles), 12 (square), 13 (kite), 15 (rhombus), 24 (general) and 26 (general). The learners also made errors concerning the properties of isosceles triangles (question 17) and equilateral triangles (question 25).

Between 85 and 90% of the learners did not recognise the hierarchical classification of quadrilaterals in questions 19, 20 and 24.

The typical errors found in the multiple-choice questions were very similar to the error patterns in the open-ended questions. The error patterns found in the open-ended questions are summarised in the section below.

6.3 TYPICAL ERRORS FOUND IN THE OPEN-ENDED QUESTIONS

Only 60 of the learners' open-ended question sections of the test were purposively selected for scanning and coding for the typical errors. Graphs indicating the number of quotations (number of times the error occurred) and the results are discussed under the following error categories:

- terminology and basic concepts;
- angles and parallel lines;
- quadrilaterals;
- triangles, congruency and similarity; and
- proof questions.

6.3.1 Terminology and Basic Concepts

In order to be able to communicate effectively, learners need to know the basic terminology of geometry. In the section on the multiple-choice questions, the results indicated that the learners made many errors concerning terminology and basic concepts. The results of the open-ended section seem to confirm this.

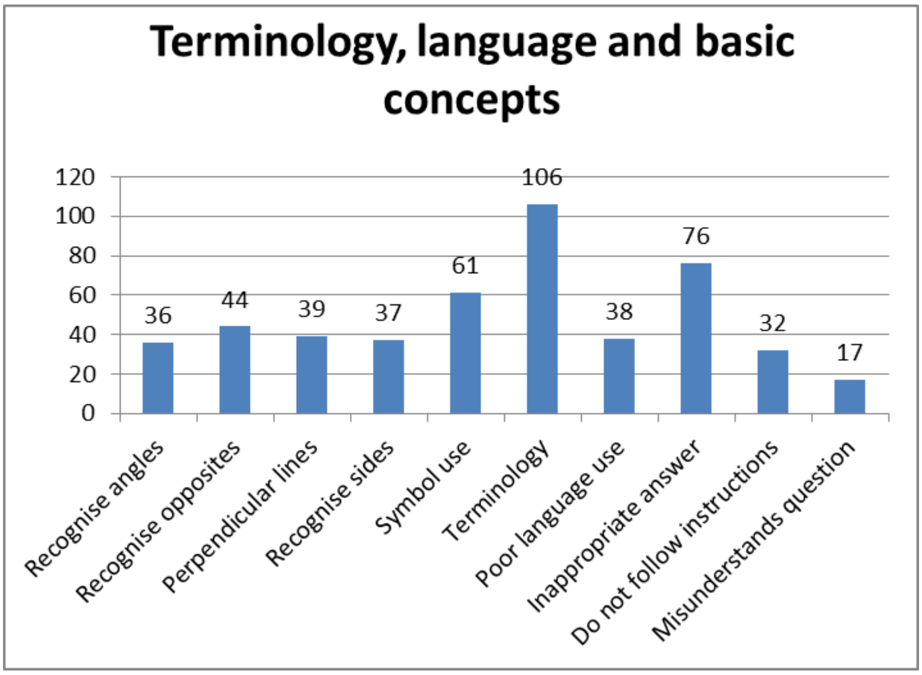


Figure 6.2: Graph with number of quotations associated with terminology, language and basic concepts

From the graph, one can conclude that terminology, symbols and inappropriate answers (which may also be due to not understanding the question or the terminology in the question) were the areas where the most errors were found.

Examples of errors in these categories are given below:

6.3.1.1 Terminology

The table below provides a few examples of the incorrect terminology.

Table 6.4: Level of knowledge of terminology

Correct terminology	Examples of incorrect terminology
Alternate interior angles	*Alternative or alternating angles*
Exterior angle of triangle	*Co-exterior angles of triangle*
Perpendicular lines	*Parallel lines*
Equilateral triangle	*Quadrilateral triangle*
Common angle	*Common holder*

Learners often used the geometric terms in completely inappropriate settings. This could indicate that they knew the terms (words) but had no idea of the meaning or of how to use them.

6.3.1.2 Inappropriate answers:

This section seems to be closely tied to terminology but was given a separate code because there were errors that did not seem to be related to the terminology as the examples below illustrate.

Table 6.5: Examples of inappropriate responses

Question	Examples of inappropriate answers
What type of quadrilateral is it?	*Yes*
	A triangle angle
	Corresponding
What is the ratio of the sides?	*180°*
	= 90° or 180° = 360°
	Hypotenuse
Are the triangles similar or congruent?	*They are not the same and the other is a circle*
How big is angle A?	*Because it is a special space*

6.3.1.3 Poor use of language

Most of the learners in the township are not taught in their home language. The township learners made 87% of the errors labelled as 'poor use of language'. A few examples are given below.

- *"they run next each other"*
- *"It is because they are touch each other"*
- *"Because they in the same way"*
- *"Because I miltiple the numbers that are there. So I got 80° that is not there"*
- Is the quadrilateral a square? *"Yes because they follow each other."*

6.3.1.4 *Recognising angles, sides or opposites:*

It was surprising to see that 41.7% of the 60 learners whose papers were coded for errors could not mark an angle or a side correctly and 51.7% could not identify opposites correctly. This was surprising because angles and sides should have been dealt with from grade 3 in primary school.

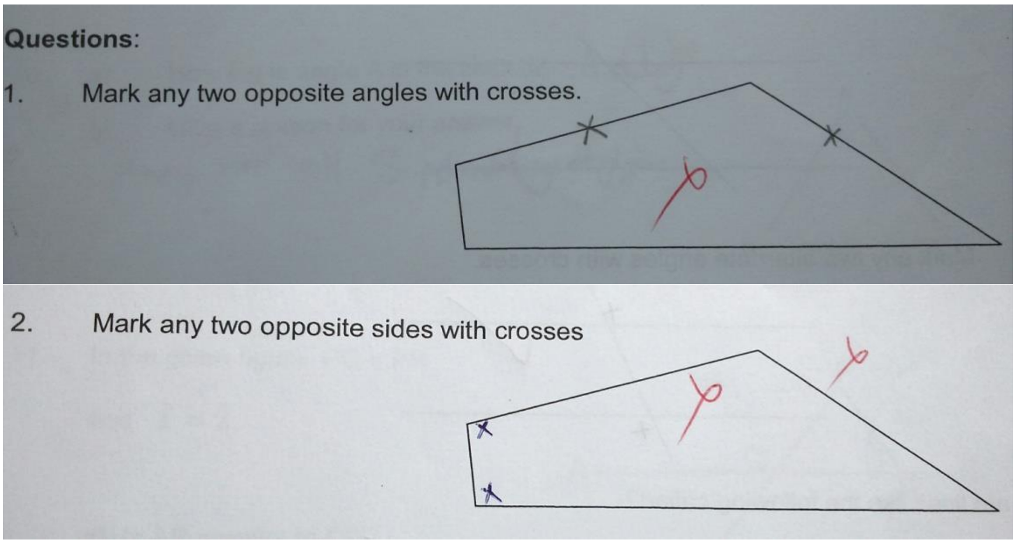


Figure 6.3: Examples of answers showing errors concerning angles and sides

Many of the learners marked adjacent sides or angles instead of opposites.

The poor knowledge of basic concepts concerning angles and sides also increases the number of errors in other questions. The learners who had Q1 (recognise opposite angles) and Q2 (recognise opposite sides) incorrect tended to make mistakes with angles and sides throughout the test.

Figure 6.3 is an example of a proof question (Q16) in which the learner confused the sides and angles.

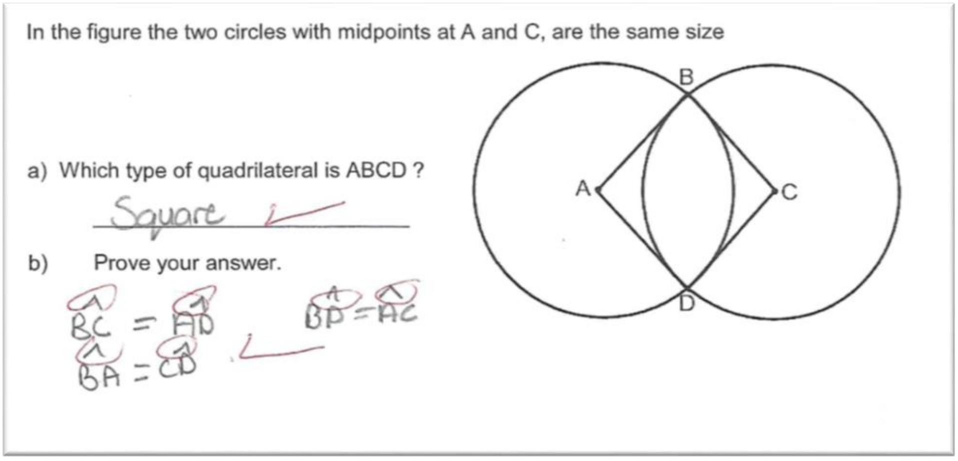


Figure 6.4: Example of a more difficult question with angle and side confusion

6.3.2 Angles and Parallel lines

Other errors concerning angles such as supplementary, exterior angles of triangles and angles formed by the transversal of parallel lines were common.

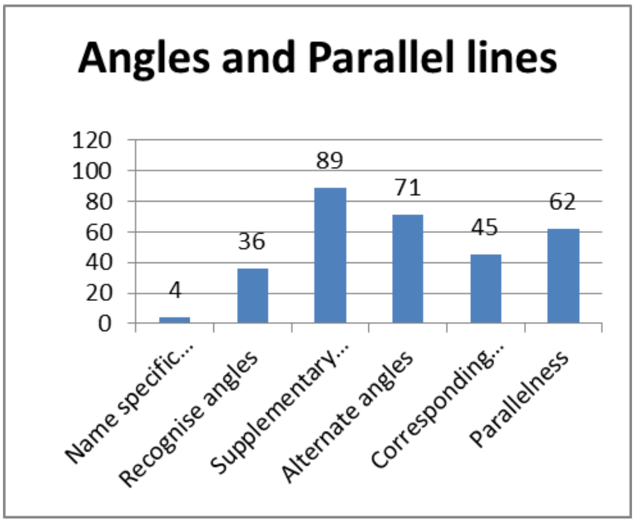


Figure 6.5: Graph with number of error quotations associated with angles and parallel lines

6.3.2.1 Angles formed by the transversal of parallel lines

Errors concerning the confusion of alternate interior and corresponding angles as well as the positioning of these angles on a sketch were common.

Question 9 was selected for further investigation during the interviews as 66,7% of the learners made errors in answering the question (see the example below).

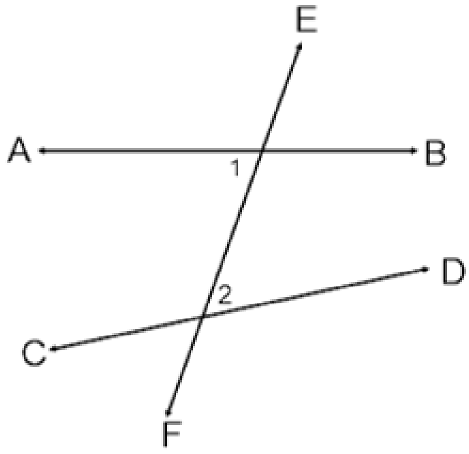
Example:

Q9. Two lines AB and CD are crossed by another line EF as shown in the sketch.

a) If the position of line CD is changed so that $\hat{1} = \hat{2}$, will CD be parallel to AB?

YES / NO

b) Give reasons for your answer.



The correct answer is:

9a) Yes

9b) If the alternate interior angles, $\hat{1} = \hat{2}$, are equal, the lines are parallel.

The answers below represent some of the typical incorrect answers found to this question.

Table 6.6: Typical errors related to question 9

Q 9a) Select yes or no	Q 9b) Reason for answer in a)
No	Angles cannot change
Yes	The line will be straight
Yes	The lines are parallel
Yes	Alternate interior angles mean parallel

Other typical errors concerning parallel lines were:

- Lines that are parallel "*should be equal in length* "
- Only have alternate interior angles if the lines are parallel.
- Because angles are equal they must be corresponding.

- "*Parallel because straight* "(parallel if lines are horizontal)
- Parallel lines "*must have intersecting line"*
- confuse alternate interior, corresponding and vertically opposite angles

From the answers above and the answers in the table, one can conclude that learners do not have a good understanding of the characteristics of parallel lines.

6.3.2.2 Supplementary angles

Of the 60 learners, 91.7% incorrectly identified, named or used supplementary angles in the open-ended questions. Supplementary angle errors had the greatest occurrence.

The typical errors that were found:

- learners did not recognise two adjacent angles on a straight line as being supplementary
- learners mistook supplementary angles for being vertically opposite.
- learners could not use supplementary angles to prove that lines are perpendicular

6.3.3 Quadrilaterals

The two most common errors concerning quadrilaterals were related to the properties and hierarchy. Learners could either not identify properties in a quadrilateral or use these properties to identify the quadrilaterals. This is reflected in Figure 6.5 where one can see that a smaller number of errors were related to the better-known square and rectangular shapes.

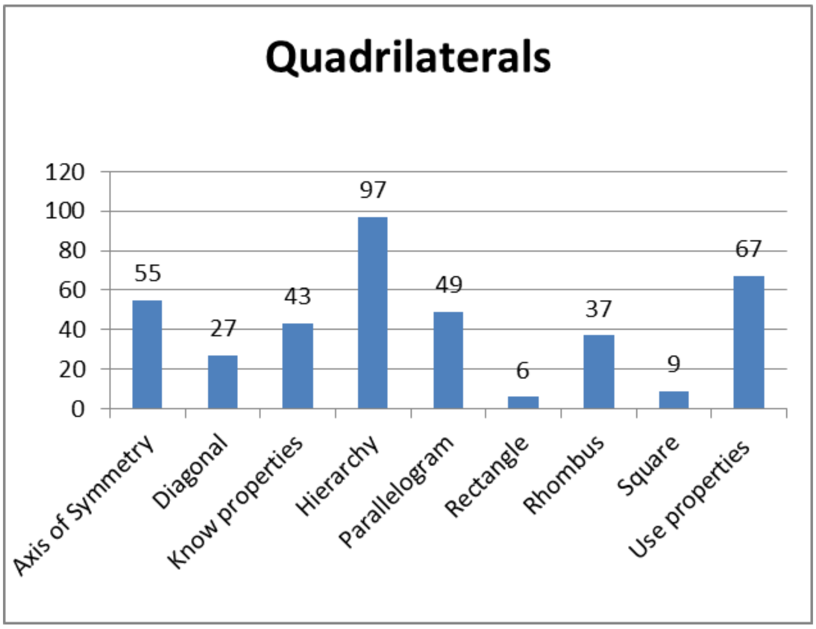


Figure 6.6: Graph indicating the number of quotations associated with quadrilaterals

Learners tend to confuse an axis of symmetry with a diagonal. They also could not describe an axis of symmetry. The table below provides some of the incorrect reasons for the question on the diagonal of a parallelogram.

Table 6.7: Analysis of responses to question 21b

Question 21b	Incorrect reasons
Will the diagonal of the parallelogram also be an axis of symmetry? Give a reason.	*Yes. It is the same length*
	Yes. It bisects the angles
	Yes. It bisects 90°
	Yes. Both sides are equal.
	Yes. Can be a line of symmetry but one triangle is upside down.
	No. It does not bisect the shape
	No. Because the lines are parallel.

In question 16 the orientation of the shape played a role in 18.3% of the learners not recognising the rhombus.

The hierarchical classification of quadrilaterals was deficient in 95% of grade 9 answer sheets that were coded. This was further investigated in the interviews.

6.3.4 Triangles, congruency and similarity

The proof of the congruency and similarity of triangles is introduced in grade 9 in the curriculum. Learners seemed to know the steps or basic format of the congruency proofs but used incorrect or irrelevant reasons and terminology. The questions concerning proof of congruency were more successfully answered than the similarity proof. Of the learners tested in this study, 86.7 % could not correctly calculate the ratio of the sides of the two triangles. Many learners confused similarity and congruency.

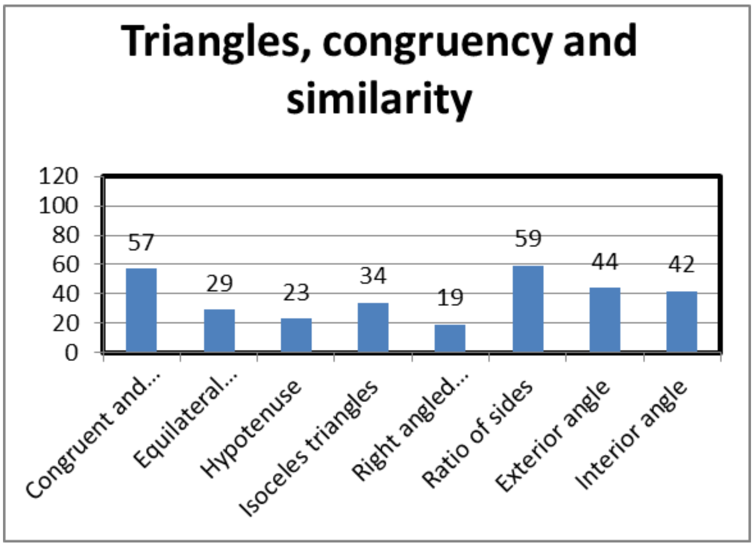


Figure 6.7: Graph with number of quotations associated with triangles, congruency and similarity

Some of the typical errors or misconceptions concerning triangles were that the learners:

- confused isosceles and equilateral triangles
- cannot identify the hypotenuse in a right-angled triangle especially if the triangle is not in the standard position.
- believed that *"every unknown angle in a triangle is 60°"*
- do not recognise that an isosceles triangle can also be right angled. A similar misconception was found by Atebe & Schäfer (2008) in their study of grade 10 learners.

- Learners do not recognise the hierarchy of triangles e.g. that an equilateral triangle is an isosceles triangle with all sides being equal.

A less common but interesting phenomenon was that a few learners used number patterns in an attempt to calculate the sizes of the angles.

6.3.5 Proof questions

The answering of proof questions was problematic for most learners. Only 6.7% had full marks for the congruency proof. Two-thirds of the learners did at least attempt a proof in question 21 on congruency of the triangles in a parallelogram. Some of them knew the format of a congruency proof but did not score any marks because they either did not supply reasons for the statements or the reasons were incorrect. This might indicate that they are already learning higher level knowledge without understanding the basic concepts (Crowley, 1987). Numerous errors were due to learners confusing sides and angles.

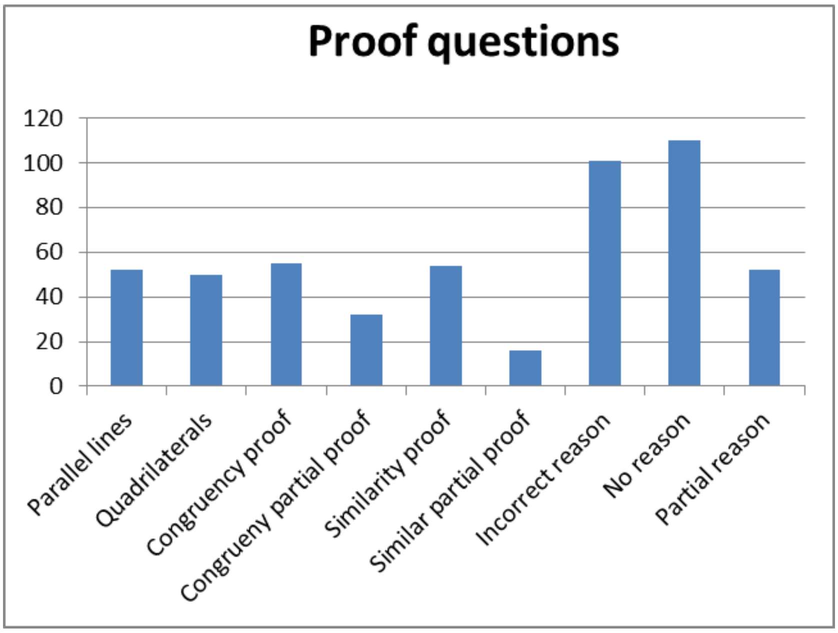


Figure 6.8: Graph with number of quotations associated with proof questions on parallel lines, quadrilaterals and triangles

In the geometry section of the diagnostic report of the grade 12 NSC examination in 2014 and 2015, one of the biggest problem areas in their proof answers were also the reasons given by learners (Department of Basic Education, 2014b, 2015a).

The proof questions on parallel lines and quadrilaterals were also poorly answered. Many of the learners did not attempt a proof but simply answered the question with a sentence stating that the lines were parallel or that it was a certain type of quadrilateral.

6.4 TYPICAL ERRORS INVESTIGATED WITH INTERVIEWS

A few of the errors were selected for probing during the six semi-structured interviews. The purpose of the interviews was to gain more insight into some of the typical errors and not to confirm the Van Hiele levels.

The results of the interviews are discussed under similar headings to those used in the previous two sections on the test results.

6.4.1 Terminology and basic concepts

During the interviews, some of the learners avoided using the terms and would describe or use gestures to answer the questions.

The learners were given various triangles, quadrilaterals, pentagons and hexagons. They were asked to sort the shapes and then explain how they sorted them. Four of the six used the number of sides or angles when they grouped them but two of them sorted them further into smaller groups with different quadrilaterals (e.g. squares, rectangles, etc separately) and triangles (e.g. isosceles, right-angled, etc.) in separate groups. It was interesting to note that most of the learners turned the shapes to the standard position whilst sorting them. Burger and Shaughnessy (1986) noted that this helps learners to identify shapes.

Three of the six interviewees pointed to adjacent sides/angles when asked to point out opposites. All learners could point out angles and sides in figures when asked but two of them used the words “angles” and “sides” incorrectly (see example below).

Learner: Turned all of the shapes into standard positions before placing them in the groups.

Researcher: “*Can you tell me how you decided to group them?”*

Learner: (Points to various groups.) “*These shapes are different*”

Researcher: *"How are they different?"*

Learner: (Points to 90° triangles.) "*This one has 90*". (Then points to isosceles triangles.) "*This **side** is 60, 60, 60*." (Incorrectly referring to angles as sides and does not use the word degrees.)

Researcher: "*Are all the angles 60° in these triangles?"*

Learner: "*Uhm…. no*." (Points to base angles.) "*These are 60, 60 and this one is*…" (indicates a sharper point using his two hands.)

Researcher: "*Do you know what this triangle is called?*"

Learner: "*Uhm… I cannot remember*."

Other terms that the learners had problems with during the interviews were: hypotenuse, equilateral, similar and congruent and the less familiar quadrilaterals such as rhombus, kite and parallelogram. The learners had no problems with naming the more familiar squares and rectangles.

Right angled triangles in different orientations were given to the learners.

Researcher: "*Yes, all these triangles have a 90° angle. They are called right-angled triangles. Do you know what a hypotenuse is?"*

Learner: *"Uhm…(*laughs*)…(*moves triangles around*)…no, I cannot remember"*

Researcher: "*It is the side opposite the 90° angle."* (Pointing to the 90°angle in a triangle)
"Can you point to the hypotenuse in this triangle?"

Learner: "*Opposite the 90°?"*

Researcher: "*Yes."*

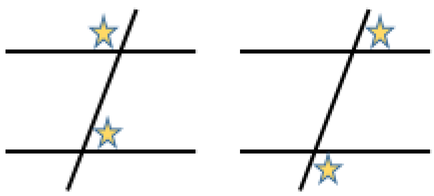
Learner: "*When you put it so…….(*long pause)…..*when you put it…(*long pause and then learner gestures to one right-angled side that is slanted)."

6.4.2 Parallel lines

The evidence from the open-ended questions suggested that the learners had misconceptions concerning alternate interior angles and parallel lines. Therefore the interviews were used to gain a little more insight into how the learners viewed alternate interior angles and parallel lines.

All the learners were asked how they would describe parallel lines to someone over the telephone. This was done to determine if they could use the correct terminology and mathematics vocabulary. The learners all found it difficult to describe parallel lines without using gestures. Only after prompting were two learners able to correctly describe them as "*two lines that are an equal distance apart"* and *"two lines that do not meet"*. The others incorrectly said "*lines are parallel because they are going straight*" and "*must have a line that crosses them*" or used more imprecise words "*they are far from each other"* or "*they are placed the same".*

Five of the learners had the misconception that lines need to be parallel before you could name alternate interior angles. Other errors concerning alternate interior angles are summarised below:

- One learner referred to them as "*alternative angles*".
- 

  This error was found in the tests as well. Three of the learners could not point out the angles correctly.
- "*alternate angles add up to 180°*"
- confused corresponding and alternate interior angles and used the terms interchangeably.

In the open- ended questions, one question (Q9) gave two lines that were not parallel (see section 6.3.2). The learners seemed to ignore this fact and treated them as if they were parallel when they answered this question. Sticks were used to investigate this presumption. During the interviews, lines were not given as parallel but they were asked whether they would be parallel if the alternate interior angles were equal. The learners seemed confused by the question and still ignored the fact that the lines had to be proven parallel and not the angles. Two learners placed the sticks parallel to each other and then stated that "t*he alternate angles are now equal*". Three of them said that the alternate interior angles would be equal because they can see a "Z". One learner just stated that she did not know. These answers may be an indication that the learners were not exposed to situations where they were asked to prove lines parallel that were not drawn parallel to each other.

6.4.3 Quadrilaterals

Very few grade 9's could apply the hierarchical classification of quadrilaterals in the tests. This finding is similar to the findings of Atebe and Schäfer (2008) in their study of grade 10 learners. Therefore I included a question on hierarchy in the interviews.

During the interviews, I questioned the learners on whether a rectangle could be regarded as a parallelogram. Initially, they all said that a rectangle cannot be a parallelogram.

Researcher: *"Do you think that the rectangle could be seen as a special case of a parallelogram?"*

Learner A: "*No, I will separate them because this one is slanted."*

Learner B: *"No the parallelogram does not have 90° angles."*

After I asked them to name and compare the properties of the rectangle and parallelogram, two agreed that a rectangle could be a special case of the parallelogram. After I explained the hierarchy of a rectangle and a parallelogram, they all acknowledged that they had not been taught the hierarchical classification in school. From their answers, I could gather that they had only been given what De Villiers (2012) refers to as partitional definitions.

6.5 SUMMARY OF THE CHAPTER

The numerous errors encountered contribute to the poor performance of the grade 9 learners (discussed in chapter 5) on the Van Hiele levels of geometric thought.

The purpose of the second phase of the research was to answer the second research question: *What are the typical errors that the grade 9 learners make in geometry?*

The typical geometry errors made by the learners in the tests were analysed and then further investigated in the interviews. The errors uncovered in the multiple-choice and open-ended questions, as well as the results of the interviews, were arranged into categories.

The following errors seemed to re-occur the most and were regarded to be the typical errors:

- Terminology, language and basic concept errors
- Confusion of angles (alternate interior-, supplementary-, vertically opposite and corresponding angles)
- Poor recognition of shapes if the orientation changed
- Poor understanding of the properties of the quadrilaterals
- Lack of ability to use the hierarchical classification quadrilaterals and triangles
- Confusion of isosceles and equilateral triangles
- Uncertainty about properties of parallel lines
- Incorrect reasons or no reasons in proof questions
- Lack of ability to do proof questions.

Although the results of the multiple-choice and open-ended sections were analysed differently the same error patterns emerged. The combination of the results of the two sections and the interviews gave a much better indication of the typical errors. The errors were then compared to the Van Hiele levels of the learners to establish whether they are linked. This is discussed in the next chapter.

CHAPTER 7

LINK BETWEEN THE VAN HIELE LEVELS AND TYPICAL ERRORS

7.1 INTRODUCTION

This section merges the two previous chapters and may seem like a repetition of the previous chapter. However, some repetition of categories is necessary to establish the link between the errors and the Van Hiele levels. A combination of the data collected in the first two phases of the study was used to answer the final research question: *Is there a link between the Van Hiele level of geometric thought and the typical errors that the grade 9 learners make?*

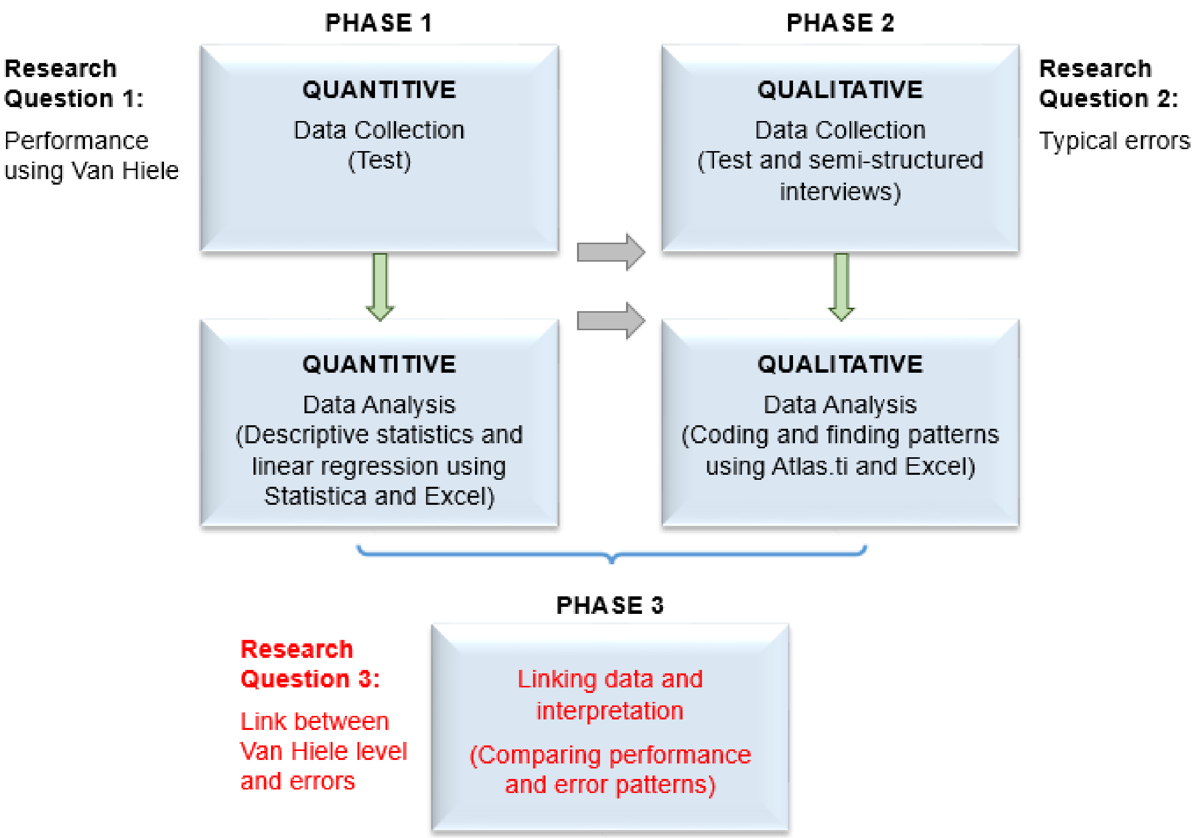


Figure 7.1: Illustration of where phase 3 fits into research design.

This chapter compares the Van Hiele levels with the typical errors using the same error categories as used in chapter 6 namely, terminology and basic concepts, orientation, quadrilaterals, angles and parallel lines, triangles and proof questions. The learners were grouped according to the highest Van Hiele level on which they achieved more than 60%, as was determined during the first phase of the study and discussed in chapter 5.

7.2 ERRORS AND LINKS

7.2.1 Number of errors

The graphs below (figures 7.2 and 7.3) show the general tendency of the number of errors or incorrect answers per Van Hiele level. The average number of errors per learner decreased as the Van Hiele levels increased. This result was expected.

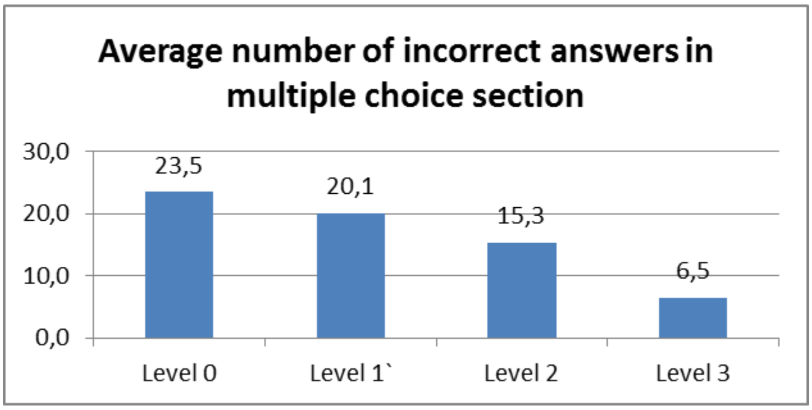


Figure 7.2: Average number of incorrect answers per learner in the multiple-choice questions

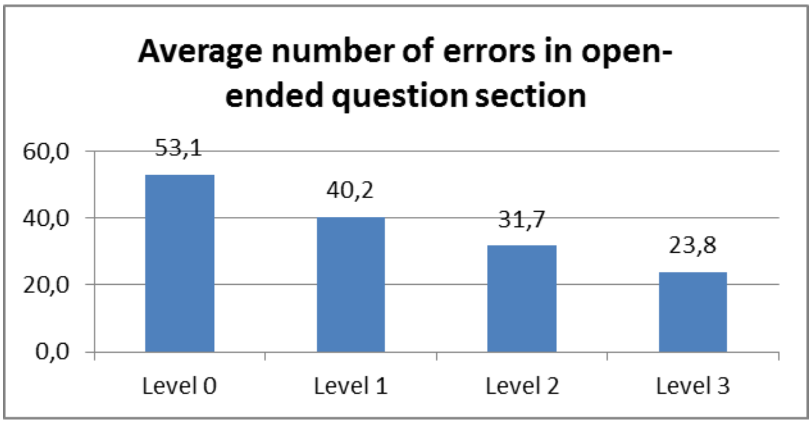


Figure 7.3: Average number of errors per learner as found in the open-ended questions

7.2.2 Link between errors and Van Hiele levels

Before the results are discussed it is necessary to describe how the decisions were made with regards to the linking of the **multiple-choice questions** to the Van Hiele levels.

Firstly, it was decided that a big difference in the percentages of incorrect answers could indicate a link between an error and a Van Hiele level. Secondly, the error was also linked to a level if none of the other levels seemed prone to making the error

The data obtained from coding the errors in the sample of 60 open-ended questions answer sheets were quantified with the aid of the analysis function of Atlas.ti. The linking of the errors to the Van Hiele levels was done by comparing the average number of quotations per learner for each Van Hiele level.

The thinking processes involved for each Van Hiele level (Chapter 2) was kept in mind throughout the comparison between the Van Hiele levels of the learners and the typical errors.

7.2.2.1 Terminology and basic concepts

A summary of the results of the multiple-choice questions is displayed in the table and that of the open-ended questions on the graphs below. The percentages of incorrect answers in the multiple-choice questions do not necessarily reflect all the incorrect choices made for that question but the percentage which is regarded as relating to terminology, basic concepts and language errors.

Table 7.1: Percentages of incorrect answers in multiple-choice questions

Terminology and basic concepts in multiple-choice questions						
Question No	Question contents (in bold) and comments about the levels	% learners that had incorrect answers				Errors linked to level
		Level 0	Level 1	Level 2	Level 3	
3.	**Identify lines that look parallel** On Level 0 many marked the lines  as parallel or marked all horizontal lines as parallel	34.6	10.3	0.0	0.0	0
5.	**Alternate interior angles terminology** Very few level 0 and 1 correctly identified alternate interior angles and many also still confused alternate interior and corresponding angles on level 2	40.8	41.0	35.0	0.0	0 to 2
7.	**Name supplementary angles on a straight line** Most Level 0 cannot name supplementary correctly but level 1 and 2 also still make this error	92.4	64.1	45.0	25.0	0 to 2
14.	**Identify supplementary angles** All levels had problems with identifying supplementary angles. The selection of answers indicated problems with the terminology	75.6	61.5	65.0	25.0	0 to 2
9.	**Recognise vertically opposite angles** Level 0 confuses alternate interior and vertically opposite angles. Level 1 still has 20.5% who confuse the two. This might be because they are between levels.	45.0	28.2	5.0	0.0	0
16.	**Number of acute angles in a right angled triangle** Level 0 and 1 had poor understanding of term 'acute'	79.2	78.9	35.0	0.0	0 to 2
17.	**Recognising exterior angle of triangle** Level 0 but also level 1 had poor understanding of the terminology	73.3	53.8	5.0	0.0	0 and 1
21.	**Similarity and congruency** Levels 0-2 performed poorly in distinguishing between similar and congruent statements. Level 3 performed reasonably.	76.3	76.9	65.0	0.0	0 to 2

The percentages in the table above indicated that most of the errors concerning terminology may be linked to the learners on Van Hiele levels 0 to 2.

There were significant differences of more than 30% between levels 2 and 3 in recognising alternate interior and supplementary angles.

A significant difference of 43,9% was found between levels 1 and 2 for the acute angles and 48.8% for exterior angles. This implies that these errors are linked to level 1.

Congruency and similarity terminology had the largest difference - 65% - between levels 2 and 3. Congruency and similarity are a new topic that is dealt with in grades 7–9 (Department of Basic Education, 2011b) which may indicate that level 2 learners were still not confident regarding this topic.

The results of the open-ended questions are demonstrated in the graph below. Once again the terminology errors seem to be linked to levels 0 to 2. The use of symbols seems to be problematic for the level 2 learners. The smaller number of symbol errors in level 0 and 1 may be the result of those learners avoiding the use of symbols by writing out the words as was found in the analysis of the question papers.

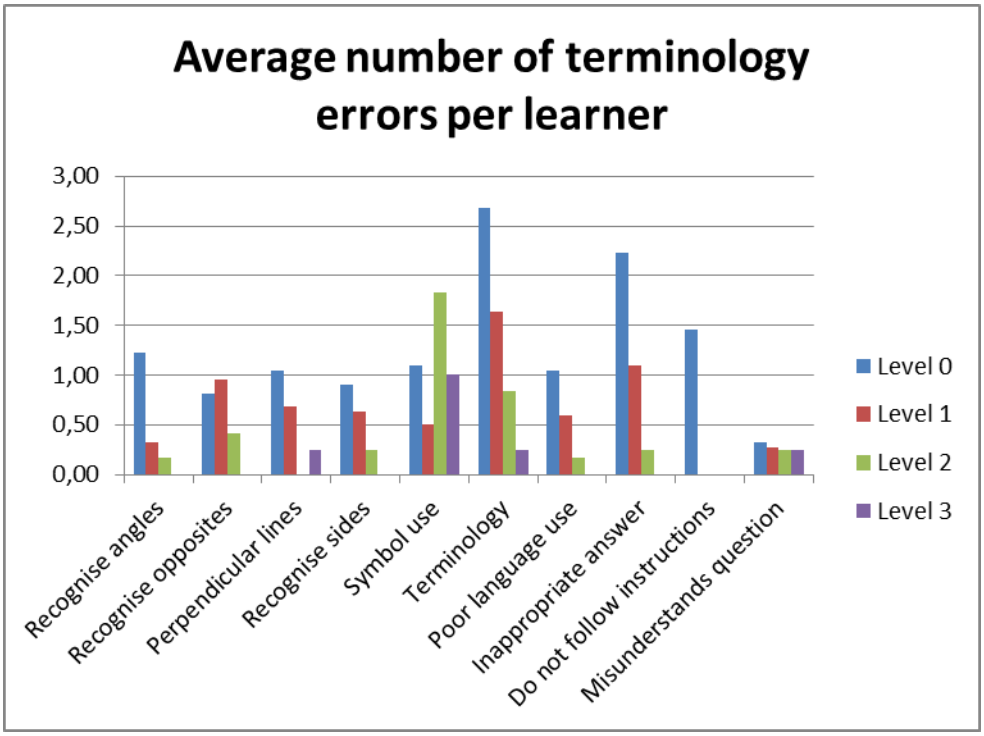


Figure 7.4: Average number of terminology errors per level as found in the open-ended questions.

The graph indicates that levels 0 and 1 learners seem to have a greater language challenge than the other levels. The language challenge for levels 0 and 1 was also evident in the interviews.

During the interviews, learners were asked about the reasons for sorting shapes into different groups.

Researcher: (Pointing to a rectangle and parallelogram) *"Why did you put these shapes into two groups?"*

Learner (level 0): "*I separate this one from this one because….. because…uhm.. this one is like it is falling" (*points to the parallelogram)

Learner (level 1): "*I put this one here because it is a rhombus."* (points to a parallelogram)

During the interviews, the learner on level 2 named the shapes correctly after prompting. The level 3 learner used the correct terminology in all of the questions.

According to De Villiers (2012), the learning of new terminology is especially important for progression from level 2 to level 3. The results given above seem to agree with De Villiers as there is a greater difference in the number of terminology, basic concept and language errors between levels 2 and 3.

7.2.2.2 Orientation

As can be seen from the data on the multiple-choice questions in the table, learners on level 0 to 2 seem to have problems identifying rectangles that are not in the standard position. Alternate interior angles with parallel lines in a different orientation e.g. upright position, are also not identified that easily by level 0 learners.

Table 7.2: Errors in identifying geometric shapes

Orientation in multiple-choice questions						
Question No	**Question contents (in bold) and comments in different levels**	% learners that had incorrect answers				**Errors linked to level**
		Level 0	Level 1	Level 2	Level 3	
2	**Identify 2 triangles amongst other shapes** Some of level 0 did not recognise triangles if not in standard position but 32,6 % of level 1 incorrectly identified the concave quadrilateral as a triangle.	25.6	0.0	5.0	0.0	None
4.	**Identify rectangles in different orientations** Level 0 errs with rectangles in different orientations but level 1 and 2 are still not confident.	62.6	35.9	40.0	25.0	0 to 2
5.	**Recognise correct alternate interior angles** Level 0 and 1 incorrectly identify alternate interior angles with vertical parallel lines	44.6	23.1	5.0	0.0	0
19.	**Hierarchy of quadrilaterals** 18,3 % of the level 0 errors were due to the orientation of the sketches.	18.3	7.7	0.0	0.0	None

In the open-ended questions, only one question investigated an orientation change. The orientation change of a rhombus was investigated in question 16. It was linked to the level 0 as 52.4% of the level 0 learners made an orientation error in comparison to 23.8% on level 1 and 0 % in level 2 and 3.

In the interviews, the orientation of right angled triangles was changed when the learners were asked to identify the hypotenuse. Only the learner on level 3 answered all questions on the hypotenuse correctly. The learner on level 2 made a mistake but could correct himself when he was asked to reconsider his answer.

7.2.2.3 Angles and parallel lines

The errors concerning angles and parallel lines are discussed below. A summary of the results of the multiple-choice questions is displayed in the table and the open-ended questions on the graph.

Table 7.3: Errors in identifying angles and lines

Angles and parallel lines in multiple-choice questions						
Quest No	Question contents (in bold) and comments in different levels	% learners that had incorrect answers				Errors linked to level
		Level 0	Level 1	Level 2	Level 3	
5.	**Recognise correct alternate interior angles** Many level 0 and 1 incorrectly identify alternate interior angles and many level 2 also confused alternate interior and corresponding angles	85.4	64.1	35.0	0.0	0 and 1 but also
6.	**Identify right angles** Many level 0 incorrect and ~ 18,5% also confuses right angles and vertically opposite angles	70.0	33.3	25.0	0.0	0
7.	**Name supplementary angles on a straight line** Level 0 and 1 has terminology problem and cannot name correctly but level 2 also still makes this error	92.4	64.1	45.0	25.0	0 to 2
14.	**Identify supplementary angles** Levels 1 to 3 had problems with identifying supplementary angles.	75.6	61.5	65.0	25.0	0 to 2
16.	**Number of acute angles in right-angled triangle** Level 0 and 1 had poor understanding of term acute	79.2	78.9	35.0	0.0	0 and 1 but also 2
17.	**Recognising exterior angle of triangle** Level 0 but also level 1 did not recognise the exterior angle	73.3	53.8	5.0	0.0	0 and 1

Angles and parallel lines in multiple-choice questions						
Quest No	Question contents (in bold) and comments in different levels	% learners that had incorrect answers				Errors linked to level
		Level 0	Level 1	Level 2	Level 3	
23.	**Parallel lines** Learners had to select the false answer. Many did not read and marked the true answer or did not understand the question and marked the longest answer.	72.5 Mark long answ 29.8	84.6 Mark long answ 38.5	75.0 Mark long answ 40.0	75.0	All
28.	**Reasoning with parallel lines** Poor reasoning using the properties of parallel lines by level 0 to 2. This was also a negative question.	81.7	71.8	65.0	25.0	0 to 2

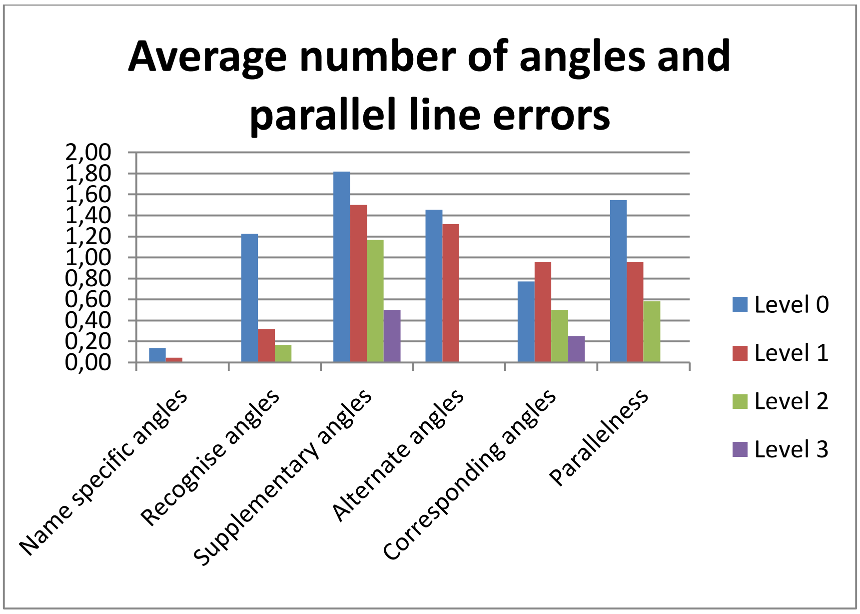


Figure 7.5: Average number of angle and parallel line errors found in the open-ended questions

The 43.9% difference between level 0 and 1 in the percentage of errors concerning the right angles was high. This indicates that identifying right angles was linked to level 0 but that one-third of level 1 learners were also prone to making mistakes.

The incorrect naming of the exterior angle of a triangle is linked to level 0 and 1.

The incorrect indication of alternate interior angles was encountered in the multiple-choice questions (Q 4), the open-ended questions (Q 6b and 11) and during the interviews with learners (also see chapter 6). After the multiple-choice and open-ended questions were compared, this error was linked to level 0 and 1. However, level 2 learners were still very prone to making mistakes. Although orientation changes of parallel lines may not have such a big influence on the identifying of alternate interior angles, the positioning of the two angles does have a greater impact on levels 0 and 1.

The errors with supplementary angles were linked to levels 0 to 2 in the multiple-choice and open-ended questions.

The multiple-choice questions on parallel lines required an interrelationship between properties. The biggest percentage difference was between levels 2 and 3 for question 28, but even the level 3 learners made many incorrect choices in question 23. These two questions were set in the negative and it was wondered whether this may have affected the outcome. However, in the open-ended questions on the proof of parallel lines, questions 9 and 11, most of the errors were also linked the level 0 to 2 learners. This was also confirmed during the interviews.

7.2.2.4 Quadrilaterals and hierarchy

The errors concerning quadrilaterals and hierarchy are discussed below. A summary of the results of the multiple-choice questions is displayed in the table and the open-ended questions on the graph.

Table 7.4: Summary of errors made in the multiple-choice questions

Quadrilaterals and hierarchy in multiple-choice questions						
Question no	Question contents (in bold) and comments in different levels	% learners that had incorrect answers				Errors linked to level
		Level 0	Level 1	Level 2	Level 3	
4.	**Recognise rectangles in different orientations between other shapes** 62.6% of level 0 only recognised the rectangle in the upright position and 19.1% did not know the meaning of the term rectangle.	81.7	35.9	50.0	25.0	0 and 2
10.	**Diagonals of rectangle** Level 0 and 1 could not identify equal diagonals	55.0	56.4	20.0	0.0	0 and 1

Quadrilaterals and hierarchy in multiple-choice questions						
Question no	Question contents (in bold) and comments in different levels	% learners that had incorrect answers				Errors linked to level
		Level 0	Level 1	Level 2	Level 3	
12.	**Properties of square** Level 0 and 1 have a poor grasp of the properties	70.0	56.4	25.0	0.0	0 and 1
13.	**Properties of kite** All levels but especially level 0 and 1 did poorly in this question most probably because the kite is less known and not as commonly mentioned in the lower grades	89.1	84.6	50.0	50.0	All
15.	**Properties of rhombus** A poor understanding of the rhombus also agrees with the findings in open-ended questions. Very interesting that % increases as levels increase. However, all the level 3 had it correct	71.0	87.2	95.0	0.0	0 to 2
24.	**Interrelation of properties of quadrilateral** Question was poorly answered by levels 0-2. Question was also set in the negative.	78.2	64.4	60.0	0.0	0 to 2
26.	**Interrelation of properties of quadrilateral** Level 0 and 1 may not be able to interrelate properties of quadrilaterals.	74.6	71.8	30.0	0.0	0 and 1
8.	**Hierarchy of rectangle** All levels err with regard to hierarchy except level 3	78.6	76.9	60.0	0.0	0 to 2
19.	**Hierarchy of quadrilaterals** Level 0 to 2 had a poor understanding of hierarchy of quadrilaterals which is confirmed with open-ended question section. 18,3 % of the level 0 errors were due to the orientation of the sketches.	91.6	89.7	90.0	0.0	0 to 2
20.	**Hierarchy of quadrilaterals and triangles mixed** All levels did poorly which could indicate poor understanding of hierarchy	91.6	89.7	84.2	75.0	All
29.	**Hierarchy of quadrilaterals** Poor understanding of the hierarchy of quadrilaterals	86.2	84.2	85.0	25.0	0 to 2

In the open-ended questions, the difference between the numbers of errors made by the lower levels in comparison to level 3 was not as pronounced as in the multiple-choice questions.

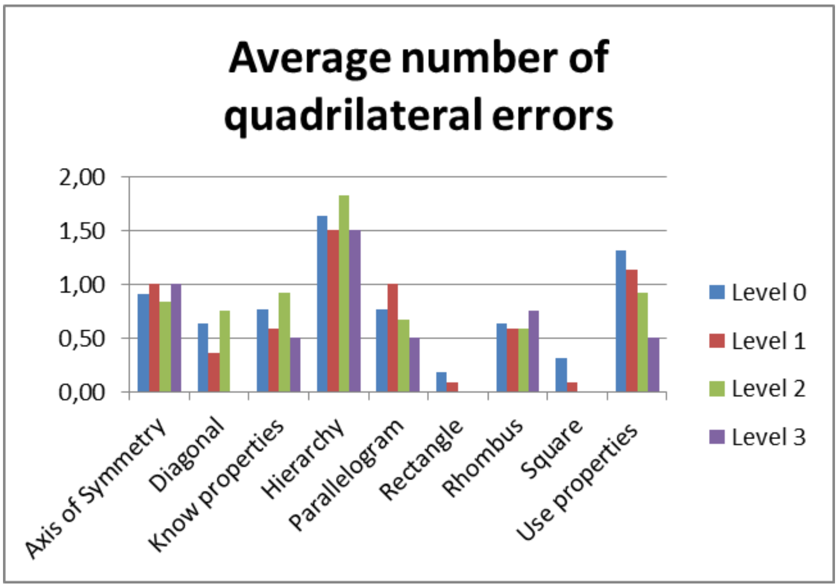


Figure 7.6: Average number of quadrilateral errors found in the open-ended questions

The multiple-choice question 4, that is based on identifying rectangles amongst other quadrilateral shapes, showed a 45.8% difference in the percentages between levels 0 and 1(see table above). If this percentage is compared with the section on orientation of shapes above, one could conclude that there is a link between the learners' level of geometric thought and the number of errors concerning the identification of rectangles when the orientation is changed. In the interviews, the level 3 learner had a better understanding of the properties of the quadrilaterals than the other 5 learners on levels 0 to 1. This tends to confirm the test results given in the table and graph above.

The evidence in table 7.3 and graph 7.5 suggests that errors with respect to the properties of the well-known shapes such as rectangles and squares can be linked to level 0 and 1. The errors of the less known shapes such as the kites and rhombi were linked to the learners on level 0 to 2 while errors concerning kites were also linked to level 3 learners. The graph indicates that learners in all levels made errors concerning an axis of symmetry.

From table 7.3, the graph 7.5 and the interviews it follows that learners in all the levels, 0 to 3, made numerous errors concerning the hierarchy of quadrilaterals and triangles. Therefore there was no clear link between the hierarchy of quadrilaterals and the levels of geometric thought. It seemed as if this type of classification was unknown to all the learners.

7.2.2.5 Triangles, similarity and congruency

The errors concerning triangles, similarity and congruency are discussed below. A summary of the results of the multiple-choice questions is displayed in the table and the open-ended questions on the graph.

Table 7.5: Summary of triangles, similarity and congruency in multiple-choice questions

Triangles, similarity and congruency in multiple-choice questions						
Question No	Question contents (in bold) and comments in different levels	% learners that had correct answer				Errors linked to level
		Level 0	Level 1	Level 2	Level 3	
18.	**Similarity if corresponding angles are equal** Most of level 0 and 1 errors were due to confusing similar and congruent, but many level 0 and 1 learners also confused sides and angles in triangles	62.3	53.8	20.0	0.0	0 and 1
21.	**Similarity and congruency** All levels, except level 3, performed poorly in distinguishing between similar and congruent statements	76.3	76.9	65.0	0.0	0 to 2
27.	**Similar and congruent hierarchy** Very few learners knew that congruent triangles are also similar	76.9	92.1	80.0	75.0	All
31.	**Congruency and similarity of triangles using circle radii** Level 0 and 1 performed poorly when properties had to be deduced	82.3	76.9	55.0	25.0	0 to 2
11.	**Properties of isosceles triangle** Level 0 had between 10- 27.9 % for all options in this question thus indicating a poor grasp of terminology. Level 1 confused equilateral and isosceles triangles	72.1	53.8	10.0	0.0	0 and 1

Triangles, similarity and congruency in multiple-choice questions						
25.	**Equilateral triangle** Learners seem to have a poor understanding of equilateral triangles and the terminology thereof. Even level 3 found this question more difficult.	86.9	84.2	90.0	50.0	All
17.	**Recognising the exterior angle of a triangle** Level 0 but also level 1 had poor understanding of terminology	73.3	53.8	9.0	0.0	0 and 1

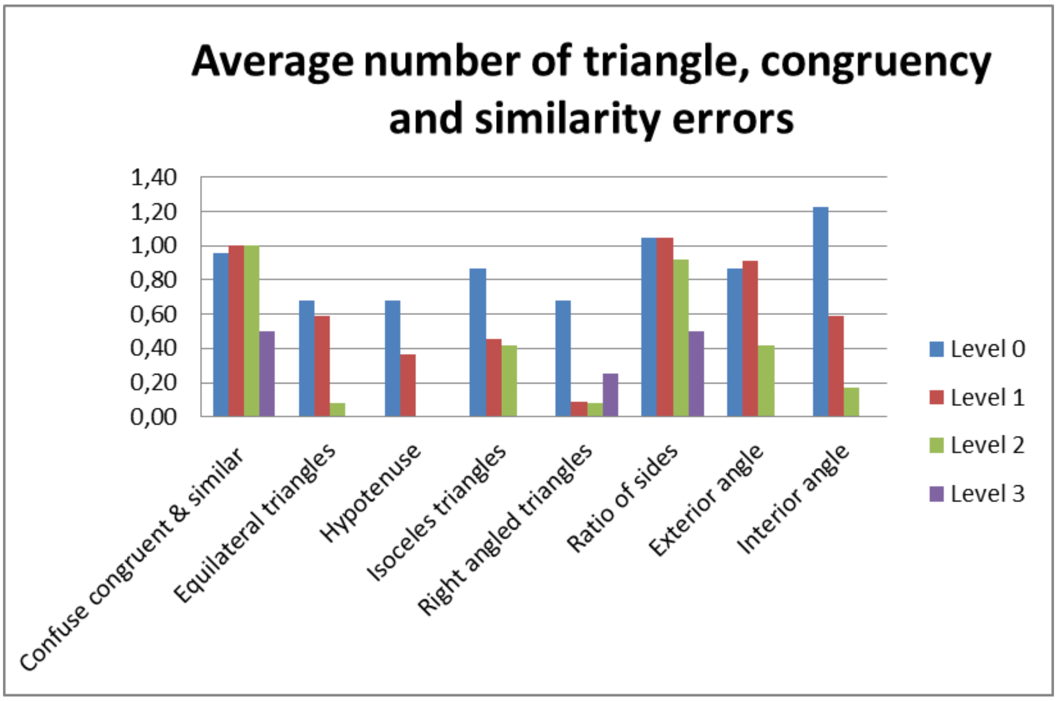


Figure 7.7: Average number of triangle, similarity and congruency errors found in the open-ended questions

When the results of the multiple-choice and open-ended questions are compared with the data from the interviews it appears that the errors concerning the properties of isosceles, equilateral and right-angled triangles are linked to levels 0 and 1. However as soon as the properties of the equilateral triangles were interrelated to each other, the level 2 and 3 learners also made numerous mistakes.

The errors concerning the basic concepts of exterior angles of a triangle and the hypotenuse were linked to levels 0 and 1.

The errors concerning similarity and congruency of triangles affected the learners in levels 0 to 2 in both the multiple-choice and open-ended questions.

7.2.2.6 Proof questions

Proof questions were only asked in the open-ended section of the test. The proof questions seemed problematic for all levels. In the open-ended questions, learners were asked to prove that lines were parallel, triangles similar and congruent and to prove that a quadrilateral was a rhombus.

On the graph below the errors concerning the proof of the quadrilateral and congruent triangles followed a descending pattern from level 0 to 3. The highest number of errors was linked to the reasons used for the statements that learners made. It is interesting to note that the number of incorrect errors is much lower on level 3 but that the level 3 learners tended to leave out the reasons for their answers or write only partial reasons. The learners at levels 0 and 1 also left out many of the reasons.

The results reported in chapter 6 also indicated that many learners did not attempt the proof questions but simply stated in a sentence that, for example, the lines were parallel. After comparing the papers of the learners that were coded using Atlas.ti, this phenomenon was linked to the levels 0 and 1 learners. Two of the level 0 learners also redrew the sketches and tried to prove congruency using the fitting of triangles onto each other.

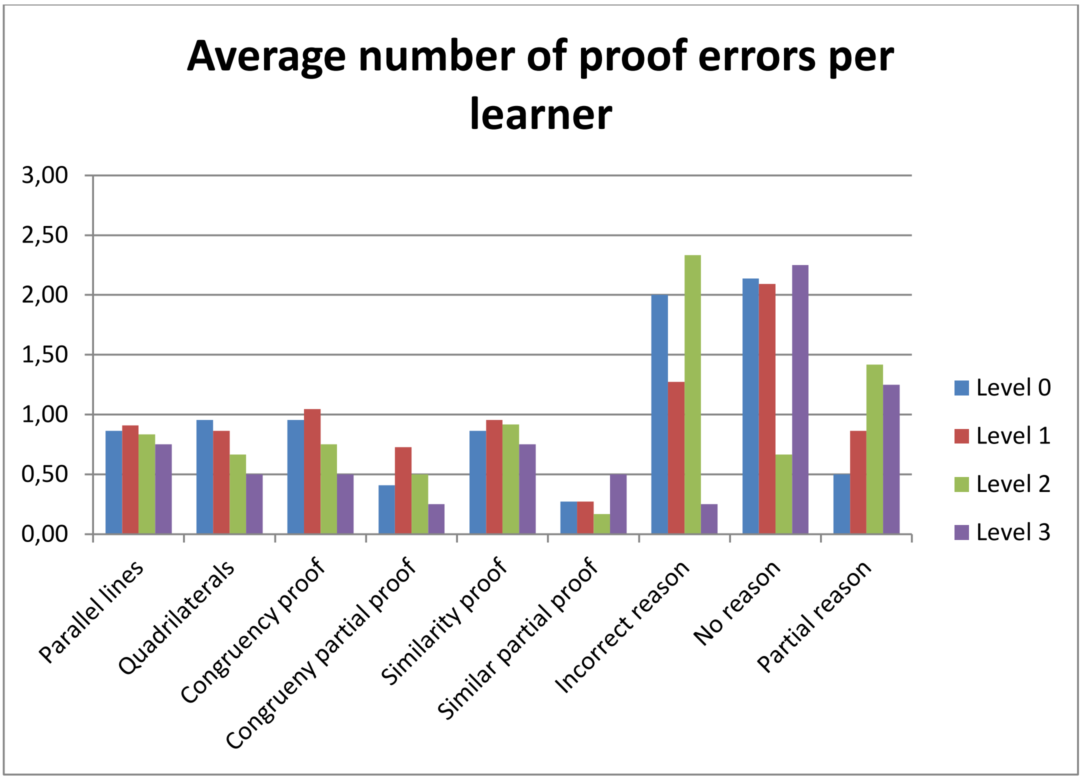


Figure 7.8: Average number of errors concerning proof questions found in the open-ended questions

7.3 SUMMARY OF THE CHAPTER

The purpose of the third phase of the research was to answer the third research question: *Is there a link between the Van Hiele level of geometric thought and the typical errors that the grade 9 learners make?*

The data from the first and second phase were merged to answer this question. The results were discussed under the headings of the error categories that were used in chapter 6. Although the content of most of the questions overlapped, some topics were emphasised in the multiple-choice section and others in the open-ended section. However, when the different sections of the test were compared they tended to confirm each other's results.

According to the Van Hiele levels of geometric thought learners at level 1 (Visualisation) should be able to name and identify quadrilaterals. At level 2 (Analysis) they should recognise the properties of quadrilaterals and at level 3 (Informal deduction) they should be able to formulate relationships between the properties of quadrilaterals (Van De Walle, 2004). If the results from the multiple-choice questions are combined with the open-ended questions one might conclude that there is a link between many of the errors and the predominant Van Hiele level of the learners.

The following chapter will discuss the findings and conclusion of the study.

CHAPTER 8

CONCLUSION

8.1 INTRODUCTION

International and national studies have indicated that the geometry performance of the South African learners is indeed dismal (Department of Basic Education, 2014, 2015; Mullis, Martin, Foy & Arora, 2012). The problem that led the researcher, to begin with this study was the poor performance of learners in geometry encountered during the 25 years of teaching mathematics and the experience that, in spite of hard work, many learners did not seem to progress in geometry.

The aim of this study was to investigate the link between the grade 9 performance according to the Van Hiele levels of geometric thought and the learners' typical errors in the two schools.

In this chapter, a summary of the research and the findings and conclusions of the study are discussed. The limitations, recommendations and avenues for further research are also presented.

8.2 SUMMARY OF THE RESEARCH

In the 1950's, two Dutch researchers, Pierre and Dina Van Hiele, experienced that their learners did not seem to perform as expected (Van Hiele, 1986). Their study of the problem led them to develop a theory on the levels of geometric thought. The Van Hiele theory formed the theoretical background of this study.

According to the theory, learners' progress through sequential levels of geometric thought as a result of instruction and geometrical experiences. If learners and instruction are at different levels of thought, the learners may experience a miscommunication. This miscommunication could lead to misconceptions that may become evident in the errors they make.

The exploring of errors can be used to make teaching and learning much more efficient and an environment should be created in classrooms where errors are seen as positive. The errors can also be used to direct future teaching to the areas of need, therefore utilising the class time more efficiently. If the errors and the Van Hiele levels are linked, the errors could also be used to guide the teacher to discovering the learners' levels of geometric thought.

This study used a mixed method design that combined quantitative and qualitative methodology to answer the following research questions:

- What is the level of geometric thought of grade 9 learners according to the Van Hiele theory?
- What are the typical errors that the grade 9 learners make in geometry?
- Is there a link between the Van Hiele level of geometric thought and the typical errors that the grade 9 learners make?

The study was organised into three phases that were linked to the three research questions:

- Phase 1: The setting of a test, the collection and analysis of the quantitative data generated using the test.
- Phase 2: The collection and analysis of qualitative data by means of coding the errors in the test and interviews.
- Phase 3: A merging of the two sets of data in order to draw the conclusions about a link between the errors and the Van Hiele levels.

8.3 SUMMARY OF THE FINDINGS

In this section, the findings will be described under headings related to each research question.

8.3.1 Performance according to the Van Hiele levels

The learners' predominant Van Hiele level was evaluated using a test consisting of a multiple-choice and open-ended question section. The descending trend in the average percentages that the learners scored in the levels was expected as this descending trend in the average performance was found in other studies using similar methods (Atebe and Schäfer, 2011; Feza and Webb, 2005; Khembo, 2011; Siyepu, 2005; Van Putten, 2008).

Although it was borne in mind that a fixed level cannot be assigned to a learner, a predominant level of thought was necessary, to determine the link between the Van Hiele levels and the errors. An average of more than 60% on a Van Hiele level was regarded in this study as an indication that learners would be able to reason comfortably on that level, and the highest level in which they acquired more than 60% was regarded as their predominant level. Although it was expected that there would be more learners on the lower levels, the high percentage (66.5%) of learners who did not acquire more than 60% on level 1 was not expected. Only 20.9% acquired more than 60% on level 1 and 10.6% acquired more than 60% on level 2.

The performances of the two schools were also compared. The township schools' results were significantly weaker than the suburban school especially in the open-ended questions (Cohen's d= 1.23 for L1 and 0.97 for L2). English is not the home language for most of the township learners and although this was not investigated in this study, one could ask whether language may have played a role in the weak results?

In the studies by Usiskin (1982) and Van Putten (2008) using only multiple-choice tests, some learners passed a higher level without passing the preceding level. This phenomenon was not found in this study. None of the learners in this study had more than 60% in a level in which the preceding level was less than 60%. The combination of the two sections (multiple-choice and open-ended questions) and the greater total number of questions used could have led to this result.

In answer to the first research question, the performance of the grade 9 learners according to the Van Hiele levels was determined and the findings implied that the grade 9 learners' level of geometric thought was very low.

8.3.2 Error Analysis

Although the results of the multiple-choice and open-ended sections were analysed differently the same error patterns emerged. The combination of the results of the two sections strengthened the findings on typical errors. The interviews further confirmed the findings of the tests.

Many of the errors were as results of errors in terminology and basic concepts. One basic concept error that stood out because it was not mentioned in the reports of other studies and was also not expected to be found in grade 9, was the confusion of sides and angles.

Errors such as the poor use of language, failure to follow instructions, misunderstanding questions and inappropriate answers could indicate language difficulties. These errors were more prominent in the township school's tests but less prominent in the interviews with the township learners.

The importance of combining the different types of data collection instruments was realised when the researcher analysed and compared the interview data to the test data. It was noticed that some learners seemed to have better verbal skills than written skills when their verbal answers in the interviews were compared to their written answers in the tests. The opposite was also found as one learner seemed to be more anxious during the interview and made more mistakes compared to the written test.

In answer to the second research question, one can conclude that the following are the typical errors that were found in the analysis of the tests:

- Terminology, language and basic concept errors
- Confusion of angles (alternate interior-, supplementary-, vertically opposite and corresponding angles)
- Poor recognition of shapes if the orientation changed
- Poor understanding of the properties of the quadrilaterals
- Lack of ability to use the hierarchical classification for quadrilaterals and triangles
- Confusion of isosceles and equilateral triangles
- Uncertainty about the properties of parallel lines
- Incorrect reasons or no reasons given in proof questions
- Lack of ability to do proof questions.

8.3.3 Link between errors and the Van Hiele levels

The pattern of errors found in the results is similar to the thinking patterns that were set out for the Van Hiele levels. According to the Van Hiele levels of geometric thought, learners at level 1 (Visualisation) should. be able to name and identify shapes. At level 2 (Analysis) they should recognise the properties of shapes and at level 3 (Informal deduction) they should be able to formulate relationships between properties of shapes (Van De Walle, 2004).

The errors concerning terminology and orientation of the shapes were found to be linked to the learners on levels 0 and 1. The exceptions were the terminology involving parallel lines, similarity and congruency which were linked to level 2 learners.

There seems to be a link between the levels 0 and 1 for the properties of the more well-known shapes such as the rectangles and square. The errors concerning the properties of the less known shapes such as rhombi and kites were linked to level 2. However, the learners of all levels made errors concerning the axis of symmetry and hierarchy of a quadrilateral. The errors of those two topics, therefore, were not linked to a specific level.

The errors concerning the interrelation of properties were found to be more common in levels 0 to 2, which agrees with the description of skills expected by a learner thinking at Van Hiele level 3. The proof questions were poorly answered by the learners in levels 0 to 2, which once again corresponds with the description of the skills expected of a learner on level 3.

Only 5 questions (questions 12, 21, 23, 24 and 28) in the multiple-choice section were set in the negative. All these questions were poorly answered by the lower Van Hiele levels. It was speculated that the learners' errors concerning the negative questions could be ascribed to carelessness. Learners do not seem to fully read the question.

In answer to the third research question, it can be concluded that most of the errors could be linked to the Van Hiele levels.

8.4 LIMITATIONS

On reflection the study may have been restricted by the test questions, the decision of the placement of learners on the levels, the overinflating of counts and the small sample size of level 3 learners.

8.4.1 The test questions

The setting of the test formed a crucial part of this study. The question of what to include and how to word the questions, to set up a valid test for the Van Hiele levels was not as easy as the researcher had originally anticipated. After studying the literature and from email correspondence with professor Zalman Usiskin, it was realised that there is a grey area as to what to include in a Van Hiele test.

The level of difficulty of the questions also plays a crucial role and the researcher endeavoured to set the questions on the same level of difficulty. However, after analysing the learners' responses to the questions, it was realised that the questions on a specific Van Hiele level were not equally difficult. The wording or sketches in the question could have impacted on the level of difficulty. The content of the questions also played a role in the difficulty of the questions as learners may have been less confident when answering questions on new topics. Therefore the difficulty of the questions may have played a role in the calculation of the scores for the different levels. An item-response analysis e.g. Rasch analysis, after the pilot test might have helped in the evaluation of this aspect of the test.

In multiple-choice tests, answers are often selected by guess work. The open-ended question section was an attempt to reduce the effect of guessing. However, the yes/ no questions in the open-ended may have encouraged guessing. Learners were expected to supply reasons for their selection but many learners did not supply reasons for their yes/no answers and it was difficult to assess how many learners had guessed the answers. Therefore it was decided to mark the answers without a reason as incorrect. The impact of this change was difficult to assess.

8.4.2 Placement on levels

The researcher also came to realise that learners cannot be accurately placed at a certain level but one may at most get an indication of how they think about geometry for the sections of content in the test at that time. For the comparison of the errors with the Van Hiele levels however, it was necessary to decide on a predominant level for each learner. Although a combination of the results of multiple-choice and open-ended questions was used in determining an average for each learner to achieve greater reliability and validity, it may be that the predominant level for some learners was incorrect.

8.4.3 Overinflating of counts in the open-ended answers

Creswell and Plano Clark (2007) mentioned the issue of overinflated counts. Overinflated counts are explained as a higher number of occurrences of certain ideas due to respondents repeating themselves. In this study this was identified due to the same error occurring more than once in a question. This was reduced in this study by counting a repetition of a certain error in a specific question. However, there was still an overinflating of counts due to certain types of questions occurring more than once in the question paper. The researcher could have minimised this effect by inserting more codes that were linked to the different questions and content of the questions.

8.4.4 Small sample size on level 3

Only 4 learners of the 194 managed to acquire more than 60% on level 3. This resulted in a very small sample for level 3 compared to 160 learners on levels 0 and 1, which may have had an influence on the percentages that were calculated and used in the linking of the errors to the Van Hiele levels.

8.5 CONCLUSIONS AND RECOMMENDATIONS

In this section contains recommendations for teaching practice as well as for further research.

8.5.1 Recommendations for practice

Examples in different orientations

Teachers should make use of examples in various orientations when they teach geometry. The teachers could make use of more textbooks and also use examples from real life situations to help learners to recognise the figures that are not in the standard position.

Progression in geometry needs time whilst having guided instruction

One of the factors that have a large impact on the effective teaching of geometry in South Africa is the time allocated to teaching geometry. Studies by Atebe and Schäfer (2011) and Sibanda (2012) found that the curriculum was too wide to be covered in the time allocated. Although geometry constitutes a sixth of the marks, only one tenth of the time is allocated to teaching geometry in the departmental work schedules.

The Department of Basic Education and the teachers should allocate more time in the schedules to the teaching of geometry.

Basic concepts

Teachers should make sure that the learners understand the basic concepts and terminology before progressing to more difficult questions.

Errors

The teachers and learners should view errors more positively and spend more time exploring them. Diagnosing and exploring errors can be a very powerful educational but also a mathematical tool, therefore errors should not be regarded only as a negative aspect in terms of “something has gone wrong” but an opportunity to explore ways of thinking.

Van Hiele theory

It may help teachers if they were more aware of the Van Hiele levels of geometric thought and the progression of learners through the levels.

8.5.2 Recommendations for further research

Van Hiele test for learners

From this study, and especially after discussion of the findings with the teachers, the researcher realised that it would be a great help if there was a standardised test that the teachers could use to give them an indication of how to plan their instruction and to help them to know which learners are behind and need more attention. Further research is needed to assess the needs of the teachers and to develop tests that would be easy to administer and mark but also supply reliable and valid results.

Language

Although this study set out during the first phase to measure the learners' level of geometric thought, the learners' understanding of English was not taken into account. However, many language-related problems were encountered. Therefore the question may be asked: Are the learners performing poorly in the test because they are challenged when reading and writing English? Another question that may be asked is whether their verbal language is better than their written language in mathematics? These questions need further investigation.

Spatial ability

The spatial ability of learners was not investigated in this study but might impact on their understanding/progression through Van Hiele level 1 (visualisation). It would be interesting to assess the effect of an improvement in the South African learners' spatial ability and their Van Hiele levels.

Errors

The third question of this study was on establishing a link between errors and the Van Hiele levels. Two follow-up questions for further research are:

- How can errors be used to help teachers and learners identify the Van Hiele levels of geometric thought and teach more effectively?
- Will the use of errors help to improve the Van Hiele levels of geometric thought of the learners?

8.6 FINAL WORD

Throughout the process of this study, the researcher has learnt to appreciate the dedication and enthusiasm of so many scholars concerning education and geometry through the ages

In South Africa, the teaching and learning of mathematics experience many challenges. The efficient use of error analysis in combination with an understanding of how the learners think in geometry might be worth considering as a useful tool in improving the geometry performance of our learners.

.

REFERENCE LIST

Abu, M. S., & Abidin, Z. Z. (2013). Improving the Levels of Geometric Thinking of Secondary School Students Using Geometry Learning Video based on Van Hiele Theory. *International Journal of Evaluation and Research in Education*, *2*(1), 16–22.

Alex, J. K. (2012). *The Evaluation of an Instructional Framework using the Van Hiele Levels for Learning and Teaching Geometry: A Study in Five Rural Senior Secondary Schools*. Unpublished Doctor of Education thesis. Walter Sisulu University, Mthatha, South Africa.

Alex, J. K., & Mammen, K. J. (2012). A Survey of South African Grade 10 Learners' Geometric Thinking Levels in Terms of the Van Hiele Theory. *Anthropologist*, *14*(2), 123–129.

Allen, D. G. (2007). *Student Thinking Lesson 1 Misconceptions in mathematics*. Texas TX7843-3368.

Anderson, L. W. (1999). *Rethinking Bloom's Taxonomy: Implications for Testing Assessment*. Syracuse, New York.

Ashlock, R. B. (2006). *Error Patterns in Computation- Using Error Patterns to Improve Instruction*. (L. A. Montgomery, Ed.) (9 th). Upper Saddle River, N.J: Pearson Education Inc.

Atebe, H. U., & Schäfer, M. (2008). 'As soon as the four sides are all equal, then the angles must be 90° each'. Children's misconceptions in geometry. *African Journal of Research in Mathematics, Science and Technology Education*, *12*(2), 47–65.

Atebe, H. U., & Schäfer, M. (2010a). Beyond Teaching Language: Towards Terminological Primacy in Learners' Geometric Conceptualisation. *Pythagoras*, *71*(July), 53–64.

Atebe, H. U., & Schäfer, M. (2010b). Research evidence on geometric thinking level hierarchies and their relationship with students' mathematical performance. *Journal of Science Teachers Association of Nigeria*, *45*(1), 75–84.

Atebe, H. U., & Schäfer, M. (2011). The nature of geometry instruction and observed learning-outcomes opportunities in Nigerian and South African high schools. *African Journal of Research in Mathematics, Science and Technology Education*, *15*(2), 191–204.

Autrey, A., Burroughs, M., & Fertig, H. (2013). Student Struggles : Solving for Missing Angle Measures. *Mathematics Teaching*, *234*, 14–18.

Ball, D. L., & Friel, S. N. (1991). Implementing the ‘ Professional Standards for Teaching Mathematics ’: What’s All This Talk about ‘ Discourse ’? *The Arithmetic Teacher*, *39*(3), 44–48.

Bansilal, S., Brijlall, D., & Mkhwanazi, T. (2014). An exploration of the common content knowledge of high school mathematics teachers. *Perspectives in Education*, *32*(1), 34–50.

Battista, M. T. (2007). The Development of Geometric and Spatial Thinking. In F. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 843–905). Reston, VA: National council of Teachers of Mathematics.

Berger, M., Bowie, L., & Nyaumwe, L. (2010). Taxonomy Matters : Cognitive Levels and Types of Mathematical Activities in Mathematics Examinations. *Pythagoras*, *71*(July), 30–40.

Bhaskar, P. V. (2014). A Diagnostic Study of Common Errors Committed by VIII Class Students in Geometry. *Indian Journal of Research*, *3*(1), 142–145.

Biber, C., Tuna, A., & Korkmaz, S. (2013). The Mistakes and the Misconceptions of the Eighth Grade Students on the Subject of Angles. *European Journal of Science and Mathematics Education*, *1*(2), 50–59.

Bishop, A. J. (1980). Spatial Abilities and Mathematics Education : A Review. *Educational Studies in Mathematics*, *11*(3), 257–269.

Blanco, L. J. (2001). Errors in the Teaching / Learning of the Basic Concepts of

Geometry. *International Journal for Mathematics Teaching and Learning*, *24*(May), 1–11.

Bleeker, C., Stols, G., & Van Putten, S. (2013). The Relationship between Teachers' Instructional Practices and their Learners' Level of Geometrical Thinking. *Perspectives in Education*, *31*(3), 66 – 78.

Borasi, R. (1987). Exploring Mathematics through the Analysis of Errors *. *For The Learning of Mathematics*, *3*(November), 2–8.

Briggs, D. C., Alonzo, A. C., Schwab, C., & Wilson, M. (2006). Diagnostic Assessment With Ordered Multiple-Choice Items. *Educational Assessment*, *11*(1), 33–63.

Brown, S. (2002). What to do about John? In P. Schwardtz & G. Webb (Eds.), *Assessment Case Studies, Experience and Practice from Higher Education* (First, p. 196). London: Kogan Page Limited.

Burger, W. F., & Shaughnessy, M. J. (1986). Characterizing the Van Hiele Levels of Development in Geometry. *Journal for Research in Mathematics Education*, *17*(1), 31–48.

Carcary, M. (2009). The Research Audit Trial – Enhancing Trustworthiness in Qualitative Inquiry. *Journal Of Business Research Methods*, *7*(1), 11–24.

Carcary, M. (2011). Evidence Analysis Using CAQDAS : Insights From a Qualitative Researcher. *The Electronic Journal of Business Research Methods*, *9*(1), 10–24.

Clements, D. H. (2003). Teaching and Learning Geometry. In J. Kilpatrick, W. Martin, & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 151 – 178). Reston, VA: National Council of Teachers.

Clements, D. H., & Battista, M. T. (1992). Geometry and Spatial Reasoning. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 420–464). New York: Macmillan Publishing Company.

Cohen, L., Manion, L., & Morrison, K. (2007). *Research Methods in Education* (6 th). New York: Routledge.

Cranfield, C. G. (2001). *An Investigation into how Grade 12 Students Understand and Solve Geometric Problems*. Unpublished master's dissertation, University of Cape Town, South Africa.

Creswell, J. W., & Plano Clark, V. L. (2007). *Designing and Conducting Mixed Methods Research*. (L. C. Shaw, K. Greene, D. Santoyo, & C. Chilton, Eds.) (1 st). Thousand Oaks: Sage Publications Inc.

Crowley, M. L. (1987). The Van Hiele Model of the Development of Geometric Thought. In M. Montgomery Lindquist (Ed.), *Learning and Teaching Geometry, K - 12 1987 Yearbook of the National Council of Teachers of Mathematics* (pp. 1–16). Reston, VA: National Council of Teachers of Mathematics.

Crowley, M. L. (1990). Criterion-Referenced Reliability Indices Associated with the van Hiele Geometry Test. *Journal for Research in Mathematics Education*, *21*(3), 238–241.

Davey, G., & Holliday, J. (1992). Van Hiele: Guidelines for Geometry. *Australian Mathematics Teacher*, *48*(2), 26 – 29.

Daymunde, L. K. H. (2010). *Test Error Analysis in Mathematics Education: A Mixed Method Study*. Unpublished doctoral thesis, University of Georgia, Athens, Georgia.

De Villiers, M. (1994). The Role and Function of a Hierarchial Classification of Quadrilaterals. *For The Learning of Mathematics*, *14*(1), 11–18.

De Villiers, M. (1996). The Future of Secondary School Geometry. In *SOSI Geometry Imperfect Conference* (p. 35). Pretoria: UNISA.

De Villiers, M. (1998). To teach definitions in geometry or teach to define? In *Proceedings of the 22nd International Conference for the Psychology of Mathematics Education* (Vol. 2, pp. 248–255).

De Villiers, M. (2012). Some Reflections on the Van Hiele theory. In *National Mathematics Congress* (pp. 21–23). Swakopmund, Namibia.

De Villiers, M., & Njisane, R.. (1987). The Development of Geometric Thinking among

Black High School Pupils in Kwazulu Natal. In *Conference of the International Group for the Psychology of Mathematics Education* (pp. 117–123). Montreal.

De Vos, A. S., Strydom, H., Fouché, C. B., & Delport, C. S. L. (2007). *Research at Grass Roots for the Social Sciences and Human Service Professions*. (R. Odendaal, Ed.) (3 rd). Pretoria: Van Schaik Publishers.

Department of Basic Education. (2011a). *Curriculum and assessment Policy Statement Further Education and Training Phase Grades 10 - 12*. Pretoria: Government Printing Works.

Department of Basic Education. (2011b). *Curriculum and Assessment Policy Statement Grades 7-9 MATHEMATICS*. Pretoria: Government Printing Works.

Department of Basic Education. (2011c). *National Curriculum Statement Curriculum and Assessment Policy Statement: Foundation Phase Grades 1 - 3*. Pretoria: Government Printing Works.

Department of Basic Education. (2014a). *National Senior Certificate 2014 Schools Subject Report Mpumalanga*. Pretoria: Government Printing Works.

Department of Basic Education. (2014b). *National Senior Certificate Examination 2014 Diagnostic Report*. Pretoria: Government Printing Works.

Department of Basic Education. (2014c). *Report on the Annual National Assessment of 2014 gr 1 to 6 & 9*. Pretoria: Government Printing Works.

Department of Basic Education. (2014d). *The Annual National Assessment of 2014- Diagnostic Report Intermediate and Senior Phases Mathematics*.Pretoria: Government Printing Works.

Department of Basic Education. (2015a). *2015 National Senior Certificate Examination Diagnostic Report*. Pretoria: Government Printing Works.

Department of Basic Education. (2015b). *National Senior Certificate Examination Report 2015*. Pretoria: Government Printing Works.

Department of Education. (2008). *Education for All (EFA) Country Report : South*

Africa. Pretoria.

Dicky, N. G. (2011). Indonesian primary teachers' mathematical knowledge for teaching geometry: implications for educational policy and teacher preparation programs. *Asia-Pacific Journal of Teacher Education*, *39*(2), 151–164. doi:10.1080/1359866X.2011.560648

Dindyal, J. (2007). The Need for an Inclusive Framework for Students ' Thinking in School Geometry. *The Montana Mathematics Enthusiast*, *4*(1), 73–83.

Driscoll, D. L., Salib, P., & Rupert, D. J. (2007). Merging Qualitative and Quantitative Data in Mixed Methods Research : How To and Why Not. *Ecological and Environmental Anthropology*, *3*(1), 18–28.

Ellerton, N. F., & Clements, M. A. (1996). Newman Error Analysis: A Comparative Study Involving Year 7 Students in Malaysia and Australia Nerida F. In P. Clarkson (Ed.), *Technology in Mathematics Education* (pp. 186–193). Melbourne: Mathematics Education Research Group of Australasia.

Ellerton, N. F., & Olson, J. (2005). The Assessment Dilemma: Correct Answers with No Understanding and Incorrect Answers with Some Understanding. In H. S. Dhindsa, I. J. Kyeleve, O. Chukwu, & J. S. H Perera (Eds.), *Future Directions in Science, Mathematics and Technical Education* (pp. 226–235). Brunei: University Brunei Darussalam.

Even, R., & Tirosh, D. (2008). Teachers knowledge and understanding of students' mathematical learning and thinking. In L. D. English (Ed.), *Handbook of International Research in Mathematics Education* (pp. 202 – 222).

Fennema, E., & Franke, M. L. (1992). Teachers' Knowledge and its Impact. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 147–164). National Council of Teachers of Mathematics.

Feza, N., & Webb, P. (2005). Assessment standards, Van Hiele levels, and Grade Seven Learners' Understandings of Geometry. *Pythagoras*, *62*, 36–47.

French, D. (2004). *Teaching and Learning Geometry* (1st). London: Continuum

International Publishing Group.

Friese, S. (2012). *Qualitative Data Analysis with Atlas.ti* (1st). London: SAGE Publications Ltd.

Frykholm, J. A. (1994). *External Variables as Predictors of Van Hiele Levels in Algebra and Geometry Students*. Madison:University of Wisconsin.

Fuys, D. J., Geddes, D., & Tischler, R. (1988). The Van Hiele Model of Thinking in Geometry among Adolescents. *Journal for Research in Mathematics Education*, *3*(Monograph), 1–196.

Fuys, D. J., & Liebov, A. K. (1997). Concept Learning in Geometry. *Teaching Children Mathematics*, *3*(5), 248–251.

Gardee, A., & Brodie, K. (2015). A teacher's engagement with learner errors in her Grade 9 mathematics classroom. *Pythagoras*, *36*(2), 1–9.

Gonzalez, G., & Herbst, P. (2006). Competing Arguments for the Geometry Course: Why Were American High School Students Supposed to Study Geometry in the Twentieth Century? *The International Journal for the History of Mathematics Education*, *1*(1), 7–34.

Grant, D. (2013). W Cape ED Quintile Media release: Background to the National Quintile System [Media release by Minister of Education of Western Cape]. Western Cape Government.

Greene, J. C., Caracelli, V. J., & Graham, W. F. (1989). Toward a Conceptual Framework for Mixed-Method Evaluation Designs. *Educational Evaluation and Policy Analysis*, *11*(3), 255–274.

Gutiérrez, A., & Jaime, A. (1998). On the Assessment of the Van Hiele Levels of Reasoning. *Focus on Learning Problems in Mathematics*, *20*(2 & 3), 27–46.

Gutiérrez, A., Jaime, A., & Fortuny, J. M. (1991). An alternative paradigm to evaluate the acquisition of the van Hiele levels. *Journal for Research in Mathematics Education*, *22*, 237–251.

Hansen, A. (2011). *Children's Errors in Mathematics - Understanding Common Misconceptions in Primary Schools*. (A. Hansen, Ed.) (2 nd). Exeter: Learning Matters Ltd.

Heinze, A., & Reiss, K. (2007). Mistake-Handling Activities in the Mathematics Classroom: Effects of an In-Service Teacher Training on Students' Performance in Geometry. In J. H. Woo, H. C. Lew, K. S. Park, & D. Y. Seo (Eds.), *31 st Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 9–16). Seoul.

Hendricks, W. W. (2012). *The Effect of Collective Efficacy on the Introduction of a New Curriculum by Mathematics Teachers*. Unpublished Doctor of Philosophy thesis, Nelson Mandela Metropolitan University, Port Elizabeth, South Africa.

Henning, E., & Dampier, G. (2012). Linguistic Liminality in the Early Years of School : Urban South African Children ' betwixt and between ' Languages of Learning. *South African Journal of Childhood Education*, *2*(1), 100–119.

Hill, H. C., Sleep, L., Lewis, J. M., & Ball, D. L. (2007). Assessing Teachers' Mathematical Knowledge. In F. K. J. Lester (Ed.), *Second Handbook on Research Mathematics Teaching and Learning* (pp. 111 – 155). United States: Information Age Publishing Inc.

Howie, S. (2002). *English Language Proficiency and Contextual Factors Influencing Mathematics Achievement of Secondary School Pupils in South Africa*. Unpublished doctoral thesis, University of Twente, Enschede, Netherlands.

Howie, S., & Plomp, J. (2003). Language Proficiency and Contextual Factors Influencing Secondary Students' Performance in Mathematics in South Africa. In *Annual Meeting of the American Educational Research Association* (p. 23). Chicago.

Howse, T. D., & Howse, M. E. (2014). Linking the van Hiele Theory to Instruction. *Teaching Children Mathematics*, *21*(5), 304–313.

Human Sciences Research Council. (2011). *Highlights from TIMSS 2011: The South African perspective*. Pretoria: Human Sciences Research Council.

Johnson, R. B., & Onwuegbuzie, A. J. (2004). Mixed Methods Research: A Research Paradigm Whose Time Has Come. *Educational Researcher*, *October*, 15–26.

Johnstone-Wilder, S., Mason, J., Begg, A., Faux, G., Lopez-Real, F., & Wake, G. (2007). *Developing Thinking in Geometry*. (S. Johnstone-Wilder & J. Mason, Eds.) (1st ed.). London: Paul Chapman Publishing.

Jones, B. L., Collis, K. F., & Watson, J. M. (1993). Towards a Theoretical Basis for Students' Alternative Frameworks in Science and for Science Teaching. *Research in Science Education*, *23*(1), 126 – 135.

Jones, K. (2002). Issues in the teaching and learning of geometry. In L. Haggarty (Ed.), *Aspects of Teaching Secondary Mathematics: Perspectives on practice* (pp. 121–139). London: RoutledgeFalmer.

Kembitzky, K. A. (2009). *Addressing Misconceptions in Geometry through Written Error Analyses*. Unpublished Doctor of Philosophy thesis, The Ohio State University, Columbus, United States of America.

Khembo, E. (2011). *An Investigation into Grade 6 Teachers' understanding of Geometry According to the Van Hiele level of Geometric Thought*. Unpublished Master of Education dissertation, University of the Witwatersrand, Johannesburg, South Africa.

Kim, J. S. (2011). *Preservice Teachers' Knowledge of Content and Students in Geometry*. Unpublished Doctor of Philosophy thesis, University of Georgia, Athens, Georgia.

Krefting, L. (1991). Rigor in Qualitative Research: The Assessment of Trustworthiness. *The American Journal of Occupational Therapy*, *45*(3), 214–222.

Labinowicz, E. (1980). *The Piaget Primer. Thinking, Learning, Teaching*. Menlo Park, California: Addison-Wesley Publishing Company.

Lehrer, R., Jenkins, M., & Osana, H. (1998). Longitudinal Study of Children's Reasoning about Space and Geometry. In R. Lehrer & D. Chazan (Eds.), *Designing Learning Environments for Developing Understanding of Geometry* *and Space* (p. 504). New York: Routledge.

Long, C., Dunne, T., & De Kock, H. (2014). Mathematics, Curriculum and Assessment : The Role of Taxonomies in the Quest for Coherence. *Pythagoras*, *35*(2), 1–14.

Luneta, K. (2015). Understanding Students' Misconceptions : An Analysis of fFnal Grade 12 Examination Questions in Geometry. *Pythagoras*, *36*(1), 1–11.

Ma, L. (1999). *Knowing and Teaching Elementary Mathematics. Teachers' Understanding of Fundamental Mathematics in China and the United States*. London: Lawrence Erlbaum Associates.

Marchis, I. (2012). Preservice Primary School Teachers' Elementary Geometry Knowledge. *Acta Didactica Napocensia*, *5*(2), 33–40.

Mariotti, M. A. (2004). Reasoning, proof and proving in mathematics education Regular Lecture based on the work of Survey Team 2. In *10th International Congress on Mathematical Education (ICME-10)* (p. 19). Copenhagen.

Martin, J. D. (2007). *Children's Understanding of Area of Rectangular Regions and Volume of Rectangular Shapes and the Relationship of These Measures to Their Linear Dimensions*. Unpublished dissertation, Tufts University, Somerville, Massachusetts, United States of America.

Mason, M. (2002). The Van Hiele Levels of Geometric Understanding. In *Professional Handbook for Teachers* (Vol. Geometry: pp. 4–8). McDougal Littell Inc.

May, T. (2011). *Social Research: Issues, Methods and Process* (4th ed.). Maidenhead, Berkshire: Open University Press.

Mayberry, J. (1983). The Van Hiele Levels of Geometric Thought in Undergraduate Pre-service Teachers. *Journal for Research in Mathematics Education*, *14*(1), 58–69.

McBride, B., & Carifio, J. (1995). Empirical Results of Using an Analytic versus Holistic Scoring Method to Score Geometric Proofs: Linking and Assessing Greeno, Bloom and Van Hiele. In *Annual Meeting of the American Educational Research*

Association (p. 56). San Francisco: ERIC.

McEamara, J., & Shaughnessy, M. M. (2011). Student errors : What can they tell us about what students DO understand ? Retrieved 29 May 2015, from www.mathsolutions.com

Melis, E. (2004). Erroneous Examples as a Source of Learning in Mathematics. In P. Kinshuk & D. G. Samson (Eds.), *IADIS International Conference: Cognitive and Exploratory Learning in the Digital Age* (pp. 311–318). Lisbon, Portugal.

Merriam, S. B. (1991). *Case Study Research in Education: A Qualitative Approach* (1 st). San Francisco: Jossey- Bass Publishers.

Merriam, S. B. (1998). *Qualitative Research and Case Study Applications in Education: Revised and Expanded from Case Study Research in Education*. San Francisco: Jossey-Bass Publishers.

Mji, A., & Makgato, M. (2006). Factors Associated with High School Learners' Poor Performance: A Spotlight on Mathematics and Physical Science. *South African Journal of Education*, *26*(2), 253–266.

Mogari, D. (2003). A Relationship between Attitude and Achievement in Euclidean Geometry of Grade 10 Pupils. *African Journal of Research in Mathematics, Science and Technology Education*, *7*, 63–72.

Moss, J., Hawes, Z., Naqvi, S., & Caswell, B. (2015). Adapting Japanese Lesson Study to Enhance the Teaching and Learning of Geometry and Spatial Reasoning in Early Years Classrooms : A Case Study. *ZDM:The International Journal on Mathematics Education*, *47*(3), 377–390. doi:10.1007/s11858-015-0679-2

Mulligan, J. (2015). Looking Within and Beyond the Geometry Curriculum: Connecting Spatial Reasoning to Mathematics Learning. *ZDM:The International Journal on Mathematics Education*, *47*(3), 511–517. doi:10.1007/s11858-015-0696-1

Mullis, I. V. S., Martin, M. O., Foy, P., & Arora, A. (2012). *Results in Mathematics* (1st ed.). Chestnut Hill, MA: TIMSS & PIRLS International Study Center.

National Planning Commission. (2012). *National Development Plan 2030 Our Future*

- *make it work*. Pretoria: Sherino Printers.

Ndlovu, M., & Mji, A. (2012). Pedagogical Implications of Students' Misconceptions about Deductive Geometric Proof. *Acta Academia*, *44*(3), 175–205.

Oberdorf, C. D., & Taylor-Cox, J. (1999). Shape Up ! Uncovering the Roots of Misconceptions. *Teaching Children Mathematics*, *5*(6), 340–345.

Olivier, A. (1992). Handling Pupils' Misconceptions. In M. Moodley, R. Njisane, & N. Presmeg (Eds.), *Mathematics Education for Pre-Service and In-Service* (pp. 193–209). Pietermaritzburg: Shuter & Shooter.

Opdenakker, R. (2006). Advantages and Disadvantages of Four Interview Techniques in Qualitative Research. *Forum:Qualitative Social Research*, *7*(4), 1–10.

Oxford University Press. (2016). Oxford Dictionaries Language Matters. Retrieved 23 March 2016, from www.oxforddictionaries.com/definitions

Ozerem, A. (2012). Misconceptions In Geometry And Suggested Solutions For Seventh Grade Students. *Procedia - Social and Behavioural Sciences*, *55*, 720–729.

Padmavathy, R. D. (2015). Diagnostic of Errors Committed by 9 th Grade Students in Solving Problems in Geometry. *International Journal for Research in Education*, *4*(1), 28–31.

Pegg, J. (1992). Students' Understanding of Geometry: Theoretical Perspectives. In B. Southwell, B. Perry, & O. Kaye (Eds.), *15 th annual conference of Mathematics Education Research Group of Australasia* (pp. 18–34). Sydney: MERGA.

Pegg, J. (1997). Broadening the Descriptors of van Hieles' Levels 2 and 3. In F. Biddulph & K. Carr (Eds.), *20 th Annual Conference of Mathematics Educational Research Group of Australasia* (pp. 391–396).

Pegg, J., & Davey, G. (1989). Clarifying Level Descriptors for Children's Understanding of Some Basic 2-D Geometric Shapes. *Mathematics Education Research Journal*, *1*(1), 16–27.

Pistorius, P. (1982). *Gister en Vandag in die Opvoeding* (9 th). Potchefstroom: Pro Rege-Pers Beperk.

Pusey, E. L. (2003). *The Van Hiele Model of Reasoning in Geometry: A Literature Review*. Unpublished Master of Science dissertation, North Carolina State University, Raleigh, United States of America.

Rach, S., Ufer, S., & Heinze, A. (2013). Learning from Errors: Effects of Teachers Training on Students' Attitudes Towards and their Individual Use of Errors. *PNA*, *8*(1), 21–30.

Radatz, H. (1979). Error Analysis in Mathematics Education. *Journal for Research in Mathematics Education*, *10*(3), 163–172.

Rodriquez, M. C., & Haladyna, T. M. (2013). Writing Selected-Response Items for Classroom Assessment. In J. H. McMillan (Ed.), *Sage Handbook of Research on Classroom Assessment* (pp. 293–311). Thousand Oaks: SAGE Publications, Inc.

Roux, A. (2005). The Influence of Language Proficiency on Geometric Thinking. *SA-eDUC Journal*, *2*(1), 1–9.

Ryan, J., & McCrae, B. (2005). Assessing Pre-service Teachers' Mathematics Subject Knowledge. *Mathematics Teacher Education and Development*, *7*, 72–89.

Saito, M. (2011). *Trends in Gender Equality in Learning Achievement in Southern and Eastern Africa: Exploration of Characteristics of Educational Environment and Curriculum Areas*. *An IIEP evidence-based Policy Forum*. Paris.

Salifu, I., & Agbenyega, J. S. (2013). Teacher Motivation and Identity Formation : Issues Affecting Professional Practice. *MIER Journal of Educational Studies, Trends and Practices*, *3*(1), 58–74.

Schleppenbach, M., Flevares, L. M., Sims, L. M., & Perry, M. (2007). Teachers' Responses to Student Mistakes in Chinese and U.S. Mathematics Classrooms. *The Elementary School Journal*, *108*(2), 131–147.

Seale, C. (1999). Quality in Qualitative Research. *Qualitative Inquiry*, *5*(4), 465–478.

Senk, S. L. (1982). Achievement in Writing Geometry Proofs. In *Annual Meeting of the American Educational Research Association* (p. 30). New York.

Senk, S. L. (1985). How Well Do Students Write Geometry Proofs? *Mathematics Teacher*, *78*(6), 448–456.

Senk, S. L. (1989). Van Hiele Levels and Achievement in Writing Geometry Proofs. *Journal for Research in Mathematics Education*, *20*(3), 309–321.

Sibanda, B. (2012). *The Perceptions of Educators and Subject Advisors on the Bearing that the South African Mathematics Curriculum Design has on Underachievement in Schools in the King Williams Town Education District of the Eastern Cape*. Unpublished Doctor of Philosophy thesis, University of Fort Hare, East London, South Africa.

Siyepu, S. W. (2005). *The Use of Van Hiele's Theory to Explore problems encountered in Circle Geometry: A Grade 11 Case Study*. Unpublished Master of Education dissertation, Rhodes University, Grahamstown, South Africa.

Smith, E., & De Villiers, M. (1989). A Comparative Study of Two Van Hiele Testing Instruments. In *Conference for the Psychology of Mathematics Education* (p. 8). Paris.

Spaull, N. (2013). *South Africa's Education Crisis : The quality of education in South Africa 1994-2011* (Vol. 27). Johannesburg.

Spychiger, M., Kuster, R., & Oser, F. (2006). Dimensionen von Fehlerkultur in der Schule und deren Messung. Der Schülerfrageborgen zur Fehlerkultur i Unterricht für Mittel- und Oberstrufe. *Schweizerische Zeitschrift Für Bildungswissenschaften*, *28*(1), 87–110.

Stols, G., Mji, A., & Wessels, D. (2008). The Potential of Teacher Development with Geometer's Sketchpad. *Pythagoras*, *68*(December), 15–21.

Taylor, N., Fleisch, B., & Shindler, J. (2008). *Changes in Education since 1994*. Pretoria: Policy Unit of the Presidency.

Teppo, A. (1991). Van Hiele Levels of Geometric thought revisited. *The Mathematics*

Teacher, *84*(3), 210–221.

Thomas, G. (2013). *How To Do Your Research Project - A Guide for Students in Education and Applied Social Sciences*. (J. Seaman, Ed.) (2 nd). London: Sage Publications Inc.

Unal, H., Jakubowski, E., & Corey, D. (2009). Differences in Learning Geometry among High and Low Spatial Ability Pre-service Mathematics Teachers. *International Journal of Mathematical Education in Science and Technology*, *40*(8), 997–1012.

Usiskin, Z. (1982). *Van Hiele levels and achievements in secondary school geometry*. *(Final Report of the Cognitive Development in Secondary School Geometry Project.)* Chicago, Illinois: University of Chicago.

Vaiyavutjamai, P., & Clements, M. A. (2006). Effects of Classroom Instruction on Student Performance on, and Understanding of, Linear equations and Linear Inequalities. *Mathematical Thinking and Learning*, *8*(2), 113–147.

Van De Walle, J. A. (2004). *Elementary and Middle School Mathematics: Teaching Developmentally*. (S. Dragin, Ed.) (5 th). Boston: Pearson Education Inc.

Van der Sandt, S., & Nieuwoudt, H. D. (2003). Grade 7 Teachers' and Prospective Teachers' Content Knowledge of Geometry. *South African Journal of Education*, *23*(3), 199–205.

Van Hiele, P. M. (1985). The Child's Thought and Geometry. In *English translation of selected writings of Dina van Hiele-Geldof and Pierre M Van Hiele* (pp. 243–252). Brooklyn, Brooklyn College.

Van Hiele, P. M. (1986). *Structure and Insight: A Theory of Mathematics Education*. (H. Beilin, Ed.)*Orlando* (United Kin). Orlando: Academic Press, Inc.

Van Putten, S. (2008). *Levels of Thought in Geometry of Pre-service Mathematics Educators according to the van Hiele Model*. Unpublished Master of Education dissertation, University of Pretoria, South Africa.

Venkat, H., & Adler, J. (2012). Coherence and Connections in Teachers' Mathematical

Discourses in Instruction. *Pythagoras*, *33*(3), 1–8.

Venkat, H., & Spaull, N. (2014). *What do we know about Primary Teachers' Mathematical Content Knowledge in South Africa ? An analysis of SACMEQ 2007*. Stellenbosch: University of Stellenbosch.

Vighi, P., & Marchini, C. (2011). A Gap between Learning and Teaching Geometry. In *Proceedings of the 7th Congres of the European Society of Mathematics Education* (p. 10). Rzeszow: University of Rzeszow.

Watson, C. (2012). *A Comparison of Van Hiele Levels and Final Exam Grades of Students at The University of Southern Mississippi*. Unpublished thesis submitted in partial fulfilment of the degree of Bachelor of Science, University of Southern Mississippi, Hattiesburg, United States of America.

White, A. L. (2010). Numeracy, Literacy and Newman's Error Analysis. *Journal of Science and Mathematics Education in Southeast Asia*, *33*(2), 129–148.

Wijaya, A., Van den Heuvel-Panhuizen, M., Doorman, M., & Robitzsch, A. (2014). Difficulties in Solving Context-based PISA Mathematics Tasks : An Analysis of Students' Errors. *The Mathematics Enthusiast*, *11*(3), 555–584.

Wilson, M. (1990). Measuring a Van Hiele Geometry Sequence: A Reanalysis. *Journal for Research in Mathematics Education*, *21*(3), 230–237.

Wu, D., & Ma, H. (2006). The Distributions of Van Hiele Levels of Geometric Thinking Among 1 st through 6 th Graders. In J. Novotna, H. Moraova, M. Kratka, & N. Stehlikova (Eds.), *International Group for the Psychology of Mathematics Education* (Vol. 5, pp. 409–416). Prague: PME.

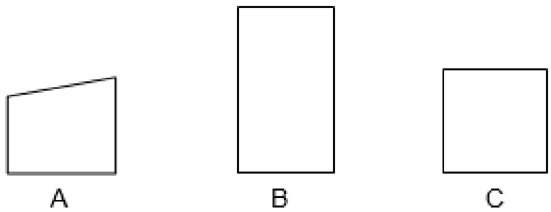
Appendix A: Grade 9 Geometry Test

Grade 9 Geometry Multiple Choice Section **C. Steyn**

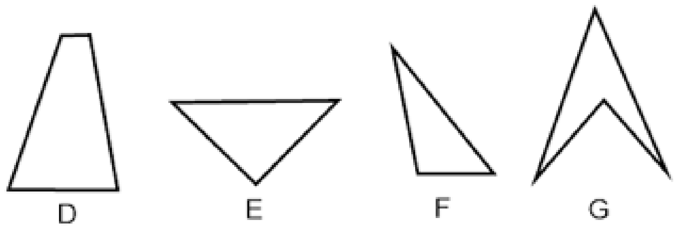
Instructions: 1. Read each question carefully. 2. Choose only one answer that you think is correct and make a cross in pencil over the corresponding number in the squares on the answer sheet. There is **<u>only one</u>** correct answer. 3. If you want to change your answer erase your original answer and clearly mark the new answer on your answer sheet. 4. There are 32 questions in this section and you have 40 minutes to complete it.

Questions:

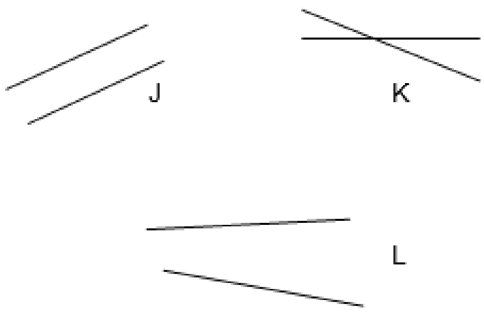
1. Which of these are squares?
   a) A
   b) B
   c) C
   d) B and C
   e) all are squares



2. Which of these are triangles?
   a) only D
   b) only E
   c) only F
   d) E and F
   e) E, F and G



3. Which of the pairs of lines look parallel?
   a) J
   b) K
   c) J and L
   d) All of them
   e) None of them



4. Which of these are rectangles?

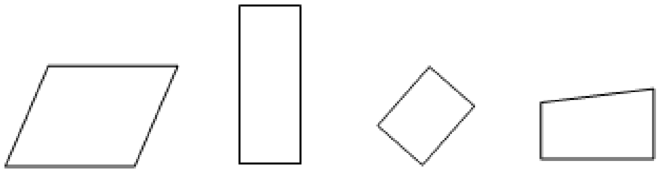
a) P and S

b) Q

c) Q and R

d) R

e) All of them



P Q R S

5. Which of the following sketches have alternate interior angles marked with an **x**?

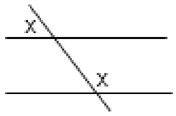
a) None of them

b) only T

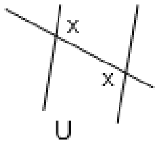
c) only U

d) only W

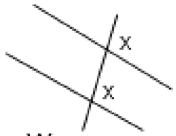
e) T and U



T



U



W

6. In which figure/s are the angles marked with an **x** right angles?

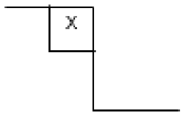
a) A and C

b) only A

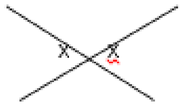
c) only B

d) only C

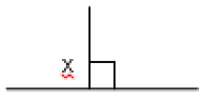
e) All of them



A

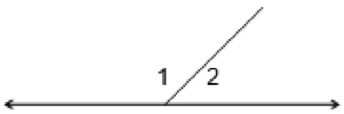


B



C

7. Angle A_1 and A_2 are ?



A

a) complementary

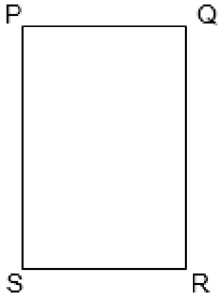
b) supplementary

c) corresponding

d) vertically opposite

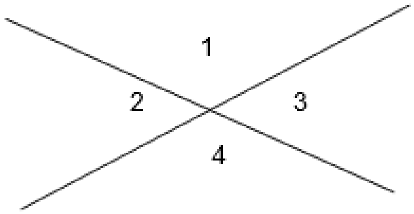
e) none of the above

8. PQRS is a rectangle because ......



a) it looks like a door
b) it is a parallelogram with all the angles 90°
c) both pairs of opposite sides are parallel
d) the sum of the angles equals 360°
e) none of the above

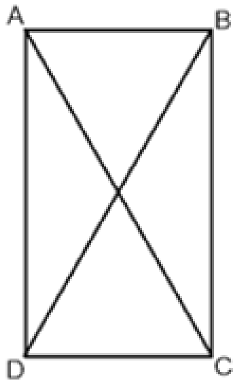
9. The two angles $\hat{1}$ and $\hat{4}$ in the sketch are called



a) supplementary
b) complementary
c) alternate interior
d) vertically opposite
e) all of the above

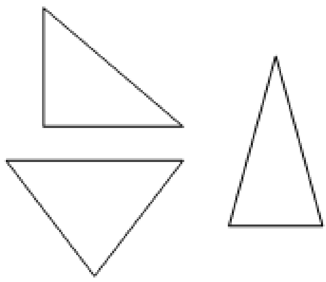
10. ABCD is a rectangle.

Which relationship is true in all rectangles?



a) AC and CD are equal in length
b) AC and BD are equal in length
c) AD and BC are perpendicular
d) AD and BD are equal in length
e) AC and BD are perpendicular

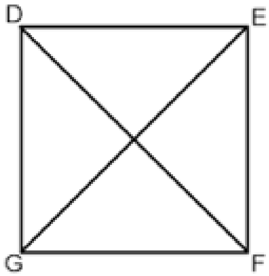
11. Here are three examples of isosceles triangles. Which statement is **true** for all isosceles triangles?



a) All three the angles are acute
b) All angles are equal to 60°
c) At least two angles must be equal
d) All three sides have the same length
e) At least one angle must be 90°

12. In the square DEFG,

DF and EG are the diagonals



Which of the statements a) – d) is **not true** in every square?

a) There are four right angles

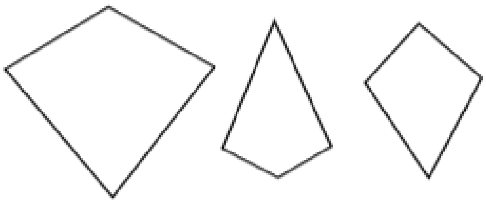
b) The two pairs of opposite sides are parallel

c) The diagonals have the same length

d) The diagonals bisect the angles

e) All the above statements are true for all squares

13. Here are three examples of a kite. A kite is a four sided figure with two pairs of adjacent sides equal.



Which of the following is **true** for all kites?

a) The two diagonals have the same length

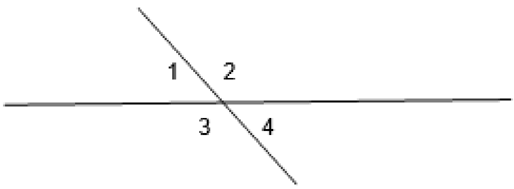
b) Each diagonal bisects two angles of a kite

c) The two diagonals are perpendicular

d) The opposite angles are equal in size

e) All the above statements are true for all kites

14. Which angles in the following are supplementary?



a) The sum of all four angles

b) 1 and 4

c) 2 and 3

d) 1 and 2

e) None

15. If you are given the following properties, which type of quadrilateral will they refer to?

Both pairs of opposite sides are parallel; all four sides have the same length; the diagonals bisect each other.

a) rectangle

b) square

c) rhombus

d) kite

e) none of the above

16. What is the maximum number of acute angles that you will find in a right angled triangle?

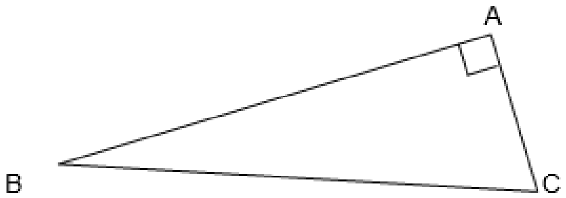
a) 1

b) 2

c) 3

d) none

e) Cannot give an exact number



17. How big is the angle at A and which reason is correct?

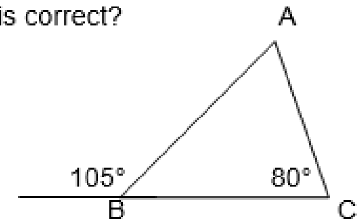
a) 25°, exterior angle of triangle

b) 25°, corresponding angles

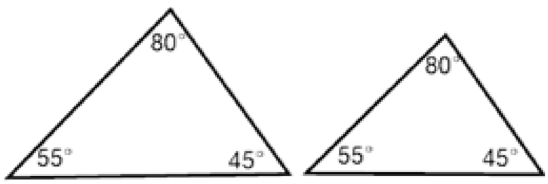
c) 75°, supplementary angles

d) 105°, alternate interior angles

e) 80°, vertically opposite angles



18. Consider the information shown on the sketch. These two triangles are similar because…



a) They look similar

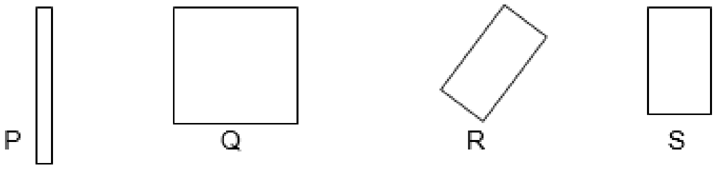
b) The corresponding sides are equal

c) The corresponding angles are equal

d) There are two sides and an enclosed angle equal

e) They are not similar

19. Which of these figures can be called rectangles?



a) All can

b) Only P

c) Only R

d) Only S

e) Only P, R and S

20. Which is **true**?

a) All similar triangles are also congruent

b) All properties of all parallelograms are properties of rectangles

c) All properties of parallelograms are properties of squares

d) All properties of squares are properties of all parallelograms

e) All isosceles triangles are similar

21. What do all congruent triangles have that some similar triangles do not have?

a) Corresponding sides are equal

b) Diagonals equal in length

c) Opposite sides parallel

d) Corresponding angles are equal

e) None of the above is true

22. Here are two statements about a figure. **Statement 1: Figure A is a circle**

Statement 2: Figure A is a square

a) Both are true

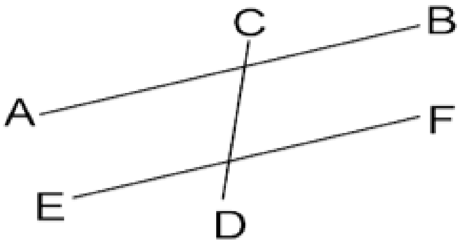
b) They cannot both be true

c) If 1 is true then 2 is also true

d) If 1 is false then 2 is true

e) None of the above is correct

23. Which statement below regarding the line segments AB, CD and EF is **not true**?



a) If there are corresponding angles, the lines AB and EF will always be parallel.

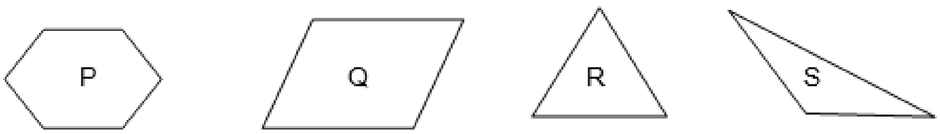
b) If alternate interior angles are equal the lines AB and EF will always be parallel

c) If co-interior angles are supplementary the lines AB and EF are always parallel

d) If the corresponding angles are equal the lines AB and EF will be parallel

e) If the alternate interior angles are equal the lines AB and EF will not necessarily be equal in length

24.



Which statement below is **true**?

a) R is not a quadrilateral because none of its angles are obtuse

b) Q is not a quadrilateral because its diagonals are not equal in length

c) P is not a quadrilateral because it can be divided into 6 equal triangles

d) S is not a quadrilateral because its angles add up to 180°

e) None of the figures can be classified as quadrilateral

25. If all the angles in two triangles are equal then.......

a) the sum of both their interior angles will be 360°

b) their sides are in proportion

c) each of their angles equal to 60°

d) a), b) and c) are correct and they are congruent

e) a), b) and c) are correct

26. Which statement concerning **all** quadrilaterals is correct?

a) Opposite angles are equal

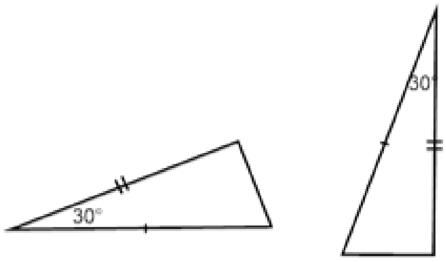
b) Opposite sides are equal

c) The interior angles add up to 360°

d) At least one pair of opposite sides are parallel

e) Both pairs of opposite sides are parallel

27. The following triangles are ......



a) similar because all corresponding sides are equal

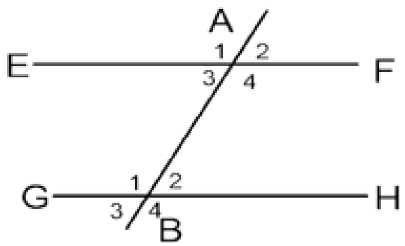
b) congruent because they look alike

c) similar because they look alike

d) congruent because they are similar

e) similar because they are congruent

28. Given $\hat{A}_3 = \hat{B}_2$. Which of the statements will be **false**?



Statement 1: EF will be parallel to GH

Statement 2: $\hat{A}_1 = \hat{B}_4$

Statement 3: AB intersects EF and GH

Statement 4: $\hat{A}_2 + \hat{B}_4 = 180°$

a) All statements are false

b) None of the statements are false

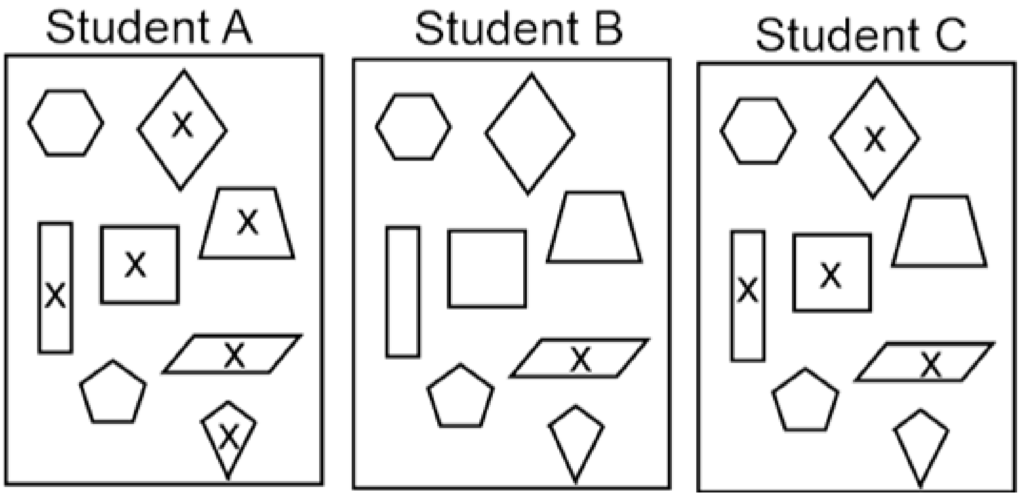
c) Statement 2 and 4 are false

d) Statement 2, 3 and 4 are false

e) Statement 4 is false

29. Three students were asked to mark all the parallelograms with an **x.**

Their answers are given below



Which student gave the correct answer?

a) A

b) B

c) C

d) All of them

e) None of them

30. What is an axiom?

a) a statement that must be proven before it can be used

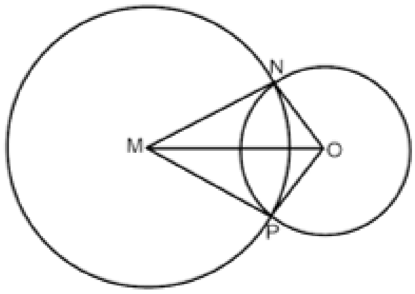
b) a fundamental assumption that serves as the basis for deduction of theorems

c) it is a well known theorem

d) It is a statement with many different proofs

e) All of the above

31. Two circles with centres M and O with radii of different lengths intersect at N and P. A four sided figure MNOP is formed. The two triangles MNO and MPO will be ....



a) not enough information is given

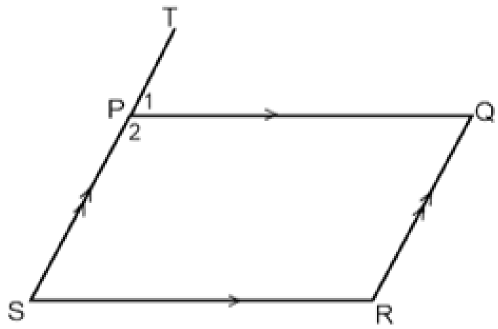
b) similar because all the sides are equal

c) similar because the corresponding sides are in proportion

d) congruent because the corresponding sides are equal

e) both b) and c) are true

32. Examine three statements concerning the figure below.



Statement 1: $\widehat{P_1} = \widehat{S}$

Statement 2: PQRS is a kite

Statement 3: $\widehat{P_2} = \widehat{R}$

Which of the following is **not true**?

a) Statements 1 and 3 are true if statement 2 is false

b) Statements 1 and 3 are false if statement 2 is true

c) Statement 2 is true if statement 3 is true

d) Statement 2 is false if statement 1 and 3 is true

e) Statement 1 is false if statement 2 is true

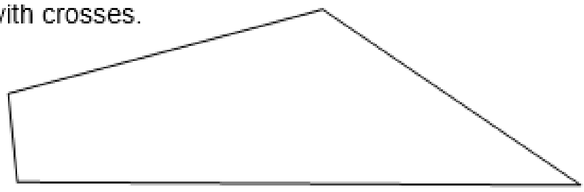
Open- ended question section

Instructions:

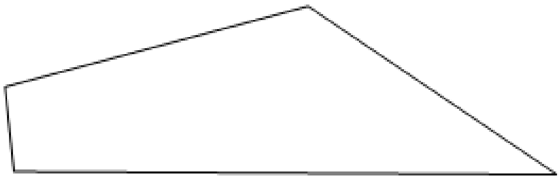
1. Read each question carefully and think carefully before you answer.
2. Please write down all your answers **on the question paper**. If you made a mistake and would like to use more paper to correct the answer ask the invigilator. Make very sure that you number your answer very clearly on the paper and write your name on the extra page. The extra page must be handed in with this question paper.
3. There are 22 questions in this section and you have 50 minutes to complete it.
4. Give full reasons where asked. Underline a **YES** or **NO** when required.

Questions:

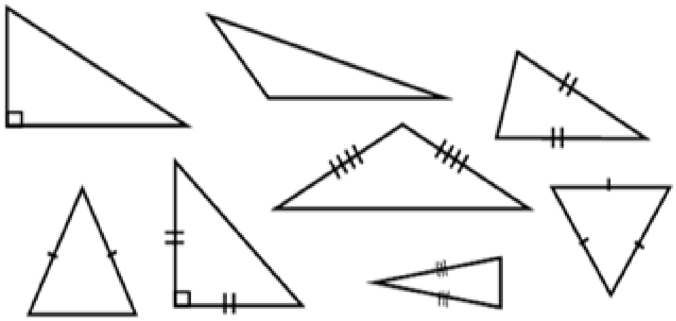
1. Mark any two opposite angles with crosses.



2. Mark any two opposite sides with crosses



3. Mark all the isosceles triangles with crosses

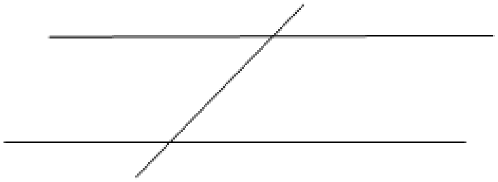


11

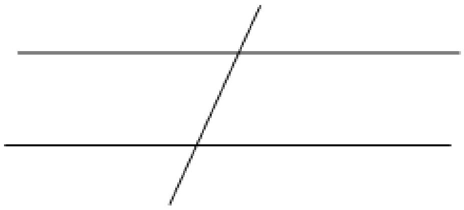
4. Draw any right angled triangle. Mark the hypotenuse on your triangle with a cross.

5. Draw any quadrilateral.

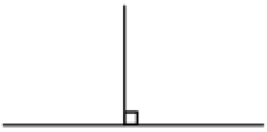
6. a) Mark any two corresponding angles with crosses.



b) Mark any two alternate interior angles with crosses.

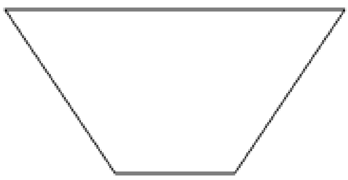


7. What are lines like the following called?



Answer:____________________

8. Draw a diagonal for the figure below. Then answer the questions.



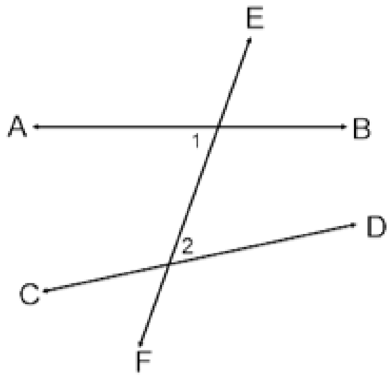
a) Will the diagonal also be a line of symmetry?

YES/NO

b) Give a reason for your answer

12

9. Two lines AB and CD are cut by another line EF as shown in the sketch.



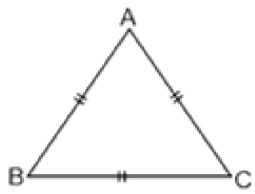
a) If the position of line CD is changed so that $\hat{1} = \hat{2}$ will CD be parallel to AB?

YES / NO

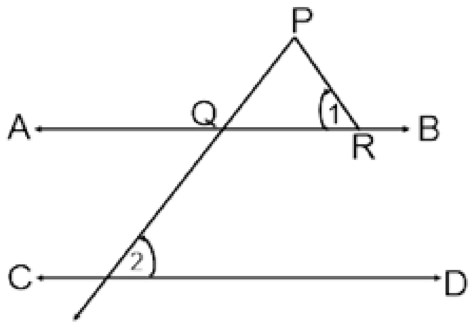
b) Give reasons for your answer.

10. a) How big is angle A in the sketch? ________

b) Give a reason for your answer.



11. In the given figure PQ = PR and $\hat{1} = \hat{2}$.



a) Is AB parallel to CD?

YES / NO

b) Give reasons for your answer.

12. A line PQ meets another line AB such that $A\hat{Q}P = B\hat{Q}P$.

a) Is $PQ \perp AB$? **YES / NO**

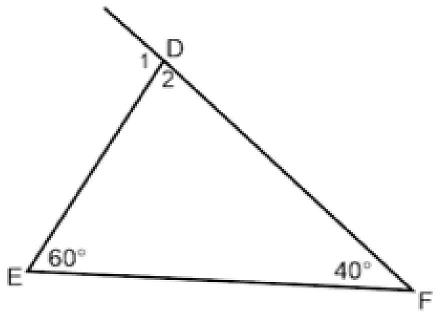
b) Give reasons for your answer.

13. a) How big is $\widehat{D}_2$ in the figure?

The figure is a scale drawing.

____

b) Write down how you got your answer? Give reasons.



c) Write down two methods in which $\widehat{D}_1$ can be determined (give reasons).

Method 1

Method 2

14

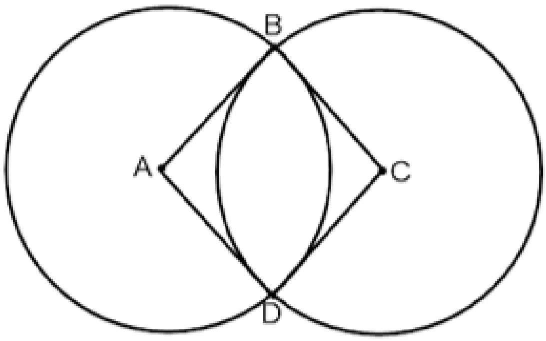
14. a) Is every square a rectangle? **YES / NO**

b) Give reasons for your answer.

15. a) Is every rectangle a parallelogram? **YES / NO**

b) Give reasons for your answer.

16. In the figure the two circles with midpoints at A and C, are the same size



a) Which type of quadrilateral is ABCD ?

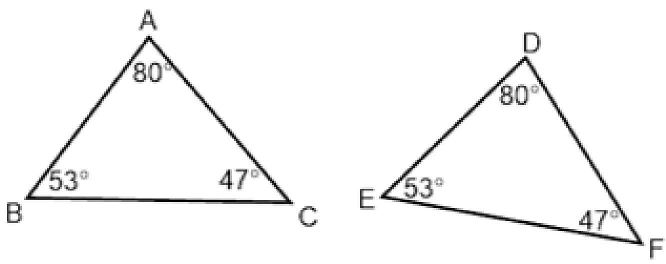
____

b) Prove your answer.

7. a) Are the two triangles similar or congruent ?

____

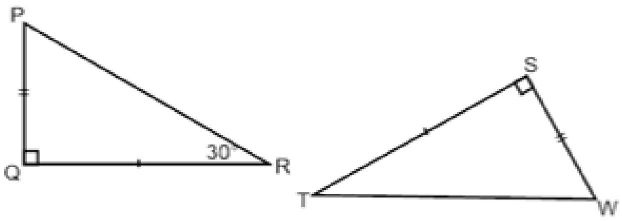
b) Prove your answer.



15

18. In the triangles PQ = SW

and QR = ST.



a) How big is angle P?

____

b) How did you get your answer?

c) How big is angle T? ____________

d) How did you get this answer? Show all the reasons.

19. The four angles A, B, C and D of a certain quadrilateral ABCD are all equal.

a) Is the quadrilateral definitely a square? **YES / NO**

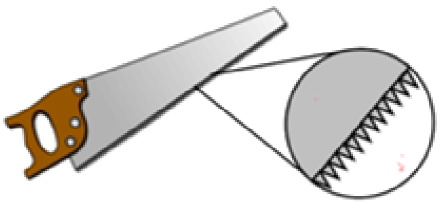
b) Give a reason for you answer.

20. Complete the following sentence:

The cutting side of the blade of a saw consists of small triangles.

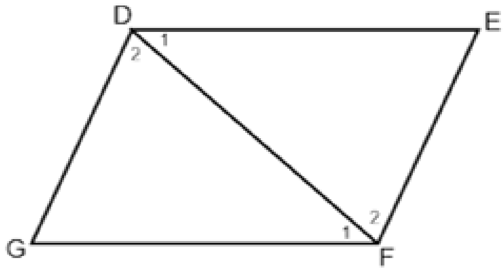
These triangles are (congruent/similar) because ____________________________

____



 16

21. The diagonal DF divides the parallelogram DEFG into two triangles.



a) Prove that the two triangles are congruent. Show all the reasons.

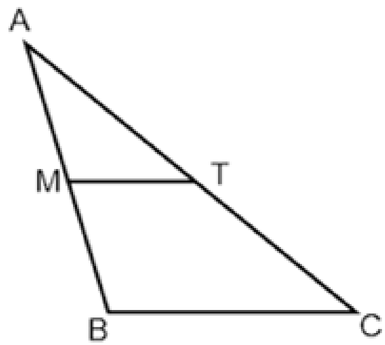
b) Will the diagonals always divide any parallelogram into congruent triangles?

YES / NO

Give a reason for your answer.

22. In the triangle M is the midpoint of AB and MT is parallel to BC.

a) Prove that Δ AMT is similar to Δ ABC.



b) What is the ratio of the sides in the two triangles?

c) Show how you got your answer in b).

ANSWER SHEET for Multiple choice section

Name: ____ Class: ____________

1	a	b	c	d	e
2	a	b	c	d	e
3	a	b	c	d	e
4	a	b	c	d	e
5	a	b	c	d	e
6	a	b	c	d	e
7	a	b	c	d	e
8	a	b	c	d	e
9	a	b	c	d	e
10	a	b	c	d	e
11	a	b	c	d	e
12	a	b	c	d	e
13	a	b	c	d	e
14	a	b	c	d	e
15	a	b	c	d	e
16	a	b	c	d	e
17	a	b	c	d	e
18	a	b	c	d	e
19	a	b	c	d	e
20	a	b	c	d	e
21	a	b	c	d	e
22	a	b	c	d	e
23	a	b	c	d	e
24	a	b	c	d	e
25	a	b	c	d	e
26	a	b	c	d	e
27	a	b	c	d	e
28	a	b	c	d	e
29	a	b	c	d	e
30	a	b	c	d	e
31	a	b	c	d	e
32	a	b	c	d	e

Level	Number Correct	% Correct
1		
2		
3		
4		
Total		

18

Appendix B: Interview questions to uncover the common errors in geometry

Semi-structured Interview questions

QUADRILATERALS AND TRIANGLES

1. Here are a few figures. Can you please separate them into groups of shapes that you think they belong to.

2. Please explain how you decided to group like you did.

3. Are all these triangles the same? If not show me what the differences are. If you had to make up a rule to sort them what would it be?

4. Are all these quadrilaterals the same? If not show me how they differ.

5. (Show a rectangle) What is this shape called? Describe as many properties of this figure as you can remember.

   (Show a parallelogram) Which of the properties of the rectangle are the same as the parallelogram?

   Can we say that a rectangle is a parallelogram? Why? / Why not?

6. Read the following aloud.

   " If four angles of a quadrilateral are equal, which quadrilateral will it be?"

   Do you understand the words? What shape could this be?

7. Show me an angle in this figure. Why is this an angle? Now name two opposite angles.

   Show me a side in this figure. Why is this a side? Now name two opposite sides.

8. Can you show me this hypotenuse in this triangle? What do you know about the hypotenuse?

PARALLEL LINES

9. Here we have two lines. Are they parallel?

   How do you know this?

10. If these lines are intersected by another line, can you show me all the angles that are equal and tell me why they are equal?

11. Will there be alternate angles in this figure? Why? / Why not?

12. (Question 11 in open-ended section of test)

ANGLES

13. Show me all the vertically opposite angles in the following figures?

    How do you know when angles are vertically opposite?

CONGRUENCY

14. When are triangles congruent? How can we prove that they are congruent?

    Will these two triangles be congruent? Why?

Appendix C: Consent by Education Department

Province of the
EASTERN CAPE
EDUCATION

Port Elizabeth District
Ethel Valentine Building • Sutton Road • Sidwell • Port Elizabeth • Eastern Cape
Private Bag X3931 • North End • Port Elizabeth • 6056 • REPUBLIC OF SOUTH AFRICA
Tel: +27 (0)41-4034400 • Fax: +27 (0)41-4510193 • Website: www.ecdoe.gov.za

Enquiries: Dr Nyathi Ntsiko   Email: nyathi.ntsiko@edu.ecprov.gov.za

Mrs C. Steyn
Researcher
c/o Dr T. Morar
Supervisor
Nelson Mandela Metropolitan University
Port Elizabeth
Email: tulsi.morar@nmmu.ac.za // steyn.carine@gmail.com

Dear Ms Steyn

REQUEST FOR PERMISSION TO CONDUCT RESEARCH IN DEPARTMENTAL SCHOOLS: PORT ELIZABETH

I refer to your letter dated 15 July 2015.

Permission is hereby granted for you to conduct your research on the following conditions:

1. Your research must be conducted on a voluntary basis.
2. All ethical issues relating to research must be honoured.
3. Your research is subject to the internal rules of the school, including its curricular programme and its code of conduct and must not interfere in the day-to-day routine of the school.

Kindly present a copy of this letter to the principal as proof of permission.

I wish you good luck in your research.

Yours faithfully

DR NYATHI NTSIKO
DISTRICT DIRECTOR: PORT ELIZABETH
/ab

20 July 2015
building blocks for growth

PGDP EASTERN CAPE
Ikamva eliqaqambileyo!

Appendix D: Ethics Clearance NMMU

• PO Box 77000 • Nelson Mandela Metropolitan University
• Port Elizabeth • 6031 • South Africa • www.nmmu.ac.za

Nelson Mandela Metropolitan University

for tomorrow

Chairperson: Research Ethics Committee (Human)
Tel: +27 (0)41 504-2235

Ref: [H15-EDU-ERE-017 /Approval]

Contact person: Mrs U Spies

7 September 2015

Dr T Morar
Faculty: Education
South Campus

Dear Dr Morar

PERFORMANCE AND ERROR ANALYSIS USING THE VAN HIELE LEVELS OF GEOMETRIC THOUGHT – A GRADE 9 CASE STUDY

PRP: Dr T Morar
PI: Ms C Steyn

Your above-entitled application served at Research Ethics Committee (Human) for approval.

The ethics clearance reference number is **H15-EDU-ERE-017** and is valid for three years. Please inform the REC-H, via your faculty representative, if any changes (particularly in the methodology) occur during this time. An annual affirmation to the effect that the protocols in use are still those for which approval was granted, will be required from you. You will be reminded timeously of this responsibility, and will receive the necessary documentation well in advance of any deadline.

We wish you well with the project. Please inform your co-investigators of the outcome, and convey our best wishes.

Yours sincerely

CB Cilliers

Prof C Cilliers
Chairperson: Research Ethics Committee (Human)

cc: Department of Research Capacity Development
Faculty Officer: Education

Appendix E: List of all codes used in Atlas.ti

Code-Filter: All

HU: Analysis of errors recoding

File: [C:\Users\C Steyn\Documents\Scientific Software\ATLASti\TextBank\Analysis of errors recoding.hpr7]

Edited by: Super

Date/Time: 2016-06-21 13:00:09

A_Angle name specific

A_Angles

A_Suppl Angle

AA_Recoded

BC_Calculation wrong

BC_Opposite

BC_Perpendicular lines

BC_Poor language

BC_Sides

BC_Symbols

BC_Terminology

CM_Careless calculation

CM_Careless omitted

Inappropriate answer

Instructions_do not follow

Misunderstands question

No Answer

Orientation Changes

P_Alternate angles

P_Corresponding angles

P_Parallelness

P_Proof

Q_Axis Symmetry_know

Q_Diagonal- draw

Q_Give properties

Q_Hierarchy

Q_Kite

Q_Parallelogram

Q_Proof

Q_Rectangle

Q_Rhombus

Q_Square

Q_Use prop to name

R_Incorrect reason

R_No reason

R_Partial reason

R_Statement_Incorrect

R_Statement_Incorrect Order

R_Statement_No

R_Statement_Partial

Ratios

SC_Congruent_No idea

SC_Congruent_No proof

SC_Congruent_Only Steps

SC_Congruent_Similar_Confusion

SC_Similar_No idea

SC_Similar_No proof

SC_Similar_Only Steps

Scores T2

Tri _Calculation_Ext angle

Tri _Equilateral

Tri_Calculation_Int angle

Tri_Hypotenuse

Tri_Isoceles

Tri_Right Angled

VH_Lev 0

VH_Lev 1

VH_Lev 2

VH_Lev 3

Appendix F: Tables with data collected in each question of the multiple choice section

The tables below are only for Highest level = 0

Q1

		Frequency	Percent	Valid Percent
Valid	a	2	1.5	1.6
	b	4	3.1	3.1
	c	107	81.7	82.9
	d	11	8.4	8.5
	e	5	3.8	3.9
	Total	129	98.5	100.0
Missing	System	2	1.5	
Total		131	100.0	

Q2

		Frequency	Percent	Valid Percent
Valid	b	33	25.2	25.6
	c	2	1.5	1.6
	d	52	39.7	40.3
	e	42	32.1	32.6
	Total	129	98.5	100.0
Missing	System	2	1.5	
Total		131	100.0	

Q3

		Frequency	Percent	Valid Percent
Valid	a	85	64.9	65.4
	b	11	8.4	8.5
	c	25	19.1	19.2
	d	3	2.3	2.3
	e	6	4.6	4.6
	Total	130	99.2	100.0
Missing	System	1	.8	
Total		131	100.0	

Q4

		Frequency	Percent	Valid Percent
Valid	a	16	12.2	12.2
	b	82	62.6	62.6
	c	24	18.3	18.3
	d	4	3.1	3.1
	e	5	3.8	3.8
	Total	131	100.0	100.0

Q5

		Frequency	Percent	Valid Percent
Valid	a	3	2.3	2.3
	b	58	44.3	44.6
	c	19	14.5	14.6
	d	24	18.3	18.5
	e	26	19.8	20.0
	Total	130	99.2	100.0
Missing	System	1	.8	
Total		131	100.0	

Q6

		Frequency	Percent	Valid Percent
Valid	a	39	29.8	30.0
	b	34	26.0	26.2
	c	24	18.3	18.5
	d	25	19.1	19.2
	e	8	6.1	6.2
	Total	130	99.2	100.0
Missing	System	1	.8	
Total		131	100.0	

Q7

		Frequency	Percent	Valid Percent
Valid	a	8	6.1	6.1
	b	10	7.6	7.6
	c	45	34.4	34.4
	d	61	46.6	46.6
	e	7	5.3	5.3
	Total	131	100.0	100.0

Q8

		Frequency	Percent	Valid Percent
Valid	a	22	16.8	16.8
	b	28	21.4	21.4
	c	52	39.7	39.7
	d	18	13.7	13.7
	e	11	8.4	8.4
	Total	131	100.0	100.0

Q9

		Frequency	Percent	Valid Percent
Valid	a	8	6.1	6.1
	b	7	5.3	5.3
	c	40	30.5	30.5
	d	72	55.0	55.0
	e	4	3.1	3.1
	Total	131	100.0	100.0

Q10

		Frequency	Percent	Valid Percent
Valid	a	13	9.9	9.9
	b	59	45.0	45.0
	c	32	24.4	24.4
	d	9	6.9	6.9
	e	18	13.7	13.7
	Total	131	100.0	100.0

Q11

		Frequency	Percent	Valid Percent
Valid	a	14	10.7	10.9
	b	20	15.3	15.5
	c	36	27.5	27.9
	d	36	27.5	27.9
	e	23	17.6	17.8
	Total	129	98.5	100.0
Missing	System	2	1.5	
Total		131	100.0	

Q12

		Frequency	Percent	Valid Percent
Valid	a	23	17.6	17.7
	b	25	19.1	19.2
	c	14	10.7	10.8
	d	29	22.1	22.3
	e	39	29.8	30.0
	Total	130	99.2	100.0
Missing	System	1	.8	
Total		131	100.0	

Q13

		Frequency	Percent	Valid Percent
Valid	a	40	30.5	31.0
	b	20	15.3	15.5
	c	14	10.7	10.9
	d	32	24.4	24.8
	e	23	17.6	17.8
	Total	129	98.5	100.0
Missing	System	2	1.5	
Total		131	100.0	

Q14

		Frequency	Percent	Valid Percent
Valid	a	32	24.4	24.4
	b	31	23.7	23.7
	c	23	17.6	17.6
	d	32	24.4	24.4
	e	13	9.9	9.9
	Total	131	100.0	100.0

Q15

		Frequency	Percent	Valid Percent
Valid	a	20	15.3	15.3
	b	65	49.6	49.6
	c	38	29.0	29.0
	d	7	5.3	5.3
	e	1	.8	.8
	Total	131	100.0	100.0

Q16

		Frequency	Percent	Valid Percent
Valid	a	34	26.0	26.2
	b	27	20.6	20.8
	c	23	17.6	17.7
	d	14	10.7	10.8
	e	32	24.4	24.6
	Total	130	99.2	100.0
Missing	System	1	.8	
Total		131	100.0	

Q17

		Frequency	Percent	Valid Percent
Valid	a	35	26.7	26.7
	b	33	25.2	25.2
	c	26	19.8	19.8
	d	18	13.7	13.7
	e	19	14.5	14.5
	Total	131	100.0	100.0

Q18

		Frequency	Percent	Valid Percent
Valid	a	15	11.5	11.5
	b	40	30.5	30.8
	c	49	37.4	37.7
	d	14	10.7	10.8
	e	12	9.2	9.2
	Total	130	99.2	100.0
Missing	System	1	.8	
Total		131	100.0	

Q19

		Frequency	Percent	Valid Percent
Valid	a	11	8.4	8.4
	b	3	2.3	2.3
	c	9	6.9	6.9
	d	24	18.3	18.3
	e	84	64.1	64.1
	Total	131	100.0	100.0

Q20

		Frequency	Percent	Valid Percent
Valid	a	46	35.1	35.1
	b	16	12.2	12.2
	c	11	8.4	8.4
	d	18	13.7	13.7
	e	40	30.5	30.5
	Total	131	100.0	100.0

Q21

		Frequency	Percent	Valid Percent
Valid	a	31	23.7	23.7
	b	30	22.9	22.9
	c	28	21.4	21.4
	d	30	22.9	22.9
	e	12	9.2	9.2
	Total	131	100.0	100.0

Q22

		Frequency	Percent	Valid Percent
Valid	a	26	19.8	19.8
	b	36	27.5	27.5
	c	19	14.5	14.5
	d	33	25.2	25.2
	e	17	13.0	13.0
	Total	131	100.0	100.0

Q23

		Frequency	Percent	Valid Percent
Valid	a	36	27.5	27.5
	b	24	18.3	18.3
	c	18	13.7	13.7
	d	14	10.7	10.7
	e	39	29.8	29.8
	Total	131	100.0	100.0

Q24

		Frequency	Percent	Valid Percent
Valid	a	24	18.3	18.6
	b	20	15.3	15.5
	c	28	21.4	21.7
	d	28	21.4	21.7
	e	29	22.1	22.5
	Total	129	98.5	100.0
Missing	System	2	1.5	
Total		131	100.0	

Q25

		Frequency	Percent	Valid Percent
Valid	a	52	39.7	40.0
	b	10	7.6	7.7
	c	28	21.4	21.5
	d	23	17.6	17.7
	e	17	13.0	13.1
	Total	130	99.2	100.0
Missing	System	1	.8	
Total		131	100.0	

Q26

		Frequency	Percent	Valid Percent
Valid	a	27	20.6	20.8
	b	31	23.7	23.8
	c	33	25.2	25.4
	d	16	12.2	12.3
	e	23	17.6	17.7
	Total	130	99.2	100.0
Missing	System	1	.8	
Total		131	100.0	

Q27

		Frequency	Percent	Valid Percent
Valid	a	64	48.9	49.2
	b	9	6.9	6.9
	c	13	9.9	10.0
	d	14	10.7	10.8
	e	30	22.9	23.1
	Total	130	99.2	100.0
Missing	System	1	.8	
Total		131	100.0	

Q28

		Frequency	Percent	Valid Percent
Valid	a	13	9.9	9.9
	b	24	18.3	18.3
	c	30	22.9	22.9
	d	18	13.7	13.7
	e	46	35.1	35.1
	Total	131	100.0	100.0

Q29

		Frequency	Percent	Valid Percent
Valid	a	28	21.4	21.5
	b	70	53.4	53.8
	c	18	13.7	13.8
	d	9	6.9	6.9
	e	5	3.8	3.8
	Total	130	99.2	100.0
Missing	System	1	.8	
Total		131	100.0	

Q30

		Frequency	Percent	Valid Percent
Valid	a	33	25.2	25.4
	b	33	25.2	25.4
	c	18	13.7	13.8
	d	30	22.9	23.1
	e	16	12.2	12.3
	Total	130	99.2	100.0
Missing	System	1	.8	
Total		131	100.0	

Q31

		Frequency	Percent	Valid Percent
Valid	a	16	12.2	12.3
	b	22	16.8	16.9
	c	31	23.7	23.8
	d	23	17.6	17.7
	e	38	29.0	29.2
	Total	130	99.2	100.0
Missing	System	1	.8	
Total		131	100.0	

Q32

		Frequency	Percent	Valid Percent
Valid	a	35	26.7	27.1
	b	25	19.1	19.4
	c	17	13.0	13.2
	d	37	28.2	28.7
	e	15	11.5	11.6
	Total	129	98.5	100.0
Missing	System	2	1.5	
Total		131	100.0	

The tables below are only for Highest level = 1

Q1

		Frequency	Percent	Valid Percent
Valid	c	38	97.4	97.4
	d	1	2.6	2.6
	Total	39	100.0	100.0

Q2

		Frequency	Percent	Valid Percent
Valid	d	38	97.4	97.4
	e	1	2.6	2.6
	Total	39	100.0	100.0

Q3

		Frequency	Percent	Valid Percent
Valid	a	35	89.7	89.7
	c	4	10.3	10.3
	Total	39	100.0	100.0

Q4

		Frequency	Percent	Valid Percent
Valid	b	14	35.9	35.9
	c	25	64.1	64.1
	Total	39	100.0	100.0

Q5

		Frequency	Percent	Valid Percent
Valid	a	1	2.6	2.6
	b	9	23.1	23.1
	c	14	35.9	35.9
	d	7	17.9	17.9
	e	8	20.5	20.5
	Total	39	100.0	100.0

Q6

		Frequency	Percent	Valid Percent
Valid	a	26	66.7	66.7
	b	7	17.9	17.9
	c	3	7.7	7.7
	d	3	7.7	7.7
	Total	39	100.0	100.0

Q7

		Frequency	Percent	Valid Percent
Valid	a	4	10.3	10.3
	b	14	35.9	35.9
	c	5	12.8	12.8
	d	11	28.2	28.2
	e	5	12.8	12.8
	Total	39	100.0	100.0

Q8

		Frequency	Percent	Valid Percent
Valid	a	2	5.1	5.1
	b	9	23.1	23.1
	c	24	61.5	61.5
	d	1	2.6	2.6
	e	3	7.7	7.7
	Total	39	100.0	100.0

Q9

		Frequency	Percent	Valid Percent
Valid	a	2	5.1	5.1
	c	8	20.5	20.5
	d	28	71.8	71.8
	e	1	2.6	2.6
	Total	39	100.0	100.0

Q10

		Frequency	Percent	Valid Percent
Valid	b	17	43.6	43.6
	c	10	25.6	25.6
	d	5	12.8	12.8
	e	7	17.9	17.9
	Total	39	100.0	100.0

Q11

		Frequency	Percent	Valid Percent
Valid	a	9	23.1	23.1
	b	6	15.4	15.4
	c	18	46.2	46.2
	d	4	10.3	10.3
	e	2	5.1	5.1
	Total	39	100.0	100.0

Q12

		Frequency	Percent	Valid Percent
Valid	a	4	10.3	10.3
	b	6	15.4	15.4
	c	4	10.3	10.3
	d	8	20.5	20.5
	e	17	43.6	43.6
	Total	39	100.0	100.0

Q13

		Frequency	Percent	Valid Percent
Valid	a	9	23.1	23.1
	b	6	15.4	15.4
	c	6	15.4	15.4
	d	9	23.1	23.1
	e	9	23.1	23.1
	Total	39	100.0	100.0

Q14

		Frequency	Percent	Valid Percent
Valid	a	9	23.1	23.1
	b	6	15.4	15.4
	c	6	15.4	15.4
	d	15	38.5	38.5
	e	3	7.7	7.7
	Total	39	100.0	100.0

Q15

		Frequency	Percent	Valid Percent
Valid	a	2	5.1	5.1
	b	29	74.4	74.4
	c	5	12.8	12.8
	d	1	2.6	2.6
	e	2	5.1	5.1
	Total	39	100.0	100.0

Q16

		Frequency	Percent	Valid Percent
Valid	a	7	17.9	18.4
	b	8	20.5	21.1
	c	11	28.2	28.9
	d	1	2.6	2.6
	e	11	28.2	28.9
	Total	38	97.4	100.0
Missing	System	1	2.6	
Total		39	100.0	

Q17

		Frequency	Percent	Valid Percent
Valid	a	18	46.2	46.2
	b	7	17.9	17.9
	c	9	23.1	23.1
	d	4	10.3	10.3
	e	1	2.6	2.6
	Total	39	100.0	100.0

Q18

		Frequency	Percent	Valid Percent
Valid	a	5	12.8	12.8
	b	7	17.9	17.9
	c	18	46.2	46.2
	d	6	15.4	15.4
	e	3	7.7	7.7
	Total	39	100.0	100.0

Q19

		Frequency	Percent	Valid Percent
Valid	a	4	10.3	10.3
	c	1	2.6	2.6
	d	3	7.7	7.7
	e	31	79.5	79.5
	Total	39	100.0	100.0

Q20

		Frequency	Percent	Valid Percent
Valid	a	11	28.2	28.2
	b	6	15.4	15.4
	c	4	10.3	10.3
	e	18	46.2	46.2
	Total	39	100.0	100.0

Q21

		Frequency	Percent	Valid Percent
Valid	a	9	23.1	23.1
	b	7	17.9	17.9
	c	6	15.4	15.4
	d	9	23.1	23.1
	e	8	20.5	20.5
	Total	39	100.0	100.0

Q22

		Frequency	Percent	Valid Percent
Valid	a	10	25.6	26.3
	b	6	15.4	15.8
	c	1	2.6	2.6
	d	15	38.5	39.5
	e	6	15.4	15.8
	Total	38	97.4	100.0
Missing	System	1	2.6	
Total		39	100.0	

Q23

		Frequency	Percent	Valid Percent
Valid	a	6	15.4	15.4
	b	5	12.8	12.8
	c	6	15.4	15.4
	d	7	17.9	17.9
	e	15	38.5	38.5
	Total	39	100.0	100.0

Q24

		Frequency	Percent	Valid Percent
Valid	a	5	12.8	12.8
	b	4	10.3	10.3
	c	6	15.4	15.4
	d	10	25.6	25.6
	e	14	35.9	35.9
	Total	39	100.0	100.0

Q25

		Frequency	Percent	Valid Percent
Valid	a	8	20.5	21.1
	b	6	15.4	15.8
	c	7	17.9	18.4
	d	11	28.2	28.9
	e	6	15.4	15.8
	Total	38	97.4	100.0
Missing	System	1	2.6	
Total		39	100.0	

Q26

		Frequency	Percent	Valid Percent
Valid	a	6	15.4	15.4
	b	6	15.4	15.4
	c	11	28.2	28.2
	d	6	15.4	15.4
	e	10	25.6	25.6
	Total	39	100.0	100.0

Q27

		Frequency	Percent	Valid Percent
Valid	a	18	46.2	47.4
	b	3	7.7	7.9
	c	7	17.9	18.4
	d	7	17.9	18.4
	e	3	7.7	7.9
	Total	38	97.4	100.0
Missing	System	1	2.6	
Total		39	100.0	

Q28

		Frequency	Percent	Valid Percent
Valid	a	3	7.7	7.7
	b	11	28.2	28.2
	c	11	28.2	28.2
	d	1	2.6	2.6
	e	13	33.3	33.3
	Total	39	100.0	100.0

Q29

		Frequency	Percent	Valid Percent
Valid	a	8	20.5	21.1
	b	17	43.6	44.7
	c	6	15.4	15.8
	d	2	5.1	5.3
	e	5	12.8	13.2
	Total	38	97.4	100.0
Missing	System	1	2.6	
Total		39	100.0	

Q30

		Frequency	Percent	Valid Percent
Valid	a	13	33.3	36.1
	b	4	10.3	11.1
	d	12	30.8	33.3
	e	7	17.9	19.4
	Total	36	92.3	100.0
Missing	System	3	7.7	
Total		39	100.0	

Q31

		Frequency	Percent	Valid Percent
Valid	a	4	10.3	10.3
	b	3	7.7	7.7
	c	9	23.1	23.1
	d	9	23.1	23.1
	e	14	35.9	35.9
	Total	39	100.0	100.0

Q32

		Frequency	Percent	Valid Percent
Valid	a	9	23.1	23.1
	b	4	10.3	10.3
	c	1	2.6	2.6
	d	22	56.4	56.4
	e	3	7.7	7.7
	Total	39	100.0	100.0

The tables below are only for Highest level = 2

Q1

		Frequency	Percent	Valid Percent
Valid	c	19	95.0	95.0
	d	1	5.0	5.0
	Total	20	100.0	100.0

Q2

		Frequency	Percent	Valid Percent
Valid	b	1	5.0	5.0
	d	19	95.0	95.0
	Total	20	100.0	100.0

Q3

		Frequency	Percent	Valid Percent
Valid	a	20	100.0	100.0

Q4

		Frequency	Percent	Valid Percent
Valid	b	8	40.0	40.0
	c	10	50.0	50.0
	e	2	10.0	10.0
	Total	20	100.0	100.0

Q5

		Frequency	Percent	Valid Percent
Valid	b	1	5.0	5.0
	c	13	65.0	65.0
	e	6	30.0	30.0
	Total	20	100.0	100.0

Q6

		Frequency	Percent	Valid Percent
Valid	a	15	75.0	75.0
	b	5	25.0	25.0
	Total	20	100.0	100.0

Q7

		Frequency	Percent	Valid Percent
Valid	a	1	5.0	5.0
	b	11	55.0	55.0
	c	1	5.0	5.0
	e	7	35.0	35.0
	Total	20	100.0	100.0

Q8

		Frequency	Percent	Valid Percent
Valid	b	8	40.0	40.0
	c	8	40.0	40.0
	d	3	15.0	15.0
	e	1	5.0	5.0
	Total	20	100.0	100.0

Q9

		Frequency	Percent	Valid Percent
Valid	b	1	5.0	5.0
	d	19	95.0	95.0
	Total	20	100.0	100.0

Q10

		Frequency	Percent	Valid Percent
Valid	a	2	10.0	10.0
	b	16	80.0	80.0
	c	1	5.0	5.0
	e	1	5.0	5.0
	Total	20	100.0	100.0

Q11

		Frequency	Percent	Valid Percent
Valid	a	1	5.0	5.0
	c	18	90.0	90.0
	d	1	5.0	5.0
	Total	20	100.0	100.0

Q12

		Frequency	Percent	Valid Percent
Valid	a	2	10.0	10.0
	b	2	10.0	10.0
	d	1	5.0	5.0
	e	15	75.0	75.0
	Total	20	100.0	100.0

Q13

		Frequency	Percent	Valid Percent
Valid	b	3	15.0	15.0
	c	10	50.0	50.0
	d	3	15.0	15.0
	e	4	20.0	20.0
	Total	20	100.0	100.0

Q14

		Frequency	Percent	Valid Percent
Valid	a	6	30.0	30.0
	b	2	10.0	10.0
	c	1	5.0	5.0
	d	7	35.0	35.0
	e	4	20.0	20.0
	Total	20	100.0	100.0

Q15

		Frequency	Percent	Valid Percent
Valid	a	1	5.0	5.0
	b	17	85.0	85.0
	c	1	5.0	5.0
	e	1	5.0	5.0
	Total	20	100.0	100.0

Q16

		Frequency	Percent	Valid Percent
Valid	a	2	10.0	10.0
	b	13	65.0	65.0
	c	4	20.0	20.0
	e	1	5.0	5.0
	Total	20	100.0	100.0

Q17

		Frequency	Percent	Valid Percent
Valid	a	19	95.0	95.0
	b	1	5.0	5.0
	Total	20	100.0	100.0

Q18

		Frequency	Percent	Valid Percent
Valid	c	16	80.0	80.0
	d	3	15.0	15.0
	e	1	5.0	5.0
	Total	20	100.0	100.0

Q19

		Frequency	Percent	Valid Percent
Valid	a	2	10.0	10.0
	e	18	90.0	90.0
	Total	20	100.0	100.0

Q20

		Frequency	Percent	Valid Percent
Valid	a	9	45.0	47.4
	b	4	20.0	21.1
	c	3	15.0	15.8
	d	2	10.0	10.5
	e	1	5.0	5.3
	Total	19	95.0	100.0
Missing	System	1	5.0	
Total		20	100.0	

Q21

		Frequency	Percent	Valid Percent
Valid	a	7	35.0	35.0
	b	4	20.0	20.0
	c	3	15.0	15.0
	d	2	10.0	10.0
	e	4	20.0	20.0
	Total	20	100.0	100.0

Q22

		Frequency	Percent	Valid Percent
Valid	a	1	5.0	5.3
	b	12	60.0	63.2
	d	4	20.0	21.1
	e	2	10.0	10.5
	Total	19	95.0	100.0
Missing	System	1	5.0	
Total		20	100.0	

Q23

		Frequency	Percent	Valid Percent
Valid	a	5	25.0	25.0
	c	5	25.0	25.0
	d	2	10.0	10.0
	e	8	40.0	40.0
	Total	20	100.0	100.0

Q24

		Frequency	Percent	Valid Percent
Valid	a	2	10.0	10.0
	b	1	5.0	5.0
	c	4	20.0	20.0
	d	8	40.0	40.0
	e	5	25.0	25.0
	Total	20	100.0	100.0

Q25

		Frequency	Percent	Valid Percent
Valid	a	5	25.0	25.0
	c	3	15.0	15.0
	d	10	50.0	50.0
	e	2	10.0	10.0
	Total	20	100.0	100.0

Q26

		Frequency	Percent	Valid Percent
Valid	a	1	5.0	5.0
	c	14	70.0	70.0
	d	3	15.0	15.0
	e	2	10.0	10.0
	Total	20	100.0	100.0

Q27

		Frequency	Percent	Valid Percent
Valid	a	9	45.0	45.0
	b	5	25.0	25.0
	c	1	5.0	5.0
	d	1	5.0	5.0
	e	4	20.0	20.0
	Total	20	100.0	100.0

Q28

		Frequency	Percent	Valid Percent
Valid	a	1	5.0	5.0
	b	7	35.0	35.0
	c	6	30.0	30.0
	e	6	30.0	30.0
	Total	20	100.0	100.0

Q29

		Frequency	Percent	Valid Percent
Valid	a	4	20.0	20.0
	b	11	55.0	55.0
	c	3	15.0	15.0
	d	1	5.0	5.0
	e	1	5.0	5.0
	Total	20	100.0	100.0

Q30

		Frequency	Percent	Valid Percent
Valid	a	4	20.0	20.0
	b	8	40.0	40.0
	d	3	15.0	15.0
	e	5	25.0	25.0
	Total	20	100.0	100.0

Q31

		Frequency	Percent	Valid Percent
Valid	c	3	15.0	15.0
	d	9	45.0	45.0
	e	8	40.0	40.0
	Total	20	100.0	100.0

Q32

		Frequency	Percent	Valid Percent
Valid	a	3	15.0	15.0
	b	2	10.0	10.0
	c	2	10.0	10.0
	d	11	55.0	55.0
	e	2	10.0	10.0
	Total	20	100.0	100.0