

# THE ECONOMETRICS OF AIRLINE NETWORK MANAGEMENT

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## ABSTRACT

The task of airline network management is to develop new flight schedule variants and evaluate them in terms of expected passenger demand and revenue. Given the industry's trend towards global co-operation, this is especially important when evaluating the potential synergies with alliance partners. From the econometric point of view, this task represents a discrete choice modeling problem in which the analyst has to account for a large number of dependent alternatives. In this paper we discuss the applicability of both standard models and recently proposed alternatives to the airline network management task. We identify their drawbacks and introduce a new specification. The superiority of the new model is demonstrated both in a simulation study and in a real-world application using airline bookings data.

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## 1 INTRODUCTION <sup>1</sup>

The creation of worldwide alliances such as the British Airways/American Airlines joint venture "One World" and the cooperation between United Airlines, Lufthansa and other major carriers, known as "Star Alliance", emphasized the importance of the network perspective in the airline industry. Rather than perceiving a timetable to be a collection of isolated routes, airlines have realized that their schedules and those of their alliance partners represent a complex network of city-pair connections in origin and destination markets. The number of markets that can be served in this network is a multiple of the limited number of routes that can be offered by any single airline. The potential synergies that airline alliances can generate are therefore mainly caused by the significant additional number of markets that become accessible by harmonizing the alliance partners' schedules. Even before the advent of international joint ventures many carriers already increased their portfolio of served markets by processes called hubbing and banking, i.e. by scheduling incoming and outgoing flights at the airline's home airport (hub) in a way that enables them to penetrate a variety of profitable transfer markets.

The task of an airline network management department is to create and evaluate flight schedule scenarios in terms of expected passenger demand and revenue. Each schedule redesign or the creation of a joint alliance schedule generates alternative options for passengers wishing to travel from an origin to their desired destination (O&D itineraries). Passengers chose direct or transfer connections from the set of offered itineraries. Transfer itineraries may invoke one or more stops, and the legs that build a feasible connection may be offered by the same (online) or different (interline) carriers. Assuming that the utility of an offered itinerary is dependent on the characteristics of the alternative, and that a customer chooses an itinerary that provides her maximum utility, a tailor-made environment is obviously available for

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the application of a discrete choice model. For strategic decisions concerning network design and/or alliance evaluation, the estimated choice probabilities are conceived as market share estimates. Multiplying these by (externally estimated) total O&D market demand and ticket price the network management department is able to provide an estimate of the revenue and passenger volume that is implied by any given schedule scenario.

In this paper we assess the applicability of the discrete choice models that have been discussed in the literature to the alliance and schedule evaluation problem. We introduce a new multinomial probit (MNP) specification that is motivated by the requirements of airline network management, but straightforwardly applicable to other discrete choice situations where the analyst faces the problem of accounting for a large number of dependent alternatives.

The application of discrete choice models in the airline industry is facilitated by the commercial availability of worldwide airline schedules. Using timetable information it is straightforward (albeit cumbersome) to construct the set of relevant itineraries from which a passenger can choose. The attributes of the alternatives can also be constructed from the schedules (e.g. elapsed time, type of aircraft) or obtained from external sources (e.g. prices). Since a large fraction of the passenger demand is recorded in computer reservation systems, whose operators sell these data to airlines, the estimation of discrete choice models can be based on high quality data. Commercial software companies, e.g. SABRE decision technologies have specialized in the implementation of schedule evaluation tools that are based on discrete choice models.

Besides the unique data availability, the requirements that a successful model has to meet are high. The number of alternatives, in terms of offered connections on an O&D market, can be quite large. Whilst the multinomial logit model (MNL) has no problems dealing with large sets of alternatives its application is not appropriate since the independence of irrelevant alternatives (IIA) assumption is violated at a critical level. In fact, the blue bus/red bus paradox, the familiar textbook example to illustrate the IIA

problem, becomes a reality in many airline markets: On a route with heavy competition between airlines it is often the case that the aircrafts of two carriers will leave from an airport at almost the same time for the same destination. Assuming independence of the utilities in such a situation is definitely a bad idea. Hence, a discrete choice model is required that will allow for dependence between alternatives and that can cope with a situation in which the number of alternatives can become quite large. We will discuss the applicability of recently proposed discrete choice models for non-IIA situations and show that most approaches have to be discarded as a result of their restrictive assumptions. After formulating an adapted version of the Generalized Autoregressive (GAR) probit model advanced by Bolduc and Ben-Akiva (1991), Bolduc (1992) and Ben-Akiva and Bolduc (1996) in greater detail, we introduce a new MNP specification that perfectly meets the requirements of airline network management. As in Yai, Iwakura and Morichi's (1997) model our approach builds on a pure attribute based specification of the utility covariance matrix. Yai, Iwakura and Morichi's specification, however, is designed for the specific route planning problem that they investigate, limiting its extension to other fields. The advantage of our approach is that it is applicable to any discrete choice problem in which one has to account for a large number of dependent alternatives.

The paper is organized as follows. In the Section 2 we will outline the econometric formulation of the network management problem (Section 2.1), discuss the applicability of several discrete choice models that were proposed in the literature (Section 2.2) and adapt the GAR-MNP for airline network management purposes (Section 2.3). In Section 2.4 we present our new MNP specification. The results of a simulation study, designed to assess the performance of the alternative approaches, are discussed in Section 3. Empirical applications and comparisons of the performance of the models are presented in Section 4. We conclude in Section 5.

## 2 ECONOMETRIC MODELS

### *2.1 Discrete choice modeling in airline network management*

In the sequel we introduce some basic notation, and outline the requirements a discrete choice model must satisfy if it is to be successfully employed for schedule and alliance evaluation. For an O&D market  $d$ ,  $d = 1, \dots, D$ , where  $D$  is the number of O&Ds,  $J_d$  alternative itineraries can be constructed.  $J = \sum_{d=1}^D J_d$  denotes the total number of itineraries. We are interested in estimating the probability that a passenger  $n$ ,  $n = 1, \dots, N$  chooses alternative  $i$ ,  $i = 1, \dots, J$  where  $N = \sum_{d=1}^D N_d$  is the total number of individuals in the sample, and  $N_d$  is the number of passengers deciding among the alternatives in market  $d$ .

We assume that passengers who want to travel from origin O to destination D are not interested in itineraries offered for other markets.<sup>2</sup> To take this into account we define the function  $g(n)$  that assigns an individual  $n$  to her O&D market  $d$ . Analogously,  $f(i)$  is a function that assigns alternative  $i$  to the O&D market to which it belongs to. Hence, the probability that individual  $n$  chooses alternative  $i$  can only be different from zero if  $g(n) = f(i)$ . We assume that all individuals assigned to a specific O&D market observe the same set of alternatives. This is justified by the passenger's real-life travel decision: Computer Reservation Systems (CRS) display all the possible connections that are offered for an O&D market. Any traveler has access to these CRS screens via her travel agency. To be precise, we deal with the following probabilistic choice setting

$$y_{in} = \begin{cases} 1 & \text{if } u_{in} \geq u_{jn} \forall j \in \{j = 1, \dots, J \mid f(i) = f(j)\} \text{ and } f(i) = g(n) \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

$$p_{in} = P(y_{in} = 1),$$

where  $y_{in}$  is the observed choice of the individual  $n$  and  $u_{in}$  is the utility that alternative  $i$  provides for individual  $n$ .

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<sup>2</sup>This is a simplifying assumption. It does not take into account, for instance, that an individual living between two airports can choose one as the origin of her trip if the desired destination is offered at both airports.

For simplicity, assume a linear specification

$$u_{in} = x_i' \beta + \varepsilon_{in}. \quad (2)$$

$x_i$  is a  $(K \times 1)$  vector of attributes describing alternative  $i$  that may contain alternative-specific constants and alternative-specific covariates.  $\beta$  is a  $(K \times 1)$  parameter vector. The alternative models discussed below imply different specifications of the random utility  $\varepsilon_{in}$ . We assume that the vector  $\beta$  is identical for a subset of the O&D markets (e.g. domestic, short-haul and long-haul market). The assumptions above imply independence between utilities of itineraries on different O&D markets

$$\text{cov}(\varepsilon_{in}, \varepsilon_{jn}) = 0 \quad \forall f(i) \neq f(j). \quad (3)$$

In the context of airline network management the covariance matrix assumed for the random utility component  $\varepsilon_{in}$  differs fundamentally from the one that is employed for the typical commuter problem (Albright, Lerman and Manski 1977, Horowitz, Sparman and Daganzo 1982, Ben-Akiva 1985, Keane 1992, Bolduc 1999). In the latter framework, physically different alternatives are identified by logical names describing the mode of transport, e.g. bus, car or shared ride. This procedure is referred to as nominal identification. The fact that the bus that consumer  $n$  living in region  $A$  chooses is not the same vehicle that consumer  $m$  living in region  $B$  selects does not matter. Although the two buses can have different attribute levels (e.g. prices), the covariances of the utilities of each bus and car alternative and each bus and shared ride alternative respectively are assumed to be identical.<sup>3</sup>

The application of the nominal identification principle is not useful for airline network management purposes: In a typical O&D market one will find a large number of itineraries possessing different attribute levels, e.g. a British Airways nonstop flight departing at 8:00 a.m. with an elapsed time of 6:30 hours, or a Lufthansa/United Airlines interline connection departing at 8:30 a.m. with an elapsed time of 8:00 hours. Although nominal

<sup>3</sup>If certain alternatives are not available for a specific individual then the reduced covariance matrix can be generated by just deleting the corresponding rows and columns in the full covariance matrix.

identification would be straightforward, e.g. by distinguishing direct flights, online connections, interline connections etc., this is not helpful. For schedule evaluation purposes an airline is interested in the choice probability of each itinerary, especially in self-offered direct flights and online connections. This implies that it is necessary to account for a specific covariance matrix for each O&D market. Given the independence assumption (3) we have to deal with a block-diagonal, but otherwise unrestricted covariance matrix. A schedule modification alters the set of relevant itineraries, which changes some or all of the covariance matrix blocks. A model that will be successfully applicable in airline network management must be able to cope with the obvious incidental parameter and identification problems.

## 2.2 Applicability of standard discrete choice models

The most simple specification that can be considered is the Multinomial Logit model (MNL). The MNL is based on the assumptions that  $\varepsilon_{in}$  is i.i.d. and follows a Gumbel distribution. Applied to the framework outlined in the previous subsection we obtain the probability that individual  $n$  chooses alternative  $i$ :

$$p_{in} = \frac{\exp(x'_i\beta)}{\sum_{j \in N_j} \exp(x'_j\beta)}, \text{ where } N_j = \{j = 1, \dots, J \mid f(i) = f(j)\} \quad (4)$$

The MNL's computational simplicity comes at the cost of the restrictive assumption of independence of irrelevant alternatives (IIA). The IIA assumption implies that the ratio of the choice probabilities of any two alternatives does not depend on the others:

$$\frac{p_{in}}{p_{jn}} = \frac{\exp(x'_i\beta)}{\exp(x'_j\beta)} = \exp(x'_i\beta - x'_j\beta) \quad (5)$$

In airline network management the IIA assumption is violated at a critical level. As is often the case on contested routes the planes of two competitors depart at almost the same time and for the same destination. One should expect that the joint market share on the city-pair connections is lower compared to a situation where the two planes start with some hours departure time difference.<sup>4</sup>

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<sup>4</sup>This argument assumes the absence of capacity restrictions

The nested logit model (NL), the cross correlated logit model (CCL) and other models of the General Extreme Value family (GEV) have been proposed for situations, in which the IIA assumption cannot be maintained (Williams 1977, McFadden 1978, Ortuzar 1982). For a simple two-level NL model the random utility  $\varepsilon_{in}$  is divided into a part that is common to alternatives that belong to the same group and a remaining unobserved utility  $e_{in}$ :

$$\varepsilon_{in} = \varepsilon_{g(i),n} + e_{in}. \quad (6)$$

The approach can easily be extended to a multi-level model by further grouping the alternatives within a group and further dividing the error components. A generalization of the NL is the CCL model proposed by Williams and Ortuzar (1982). This approach allows for interaction terms in the covariance matrix and does not require the hierarchical structuring that has to be imposed for NL. However, the model is highly complex and inconsistent with utility maximization, as conceded by the authors.

The core problem of all tree structured models, however, is that a hierarchical structure of the decision process has to be assumed. In the case of the nests being based on continuous variables one has to cut the range of attribute values into pieces. This can lead to implausible discontinuities of choice probabilities, e.g. when changing the departure time of a flight and thereby shifting it from one nest to another. Furthermore, the IIA problem remains present on the level of the nests. As an example, consider a simple one-level NL model where the departure weekdays are chosen as the nesting criterion. The covariance between an alternative that belongs to a specific nest (e.g. a Tuesday departure) and any other alternative that belongs to a different nest (e.g. a Monday departure or Friday departure) is equal. For the weekday nesting this is obviously a very doubtful assumption: It is much more reasonable to assume that a Tuesday departure is conceived as being more similar (in terms of unobserved utility) to a Monday departure than it is to a Friday departure. Bhat (1997) investigates more flexible NL specifications that aim to provide a solution to this problem by introducing covariance heterogeneity between nests based on the individual's characteris-



tics. Yet, because of the lack of information about individuals, this approach is not suitable for airline network management. The same holds true for the MNL-approach recently proposed by Ivaldi and Viauroux (1999).

The Multinomial Probit model (MNP) is the natural tool to be applied to non-IIA problems. Recent work on simulation based methods has helped greatly to solve the numerical problems associated with the evaluation of the multidimensional integrals required to compute the choice probabilities. Both McFadden (1989), Börsch-Supan and Hajivassiliou (1993), Hajivassiliou and McFadden and Ruud (1996) have introduced simulation based methods that permit MNP modeling of choice problems with a larger number of alternatives.

The second problem associated with the MNP is caused by the abundance of covariance elements that have to be accounted for if the number of alternatives is large. The identifying restrictions needed for MNP application have been extensively discussed in the literature (see Albright, Lerman and Manski (1977) Horowitz, Sparman and Daganzo (1982), Dan-sie (1985), Bolduc (1992), Bunch (1991), Horowitz (1991), Keane (1992)). Bunch (1991) argues that the identifying restrictions imply assumptions which are equivalent to choosing among hierarchical structures in GEV-type models mentioned above. Horowitz (1991) and Bunch (1991) conclude that it is questionable whether the performance of MNP is superior to GEV-type models.

Horowitz (1991) has pointed to another problem associated with MNP: Covariances between new alternatives are unknown, therefore the MNP is inadequate for forecasting purposes. Since market share forecasts for itineraries that are newly generated by schedule redesigns are crucial in airline network management this seems to be a devastating critique. We will show in the following that all is not lost for MNP, arguing that a sparse parameterization of the utility covariance matrix is the key to the solution. Hausman and Wise (1978) were the first to discuss the role of modeling covariances in the MNP model, and subsequent work is built heavily on their basic ideas.

It is therefore helpful to review their framework and its critique. In the Hausman and Wise model the utility is specified as

$$u_{in} = x'_{in}\bar{\beta} + x'_{in}\tilde{\beta}_n + e_{in}, \quad (7)$$

where  $\bar{\beta}$  contains the average taste parameters and  $\tilde{\beta}_n$  represents individual taste variations which are assumed to be iid.  $N(0, I)$ . The errors  $e_{in}$  are assumed to be i.i.d.  $N(0, 1)$ .

Yai, Iwakura and Morichi (1997) criticize that this approach leads to covariances that are proportional to the product of the attributes of the two alternatives. This is a crucial critique in the context of airline network management: One does not expect two itineraries to be more similar just because, for instance, they both start in the evening instead of in the morning. Yet, this is exactly what Hausman and Wise's approach would imply if we used the departure time as a covariate in (7). Based on their critique Yai, Iwakura and Morichi (1997) introduce the concept of structured covariances in MNP modeling. Their approach towards the modeling of dependencies between alternatives is closely linked to the peculiarities of the route choice problem that they analyze: The covariance of two routes is determined by their common transfer stations and the common parts of the route. Yet, this approach is a rare example of a pure attribute based specification of the covariance structure. In Section 3 we will present a more general approach that contains Yai, Iwakura and Morichi (1997) model as a special case.

### *2.3 GAR-MNP adapted for airline network management*

In this section we will present an adaption of the Generalized Autoregressive (GAR) MNP that was introduced by Bolduc and Ben-Akiva (1991) and Bolduc (1992). Ben-Akiva and Bolduc (1996) extend the concept to a general factor analytic approach, which contains GAR and other specifications as special cases. After outlining its basic idea, we adapt the GAR approach to enable its application in network management and discuss some of its shortcomings.

In Ben-Akiva and Bolduc's (1996) general factor analytic approach, the stochastic utility component is decomposed into an independent random

variable  $\nu_{in}$  and a covariance generating component  $\vartheta_{in}$ :

$$\varepsilon_{in} = \vartheta_{in} + \nu_{in} \quad (8)$$

The error term  $\nu_{in}$  is assumed to be i.i.d., either following a normal or a Gumbel distribution.  $\vartheta_{in}$  is defined via a factor structure

$$\vartheta_n = F_n \zeta_n, \quad (9)$$

where  $\vartheta_n = \vartheta_{1n}, \dots, \vartheta_{L_n n}$  with  $L_n$  being the number of alternatives in the choice set of individual  $n$ .  $\zeta_n \sim N(0, I_M)$  is a  $(M \times 1)$  random vector that is i.i.d. multivariate normal, where  $M \leq L_n$ .  $F_n$  is a  $(L_n \times M)$  matrix for which Ben-Akiva and Bolduc (1996) propose four specifications. Among these only one, the heteroscedastic specification, is suitable for airline network management. Assume

$$\vartheta_n = \rho W_n \vartheta_n + T \zeta_n. \quad (10)$$

where  $T$  is a diagonal matrix of alternative-specific standard deviations and  $(-1 < \rho < 1)$ . The parameter  $\rho$  accounts for the overall strength of dependence between alternatives.  $W_n$  is a  $(L_n \times L_n)$  weighting matrix. Rewrite (10) as

$$\vartheta_n = (I - \rho W_n)^{-1} T \zeta_n. \quad (11)$$

Bolduc (1992) proposes the choice of the  $i, j$ 'th element of  $W_n$  as

$$w_{ij,n} = \begin{cases} \frac{w_{ii,n}^*}{\sum_{k=1}^{L_n} w_{ik,n}^*}, & \text{if } i \neq j \\ 0, & \text{otherwise} \end{cases}, \quad (12)$$

where  $w_{ij,n}^*$  is inversely related to the similarity or proximity of the alternatives  $i$  and  $j$ . As proposed by Bolduc (1992),  $w_{ij,n}^*$  can be defined as a Boolean matrix. This is useful for standard problems using nominal identification, but cannot be applied in an airline network management context. As an alternative, Bolduc (1992) suggested a distance function such as

$$w_{ij}^* = (\Delta_{ij})^{-\lambda} \quad (13)$$

where  $\lambda > 0$  and  $\Delta_{ij}$  is a distance measure between  $i$  and  $j$ . In the following we will adopt these basic ideas in order to provide a GAR variant that is applicable in airline network management.

Consider the following specification for the covariance generating component  $\vartheta_n$ :

$$\vartheta_n = \rho W \vartheta_n + T \zeta_n. \quad (14)$$

Since individual-specific data are not used for airline management the suffix  $n$  is dropped for the weighting matrix. Furthermore, as nominal identification has been discarded, we cannot assign specific standard deviations to the alternatives. Hence, we have  $T = \sigma_0 I$ . Consequently (14) reduces to

$$\vartheta_n = \rho W \vartheta_n + \sigma_0 \zeta_n, \quad (15)$$

or, equivalently,

$$\vartheta_n = \sigma_0 (I - \rho W)^{-1} \zeta_n.$$

Conceiving  $\Delta_{ij}$  in (13) as a general measure for the proximity of the alternatives in terms of one or more attributes, we propose to use the following weighting function:

$$w_{i,j}^* = \exp\left(-\sum_{p=1}^P \alpha_p |z_{i,p} - z_{j,p}|\right), \quad (16)$$

where  $z_i$  and  $z_j$  are  $(P \times 1)$  vectors of attributes that account for the similarity of two alternatives.<sup>5</sup>  $\alpha_p$  are parameters to be estimated. The weighting function (16) is preferred to (13) because it is also defined for  $z_{i,p} = z_{j,p}$ .<sup>6</sup> This specification yields an MNP in which the covariance matrix depends on the parameters  $\rho$ ,  $\sigma_0$  and  $\alpha = (\alpha_1, \dots, \alpha_p)$ .

Three drawbacks of the GAR approach are induced by the normalization in equation (12) which is nevertheless necessary to ensure that the regularity conditions required for consistency and asymptotic normality of the ML estimator are satisfied (Bolduc and Ben-Akiva, 1991). First, the covariance of any two alternatives  $i$  and  $j$  depends on the presence of a third alternative which is incompatible with the marginalization property of the multivariate normal distribution. Second, the normalization induces counterintuitive effects. If one assumes a block-diagonal weighting matrix within an airline

<sup>5</sup>Note that the block-diagonal covariance matrix implies that  $w_{i,j}^*$  for  $f(i) \neq f(j)$ .

<sup>6</sup>Whilst a number of alternative weighting functions were tested (16) turned out to be the most robust.

O&D market, the covariance structure implied by the GAR model would be invariant to a multiplication of any block of the weighting matrix. Third, it is possible that a closer proximity between two alternatives implies a higher joint choice probability. However, this is contradictory to the effect that similarity is expected to exert on choice probabilities: If for an alternative  $i$  the normalized weights  $w_{ik}$  (one row of  $W$ ) possess large size differences, then the variance of alternative  $i$ 's utility is higher compared to a situation in which the weights are equally distributed. This yields a higher choice probability, *ceteris paribus*.

#### 2.4 An alternative MNP specification

In this chapter we will introduce an alternative approach towards modeling similarities between alternatives. The basic idea is to decompose the random utility  $\varepsilon_{in}$  into error components that are related to attributes describing the alternatives. For simplicity we refer to the standard linear utility specification

$$u_{in} = x_i' \beta + \varepsilon_{in}. \quad (17)$$

As for GAR and NL,  $\varepsilon_{in}$  is composed into an i.i.d.  $N(0,1)$  error  $\nu_{in}$  and an attribute-dependent component  $\vartheta_{in}$ :

$$\varepsilon_{in} = \vartheta_{in} + \nu_{in} \quad (18)$$

Let  $z_i = (z_{i1}, z_{i2}, \dots, z_{iP})$  be a vector of attributes describing alternative  $i$ , where  $z_i \subseteq x_i$ .  $\vartheta_{in}$  is linearly decomposed into  $P$  error components. Each of those is associated with a specific attribute in  $z_i$

$$\vartheta_{in} = \vartheta_{in1} + \vartheta_{in2} + \dots + \vartheta_{inP}, \quad (19)$$

where  $\vartheta_{in1}$  is the error associated with the attribute  $z_{i1}$ ,  $\vartheta_{in2}$  is associated with  $z_{i2}$ , and so on.<sup>7</sup> We assume that the random utility  $\vartheta_{ink}$  is associated with the level of the  $k$ 'th attribute, i.e.  $\vartheta_{ink} = \vartheta_{jnk}$  if  $z_{ik} = z_{jk}$ . This specification is related to the Hausman and Wise model (7) in the sense

<sup>7</sup>In general there could also be error components that are related to two or more attributes, but for simplicity we take a linear approach here.

that we account for unobserved individual-specific deviations of the utility that are associated with some attributes. However, we do not assume the restrictive linear relation of the random coefficients and attribute levels as is the case in (7). In the following, we will propose three basic variants of (19) by distinguishing three types of attributes, non-ordered and ordered categorial and continuous.

Let  $z_i^b$  be a dichotomous attribute and  $\vartheta_{in}^b$  the random utility that is associated with this attribute. Let  $\xi_n^{b,1}$  ( $\xi_n^{b,0}$ ) denote the random utility component that is associated with an alternative where the dichotomous variable  $z_i^b$  equals one (zero). Consider the following specification:

$$\vartheta_{in}^b = \delta_{z_i^b,0} \cdot \xi_n^{b,0} + \delta_{z_i^b,1} \cdot \xi_n^{b,1}. \quad (20)$$

$\xi_n^{b,0}$  and  $\xi_n^{b,1}$  denote i.i.d. random variables,  $\xi_n^{b,0} \sim N(0, \sigma_0^2)$  and  $\xi_n^{b,1} \sim N(0, \sigma_1^2)$  where  $cov(\xi_n^{b,0}, \xi_n^{b,1}) = 0$ .  $\delta_{s,t}$  is the Kronecker symbol:

$$\delta_{s,t} = \begin{cases} 1 & \text{for } s = t \\ 0 & \text{otherwise} \end{cases} \quad (21)$$

This implies:

$$\begin{aligned} cov(\vartheta_{in}^b, \vartheta_{jn}^b) &= cov(\delta_{z_i^b,0} \cdot \xi_n^{b,0}, \delta_{z_j^b,0} \cdot \xi_n^{b,0}) + cov(\delta_{z_i^b,1} \cdot \xi_n^{b,1}, \delta_{z_j^b,1} \cdot \xi_n^{b,1}) \\ &= \delta_{z_i^b,0}, \delta_{z_j^b,0}, cov(\xi_n^{b,0}, \xi_n^{b,0}) + \delta_{z_i^b,1}, \delta_{z_j^b,1}, cov(\xi_n^{b,1}, \xi_n^{b,1}) \\ &= \delta_{z_i^b,0}, \delta_{z_j^b,0}, \sigma_0^2 + \delta_{z_i^b,1}, \delta_{z_j^b,1}, \sigma_1^2 \end{aligned} \quad (22)$$

In words, the covariance matrix is non-zero only if the two alternatives  $i$  and  $j$  take on the same level of the dichotomous attribute  $z_i^b$ .  $\sigma_0^2$  and  $\sigma_1^2$  are parameters to be estimated. To illustrate this specification, consider a binary indicator that equals one if the carrier that offers an itinerary is "American Airlines" and zero if not. The dummy indicator "American Airlines" is assumed to enter the systematic utility as an explanatory variable. However, since people have different experiences when travelling with the airline, it is important to account for individual specific deviations from the average utility: A passenger having experienced an enjoyable (unpleasant) flight with American Airlines will assign a higher (lower) utility to all alternatives operated by this carrier. The binary specification can be extended straightforwardly to deal with polytomous non-ordered variables.

The above specification can also be extended to deal with ordered polytomous variables. Let  $z_i^o$  denote such an attribute taking on the values  $m = 1, 2, \dots, M$ . We define a  $(M \times 1)$  random vector  $\xi_n^o = (\xi_{1n}^o, \dots, \xi_{Mn}^o) \sim N(0, \sigma^2 I_M)'$ .  $\xi_{mn}^o$  can be interpreted as the intrinsic unobserved utility deviation that is related to the attribute level  $m$ . Consider the following specification of a spatial moving average process:

$$\vartheta_{in}^o = \sum_{m=1}^M \xi_{mn}^o \cdot A(m, z_i^o; \lambda). \quad (23)$$

$A(t, s; \lambda)$  is an amplitude function weighting the intrinsic errors  $\xi_{mn}^o$ . For obvious reasons we choose an amplitude function that decreases as the distance between  $z_i^o$  and  $m$  grows. Symmetric functions such as

$$A(m, z_i^o; \lambda) = \Gamma \cdot \exp\left(-\frac{(m - z_i^o)^2}{2\lambda^2}\right) \quad (24)$$

or

$$A(m, z_i^o; \lambda) = \Gamma \cdot (1 + \lambda|m - z_i^o|)^{-2} \quad (25)$$

are obvious candidates, but periodic functions may also be suitable.  $\Gamma$  is a normalization constant such that  $\text{var}(\vartheta_{in}^o) = \sigma^2$ . The covariance between  $\vartheta_{in}^o$  and  $\vartheta_{jn}^o$  is given by

$$\text{cov}(\vartheta_{in}^o, \vartheta_{jn}^o) = \sigma_r^2 \sum_{m=1}^M A(m, z_i^o; \lambda) A(m, z_j^o; \lambda). \quad (26)$$

Details of the derivation are deferred to the appendix. Choosing  $A(m, z_i^o; \lambda) = \delta_{m, z_i^o}$  reduces the covariance formula (26) to (22).

To illustrate the ordered case, consider the following example: When modeling passenger choice in airline network management one sometimes includes weekday dummies as explanatory variables in order to account for a passenger's weekday preferences. The above specification allows us to account for individual-specific deviations from the mean day of week departure preferences. For some O&D markets in which only a few itineraries per week are offered (e.g. on exotic intercontinental routes), a passenger that is familiar with this constrained supply situation may ask for a flight "in the middle of the week" rather than for a connection on his preferred departure day (e.g. Wednesday). If no connection is available on Wednesday our

passenger might prefer a Tuesday or Thursday flight to a flight on Saturday as these days are still in or close enough to his or her favored departure date/time window. This is precisely what is implemented by the spatial MA specification (23) which implies that the random utility component associated with the itineraries' departure day is also dependent on the intrinsic random utilities of the neighboring days.

Transferring these ideas to the continuous case is straightforward. Let  $z_i^c$  denote a continuous attribute of the alternative  $i$  that can take values of an interval  $I \subset \mathbb{R}$ . We define the stochastic process  $\{\xi_n^c(y), y \in I\}$  that generates a random variable for each  $t \in T$ . For the sake of simplicity we assume a white noise process with variance  $\sigma^2$ . An obvious continuous attribute that has to be considered when modeling discrete choice in airline network management is the departure time associated with an itinerary. Departure time preferences, or, equivalently, intra-day or intra-week seasonalities, play an important role in explaining the utilities that are assigned to an itinerary. Again,  $\xi_n^c(y)$  can be conceived as a random variable that accounts for deviations from the mean utility associated with a certain departure time. Following the same logic as for the ordered case, we can specify a continuous version of the two-sided MA process introduced in equation (23):

$$\vartheta_{in}^c = \int \xi_n^c(y) A(y, z_i^c; \lambda) dy. \quad (27)$$

In appendix A we derive that the covariance of  $\vartheta_{in}^c$  and  $\vartheta_{jn}^c$  can be written as:

$$\text{cov}(\vartheta_{in}^c, \vartheta_{jn}^c) = \sigma^2 \int A(y, z_i^c; \lambda) A(y, z_j^c; \lambda) dy. \quad (28)$$

In the following we will use the "normal" weighting function for the two-sided MA process:

$$A(y, z; \lambda) = \sqrt{\sqrt{\frac{2}{\pi}} \frac{1}{\lambda}} \exp\left(-\frac{(y-z)^2}{\lambda^2}\right) \quad (29)$$

$\sigma^2$  is the amplitude and  $\lambda$  denotes the width of the weighting function. By choosing this weighting function we ensure that  $\text{var}(\vartheta_{in}^c) = \sigma^2$ . Straightforward algebra yields:

$$\text{cov}(\vartheta_{in}^c, \vartheta_{jn}^c) = \sigma^2 \cdot \exp\left(-\frac{(z_j^c - z_i^c)^2}{2\lambda^2}\right), \quad (30)$$



$\sigma^2$  and  $\lambda$  are additional model parameters that have to be estimated. The dichotomous, ordered and continuous specifications can easily be combined under the independence assumption for the intrinsic errors. It is therefore only natural to refer to this model as the Attribute Based Covariance-MNP (ABC-MNP). It does not require a-priori decisions about the hierarchy of the decision process, and can be parameterized parsimoniously enough to be empirically tractable. Furthermore, the inclusion of new alternatives poses no problems as long as their attributes are known in advance. Note that all parameters in the ABC-MNP are identified. Since we restrict  $\nu_{in}$  to be i.i.d., and  $N(0, I)$  a multiplication of the covariance matrix with a scalar that only effects the covariance parameters is impossible. An alternative restriction would have been  $cov(u_{1n}, u_{1n}) = 1$ . Both restrictions have  $\Sigma = I$  as limiting cases. The choice of the first restriction is a natural extension of our derivation. Hence, the ABC-MNP defies Bunch's (1991) and Horowitz' (1991) fundamental critique of the MNP.

### 3 SIMULATION STUDY

In this Section we present the results of a Monte Carlo Study that is designed to compare the performance of the discrete choice models discussed and introduced in the previous Sections. The simulated data generating process mimics the discrete choice behavior that one has to account for in airline network management. For reasons of computational tractability we only consider two O&D markets, i.e.  $D = 2$ . On each market 10 itineraries are offered to the passengers. The offered itineraries are chosen to represent one day of the week of two typical O&D markets by allowing for a higher density of connections in the morning and in the evening. In each market both nonstop and transfer connections are offered. Table 1 shows the resulting itineraries. O&D market 1 contains some flights that leave almost simultaneously (alternative 1 and 2 or alternatives 3, 4 and 5). In market two the itineraries are more evenly spread throughout the day.

insert table 1 about here

In order to model passenger choice we assume a simple decision process. Only one attribute is assumed to enter the systematic part of the utility. This variable is one for all simulated nonstop itineraries and zero for all connections. The utility coefficient  $\beta$  is set to one and is assumed to be equal for the two markets. The simulated data generating processes (DGP) produce random utilities that are distributed multivariate normal with covariance matrix  $\Sigma$ .  $\Sigma$  takes on the idiosyncratic block diagonal structure outlined in Section 2.1,

$$\Sigma = \begin{pmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{pmatrix}. \quad (31)$$

The DGP imply five different covariance matrices. With the exception of DGP 1 the covariance of two alternatives depends on their proximity in terms of their departure times  $t_i$  and  $t_j$ . The covariance generating functions are specified as follows:

$$\text{DGP 1 (independent): } \sigma_{ij} = \delta_{ij}$$

$$\text{DGP 2 (normal): } \sigma_{ij} = \phi \exp\left(-\frac{(t_i - t_j)^2}{2\gamma^2}\right) + \delta_{ij}$$

$$\text{DGP 3 (triangular): } \sigma_{ij} = \begin{cases} \phi \left(1 - \frac{|t_i - t_j|}{\gamma}\right) + \delta_{ij} & \text{if } |t_i - t_j| \leq \gamma \\ 0 & \text{otherwise} \end{cases}$$

$$\text{DGP 4 (rectangular): } \sigma_{ij} = \begin{cases} \phi + \delta_{ij} & \text{if } |t_i - t_j| \leq \gamma \\ 0 & \text{otherwise} \end{cases}$$

$$\text{DGP 5 (exponential): } \sigma_{ij} = \phi \exp\left(-\frac{|t_i - t_j|}{\gamma}\right) + \delta_{ij}$$

where  $\delta_{ij}$  is the Kronecker symbol defined above. For all DGP equation we have  $\sigma_{ij} = 0$  if  $f(i) \neq f(j)$ . The parameters  $\gamma$  and  $\phi$  are chosen such that positive definiteness of  $\Sigma$  is ensured. For DGP 2-5  $\phi$  is set to 4. The parameter  $\gamma$  is set to 50 for the normal case, 120 for the rectangular case, 150 for the triangular case and 30 for the negative exponential function. Figure 1 depicts the the implied covariance of two alternatives with respect to departure time differences. With the exception of DGP 1, the covariance between two alternatives decreases with increasing departure time differences.

insert figure 1 about here

Multivariate normal random variables that are needed to compute the stochastic utilities are generated using a Gibbs sampling procedure.<sup>8</sup> In 100 replications of the process we simulate the random utilities of 1000 passengers for each O&D market. Assuming the standard probabilistic choice behavior (see (1)), we then calculate the resulting market shares of the itineraries. Having obtained the simulated passenger distribution, we estimate a standard Multinomial Logit, an independent MNP, the adapted GAR-MNP, and the ABC-MNP. For the GAR-MNP we use the Logit Kernel estimator (LKE) that assumes a Gumbel distribution of  $\nu_{in}$  in equation (5). 1000 draws are used for the LKE. The independent and the ABC-MNP employ the Geweke, Hajivassiliou-Keane (GHK) simulator with 20 replications and 100 realizations. For each model, the estimated choice probabilities (=estimated market shares) are compared against the empirical (simulated) market shares. To measure the forecasting accuracy, we compute the root mean squared error,  $RMSE = \sqrt{\frac{1}{20 \cdot 100} \sum_{r=1}^{100} \sum_{i=1}^{20} (\hat{p}_{ir} - p_{ir})^2}$ , and the mean absolute error  $MAE = \frac{1}{20 \cdot 100} \sum_{r=1}^{100} \sum_{i=1}^{20} |\hat{p}_{ir} - p_{ir}|$ , where  $\hat{p}_{ir}$  is the choice probability estimate for itinerary  $i$  in replication  $r$  and  $p_{ir}$  the simulated (empirical) market share of itinerary  $i$  in replication  $r$ . To illustrate the distribution of the forecasting errors that are produced by the competing models, figure 2 depicts the corresponding kernel densities.

insert figure 2 about here

insert table 2 about here

Table 2 clearly shows that for DGP 2 to DGP 5 the ABC-MNP model outperforms the other approaches in terms of forecasting accuracy. The kernel estimates in figure 3 underline this result graphically. This is an expected result for DGP 2 and DGP 3, since these DGPs correspond closely to the ABC-MNP. However, the performance of the ABC-MNP is also outstanding for the DGP 4 and 5, though these DGPs do not correspond closely to the

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<sup>8</sup>The Gibbs sampling procedure is based on a Markov chain that utilizes univariate truncated normal densities to construct conditional variates and has the truncated multivariate normal as its limiting distribution (Hajivassiliou, 1992)

ABC-MNP. Both the GAR- and ABC-MNP can adapt satisfactorily to an independent situation<sup>9</sup>.

## 4 EMPIRICAL APPLICATION

### 4.1 Data

The source for airline demand data are four major computer reservation systems (CRS).<sup>10</sup> The raw CRS data contain each booking as a separate record, but the origin and destination itineraries still have to be extracted from the data set. We use standard rules that identify an origin and destination trip from the raw data. These rules encompass the maximum allowed stay at an airport that is required to distinguish the destination of a trip from a mere transfer stay. Identical O&D itineraries are identified across the CRS and then consolidated in order to form the total passenger demand for an itinerary. It is important to note that CRS data do not account for flown but only for booked passengers.<sup>11</sup> If one could use ticket information, these problems would be circumvented, but the airlines have access to their own tickets only. The main advantage of using CRS data is that since the CRSs include bookings for all carriers they give a representative picture of the demand structure for most markets.

The network management department of Austrian Airlines provided booking data from the major CRS operating systems Amadeus, Galileo, Sabre and Worldspan, covering the period of October 1 to October 31, 1996.

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<sup>9</sup>For the sake of brevity, we do not compare the efficiency and bias of the estimation of the utility parameter  $\beta$ . However, one result is noteworthy and is important for the interpretation of the estimation results presented in the empirical Section. For the DGP 2, the average estimated utility parameter produced by the independent MNP is 0.40 (standard deviation 0.03). The average ABC-MNP estimated  $\beta$  is 0.97 (standard deviation: 0.21). At first sight one could conceive this as a confirmation of the hypothesis that neglecting dependencies between alternatives will lead to biased utility parameter estimates. Note however, that this deviation is also due to the identifying restrictions that are necessary for the independent MNP formulation: When estimating the independent MNP we restrict the random utility variances for each alternative to unity. DGP 2, however, generates a random utility variance (homoskedastic) which is five. The independent MNP accommodates to the identifying variance restrictions by reducing the utility parameters.

<sup>10</sup>Major CRS operators are Amadeus, Apollo, Galileo, Sabre, Worldspan.

<sup>11</sup>The difference is caused by so called no-shows - people that book multiple flights in order to ensure their booking, but do not show up - and so called go-shows, passengers that buy their tickets directly and whose bookings are not recorded in the CRS.

To ensure homogeneity of the markets we focussed on a subset of short-haul domestic O&D markets. The first selection criterion required that Lufthansa (IATA two letter code LH) and Deutsche BA (IATA two letter code DI) offered nonstop or connecting flights at least twice a day. In order to reduce computational needs, we restricted our attention to Monday departures that were consolidated, so that they represent a standard weekday. The selection criteria resulted in 10 O&D markets with 229 itineraries attracting non-zero passenger demand. All origin and departure cities are located in Germany.<sup>12</sup> The largest (smallest) number of alternatives in an O&D market was 36 (5). 142 alternatives were nonstop or online connections offered by Lufthansa, 68 alternatives were nonstop or online connections operated by Deutsche BA. The remaining alternatives are direct, online or interline connections offered by other carriers. The total number of booked passengers is 32,246. The total market share of Lufthansa is 64%. The market share of Deutsche BA amounts to 34%.

#### *4.2 Model specification*

The set of explanatory variables that enter the systematic part of the utility function is to a large extent standard in airline network management. As outlined above our focus is on attributes that describe the itineraries offered. If an itinerary takes the passenger from his origin to his destination without requiring the passenger to change planes then the utility of this itinerary will be higher than a connection which requires the passenger has to change planes. In addition, not having to change planes leads to a shorter elapsed time and yields a higher utility *ceteris paribus*. Another important factor is the role that airline image plays in passenger choice.

Accounting for departure time preferences is a crucial issue when modeling airline passenger demand. In the process of schedule redesign, a flight is often moved within the day and it is important to estimate the preferences

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<sup>12</sup>Furthermore, changes in flight numbers during the four week period were accounted for, and marketing flights were mapped to their corresponding operating flights. For marketing reasons, operating flights can be sold under two or more flight numbers (code-sharing). The additional non-operating entries in the schedule are referred to as marketing flights.

that are associated with a specific departure time correctly. A departure in the middle of the day is typically inconvenient for business travellers. Early morning and late afternoon fits their schedules better. We advocate a Fourier series approach to model the intra-day pattern (diurnality) of departure time preferences which requires only a few additive terms to fit meaningful diurnality functions. The following specification is employed:

$$F_Q(t_i; \gamma, \phi) = \sum_{q=1}^Q \gamma_q \sin\left(\frac{2\pi q}{T} t_i + \phi_q\right), \quad (32)$$

where  $t_i$  stands for departure time, and  $\gamma = (\gamma_1, \dots, \gamma_Q)'$  and  $\phi = (\phi_1, \dots, \phi_Q)'$  are parameters that have to be estimated.  $T$  is the maximum departure time possible which is set to 1440 (minutes of a day)

For all of the estimated models the same specification for the systematic utility is employed. Write the basic specification (1) as  $u_{in} = v_i + \varepsilon_{in}$ , where  $v_i = x_i' \beta$ , then we have

$$v_i = \beta_1 \cdot N_i + \beta_2 \cdot E_i + \beta_3 \cdot LH_i + \beta_4 \cdot DI_i + F_3(t_i; \gamma_1, \gamma_2, \gamma_3, \phi_1, \phi_2, \phi_3). \quad (33)$$

$E_i$  denotes the elapsed time if the itinerary  $i$  is a nonstop connection, and is zero otherwise. The binary indicator  $N_i$  is one for a nonstop connection and zero if not.

For the covariance generating components in the GAR- and ABC-MNP the following specification is chosen. For the GAR-MNP,

$$w_{i,j}^* = \exp\left(-\left(\alpha_1 \cdot \frac{|t_i - t_j|}{100} + \alpha_2 \cdot |LH_i - LH_j|\right)\right) \quad (34)$$

is used as the weighting function (see section 2.3). For the ABC-MNP the covariance generating component is

$$\vartheta_{in} = \int_T \xi_n^c(y) A(y, t_i; \lambda_1) dy + \delta_{LH_i,0} \cdot \xi_n^{b,0} + \delta_{LH_i,1} \cdot \xi_n^{b,1}, \quad (35)$$

where  $var(\xi_n^c(t_i)) = \sigma_1^2$  and  $var(\xi_n^{b,0}) = var(\xi_n^{b,1}) = \sigma_2^2$ . This implies the covariance matrix entries

$$cov(u_{in}, u_{jn}) = \delta_{ij} + \sigma_1^2 \exp\left(\frac{(t_i + t_j)^2}{2\lambda_1^2}\right) + \delta_{LH,DI} \sigma_2^2. \quad (36)$$

### 4.3 Estimation results

The maximum likelihood estimation results for the MNL, Independent MNP, GAR-MNP and ABC-MNP are contained in table 3. For the GAR- and ABC-MNP the configuration of the LKE and GHK simulators are the same as in Section 3.

insert table 3 about here

For all specifications the estimates of the parameters that appear in the systematic utility equation (33) have the expected sign with small standard deviations.<sup>13</sup> The shape of the estimated departure time preference function is as is expected of a typical Monday: There is a high utility associated with an early morning departure and a utility peak in the evening (see figure 3). It is idiosyncratic of a Monday that the morning peak is higher. For Friday departures one would obtain the inverse pattern.

insert figure 3 about here

The model parameters that have to take on positive values in order to generate non-zero covariances between alternatives are  $\rho$  and  $\sigma_0$  in the GAR- and  $\sigma_1^2$  and  $\sigma_2^2$  in the ABC-MNP. Table 3 shows all of these parameters to be different from zero at 1% significance. Two goodness-of-fit measures are used to assess the ability of the models to explain and predict itinerary passenger demand. Table 3 reports the passenger weighted RMSE and the MAE associated with an in-sample forecast of the itinerary passenger demand. Both the RMSE and MAE show a clear improvement in the goodness of fit produced by the two models that account for dependencies between alternatives. Applying the modified Likelihood Ratio statistic proposed by Horowitz (1983), it becomes evident that both the independent MNP and GAR-MNP are rejected in favor of the ABC-MNP. Table 4 reports the detailed results.

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<sup>13</sup>The differences in the magnitude of the utility parameters between the I-MNP and ABC-MNP are expected. See Section 3.

insert table 4 about here

For both the GAR- and ABC-MNP departure time differences generate similarity between alternatives. The crucial question is whether the drawbacks of the adapted GAR model (see Section 2.3) matter when the model is applied in network management. The following scenario is designed to emphasize the importance of modeling the similarities between offered itineraries when evaluating schedule revisions. Table 5 displays a subset of itineraries that were used for estimation.

insert table 5 about here

We use the ML estimates for the GAR- and ABC-MNP and compute an estimate of the covariance and correlation matrix that is implied by the two models. Suppose that Airline 1 decides to introduce a new flight which departs at the same time as itinerary number 9 offered by Airline 2. Both nonstop connections depart at 11:05 a.m. and have an implied elapsed time of 65 minutes. In Section 2.3 we showed that the GAR-MNP implies that the introduction of a new alternative will produce a completely new covariance matrix. For the ABC-MNP this only requires that a new row and column be inserted, while the other covariance and variance elements remain unchanged. Tables 6, 7 and 8 contain the implied covariance matrices before and after the introduction of the new flight for the ABC-MNP<sup>14</sup> and the GAR-MNP.

insert table 6 about here

insert table 7 about here

insert table 8 about here

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<sup>14</sup>Since the remaining part of the covariance matrix does not change we only show the matrix after the introduction of the new flight



It is important to recognize the significant increase in the variance associated with itinerary 9. Using the parameter estimates from table 3 we can now estimate the relative change in choice probabilities that are induced by the introduction of the new flight. Figure 4 depicts the outcome for the MNL, GAR-MNP and the ABC-MNP.

insert figure 4 about here

The poor performance of the MNL is obvious and a consequence of the IIA assumption. The relative change of choice probabilities is identical for all alternatives. By contrast, the ABC-MNP result is much more plausible with itinerary number 9 suffering the largest relative reduction in choice probability. Although alternatives with departure times close to alternative 9 also experience a loss in their choice probabilities, the choice probabilities of alternatives with departure times in the morning or the evening are hardly affected at all. The relative change in the choice probabilities is very different in the GAR-MNP: The market share of alternative 9 is increased after the introduction of the new flight. The potential reasons for this implausible effect have been outlined in Section 3. Tables 6 and 7 reveal that the variance of alternative 9 has increased leading to the higher choice probability, *ceteris paribus*. This variance increase clearly more than offsets the negative effect on choice probability that is exerted by the non-zero covariance of itinerary 9 and the newly introduced alternative. One reason for this is that we cannot account for alternative-specific error variances in our GAR-adapton. Admittedly, this greatly reduces the flexibility of the original GAR-MNP, but the restriction is inevitable when the model is applied for network management purposes.

## 5 CONCLUSIONS AND OUTLOOK

With the advent of large international alliances, the network perspective has become even more important for the airline industry. Alliance flight schedules are conceived as a complex network of offered connections that

passengers use to travel from their origins to their desired destinations. Discrete choice models play a key role in the industry when assessing alliance synergies or simply evaluating alternative schedules in terms of expected (network) passenger demand and revenue. An econometrician in the airline industry enjoys privileged data availability. First, historical passenger demand data can be obtained from Computer Reservation Systems. Second, the commercial availability of worldwide schedules makes it possible to obtain an exact view of the supply side of each O&D market.

Three idiosyncrasies of airline network management have been emphasized: First, the omnipresence of the independence of alternatives problem: In a competitive market it is likely that the planes of two carriers will start at almost the same time to the same destination. Secondly, nominal identification must be discarded, since it is insufficient to estimate the choice probability of e.g. all nonstop, interline and online connections, analogously to the standard commuter problem. In airline network management one has to deliver choice probabilities on the level of the offered itineraries. However, the number of alternative itineraries one has to account for can become quite large. Hence, the applicability of the Multinomial Probit model, the generic solution to non-IIA problems, is questionable, because of computational demand and unsolved identification issues. Third, individual passenger information is not of interest for discrete choice modeling in airline network management. Instead, the focus is on schedule related attributes (elapsed time, departure time, etc.). By reallocating flights and changing planes during the schedule design process, these attributes form the strategic instruments that influence passenger demand.

Our econometric contribution is the formulation and estimation of a MNP model that perfectly meets the requirements of airline network management. The specification is suitable for discrete choice modeling in non-IIA situations in which the analyst has to account for a large number of alternatives, and where the focus is on using attribute related covariates. The MNP model that we propose is referred to as the Attribute Based Covariance-MNP since we allow for random utility deviations that are asso-

ciated with the attributes of alternatives. The ABC-MNP adopts elements both from random coefficient models along the lines of Hausman and Wise (1978) and the GAR-MNP. In a simulation study we showed that the ABC-MNP clearly outperforms other discrete choice specifications.

In an empirical application we have demonstrated the ABC-MNP's practical applicability for airline network management, and compared its performance with that of the GAR-MNP and standard discrete choice models. In a simulation study have shown that the ABC-MNP outperforms both the Multinomial Logit and the Independent MNP. The new model defies Horowitz' (1991 and Bunch's (1991) critique, who questioned the superiority of the Multinomial Probit over the Nested Logit model. In addition to the airline network management task, the ABC-MNP is also applicable to related discrete choice problems in transportation and marketing research.

Besides its advantages, the ABC-MNP also implies one major drawback. Compared to GEV models and the GAR-MNP the parameter estimation is more computer intensive. We intend to apply the method of simulated scores (MSS) as proposed by Hajivassiliou and McFadden (1998) in order to cope with this problem. We expect MSS to provide a significant reduction of the computational burden.

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A COVARIANCES IN THE ABC-MNP: ORDERED AND CONTINUOUS  
ATTRIBUTES

Let  $z_i^o = 1, \dots, M$  be an ordered polytomous attribute of an alternative  $i$  and  $\vartheta_i^o$  be the error component related to that attribute (we drop the index  $n$  for the sake of brevity of notation).  $\vartheta_i^o$  is defined as

$$\vartheta_i^o = \sum_{m=1}^M \xi_m^o \cdot A(m, z_i^o; \lambda),$$

where  $A(m, z_i^o; \lambda)$  is an amplitude function and  $\xi_m^o$  a random variable representing a white noise process. We have

$$\begin{aligned} E(\xi_i^o) &= 0 \\ E(\xi_i^o \xi_j^o) &= \sigma^2 \cdot \delta_{i,j} \end{aligned} \tag{A.1}$$

In order to derive (26) we use:

$$\text{cov}\left(\sum_i \xi_i^o, \sum_j \xi_j^o\right) = \sum_{i,j} \text{cov}(\xi_i^o, \xi_j^o) \tag{A.2}$$

$$\text{cov}(c \cdot \xi_i^o, \xi_j^o) = c \cdot \text{cov}(\xi_i^o, \xi_j^o) \tag{A.3}$$

$$\sum_m \delta_{m,m'} \cdot A(m, m') = A(m, m), \tag{A.4}$$

where  $c$  denotes an arbitrary constant and  $\delta_{m,m'}$  the Kronecker symbol. We have

$$\text{cov}(\xi_i^o, \xi_j^o) = E(\xi_i^o \xi_j^o) - E(\xi_i^o)E(\xi_j^o) = \sigma^2 \cdot \delta_{i,j} - 0 \cdot 0 = \sigma^2 \cdot \delta_{i,j} \tag{A.5}$$

$$\begin{aligned} \text{cov}(\xi_i^o, \xi_j^o) &= \text{cov}\left(\sum_m \xi_m^o A(m, z_i^o; \lambda), \sum_{m'} \xi_{m'}^o A(m', z_j^o)\right) \\ &= \sum_m \sum_{m'} \text{cov}(\xi_m^o A(m, z_i^o; \lambda), \xi_{m'}^o A(m', z_j^o)) \quad \text{using A.2} \\ &= \sum_m \sum_{m'} A(m, z_i^o; \lambda) A(m', z_j^o) \text{cov}(\xi_m^o, \xi_{m'}^o) \quad \text{using A.3} \\ &= \sum_m \sum_n A(m, z_i^o; \lambda) A(m', z_j^o) \sigma^2 \delta_{m,m'} \quad \text{using A.5} \\ &= \sigma^2 \sum_m A(m, z_i^o; \lambda) A(m', z_j^o) \quad \text{using A.4} \end{aligned}$$

In the following we derive the covariance of the random utility  $\vartheta^c(y)$  and  $\vartheta^c(z)$  with  $y, z \in I$ .  $\vartheta^c(y)$  is defined in a similar way to equation (27), but



we have dropped the subscript  $i$ , since only the level of the variable  $y$  is needed, i.e. consider  $y = x_i^c, z = x_j^c$ :

$$\vartheta^c(y) = \int \xi^c(z)A(y, z)dz \quad (\text{A.6})$$

$\{\xi^c(y), y \in I\}$  can be described using Dirac's delta function  $\delta(y - z)$ :

$$\begin{aligned} E(\xi^c(y)) &= 0 \\ E(\xi^c(y), \xi^c(z)) &= \sigma^2 \cdot \delta(y - z) \end{aligned} \quad (\text{A.7})$$

We will make use of the properties of the Delta function:

$$\int dy \int dz F(y)F(z)\delta(y - z) = \int dy F(y)^2. \quad (\text{A.8})$$

where  $F(y)$  is an arbitrary function of  $y$ .

By changing the order of integration for the calculation of the expectation and the integral defined in (A.6), we have

$$\begin{aligned} E(\vartheta^c(y)) &= E\left(\int \xi^c(z)A(y, z)dz\right) \\ &= \int E(\xi^c(z))A(y, z)dz \\ &= \int 0 \cdot A(y, z)dz \\ &= 0 \\ \\ E(\vartheta^c(y)\vartheta^c(s)) &= E\left(\int \xi^c(z)A(y, z)dz \int \xi^c(s')A(s, s')ds'\right) \\ &= \int \int dz ds' A(s, s')A(y, z)E(\xi^c(s')\xi^c(z)) \\ &= \int \int dz ds' A(s, s')A(y, z)\sigma^2\delta(s' - z) \\ &= \sigma^2 \int dz A(s, z)A(y, z) \\ \\ cov(\vartheta^c(y), \vartheta^c(s)) &= \sigma^2 \int dz A(s, z)A(y, z). \end{aligned} \quad (\text{A.9})$$

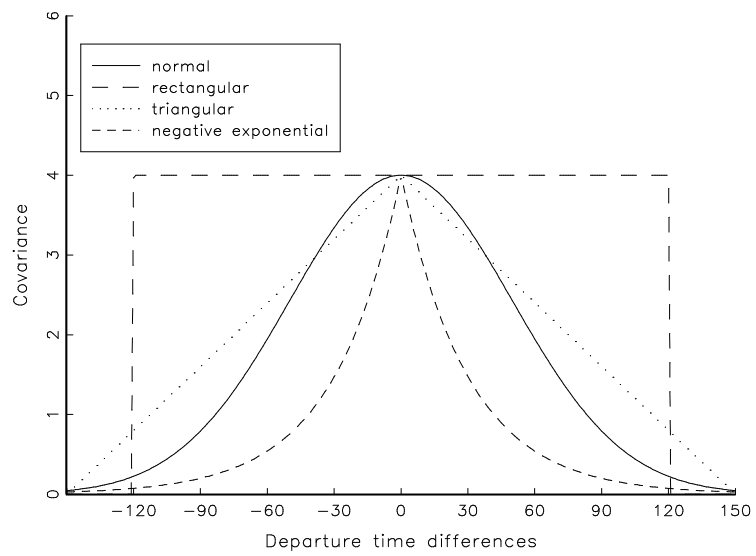


FIGURE 1.— Departure time difference dependent covariances

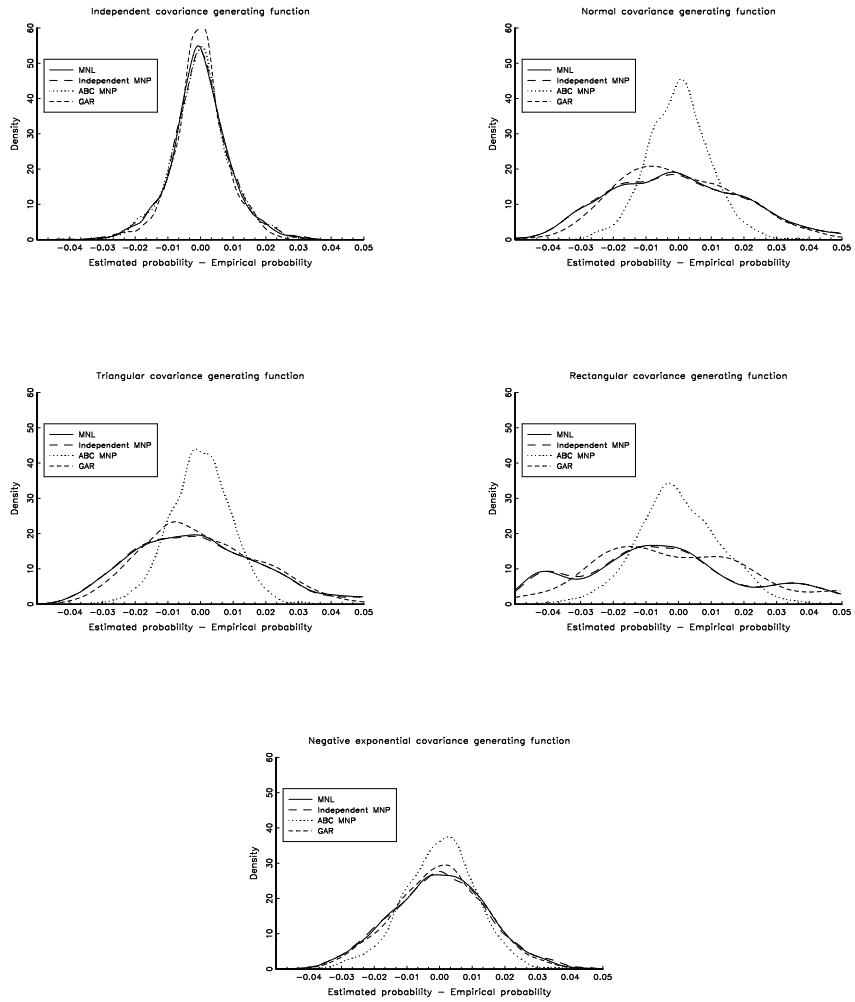


FIGURE 2. — Monte Carlo Results<sup>a</sup>: Density plots for difference between actual vs. forecasted market shares

<sup>a</sup> Note: Gaussian kernels with bandwidth as proposed proposed by Silverman (1986) p. 48.

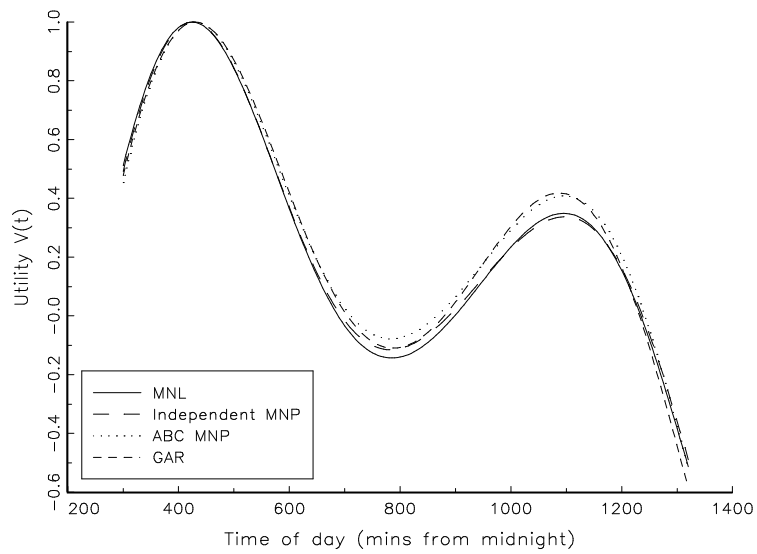


FIGURE 3.— Diurnality: Time of day preferences

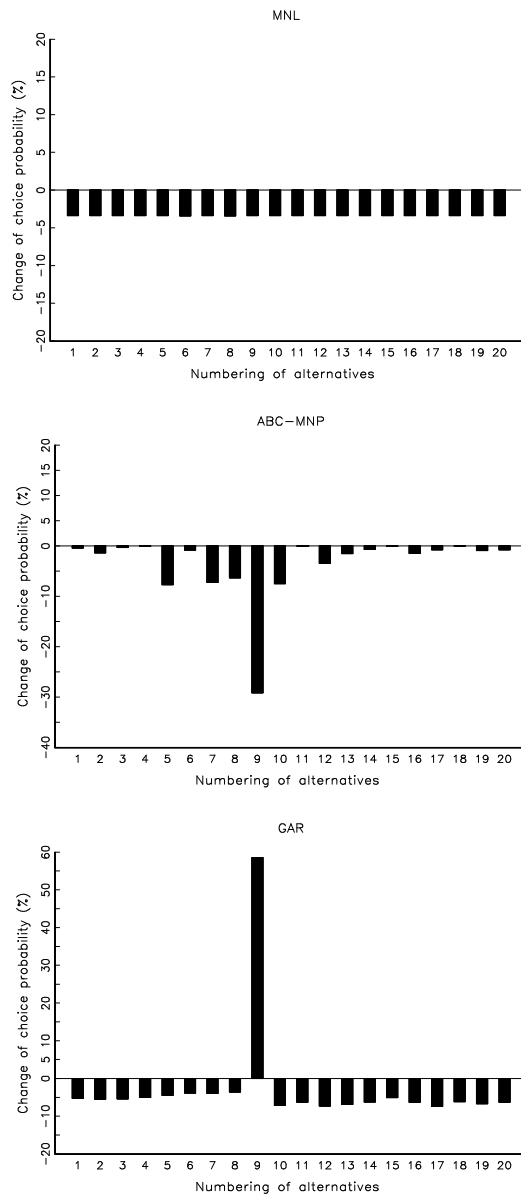


FIGURE 4.— Change of choice probabilities

TABLE 1  
SAMPLE DESIGN MONTE CARLO STUDY

O&D Market 1			O&D Market 2		
Alternative	Nonstop- Indicator	Dep. Time (min. after midnight)	Alternative	Nonstop- Indicator	Dep. Time (min. after midnight)
1	1	390	1	1	390
2	1	390	2	1	420
3	0	480	3	1	450
4	1	485	4	0	600
5	0	500	5	0	620
6	1	735	6	0	800
7	0	900	7	0	960
8	1	1120	8	0	1140
9	1	1140	9	1	1170
10	1	1200	10	1	1200

TABLE 2  
MONTE CARLO RESULTS

DGP	Model	Mean	Std.Dev.	Skewness	Kurtosis	RMSE	MAE
1	MNL	0.0000	0.0090	0.0001	0.0001	0.0090	0.0070
	I-MNP	0.0000	0.0092	0.0001	0.0001	0.0090	0.0070
	ABC-MNP	0.0000	0.0092	0.0002	0.0001	0.0090	0.0070
	GAR	0.0000	0.0082	-0.0001	0.0000	0.0080	0.0060
2	MNL	0.0000	0.0206	0.0047	0.0010	0.0210	0.0170
	I-MNP	0.0000	0.0209	0.0062	0.0011	0.0210	0.0170
	ABC-MNP	0.0000	0.0096	0.0001	0.0001	0.0100	0.0070
	GAR	0.0000	0.0183	0.0030	0.0006	0.0180	0.0150
3	MNL	0.0000	0.0203	0.0091	0.0011	0.0200	0.0160
	I-MNP	0.0000	0.0206	0.0108	0.0012	0.0210	0.0160
	ABC-MNP	0.0000	0.0093	-0.0001	0.0000	0.0090	0.0070
	GAR	0.0000	0.0176	0.0035	0.0005	0.0180	0.0140
4	MNL	0.0000	0.0319	0.0644	0.0085	0.0320	0.0240
	I-MNP	0.0000	0.0322	0.0707	0.0092	0.0320	0.0240
	ABC-MNP	0.0000	0.0128	0.0005	0.0002	0.0130	0.0100
	GAR	0.0000	0.0251	0.0132	0.0023	0.0250	0.0200
5	MNL	0.0000	0.0144	0.0001	0.0003	0.0140	0.0120
	I-MNP	0.0000	0.0148	0.0006	0.0003	0.0150	0.0120
	ABC-MNP	0.0000	0.0111	-0.0004	0.0001	0.0110	0.0090
	GAR	0.0000	0.0138	0.0000	0.0002	0.0140	0.0110

$R = 100$  replications,  $J = 20$  itineraries.

Mean AE (mean absolute error):  $\frac{1}{RJ} \sum_{r=1}^R \sum_{i=1}^J |\hat{p}_{ir} - p_{ir}|$

RMSE (root mean squared error):  $\left[ \frac{1}{RJ} \sum_{r=1}^R \sum_{i=1}^J (\hat{p}_{ir} - p_{ir})^2 \right]^{0.5}$

TABLE 3  
ESTIMATION RESULTS

	MNL	I-MNP	GAR-MNP	ABC-MNP
$\beta_1$	6.25 (0.233)	2.67(0.109)	6.55 (0.273)	4.16 (0.235)
$\beta_2$	-0.03 (0.003)	-0.01 (0.001)	-0.03 (0.003)	-0.01 (0.003)
$\beta_3$	0.96 (0.065)	0.45 (0.061)	1.00(0.072)	1.07 (0.067)
$\beta_4$	0.87 (0.063)	0.41 (0.063)	0.82(0.070)	0.77 (0.056)
$\gamma_1$	0.61 (0.065)	0.32 (0.065)	0.92 (0.078)	4.01 (0.334)
$\gamma_2$	0.76 (0.053)	0.39 (0.026)	1.08 (0.079)	3.67 (0.244)
$\gamma_3$	0.10 (0.027)	0.06 (0.014)	0.11 (0.038)	0.32 (0.043)
$\phi_1$	3.48 (0.098)	3.45 (0.093)	3.43 (0.085)	3.13 (0.016)
$\phi_2$	2.73 (0.035)	2.71 (0.035)	2.74 (0.036)	2.60 (0.011)
$\phi_3$	1.87 (0.140)	1.81 (0.119)	2.01 (0.186)	1.32 (0.116)
$\rho$			0.63 (0.020)	
$\sigma_0$			1.07 (0.100)	
$\alpha_1$			6.74 (-0.422)	
$\alpha_2$			2.77 (-0.220)	
$\sigma_1^2$				37.69 (5.833)
$\lambda_1$				144.59 (6.02)
$\sigma_2^2$				1.24 (0.162)
$\log \mathcal{L}$	-95500.78	95461.77	-95084.84	-94891.32
MAE	0.285	0.283	0.267	0.253
RMSE	0.836	0.847	0.695	0.68

MAE (mean absolute error):  $\left[ \frac{1}{N} \sum_{i=1}^J |\hat{N}_i - N_i| \right]$

RMSE (root mean squared error):  $\left[ \frac{1}{N} \sum_{i=1}^J \left( \frac{\hat{N}_i - N_i}{N_i} \right)^2 \right]^{0.5}$

$N$  is the total number of passengers,  $N_i$  the number of passengers on itinerary  $i$

$\hat{N}_i = \hat{p}_i N_{d(i)}^D$  is the estimated demand for itinerary  $i$ , where  $\hat{p}_i$  is the estimated choice probability and  $N_{d(i)}^D$  is the total passenger volume on O&D market  $d(i)$ .

Robust standard errors in parantheses



TABLE 4  
MODIFIED LIKELIHOOD RATIO STATISTICS

	LR	MLR
GAR-MNP vs. MNL	832	414
ABC-MNP vs. I-MNP	1141	569
ABC-MNP vs. GAR-MNP		192

Modified LR statistic (MLR) as proposed by Horowitz (1983) and Horowitz et al. (1986)

TABLE 5  
 FLIGHTS IN EXEMPLARY MARKET

$i$	Dep. time	Nonstop	Elap. time	Airline 1	Airline 2
Original example market					
1	06:25:00	1	65	1	0
2	06:30:00	1	65	0	1
3	07:55:00	1	65	0	1
4	08:00:00	1	65	1	0
5	09:15:00	1	65	0	1
6	09:15:00	0	185	1	0
7	09:45:00	1	65	1	0
8	09:45:00	0	185	1	0
9	11:05:00	1	65	1	0
10	13:20:00	1	70	1	0
11	14:20:00	1	65	1	0
12	14:25:00	1	65	0	1
13	15:50:00	1	65	0	1
14	16:15:00	1	65	1	0
15	17:35:00	1	65	1	0
16	17:40:00	1	65	0	1
17	18:25:00	1	65	1	0
18	19:05:00	1	65	0	1
19	19:25:00	1	65	1	0
20	20:50:00	1	65	0	1
Inserted flight:					
9+	11:05:00	1	65	0	1

TABLE 6  
COVARIANCE MATRIX IMPLIED BY ABC-MNP

	1	2	...	5	6	7	8	9	10	11	12	13	14	...	20	9+
1	40															
2	38	40														
3	31	33	...													
4	32	31	...													
5	19	21	...	40												
6	20	20	...	38	40											
7	16	15	...	37	38	40										
8	16	15	...	37	38	39	40									
9	7	6	...	28	29	34	34	40								
10	2	1	...	9	10	14	14	26	40							
11	1	0	...	4	5	7	7	16	36	40						
12	0	1	...	5	4	6	6	14	34	38	40					
13	0	1	...	2	1	2	2	5	22	31	33	40				
14	1	0	...	1	2	2	2	5	19	29	28	37	40			
15	1	0	...	0	1	1	1	2	9	16	16	29	34	...		
16	0	1	...	1	0	0	0	1	7	14	16	29	32	...		
17	1	0	...	0	1	1	1	2	5	10	10	21	26	...		
18	0	1	...	1	0	0	0	0	2	5	7	16	19	...		
19	1	0	...	0	1	1	1	1	3	5	4	12	17	...		
20	0	1	...	1	0	0	0	0	0	1	2	6	6	...	40	
9+	6	7	...	29	28	32	32	38	24	15	16	7	4	...	1	40

TABLE 7  
COVARIANCE MATRIX IMPLIED BY GAR-MNP

	1	2	...	5	6	7	8	9	10	11	12	13	14	...	20
1	1.8														
2	1.1	1.8													
3	0	0	...												
4	0	0	...												
5	0	0	...	1.5											
6	0	0	...	0.8	1.5										
7	0	0	...	0.3	0.3	1.5									
8	0	0	...	0.3	0.3	0.8	1.5								
9	0	0	...	0.2	0.2	0.6	0.6	1.1							
10	0	0	...	0	0	0	0	0	1.1						
11	0	0	...	0	0	0	0	0	0	1.8					
12	0	0	...	0	0	0	0	0	0	1.1	1.7				
13	0	0	...	0	0	0	0	0	0	0	0	1.7			
14	0	0	...	0	0	0	0	0	0	0	0	1	1.7		
15	0	0	...	0	0	0	0	0	0	0	0	0	0	...	
16	0	0	...	0	0	0	0	0	0	0	0	0	0	...	
17	0	0	...	0	0	0	0	0	0	0	0	0	0	...	
18	0	0	...	0	0	0	0	0	0	0	0	0	0	...	
19	0	0	...	0	0	0	0	0	0	0	0	0	0	...	
20	0	0	...	0	0	0	0	0	0	0	0	0	0	...	1.1

TABLE 8  
COVARIANCE MATRIX IMPLIED BY GAR-MNP (AFTER ADDING NEW  
FLIGHT)

	1	2	...	5	6	7	8	9	10	11	12	13	14	...	20	9+
1	1.8															
2	1.1	1.8														
3	0	0	...													
4	0	0	...													
5	0	0	...	1.5												
6	0	0	...	0.8	1.5											
7	0	0	...	0.3	0.3	1.5										
8	0	0	...	0.3	0.3	0.8	1.5									
9	0	0	...	0	0	0	0	1.8								
10	0	0	...	0	0	0	0	0	1.1							
11	0	0	...	0	0	0	0	0	0	1.8						
12	0	0	...	0	0	0	0	0	0	1.1	1.7					
13	0	0	...	0	0	0	0	0	0	0	0	1.7				
14	0	0	...	0	0	0	0	0	0	0	0	1	1.7			
15	0	0	...	0	0	0	0	0	0	0	0	0	0	...		
16	0	0	...	0	0	0	0	0	0	0	0	0	0	...		
17	0	0	...	0	0	0	0	0	0	0	0	0	0	...		
18	0	0	...	0	0	0	0	0	0	0	0	0	0	...		
19	0	0	...	0	0	0	0	0	0	0	0	0	0	...		
20	0	0	...	0	0	0	0	0	0	0	0	0	0	...	1.1	
9+	0	0	...	0	0	0	0	1.1	0	0	0	0	0	...	0	1.8