INVESTIGATING THE RELATIONSHIP BETWEEN MATHEMATICAL KNOWLEDGE FOR TEACHING AND SELF-EFFICACY OF PRE-SERVICE MATHEMATICAL LITERACY TEACHERS

By

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DECLARATION

I, *Nicola Stephanie van Zyl*, hereby declare that the dissertation for *M Ed* is my own work and that it has not previously been submitted for assessment or completion of any postgraduate qualification to another University or for another publication.

Nicola Stephanie van Zyl

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ABSTRACT

Although a good understanding of mathematical content knowledge is essential for effective mathematics teaching, this might not be enough. Mathematical knowledge for teaching (MKT) requires a kind of depth and detail special to teaching, and involves mathematical reasoning as well as thinking from a learners' perspective. Educational outcomes are also influenced by teachers' self-efficacy beliefs regarding their ability to teach effectively.

This study was an investigation into the relationship between pre-service teachers' mathematical knowledge for teaching (MKT) and their mathematical self-efficacy with regard to MKT. Participants in the study were 137 BEd (FET) students at Nelson Mandela Metropolitan University, specializing in Mathematical Literacy as teaching subject. The quantitative data used for the study were gathered using a questionnaire on MKT for the topics number concepts and operations. This questionnaire was designed by Deborah Ball's Michigan research team, to which I added a question on self-efficacy for every item.

An analysis of the data gathered from the questionnaire reveals interesting and disturbing trends. The results suggest that, in more than 80% of the cases, respondents were either completely sure their answer was correct, or tended to think their answer was correct, indicating high levels of self-efficacy. Since only about 40% of answers were in reality correct, this indicates that participants believed their answer to be correct, although their interpretation of the mathematical knowledge for teaching involved was incorrect. Hence: they don't know that they don't know!

The results of this study suggest that there is a need for educators of teachers to help improve prospective mathematical literacy teachers' mathematical knowledge for teaching. Pre-service teachers should be taught to use cognitive skills that will raise the likelihood of improved learner understanding. For this, robust understanding of the fundamental mathematics involved is needed, as well as high levels of self-efficacy with regard to the teaching of mathematics.

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Introduction

1.1 BACKGROUND

Every year when the Department of Education (DoE) announces the Matric results, mathematics is singled out as one of the problematic subjects. One of the consequences is that the quality of mathematics teaching tends to come under scrutiny, with the perception that the quality of teaching depends on teachers' knowledge of the subject. Stigler and Hiebert (1999, p. 5) observe that the focus must turn to teachers in order to improve teaching and that "the purpose of good teaching is good learning". They argue that the solution to the problem of improving teaching lie with the teachers. Several researchers reason that a teacher's knowledge of his or her subject is one of the most important requirements for good teaching (Ball, Hill & Bass, 2005; Lannin et al., 2013; She, Lan & Wilhelm, 2011; Shulman, 1987). Verloop, Van Driel and Meijer (2001) reviewed studies on teacher knowledge and argue that teacher education will be improved with a better understanding of teacher knowledge. Kleickmann et al. (2013, p. 90) claim that "one of the main challenges for research on teacher education lies in the assessment of teacher knowledge". Ball and Bass (2000, p. 86) argue that teachers need "…a kind of responsibility to subject matter…" in order to successfully deal with the challenges of diverse classrooms.

Research suggests that in school mathematics, self-efficacy has an effect on academic performance irrespective of the level of intellectual ability. Prospective teachers' self-efficacy beliefs have an effect on their performance in the classroom and their efficacy as teachers (Michaelides, 2008). Self-efficacy relates to a person's belief that he or she is capable of accomplishing something with a certain degree of success. Beliefs in personal efficacy impact on motivation, choices, effective functioning, responding to difficulty and susceptibility to stress and depression (Bandura, 1994).

1.2 THEORETICAL FRAMEWORK

The three theoretical constructs that have been identified as framing this study are Pedagogical Content Knowledge, Mathematical Knowledge for Teaching, and Self-efficacy.

Shulman (1987, p. 8) identified Pedagogical Content Knowledge (PCK) as one of seven categories of teacher knowledge: "…content knowledge, general pedagogical knowledge, curriculum knowledge, pedagogical content knowledge, knowledge of learners and their characteristics, knowledge of educational contexts and knowledge of educational ends, purposes and values". Of these seven categories, it was PCK that appeared to generate the greatest interest amongst researchers (Hurrell, 2013). PCK is a special kind of teacher knowledge connecting content and pedagogy (Ball, Lubienski & Mewborn, 2001). While various studies have suggested a link between teachers' PCK and their learners' academic achievement, this has not been demonstrated satisfactorily, possibly because properly validated instruments for measuring PCK have not yet been developed (Ball, Thames & Phelps, 2008; Lanin et al., 2013).

Mathematical Knowledge for Teaching (MKT) focuses on one aspect of mathematics teacher knowledge that has been investigated during the past few years. Ball et al. (2001) define the construct MKT as covering the cognitive domains Subject Matter Knowledge (SMK) and PCK. They suggest that MKT is the construct that relates content knowledge to the practice of teaching.

Self-efficacy is a person's belief in his or her ability to achieve a certain aim (Michaelides, 2008). In his theory of self-efficacy, Bandura (1977) proposes that the first years of teaching could be critical to the long-term development of teacher efficacy. Meiring (2010, p. 47) mentions that self-efficacy "…is a motivational construct and it is not linked to competence". According to Nicolaidou and Philippou (2003, p. 3), self-efficacy plays a big part in the choices people make, and "…play an essential part in achievement motivation".

1.3 STATEMENT OF THE PROBLEM

As a teacher educator, involved with teaching both mathematics as well as mathematics and mathematical literacy method, I have realised that, although the Bachelor of Education Further Education and Training (BEd FET) students might pass the advanced mathematics modules taken in their second and third years, they often struggle with the mathematics required at secondary school level. This could affect their belief in their own ability to be able to teach effectively. When teaching the mathematics method module, I have often found that students have serious misconceptions and inadequate deeper understanding of the fundamental mathematics involved in school-level mathematics problems. This includes the students specialising in mathematical literacy, who take a course in Mathematics I during their third year. Not enough is known about the strength of these students' understanding of the mathematics needed to teach (their MKT) and their beliefs in their own ability to teach effectively (their self-efficacy). Ball et al. (2001) argue that the dominant reason for the lack of mathematical proficiency among learners seems to be the lack of MKT needed by teachers for teaching mathematics effectively. Possessing effective mathematical knowledge for teaching implies the teacher needs to understand not only that something is true, but also to understand why it is true, and the teacher has to be able to convey this knowledge to learners (Shulman, 1986).

Correct mathematical knowledge is not the only requirement for effective teaching. Pre-service teachers' self-efficacy beliefs concerning their own MKT might have a positive or a negative influence on the efficacy of their teaching (Bandura, 1977). Self-efficacy has been described as a construct that "…has to do with self-perception of competence rather than actual level of competence" (Oh, 2011, p. 236). Faulty beliefs of competence may have consequences for the options teachers choose in their teaching or in the rigour employed in the preparation of their teaching. Oh (2011) reports that individuals (referring to pre-service teachers in the context of this study) often overestimate or underestimate their actual capabilities. This highlights the problem investigated in this current study – the alignment between MKT and self-efficacy. The assumption is that pre-service mathematics and mathematical literacy teachers' self-efficacy beliefs in their MKT need to be aligned with their actual MKT in order for them to be effective mathematics teachers.

1.4 RESEARCH AIMS AND RESEARCH QUESTIONS

In the past, research in mathematics education has often focused on the constructs, PCK, as defined by Shulman (1986), and MKT, as defined by Ball et al., (2001). Research into self-efficacy, as defined by Bandura (1977), focused on teachers' efficacy beliefs and beliefs about teaching and learning (Lannin et al., 2013). This current study is an effort to investigate the possible alignment, or non-alignment, between the prospective mathematical literacy pre-service teachers' MKT and their self-efficacy beliefs about their mathematical knowledge needed for teaching.

According to Ball et al. (2001), pre-service mathematics teachers might have confidence in their own interpretation of the MKT needed to best teach the topic, while they might be doing it using inappropriate methods of instruction. The purpose of this study is, first to determine the level of MKT of the FET mathematics students, and second to determine whether their self-efficacy beliefs are aligned with their MKT as measured by the questionnaire used in this study.

Research instruments used in research into mathematical self-efficacy generally measure a person's self-efficacy beliefs in being able to do mathematics successfully (Zimmermann, Bescherer & Spannagel, 2010). Education research, however, has not focused on the construct MKT with regard to self-efficacy of pre-service teachers. Since self-efficacy beliefs are generally related to domain and topic (Pajares & Miller, 1995), the intention is that the instrument used in this study, together with the self-efficacy questions for each item, could give an indication of the alignment between self-efficacy beliefs about MKT and outcomes of successful presentations of MKT with respect to the relevant topic.

The study was guided by the following research question:

Does a relationship exist between mathematical knowledge for teaching and self-efficacy of pre-service mathematical literacy teachers?

Two sub-questions needed to be asked in order to address this issue:

- What is the scope of the MKT of the different year groups of FET students specializing in mathematical literacy, on the topic of number concepts and operations, as measured by the survey questionnaire?
- What is the participants' self-efficacy with respect to their MKT for each item of the questionnaire?

Addressing the first question gives an indication of the students' MKT as well as an indication of the development (or not) of MKT during the four years of study. Addressing the second question indicates the students' self-efficacy beliefs regarding their own MKT. These two questions help to investigate the possible existence of a relationship between the students' real MKT and their self-efficacy beliefs regarding this MKT.

1.5 RESEARCH DESIGN AND METHODOLOGY

Since my field of interest is the methodology linked to the training of prospective mathematics teachers, a quantitative research approach, using a validated research instrument, was chosen for the research project. To answer the first sub-question regarding participants' MKT, a questionnaire was used that is part of a set of measures developed by a research team from the University of Michigan, led by Deborah Ball. These measures are known as the 'learning mathematics for teaching measures of mathematical knowledge for teaching' (LMT, 2012). The questionnaire chosen for the current study is called the 'Number concepts and operations Content Knowledge scale' which was developed by the Michigan team (LMT, 2012; Ball, 2003; Hill, Ball & Schilling, 2004; Schilling, Blunk & Hill, 2007). The mathematics topics covered by the questionnaire are number concepts and operations.

The second sub-question was answered by adding an extra component, concerning self-efficacy beliefs with regard to the MKT answer given, to each item of the questionnaire. Data gathered from the questionnaire were analysed using both descriptive and inferential statistical methods in order to answer these research questions, as well as to answer the primary question regarding the relationship between MKT and self-efficacy.

A convenience sample of 137 students from Nelson Mandela Metropolitan University (NMMU) were used as participants in this study. These were all students enrolled in the BEd. (FET) course specialising in mathematical literacy education, from all four years of study.

In this study, the principles of informed consent were observed. All participation by students was voluntary, and participants' anonymity and confidentiality was ensured. Before completing the questionnaire, participants were informed of the objectives of the study. No individual students' MKT was disclosed. The findings regarding misconceptions and lack of MKT with regard to specific topics, as well as information about their efficacy beliefs, might benefit future students if the results of the study are used to inform the method curriculum.

1.6 DEFINITION OF TERMS AND CONCEPTS

The constructs PCK, MKT and self-efficacy are the concepts underpinning this study. Pedagogical content knowledge (PCK) as a construct was defined by Shulman (1986, p. 9) as knowledge "…which goes beyond knowledge of subject matter per se to the dimension of subject matter knowledge for teaching". He argues that PCK is the individual form of content knowledge that represents the aspects of content most relevant to teaching. He explains the teacher's PCK as: the teacher knowing and using a variety of different interpretations of a topic. This includes similarities, comparisons and explanations that can be implemented to make a given topic understandable to a learner. Shulman's notion of PCK (1986) refers to the blending of both knowledge of the subject matter and the pedagogic knowledge needed to teach the subject.

Research, similar to Shulman's, was conducted by a group of researchers from the University of Michigan, who studied the lack of mathematical proficiency of American adults (Ball et al., 2001). This team of researchers started two projects investigating PCK. These were the Mathematics Teaching and Learning to Teach Project (MTLT) and the Learning Mathematics for Teaching Project (LMT) (Ball et al., 2005). The focus of these projects was on "building a map of usable professional knowledge of subject matter" (Ball et al., 2013, p. 11). For researchers and academics, a central contribution of this research team lies in their development of measurement tools for assessing teachers' mathematical knowledge for teaching (MKT). Their research has played a role in the process of investigating the relationship between a teachers' MKT and their learners' performance in mathematics (LMT, 2012). The framework for MKT presented by Ball et al. (2008) explains the concept of MKT in detail, using sub-domains and their measures. This framework broadens Shulman's idea that knowledge for teaching includes specialised knowledge of content, as Ball et al. (2001) define PCK as being a sub-domain of the construct MKT.

Researchers in psychology and education ascribe the concept of teacher efficacy to Bandura's (1977) social cognition theory. In this theory, self-efficacy has to do with an individual's own perception of competence and not with his or her actual level of competence. Furthermore, teachers' self-efficacy beliefs are associated with their conduct, enthusiasm, planning and creativeness in teaching, as well as commitment to teaching. Social cognitive theory suggests that self-efficacy beliefs, rather than actual ability, more accurately predict a teacher's performance (Michaelides, 2008). Bandura (1977) argues that students with high self-efficacy beliefs are more willing to attempt difficult tasks. They are willing to apply a high degree of effort in order to complete a task, they ascribe failure to things which they can control, and are not inclined to apportion blame elsewhere. Self-efficacious individuals overcome obstacles more readily in order to realise their ideals. In direct contrast, persons with low levels of self-efficacy have little belief in their own capability to succeed (Margolis & McCabe, 2006).

A teacher may believe that a required outcome is reached by a specific course of action, but if the teacher doubts his or her ability to successfully perform this course of action, the outcome might not be achieved (Bandura, 1977). Self-efficacy directly influences a teacher's performance in, for instance, the choice of actions and examples in teaching a new topic, the amount of effort put into the preparation and teaching, and the manner in which he or she copes with learners' problems of understanding (Bandura 1977).

Self-efficacy can be associated with MKT, since teacher-efficacy impacts on important educational outcomes such as a teacher's commitment to excellence and his or her enthusiasm for teaching. Self-efficacy also impacts on learner achievement and learners' beliefs in their own capability. In the same way that learners' self-efficacy beliefs have an influence on their academic success, teachers' self-efficacy beliefs with respect to their MKT influence their effectiveness as teachers (Tschannen-Moran & Woolfolk Hoy, 2001).

This current study investigates the possible existence of a relationship between the constructs MKT and self-efficacy. The focus of this study is on teachers' beliefs about the knowledge they need for teaching a specific mathematical topic, and their self-efficacy beliefs regarding this knowledge.

1.7 RATIONALE FOR THE STUDY

Teachers of mathematics and mathematical literacy should have more than just the mathematical knowledge that educated adults are expected to have. They should understand "…the insides of ideas, their roots and connections, their reasons and ways of being represented" (Ball, 2003, p. 8). MKT is described by Ball et al. as "…the mathematical knowledge that teachers need to carry out their work as teachers of mathematics" (Ball et al., 2013, p. 4). Ball (2003, p. 8), when identifying the teaching of mathematics as "a serious and demanding arena of mathematical work", argues the need for a considered and continuous effort to identify the mathematical knowledge needed for effective teaching of mathematics. To improve learners' conceptual understanding, the mathematics teacher needs to be able to explain effectively, for instance, the underlying principle of the concept of the commutative property of numbers – "why is it true that 10 baskets with 25 apples each, contain the same amount of apples as 25 baskets with 10 apples each?"

When lecturers design the method module for teaching mathematical literacy, they should consider what mathematical knowledge is needed for effective teaching of the curriculum. The subject Mathematical Literacy was defined as a "…subject driven by life-related applications of mathematics that must develop learners' ability and confidence to think numerically and spatially in order to interpret and critically analyse everyday situations and to solve problems" (Graven & Buytenhuys, 2011, p. 2). However, there are a multitude of underlying mathematical concepts that teachers not only need to know, but need to know how to teach for better understanding in the Intermediate and Senior phases (IP and SP) of the school curriculum (Siemon et al., 2014). Educators of mathematics teachers have the important responsibility of pre-service teacher training, and are concerned about the effectiveness of the methodology courses in developing the student teachers' MKT. Ball et al. (2013) reason that MKT needs to be the foundation of curriculum-design for the method module of pre-service mathematics teacher training. Research done in this study might be beneficial to this cause.

Although teacher knowledge improves teacher effectiveness, outcomes are also affected by teacher confidence and teacher self-efficacy beliefs. Siemon et al, (2014) identify teachers' knowledge and beliefs about mathematics, and the teaching and learning of mathematics as a primary factor in the failure to improve learners' performance in mathematics. According to Cerit (2010, p. 69) "determining the level of pre-service teachers' self-efficacy belief may contribute to foresee how they will behave during in-service training". This may also be important in terms of the efficiency of teacher training programmes.

1.8 BRIEF OVERVIEW OF THE CHAPTERS

This chapter has given a brief description of the research problem and the research questions, as well as the rationale behind the study and the objectives of this study.

Chapter two provides a comprehensive overview of the literature relevant to this study. Literature on teacher knowledge, with specific reference to PCK and MKT, is discussed, followed by a discussion of the literature on teacher knowledge and teacher self-efficacy. A review of research done in these fields is given, as well as an overview of methods of measurement of MKT and self-efficacy.

The methodology and research design of the study is discussed in Chapter 3, and gives motivations for the methods employed. The sample and research instrument used is described, and a summary of the data collection methods and data analysis is given. Limitations and ethical considerations are also touched upon.

In Chapter 4, results obtained from the questionnaire on MKT and self-efficacy are analysed and discussed critically, with reference to the research questions underpinning the study.

The last chapter highlights the most important findings of the study, implications for development of teachers' mathematical knowledge for teaching are mentioned, and recommendations for further research are given.

1.9 SUMMARY

The research problem under investigation in this study is the relationship between MKT and self-efficacy. In this chapter, the research problem was identified after some background was given and the theoretical framework briefly discussed.

With the aim of answering the research question as well as the sub-questions, the quantitative research method and the research instrument that was used, was discussed. The constructs underpinning this research was defined and concisely explained.

This study has as its principal aim the desire to investigate the existence of a relationship between the FET students' MKT and their self-efficacy beliefs regarding their own MKT. If students have an erroneous perception of their own efficacy to teach effectively, it could have unfavourable consequences for the learners that will be in their classrooms.

In the next chapter, some of the literature on the constructs PCK, MKT and self-efficacy is reviewed.

Chapter 2

Literature review

2.1 INTRODUCTION

The results of international studies and South African education reports have repeatedly shown that the state of mathematics and science education in South Africa is in crisis. Because of this, teacher training of prospective mathematics and science teachers is a matter of the highest importance for education in South Africa (Rollnick & Mavhunga, 2015; Adler & Davis, 2006).

In the Curriculum and Assessment Policy Statement (CAPS) of the South African Department of Basic Education, one of the specific aims of the teaching of mathematics in South African Schools is to develop problem solving and cognitive skills.

> *Teaching should not be limited to **"how"** but should* **rather** *feature the **"when"** and **"why"** of problem types. Learning procedures and proofs without a good understanding of why they are important will leave learners ill-equipped to use their knowledge in later life* (DBE, 2011, p. 8).

This suggests that the curriculum itself is underpinned by a philosophy that supports the teaching and learning of mathematics. This might imply that the onus for the crisis in mathematics education might lie with teachers and teacher knowledge, rather than with the curriculum itself.

This study seeks to investigate the alignment between pre-service mathematics teachers' Mathematics Knowledge for Teaching (MKT) and their self-efficacy beliefs regarding their MKT. In this chapter the focus is on a review of the existing literature on MKT and literature on self-efficacy related to this study. Views of seminal scholars in the fields of mathematical teacher knowledge as well as beliefs about the teaching of mathematics are considered and discussed. Teachers' mathematical knowledge for teaching (MKT) and their self-efficacy regarding this MKT are the constructs that form the theoretical framework of this study. Teacher knowledge and teacher self-efficacy beliefs are discussed in some detail. The section about teacher knowledge considers the definitions and different models of pedagogical content knowledge and of mathematical knowledge for teaching. In the section about teacher self-efficacy beliefs, attention is paid to definitions and models of the constructs teacher efficacy and teacher self-efficacy. These sections are followed by a brief discussion of the link between teachers' mathematical knowledge for teaching and their self-efficacy beliefs. The chapter concludes with a review of some of the research done in these fields, and a discussion of the ways in which the constructs MKT and self-efficacy are measured.

2.2 TEACHER KNOWLEDGE

Fennema and Franke (1992) view teacher knowledge as knowledge that can only be properly understood from the perspective of the milieu in which teachers work. This milieu includes the educational system in a specific country – its aims, the curriculum and associated materials, and the assessment system. It also includes the specific school where the teaching takes place, and its practices and beliefs (Petrou & Golding, 2011).

Researchers vary in their beliefs about existing aspects of teacher knowledge - aspects such as the foundation of teacher knowledge and the depth and structure of teacher knowledge (Mosvold & Fauskanger, 2013). When considering the knowledge base of teaching, Shulman (1987) argues that

> *The key to distinguishing the knowledge base of teaching lies at the intersection of content and pedagogy, in the capacity of a teacher to transform the content knowledge he or she possesses into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students* (Shulman, 1987, p. 15)

According to Siemon et al. (2014, p. 53) "teaching is neither simply common sense nor something that some people are born able to do". Gitomer and Zisk (2015) argue that teacher knowledge has always been a prerequisite for successful teaching, and that teacher knowledge is closely related to teaching ability. They maintain that teachers should not only have the subject-specific content knowledge that supports instruction and learning, but should be able to integrate content knowledge with pedagogical knowledge. Teachers should know and understand individual learning, and should be able to share their own knowledge in ways that will increase learner understanding (Gitomer & Zisk, 2015).

2.2.1 Mathematical proficiency

Research on learning mathematics include investigation into mathematical proficiency. Kilpatrick, Swafford and Findell (2001) formulated the aims of mathematics learning by defining mathematical proficiency as having five intertwining strands. These five strands are: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. Together these strands form a framework for discussing the knowledge and beliefs that comprise mathematical proficiency. Conceptual understanding is an understanding of concepts, operations and relations, which often result in students realising similarities and connections between related facts. Procedural fluency is knowledge of the use of procedures (when and how to use) and skill in accurate and efficient computation. Strategic competence is the ability to verbalise and solve mathematical problems, and is jointly supportive of the previous two strands. Adaptive reasoning refers to the ability to think logically and to be able to reflect on and justify reasoning. The possession of a productive disposition implies having the ability to see the sense and usefulness of mathematics, combined with persistence and a self-efficacy belief in ability (Kilpatrick et al., 2001).

It is important to note that Kilpatrick et al. (2001) include confidence in mathematical ability as part of mathematical proficiency. When a teacher has a productive disposition, this can be seen as having the ability to recognise the sense in mathematics, seeing it as both useful and worth doing. Philipp (2007) argues that this implies a belief that constant and continued effort is productive, and emphasises the importance of a positive belief in one's own mathematical ability. According to Phillip (2007, p. 309) "…proficiency in mathematics has affective aspects". In the context of this study, it suggests the confidence in one's own ability for effective teaching. This highlights the importance of the questions on self-efficacy which accompany the MKT-items of the questionnaire.

2.2.2 Procedural and conceptual knowledge

Researchers into cognitive psychology and artificial intelligence relates the distinction between "knowing that" and "knowing how" to a distinction between declarative (or conceptual) knowledge and procedural knowledge. Gitomer and Zisk (2015) mention that declarative knowledge can be described as "descriptive and use-independent", representing concepts, ideas and theories, and, in contrast, procedural knowledge as "prescriptive and use-specific", representing goals, situations and actions (Gitomer & Zisk, 2015, p. 6). This interpretation can be described by looking at an everyday example: a person might know how to ride a bicycle, but could not be able to teach another person how to ride the bicycle. This example illustrates the distinction between practical and theoretical knowledge. In mathematics teaching and learning, this distinction represents the difference between knowledge where relevant algorithms are used, and a deeper understanding of mathematics, where the use of algorithms are supplemented by understanding how and why these algorithms work (Johannsdottir, 2013). This is the difference between rules without reason, and knowing both what to do and why, as highlighted in the quote from the CAPS document (section 2.1 on page 10).

Procedural knowledge does have a place in, for instance, the need to recognise number names and symbols, and knowing rules, algorithms and procedures. Apart from this, learning mathematics involves conceptual understanding, which builds on important ideas and various representations, and where relationships among concepts are as important as facts. Conceptual understanding can be reinforced by shared discussion, rich and challenging tasks, and personal success (Siemon et al., 2014).

Research into the mathematical ideas and conceptions of mathematics learners, have shown that learning mathematics is multifaceted, time-consuming and often not straight-forward. These results point to the realisation that the way in which learners build their mathematical ideas and concepts are often different from the way teachers think it is done (Even & Tirosh, 2008).

2.2.3 Mathematical teacher knowledge

Hodgen (2011) argues that teacher knowledge is rooted in practical teaching and cannot be successfully defined abstractly, as attempting this will effectively not capture its dynamic nature. Teachers' mathematical knowledge can be described as "a dynamic, contextualized and active process of knowing, rather than the more static, abstract and passive notion of knowledge" (Hodgen, 2011, p. 29).

Researchers into MKT agree that teachers need to have a deep understanding and knowledge of the mathematics they teach and they must also be able to explicate this knowledge in their teaching ability (Shulman 1987; Ball et al., 2005; Hill, Ball & Schilling, 2008). Teachers have to know the key concepts, skills and strategies underpinning the mathematics they are teaching. Moreover, teachers should have a deep understanding of the links between concepts, the potential different levels of conceptual difficulty and what the best approach might be to teaching that concept (Siemon et al., 2014). Mathematics teachers' mathematical knowledge should be such that they have a detailed knowledge of the subject matter to be taught. They should also have knowledge of more advanced mathematics, in order to have the perspective needed for deeper understanding of the mathematics, which would in turn facilitate learning by learners. Baumert et al. (2010) claim that teachers who lack mathematics content knowledge are less equipped to explain and represent topics in ways that make sense to learners.

There is disagreement about exactly what knowledge is needed for teaching mathematics effectively (Hodgen, 2011). Hodgen (2011) refers to several studies which suggest that teachers' poor subject knowledge leads to poor learner performance, but maintains that the actual knowledge teachers need to teach mathematics effectively has not been sufficiently identified. From research it would appear that increased academic knowledge of mathematics does not necessarily ensure increased learner performance. The defining link between teachers' mathematical knowledge and mathematics teaching outcomes has not been properly established (Hodgen, 2011).

2.2.4 Pedagogical content knowledge

Pedagogy is defined as the principles, practice or profession of teaching (Collins English Dictionary, 2015). By pedagogical content knowledge is meant an "interaction of pedagogical knowledge and content knowledge [that] together causes a metamorphism and fusion of both of these knowledge types into a new understanding" (Wood, 2003, p. 50).

At a time when researchers emphasised the broad aspects of teaching, without paying particular attention to the specific content matter of the subject, Shulman (1987) proposed the construct PCK, which he defined in the following way:

> *Pedagogical content knowledge identifies the distinctive bodies of knowledge for teaching. It represent the blending of content and pedagogy into an understanding of how particular topics, problems or issues are organised, represented and adapted to the diverse interests and abilities of learners, and presented for instruction* (Shulman, 1987, p. 8).

Shulman suggested that the content knowledge and preparation of teachers should be studied in more depth (Stylianides & Delaney, 2011). Shulman (1986) argues that, in conjunction with the basic and general pedagogical skills required for teaching, attention should be paid to knowledge of how to teach subject content. He called this new direction a "missing paradigm", referring to the fact that a teacher's content knowledge and general pedagogical abilities are considered to be complementary requirements for teaching. He argued that "mere content knowledge is likely to be as useless pedagogically as content-free skill" (Shulman, 1986, p. 8). According to Shulman, these two aspects of teacher capabilities – content knowledge and pedagogical ability - need to be blended. Teachers' "knowledge base must deal with the purposes of education as well as the methods and strategies of educating" (Shulman, 1987, p. 13). Siemon et al. (2014) suggest that the main influence of Shulman's conception of PCK is drawing attention to the notion that knowing and really understanding the subject matter is not enough for effective teaching and learning to take place.

In his quest to understand the relationship between "the complexities of teacher understanding and transmission of knowledge" (Shulman, 1986, p. 9), Shulman proposed a theoretical framework for the domains and categories of teacher knowledge, and suggested a differentiation between three categories of teachers' content knowledge namely: subject matter knowledge, curricular knowledge and pedagogical content knowledge

Shulman (1986) argues that PCK is the individual form of content knowledge that represents the aspects of content most relevant to teaching. He explains the teacher's PCK as the teacher knowing and using a variety of different interpretations of a topic, including similarities, comparisons and explanations that can be implemented to make a given topic understandable to a learner. Curricular knowledge, on the other hand, includes knowledge of the various instruction materials and programs available for teaching a topic.

With regard to the model for pedagogical reasoning and action that Shulman (1987) proposed (Figure 2.1), he suggests that the activities of comprehension, transformation, instruction, evaluation and reflection form a cycle through which effective teaching and learning takes place. He explains that teaching begins with a teacher's own understanding of the subject matter to be taught, as well as the way this should be done. Teaching then continues through a number of actions that provide the learners with instructions and opportunities for learning, ending with new comprehension by both teacher and learner (Shulman, 1987).

A model of pedagogical reasoning and action

Comprehension

Of purposes, subject matter structure, ideas within and outside the discipline

Transformation

Preparation: critical interpretation and analysis of texts, structuring and segmenting, development of a curricular repertoire, and clarification of purposes

Representation: use of a representational repertoire which includes analogies, metaphors, examples, demonstrations, explanations, and so forth

Selection: choice from among an instructional repertoire which includes modes of teaching, organizing, managing and arranging

Adaptation and Tailoring to Student Characteristic: consideration of conceptions, preconceptions, misconceptions, and difficulties, language, culture and motivation, social class, gender, age, ability, aptitude, interests, self-concepts, and attention

Instruction

Management, presentations, interactions, group work, discipline, humour, questioning, and other aspects of active teaching, discovery of inquiry instruction, and the observable forms of classroom teaching

Evaluation

Checking for student understanding during interactive teaching

Testing student understanding at the end of lessons or units

Evaluating one's own performance, and adjusting for experiences

Reflection

Reviewing, reconstructing, re-enacting and critically analysing one's own and the class' performance, and grounding explanations in evidence

New Comprehensions

Of purposes, subject matter, students, teaching and self

Consolidation of new understandings, and learnings from experience

Figure 2.1: Shulman's Model of Pedagogical Reasoning and Action (Shulman 1987, p. 15)

Shulman's definition of PCK as knowledge "…which goes beyond knowledge of subject matter per se to the dimension of subject matter knowledge for teaching" (Shulman, 1986, p. 9) immediately interested researchers, and many attempts across various domains have been made to explain the construct in more detail (Ball et al., 2008). Shulman's idea of pedagogical content knowledge has been one of the more influential areas of research for nearly three decades, and has been cited extensively in the academic literature. Ball et al. (2008, p. 393) argue that "it is the breadth of literature on pedagogical content knowledge that highlights the term's heuristic value as a way of conceptualizing teacher knowledge". Stylianides and Delaney (2011) suggest that the attractiveness of PCK as construct possibly results from the way in which content knowledge and the practice of teaching has been combined, thus moving into the realm of praxis.

Although various definitions of PCK have been proposed, some characteristics emerged that were common to all of them. All the definitions placed PCK central to a specific topic and domain and the definitions all underscore the fact that PCK is specifically concerned with the the actual work of teaching (Gitomer & Zisk, 2015).

Fennema and Franke (1992) suggest that the effective teacher should have insight into the way that students think and learn, and that this knowledge is central to effective teaching. The framework designed by Fennema and Franke (Figure 2.2) consists of the components: knowledge of content, pedagogy, and learners' cognition, and knowledge of teachers' beliefs.

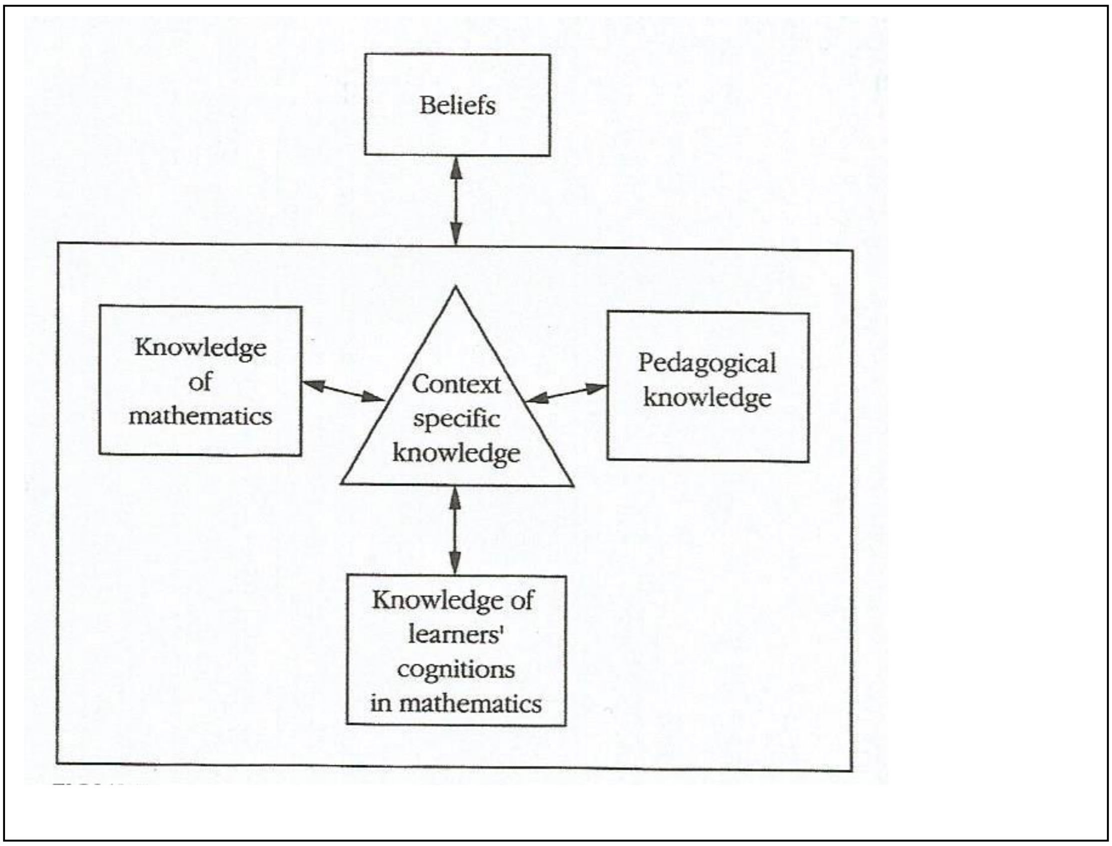

Figure 2.2: Teachers' knowledge: developing in context (reproduced in Rowland & Ruthven, 2011, p. 13)

The PCK model of Fennema and Franke (1992) is an expansion of Shulman's view of teachers' knowledge for teaching, and their definition of subject matter knowledge reflects this. However, interactions inside the classroom is central to their model, and Fennema and Franke (1992) argue that Shulman's model does not allow for the dynamic nature of knowledge, since it ignores synergy in the classroom which often impacts on teacher knowledge. They reason that the cognitive processes of learners and the typical patterns of learner understanding, as well as the common errors learners make and the misconceptions learners have, are central to effective teaching. Furthermore they state that the teacher should be able to anticipate which aspects learners will find difficult or easy, and should have the ability to interpret learners' understanding or lack of understanding whilst busy teaching.

A model involving the so-called knowledge quartet was developed in a research project: "Subject Knowledge in Mathematics (SKIMA)", and identified four categories of teacher actions inside a classroom (the 'quartet'), shown in Table 2.1. This research project was undertaken by Rowland, Huckstep and Thwaites (2005) and makes use of Shulman's theoretical framework. Their model is similar to Fennema and Franke's model since it categorises classroom situations which involve the emergence of mathematical knowledge in teaching (Thwaites, Jared & Rowland, 2011; Rowland & Ruthven, 2011).

The aim of the knowledge quartet framework was to promote reflection on both teaching and teacher knowledge in primary schools. The framework was developed to offer a structure for reviewing lessons, in order to develop teachers' MKT through reflection. The researchers were interested in what the primary school teachers know and believe, and in ways these skills can be enhanced. Their model offers "an empirically based conceptual framework for lesson review discussions" (Turner & Rowland, 2011, p. 197). In this framework, classroom actions in mathematic lessons are categorised, mostly with regard to the subject matter being taught, as well as the teachers' knowledge involved (Thwaites et al., 2011). Thwaites et al. (2011) developed a coding system of some aspects of teachers' actions that they considered to be important in preparing and presenting a lesson. Eighteen codes were identified, and then grouped into four categories which form the cornerstones or 'members' of the knowledge quartet namely: foundation, transformation, connection and contingency (Table 2.1).

Table 2.1: The knowledge quartet – dimensions and contributory codes (Thwaites et al, 2011, p. 86)

Dimension	Contributory codes
Foundation: knowledge and understanding of mathematics per se and of mathematics-specific pedagogy, beliefs concerning the nature of mathematics, the purposes of mathematics education, and the conditions under which students will best learn mathematics	awareness of purpose; adheres to textbook; concentration on procedures; identifying errors; overt display of subject knowledge; theoretical underpinning of pedagogy; use of mathematical terminology
Transformation: the presentation of ideas to learners in the form of analogies, illustrations, examples, explanations and demonstrations	choice of examples; choice of representation; use of instructional materials; teacher demonstration (to explain a procedure)
Connection: the sequencing of material for instruction, and an awareness of the relative cognitive demands of different topics and tasks	anticipation of complexity; decisions about sequencing; making connections between procedures; making connections between concepts; recognition of conceptual appropriateness
Contingency: the ability to make cogent, reasoned and well-informed responses to unanticipated and unplanned events	deviation from agenda; responding to students' ideas; use of opportunities; teacher insight during instruction

The knowledge quartet is instrumental in getting teachers to reflect on how different pedagogical methods might improve the quality of their teaching and to focus on the specific PCK involved in their teaching of a topic (Turner & Rowland, 2011). Turner and Roland (2011) argue that "such knowledge and beliefs inform pedagogical choices and strategies in a fundamental way" (Rowland & Ruthven, 2011, p. 18). Taken as a whole, the foundation category does involve beliefs on personal constructs, and it is possible that this might involve the personal construct self-efficacy.

Thwaites et al. (2011) began a review of the knowledge quartet which was aimed at secondary school pre-service teachers. The difference between these pre-service teachers and their primary school colleagues lies in the fact that the secondary school pre-service teachers were all mathematics specialists and had specialist mathematics assistance during the time they were at schools doing their practicum. The researchers found that some of the codes within the knowledge quartet framework had to be adapted to be suitable for secondary mathematics teaching (Thwaites et al., 2011).

Lannin et al. (2013) presented a model of PCK adapted from, but differing with, the Shulman model (Figure 2.3). They argue that the connection between knowledge and beliefs should also be defined, and when discussing PCK they address both the knowledge and the beliefs teachers have about teaching and learning mathematics. Lannin et al. (2013, p. 406) view teacher knowledge as "…beliefs that are justified in the mind of the individual teacher". They made use of Philipp's (2007, p. 259) definition of knowledge, where Philipp argues that knowledge is "…beliefs held with certainty or justified true belief". From their use of Philipp's definition of knowledge it is evident that they do reflect on teachers' beliefs about teaching and learning mathematics. However, they do not reflect on the teachers' self-efficacy beliefs regarding their own MKT, and the justification of these beliefs.

In the model they propose, (Figure 2.3) they identify two aspects of PCK: knowledge of curriculum and knowledge of assessment on the one hand, and knowledge of instructional strategies and knowledge of student understanding of mathematics on the other hand. This model corresponds with the right-hand side (PCK) of the model of Ball et al. (2008), which is discussed in the next section. Ball et al.'s (2008) knowledge of content and students (KCS) correlates with *knowledge of student understanding of mathematics*, while knowledge of content and teaching (KCT) is similar to *knowledge of instructional strategies for mathematics.* Both models also include knowledge of curriculum.

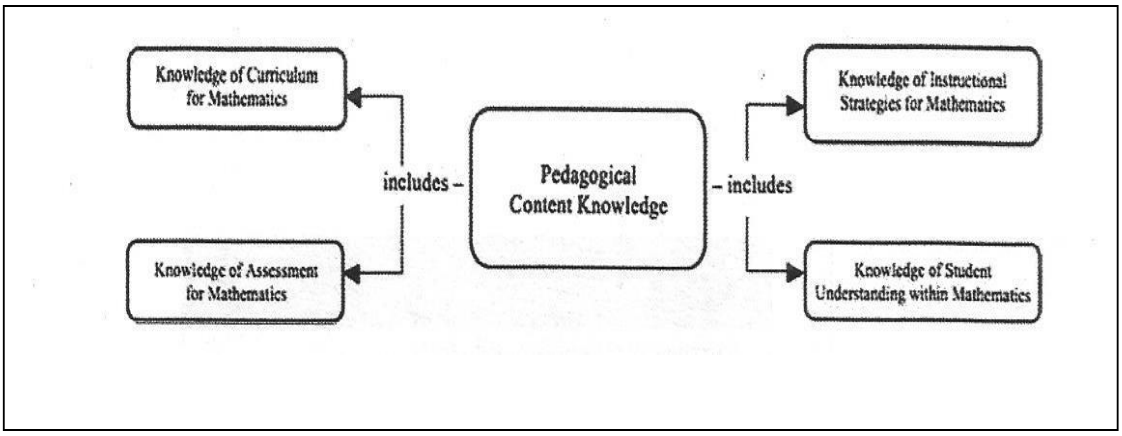

Figure 2.3: A model of PCK for teaching mathematics (Lannin et al., 2013 p. 406)

2.2.5 Mathematics knowledge for teaching

The research team of Ball et al. (2001), sometimes called the Michigan team, studied the lack of mathematical proficiency of American adults. Their focus was specifically on the work of teaching mathematics, with the aim to determine what teachers need to do in a classroom when teaching a topic. They wanted to recognize the precise knowledge needed to perform the task of teaching mathematics (Ball et al., 2013), and observe that "…we want to understand the mathematical reasoning that underlies the decisions and moves made in teaching" (Ball et al., 2008, p. 403). In order to have better insight into the content knowledge specific to teaching mathematics, the Michigan team started two projects investigating knowledge for teaching. These were the Mathematics Teaching and Learning to Teach Project (MTLT) and the Learning Mathematics for Teaching Project (LMT) (Petrou & Golding, 2011). These projects focused on mapping subject matter knowledge for teaching mathematics.

By studying the work of teaching in detail, Ball et al. (2001) first tried to determine the exact teaching actions needed in the classroom when teaching a specific topic, and second, to identify the knowledge needed to carry out these tasks effectively. In their effort to examine exactly what is included in teaching specific mathematics content knowledge, Ball et al. (2001) developed the encompassing construct: mathematical knowledge for teaching (MKT). The model they propose expands Shulman's conception of PCK, since it defines MKT as consisting of two domains, namely: subject matter knowledge (SMK) and PCK, as shown in Figure 2.4. They defined MKT as differing from PCK, arguing that there are facets of SMK that do not necessitate the need for pedagogical knowledge. These are aspects of SMK that include specialised teacher tasks - tasks that relate to practice but that do not require knowledge of teaching or of students. It includes tasks such as determining the validity of a mathematical argument or selecting an appropriate example to illustrate a specific mathematical idea (Ball et al., 2001).

Evaluating a learner's non-standard approach is an aspect of mathematical knowledge important for teaching, but it is an aspect that does not require knowledge about students or teaching. Although it does enhance learning, it only requires knowledge of mathematics and not of pedagogy. Ball et al. (2013) defend their framework, which considers PCK and SMK, to be separate divisions of MKT, by arguing that many of the collective teaching tasks, although needing extensive mathematical knowledge, essentially do not require knowledge of teaching and knowledge of students.

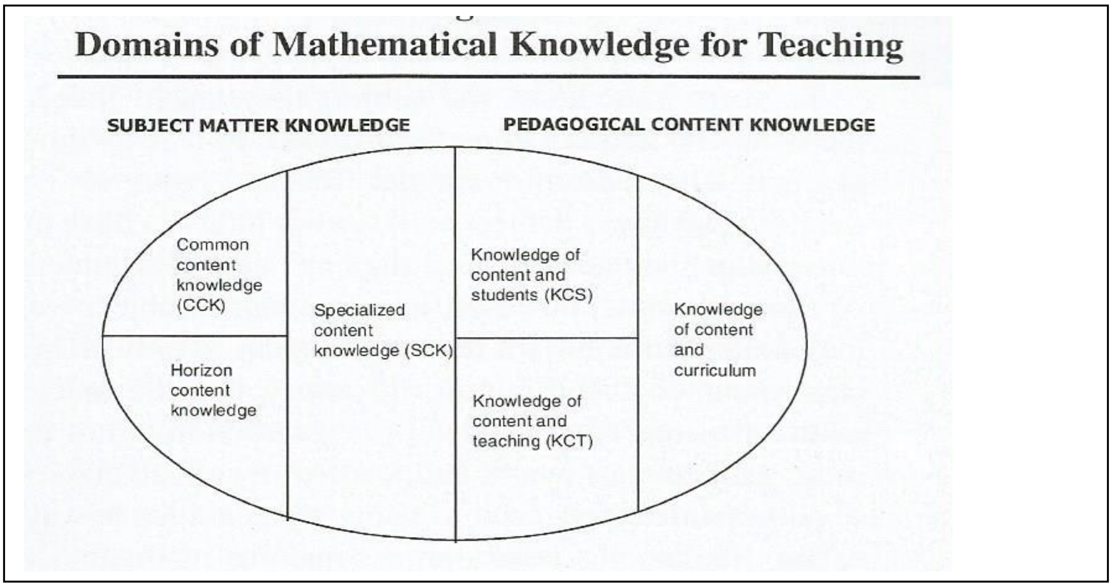

Figure 2.4: Mathematical knowledge for teaching (Ball et al., 2008, p. 403).

By forming their conceptualisation of MKT, the Michigan team attempted to develop the fundamentals of MKT and to explain the sub-domains in detail. Their model of MKT (Figure 2.4) illustrates the relationship between the constructs SMK and PCK, dividing both into three categories. The left side (SMK) of the oval contains portions considered to be different from PCK as it was defined by Shulman. The sub-domain SMK includes the categories common content knowledge (CCK), knowledge at the mathematical horizon, and specialised content knowledge (SCK).

CCK is mathematical knowledge that is known in common to other individuals (not teachers) who know and use mathematics. This knowledge domain includes the ability to solve problems and to evaluate answers.

Horizon content knowledge is an understanding of the connection between mathematical topics included in the whole of the mathematics curriculum for schools (Ball et al., 2008). It is knowledge of the mathematics that has already been done and the mathematics that will be done in the near future. Horizon knowledge is described by Ball et al. (2008, p. 403) as "…an awareness of how mathematical topics are related over the span of mathematics included in the curriculum".

Ball et al. (2013, p. 9) define SCK as "…mathematical knowledge beyond that expected of any well-educated adult, but not yet requiring knowledge of students or teaching". SCK is mathematical knowledge and not pedagogy (Ball et al., 2008). It is the specialised knowledge that a teacher needs for teaching and uses in the classroom. It is mathematical knowledge not necessarily known to all mathematicians, such as knowing how to evaluate a learner's non-standard approach or procedure. SCK is knowledge special to teaching (Ball et al., 2008).

The right-hand side of the oval in Figure 2.4, the PCK half, includes knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum. The two constructs, knowledge of content and students (KCS), and knowledge of content and teaching (KCT) correspond to the two elements of PCK as defined by Shulman. As mentioned before, KCS and KCT as described by Ball et al. (2008) matches elements of the model of Lannin et al. (2013).These elements are the conceptions and preconceptions learners have, and a teacher's knowing how to present a topic in such a way that it makes sense to learners (Shulman, 1986). KCS combines experience with learners and knowledge of their thinking. This knowledge may help a teacher to anticipate what topics or problems learners might find easy or difficult, and will help a teacher hear and respond appropriately to learners' thinking. KCT is knowledge that combines knowing about mathematics and knowing about teaching (Ball et al., 2008). This includes knowing how to sequence particular content, and includes knowledge on how to use learners' thinking to make a remark. Curriculum knowledge is knowledge of the exact subject matter that should be taught to a specific group of learners, and includes national documents such as policy and curriculum, as well as school-specific requirements in teaching and assessment (Cogill, 2008).

Ball et al. (2008) reiterates that the boundaries they describe in their model (Figure 2.4) are not fixed. As an example, they mention the topic of decimals. Decimals as a topic covers the ordering of decimals (CCK), the choosing of an appropriate example of a list of decimals to be ordered – a list that will highlight the fundamental mathematical concepts (SCK), and also the identification of decimals that could cause problems to learners (KCS). Teachers also have to decide on how best to address these problems (KCT). Because of their dynamic interaction, it is not always easy to identify boundaries between these sub-topics and teacher actions (Ball et al., 2008).

The construct MKT can be illustrated by using an example of a learner question that combines the need for SMK as well as PCK. A teacher might be asked a question involving operations on fractions, such as "*why can you multiply to multiply, but not add to add?*" When multiplying fractions, you multiply the numerators and multiply the denominators, but when adding fractions, you do not simply add the numerators and add the denominators. $\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$ but $\frac{a}{b}+\frac{c}{d}\neq\frac{a+c}{b+d}$ (Hodgen, 2011, p. 27). Answering this question combines the need for SMK as well as PCK, as this is not only a mathematical question, but it is also a pedagogical question. Mathematically this question addresses the algorithms for the arithmetic involving fractions and pedagogically the teacher must enable learners to see why the algorithm works as it does (Hodgen, 2011). This illustrates the meaning of MKT.

The above question from a learner about operations on fractions differentiates between two of the categories of SMK, where CCK is knowing how to add fractions, and SCK is knowing the underpinning mathematics. Some of the 'big' ideas of school mathematics are at the heart of this problem on fractions (Hodgen, 2011). The fundamental mathematics involved here lies in the nature of rational numbers, since rational numbers are defined as the division of integers, and in the relationship between multiplication and addition, as well as in the representation of rational numbers (Hodgen, 2011). These ideas are part of the teacher's common content knowledge, and the teacher should have the ability to explain these concepts effectively (pedagogically) to learners (Hodgen, 2011). The teacher's knowledge of content and of the student helps the teacher to explain to learners that fractions differ fundamentally from integers, and, although fractions can be indicated on a number line, they do not have the characteristics of counting numbers. Fractions are typically used to denote the relationship between two integers – "fractions are relative, not absolute" (Siemon et al., 2014, p. 363). Fractions should not be described by the teacher as writing "…one number over another number with a horizontal bar in between them" (Sieman et al., 2014, p. 371). Here the teacher's specialised knowledge for teaching comes into play. Learners should also understand that fractions can be represented in many different and equivalent ways. Musser, Burger and Peterson (2011) emphasise the importance of the teacher understanding the system of whole numbers (integers), since whole numbers play an important part in understanding the concept of fractions.

Shulman (2015) recalls that Ball and Lampert had argued that the strategic knowledge which he had suggested as one of the essentials of PCK, should be understood as an active construct describing methods teachers use when they teach specific subjects to specific learners in particular situations. This argument highlights the notion of Ball et al. that PCK is a construct alongside SMK in the definition of MKT. Ball et al. (2013, p. 8) argue that the aspects of SMK "that need to be uncovered, mapped, organised and included in mathematics courses for teachers" are aspects of SMK and not of PCK.

In Ball et al.'s (2005) conceptualisation of MKT, the importance of teachers' beliefs in their teaching and in their ability to teach are not taken into account. However, Ball (1988) remarks that teachers' beliefs about teaching mathematics may be affected by what they experience in the classroom. Their teaching may also be influenced by their beliefs about learners. This includes "what they teach, in what ways, to whom, and how they think about their students' success or failure in learning mathematics" (Ball, 1988, p. 13).

2.3 TEACHER SELF-EFFICACY BELIEFS

Researchers in psychology and education ascribe the concept of teacher self-efficacy to Bandura's (1977) social cognition theory. Social cognitive theory suggests that, for example, in the teaching of school mathematics, self-efficacy beliefs, rather than actual ability, more accurately predict a teacher's performance, since self-efficacy has to do with self-perception of competence rather than actual level of competence (Bandura, 1977; Michaelides, 2008). In the context of this current study, the question being investigated is whether the beliefs that students have in their own ability to teach a certain topic, agree with the teaching-strategy that teaching experts consider to be the best approach for the teaching of that topic.

Bandura (1977) defines self-efficacy beliefs as a person's belief in his or her ability to achieve a certain aim, and argues that self-efficacy differs from self-confidence because it is not confidence in general, but is confidence about ability, and is directed at a specific situation or context. Silverman and Davis (2009, p. 3) maintain that teacher self-efficacy is "theoretically and empirically" different from constructs such as self-concept, self-esteem, locus of control and sense of responsibility. Teachers' self-concepts and self-esteem are "broad, descriptive mental representations" (Silverman & Davis, 2009, p. 3) in the mind of the teacher, related to their performance in the classroom. This is distinct from self-efficacy, which is considered to be related to task-specific judgments of competency. Where self-esteem is considered to be a favourable impression of oneself, self-efficacy is more a belief in one's own capability to perform a given task successfully (Tschannen-Moran, Woolfolk Hoy & Hoy, 1998). A self-concept statement could only go down to: "Are you a good math student?" while a self-efficacy statement is much more exact: "Can you solve this specific mathematical problem?" (Bong &

Skaalvik, 2003). In the context of this current study, the self-efficacy statement relates to the question "How confident are you about your ability to identify the best way to teach this topic?" This study endeavours to understand the strength of pre-service teachers' self-efficacy beliefs regarding their mathematical knowledge for teaching certain topics of the curriculum.

Self-efficacy has a strong correlation with academic outcomes such as: the ability to problem solving, feelings about mathematics and anxiety about mathematics (Michaelides, 2008). Research has shown (Zimmerman et al., 2010) that self-efficacy contributes to academic performance in school mathematics regardless of the level of intellectual capability. Zimmerman et al. (2010) remark that, in view of the fact that self-efficacy beliefs are related to particular domains and tasks within that domain, measurement of self-efficacy should be related to specific tasks and domains. In the context of this study, 'measurement' of self-efficacy is linked to each MKT item, as is described in Chapter 3, Section 3.3.1.3.

Researchers have increasingly focused on the beliefs of pre-service teachers, since creating a firm foundation for future beliefs is more likely to be achieved in the formative years of pre-service training. According to Woodcock (2011), research suggest that professional development courses for in-service teachers impact more upon teachers possessing a higher level of self-efficacy, since these teachers are more willing to attempt new actions and methods in the classroom. It is, therefore, necessary that pre-service teachers have high levels of self-efficacy by the time they start their own in-service practice. Pre-service teachers have had 12 or more years of experience as learners in a classroom, and have made decisions about 'good' or 'bad' teachers. These observations might have an impact on their teaching efficacy, since beliefs that have been held for a long time are extremely difficult to change (Woodcock, 2011; Raths, 2001).

The connection between teacher efficacy and teacher self-efficacy has been extensively studied in recent years (Siegle & McCoach, 2007; Oh, 2011; Swan, Wolf & Cano, 2011; Gur, Cakiroglu & Capa Aydin, 2012). While self-efficacy is a task and topic specific construct, efficacy is a more general construct. In studies on teacher efficacy, pre-service and/or in-service teachers were asked to judge how confident they were in dealing with problems, mainly related to dimensions like instructional strategies, classroom management and student engagement, as well as general teaching skills. Questions such as "How much can you do to calm a student who is disruptive or noisy?" were asked, and relate to teacher efficacy. Many of the studies that were done used the Teachers' Sense of Efficacy Scale (TSES), developed by Tschannen-Moran et al. (1998). In a study conducted by Jaafar and Ayub (2010), seeking students' mathematics self-efficacy, university students were asked to answer questions like "I am confident that I can accomplish the task given" and they had to rate their confidence on a 4-point Likert scale ranging from strongly disagree (1) to strongly agree (4) (Jaafar & Ayub, 2010, p. 521). These were questions on self-efficacy, since the questions were specific to a given task.

Usher and Pajares (2009, p. 98) report on surveys that were done on feelings about mathematics, with questions such as "I start to feel stressed out as soon as I begin my math work" In contrast to this, in a survey done by Zimmerman et al. (2010, p. 4), pre-service teachers' self-efficacy were measured when they were asked questions on how confident they feel about their ability to solve a specific mathematical problems, posing questions such as: "I am confident to solve the systems of equations with $x + y = 7$ and $xy = 30$".

According to Pampaka and Williams (2010), in self-efficacy studies, items are often offered in the form of a 4-point Likert-type scale, where participants choose their level of confidence in their ability to achieve a specific outcome. Pampaka and Williams report on the construction of a mathematics self-efficacy instrument used to measure higher education students' perceived self-efficacy in mathematics. Their items were based on specific mathematical competencies such as costing a project or graphing experimental data, and purely mathematical questions such as solving an equation for *x*, as well as questions on basic and complex calculus and questions on problem solving and modelling of real situations. Participants were asked to answer MKT-related questions by rating, on a Likert scale, their confidence in their ability to answer the mathematics question. They were specifically instructed not to solve these problem itself, but just to evaluate their own self-efficacy with regard to solving each problem (Pampaka et al., 2007; Pampaka & Williams, 2010). In these studies, with questions related to solving specific mathematical problems, the participants were told to just consider their self-efficacy with regard to their ability to solve the problem, without actually answering the problem.

Lannin et al. (2013) argue that both knowledge and the beliefs teachers have about teaching and learning are addressed when aspects of PCK are researched and that researchers should endeavour to clarify the relationship between knowledge and beliefs. As discussed earlier, they view teacher knowledge as "beliefs that are justified in the mind of the individual teacher" (Lannin et al., 2013, p. 406). When Fauskanger and Mosvold (2013) discuss the MKT measures of Ball et al. (LMT, 2012), they mention that Ball et al.'s framework does not recognise the importance of teacher beliefs. They argue that researchers have started to realise the importance of teachers' beliefs, and that some researchers propose that knowledge and beliefs are of equal importance. This was the rationale behind this study, in the sense that this study attempts to investigate the relationship between prospective teachers' MKT and their self-efficacy beliefs regarding their own MKT.

Bandura theorised that a person's self-efficacy beliefs develop through four major sources, these being performance accomplishments, vicarious experience, verbal persuasion and physiological states (Bandura, 1977). In Bandura's model, (Figure 2.5), self-efficacy is the cause of behaviour but not vice-versa (Williams, 2010).

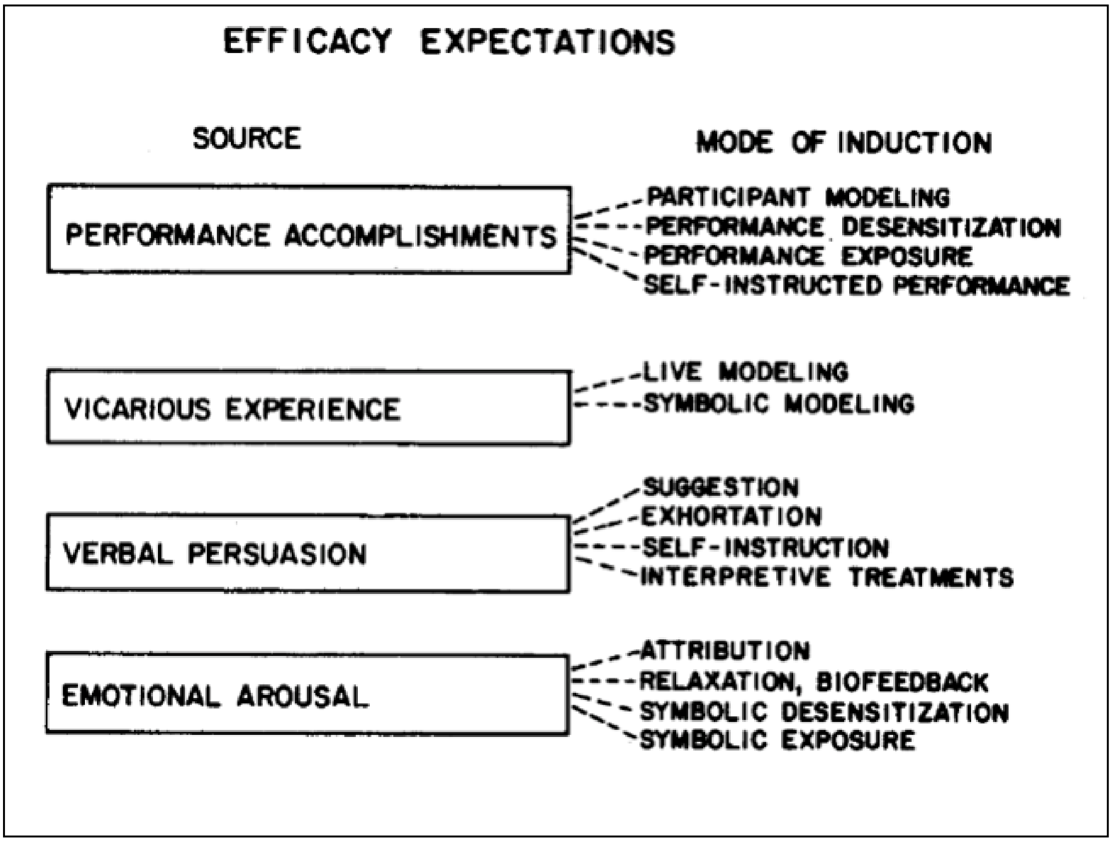

Figure 2.5: Bandura's model of self-efficacy (Bandura, 1977, p. 195)

Performance accomplishments (also called mastery experiences) are an important factor in a teacher's sense of efficacy (Bandura, 1977; Woolfolk Hoy, 2000). If the teacher has been able to make a difference in a learner's understanding, this has a very positive effect on the teacher's own sense of self-efficacy, and contributes to future proficiency. The first years of teaching could have an important influence on the long-term development of teacher efficacy (Woolfolk Hoy, 2000). Successful prior teaching experience, another mastery experience, can also serve as an influential source of self-efficacy beliefs (Fives & Buehl, 2010).

Pre-service teachers' efficacy beliefs could be built through the vicarious experience of observing someone with whom they can identify, and comparing themselves to this individual, such as another teacher presenting a particularly successful lesson or a classmate presenting an abridged practice lesson in the class. By seeing others like themselves succeeding at a teaching task, they evaluate their own capabilities (Usher & Pajares, 2009). They measure themselves against classmates, peers or adults, making judgments about their own efficacy. Pre-service teachers' twelve years as learners, observing teachers in a classroom, is another vicarious experience influencing their efficacy beliefs (Usher & Pajares, 2009). Vicarious experiences have a big impact on the self-efficacy of persons who have few experiences (Gur et al., 2012).

Bandura (1977) describes verbal persuasion as social encouragement received from others, arguing that, through persuasion, people are led to believe they can cope with a task they consider to be difficult. Positive feedback affects a person's self-efficacy beliefs positively (Gur et al., 2012). A teacher's sense of efficacy can be boosted by supportive comments from learners' parents, other teachers or colleagues within the school setting. This can contribute to an influential stimulus in self-efficacy, leading to attempts to try harder (Woolfolk Hoy, 2000). The effect of verbal persuasion is more permanent if it is combined with experiences of actual success (Siegle & McCoach, 2007).

Self-efficacy beliefs are also influenced by emotional and physiological states such as nervousness, strain, fatigue and temperament (Bandura, 1977). Anxiety, in particular, can have a very negative impact on a person's sense of self-efficacy (Schulze & Schulze, 2006). Increasing a person's physical and emotional well-being and reducing negative emotional states, strengthens self-efficacy (Usher & Pajares, 2009).

In their research on teacher self-efficacy, Tschannen-Moran et al., (1998, p. 203) attempted to study "the conceptual underpinnings of teacher efficacy". In their model (Figure 2.6), they acknowledge the four sources of influence on efficacy as proposed by Bandura (1977), but add performance as a new source of efficacy information. The inclusion of performance as a further source of efficacy information gives their model a cyclical nature, which the model of Bandura (1977) does not have. Silverman and Davis (2009, p. 2) explain that the model of Tschannen-Moran et al. (1998) identifies "ways in which efficacy judgments result as a function of the interaction between teachers' analysis of teaching task in context and their teachers' assessment of their personal teaching capabilities as they relate to the task". Tschannen-Moran et al. (1998) point out that teachers, when evaluating their self-efficacy, should consider the teaching task and its context, as well as their own mastery of the specific contents. All of these have an impact on self-efficacy, and self-efficacy beliefs are influential because of their cyclical nature. When a teacher has performed a specific task with great efficiency, this causes an increased belief in future self-efficacy. This progression gradually equalises to form a stable set of efficacy-beliefs (Tschannen-Moran et al., 1998).

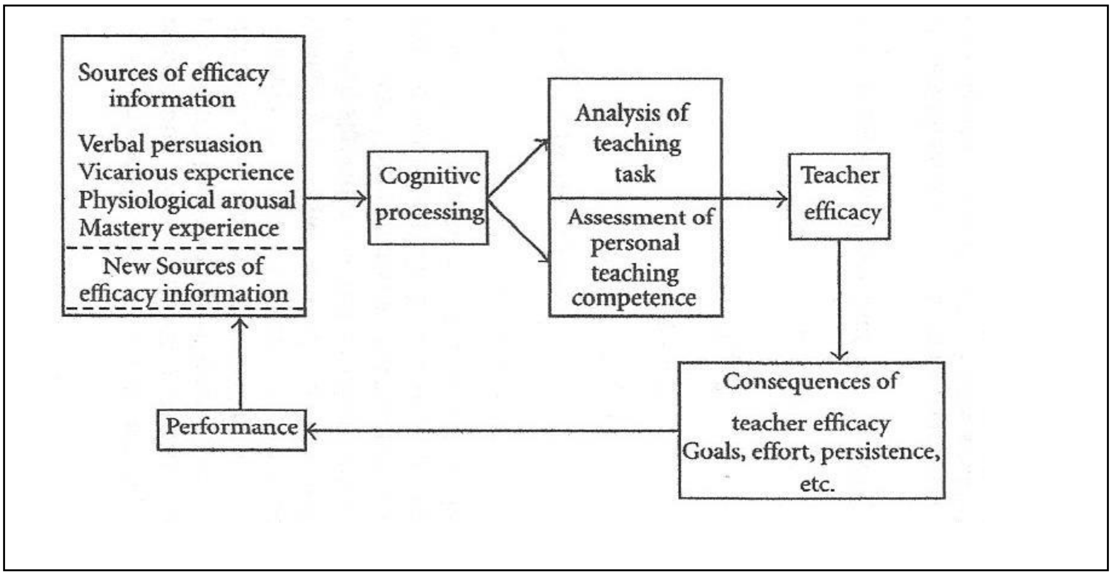

Figure 2.6: The cyclical nature of teacher efficacy – (Tschannen-Moran et al 1998)

In agreement with Tshannen-Moran et al.'s model, Williams (2010) comments that Bandura's model of self-efficacy, where self-efficacy is the cause of behaviour outcomes but not vice-versa, has been challenged by studies which showed the effect of expected outcomes on self-efficacy ratings. Williams' (2010) argument that self-efficacy is influenced by outcome expectancies has been negated by Bandura, but Williams maintains that the negation of the causal influence of expected outcomes on self-efficacy has been the cause of an inordinate focus on self-efficacy as opposed to the influence that expected outcomes have on self-efficacy beliefs.

2.4 LINK BETWEEN MKT AND SELF-EFFICACY

Even though teachers might be capable mathematicians, they might only be effective teachers if they are able to transform and communicate their mathematical knowledge in ways that promote learner understanding. "Effectiveness in teaching resides not simply in the knowledge a teacher has accrued but how this knowledge is used in the classrooms" (Hill, Rowan & Ball, 2005, p. 375). Teachers need to be able to listen to the learners, correctly interpreting the learners' understanding as well as responding in an appropriate and effective manner to pre-conceptions and misconceptions learners might have (Shulman, 1986, 1987; Ball et al., 2001, 2005; Hill et al., 2005). Succinctly put, teachers need MKT to be effective as teachers of mathematics. "Teachers highly proficient in mathematics will help others learn mathematics only if they are able to use their own knowledge to perform the tasks they must enact as teachers" (Hill et al, 2005, p. 376). They might not be able to perform the teaching tasks of 'hearing' students, selecting and using good assignments and managing classroom discussions if they lack in self-efficacy regarding their own teaching ability.

However, teachers' MKT is not the only important factor in effective teaching. Teachers' beliefs, specifically teachers' self-efficacy beliefs regarding their ability to teach effectively, influence educational outcomes such as teachers' commitment to excellence and learner achievement (Bandura, 1977). In the same way that learners' self-efficacy beliefs have an influence on their academic success, teachers' self-efficacy beliefs with respect to their MKT influence their effectiveness as teachers (Tschannen-Moran & Woolfolk Hoy, 2001).

2.5 RESEARCH REVIEW

In this section, some of the extensive research that has been done into PCK and MKT, as well as into self-efficacy, is discussed. Researchers endeavoured to expand the notion of PCK, and to investigate exactly what the knowledge is that teachers need in order to teach effectively (Silverman & Thompson, 2008). Research into self-efficacy focused on the development, context and measure of the construct, and the effect of self-efficacy on classroom learning (Silverman & Davis, 2009)

2.5.1 International research

Research studies have often focused on the PCK of pre-service and in-service teachers (Hill et al., 2005; Hill et al., 2008; Hodgen, 2011; Borowski et al., 2011; Pepin, 2011). Morris, Hiebert and Spizer (2009) believe that the analysis of teaching skills should be coupled with aspects of MKT. "The concept of MKT provides the most promising current answer to the longstanding question of what kind of content knowledge is needed to teach mathematics well" (Morris et al., 2009, p. 492). They argue that this will define some of the skills and content that pre-service graduates can, and should, acquire. Teacher training programmes might be enriched by the inclusion of the development of these competencies in the curriculum. Morris et al., (2009) argue that the competencies needed for unpacking learning goals into their constituent parts, are competencies that can be taught in teacher education programmes, and that these are competencies essential to the analysis of mathematics teaching. They hold that teacher training programmes should be structured in a way that will "prepare prospective teachers to learn from teaching when they enter the profession" (Hiebert, Morris, Berk & Jansen, 2007, p. 48)

Shulman (1987) studied knowledge growth in teaching with the goal to ascertain the teacher conduct and approaches that would most likely lead to success in learning. Effective teachers were asked to explain key ideas in topics using metaphors, diagrams and explanations that were based on the content and that learners could relate to. Shulman found that an important aspect of a pre-service teacher's training should be the ability to reason about the teaching of specific topics (Shulman, 1987).

From research conducted by Ball et al. (2001), the conclusion was that the dominant reason for the lack of improvement in mathematical proficiency among learners seems to be the lack of teachers' pedagogical content knowledge needed for teaching mathematics. A very important contribution by Ball's Michigan research team lies in their development of measurement tools for teachers' mathematical knowledge for teaching, called the learning mathematics for teaching (LMT) measures. Their research has played an important role in the process of investigating the relationship between teachers' MKT and their students' performance in mathematics (Hill et al., 2005).

An, Kulm, and Wu (2004) compared the PCK of middle school mathematics teachers in China to the PCK of teachers in the United States of America. Their results indicate that, while both groups of teachers had good levels of PCK, the main differences were in their interpretation of the components of teaching for understanding. While the Chinese teachers focused on conceptual understanding, the teachers in the US used concrete models to help correct misconceptions. This suggests that relational understanding enhances performance. Teachers who have confidence in their own ability to teach effectively, are better able to promote learner understanding.

2.5.2 The South African context

The Quantum Project is a research project focussing on the development of quality mathematical education for teachers in South Africa. Adler and Davis (2006) speak of "opening another black box" when researching the importance of mathematical knowledge in mathematics teacher education. The key goal of the Quantum project was "…an elaboration of mathematical knowledge for teaching, theoretically and methodologically" (Adler & Davis, 2006, p. 272). The methodology developed by the researchers from the Quantum Project allowed them to describe what and how mathematics is founded in the practice of teacher education. Their study contributes to research on PCK, in studying the structure of mathematics for teaching in post-apartheid South Africa.

Venkatankrishnan and Van Jaarsveld (2014) identified some gaps in the model proposed by Ball et al. (2008), suggesting that the model should be adapted by putting a bigger emphasis on the construct specialised content knowledge (SCK) relative to knowledge of content and students (KCS), and knowledge of content and teaching (KCT). They hold that MKT should not be divided exactly in half, but that subject matter knowledge (SMK) should be about two-thirds of the whole of MKT.

When investigating the PCK of some teachers in the Eastern Cape, Stewart (2009) found the teachers' PCK to be superficial, with limited ability to address misconceptions. She concludes that the relationship between teachers' PCK and learner achievement needs to be further investigated. Ijeh and Nkopodi (2013) used a qualitative research method to develop a theoretical model for investigating the PCK of teachers in South Africa and Zimbabwe. Their results showed that experienced teachers showed higher levels of PCK, which strengthens the outcome to be expected, since experience should promote an increase in PCK.

Regarding self-efficacy, Austin (2010) reported increased levels of self-efficacy between students participating in a study that she did as part of research for a D Phil degree. She investigated the effects of a values-based approach to teaching and learning, and concluded that the data showed a values-based approach to teaching could be used effectively by mathematics teachers (Austin, 2010). Her study was founded on the view that teaching is about commitment in sharing learning and evolving groups of inspired students. This study has value, since it promotes higher levels of self-efficacy in pre-service teachers which would lead to increased teacher efficiency.

2.6 MEASURING MKT AND SELF-EFFICACY

Traditionally, a teacher's proficiency is measured using written tests on basic mathematical ability. According to Hill et al. (2005), by employing this method of teacher assessment, important factors that produce quality teaching may well go unnoticed. Although mathematical content knowledge is important for teaching proficiency, the crucial factor for effective teaching is "how this knowledge is used in the classrooms" (Hill et al., 2005, p. 376). Shulman (1986) makes a plea that the traditional assessment of mathematical content knowledge was not an effective tool for the measurement of teachers' proficiency as mathematics teachers. He argues that the tests used during the nineteenth century did not succeed in eliminating incompetence. These were tests which included direct mathematical and other subject knowledge as well as general knowledge questions, and where only 50 out of a possible 1000 points were linked to pedagogical practice (Shulman 1986). On the other hand, the emphasis should not only be on correct classroom actions which was the 'new' direction taken in the 1980's (Shulman, 1986). In these more recent tests the emphasis was on capacity to teach, with no inclusion of subject content knowledge. The important point is that "teachers have and employ a distinct set of content and professional knowledge when engaging in the work of teaching" (Gitomer & Zisk, 2015, p. 24), and as such, the knowledge defined by the constructs MKT, PCK and SMK, as delineated by Ball et al. (2008), is knowledge that assists teachers to teach effectively. Hill et al. (2005) argue that "measuring quality teachers through performance on tests of basic verbal or mathematics ability may overlook key elements in what produces quality teaching" (Hill et al., 2005, p. 375). Measurement of MKT should focus on the work of teaching in order to judge teachers' professional knowledge for teaching mathematics (Gitomer & Zisk, 2015). This underscores the worth of questionnaires that integrate mathematics with classroom situations, as does the questionnaire of Ball et al., used in the current study.

A wide range of measurement tools have been used in the research about MKT. This includes multiple-choice items, cognitive study tasks and constructed-response items. In most cases classroom situations were described where teachers were interviewed to elicit responses. However, Hodgen (2011) argues that, as the real test of a teacher's MKT can only be measured inside an active classroom, neither interviews nor questionnaires can fully capture relevant MKT, while Corcoran and Pepperell (2011) propose the value of lesson study in developing ways to improve MKT. Videos of teaching examples have also been used to prompt discussions on teaching practice (Rowland et al., 2005; Watson & Barton, 2011). Few large-scale surveys have been done, due to the difficulty of implementation and the relatively high cost. In general, small in-depth studies have proven to be both practical and important (She et al., 2011).

An important and often-quoted large-scale research study is the international teacher education and development study in mathematics (TEDS-M). The TEDS-M study was an effort to examine the content knowledge and mathematical knowledge for teaching (MKT) for teachers in different countries, using instruments containing mostly multiple-choice items (Gitomer &

Zisk, 2015). The study consisted of three intersecting sub-studies, of which the third of the sub-studies has bearing on my research project. In this sub-study, researchers studied the effect of teacher training in generating knowledgeable mathematics teachers, as well as the problems of teacher education and learning. Data were collected through surveys and focused field studies. The aim was to encourage cross-national discussion amongst leaders and policy-makers in mathematics education. Botswana was the only African country that participated in this study (Tatto, Schwille & Rodrigues, 2005).

The questionnaire of Ball et al., used in this study, is one of the most cited attempts to measure MKT. The measures developed by the Michigan team (Hill et al., 2004) are directly applicable to teaching mathematics and to helping researchers evaluate teachers' common content knowledge (CCK) and specialised content knowledge (SCK) (Gitomer & Zisk, 2015). The rationale behind the development of these measures were that "the actual mathematical content that teachers must know to teach has yet to be precisely mapped" (Hill et al., 2004, p. 13). Research on education, where surveys were used, mostly measured teachers' general cognitive ability and their knowledge related to the actual job of teaching (Rowan, Schilling, Ball & Miller, 2001). Hill et al. (2004, p. 13) report that the conjectures about "the potential organisation of such knowledge" made by researchers such as Shulman, served to initiate their investigation. The Michigan team's question was "What mathematical knowledge is needed to help learners learn mathematics" (Hill et al., 2004, p. 15). Ball (2003) argues that this question actually has three central principles. First, teachers' knowledge should be much more than just the mathematical knowledge known to the average educated adult. Second, this mathematical knowledge differs from the mathematics used by professionals such as engineers, physicist or architects, since it constitutes solving problems such as interpretation and analysis of learner errors and misconceptions, choosing relevant representations for teaching or choosing suitable definitions to use. Third, the teachers' mathematical knowledge must be usable to solve these teaching problems, offering well-defined explanations and well-chosen examples and problems.

Hill et al. (2004) initially wrote and piloted a large number of multiple-choice items in the field of teaching elementary mathematics. These first questions were from the curricular domains of number concepts, place value and operations, since these were topics that formed an integral part of the elementary mathematics curriculum. Within these domains they identified content knowledge and knowledge of students' thinking, and the mathematical content areas as well as their related teaching tasks formed the base of the questions (Rowan et al, 2001). Each questionnaire item linked classroom scenarios with multiple-choice questions about the situation. "Decisions on correct and incorrect responses to particular questions were based on research on teaching and learning in the "fine-grained" curricular domains" (Rowan et al, 2001, p. 5).

These initial efforts to map MKT were later augmented by including the mathematical content areas of patterns, functions and algebra, and "mathematics educators, mathematicians, professional developers, project staff and former teachers" were asked to write items (Hill et al., 2005, p. 387). The multiple-choice format made larger-scaled studies possible, since it reduced cost. Writers were requested to encapsulate the items into knowledge any competent individual using mathematics should have (CCK) and the specialised knowledge that teachers should have (SCK). The difference between these two constructs are illustrated by the following two sample items.

In Example 1 (Figure 2.7), teachers have to find the value of x if $10^x = 1$ .The knowledge needed here, is common content knowledge (CCK) – knowledge used in the classroom, but also knowledge that many adults and all mathematicians know. This knowledge is not special to teaching.

Example 1 (Hill et al, 2005, p. 401)

Mr. Allen found himself a bit confused one morning as he prepared to teach. Realising that 10 to the second power equals 100 ($10^2 = 100$), he puzzled about what power of 10 equals 1. He asked Ms Berry next door. What should she tell him? (Mark one answer).

a) 0

b) 1

c) Ten cannot be raised to any power such that 10 to that power equals 1

d) -1

e) I'm not sure.

Figure 2.7: Example 1 (Hill et al, 2005, p. 402)

Example 2 (LMT, 2012, p. 5)

3. Imagine that you are working with your class on multiplying large numbers. Among your students' papers, you notice that some have displayed their work in the following ways:

Student A	Student B	Student C
35	35	35
x 25	x25	x 25
125	175	25
+75	+700	150
875	875	100
		+600
		875

Which of these students would you judge to be using a method that could be used to multiply any two whole numbers?

	Method would work for all whole numbers	Method would NOT work for all whole numbers	I'm not sure
a) Method A	1	2	3
b) Method B	1	2	3
c) Method C	1	2	3

Figure 2.8: Example 2 (LMT, 2012, p.5)

In Example 2 (Figure 2.8), three different methods of solving a multi-digit multiplication problem are given. Teachers have to decide whether these methods could be used for any two integers. In this case, teachers should inspect each step in the calculations, using their mathematical knowledge to decide whether it makes sense and whether it is a method that can be used in general. This is not a task mathematicians in other fields where mathematics is used, usually have to do. Hill et al. (2005) argue that it is a purely mathematical task, and requires no pedagogy. The Michigan team called this knowledge specialised content knowledge (SCK).

A study of interest was done by Copur-Gencturk and Lubienski (2013) in which the MKT of 24 teachers was measured using two measures namely: the LMT measures developed by Ball et al., and the DTAMS (2004) measures (Diagnostic mathematics assessments for middle school teachers) developed by the University of Louisville centre for research in mathematics and science teacher development. They compared the LMT and DTAMS measures for detecting gains teachers made in a mathematics content course aimed at real-world applications compared to a hybrid course blending mathematics content and pedagogy. Their results show large gains by teachers on both measures after taking the course. However, the patterns of this change differ, showing that "these measures capture substantially different aspects of mathematics knowledge" (Copur-Gencturk & Lubienski, 2013, p. 211). Their study emphasises the importance of teachers' MKT, which is considered mathematical knowledge different from 'everyday' mathematical knowledge. The results of the study suggest that the LMT measures are a better choice for investigating the MKT of student teachers in combined content-method courses for elementary pre-service teachers, while the DTAMS measure better evaluates MKT in general mathematics courses. They argue that the results of this study indicate that a combined content-method hybrid course, integrating content and pedagogy, are of importance to academics designing professional development programs for teachers, since, in their research this type of course had a significant impact on teachers' specialised knowledge for teaching (Copur-Gencturk & Lubienski, 2013)

Although the literature has some studies where the Michigan team's MKT measures are used (Copur-Gencturk & Lubienski, 2013; Johannsdottir, 2013; Fauskanger and Mosvold, 2013; Johnson, 2011), no studies have been found that directly link answering the MKT questions with perceived self-efficacy. The study by Fauskanger and Mosvold (2013), uses the MKT questionnaire of Ball et al., but participants discuss their perception of their own MKT without answering the items of the questionnaire.

2.7 SUMMARY

The definitions, models and research described in this chapter accentuate first, the importance of mathematical knowledge for teaching as an integral part of preparing students to be teachers. Second, self-efficacy as construct is explained, as well as the importance of positive self-efficacy beliefs for promoting teaching and learning. Drawing a comparison between pre-service teachers' topic-specific MKT and their self-efficacy beliefs with respect to this MKT gives an indication of how ready they are to teach. In this current study, pre-service teachers are asked to answer each specific question related to MKT. After answering the MKT-related question, they rate their own confidence with regard to the correctness of their answer.

Student-teachers should be well-grounded in the fundamentals of mathematics, as well as the other important aspects as delineated by Lannin et al. (2013). They should have in-depth knowledge of the ‘how’, but most importantly the ‘why’ of every important topic in the school curriculum. Only then will there be a marked improvement in the standard of mathematics knowledge of learners.

In the next chapter, the research design and research method used in this study is discussed. This includes the methodology and limitations of the study, as well as the ethical considerations.

Chapter 3

Methodology

3.1 INTRODUCTION

The purpose of this research study is to determine the relationship between the mathematical knowledge for teaching (MKT) and the self-efficacy regarding this MKT, of the BEd (FET) students specialising in mathematical literacy at the Nelson Mandela Metropolitan University (NMMU). This chapter provides an outline of the research design and methodology used in the study. This includes a discussion of the research approach and research paradigm in which the study is situated, with justification for the particular approach and method used. A quantitative research approach is discussed, as well as the frameworks used in educational research, with a focus on the positivist framework in which this study is situated. The reason for the methodology used is explained, and the research instrument is explained in detail, also mentioning issues of reliability and validity. This is followed by noting some of the methodological limitations of the study and addressing the ethical considerations.

3.2 RESEARCH DESIGN

The design of a research study is the logic that links the data to be collected to the initial questions of the study. Johnson and Christensen (2012) hold that research is structured through research design, which clarifies how the main parts of the study are incorporated into the investigation of the research problems. According to Trochim (2006), the research design preserves the unity of the research project, describing the strategy followed by the researcher in order to answer the research question.

The availability of data and measurement of concepts and ethical issues should inform the research design (Kumar, 2011). The research questions and research method for this study were decided upon by considering the researcher's field of interest and its relevance to the methodology courses taught to pre-service teachers. The availability of a properly validated research instrument, as well as the availability of participants for the research project, were factors taken into account in the choice of design. In developing the research questions, the common characteristics of research questions highlighted by Verma and Mallick (2004) were considered. “Good research questions will share some common characteristics. They will:

- Go to the heart of the research problem being addressed;
- Be simply and clearly expressed; and
- Be answerable, using the tools at the researcher’s disposal.” (Verma & Mallick, 2004, p. 140).

3.2.1 Research framework

According to Botha (2011), the conceptual framework of a research study allows a researcher to explain why a specific path of action is pursued, based on the experience of others or on what the researcher personally would like to explore or discover. Botha (2011) mentions the view of various seminal scholars when she argues that the philosophical and theoretical basis of a research study forms its conceptual framework, connecting the problem statement, the literature review, the methodology and the data collection and analysis of the study, giving consistency to the empirical investigation.

3.2.1.1 Philosophical framework

This research study is premised on Fennema and Franke’s (1992) belief that it is insufficient to only focus on teachers’ pedagogical content knowledge to determine the effectiveness of the teaching - all aspects of teacher knowledge should be considered. This includes the context of the educational system, the aims of mathematics education, the curriculum and its associated materials (such as textbooks) and the relevant assessment system. All these factors have an impact on the effectiveness of the teacher in the classroom (Fennema & Franke, 1992). The belief exists that a pre-service teacher’s mathematical knowledge for teaching develops as he or she progresses through the levels of training, and even after the completion of postgraduate training (Fennema & Franke, 1992; Lannin et al., 2013). This belief is a further basis for the current study.

3.2.1.2 Theoretical framework

According to Agherdien (2007), the theoretical framework of a research study is a vital tool for researchers, giving a viewpoint from which to consider and create knowledge, as well as informing research design. Agherdien (2007) argues that it is desirable that researchers, when studying a phenomenon, utilise theories that have withstood the test of time; such theories becoming part of the researcher's tacit and explicit knowledge. When analysing and interpreting the data, this theory is used to promote understanding; hence the theory becomes the lens through which the results are viewed. Agherdien, (2007, p. 27) mentions that "theoretical frameworks are epistemological guides for research" and that the theoretical framework helps in "situating research within existing theory" (Agherdien, 2007, p. 29). The worth of theory is that it provides a better understanding of, and has the ability to help in solving problems of scholarship, since prior research can be seen as forming a platform for any new research study. The theory provides a position from which data can be viewed. The research study's literature review is also guided by the theoretical framework, assisting the researcher to understand what research has been done and what still needs to be done about the research topic (Agherdien, 2007). The theory also enables a justification of the importance or usefulness of the research.

Shulman's (1986) theory about pedagogical content knowledge, and the theory of Ball et al. (2001) regarding mathematical knowledge for teaching, as well as the theory of Bandura (1977) and Tschannen-Moran et al. (1998) regarding self-efficacy, provide the theoretical framework for this study. These theories assist in answering the research questions. Using MKT as the theoretical framework allows the study to focus on questions directly related to the knowledge base of a teacher, while using self-efficacy as a framework allows the researcher to investigate the relationship between the constructs MKT and self-efficacy.

3.2.2 Research approach

In educational research, the main types of research approaches are quantitative -, qualitative - or mixed-methods approaches. Pure quantitative research depends on the collection of quantitative or numerical data, and focuses on the testing of hypotheses and theories (Johnson & Christensen, 2012). In quantitative research, the focus is on one or more (but often limited in number) contributing factors (or variables) at the same time. Factors that are not being studied are held constant (Johnson & Christensen, 2012). In quantifying and attempting to clarify a relationship between two or more variables, quantitative research considers measuring and analysing the connecting bond between variables (Denzin & Lincoln, 2000). A quantitative variable is a variable that differs in amount or degree. Data are generally reduced to numbers, and attitudes are usually measured using rating scales. The quantitative variables in this research study are the constructs MKT and self-efficacy.

Standardised questionnaires and other quantitative measuring tools can be used to carefully measure what is researched without the researcher being directly involved with the participants. After participants have provided their responses, the researcher typically calculates and reports an average and other relevant descriptive and inferential statistical information for the group of participants (Johnson & Christensen, 2012).

In descriptive research, quantitative research methods are used when seeking a quantitative answer or a numerical change in a situation. Quantitative methods can also be used in inferential research to attempt an explanation of a phenomenon, and is particularly suitable for testing a hypothesis, although it is not suitable for the development of hypotheses or theories (Muijs, 2011).

3.2.3 Research paradigms

A research paradigm indicates an academic model that has been generally accepted and which is responsible for the framework in which research is positioned. Arthur, Waring, Coe and Hedges (2012) report that the use of the word paradigm comes from the philosopher Thomas Kuhn who considers the word paradigm to outline a specific worldview, describing it as shared understandings within a community of scientists. Cohen, Manion and Morrison (2011) defines the term paradigm as the motivation or philosophical purpose for undertaking a study, while Johnson and Christensen (2012) see it as being based on the shared perceptions, collective expectations and beliefs of researchers, which influences the paradigm in which the research project is situated.

Paradigms are largely categorised by the type of methodology employed by the researcher as a qualitative or a quantitative paradigm (Stewart, 2009). In educational research, the important paradigms are the positivist/post-positivist paradigm, the interpretivist/constructivist paradigm, the transformative paradigm and the pragmatic paradigm (Mackenzie & Knipe, 2006). In positivist and post-positivist research, the research approach is mostly through quantitative research methods of data collection and analysis (Mackenzie & Knipe, 2006). The research tools used are surveys, questionnaires, scales and experiments

Stewart (2009) quotes Guba and Lincoln’s argument that differences between paradigms are not necessarily insurmountable, adding that Guba and Lincoln reason that it is possible to mix elements of paradigms. Research methods can cross paradigm boundaries, which reinforces that the research approach, its data collection and methods of analysis, should be determined by the research question.

The interpretive-constructivist paradigm is usually allied to qualitative or mixed-methods research approaches such as case studies, phenomenological and hermeneutic research, and was developed as a reaction to positivism (Mack, 2010). The transformative paradigm uses qualitative research into topics such as feminism or critical theory as well as emancipatory research, while the pragmatic research paradigm focuses on real world and problem-centred situations, such as the consequences of actions.

Positivist researchers aim to describe a phenomenon or a reality using direct observation and measurement as research tools. Measuring and analysing the connecting bond between variables is typical of a positivistic paradigm (Stewart, 2009). In post-positivist research, it is assumed that the research is shaped by well-developed theories and also by the theory being tested (Mack, 2010). Mack (2010) comments that, in the positivist paradigm (sometimes referred to as the scientific paradigm) the purpose of the research is to investigate a hypothesis, using statistical analysis and conclusions that can be generalised. According to Mack (2010), positivist researchers in general can be considered to be researchers who are concerned with probability more than absolute certainty. Mack (2010) defines post-positivist researchers as those experiencing the world as ambiguous, variable and multiple in its realities. Mackenzie and Knipe (2006) argue that the philosophy of the positivist researcher is to regard knowledge as reliable only if it has been gained through measurement. They hold that, in positivism studies, the role of the researcher is limited to data collection and interpretation through objective approach and the research findings are usually observable and quantifiable.

This research study is situated within the positivist paradigm, since quantitative methods are used, in order to be as objective as possible in testing the hypothesis of the existence of a relationship between the variables MKT and self-efficacy. Quantitative measures are used in the research approach, in order to determine the pre-service teachers' MKT as well as their self-efficacy beliefs regarding this MKT.

3.3 METHODOLOGY

Trochim (2006) suggests that a research study's methodology must respond to questions about the data collection and analysis. The methodology is determined by the research questions which should be used as a standard to describe the methodology of the study, to be followed by a summary of the process of data collection and analysis.

For this study, the problem being researched, is whether a relationship exists between pre-service mathematical literacy teachers' MKT and their self-efficacy beliefs regarding this MKT. In order to respond to this question, two sub-questions were posed. These were: first the question of the scope of the MKT of the different year groups of FET students on the topics of number concepts and operations, as measured by the survey questionnaire; and second the participants' self-efficacy with respect to their MKT for each item of the questionnaire.

Addressing the first sub-question gave an indication of the students' MKT. When answering the second sub-question, the focus was on the students' self-efficacy beliefs regarding their own MKT. The responses to these two research sub-questions provided an answer to the research question, since it helped to identify whether or not a relationship exists between the students' MKT and their self-efficacy beliefs regarding this MKT.

3.3.1 Research instrument

In order to address the first of the research sub-questions - assessing the pre-service teachers' level of MKT - a research instrument was used which was developed and validated by a team of researchers from the University of Michigan (LMT, 2012; Schilling et al., 2007). The questionnaire was utilised to assess the MKT of the BEd (FET) students in the four different year groups who were specialising in mathematical literacy education.

3.3.1.1 The questionnaire

Johnson and Christenson (2012, p. 162) categorise a questionnaire as a "self-report data collection instrument" that participants in a research study complete, in order to "measure many different kinds of characteristics" (Johnson & Christenson, 2012. p. 163). According to Arthur et al. (2012), questionnaires can be described as tools administered to participants from a population sample, and used to gather information. Questionnaires, paper-based or electronic, are easily usable, since they are easy to implement and are not expensive to reproduce.

3.3.1.2 Measuring MKT

The questionnaire used in this study assesses domain-specific aspects of the MKT of mathematics teachers and mathematics pre-service teachers, and is part of a set of questionnaires known as the 'Learning mathematics for teaching measures of mathematical knowledge for teaching', known as the LMT-measures (LMT., 2012), as was mentioned earlier. The LMT measures are created to measure the range of teachers' MKT reliably, and are designed to provide norm-referenced comparisons of teachers in a random sample. These questionnaires are released only to persons who have attended a special training workshop, either in person or online, with the clear understanding that a researcher is not allowed to divulge the content of the questionnaire used, hence it is not possible to give the actual questionnaire used. The researchers have, however, published a sample questionnaire with questions similar to the actual questions asked, and which may be used in a publication. This set of sample questions is given in Appendix A. After undergoing online training in the use of the instrument in August 2014, permission was granted to the researcher by the Michigan team to use their instruments in this research study, with the provision that the relevant questionnaire not be published in any form (LMT, 2012).

Arthur et al. (2012) emphasise the importance of carefully defining the construct(s) that a questionnaire is designed to assess. The Michigan team did a review of the research literature before deciding which aspects of teacher mathematical knowledge to measure. The LMT measures designed by the Michigan team (used as the data collection instrument in this study) illustrates knowledge used by teachers in classrooms, not just general mathematical content knowledge. To ensure this, items included in the questionnaires were designed to "…gauge proficiency at providing students with mathematical explanations and representations and working with unusual solution methods" (Hill et al., 2005, p. 387).

In 2001 the Michigan team started to investigate ways to assess the MKT of mathematics teachers (Hill et al., 2005). They first described the domains of teachers' content knowledge for teaching that they wanted to measure, before designing questionnaires that could be used for this purpose. They limited items of the questionnaires to the most commonly used content areas, namely: number concepts and operations, and patterns, functions and algebra (Hill et al., 2005). The designers of these questionnaires chose the topics of number concepts and operations because these topics form a significant part of the primary (intermediate phase – IP - in South Africa) and middle school (senior phase – SP -in South Africa) sections of the school curriculum, and because much research has been done on the teaching and learning of these topics. When the Michigan team started the investigation, they initially chose to include items from subdomains of both the cognitive domains of subject matter knowledge (SMK) and pedagogical content knowledge (PCK), as conceptualised in the framework described in Section 2.2.5 of Chapter 2. The subdomains they included were common content knowledge

(CCK) and specialised content knowledge (SCK) in the domain of SMK, and knowledge of mathematical content and students (KCS) in the domain of PCK.

They described CCK as knowledge used in the work of teaching "…in ways *in common with* how it is used in many other professions or occupations that also use mathematics" (Hill et al., 2008, p. 377). SCK is described as the special knowledge of teaching that teachers need. This includes accurate representation of mathematical concepts, mathematical explanations for commonly used rules and procedures, and understanding alternative solution methods. KCS is described as "content knowledge intertwined with knowledge of how students think about, know, or learn the particular content" (Hill et al., 2008, p. 375).

However, from a pilot project carried out by the Michigan team, (the designers of the LMT measures), they realised that items written in the category KCS did not always reflect on knowledge of content and students. Schilling et al. (2007) report that during interviews with participants, it became clear that many participants, when answering the items categorised as KCS-related, used mathematical reasoning, test-taking strategies or guesswork. Consequently, it was decided that only items from the CCK and SCK subdomain of MKT would be used in the LMT measures.

Mathematics educators, mathematicians, professional developers, project staff and former teachers were asked to write items for possible inclusion in the LMT measures. These items had to be in the format of multiple-choice questions for ease of scoring and scaling of large numbers of responses, and the writers were asked to try to capture two key elements of content knowledge for teaching mathematics: 'common' knowledge of mathematics and specialised knowledge in teaching mathematics to students (Hill et al., 2005). The multiple-choice questions selected by the Michigan team were aimed at separate aspects of content and pedagogical knowledge, to reflect practical teaching scenarios where teachers were asked questions about best representations and sequencing of examples, or questions about how they would respond to a learner's answer. Every item was designed to measure a single aspect of teachers' MKT (Rowan et al., 2001). The items either stand alone (stem item), or have other problems attached to it (stem and leaves) (Hill et al., 2005). Each multiple-choice question has one correct and one or more incorrect answers. Figures 3.1 and 3.2 show examples (from the released LMT measures; see Appendix A) of items with, and without leaves.

1. Ms. Dominguez was working with a new textbook and she noticed that it gave more attention to the number 0 than her old book. She came across a page that asked students to determine if a few statements about 0 were true or false. Intrigued, she showed them to her sister who is also a teacher, and asked her what she thought.

Which statement(s) should the sisters select as being true? (Mark YES, NO, or I'M NOT SURE for each item below.)

	Yes	No	I'm not sure
a) 0 is an even number.	1	2	3
b) 0 is not really a number. It is a placeholder in writing big numbers.	1	2	3
c) The number 8 can be written as 008.	1	2	3

Figure 3.1: Example of an item with leaves

2. Ms. Chambreaux's students are working on the following problem: Is 371 a prime number?

As she walks around the room looking at their papers, she sees many different ways to solve this problem. Which solution method is correct? (Mark ONE answer.)

a) Check to see whether 371 is divisible by 2, 3, 4, 5, 6, 7, 8, or 9.

b) Break 371 into 3 and 71; they are both prime, so 371 must also be prime.

c) Check to see whether 371 is divisible by any prime number less than 20.

d) Break 371 into 37 and 1; they are both prime, so 371 must also be prime.

Figure 3.2: Example of an item without leaves

The Michigan team mention that they used research on teaching and learning in the curricular domain to make decisions about the correctness of answers (Rowan et al., 2001). The LMT measures have been widely used for more than 10 years to compare the MKT of different groups of teachers and prospective teachers at different stages of teaching (Hill et al, 2008; Johannsdottir, 2013; Fauskanger & Mosvold, 2013; Johnson, 2011; Copur-Gencturk & Lubienski, 2013).

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3.3.1.3 Measuring self-efficacy

In order to address the study's second sub-question - determining the students' self-efficacy with respect to their MKT - an additional self-efficacy question was added to each MKT item of the questionnaire. According to Michaelides (2008), the aim of a self-efficacy research study is to predict performance outcomes from self-efficacy judgments. A Likert-type scale is often used for this purpose. Likert-type responses are of a closed format, where a question is asked and answers are presented on a scale from 3 to 7 or more, and are used to judge a respondent's feelings about a question (Arthur et al., 2012). Self-efficacy beliefs regarding mathematics competency are typically assessed by asking students to indicate on a Likert-scale their perceived confidence that they will be able to solve actual mathematical tasks with success (Pajares & Miller, 1997). For example, participants use the given scale to rate their confidence for being able to successfully solve a problem.

Academic self-efficacy is measured by giving or describing problems that are similar to the actual problems to be solved. The participants then judge how sure they are that they will be able to successfully answer the question or solve the problem (Bong & Skaalvik, 2003). Participants are instructed not to solve the problem itself, but just to indicate how sure they are that they are able to solve the question correctly. Bong and Skaalvik (2003) discuss self-efficacy questionnaire items where, instead of including the actual problems, only descriptions of the problems to be solved are given. These include questions such as "How confident are you that you can *successfully solve equations containing square roots*?" Michaelides (2008) mentions that the participants' confidence in their own ability is first measured, and afterwards they are given alternative (equivalent) questions to solve, where this confidence is then assessed.

For this study, the additional self-efficacy question that was added to each MKT item of the questionnaire is presented in the form of a Likert-type scale, where the participants indicate their own beliefs in the correctness of their answer to the respective/linked item on the questionnaire. The self-efficacy question of the current research study is "How sure are you that your answer is correct". The 'answer' refers to the answer to the MKT item of the questionnaire. In the current study, participants are required to first answer the MKT question, and then evaluate how sure they are about the correctness of their answer. This differs from other approaches. The rationale behind this method is based on arguments by Bandura (1977) and Michaelides (2008), which state that self-efficacy questions should correspond directly to the performance criterion being measured, and the measurement of self-efficacy should preferably be done at the same time or shortly after the measurement of the construct being investigated, in order to avoid mis-measurement of self-efficacy. Table 3.1 shows this self-efficacy question linked to each MKT question, with the three options given.

Table 3.1: Self-efficacy question added to each MKT item of the questionnaire

How sure are you that your answer is correct?		
I am not at all sure that my answer is correct.	I tend to think that my answer is correct, but I am not sure.	I am completely sure that my answer is correct

3.3.1.4 *Reliability and validity*

The reliability of a research instrument is a critical diagnostic that assesses the usefulness of the instrument. Hill et al. (2008, p. 385), defines reliability is defined as "the proportion of true score variation in the data to true score and error variation combined". Error variation may be caused by factors such as some items that do not cohere as well as was planned, items that do not help to discriminate between knowledgeable and less knowledgeable persons, or items that are not in accordance with the ability level of the persons being tested. "In general, reliability of 0.7 or above (on Cronbach's alpha) is considered adequate for instruments intended to answer research and evaluation questions using relatively large samples" (Hill et al., 2008, p. 386).

Hill et al. (2008) took extensive actions to guarantee validity and reliability of their MKT measures. They explain that after the draft items, written by the experts, had been reviewed by mathematicians and mathematics educators - both internal and external to the project, the items were piloted in California's Mathematics Professional Development Institutes. The MKT items were piloted with over 600 elementary school teachers. Hill et al. (2008) report that extensive research had been conducted to investigate whether the MKT items reliably and validly measure teachers' mathematical knowledge for teaching. This testing generated enough responses to conduct statistical analysis. It was found that the average reliability for piloted items was in the low 0.80's (Cronbach's alpha) with the occurrence of very few unsuccessful items. Specialised factor analysis was undertaken and this revealed a strong general factor in the piloted items (Hill et al., 2004). For determining this, Hill et al. (2004, p. 16) used a "program written to accommodate items linked by common stems or scenarios". The number concepts and operations form loaded strongly on the underlying dimension of knowledge of content (Hill et al., 2004), and this influenced them to consider only the CCK and SCK dimensions of MKT, and not the KCS dimension, as explained earlier.

Using information from the pilot test, Hill et al. (2004) were able to choose appropriate items from their large pool of piloted items for inclusion in the measures they were designing. These were "items that had shown desirable measurement properties, including a strong relationship to the underlying construct, a range of difficulty levels and a mix of content areas" (Hill et al. 2005, p. 388). It was found that the created assessments measure the range of teachers' MKT reliably, with internal reliability analyses (Cronbach's alpha) resulting in acceptable alpha coefficient scores ranging from 0.71 to 0.84 (Hill et al., 2004). Since the original validated MKT items were used unchanged in this research study, the reported reliability and validity of the original items were accepted as applicable in this study.

Pampaka & Williams, (2010, p. 160) reason that "validation refers to the accumulation of evidence to support validity arguments". As part of the pilot project, the Michigan team also validated their work (Hill, Dean and Goffney, 2005). The validation was done by using cognitive interviews and videotape validation, as well as an analytical study of instructional student improvement gain (Hill, Dean & Goffney, 2005). Piloted items were compared with the National Council of Teachers of Mathematics (NCTM) standards, to verify that the domains, as specified, were covered (Hill et al., 2005). The results of cognitive interviews with teachers indicated that teachers' responses were inconsistent in only about 5.9 percent of cases, which meant that there were very few cases where correct mathematical thinking led to incorrect answers, or vice versa. According to NCTM standards, "the content validity check of the entire piloted item set indicated adequate coverage across the domains of number concepts, operations, and patterns, functions and algebra" (Hill et al, 2005, pp. 388-9).

3.3.2 Data collection

The data used in the study were collected after the participants completed the questionnaire on the MKT required for the mathematical topic number concepts and operations. The data from the questionnaire were then analysed in order to answer the research questions of the study.

3.3.2.1 Participants

The population of this research study was all the BEd (FET) students in their first to fourth years of study at NMMU, and who specialise in mathematical literacy as one of their teaching subjects. All the students in years one to three were students who had qualified to write the relevant June examination paper on their module, while the fourth-year students were all busy doing their year of school-based learning.

The sample of students who participated in the research was a sample of convenience (Johnson & Christensen, 2012), since all the students were contacted via e-mail and through their lecturers, informing them about the research and asking for their participation. Those students who presented themselves and volunteered then completed the questionnaire. Table 3.2 summarises the numbers of participants (sample size) and the relevant modules they were enrolled in, as well as the total number of students registered for that module (the population size). The first and second year participants were all students taking the subject module for mathematical literacy, PFEL101 or PFEL201. The third year participants were students taking the POSD311ML module, which is the method module for mathematical literacy. The fourth year participants were the students who were doing the school-based learning component of the BEd (FET) degree in the schools, and who had completed the method module for mathematical literacy the previous year. The sample comprised about 60 percent of the population. The low number of participants in the second-year group was disappointing, since all the students were notified by e-mail of the research, and the questionnaire was written during their usual PFEL201 period, to facilitate participation. It is possible that only those students with good mathematical content knowledge and/or high levels of self-efficacy were interested to participate in the study.

Table 3.2: Summary of population sample statistics

	Module	**Number of participants in sample**	**Population total**	**Percentage of population**
Fourth year	POSD411 ML	43	63	68.3%
Third year	POSD311 ML	57	78	73.1%
Second year	PFEL201	8	48	16.7%
First year	PFEL201	29	38	76.3%
Total		**137**	**227**	**60.4%**

3.3.2.2 Choice of questionnaire

In the current study, the focus of the measures used, relate to the mathematical topics of number concepts and operations. These measures are related to the Intermediate Phase (IP) and Senior Phase (SP) curriculum, school ages 10 to 15 years. Although the study targeted pre-service teachers in the FET phase, the questionnaire was still valid. The reason for this choice was that, if teachers understand the structure of arithmetic and the fundamental concepts that are the underpinnings of mathematics, they will be able to help learners to recognise patterns and structure in mathematics. This improves learners' understanding and enjoyment of mathematics, and improves performance and achievement (Siemon et al., 2014).

According to the SA mathematics curriculum (DBE, 2011), at the end of Grade 10, learners are required to have a working knowledge of a "wide range of numbers and mathematical relationships in whatever form they appear, including equations, identities, inequalities, functions and relations" (Siemon et al., 2014, p 195). Since the work done in the intermediate phase (IP) and secondary phase (SP) are the foundations of this knowledge, the pre-service teachers' MKT related to this essential mathematical knowledge was assessed. It is possible that these FET students who have chosen mathematical literacy as their teaching subject will teach mathematics to at least the Grade 8 and 9 learners once they are employed as teachers at a school. A previous NMMU student, appointed to teach pure mathematics to Grade 9 learners at a local high school, even had to act as mentor for fourth-year students who specialised in pure mathematics, during their year of school-based learning (Morris, M., pers comm., 2015).

3.3.2.3 Completing of the questionnaire

The questionnaire used in this study consisted of both the items from the original LMT measures on number concepts and operations, compiled by the Michigan team, together with the self-efficacy questions accompanying each item. This questionnaire was handed out to the various groups of FET students specializing in mathematical literacy. Completion of the questionnaire was done with each group separately during their relevant lecture slots. The students answered the questionnaires in their classrooms under supervision of the lecturer or the researcher. Participants were informed that it was not compulsory for them to answer all the questions (Fletcher, 2013). All the data were gathered during the last weeks of the first semester of 2015.

After the questionnaires had been completed, the data were captured item by item, for each participant, using electronic spreadsheets. This initial captured data counted responses to each possible answer, including incorrect responses, as these incorrect answers might provide useful information regarding students' misconceptions. The responses to MKT items were then graded and coded as correct (1-point) or incorrect (0-points). The self-efficacy responses, related to each item, was a response to the question "How sure are you that your answer is correct?" This was coded as choices 1, 2 or 3, and captured on the same Excel spreadsheet.

For assistance with the analysis and interpretation of the data and for assistance in statistical computation, this researcher had a number of meetings with a statistician. This was done to enhance understanding of the statistics involved. The statistician is a senior lecturer at the NMMU's Unit for Statistical Consultation (USC).

3.3.3 Analysis of data

The results of the raw data were analysed for comparison, using descriptive and inferential statistical methods, and conclusions were drawn from this. Descriptive statistics were used to describe the basic features of the data, summarising the sample and the measures of central tendency. Inferential statistics were used to test the hypothesis of the existence of a relationship between MKT and self-efficacy. In this study, the MKT of first-, second-, third- and fourth-year students was measured for the mathematical topic of number concepts and operations. In addition, their self-efficacy beliefs regarding their MKT for these questions were also assessed. The results of the study are discussed in three sections in Chapter 4, in response to the two sub-questions and the main research question of the study.

The first sub-question of the study is: "What is the scope of the MKT of the different year groups of FET students on the topic of number concepts and operations, as measured by the survey questionnaire?" To answer this question, the responses from the whole questionnaire survey were examined to see if there is a significant difference in students' ability to correctly answer the different MKT questions. For comparison, the average number of correct responses to each item was calculated, as well as the average number of correct responses of each of the four year groups. For analysis, the number of correct answers for the whole group was also ranked, from the lowest number of correct responses to the most correct of responses.

In accordance with a statement by Arthur et al. (2012) about the importance of the constructs the questionnaire was designed to assess, the items of the questionnaire were divided into items testing the students' common content knowledge (CCK) and their specialised knowledge for teaching (SCK), for comparison in performance. An example of CCK would be to arrange decimals in order of magnitude, while SCK indicates, for instance, the ability to explain alternative methods of computation used by learners. The items were also divided into three mathematical sections: Numbers and operations, fractions and decimals, and ratio and proportion, to highlight topics which were in need of more attention in the method module.

The difficulty of each item of this questionnaire should be taken into account when analysing the questionnaire's responses. During the pilot phase of the questionnaire, the original designers of the questionnaire used Item Response Theory (IRT) to calculate the difficulty of each item - "IRT quantifies how well a test discriminates between teachers with various levels of PCK" (VVOB, 2014, p. 4). Item difficulty is associated with the trait level of participants - it is the trait level needed to have a 50 percent chance of answering the item correctly (Hill et al, 2008). Fur and Bacharach (cited in Johannsdottir, 2013) observe that trait level is the participants' level on the characteristic being assessed by the items. For this study, items were ranked according to their difficulty levels supplied by the designers of the questionnaire. The ranking was done from the most difficult item (ranked 1) to the easiest item (ranked 33).

Thereafter, the rankings of correct number of responses for each question were compared with the ranking of the difficulty level of each question. Difficulty ranking was on a scale from 1 to 33, with 1 being the most difficult item, while correct response ranking was on a scale from 1 to 33 with 1 being the item that had the least number of correct responses.

The Pearson correlation coefficient was calculated using the raw item difficulty and the number of correct responses. Wegner (2007, p. 418) defines Pearson's correlation coefficient as a statistical measure that "computes the correlation between two ratio-scaled (numeric) random variables" and takes on values between -1 and +1. A correlation is simply a statistical observation and does not imply a cause and effect relationship. According to Wegner, (2007, p. 418) "A low correlation does not necessarily imply that the variables are unrelated, since non-linear relationships are not measured by the Pearson relationship". Close groupings of data points on a scatter plot implies a strong linear relationship, with the correlation coefficient r being close to 1 for a positive relationship (Wegner, 2007).

Further analysis of this comparison showed that there were some instances where items that were considered to be easy, according to the categorisation of the developers, were not responded to correctly by many participants, while on the other hand, some items that were categorised as being difficult items, were answered correctly by an unexpectedly high number of participants. Some of these items were analysed and possible reasons for the discrepancies were sought. Details of this analysis are given in Chapter 4.The second sub-question was: “What are the participants’ self-efficacy with respect to their MKT for each item of the questionnaire?” Data analysis needed to answer this question, was done using cross-tabulation. Muijs (2011) describes cross-tabulation as a statistical method that compares the relationship between variables. In the case of this study, the two variables are the participants’ MKT and their self-efficacy with regard to this MKT. In essence, cross-tabulation gives a table or matrix showing the number of cases falling into each combination of the categories (Muijs, 2011). The cross-tabulation was first done using the self-efficacy choices of the total number of participant responses for each item (all possible responses). Another matrix showed the numbers and percentages of the self-efficacy choices of the number of participants who had incorrect or correct responses. An example of the cross-tabulation used for this study is given in Table 3.3, showing the way the first matrix was done for an item that had three possible choices of answer.

Table 3.3: Matrix of cross-tabulated results

Self-efficacy	**Answer 1** **Incorrect answer**	**Answer 2** **Correct Answer**	**Answer 3** **Incorrect answer**	**Total responses**
Not at all sure is correct				
Tend to think is correct				
Completely sure is correct				
Total				

The main research question was “What relationship exists between pre-service FET mathematical literacy teachers’ MKT and their self-efficacy beliefs regarding the required MKT?” To answer this question, the cross-tabulated data were analysed. In order to determine whether a significant relationship exists between the variables, the expected frequency of the number of cases for each cell was calculated, using the percentage of the row or column variables of the whole, as is explained in Chapter 4, section 4.4.3.3.

To evaluate the statistical significance of the data, a Pearson chi-squared analysis was done, and for practical significance of the data, a Cramer's V analysis was used. Various statistical tests exist which can be used to calculate the significance value or probability value of the relationship under scrutiny. In this study, the Pearson chi-square test was used. "This tests the hypothesis that the row and column variables are independent or unrelated to one another" (Muijs, 2011, p. 106). Muijs (2011) reports that in order to implement the Pearson chi-square test, the two variables being investigated must be nominal or ordinal, but not continuous. The chi-square test gives a test statistic, the degrees of freedom and a significance level or p-value. The p-value has to be as small as possible for the relationship being studied to be statistically significant (Muijs, 2011). The default value used for this study is $p < 0.05$ (a 95% confidence level). A p-value smaller than 0.05 implies that there is a less than 5 percent probability that the differences found are due to chance sample fluctuations. It does not indicate where the differences lie, or how strong the relationship between the variables are, but does indicate whether a relationship exists. While a p-value can inform the reader whether an effect exists, the p-value will not reveal the strength of the relationship (Sullivan & Feinn, 2012). Muijs (2011) argues that it would not be correct to think that a lower p-value will imply a more significant the relationship, since "the significance level is only partly determined by the strength of the relationship. It is equally determined by the sample size" (Muijs, 2011, p. 109).

Since the chi-square test only gives a statistical significance, an answer is still needed about the strength of the relationship. Therefore a different measure was needed to evaluate the strength of the relationship, also called the effect size of the relationship. Kotrlik and Williams (2003, p. 2) quote the definition for effect size given by Morse as effect size being "a measure of the degree of difference or association deemed large enough to be of practical significance". While statistical significance (given by the chi-square test) indicates whether the findings are possibly due to chance, the effect size or practical significance highlights the scale of the differences found (Sullivan & Feinn, 2012). The complete results of a research study can only be found by applying both the chi-square test and a test for effect size, of which Sullivan and Feinn (2012) regard effect size to be the more important result.

The statistical measures used to describe effect size for chi-square tests, are inter alia the Phi test and Cramer's V test (Cohen, 1988). Kotrlik and Williams, (2003, p. 6) describe the Phi-test as a "Pearson product-moment coefficient calculated on two nominal, dichotomous variables, when the categories of both variables are coded 0 and 1". Ferguson (2009) holds that the Cramer's V statistic is "typically used to represent the strength of association from chi-squared analysis" (Ferguson, 2009, p. 534). The Cramer's V test is used to describe the magnitude of the relationship between categorical variables when the contingency table involved is larger than 2x2, which implies that the number of possible values for the two variables are unequal, generating a different number of rows and columns in the data matrix.

Using both the chi-square test and the Cramer's V test was appropriate for use in this study. In the cross-tabulation of the data of this research study, the matrices formed by the cross-tabulation differed. The matrices all had three rows (the self-efficacy choices) and between two and five columns, since some of the questions only had two possible answers (incorrect and correct) while other questions had more than two, even up to five possible answers, of which only one answer was correct. This was explained in Chapter 3, section 3.3.1.2.

3.4 LIMITATIONS

A more useful result regarding the measurement of the students' MKT might have been obtained if a mixed-methods research approach had been used, such as asking participants to verbally explain their choice of answer, or to include some open-ended questions. This would have been the better choice for assessment of MKT. One of the reasons why this route was not followed, is because of the dimension that the possibility of a relationship with self-efficacy brought to this study. Open-ended questions would make it very difficult to directly compare the participants' answers with their self-efficacy choices. When using open-ended questions, the participant has to reason about possible scenarios, which often includes some uncertainty of success with the interpretation of the MKT involved, and hence the response cannot be identified as correct or incorrect, making measurement of self-efficacy beliefs very difficult.

Michaelides (2008) reports on a study where participants were presented with either multiple-choice or open-ended questions. The scores of the students doing the open-ended performance test were not as high as those of the students doing the test with multiple-choice questions. The conclusion was that the participants found the multiple-choice test easier, and that students' self-perceptions of mathematics capabilities may be less accurate than has previously been reported (Michaelides, 2008). Including open-ended question in the questionnaire of this study might have had a similar effect, compromising results about the possibility of a relationship between MKT and self-efficacy. Another reason is that, in changing the original LMT questionnaire by including open-ended questions, its reliability and validity may have been compromised.

Participants could also have been asked to complete an additional survey about their own experiences in mathematics and the teaching of mathematics. Questions on affect could have been included. All of this, however, would not have had any measurable impact on the final conclusions of the research, since the primary objective was to establish the possibility of a link between their MKT for the topics included and their self-efficacy regarding this MKT. Using the original LMT questionnaire, where each item was coupled with the self-efficacy question, produced the required insight into the answer to the research questions

The low number of participants with respect to sample size limited the scope of the study, and the possibility to generalise. Although the other year groups had a participation of more than 50 percent, very few of the second-year students were prepared to take part in the study.

3.5 ETHICAL CONSIDERATIONS

The students who participated in this research were informed about the aim of the research and their written consent was obtained before they attempted the questionnaire. Copies of the recruitment letter and the informed consent document can be found in Appendix C. Students were made aware that participation was voluntary and could at any time be interrupted, and that there were no direct benefits from taking part in the study. The benefit of the study to its participants is that any misconceptions regarding the MKT and self-efficacy beliefs of the groups and the whole sample, are to be highlighted. This will be reported in the study in a general way. It might in future form part of possible changes to the curriculum of the BEd (FET) mathematical literacy method module, in response to an argument by Smith, diSessa and Roschelle (1993, p. 115), who argue that misconceptions can impede learning and that "…instruction must confront and replace it".

The anonymity and confidentiality of all participating students were ensured, since real names were not used in the reporting; participants were given coded numbers, according to their year groups. No individual participant can, therefore, be identified by name, student number or any other identifying characteristic in any publication or shared representation of this study. The consent forms, containing names and students numbers of participants, were detached from the questionnaires and stored for safe-keeping before the data analysis was done.

In this study, the only potential for harm was if students were to be given their results for the MKT questionnaire, and through this having their own MKT being compared to the MKT of other students. Since the MKT of individual students cannot and will never be divulged, this study intends no harm to any person or any institution. Every possible attempt has been made to counteract possible harm to students or to the university. Ethics clearance for this study was received from the NMMU Research Ethics Committee (Human). The NMMU REC-H ethics clearance reference number is H15-EDU-CPD-003. (Appendix B).

All relevant data will be kept in a safe place after use in the study. Data (one copy of each) will be stored by PRP. Hard copies of data will be stored for five years for data analysis and verification. The data were used for the purpose of writing up the MEd dissertation, as well as for a future presentation, research and possible publication. No individual will be identified in any way that could link the data back to the participants in any publication or shared presentation of the study.

3.6 SUMMARY

In this chapter the objectives of the study was mentioned, which was to investigate the possible existence of a relationship between MKT and self-efficacy.

The philosophical framework, based on the beliefs of Fennema and Franke (1992), and the theoretical framework, based on the work of Shulman (1986), Ball et al. (2001) and Bandura (1977) have been explained. Research frameworks, approaches and paradigms were also discussed in general.

The quantitative nature of the research design was explained, as well as the methodology of the study. The research instrument, its origins, reliability and validity was discussed in detail. The sample and setting of the research study was described, and the methods employed in analysing the data were referred to. The detailed results of the data analysis is given in the next chapter.

Chapter 4

Analysis and discussion of the results

4.1 INTRODUCTION

The previous chapter described the methodology of this study's research into a relationship between mathematical knowledge for teaching (MKT) and self-efficacy regarding this MKT. In the first part of this chapter, the results of the MKT questionnaire completed by the students are discussed. After discussion of the MKT results, the results of the questions on self-efficacy are considered. Finally, the relationship between the two constructs is investigated by interpreting the statistical analysis that was done to investigate this relationship. This proposes to answer the main research question "Does a relationship exist between mathematical knowledge for teaching and self-efficacy of pre-service mathematical literacy teachers?"

4.2 MATHEMATICAL KNOWLEDGE FOR TEACHING

The MKT results of the different year groups of FET students, as measured by the questionnaire that was used, are discussed in this section. The focus of the discussion on MKT is first on the statistical results of the numbers of correct responses given to items. This is followed by a discussion of item difficulty and the correlation between item difficulty and numbers of correct responses. Items are divided into topics included in the mathematical domain of number concepts and operations, as well as into domains of MKT. After this, discrepancies between item difficulty and number of correct responses of some of the items are discussed, trying to identify reasons for the observed discrepancies.

The first-sub-question of the study is "What is the scope of the MKT of the different year groups of FET students on the topics of number concepts and operations, as measured by the survey questionnaire?" The results of the MKT gained from the questionnaire answer this question, since it gives an indication of the MKT of the different year groups, as well as of the whole sample of students.

4.2.1 Sample demographics

The population sample consisted of 137 BEd (FET) students at NMMU, specialising in mathematical literacy, from first to fourth year of study. Table 4.1 provides a summary of the numbers of students from the four different year groups who participated in this study. As discussed in Chapter 3, the low numbers of participation of students in some of the year groups was unfortunate.

Table 4.1: Summary of participation of different year groups

Group	Participation	Percent of total sample
Fourth years	43	31.4%
Third year	57	41.6%
Second year	8	5.8%
First year	29	21.2%
Total	137	100.0%

The items included in the questionnaire were all in the MKT domain of content knowledge, and should have been known to all the participants. All the participants in the study had previously been exposed to the mathematics included in the questionnaire, since they had studied mathematics at school, at least up to Grade 9 level. Therefore, all participants should have the mathematical knowledge needed to answer the mathematics underpinning the items. However, all students had not necessarily been exposed to the MKT involved in teaching the topic, since not all had done method modules.

The results of the second-year students will not be considered as relevant because of the size of the sample of second-year students. Only eight of the possible 48 participants (16.7 percent of all the second-year FET students specialising in mathematical literacy) volunteered to write the questionnaire, as was explained in Chapter 3.

4.2.2 Correct responses to the items of the questionnaire

In order to determine the MKT of the students, the number of correct responses given by each participant was analysed for each year group as well as for the whole sample. It should be noted that it was not compulsory for participants to respond to all items, as was stated in the written instructions. For this study, raw scores and percentages have been used to illustrate the results. In Table 4.2, the number of participants who gave the various different numbers of correct responses to all the items is summarised. Out of a possible total of 33 responses per participant, nobody had less than three or more than twenty-three correct responses. The total number of participants who had three correct responses, four correct responses, and up to 23 out of the possible 33 correct responses, was calculated. This was done for each of the different year groups, as well as for the whole sample of participants. As an example, reading from Table 4.2, correct responses to 14 of the 33 items were attained by a total of 20 participants, while in the group of third-year students, nine participants correctly answered 14 of the MKT items.

Table 4.2: Total number of correct responses to MKT items

	Number of participants with various different total number of correct responses per year-group				**Total participants with number of correct responses**
Total number of correct responses	4th years *n* = 43	3rd years *n* = 57	2nd years *n* = 8	1st years *n* = 29	***N* = 137**
3 correct responses	1	1	0	0	2

	Number of participants with various different total number of correct responses per year-group				**Total participants with number of correct responses**
Total number of correct responses	4th years *n* = 43	3rd years *n* = 57	2nd years *n* = 8	1st years *n* = 29	***N* = 137**
4 correct responses	0	1	0	0	1
5 correct responses	0	0	0	1	1
6 correct responses	1	2	0	1	4
7 correct responses	2	0	0	0	2
8 correct responses	1	1	0	1	3
9 correct responses	4	3	1	2	10
10 correct responses	2	1	0	2	5
11 correct responses	7	4	1	6	18
12 correct responses	4	7	2	3	16
13 correct responses	6	5	0	3	14
14 correct responses	5	9	0	6	20
15 correct responses	5	7	2	1	15
16 correct responses	3	9	0	1	13
17 correct responses	0	2	2	1	5
18 correct responses	2	1	0	0	3
19 correct responses	0	1	0	0	1
20 correct responses	0	2	0	1	3
21 correct responses	0	0	0	0	0
22 correct responses	0	0	0	0	0
23 correct responses	0	1	0	0	1
24 up to 33 correct responses: none					

4.2.2.1 Statistical results of numbers of correct responses

The results for correct responses to the items of the questionnaire reflected a great variety in the participants' understanding of the mathematical topics involved. The test scores tended towards a normal distribution, and the average for correct responses for all the items of the questionnaire was 39.02 percent. Table 4.3 shows a descriptive statistical comparison between the four groups, as well as the values for the whole sample. This is done in terms of the average percent of correct responses to all items.

Table 4.3: Comparison between groups in terms of average percent of correct responses

Participants	***N***	**Mean**	**Median**	**Standard deviation**	**Variance**
4th years	43	37.70	39.39	8.71	75.78
3rd years	57	40.78	42.42	10.93	119.40
2nd years	8	40.15	40.91	7.89	62.31
1st years	29	37.20	36.36	10.11	102.25
Total group	137	39.02	39.39	9.97	99.45

The results show a similar pattern for the MKT of the four different year groups. The fourth-year students are all pre-service teachers doing their school based learning. The third-years students had the best results in respect of the mean value of 40.8 percent for their MKT, and they also had the highest number of correct responses. These students have been studying teaching methods in their third year and are pre-service teachers who will start their school-based training in their fourth year. These reasons suggest why the performance of the third-year students on the MKT questionnaire was higher than that of the fourth-year group. The relatively high average score of the second-year students are possibly due to only the stronger students being willing to attempt the questionnaire. The results of the first-year students (37.2 percent correct responses) were about the same as that of the fourth-year students.

4.2.2.2 Correct responses per individual items of questionnaire

The questionnaire consisted of 16 items, and five of these items had sub-items, giving a total of 33 items that could be responded to. The total number of correct responses for each of the 33 items was calculated for the whole sample as well as for the four different year groups.

Five of the items (2, 3, 10, 14 and 16) had sub-items. Figure 4.1 shows the number of correct responses for all 16 items, using the average number of correct responses for items that had sub-items. A table giving the number of all responses is given in Appendix D. The items with sub-items had an overall higher average number of correct responses than items where only one answer was required, as can be seen from Figure 4.1, where the five highest values are for the questions that had sub-items. When responding to an item with more than one part, the participants were possibly more involved with the subject of the item and thinking about different aspects of the topic involved, hence the good response to these items.

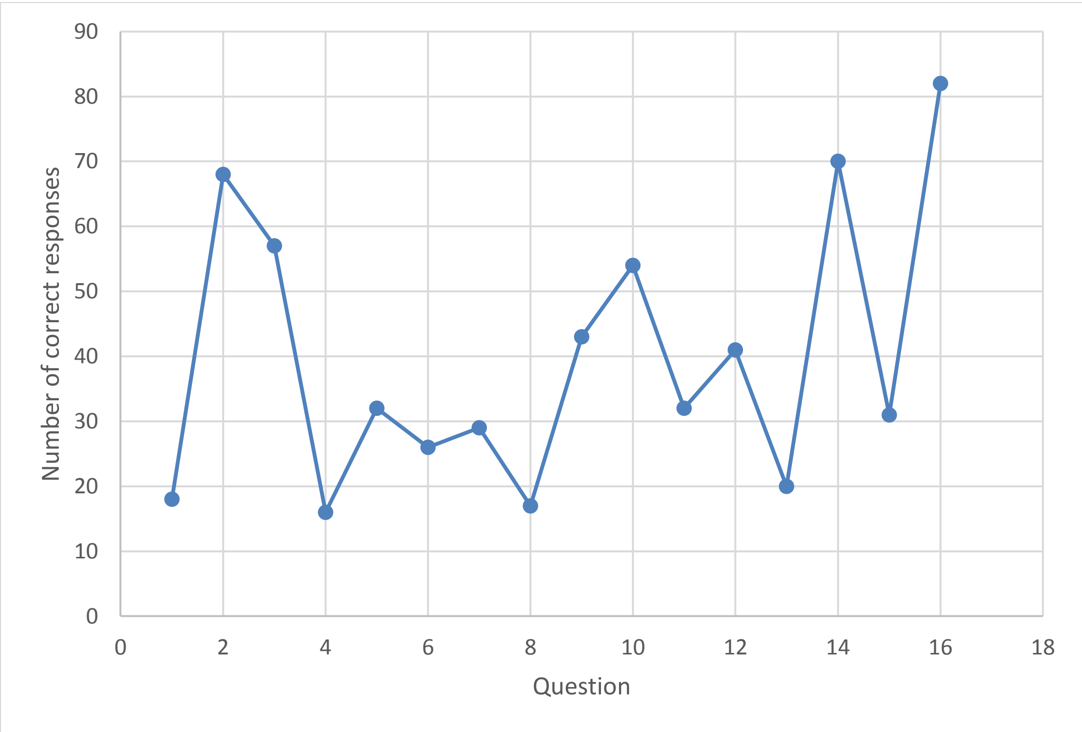
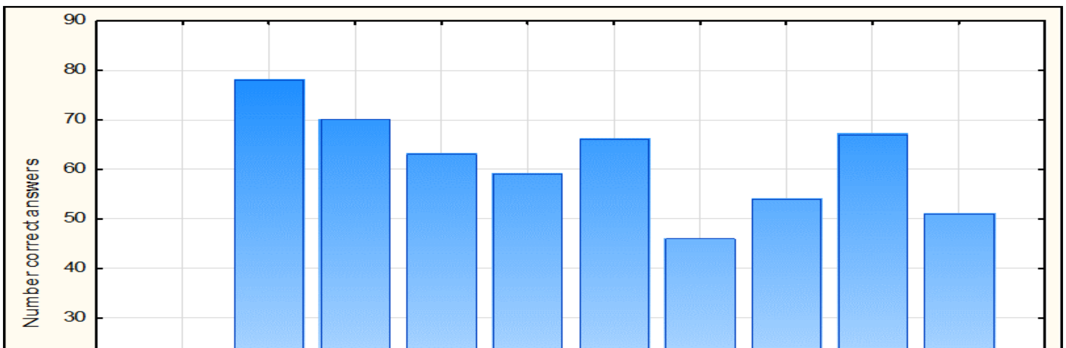


Figure 4.1: MKT results showing total number of correct responses for the 16 main items

Figure 4.2 shows bar charts illustrating the total number of correct responses per each of the 33 items for the whole sample of participants, and a discernible difference between the highest number of correct responses for the 33 different items and sub-items is evident.



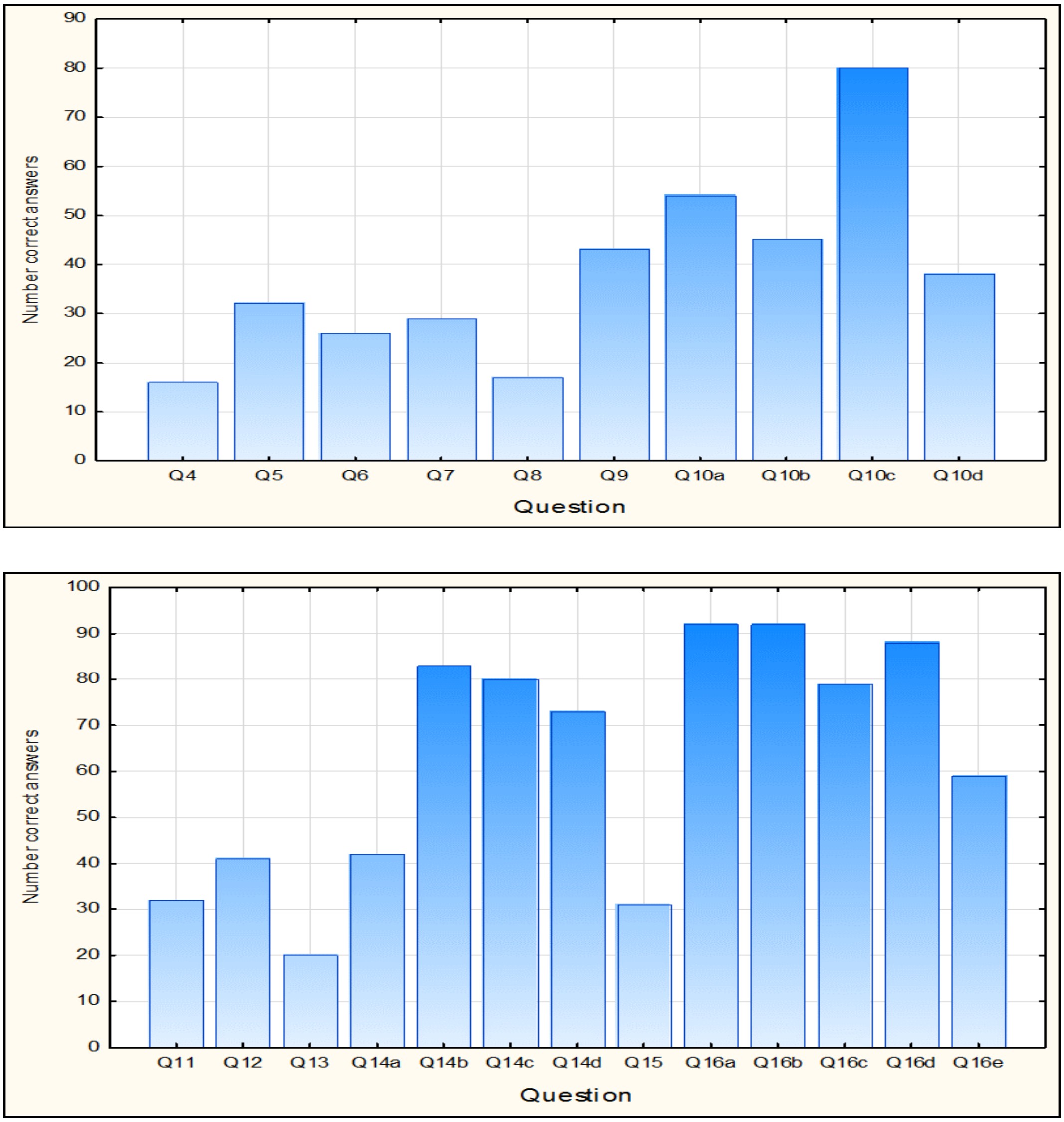


Figure 4.2: MKT results showing total number of correct responses for all 33 items

4.2.2.3 Ranking of numbers of correct responses to each item

The number of correct responses to each of the 33 items was ranked from the lowest number of correct responses for an item (ranked 1), to the highest number of correct responses (ranked 33). This was done in order to identify items that had low numbers of correct responses, as well as items that were responded to very well. An alternate ranking was also done, where the five items that had sub-items attached to them, were counted as one item each (total of 16 items) and ranked accordingly from one to 16, using the average number of correct responses as well as the average difficulty level for each of the sub-items. Appendix D shows the results giving the total number of correct responses to each of the 16 items, together with their sub-items (33 items in total) for the whole sample of 137 participants, as well as the percentage of correct responses per item. It also shows the ranking of correct responses (least number correct = 1), with first a ranking for all 33 items, sub-items indicated separately, and then a ranking for the 16 complete items. As can be seen from the numbers of responses given in the table, not all items were responded to by all participants.

4.2.3 Item difficulty

When responses are analysed, the difficulty level of each item is a factor that should be taken into account, since differences in difficulty level help to differentiate between participants. In a questionnaire such as the one used in this study, not all items have the same level of difficulty. As explained in Chapter 3, the difficulty level of each item was determined by the developers of the questionnaire during the pilot phase. The developers based item difficulty on participants having a 50/50 chance of answering the item correctly (Hill et al., 2008). The underlying assumption is that a person whose mathematical knowledge for teaching is good, is more likely to give correct responses to the items.

The item difficulty supplied by the Michigan team is based on a continuum, where an item with difficulty level categorised as zero is an item of average difficulty. Items below zero are categorised as being easier, and items above zero as being more difficult. Items with a difficulty level at or below -0.43 is considered to be very easy, and items with a difficulty level at or above +0.43 are considered difficult (Johannsdottir, 2013).

Difficulty of items is ranked from most difficult (ranked 1) to least difficult (ranked 33), since the number of correct responses for each item is also ranked from the least number of correct responses to the most. By ranking results in this way, the most difficult items should have the least number of correct responses, and the easiest items should have the most number of correct responses. The difficulty level of all items, and ranking for all 33 items as well as for the 16 main items are given in Appendix E.

A comparison between the item difficulty, as supplied by the designers, and the number of correct responses for each item, shows a strong negative relationship, with a Pearson correlation coefficient of r = -0.6589. This indicates that the higher the item difficulty, the lower the number of correct responses, which was the expected result. Figure 4.3 shows a scatterplot with numbers of correct responses per item against item difficulty. The two arrows below the scatter plot indicate the values +0.43 and -0.43 on the continuum of item difficulty. Items below -0.43 were categorised as easy, and items above +0.43 were categorised as difficult by the designers. A difficulty level of zero was categorised as average.

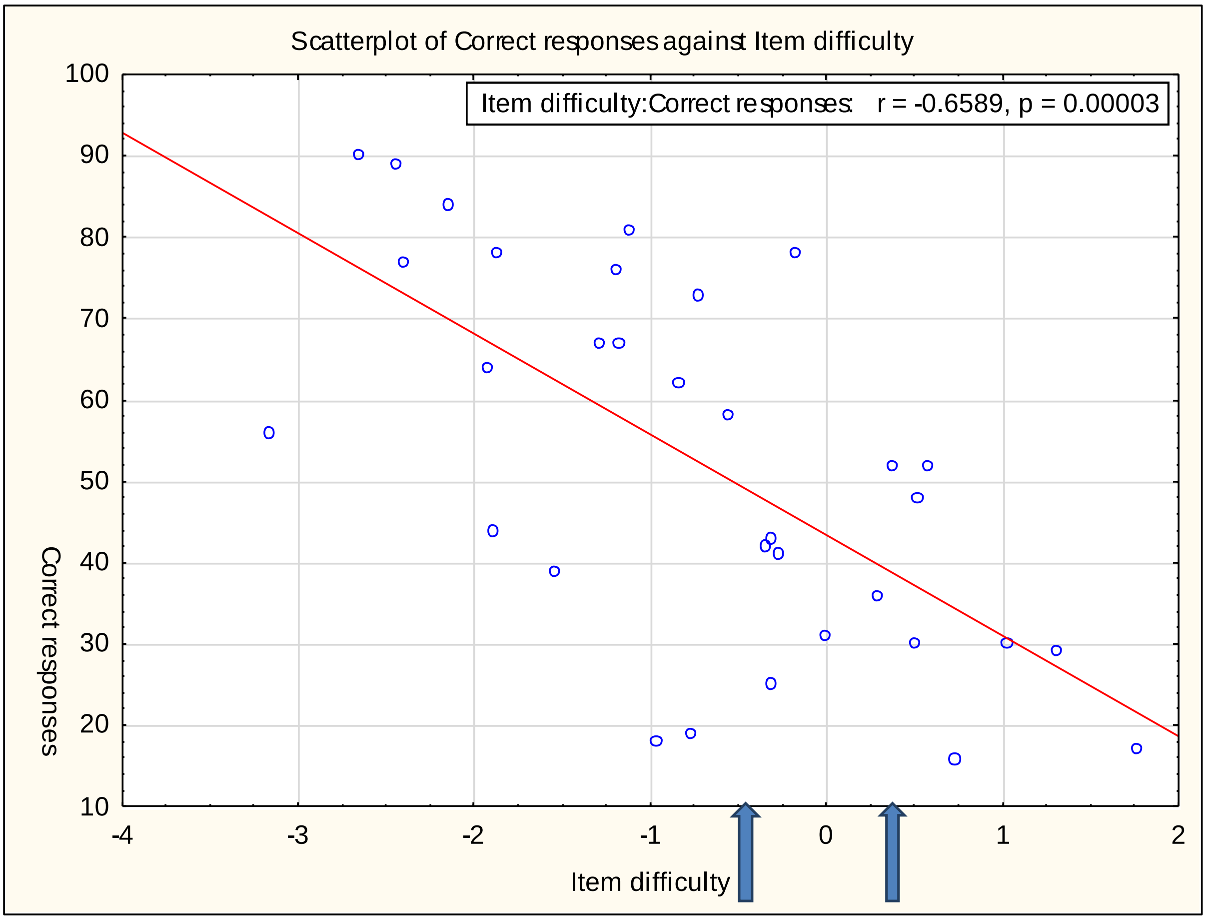


Figure 4.3: Scatter plot of correct responses against item difficulty

4.2.4 Items divided into mathematical topics

Since one aim of this study was to investigate possible changes to be effected to the Mathematical Literacy Method curriculum, it is important to identify specific problem areas in the pre-service teachers' content knowledge. For the purposes of this study, and in order to investigate performance on mathematical topics, the items, all of which were in the mathematical domain of number concepts and operations, were sorted into three related topics. These are the topics numbers and operations, fractions and decimals, and ratio and proportion.

Although the questionnaire had 33 items, five of these had sub-items, which gives a total of 16 different items to be classified as shown in Table 4.4, where the topic of each question is given.

Table 4.4: Classification of mathematical topics of the items of the questionnaire

Numbers and operations	Fractions and decimals	Ratio and proportion
Q2. Properties of rational and irrational numbers	Q3. Division of fractions	Q1. Most difficult proportion
Q4. Conventional long division	Q10. Area model of decimal multiplication	Q8. Why cross-multiplication works
Q5. Division by zero	Q11. Adding mixed numbers	Q9. Table method for profit calculation
Q6. Divisibility rules for 4	Q14. Word sums for number divided by fraction	Q12. Percentages
Q7. Simplify surds	Q16. Equivalent forms of numbers	Q13. Ratio of ingredients
		Q15. Ratio boys: girls

Using the classification of items into mathematical topics, the difficulty ranking of the 16 items was compared to the ranking of numbers of correct responses. Table 4.5 shows first the difficulty ranking of the 16 different items (divided into mathematical topics), ranked from the most difficult question to the least difficult question, and second the ranking according to the number of correct responses for that question, ranked from the least number of correct responses to the most correct responses. The final column in Table 4.5 shows the difference in rankings.

Table 4.5: Classification of mathematical topics: ranking of difficulty and number of correct responses of the 16 different items

Mathematical sub-topic	**Question**	**Difficulty ranking** x_i	**Ranking: number of correct responses** y_i	**Difference in ranking** $x_i - y_i$
		Most difficult =1	Least number of correct responses = 1	
Numbers and operations	Q2. Properties of rational and irrational numbers	12	14	-2
	Q4. Conventional long division	4	1	3
	Q5. Division by zero	7	8	-1
	Q6. Divisibility rules for 4	8	5	3
	Q7. Simplify surds	2	6	-4
Fractions and decimals	Q3. Division of fractions	11	13	-2
	Q10. Area model of decimal multiplication	6	12	-6
	Q11. Adding mixed numbers	5	9	-4
	Q14. Word sums for number divided by fraction	14	15	-1
	Q16. Equivalent forms of numbers	16	16	0
Ratio and Proportion	Q1. Most difficult proportion	13	3	10
	Q8. Why cross-multiplication works	1	2	-1
	Q9. Table method for profit calculation	9	11	-2
	Q12. Percentages	15	10	5
	Q13. Ratio of ingredients	10	4	6
	Q15. Ratio boys: girls	3	7	-4

For items with sub-items, the average (median) of the rankings of the different parts was used, as was explained in Section 4.2.2.3. These rankings are given in Appendix E. This gives a total of 16 items, as was shown in Figure 4.1. From Table 4.5, it can be seen that the items categorised as being easier, have the most correct responses, and the items categorised as more difficult have fewer correct responses. This is also reflected in the Pearson correlation coefficient, as mentioned in Section 3.3.3.

The question on equivalent forms of numbers [Q16, dealing with fractions and decimals] was the easiest question, and also had the most number of correct responses (similar rankings). The most difficult question was the question on "Why cross-multiplication works" [Q8, dealing with ratio and proportion], and this had the second least number of correct responses. The question on long division [Q4] had the least number of correct responses, and is ranked fourth most difficult by the developers. The possible reasons for this are discussed in Section 4.2.5.

A positive difference in rankings indicates an item identified by the developers of the LMT measures to be easy, but not answered as well as could be expected, such as Q12. This was a question on whether a discount of 50 percent is the same as a discount of 40 percent and a further discount of 10 percent on that discount. On the other hand, a negative difference in ranking indicates an item categorised as being more difficult, but which was answered correctly by more participants than would be expected, such as Q15, which was a question on the ratio of boys to girls in a classroom, and is a question type the mathematical literacy students are familiar with. The table shows more differences that are negative than differences that are positive which suggests that, in some instances, the more difficult items were answered better than the difficulty level indicates, or better than expected. Other possible reasons for this are discussed in Section 4.2.5.

The topic fractions and decimals showed the best results in terms of differences of rankings (no positive differences) which indicated that the participants' MKT for this topic was good. However, three of the items under the topic ratio and proportion showed high positive differences (even a difference in ranking of 10 for question 13), which pointed to rather low MKT for these items. Since fractions are intrinsically embedded in problems on ratio and proportion, the participants' knowledge of fractions did no enable them to correctly interpret the questions on ratio and proportion. Ball et al. (2001, p. 447) comment on studies showing that learners, when working with fractions, tended to overgeneralise knowledge of whole numbers, "which led to misconceptions and impoverished ideas about rational numbers".

4.2.5 Items divided into domains of MKT

Using the framework of Ball et al. (2008), discussed in Section 2.2.5, the data from the questionnaire were also analysed by considering the cognitive domains of mathematical knowledge for teaching. All the items of the questionnaire fall into the MKT domain of subject matter knowledge (SMK), but can be divided into the MKT sub-domains of specialised content knowledge (SCK) and common content knowledge (CCK). Whereas CCK is mathematical knowledge that all people working in mathematics share (For example: "what number is halfway between 2.5 and 2.55?"), SCK is knowledge unique to mathematics teachers and used in the classroom when teaching mathematics (For example: "show learners the reason why any number is divisible by 4 if the last two digits are divisible by 4"). Schilling et al. (2007) mention that the SCK items help to differentiate between teachers with and without specialised mathematical knowledge, which helps to predict success in teaching and in learner achievement. Table 4.6 gives a breakdown of the 16 items divided into the cognitive MKT domains of SCK and CCK.

Table 4.6: Items arranged by cognitive domains of SCK and CCK

Cognitive domains	
Specialised content knowledge (SCK)	**Common content knowledge (CCK)**
Q1. Most difficult proportion	Q2. Properties of rational and irrational numbers
Q3. Division of fractions	Q6. Divisibility rules for 4
Q4. Conventional long division	Q12. Percentages
Q5. Division by zero	Q16. Equivalent forms of numbers
Q7. Simplify surds	
Q8. Why cross-multiplication works	
Q9. Table method for profit calculation	
Q10. Area model of decimal multiplication	
Q11. Adding mixed numbers	
Q13. Ratio of ingredients	
Q14. Word sums for number divided by fraction	

Cognitive domains	
Specialised content knowledge (SCK)	**Common content knowledge (CCK)**
Q15. Ratio boys: girls	

Figures 4.4, 4.5 and 4.6 show the items classified by the developers as being easy or difficult, that fall into the two different domains of CCK and SCK. The ranked item difficulty, as categorised by the developers, as well as the ranking of the number of correct responses for that item, are given. These rankings were from most difficult ranked 1, to easiest ranked 33 for difficulty level, and from least number of correct responses, ranked 1, to most number of correct responses ranked 33.These rankings are given in Appendices D and E. Items not shown in these figures (Questions 5, 6, 9, 10, 11 and 14c) were neither easy nor difficult, and were categorised by the designers as being between -0.43 and +0.43 on the continuum. For items with sub-items, each sub-item is shown separately. This was necessary since the difficulty levels of some of the sub-items are very different.

Figure 4.4 shows items categorised as difficult and requiring SCK. From Figure 4.4 it can be seen that, with the exception of the item on conventional long division [Q4], all the items have a correct response ranking higher than the difficulty ranking of the items (Appendix F). This might suggest the participants' strong specialised knowledge for teaching these topics.

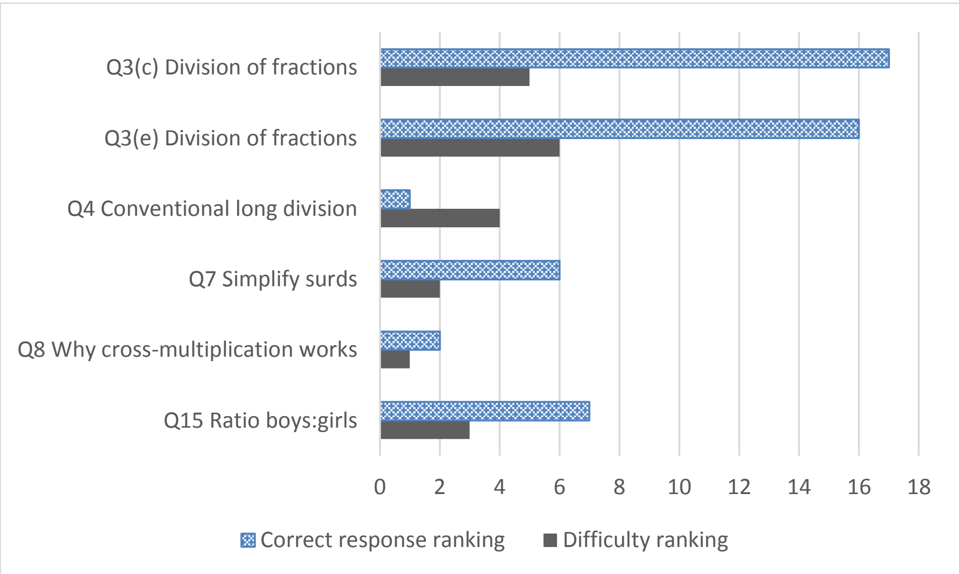


Figure 4.4: Items categorised as difficult and requiring SCK

The response to the item on conventional long division [Q4], where alternate algorithms for long division are given, suggests that the students might not fully understand how long division "works", or might not have been introduced to alternate methods of division in previous years. In Question 4, participants were given four explanations for why the conventional algorithm of the long division of two whole numbers works, with only one correct explanation. Very few correct responses were given by the participants. Ball et al. (2001, p. 446) report that many teachers do not understand that long division should be described as an "iterative process of making groups of a particular size and removing a certain number of those groups". According to Ball et al. (2001) teachers interpret division as sharing, instead of using the concept of measurement when explaining division. This might be the reason why teachers are unable to explain problems involving division by zero, division where the numerator is bigger than the denominator, or division of fractions and decimals.

Figure 4.5 shows items categorised as easy and requiring SCK. Five of the seven items presented in Figure 4.5 show a correct response ranking lower than the item difficulty ranking supplied, suggesting the paticipants' lack of specialised knowledge for teaching these topics. When comparing the sub-items of the item on division of fractions [Q3] given in Figures 4.4 and 4.5, it can be seen that the sub-items categorised as being more difficult (Q3(c) and Q3(e)), had more correct responses than the sub-items categorised as being easier sub-items (Q3(a), Q3(b) and Q3(d)). The participants' inability to correctly interpret these questions, perhaps due to language issues, could have influenced participants' performance in this question. Limitations attributed to English not being the home language of participants are discussed in Section 4.2.6.2.

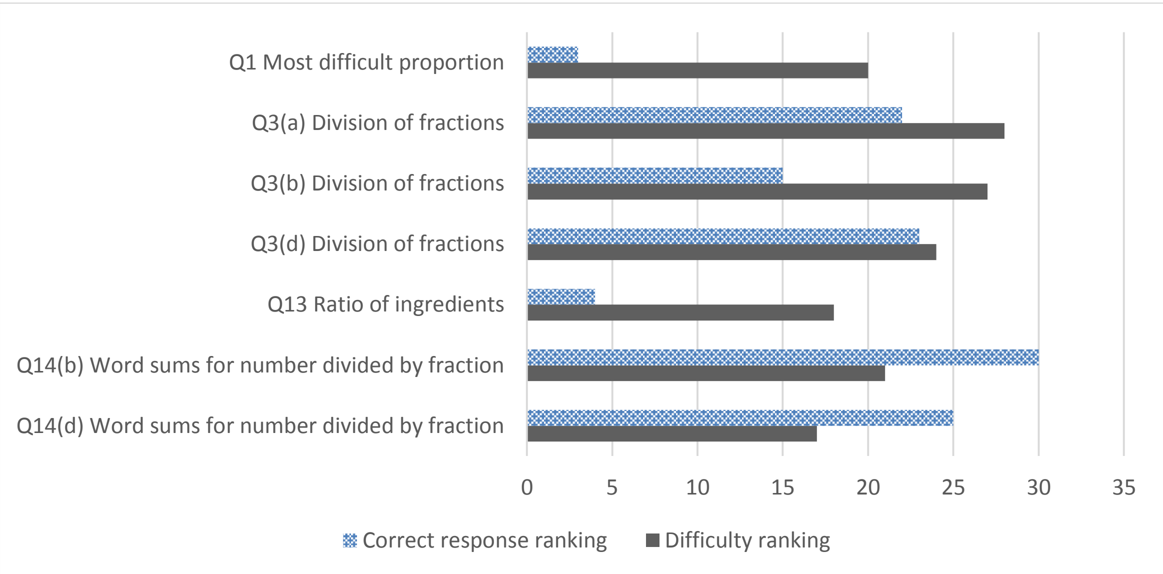


Figure 4.5: Items categorised as easy and requiring SCK

The two sub-items from Question 14 (in Figure 4.5) that show a correct response ranking higher than the item difficulty ranking supplied, are examples of the MKT required for teaching division of fractions. Question 14 is an item on word-sums for fractions, and is a multi-answer item the developers of the questionnaire categorised as easy. Participants were asked questions about writing word sums for a number divided by a fraction. Four different word sums were given, and participants had to decide whether each sum was appropriate to be used as an example. The two sub-items that were answered well [Q14(b) and Q14(d)], shown in Figure 4.5, are both examples that mathematical literacy students are familiar with, hence the good responses. Question 14(d) was also the last of the sub-items, and participants possibly understood the purpose of the question better after they had already answered three similar questions. This item is similar to Question 7 of the released items (see Appendix A).

Figure 4.6 shows the four items where CCK was required. CCK is mathematical knowledge the average educated adult should have. According to the difficulty ranking as defined by the developers, these four items were categorised as being easier items on the continuum. The two items with sub-items [Q2 and Q16] had high numbers of correct responses. The question on percentages [Q12] did not have as many correct responses as expected from the much higher difficulty level (indicating a question categorised as being easier). Possible reasons for the low number of correct responses for Q12 and Q16(e) are discussed in Section 4.2.6.

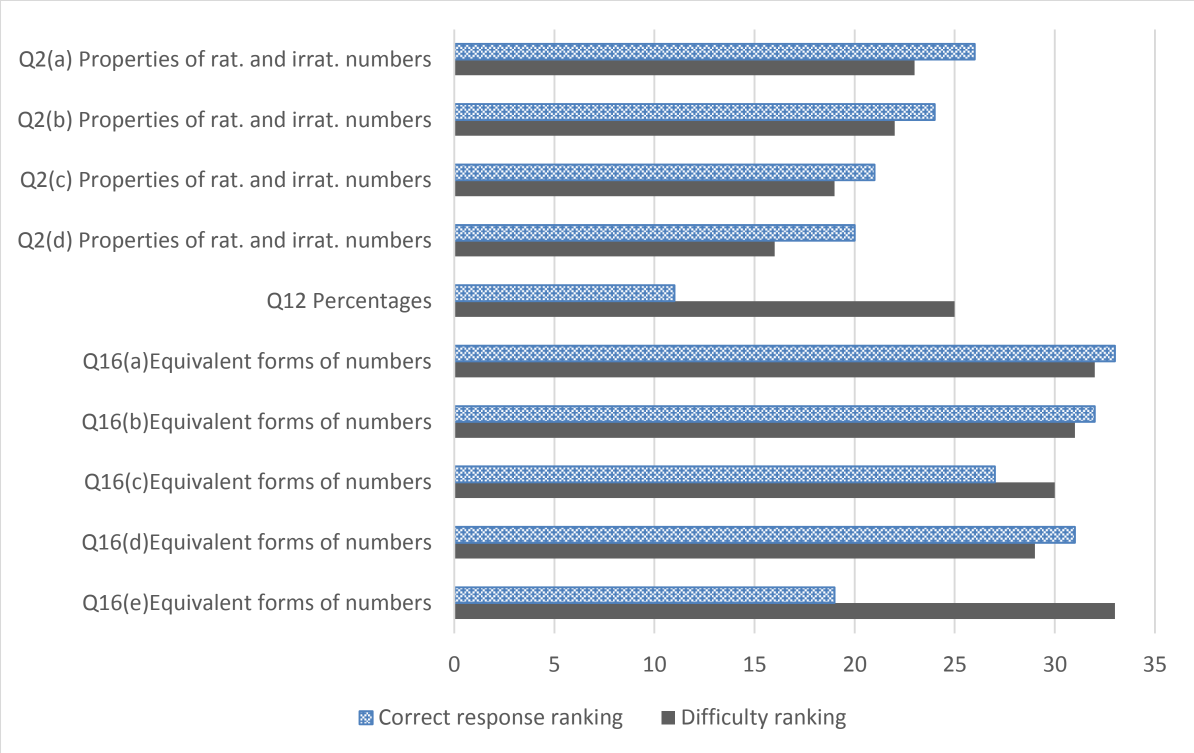


Figure 4.6: Items requiring CCK

4.2.6 Observed discrepancy between item difficulty and numbers of correct responses

For a number of items, a noticeable discrepancy exists between the level of difficulty of the item as categorised by the developers, and the performance of the participants (Appendix F). Some items categorised as difficult were answered correctly by many participants, while on the other hand, some items categorised as easy showed few correct responses. Figures 4.7 and 4.8 show instances of this phenomenon, where there is a difference of more than eight between the ranking of difficulty and the ranking of the number of correct responses. Possible reasons for these discrepancies are explored in this section.

4.2.6.1 Items categorised as more difficult answered well

In Figure 4.7 the items that are categorised as not being easy, but which were responded to well, are shown. This might indicate that participants had the required MKT for teaching these topics.

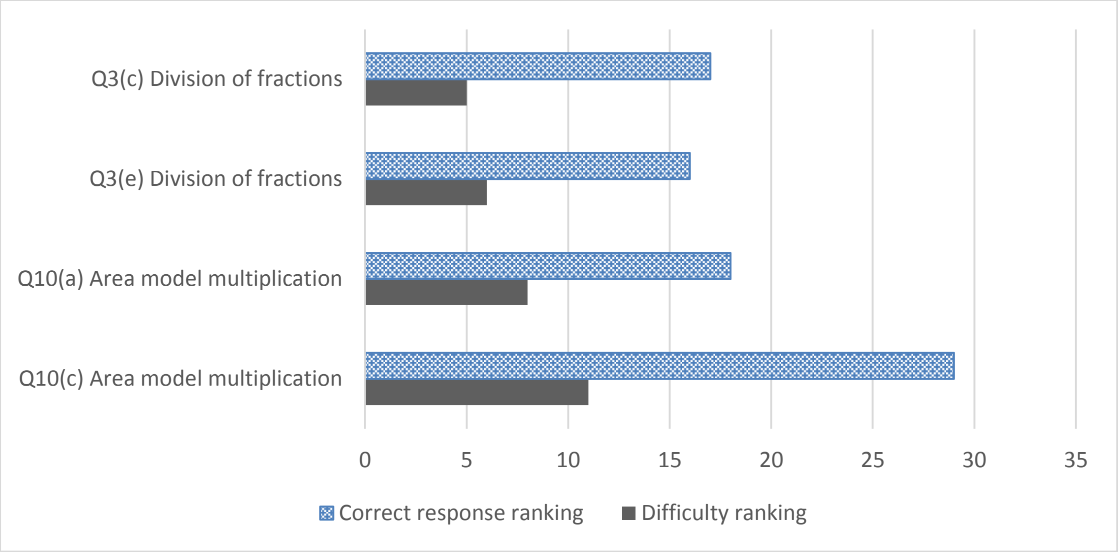


Figure 4.7: Items categorised as more difficult answered well

Figure 4.7 shows noticeable differences in ranking between difficulty level and numbers of correct responses. For all four items shown in Figure 4.7, the ranking of the number of correct responses (MKT) is much higher than the difficulty ranking, which suggest that participants were well able to respond correctly to these items that were categorised as being more difficult. The four items included in this figure were parts of two items that had sub-items [Q3 and Q10], which might be the reason for the good response ranking, as discussed in Section 4.2.2.2.

Question 10 was an item concerning the use of an area model to explain decimal multiplication. In this item, an area model representing decimal multiplication, was given. Four statements were given about the use of this model, and for each statement, participants had to choose whether the statement was true or false. Although these responses were not categorised as being especially difficult or easy by the developers (difficulty levels between -0.313 and 0.382), all the sub-items had an above average number of correct responses. Question 10(c) had the highest difference in ranking of all the items of the questionnaire - correct response ranking of 29 and difficulty ranking of 11. This might be due to the response to this item being a 'common sense' answer for the participants since in the school subject Mathematical Literacy diagrams are often used to represent the problem. This might be a possible reason why so many participants had a correct response to this specific sub-item.

These rules for working with decimals can be confusing to learners. When decimals are added, the general rule is to line up the decimal places. However, when decimals are multiplied, the rule is to move the decimal point to get whole numbers that can be multiplied. Using the area model is one way of making sense of the difference between these two rules, especially regarding multiplication of decimals. Ball et al. (2001) propose that learners should be taught to use a 10x10 grid to multiply decimals such as 0.3 and 1.7. Ball et al. (2001) argue that teachers should know more than just the correct rule for multiplying decimals. Teachers should, for instance, know how to teach the meaning of place value and the meaning of the places in a number. They emphasise that learners' knowledge of multiplication of whole numbers "might interfere with or obscure important aspects of multiplication of decimals" (Ball et al., 2001, p. 448). Question 35 of the released items (Appendix A) is comparable to Question 10 of this questionnaire.

4.2.6.2 Items categorised as being easy but not answered well

The items shown in Figure 4.8, are items that were considered easy, but where not many correct responses were given. Possible reasons for this are discussed.

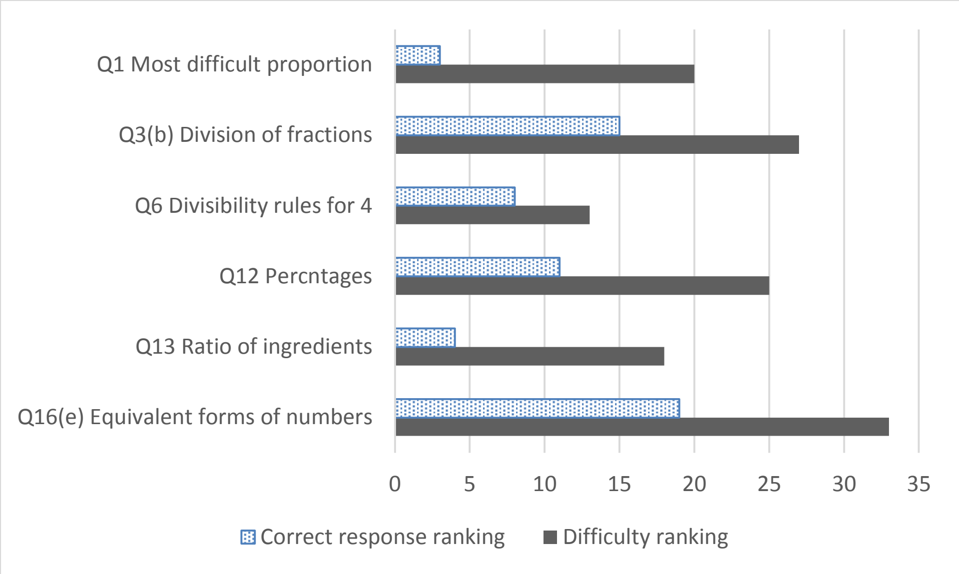


Figure 4.8: Items categorised as easy not responded to well

Question 1 was an item on 'the most difficult proportion', and had the second highest difference (17) between difficulty level and ranking of number of correct responses (Appendix F). The difficulty ranking of 20 identifies this item as being categorised as one of the easier items, while a correct response ranking as low as three is an indicator that few of the participants responded correctly to this item. In this item, participants were given three proportions to solve, each with one variable, and were informed that learners had not yet been taught the method of cross-multiplication. From these three proportions, participants had to choose the proportion they considered to be the most difficult of the three. The big difference in ranking might suggest that participants had real difficulty in making the correct decision about the MKT involved in this item. The option "all problems provide the same level of difficulty" was chosen by 68.4 percent of the participants. Although all three proportions could easily be solved using cross-multiplication, not all of the problems could as easily be solved without using cross-multiplication, hence making some proportions more difficult than others. Another reason for the low number of correct responses could be that, as this was the first item of the questionnaire, the students did not yet understand the thinking involved, not realising the necessity of focusing on the implications for teaching the topic of proportions. It is possible that participants did not yet understand the questionnaire correctly, despite the two trial questions given beforehand.

For the item on division of fractions [Q3], sub-item 3(b) shown in Figure 4.7, was categorised as an easy question. Only sub-items 3(c) and 3(e) (see Figure 4.4) were considered difficult, while the other three sub-items were categorised as very easy items; well below zero on the continuum of difficulty levels as categorised by the developers (Figure 4.5). However, the response ranking of all five sub-items of Question 3 were more or less the same. A possible reason for this anomaly could be language. The wording of the question could well have been confusing for individuals for whom English is not their mother tongue. One of the mathematics lecturers who assisted me in implementing the questionnaire mentioned that students had complaints about words whose meaning they did not understand (Walton, pers comm. 23 March 2015).

Question 6 is an item on the divisibility rule for the number ‘4’, where four different explanations are given of why the rule works. Participants had to choose the best explanation. Three of the four options given were plausible, but only one of them was the correct mathematical explanation, which was responded to correctly by 25 (19.1 percent) of participants. A fourth option, one which was conspicuously untrue since it indicated that the divisibility rule for four was similar to the divisibility rule for three, was chosen by 46 (35.1 percent) of the participants. The poor results to this relatively easy question indicates the misconceptions students have about divisibility rules, suggesting that some participants do not have conceptual understanding of the mathematical foundation of divisibility rules. Smith et al. (1993) claim that misconceptions which have become entrenched in the minds of learners, linger in the subconscious mind to the detriment of future learning.

The item on percentages [Q12] falls under the topic of ratio and proportions, since a percentage is just another way of writing a proportion. This was categorised as a very easy question. It is a well-known problem describing a scenario about a 40 percent discount plus a further 10 percent discount on that, and asking learners whether this would be the same as a 50 percent discount. Participants had to consider four hypothetical responses from learners and decide which of the solutions given showed the best understanding of the mathematics involved here. The correct learner response was chosen by 39 (29.8 percent) of the participants. The second option seemed to be a very plausible ‘mathematical’ answer, talking about the non-linearity of percentages, but which was rather ambiguous and perhaps confusing to the participants, not all of whom are proficient in English. Thirty-six percent of participants chose this learner response. On the positive side, only nine participants said that 50 percent is equal to 40 percent plus 10 percent in this scenario, which suggests that the participants do have the correct mathematical content knowledge, just not the correct mathematical knowledge for teaching this topic, or the correct understanding of the language used in this question.

Question 13 was another question on ratio and proportion (ratio of ingredients) categorised as being easy (difficulty level -0.76). In this item, participants were given different learner solutions to a problem involving ratio of ingredients in a recipe, and were asked to choose the answer that was not a valid answer. Only 19 (14.8 percent) of all participants responded correctly to this item. The incorrect option 'all strategies are valid', was chosen by 29.7 percent of students. The first option given was the valid option, but it was a learner option where an intuitive method was used to solve the problem, and involved using a method not habitually taught in a classroom. Since 28.9 percent of participants chose this option as being invalid, it suggests participants' inability to recognise alternative computational strategies that are also correct. Ball and Bass (2000) explain that teachers should know and understand that many problems have alternative methods of solution, and teachers should be able to recognise a learner's correct alternative solution to a problem. They reason that, "…given the multiple approaches produced by the students, there is a profound mathematical imperative to inspect, analyse, and reconcile them" (Ball & Bass, 2000, p. 96). Ball and Bass (2000) refer to Liping Ma's comparison of a teacher's knowledge of mathematics with the experienced taxi driver's knowledge of his city. Just as the taxi driver has to know a variety of ways to get to his passengers' destinations, the teacher has to be flexible and adaptive in his approaches to alternative strategies learners used for solving problems.

Question 16 was an item about equivalence of numbers, and had five sub-questions. The last sub-item [16(e)] was categorised by the developers as being the easiest one of all the items in the questionnaire. The paucity of correct responses to this last sub-question [Q16(e)] might be due to question-fatigue. As this sub-question was the very last item of the questionnaire, participants might not have been concentrating any longer. They had already answered four sub-questions on the same topic, mostly correctly, and perhaps just did not really think about the last item carefully enough.

4.2.7 Critical discussion of items with least numbers of correct responses

To a great extent, teachers' repertoire of strategies for teaching a topic, and alternate mathematical representations and examples that could be implemented, depend on their own conceptual understanding of the topic involved (VVOB, 2014). Of the eight items that were ranked lowest in terms of numbers of correct responses, four were on the topic of ratio and proportions, three on the topic of number concepts, and one on fractions. Table 4.7 gives an analysis of these eight items. Four of the items in Table 4.7 were categorised as difficult by the designers, and had the lowest difficulty ranking of all the items on the questionnaire. Since these items had been categorised as difficult by the developers, the low number of correct responses was not surprising. What was unexpected, however, was the low number of correct responses to Questions 1 and 13, which had been categorised as easy items, as was discussed in Section 4.2.6.

Seven of the eight items in Table 4.7 were on the cognitive MKT domain of specialised mathematical teacher knowledge (SCK), which might suggest the need for these topics to be included in the curriculum for the mathematical literacy method module.

Table 4.7: Analysis of items with least correct responses

Correct response ranking (Least correct =1)	**Question number**	**Mathematical domain and topic of item**	**Difficulty ranking (Most difficult=1)**
1	4	Numbers and operations: Conventional long division	4
2	8	Ratio and proportion: Why cross-multiplication works	1
3	1	Ratio and proportion: Most difficult proportion	20
4	13	Ratio and proportion: Ratio of ingredients	18
5	6	Numbers and operations: Divisibility rules for 4	13
6	7	Numbers and operations: Simplify surds	2
7	15	Ratio and proportion: Ratio boys :girls	3
8	11	Fractions and decimals: Adding mixed numbers	7

4.2.7.1 Questions on ratio and proportion with few correct responses

The two items on ratio and proportion that were categorised as difficult, were Question 8 (difficulty 1.76) and Question 15 (difficulty 1.02). The other two questions on ratio and proportion, Questions 1 and 13, were considered to be easy items with difficulty levels -0.97 and -0.76 respectively. While ratios are a way of comparing one quantity with another, a proportion is a statement that two given ratios are equal. Proportional reasoning is "a complex form of reasoning that depends on many interconnected ideas and strategies that develop over a long period of time" (Siemon et al., 2014, p. 193).

For Question 8, "Why cross-multiplication works", the ratio and proportion scenario that is used is that of a percentage written as a proportion. Participants were asked to identify, from five explanations given, the explanation that gives the best mathematical reason why cross-multiplication works. According to Musser et al. (2011), cross-multiplication is a method which should be presented to learners in a natural way, from an early stage. Teachers should also realise that, for some problems, cross-multiplication might not be a necessary choice of method since proportions can often be solved by algebraically reasoning about the problem. Learner misconceptions and errors might result if learners do not understand when cross-multiplication is the best method to use. Learners often mistakenly see expressions such as 10 km per hour and 10 hours per km to have the same meaning (Musser et al., 2011). Learners experience many problems with algebraic relationships such as "Peter has four times as many chocolates as Susan". Musser et al. (2011) reason that learners will often write 'four times the number of Peter's chocolates is equal to Susan's number of chocolates'. Learners will be able to interpret this statement correctly if they understand the fundamental mathematics of the concept of proportion.

Question 15 is a question on ratio, written as proportion, and was categorised by the developers as a difficult question. Participants were given responses from two learners for a question on the ratio of girls to boys in a class. From four possible statements about the correctness of these given responses, participants were asked to identify the statement that was true about the methods used by the learners in their responses. This question was a good example of testing teachers' specialised knowledge for teaching, since it involved the fundamental mathematical underpinnings of the concept of ratio and proportion.

Questions 1 and 13 were categorised as easy questions. Possible explanations for the low number of correct responses to questions 1 and 13 are discussed in section 4.2.6, which is a discussion of items categorised as easy but not responded to well. Since Question 1 was the first question of the questionnaire, participants might not have understood how the MKT should be interpreted. The low number of correct responses to Question 13 is an indication that the participants might not have the MKT needed to identify alternate strategies of solutions to a problem, as discussed previously. The reader might be interested in looking at Question 23 of the released items, (Appendix A), which is similar to Question 13.

4.2.7.2 Questions on fractions with few correct responses

Question 11 (which appears in Table 4.7), had a low number of correct responses, and was the only question on fractions that did not have sub-items. The items on fractions that had high numbers of correct responses (Questions 3, 10 14 and 16: Table 4.5) were all items that had sub-questions. Question 11 was categorised as a difficult question (difficulty 0.51), involving addition of positive and negative mixed numbers. One learner's method for solving the problem was given in the scenario, as well as her explanation for using this method. Participants had to consider four different statements about the solution given by the learner, and had to identify the one statement about the method that was true. The participants did not understand that fractions are essentially different from whole numbers, since fractions are the ratio of two integers and represent a relationship between two numbers. If learners incorrectly generalise operations with fractions from whole numbers, their response to the question "How would you add the fractions $\frac{2}{3}+\frac{3}{4}$?", could be to respond with $\frac{2}{3}+\frac{3}{4}=\frac{5}{7}$. For this misconception, a teacher could use a number line to show learners why fractions cannot be added by adding numerators and denominators separately (Musser et al., 2011).

According to Smith et al. (1993, p.121), research has shown that "middle school students' numerical knowledge of additive relations has interfered with learning various multiplicative relations such as proportional reasoning". When teaching rational numbers, teachers should promote learners' understanding of the magnitude and relationships involved by implementing a variety of representations and interpretations, rather than simply teaching learners the rules and algorithms (Musser et al. 2011). Since ratio and proportion also involve fractions, it can be concluded from the results shown in Table 4.7 that participants did not have the required MKT concerning the fundamental mathematical underpinnings concerning fractions. "Fractions are relative, not absolute" (Siemon et al., 2013, p. 363). Learners should understand the concept of numerator and denominator. However, Musser et al. (2011) caution that learners should realise that, although fractions consist of two numbers separated by a line, fractions actually represent a single number, since they represent the concept of rational numbers.

4.2.7.3 Questions on numbers and operations with few correct responses

The topic for Question 7 was the simplification of surds, and involves rational and irrational numbers. Question 7 sketches the scenario of a teacher who wants to select a problem leading to a discussion of different strategies for the simplification of surds. Four different surds are given as options to choose from, as well as a fifth option which states that all four surds would work equally well. The developers categorised this as the second most difficult question of the questionnaire. The reason for the low number of correct responses is possibly that the participants have never been taught the method of simplifying surds, and hence could not make a correct decision on the best choice of example.

The same reason is possibly why Question 4 had such a low number of correct responses. This question involved the best explanation of why the conventional algorithm for long division works, as was discussed in Section 4.2.5. Participants had probably been taught the algorithm for long division in elementary school, without any clear explanation of why it works. In the following years at school, they had simply used their calculators for division, never considering the fundamental mathematics involved. Question 4 is the only one of the eight questions from Table 4.7 that falls into the MKT domain of common content knowledge (CCK).

4.3 SELF-EFFICACY

In this section, the results of the participants' responses with respect to their self-efficacy is discussed. The discussion focuses on and analyses the choices about self-efficacy that was made. Pre-service teachers' self-efficacy with regard to the MKT of each item of the questionnaire was studied by using the method of cross-tabulation. The cross-tabulation showed the number of cases falling into each combination of the two variables MKT and self-efficacy for each item of the questionnaire.

4.3.1 Results of self-efficacy choices

When doing the cross-tabulation of the data from the questionnaire, the first cross-tabulation recorded all the incorrect responses as well as the correct responses, as this gives important information regarding participants' misconceptions of MKT. To better explain this, an example from the released items is given and discussed.

Example

Learners in Miss Nay's class have been working on putting decimals in order. Three learners – Andy, Clara and Keisha – presented 1.1; 12; 48; 102; 31.3; .676 as their ordered list. What error are these learners making? (Circle ONE answer.)

a) They are ignoring place value.

b) They are ignoring the decimal point.

c) They have forgotten the numbers between 0 and 1.

Figure 4.9: Example from released items showing three possible responses.

In this example, although (b) is given as the correct response, it would be an added advantage if the researcher were to know that, for instance, many participants chose option (c), and indicated that they were completely sure their answer was correct. This would indicate misconceptions regarding decimals, and suggest the direction supplemental instruction, in the method module, should take. In doing cross-tabulation where all the incorrect responses are shown, as was done in the first cross-tabulation, this could be identified.

As an example of the results of the questionnaire, the first cross-tabulation that was done for Question 1 is given in Table 4.8. In Table 4.8, "percentage within Q1_SE" shows the row percentage: within the specific self-efficacy choices, the percentage correct and incorrect responses. The "percentage within Q1" shows the column percentage within the correct and incorrect categories, and indicates the percentage participants choosing each one of the three self-efficacy options. Note that, for Question 1, choice 3 is the correct choice for the MKT item, and all the other choices were incorrect. If a respondent did not give the correct answer, an "incorrect" choice was indicated.

Table 4.8: Cross-tabulation for Question 1, showing all MKT choices and self-efficacy choice

		Q1				Total
		Option 1	Option 2	Option 3 Correct	Option 4	
Not at all sure correct	Count	3	6	1	4	14
	percentage within Q1_SE	21,4%	42,9%	7,1%	28,6%	100,0%
	percentage within Q1	33,3%	40,0%	5,6%	4,4%	10,5%
Tend to think is correct	Count	4	9	12	53	78
	percentage within Q1_SE	5,1%	11,5%	15,4%	67,9%	100,0%
	percentage within Q1	44,4%	60,0%	66,7%	58,2%	58,6%
Completely sure is correct	Count	2	0	5	34	41
	percentage within Q1_SE	4,9%	11,5%	12,2%	82,9%	100,0%
	percentage within Q1	22,2%	60,0%	27,8%	37,4%	30,8%
Total	Count	9	15	18	91	133
	percentage within Q1_SE	6,8%	11,3%	13,5%	68,4%	100,0%
	percentage within Q1	100,0%	100,0%	100,0%	100,0%	100,0%

From Table 4.8, it can be seen that only 133 of the 137 participants responded to Question 1. As has been mentioned before, the participants were not required to respond to all items of the questionnaire. For easier readability and analysis, the results for Q1, shown in Table 4.8 are summarized in Table 4.9, showing only the number of correct and incorrect responses, together with the self-efficacy choices.

Table 4.9: Cross-tabulation Question 1, showing incorrect and correct responses and self-efficacy choice

	Incorrect	Correct	Total
Not at all sure correct	13	1	14
Tend to think correct	66	12	78
Completely sure correct	36	5	41
Total	115	18	133

4.3.2 Analysis of self-efficacy choices

In order to attempt to analyse the results and to identify the self-efficacy choices made by the participants, the total numbers of correct and incorrect responses for each self-efficacy choice was calculated for all 33 items of the questionnaire, and the average of each cell with respect to the whole sample was found. Figure 4.10 shows a graphic summary of these calculations. It is necessary to keep in mind that, although the responses mentioned in the tables are responses to questions on mathematics, these questions are all contextualised in teaching, which means that they are responses relating to the MKT involved with teaching the specific mathematical topic.

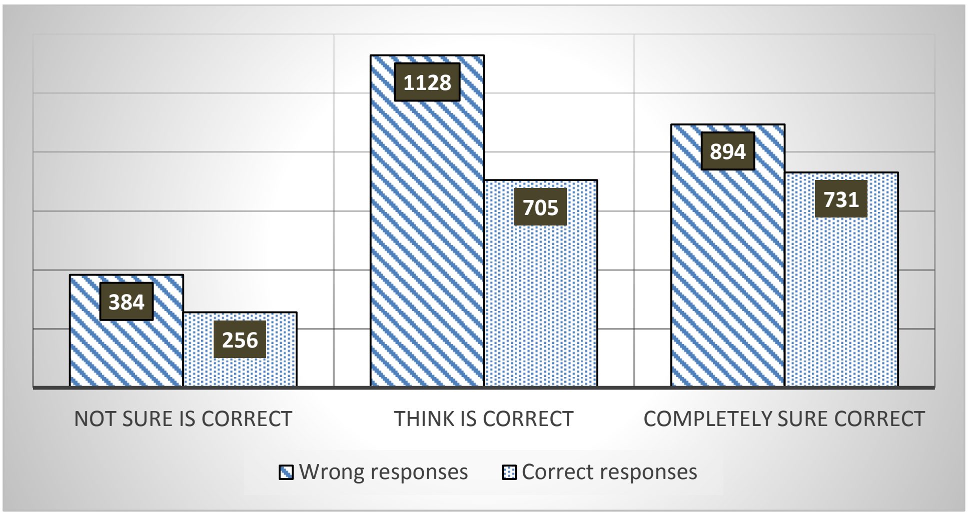


Figure 4.10: Wrong and correct responses to all items arranged into self-efficacy choices

In Table 4.10, a cross-tabulation of all incorrect and correct responses to all items as well as (in brackets) the percentages of each cell as compared to the whole sample, is given. These numbers were taken from the scores for correct responses obtained from the questionnaire.

Table 4.10: Correct and incorrect responses to all items cross-tabulated with self-efficacy responses for all items (Percentages of total)

	Incorrect responses	**Correct responses**	**Total**
Not at all sure answer is correct	384 (9.37%)	256 (6.25%)	640 (15.62%)
Tend to think answer is correct	1128 (27.53%)	705 (17.20%)	1833 (44.73%)
Completely sure answer is correct	894 (21.82%)	731 (17.84%)	1625 (39.65%)
Total	2406 (58.71%)	1692 (41.29%)	4098 (100.00%)

Reading from the column with correct responses in Table 4.10, it can be seen that 17.8% (n = 731) of the correct responses were from participants who were completely sure their answer was correct, while 17.2% (n = 705) of the correct responses were from participants who tended to think their answer was correct. This shows that, in total 1436 (84.9%) of the 1692 correct responses were from participants who were either completely sure their answer was correct or tended to think their answer was correct. However, as can also be seen by reading from the column of incorrect responses in the table, 21.8% (n = 894) of the incorrect responses were from participants who indicated they were completely sure their answer was correct, while 27.5% (n = 1128) of the incorrect responses were from participants who tended to think their answer was correct. This is an indication that a total of 2022 (84%) of the 2406 incorrect response, were from participants who were either completely sure their (incorrect) answer was correct or tended to think their answer was correct.

These results suggest that the participants had high levels of self-efficacy, since in more than 80 percent of cases participants indicated that they were either completely sure their answer was correct or tended to think their answer was correct, whether their responses had been correct or incorrect. However, only about 40 percent of participants' responses were in reality correct, which indicates that many students believed their answer to be correct, even though their interpretation of the MKT involved was incorrect. Hence: they do not know that they do not really have the appropriate mathematical knowledge for teaching the topics of number concepts and operations. They are, however, fairly confident of their ability to teach these topics correctly.

Question 5 is a question worthy of notice because such a high number of participants thought their response was correct, although they had given the incorrect response. The data for Question 5 is presented in Table 4.11. Only 31 correct responses were given, as opposed to 101 incorrect responses, so only 23.5 percent of participants gave the correct response. However, 125 (94.7 %) of all participants were completely sure their answer was correct, or tended to think their answer was correct.

Table 4.11: Data for Question 5: Division by zero

	Incorrect responses	**Correct responses**	**Total**
Not at all sure answer is correct	5	2	7
Tend to think answer is correct	38	15	53
Completely sure answer is correct	58	14	72
Total	101	31	132

This question [Q5] is a common content knowledge item of difficulty level 0, difficulty ranking 10 and correct response ranking 9. The topic of this item was division by zero. Question 1 of the released items (Appendix A) is a similar question. The high number (96) of participants who gave the incorrect response, but were either completely sure their answer was correct (58) or who thought their answer was correct (38), might indicate that the participants thought their interpretation was correct, because they saw this topic (division with zero) as a question on such an ordinary kind of knowledge, that they did not think their response could be incorrect.

Since CCK is knowledge known to the average educated individual, and “knowledge of a kind used in a wide variety of settings” (Ball et al., 2008, p. 399), the results gleaned from this item again emphasise how necessary it is for teachers to know the fundamental underpinning of the material they teach.

4.4 RELATIONSHIP BETWEEN MKT AND SELF-EFFICACY

When the relationship between two variables is studied, the two things that need to be considered is whether the relationship is statistically significant, and whether the effect size (the strength) of this relationship is large, medium or small. It was decided to use the chi squared test to calculate the significance value of the relationship between the participants' MKT and their self-efficacy, and the Cramer's V test to establish effect size, for this study.

4.4.1 Statistical significance of the relationship between MKT and self-efficacy

The Pearson chi-squared test is "a statistical measure used to test hypotheses on *patterns of outcomes* of a random variable in a population. The patterns of outcomes are based on *frequency counts* of categorical random variables" (Wegner, 2007, p. 339). The chi-squared test requires no assumptions about the shape of the distribution from which the sample was drawn. It does assume random sampling and can be applied to nominal and/or ordinal valued variables, but not to continuous variables (Wegner, 2007).

For this study, the chi-squared test is used to test whether the variables MKT and SE are associated by examining their joint patterns of outcomes, as measured by the research instrument used. The research hypothesis proposes a relationship, not due to chance sample fluctuations, between the variables MKT and self-efficacy. The null hypothesis for this study states that no relationship exists between these two variables, which means that they are unrelated, or statistically independent. The alternative hypothesis implies that a relationship exists between MKT and SE. In this study the default value that was used to test for statistical significance was set as less than 0.05, corresponding to a confidence level of 95 percent.

A test statistic (value of χ^2), the degrees of freedom and a significance level, or p-value ar given by the Pearson chi-squared test. For this study, the degrees of freedom for all items was two. Degree of freedom is calculated by finding the result of the product (number of rows minus 1) multiplied by (number of columns minus 1). The chi-squared test is based on the size of the differences between the expected frequencies and the observed frequencies, which are the frequencies actually in the cells. The formula for the test statistic, or the χ^2 value is $\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e}$ where f_o and f_e indicates the observed and the expected frequency of the counts respectively. This means that, for each cell, the difference between observed and expected frequency is squared and then divided by the expected frequency and these values for all the cells are summed. Note that expected frequency is the cell frequencies that would be expected if the two variables MKT and SE were statistically independent (Wegner, 2007). The formula that was used to calculate expected frequency is given by doing the computation: $\frac{\text{Row total} \times \text{Column total}}{\text{Total responses}}$. It is important to note that row total numbers will differ slightly from the numbers from the individual frequency analysis. The reason for this is that participants did not always respond to both variables, and are hence eliminated by the missing value statement. The number of missing cases is slightly higher in the crosstabs than it would be for the frequency analysis (Fletcher, 2013).

The p-value that is given by the chi-squared test is found by using a table or a computer programme. If the p-value lies below 5 percent, ($p < 0.05$), then there is strong sample evidence to infer that the alternative hypothesis is true. For the relationship to be statistically significant, this p-value has to be as small as possible. It can then be inferred with strong confidence that the alternative hypothesis is true, which indicates that a relationship, not due to chance, exists between MKT and self-efficacy. Any p-value above 5 percent implies that the null hypothesis cannot be rejected, implying the non-existence of a relationship.

Muijs (2011) comments that, in order to make sense of a cross-tabulation table, the percentages of the row (self-efficacy choices) or the column (incorrect or correct answer) need to be known. Remarkable differences in column-percentages across any row in a cross-tabulation indicate that a relationship exists. These percentages, for Question 1 of the questionnaire, are shown in Table 4.8.

The chi-squared analysis that was done on the research data was used to find whether any significant statistical relationship existed between the students' MKT and their self-efficacy, as measured by the questionnaire used in the study. It was found that nine of the 33 items had a value of p smaller than 0.05. As explained above, these results implied that, for these nine items, the alternative hypothesis - the hypothesis that a relationship between MKT and self-efficacy exists, could be accepted with a 95 percent degree of confidence. Thus, on nine of the 33 items of the questionnaire, the participants' choices showed a statistically significant relationship between their MKT and their self-efficacy. These nine items are identified in Table 4.13.

However, the chi-squared test merely indicates whether the relationship between MKT and self-efficacy is statistically significant. It informs that "there is a low probability that the differences we have found are due to chance sample fluctuations" (Muijs, 2011, p. 108). This analysis does not inform us about the strength of this relationship. According to Arthur et al. (2012), if the degree of the relationship between two variables is being investigated, the effect size must also be calculated.

4.4.2 Practical significance of the relationship between MKT and self-efficacy

Ferguson (2009) argues that, when chi-squared analysis has been done, the Cramer's V statistic is the best effect size statistic that should be used to represent the strength of association between the variables. The Cramer's V statistic is an effect size statistic that indicates the practical significance of the relationship. The Cramer's V analysis that was used in this study showed 29 items with small practical significance and four items with medium practical significance, while no item showed a large practical significance.

The interpretation of the Cramer's V statistic differs slightly for cases where there are only two possible answers, and cases with more than two answers. The values have the following interpretations (Cohen, 1988):

Table 4.12: Interpretation of Cramer's V statistic

	Two answers (columns)	**More than two answers**
Small practical significance	0-0.29	0-0.20
Medium practical significance	0.30-0.49	0.21-0.34
Large practical significance	0.5+	0.5+

As mentioned, Table 4.13 gives the results for the chi-squared test and the Cramer's V analysis of the nine items that show a statistically significant relationship. As can be seen from Table 4.13, four of the nine items that show statistical significance on the chi-squared test are both statistically significant and have a medium practical significance, while all the other items are of small practical significance.

Table 4.13: Results of Pearson chi squared test and Cramer's V statistic

Question	**N**	χ^2 **-value**	**p-value**	**Cramer's V**	**Practical significance**
Q2(b)Properties of rational and irrational numbers	128	16.54	0.000	0.359	Medium
Q2(d) Properties of rational and irrational numbers	129	12.210	0.002	0.308	Medium
Q3(d) Division of fractions	129	7.333	0.026	0.238	Small
Q9 Table method for profit calculation	134	14.842	0.001	0.333	Medium
Q10(a)Area model of decimal multiplication	106	8.642	0.013	0.286	Small
14(b) Word sums for number divided by fraction	126	10.651	0.005	0.291	Small
Q14(d) Word sums for number divided by fraction	126	7.850	0.020	0.250	Small
Q16(b) Equivalent forms of numbers	124	19.491	0.000	0.396	Medium
Q16(d) Equivalent forms of numbers	125	6.141	0.046	0.222	Small

4.4.3 Discussion of results for the four items that were of statistical significance and had medium practical significance

The results for the four items that were statistically significant and showed medium practical significance, are now discussed. Differences in ranking of difficulty and numbers of correct responses is shown, as well as a comparison of the column-percentages between incorrect and correct responses for the four items. Thereafter, the actual results of the four items as well as their statistical test values are given, followed by a comparison of actual (observed) frequency and expected frequency of self-efficacy responses for incorrect and correct responses to each of the four items.

4.4.3.1 Comparison of the ranking of difficulty and correct responses

All four of the statistically significant items that showed medium practical significance, were categorised as easy to very easy items. In Figure 4.11, a comparison of the ranking of the difficulty and the ranking of the number of correct responses to the four items is shown. All four items show a high degree of correlation between the rankings of difficulty and numbers of correct answers.

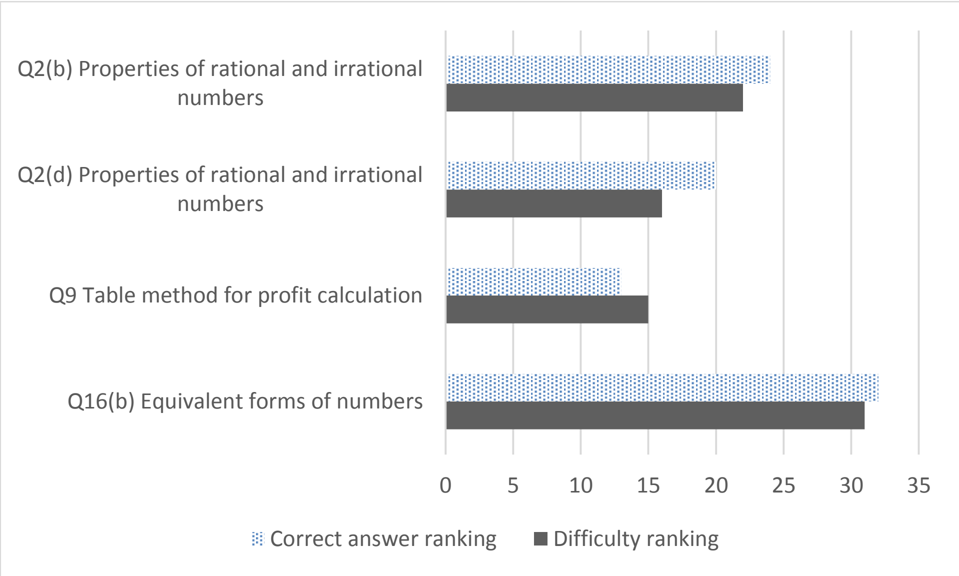


Figure 4.11: Comparison of difficulty ranking and ranking of number of correct responses for statistically significant items that showed medium practical significance

4.4.3.2 Comparison of the column percentages for the incorrect and correct responses, compared to a combination of the self-efficacy choices

The column percentage total is an important statistic for this study. Cross-tabulation of results is interpreted by comparing column-percentages across rows. A notable difference in column-percentage across any row indicates the existence of a relationship (Wegner, 2007).

A graphic representation of an analysis of the column percentages for the incorrect and correct responses, compared to the self-efficacy choices "Not sure answer is correct" and the response "Tend to think answer is correct" combined with "Completely sure answer is correct", is shown in Figure 4.12.

As can be seen from these bar charts, in all the cases a notable difference in column-percentage across the rows occurs, which indicates that a relationship exists for these four items. From this table, a disturbing picture regarding participants' self-efficacy in comparison with their actual MKT emerges. For all four items, the self-efficacy choice for participants giving the incorrect response indicates that they are sure their response is correct, or think their response is correct.

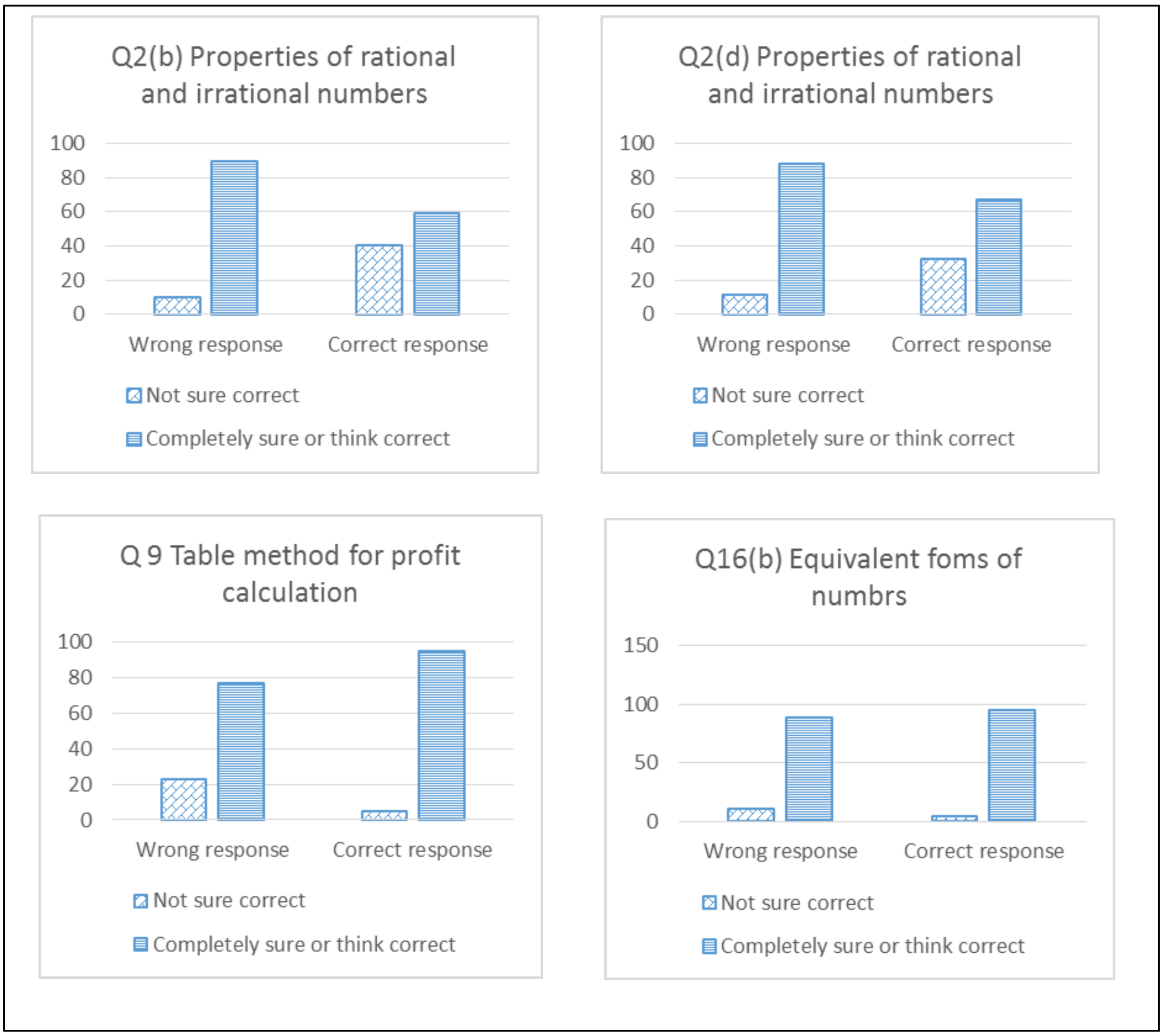


Figure 4.12: Comparison of column percentages for incorrect and correct responses for self-efficacy choices “Not sure answer is correct” and “Completely sure answer is correct or tend to think answer is correct.”

4.4.3.3 Discussion of statistical analysis and frequencies

The four items that are of medium practical significance is now discussed separately. The actual results and their statistical test values are given first, followed by a comparison of actual (observed) frequency and expected frequency of self-efficacy responses for incorrect and correct responses to each of the four items. Expected frequency was calculated as was described in Section 4.4.1.

4.4.3.4 Discussion of Questions 2(b) and 2(d)

The first two items to be discussed are sub-items of an item that is classified under the mathematical topic of number concepts. This item sketched a scenario where participants had to make true or false decisions about the correctness of some properties of rational and irrational numbers that were given. The difficulty levels were -1.177 for Q2(b) and -0,549 for Question 2(d) respectively (Appendix E), which indicates that both were classified as easier items.

For Question 2(b), the results of the questionnaire are given in Table 4.14(a) with Table 4.14(b) giving the results of the statistical analysis. Of the 137 participants, 128 responded to this item. From Table 4.14(a) it can be seen that for Question 2(b), the number of correct responses were a little more than the number of incorrect responses (n = 67 to n = 61). The self-efficacy choice of being ‘not at all sure’ that the answer was correct, was given by 40.3 percent (n = 27) of the correct responses (n = 67), while 14.9 percent (n = 10) were completely sure that their answer was indeed correct. The difficulty ranking of 22 was just lower than the ranking of 24 for number of correct responses (Appendix F). From Figure 4.12, it can be seen that 90.2 percent of the participants who had incorrect responses, indicated that they were relatively sure their response was correct. The big difference in column-percentages of the choices ‘not at all sure correct’ as well as ‘tend to think is correct’ is also important here, since it indicates that a relationship exists (Table 4.14(a)).

The p-value of zero for the Cramer’s V analysis (Table 4.14(b)) indicates that there is a strong sample evidence to infer that the alternative hypothesis is true, which indicates the existence of a relationship between participants’ MKT and their self-efficacy for this item.

Table 4.15 shows, first the actual frequency of the responses, and second, the expected frequency for Question 2(b). The expected frequency was computed as was described in Section 4.4.1 (thus the decimal numbers). For this item, only the ‘completely sure answer is correct’ self-efficacy choice shows a direct correlation between actual and expected frequency. For incorrect responses, the actual frequency was 9 and expected frequency was 9.1. For correct responses the actual frequency was 10 and expected frequency 9.9.

Table 4.14 (a): Results of MKT and self-efficacy responses for Q2(b)

Question	MKT answer		
Q2(b) Properties of rational and irrational numbers	**Incorrect**	**Correct**	**Total**
Not at all sure answer is correct Count Column percentage (incorrect or correct)	6 9.8%	27 40.3%	33
Tend to think answer is correct but not sure Count Column percentage (incorrect or correct)	46 75.4%	30 44.8%	76
Completely sure answer is correct Count Column percentage (incorrect or correct)	9 14.8%	10 14.9%	19
Total Count Column percentage (incorrect or correct)	61 100.0%	67 100.0%	128

Table 4.14 (b): Chi-squared and Cramer's V analysis for Q2(b)

χ^2	***p***	**Cramer's V**	**N**
16.540	0.00	0.359	128

Table 4.15: Actual frequency compared to expected frequency of response for Q2(b)

Question	**MKT answer**		
Q2(b) Properties of rational and irrational numbers	**Incorrect**	**Correct**	**Total**
Not at all sure answer is correct	6 15.7	27 17.3	33
Tend to think answer is correct but not sure	46 36.2	30 39.8	76
Completely sure answer is correct	9 9.1	10 9.9	19
Total	61	67	128

For Question 2(d), Table 4.16(a) gives the results from the questionnaire, and Table 4.16(b) gives the results of the statistical analysis. The number of participants for this question was 129. From Table 4.16(a) it can be seen that this item had fewer correct responses than incorrect responses. Only 13.8 percent (*n* = 8) of the 58 participants with correct responses were completely sure that their answer was correct. Of the 71 participants who had incorrect responses, 88.7 percent (*n* = 63) indicated that they were either completely sure their response was correct, or tended to think their response was correct. The difficulty ranking of 19 was again just lower than the ranking of 21 for number of correct responses, as shown in Appendix F. For this item, big differences in column-percentages of the choices 'not at all sure correct' as well as 'completely sure answer is correct' can be seen.

Table 4.16(a): Results of MKT and self-efficacy responses for Q2(d)

Question	**MKT answer**		
Q2(d) Properties of rational and irrational numbers	**Incorrect**	**Correct**	**Total**
Not at all sure answer is correct — Count Column percentage (incorrect or correct)	8 11.3%	19 32.8%	27
Tend to think answer is correct but not sure — Count Column percentage (incorrect or correct)	39 54.9%	31 53.4%	70
Completely sure answer is correct — Count Column percentage (incorrect or correct)	24 38.3%	8 13.8%	32
Total — Count Column percentage (incorrect or correct)	71 100.0%	58 100.0%	129

Table 4.16 (b): Chi-squared and Cramer's V analysis for Q2(d)

χ^2	***p***	**Cramer's V**	**N**
12.210	0.002	0.308	129

The p-value of 0.002, which lies below 1 percent, indicates a strong sample evidence that the alternative hypothesis is true, indicating the existence of a relationship, not due to chance, between participants' MKT and their self-efficacy for this item.

Table 4.17 shows the actual frequency of the responses to Q2(d) compared with the expected frequency. The strongest correlation between actual and expected frequency for this item was for the choice ‘Tended to think answer correct’.

Table 4.17: Actual frequency compared to expected frequency of response for Q2(d)

Question	**MKT answer**		
Q2(d) Properties of rational and irrational numbers	**Incorrect**	**Correct**	**Total**
Not at all sure answer is correct	8 14.9	19 12.1	33
Tend to think answer is correct but not sure	39 385	31 31.5	76
Completely sure answer is correct	24 17.6	8 14.4	19
Total	61	67	128

This item (Question 2) required CCK for all sub-items. Question 2(b) reflected on the difference in the decimal patterns of rational and irrational numbers, while Question 2(d) was a question on multiplying an irrational number by an integer. The other two sub-items for Question 2 had similar patterns of differences in ranking of difficulty and numbers of correct responses. However, there were no big differences in column percentages for any of the self-efficacy choices of the other two sub-items, and hence neither of them showed any statistical significance and small practical significance, according to the statistical measures employed.

4.4.3.5 Discussion of Question 9

Question 9 was a question on ratio and proportion, where a table with values was given, and questions were asked about this table. This item had the highest number of responses of all items, since 134 of the 137 participants responded to this item. This type of question is quite frequent in Mathematical Literacy examinations, and as such the participants deemed themselves capable of responding to this item. The number of incorrect responses ($n = 92$) was far more than the number of correct responses ($n = 42$). The difficulty ranking of 15 was just higher than the ranking of 13 for number of correct responses (Appendix F). Although only 21.4 percent ($n = 9$) of participants with correct responses were completely sure their answer was correct, 73.8 percent (n = 31) of participants indicated that they tended to think their response was correct. Of the participants who had incorrect responses, a total of 77.2 percent (n = 71) indicated that they were either completely sure their response was correct, or tended to think that their response was correct. The noteworthy indicator for this item is the big difference in column-percentages of the choice 'tend to think answer is correct'. For Question 9, Table 4.18(a) gives the results of the questionnaire, and Table 4.18(b) gives the results of the statistical analysis.

Table 4.18(a): Results of MKT and self-efficacy responses for Q9

Question		**MKT answer**		
Q9 Table method for profit calculation		**Incorrect**	**Correct**	**Total**
Not at all sure answer is correct	Count	21	2	23
	Column percentage (incorrect or correct)	22.8%	4.8%	
Tend to think answer is correct but not sure	Count	36	31	67
	Column percentage (incorrect or correct)	39.1%	73.8%	
Completely sure answer is correct	Count	35	9	44
	Column percentage (incorrect or correct)	38.0%	21.4%	
Total	Count	92	42	134
	Column percentage (incorrect or correct)	100.0%	100.0%	

Table 4.18(b): Chi-squared and Cramer's V analysis for Q9

χ^2	***p***	**Cramer's V**	**N**
14.842	0.001	0.333	134

The p-value of 0.001, again less than one percent, indicates that, for this item, a relationship exists between participants' MKT and their self-efficacy.

Table 4.19 shows the actual frequency of the responses compared with the expected frequency for Question 9. This item showed no perceptible correlation between actual and expected frequency for any of the cells

Table 4.19: Actual frequency compared to expected frequency of response for Q9

Question	**MKT answer**		
Q9 Table method for profit calculation	**Incorrect**	**Correct**	**Total**
Not at all sure answer is correct	21 15.8	2 7.2	33
Tend to think answer is correct but not sure	36 46.0	31 21.0	76
Completely sure answer is correct	35 30.2	19 13.8	19
Total	61	67	128

Question 9 required specialised content knowledge from participants. Participants were given a situation of trying to work out the profit on 90 sold tickets, given the profit made on the sale of 15 tickets. A hypothetical solution from a learner, with a table containing values for profit against number of items sold, was given, and questions were asked about the method employed by the learner when completing this table. The questions were about whether the method used by the learner was correct or incorrect, and four different interpretations were given. Although this item was not categorised as being difficult, responses indicated that the learners were not able to interpret the situation correctly, since only 31.3 percent (n = 42*)* of all the responses were correct.

4.4.3.6 Discussion of Question 16(b)

The mathematical topic for Question 16 is fractions and decimals. Five lists of three numbers written as fractions, decimals and percentages were given, and participants had to decide, for each list, whether or not the expressions were equivalent forms of the same number. These were the five easiest items of the questionnaire, with the highest numbers of correct responses.

Question 16(b) was answered by 124 of the 137 participants. A reason for the low response, despite the easiness of the item, might be because it was the last item of the questionnaire, and respondents were losing interest. The number of correct responses (89) was far higher than the number of incorrect responses (35) for this item, as seen from Table 4.20(a). The self-efficacy choice of being completely sure that the answer was correct, was given by 64.5 percent (*n* = 80) of all participants, although 34.3percent (*n* = 12) of participants responded incorrectly to this item. The difficulty ranking of 31 was about the same as the ranking of 32 for number of correct responses (Appendix F). Only 4.5 percent (*n* = 4) of the participants who had the correct response, were not sure if their answer was correct. The big difference in column-percentages for the choices 'tend to think is correct' as well as 'completely sure answer is correct' is indicative of the existence of the relationship, not due to chance, between MKT and self-efficacy for this item. Table 4.20(a) gives the results of the questionnaire, and Table 4.20(b) gives the results of the statistical analysis for Question 16(b).

Table 4.20(a): Results of MKT and self-efficacy responses for Q16(b)

Question	**MKT answer**		
Q16(b) Equivalent forms of numbers	**Incorrect**	**Correct**	**Total**
Not at all sure answer is correct Count Column percentage (incorrect or correct)	4 11.4%	4 4.5%	8
Tend to think answer is correct but not sure Count Column percentage (incorrect or correct)	19 54.3%	17 19.1%	36
Completely sure answer is correct Count Column percentage (incorrect or correct)	12 34.3%	68 76.4%	80
Total Count Column percentage (incorrect or correct)	35 100.0%	89 100.0%	124

Table 4.20 (b): Chi-squared and Cramer's V analysis for Q16(b)

χ^2	***p***	**Cramer's V**	**N**
9.491	0.00	0.396	124

The p-value of zero indicates strong evidence for the conclusion that the alternative hypothesis is true, which indicates the existence of a relationship between participants' MKT and their self-efficacy for this item.

Table 4.21 shows the actual frequency of the responses compared with the expected frequency for Question 16(b). This item showed some correlation between actual and expected frequency for the self-efficacy choice 'not at all sure answer is correct'.

Table 4.21: Actual frequency compared to expected frequency of response for Q16(b)

Question	**MKT answer**		
Q16(b) Equivalent forms of numbers	**Incorrect**	**Correct**	**Total**
Not at all sure answer is correct	4 2.3	4 5.7	33
Tend to think answer is correct but not sure	19 10.2	17 25.8	76
Completely sure answer is correct	12 22.6	68 57.4	19
Total	61	67	128

For Question 16 only common content knowledge was required. Of the five sub-items of Question 16, Question 16(d) also showed statistical significance, with a p-value of 0.046. However, the Cramer's V statistic of 0.121 indicates just a small practical significance.

In the discussion of the four items of the questionnaire that showed medium practical significance, it was shown that, for each of the four items, participants had high levels of self-efficacy, since they were completely sure their answer was correct, or tended to think their answer was correct. The responses for the self-efficacy choices of 'tend to think answer is correct' and 'completely sure answer is correct' was visibly higher than the responses for 'not at all sure answer is correct' for all four items. However, as was indicated, the participants who gave the incorrect answers were also either completely sure, or tended to think their (incorrect) answer was correct.

4.5 SUMMARY

In this chapter, the results of the study were analysed and discussed. First, the results on the MKT items of the questionnaire were investigated. The numbers of items with correct responses was given for all four-year groups as well as for the whole sample of 137 participants. Results showed that, from the total of 33 possible correct responses, no participant had less than three or more than 23 correct responses. These results were analysed statistically, and the average for correct responses were shown to be 39.02 percent, with a standard deviation of 9.97 percent. Item difficulty was discussed, and the correlation between item difficulty and numbers of correct responses were presented graphically, showing a strong negative relationship. The items with the least number of correct responses, were items on the topic of ratio and proportion. Items were further divided into mathematical topics as well as into the MKT domains of SCK and CCK. Some items which showed noticeable discrepancy between item difficulty and numbers of correct responses were discussed

Second, the results of the self-efficacy questions added to each item of the MKT questionnaire was given. The cross-tabulation of MKT answers and self-efficacy responses were given and discussed. Results showed 15.62 percent respondents indicating that they were not at all sure that their answers were correct. Of all participants, 84.89 percent indicated that they were either completely sure their answer was correct, or tended to think their answer was correct. However, when these results were combined with the correctness or incorrectness of responses, it was shown that 84 percent of participants with incorrect responses were either completely sure their answer was correct, or tended to think their answer was correct, indicating a lack of mathematical knowledge as well as of MKT for teaching the topics involved.

Third, the relationship between the MKT and the self-efficacy of respondents was investigated. The statistical and practical significance of the relationship was noted. Nine of the 33 items of the questionnaire showed a statistically significant relationship between participants' MKT and self-efficacy. Of these nine, only four items also were seen to be of medium practical significance, with no items showing high practical significance. Results from items which had the most noteworthy relationship were discussed in terms of the difficulty ranking against numbers of correct responses and comparison of the column percentages for responses to the self-efficacy choices. This was followed by separate discussions of a summary of the results of MKT and self-efficacy responses, the statistical analysis and the actual against the expected frequency of each of the four items.

In the next chapter, the conclusions drawn from these results are discussed. Recommendations are made for implementation of the results and some suggestions are made for future research.

Chapter 5

Summary, Recommendations and Conclusion

5.1 INTRODUCTION

In the previous chapter the results obtained from analysis of the data from the participant questionnaire data were discussed. This chapter concludes the dissertation by offering a summary of these results framed in terms of the research questions posed. The data and results related to the participating pre-service teachers' mathematical knowledge for teaching (MKT) are discussed, as well as their self-efficacy with regard to this MKT. This is followed by an exploration of possible relationships between the MKT and self-efficacy of the students who participated in this study. Possible recommendations for implementation of the findings of study is made, followed by suggestions for future research.

5.2 SUMMARY OF FINDINGS

The findings of the study are discussed first with regard to the MKT as measured by the questionnaire designed by the Michigan team (LMT, 2012), and second with regard to the responses on the self-efficacy questions added to each item of the questionnaire. The relationship between MKT and self-efficacy is then investigated.

5.2.1 Results with respect to MKT

Mathematical knowledge for teaching (MKT) is an important construct in mathematics teacher education. Ball et al. (2001) argue that mathematics teachers should have deep conceptual understanding of mathematics, as well as the ability to predict problems and difficulties learners might experience with a given topic. Teachers need to have the specialised knowledge needed to understand what learners are trying to do when they use unusual or alternative methods. Teachers need perception of and insight into learners' mathematical problems (Hill et al., 2008).

Ball and Bass, (2000, p. 89) describe the questionnaire used in this study as "a mathematical analysis of core activities of mathematics teaching". Scrutiny of the data from the MKT questionnaire indicated an average of 39 percent of correct responses for the whole group. Discussion of the analysed data (Chapter 4) draws attention to the participants' lack of understanding of fundamental mathematics. The reason for the high number of incorrect responses might also be attributed to misconceptions these pre-service teachers might have. This disappointing result would indicate the need for improvement of the MKT of the pre-service teachers specialising in mathematical literacy.

The MKT questionnaire's results indicated that participants seemed to have a degree of mathematical content knowledge on the topic of number concepts and operations. However, it appeared that they had difficulty in the evaluation and implementation of alternative examples or unusual methods of instruction used when teaching this topic. The results suggest that participating students did not have the required specialised content knowledge (SCK) needed for teaching mathematics, as indicated by the overall low success rate in correctly responding to the MKT questionnaire items. This is confirmed by the average of 39 percent correct responses given. Although a few of the more difficult items were responded to correctly by participants, many participants did not respond correctly to some of the easier items (Section 4.2.6). Students seem to be able to successfully answer most items that focus directly on fractions and decimals, since in almost all of these items their correct answer ranking was above the difficulty rankings of the items (Table 4.5). The items on ratio and proportion did not have as many correct responses, which indicated a lack of fundamental understanding of the mathematics that underpin the topic of fractions (Section 4.2.7).

One of the most common incorrect answers on the items of the questionnaire chosen by the participants, was the 'all of the above' response, which appeared as a choice in many of the items. According to Hill et al. (2008), this non-specific answer is often given by teachers who do not have the specialised knowledge needed to accept different explanations for learners' errors, or the insight to identify alternative ways to find solutions to problems.

The MKT results (Table 4.3) indicated only small differences between the MKT of the participants from the first-, second-, third- and fourth-years of study. The highest number of correct responses (mean value 40.78%) was from the third-year students, who are pre-service teachers doing a course in methodology. The slightly better performance of the third-year students might indicate a positive effect of the modules in methodology of mathematical literacy they take in their third year. The results of the fourth-year students, who are in the school-based-learning phase of their studies, where they spend four days of every week at schools, are lower than that of the third years (37.70%), and similar to the MKT of the first-year students (37.20%).

5.2.2 Results with respect to self-efficacy

Self-efficacy is defined by Pajares and Miller (1995) as a valuation of competence, specific to carrying out a particular task. According to Zimmerman et al. (2010), self-efficacy beliefs influence job-satisfaction, and teachers' self-efficacy beliefs are associated with their conduct, enthusiasm, planning and creativeness in teaching, as well as their commitment to teaching. Teachers' self-efficacy beliefs have a positive impact on teacher performance and enthusiasm, as well as on learner achievement (Silverman & Davis, 2009).

The results that were obtained from the self-efficacy items that complimented each of the items in the MKT questionnaire were summarised in Table 4.10. These results exhibited participants' high levels of self-efficacy regarding their own mathematical knowledge for teaching for the items included in the questionnaire. More than 80 percent of the participants indicated that they were either 'completely sure' their answer was correct (39.65%) or 'thought' their answer was correct (44.73%), although only about 40 percent of responses to the MKT items were indeed correct. This is a strong indication that, although these pre-service teachers lack the mathematical knowledge needed for teaching these topics, they do not lack confidence in their own ability to teach the topics correctly.

Michaelides (2008) reports that research on mathematics self-efficacy has shown that the confidence students have in their ability to solve problems was constantly seen to be an overestimation of their actual capabilities. Underestimation of capability inhibits the desire to be more progressive in the use of teaching methods and in experimentation, and might deter individuals from engaging in more challenging methods of instruction. According to Bandura (1994), individuals are more likely to succeed in their undertakings if their self-efficacy beliefs are higher than their actual achievement. Higher self-efficacy beliefs are more advantageous to success, since the teacher with higher levels of self-efficacy is more willing to take on challenging tasks that require higher levels of cognitive input. The high levels of self-efficacy shown by participants in this study indicate these participants' ability and even readiness to undertake the difficult task of teaching, although they still lack the content knowledge required to teach mathematics effectively.

Studies published by seminal scholars confirmed that "efficacy is highest among pre-service teachers and that this level of efficacy drops, often to a great extent, during the first year of teaching" (Woodcock, 2011, p. 25). Woodcock argues that the experiences students encounter during their year of induction into teaching have a negative influence on their initial efficacy as a teacher. The results from this study's data support Woodcock's (2011) findings, since the participants were all pre-service teachers, and exhibited high levels of self-efficacy.

5.2.3 Results with respect to relationship between MKT and self-efficacy

From analysis of the data obtained from the questionnaire, regarding MKT and self-efficacy, little strong evidence could be found for the existence of a relationship between the two constructs. In only nine of the 33 items of the questionnaire, the relationship between MKT and self-efficacy was significant ($p < 0.05$). Of these nine items, however, only four items showed medium practical significance, with a Cramer's V value of more than 0.3, while the other five showed only a small practical significance. No large practical significance was shown for any of the items of the questionnaire. This suggests that the self-efficacy of the participants is mostly independent of their MKT. As mentioned, a high level of self-efficacy was evident, but this was not justified by the low numbers of correct responses on the MKT items.

5.2.4 Limitations revisited

The instrument used in this study is effective for comparing the mathematical knowledge for teaching of different groups of students, but has not been validated to measure this knowledge effectively (LMT, 2012). Students' MKT may only reliably be measured if additional mathematics domains, such as Functions or Geometry, and additional sub-domains of PCK, such as knowledge of content and students (KCS) are also brought into play (Rowan et al., 2001). Although this was not a limitation to the present study, it would be a limitation if the intent was to measure MKT. Furthermore, Woolfolk Hoy (2000, p. 9) argues that "In order to be useful and generalisable, measures of teacher efficacy need to tap teachers' assessments of their competence across the wide range of activities and tasks they are asked to perform". These could include mathematical topic-specific questions over a wide range of mathematical topics (Zimmerman et al., 2010), and would be a more accurate indicator of overall mathematical content knowledge. It could also include classroom-related performance questions, such as "I feel confident of my ability to discipline learners", or "I feel confident that I can set a balanced assessment task".

The use of multiple-choice items limited the effectiveness of this study. Although Hill et al. (2005, p. 373) report that multiple choice items "…can both reliably discriminate among teachers and meet basic validity requirements…", my experience was that the participants did not always interpret the questions correctly, and that their MKT might actually be higher than was indicated by their responses to the multiple-choice items. Some items had as many as five sub-items, with the result that, when learners reached the last item of the questionnaire [Q16], which also had five sub-items, question fatigue might have set in. This could be seen since the last sub-question [Q16(e)] was categorised as the easiest on the questionnaire, but was not responded to as well as could be expected according to its categorisation. Many participants did not even complete Question 16 at all. Nevertheless, the findings on MKT has value as it assisted in identifying areas of mathematical knowledge that need to be revisited in the method curriculum.

A mathematics teacher needs specialised fluency with mathematical language, but this fluency with mathematical language might be impacted upon by a lack of fluency in the language of teaching and learning. In the questionnaire, the fact that many of the participants were not English home language speakers might have influenced their responses, and hence been a limitation. A person who is not fluent in English might find it difficult to understand the difference between "3 divided into 15" and "3 divided by 15. A question such as "How much is ½ of ¾?" or "How much is ¾ twelfths?" might confuse many participants for whom English is not their home language. These are technical issues of language prevalent in most South African schools, with its diversity of mother-tongue speakers. The discrepancy between item difficulty and numbers of correct responses observed in Question 3 illustrates this point. Question 3(b) had a difficulty ranking of 27 and was thus categorised as easy, but the correct response ranking was only 15, which showed that there were not as many correct responses as could reasonably have been expected (Section 4.2.5).

The low number of participants with respect to population size limited the scope of the study, and the possibility to generalise. A more representative sample would have improved the usability of the results. Time-table constraints made it necessary to administer the questionnaire at the end of the semester. The unfortunate effect of this was that some students were not motivated to participate in the study. Because participation was voluntary, the possibility exists that the participants were mostly individuals with more content knowledge than those who did not volunteer to participate. It also includes the possibility that students with high levels of self-efficacy with respect to their MKT might have been more prevalent amongst the sample. The results of this study can also not be generalised to practising teachers, because all the participants were student teachers at NMMU who have not experienced the responsibility inherent in daily classroom teaching. However, the findings could still be valuable to assist implementation of other similar studies, and in the design of the mathematical literacy method module.

5.3 RECOMMENDATIONS

Although the results of this study suggest participants' lack of correct MKT, the results reflect high levels of self-efficacy among the participants. According to Schulze and Schulze (2006), highly efficacious students are more likely to succeed at their tasks. Schulze and Schulze believe that if student-teachers have high levels of self-efficacy, they might want to improve their own MKT in order to give their future learners the best guidance and instruction they are capable of. Research on self-efficacy supports the importance of having high self-efficacy "when faced with new and challenging skills" (Schulze & Schulze, 2006, p. 107). As a lecturer of mathematical literacy pre-service teachers, I perceive this as a positive that could be focused on to increase efficacy. If students are confident of their ability to teach, it remains the responsibility of the method lecturer to ensure that these students' confidence are warranted by correct and adequate MKT.

5.3.1 Implications for existing theory

Nicolaidou and Philippou (2003, p. 4) report that research into the relationship between efficacy and achievement in mathematics found that "…self-efficacy beliefs appear to be a more important factor influencing attitudes, achievement, and educational and career choices, than other variables such as anxiety, mathematics experiences, perceptions of mathematics and self-regulation beliefs…". As was mentioned in Chapter 2 (Section 2.3), confident perceptions of self produces the required outcomes of efficiency. Michaelides (2008) argues that a strong correlation exists between self-efficacy and academic outcomes such as skill in problem-solving. According to Bong and Skaalvik (2003, p. 32) "…strong self-efficacy and positive self-concept lead students to set challenging yet attainable academic goals for themselves, feel less anxious in achievement settings, enjoy their academic work more, persist longer on difficult tasks and, overall, feel better about themselves as a person and as a student". Bong and Skaalvik (2003) argue that lecturers involved in teacher training need to employ instructional procedures that are known to enhance students' perceptions of self, to facilitate the forming of accurate yet optimistic self-efficacy beliefs in pre-service teachers.

In Bandura's view, a teacher's self-efficacy beliefs may help him or her to undertake educational activities in class that promote learning. The beliefs teachers have in their own abilities for effective teaching strongly influence their efficacy (Bandura 1994). However, Bandura (1994, p. 8) reasons that levels of self-efficacy beliefs should not be "unrealistically exaggerated". The problem arises when these beliefs are incorrect - when teachers do not doubt their ability, but simply do not have MKT to validate this ability – when they do not know that they do not know!

The results from this study indicate a paucity of fundamental mathematical knowledge in the participants. In the opinion of Ball et al. (2001), during their years at school many learners' interest in mathematics is never stimulated and Ball et al. argue that teachers do not succeed in exposing learners to the power, beauty and elegance of mathematics. The result is that learners consider mathematics to be a set of rules to which skills and procedures must be applied. Mathematical knowledge for teaching requires special skills from teachers, who should be able to use the language of mathematics properly when explaining concepts and definitions. Lecturers of pre-service mathematical literacy teachers should impart these elements of mathematical knowledge to students in the method lectures. Ball et al (2013, p. 12) comment that "explicit knowledge and skill in these areas is vital for teaching".

5.3.2 Recommendations for implementation

From the discussions on the results of the MKT questionnaire, given in Chapter 4, it follows that there exists a need for improvement of the pre-service teachers' MKT. The overall average of 39 percent of correct responses indicates a level of mastery of MKT that most mathematics educators would consider to be insufficient for effective teaching of these topics. The ways in which these topics need to be presented to school learners should be included in the method modules of the students specialising in mathematical literacy teaching. It is, however, not enough to only consider the MKT of the pre-service teachers. Teachers should also have confidence in their own ability to teach with good effect. The self-efficacy of pre-service teachers should therefore also receive attention in the method modules

5.3.2.1 Restructuring and implementation of mathematical literacy method module

In Section 2.5.1, Morris et al.'s (2009) argument about implementation of MKT in teacher training programmes were discussed. According to Hill et al. (2005), learner achievement might be improved by the implementation of efficient pre-service programmes and content-focused professional development. In the development of modules on the method of mathematical literacy teaching, an important consideration should not be what or how many courses in mathematics pre-service teachers have taken, but whether they will be able to use their mathematical knowledge effectively in their teaching. The problem of teacher knowledge comprises not only what teachers need to know, or the way in which they have to know it, but it is also a problem of how this knowledge should be taught to pre-service teachers (Ball & Bass, 2000). Stigler and Hiebert (1999) point out that the task of the lecturer who presents the method module, is to help the student teachers to learn more about teaching the specific subject. Lecturers should help student-teachers to know how to develop learning opportunities that will motivate learners and stimulate learning. It is the opinion of Stigler and Hiebert (1999) that lecturers should help their students to develop effective teaching methods. Stigler and Hiebert (1999, p. 6) argue that "if you can improve the methods that good teachers use, you will have achieved improvement that lasts".

Ball et al. (2013, p. 5) argue that the professional training student teachers receive, should be organised "…to help teachers learn the range of knowledge and skill they need in focused ways". During mathematics content courses in methodology, lecturers should focus on student teachers' conceptual understanding of the facts, formulas and algorithms involved, to promote better understanding. The student teachers should be encouraged to learn to think from a learner's viewpoint, and to recognise what they will have to do and how they will have to teach, to promote learner understanding and achievement. Imperfections in the understanding of fundamental mathematical concepts need to be addressed before MKT can be improved upon. Errors in understanding may result from overgeneralising the concepts used in prior mathematical experiences. The negative impact that the dearth of MKT of the participants will have on the effectiveness of their future teaching could be mitigated by implementation of a carefully developed mathematical literacy method course. In order to equip pre-service teachers with the MKT required to teach effectively, modules on teaching methods should be kept under the spotlight in any teacher education programme, since there is a need to "…expand their conceptual understanding during the mathematics content courses" (Ball et al., 2001, p. 450). Hill et al. (2008) maintain that expert teachers are distinguished by the relevance and detail of their insight into learners' problems of understanding. If teachers do not have the necessary mathematical knowledge they need for teaching, they will not be able to recognise different interpretations learners might have.

According to Hill et al. (2005), teachers with weak subject matter knowledge are those for whom courses in professional development are most beneficial. The participants in this study were all students who had chosen mathematical literacy as their specialisation. From their choice to specialise in mathematical literacy instead of in pure mathematics, the assumption might be made that these students are individuals who are not confident in their own mathematical knowledge or do not qualify for pure mathematics because of past history. It is thus all the more important that the modules on methodology these students complete, be constructed with all the above thoughts in mind.

5.3.2.2 Topics to be addressed in the method module

According to Ball et al. (2001), research has emphasised that teachers lack understanding of the fundamental ideas and concepts underpinning the mathematics of the curriculum. Ball et al. (2001) places propositional and procedural knowledge of mathematics central to the knowledge needed for teaching mathematics. This includes grasping the ideas behind specific topics such as, for instance, fractions or trigonometry, in order to clear up possible learner confusion. The reasoning behind procedures such as long division or factorisation of equations should be understood. In these procedures, the fundamental concepts underpinning the procedures is often buried beneath rules and algorithms. Teachers should know how to best explain concepts such as parallelism or infinity, and should understand interactions between such topics, procedures and concepts. Ball et al. (2001, p. 444) call this "substantive knowledge of mathematics". They argue that teachers, when teaching a certain mathematical topic, should understand the ideas that are connected to this topic. These are topics that should be addressed in a pre-service method module, to equip the student teachers with the necessary fundamental mathematical knowledge that will enable them to teach the topic correctly, and to be able to dispel learners' prior misconceptions. Some domains of the mathematics syllabus are often quite difficult for learners to understand correctly. This includes arithmetic with integers and fractions, or a topic such as probability or geometry. Pre-service teachers could be taught appropriate ways to approach these topics. In the method module, students should be taught specific representations and ways to develop the topic, which would facilitate learner understanding.

Fluency with basic number combinations is important for understanding multiplication and division (Musser et al., 2011). When multiplication of whole numbers and decimals is taught, teachers should have a clear understanding of multiplication as iterated addition, but multiplication is also inter alia the product of two units that produce an area. The area model for decimal multiplication used in Question 10 is an example of a representation that teachers should use in promoting understanding of multiplication. These different representations of the basic operations with numbers are all topics that could be included in the method curriculum.

Many adults, after spending 12 years at school, have misconceptions in understanding mathematics. The topic of misconceptions should be addressed separately in the method module. Smith et al. (1993, p. 124) argue: "Because they are fundamentally flawed, misconceptions themselves must be replaced". Smith et al. (1993) add that prior misconceptions are not easily replaced in the minds of students, and advise that students in teacher training programmes should be guided to understand the reasons for misconceptions, and how they might be neutralized. Method lecturers should assist pre-service teachers to develop suitable and proficient ideas which could be used in the place of prevailing misconceptions. The advice of Smith et al. (1993, p. 122) is "…to neutralize the interference of misconceptions, instruction should confront students with the disparity between their misconceptions and expert concepts…".

Siemon et al. (2014) believe that teachers need to implement many different strategies and representations when teaching fractions. This will help the learners in their future understanding of equivalent fractions, different representations of ftactions, as well as the concept of proportion. The fact that learners find the topic of proportion difficult (Musser et al., 2011) was affirmed by the items in this study on the topic of ratio and proportion. These items showed that the pre-service teachers did not have a clear understanding of proportion. Teachers need the MKT that will help them choose different examples and representations, and help them to explain reasons behind incorrect methods used by learners. These examples should include situations where proportion is involved, and should from part of the method curriculum

5.3.2.3 Role of self-efficacy in teacher instruction

Silverman and Davis (2009) suggest that teacher efficacy has been shown to be a major predictor of the ability and dedication of a teacher. According to (Silverman & Davis, p. 5), "the task of teacher education is, fundamentally, to develop competent and confident teachers". In the opinion of Nicolaidou and Philippou (2003), method modules for teacher training should be carefully structured to promote not only pre-service teachers' subject matter knowledge, but also their self-efficacy beliefs with regard to their own ability to teach their subject effectively.

According to Bandura (1994), the beginning of their professional career is an important period in the future success of teacher's careers. Bandura (1994, p. 11) mentions that their perceived self-efficacy has an important effect on the development of their "basic cognitive, self-management and interpersonal skills". Bong and Skaalvik (2003) concur, and continue by quoting several seminal scholars when they argue that determination of goals, academic aspirations and execution of tasks are strongly related to self-efficacy beliefs. This underscores the importance of a well-structured teacher training programme.

The four sources of self-efficacy proposed by Bandura (1977) can be utilised to develop and promote the pre-service teachers' self-efficacy. Mastery experiences manifest in the form of successful teaching experiences in practice teaching or in classroom teaching lessons. Vicarious experiences are gained by seeing peers, lecturers or other teachers successfully present a difficult topic. Verbal persuasion in the form of classroom discussion after a practice lesson can be valuable, and increased physical and mental well-being will improve self-efficacy. The four sources of self-efficacy mentioned by Bandura (1977) was discussed in Section 2.3.

In my study, participants showed high levels of self-efficacy. However, Silverman and Davis (2009) mention that pre-service teachers should not be allowed to start their teaching careers with a false sense of efficacy. They argue that high level of self-efficacy displayed by participants might have originated from situations unrelated to the reality and demands of running a classroom and all the problems this might entail. This has bearing on the high levels of self-efficacy among the participants indicated by the results of this study. Although Silverman and Davis might be talking about general pedagogy or classroom practice, this is just as relevant to MKT. During the method module, a discussion on this topic might be very effective. Student teachers need to be made aware of the danger of low MKT versus high self-efficacy.

5.4 SUMMARY OF CONTRIBUTIONS

Ball et al. (2001) focused on two approaches in their studies of the problem of teachers' mathematical knowledge for teaching. Their first research approach centred mainly on the amount of mathematics that teachers had studied, number of courses taken or certificates or degrees attained. Their second research approach was on the nature of teachers' knowledge, based on Shulman's notion of PCK and on their own notion of specialised mathematical knowledge for teaching, as explained in Chapter 2. This study has added a third research approach to the approaches of Ball et al. (2001), in that the dimension of teachers' self-efficacy beliefs with respect to their own specialised mathematical knowledge for teaching was investigated. This relationship had not been previously explored.

The participants in this study had to answer the MKT question of the questionnaire first and the SE choice afterwards, reflecting on their belief in the correctness of their MKT answer. Bong and Skaalvik (2003) hold that self-efficacy should be assessed before completing a task, since self-efficacy is a predictive construct, and beliefs could change when the details of the items are discussed. My study puts a different perspective on the assessment of self-efficacy. In this study, my intention was to first determine the participants' MKT, and then ask them to indicate their beliefs in the correctness of their responses.

The results of the study indicate that the prospective teachers involved in the study do not have the mathematical knowledge they need to teach the mathematics curriculum. These findings indicate the need for a course in the foundations of mathematics, where the basics of algorithms, rules and theorems are inspected, as explained in Section 5.3.2. Student-teachers need to understand the MKT involved with teaching the topics included in the mathematics curriculum, at least for Grades 8 and 9. The fundamental underpinnings of mathematics, especially the mathematics relating to the intermediate phase and senior phase levels, (IP and SP), should form an integral part of the method curriculum for training teachers in the teaching of mathematical literacy.

The self-efficacy questions used in the study pointed to a situation of high levels of self-efficacy, combined with lower levels of MKT. If teachers feel themselves confident of teaching a topic correctly, while they are not cognisant of the correct way it should be taught, learner understanding could be compromised, and this could lead the way to even fewer learners excelling in mathematics at the IP and SP phases of their school careers. According to Silverman and Davis (2009) pre-service teachers should be helped to think about the best ways to approach their teaching tasks. Silverman and Davis (2009, p. 5) reason that teachers need to think carefully about the areas where they feel more or less confident, and need to realise that "…feeling incompetent may lead them to avoid important classroom tasks". They caution that pre-service teachers could start their teaching careers "…with a false, uncalibrated sense of efficacy…" if they only experience instances of success in teaching.

5.5 SUGGESTIONS FOR FUTURE RESEARCH

This study endeavours to investigate the relationship between pre-service teachers' MKT and their self-efficiency related to this MKT, which seems to be a gap in the research that has not been addressed. Cerit (2010, p. 69) suggests that "…determining the level of pre-service teachers' self-efficacy belief may contribute to foresee how they might behave during in-service training based on self-efficacy feelings". He argues that this knowledge about pre-service teachers' self-efficacy beliefs might enhance teacher training programs.

In the questionnaire used in this study, participants were asked to consider approaches to MKT items relating to classroom scenarios, by responding to multiple-choice items. This questionnaire could be enriched by including open-ended questions. Participants could be asked why a specific answer was chosen by them, or why they thought a learner might find a specific problem more difficult than another. In future studies, MKT could also be assessed by conducting interviews and group discussions about the MKT involved, instead of merely having participants answer multiple-choice items on the mathematics for teaching inherent in the different classroom scenarios given.

The nature and development of pre-service teachers' MKT could be examined by implementing a longitudinal study, tracking individual students through their first few years of teaching. In these cases, MKT could also be assessed by lesson observation and interviews or the use video tapes.

Measures of similar difficulty as the measures involved in this study but in different mathematical domains should be used in research. In the current study, the mathematical topics of number concepts and operations were done but research should also be done into topics such as patterns, functions and algebra, as well as geometry and measurement, and statistics and probability. This would promote research into the actual mathematical content that teachers need to know. The precise mathematical content needed for effective teaching should be properly mapped and constituted. The relationship between student achievement and teacher MKT could be studied for these different mathematical knowledge domains.

More research into the development of teacher training programmes to promote better MKT is needed. Ball et al. (2001) reason that professional education and in-service training should not be intellectually superficial. The possibility of a mathematical literacy method module structured around MKT and self-efficacy should be investigated. Such a module would enable lecturers to address content knowledge and knowledge for teaching and learning almost simultaneously. This would involve a rigorous investigation into the specialized mathematical knowledge needed for teaching mathematics, as well as into how to best develop self-efficacy by implementation of the four sources described by Bandura (1977).

In research on mathematics education, questions need to be asked about the improvement of curricular materials. According to Ball et al. (2001), research has shown that ineffectual teacher training and professional development are one of the causes of the failure to improve mathematics education in schools. Baumert et al. (2010) mention that lecturers should not compromise on subject matter training in the professional teaching programmes, since this would not be beneficial to the quality of instruction and of learner achievement

Since MKT develops with experience, effective professional development requires continuous interactive support over a substantial period of time. Woolfolk Hoy (2000) reports evidence to suggest that support during the early years of teaching positively impacts and protects self-efficacy, which in turn is beneficial to teaching and learning. She suggests that research be done into the relationship between the characteristics of schools and teacher beliefs and self-efficacy. A longitudinal study that tracks individuals from their pre-service years into their teaching years, with regular interviews or questionnaires regarding their self-efficacy beliefs and experiences, would facilitate this.

Given that self-efficacy relates to effectiveness and attitude, the development of efficacy beliefs among teachers could be investigated. Oh (2011) maintains that factors that have an impact on the self-efficacy development of pre-service teachers should be studied, since strong self-efficacy beliefs help teachers in their teaching careers. Longitudinal studies mapping development of self-efficacy could be done around teacher training programmes and during the first years of teaching. Another topic worthy of research, would be to determine the true effect and impact the self-efficacy beliefs of teachers have on learner performance. Bong and Skaalvik (2003) report that not much research has been done on the strength of self-efficacy beliefs. In longitudinal studies, the effect that self-efficacy beliefs about MKT have on effective teaching and learning could be investigated.

Research about misconceptions have been done extensively, but this should also be linked to self-efficacy. It would be interesting to see whether a relationship exists between procedural mathematical misconceptions and self-efficacy. Stigler and Hiebert (2009, p. 7) suggest that researchers should "…harvest what good teachers are learning about teaching and to share what they have learnt so others can try these new approaches...To really improve teaching we must invest far more than we do now in generating and sharing knowledge about teaching". Mapping learner misconceptions for use in the method module would be worthwhile research.

5.6 . CONCLUSION

This study shows a need for better fundamental mathematics instruction as well as instruction in MKT for the pre-service teachers involved in the study. It also showed the participants' high levels of self-efficacy regarding the MKT of the topics included in the study. No discernible relationship between MKT and self-efficacy was found. This should be addressed and methods should be implemented to make the relationship more apparent.

It is important that students should have detailed knowledge of the mathematics they have to teach. Stigler and Hiebert (2009, p. 2) argue: "Even the best teachers, the ones judged the most competent, cannot be effective if the methods they are using do not promote better student learning". Teaching is a long learning curve that cannot be completely covered during a teacher training preparation programme. Students only really learn to teach when they are teachers themselves, but a good teacher training programme where serious attention is paid to subject-specific knowledge and instruction-methods, can facilitate this learning curve. A positive attitude towards mathematics and high self-efficacy levels towards the teaching of mathematics could also directly influence competence and promote learner achievement in mathematics. Prospective teachers should start their teaching careers with deep conceptual understanding of fundamental mathematics, as well as high levels of self-efficacy towards the teaching of mathematics, and there should be positive correlation between these two constructs.

The value of this study lies in that it highlights the mismatch between MKT and self-efficacy, which illustrates the need of increased levels of the mathematical knowledge needed for teaching, coupled with high levels of self-efficacy. The pre-service students that participated in the study had high levels of self-efficacy; they have been well-trained in the pedagogical aspects of teaching, and will be excellent teachers in the respect of humanising pedagogy. However, if their MKT does not improve, the mathematical knowledge of learners in our schools will not be improved. When teacher-training programmes in mathematics are not reinforced by comprehensive and effective approaches to teaching and learning, the crisis around mathematics and education in South Africa will not be resolved.

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Appendix A: Released LMT sample items

LEARNING MATHEMATICS FOR TEACHING

MATHEMATICAL KNOWLEDGE FOR TEACHING (MKT) MEASURES

MATHEMATICS RELEASED ITEMS 2008

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 Measures development supported by NSF grants REC-9979873, REC- 0207649, EHR-0233456 & EHR 0335411, and by a subcontract to CPRE on Department of Education (DOE), Office of Educational Research and Improvement (OERI) award #R308A960003.

lmt learning mathematics for teaching project

December 26, 2008

Dear Colleague:

Thank you for your interest in our survey items measuring mathematical knowledge for teaching. To orient you to the items and their potential use, we explain their development, intent, and design in this letter.

The effort to design survey items measuring teachers' knowledge for teaching mathematics grew out of the unique needs of the *Study of Instructional Improvement* (SII). SII is investigating the design and enactment of three leading whole school reforms and these reforms' effects on students' academic and social performance. As part of this research, lead investigators realized a need not only for measures which represent school and classroom processes (e.g., school norms, resources, teachers' instructional methods) but also teachers' facility in using disciplinary knowledge in the context of classroom teaching. Having such measures will allow SII to investigate the effects of teachers' knowledge on student achievement, and understand how such knowledge affects program implementation. While many potential methods for exploring and measuring teachers' content knowledge exist (i.e., interviews, observations, structured tasks), we elected to focus our efforts on developing survey measures because of the large number of teachers (over 5000) participating in SII.

Beginning in 1999, we undertook the development of such survey measures. Using theory, research, the study of curriculum materials and student work, and our experience, we wrote items we believe represent some of the competencies teachers use in teaching elementary mathematics – representing numbers, interpreting unusual student answers or algorithms, anticipating student difficulties with material. With the assistance of the University of California Office of the President[1], we piloted these items with K-6 teachers engaged in mathematics professional development. This work developed into a sister project to SII, *Learning Mathematics for Teaching* (LMT). With funding from the National Science Foundation, LMT has taken over instrument development from SII, developing and piloting geometry and middle school items.

We have publicly released a small set of items from our projects' efforts to write and pilot survey measures. We believe these items can be useful in many different contexts: as open-ended prompts which allow for the exploration of teachers' reasoning about mathematics and student thinking; as materials for professional development or teacher education; as exemplars of the kinds of mathematics teachers must know to teach. We encourage their use in such contexts. <u>However, this particular set of items is, as a group, NOT appropriate for use as an overall measure, or scale, representing teacher knowledge.</u> In other words, one <u>cannot</u> calculate a teacher score that reliably indicates either level of content knowledge or growth over time.

We ask users to keep in mind that these items represent steps in the process of developing measures. In many cases, we released items that failed, statistically speaking, in our piloting; in these cases, items may contain small mathematical ambiguities or other imperfections. If

[1] Elizabeth Stage, Patrick Callahan, Rena Dorph, principals.

you have comments or ideas about these items, please feel free to contact one of us by email at the addresses below.

These items are the result of years of thought and development, including both qualitative investigations of the content teachers use to teach elementary mathematics, and quantitative field trials with large numbers of survey items and participating teachers. Because of the intellectual effort put into these items by SII investigators, we ask that *all* users of these items satisfy the following requirements:

1) Please request permission from SII for any use of these items. To do so, contact Geoffrey Phelps at gphelps@umich.edu. Include a brief description of how you plan to use the items, and if applicable, what written products might result.

2) In any publications, grant proposals, or other written work which results from use of these items, please cite the development efforts which took place at SII by referencing this document:

Hill, H.C., Schilling, S.G., & Ball, D.L. (2004) Developing measures of teachers' mathematics knowledge for teaching. Elementary School Journal 105, 11-30.

3) Refrain from using these items in multiple choice format to evaluate teacher content knowledge in any way (e.g., by calculating number correct for any individual teacher, or gauging growth over time). Use in professional development, as open-ended prompts, or as examples of the kinds of knowledge teachers might need to know is permissible.

You can also check the SII website (http://www.sii.soe.umich.edu/) or LMT website (http://www.sitemaker.umich.edu/lmt) for more information about this effort.

Below, we present three types of released item – elementary content knowledge, elementary knowledge of students and content, and middle school content knowledge. Again, thank you for your interest in these items.

Sincerely,

Deborah Loewenberg Ball
Dean, School of Education
William H. Payne Collegiate Professor
University of Michigan

Heather Hill
Associate Professor
Harvard Graduate School of Education

Study of Instructional Improvement/Learning Mathematics for Teaching
Content Knowledge for Teaching Mathematics Measures (MKT measures)
Released Items, 2008

ELEMENTARY CONTENT KNOWLEDGE ITEMS

1. Ms. Dominguez was working with a new textbook and she noticed that it gave more attention to the number 0 than her old book. She came across a page that asked students to determine if a few statements about 0 were true or false. Intrigued, she showed them to her sister who is also a teacher, and asked her what she thought.

Which statement(s) should the sisters select as being true? (Mark YES, NO, or I'M NOT SURE for each item below.)

	Yes	No	I'm not sure
a) 0 is an even number.	1	2	3
b) 0 is not really a number. It is a placeholder in writing big numbers.	1	2	3
c) The number 8 can be written as 008.	1	2	3

2. Ms. Chambreaux's students are working on the following problem:

Is 371 a prime number?

As she walks around the room looking at their papers, she sees many different ways to solve this problem. Which solution method is correct? (Mark ONE answer.)

a) Check to see whether 371 is divisible by 2, 3, 4, 5, 6, 7, 8, or 9.

b) Break 371 into 3 and 71; they are both prime, so 371 must also be prime.

c) Check to see whether 371 is divisible by any prime number less than 20.

d) Break 371 into 37 and 1; they are both prime, so 371 must also be prime.

3. Imagine that you are working with your class on multiplying large numbers. Among your students' papers, you notice that some have displayed their work in the following ways:

Student A	Student B	Student C
35 x 25 125 +75 875	35 x25 175 +700 875	35 x25 25 150 100 +600 875

Which of these students would you judge to be using a method that could be used to multiply any two whole numbers?

	Method would work for all whole numbers	Method would NOT work for all whole numbers	I'm not sure
a) Method A	1	2	3
b) Method B	1	2	3
c) Method C	1	2	3

4. Ms. Harris was working with her class on divisibility rules. She told her class that a number is divisible by 4 if and only if the last two digits of the number are divisible by 4. One of her students asked her why the rule for 4 worked. She asked the other students if they could come up with a reason, and several possible reasons were proposed. Which of the following statements comes closest to explaining the reason for the divisibility rule for 4? (Mark ONE answer.)

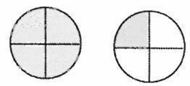
a) Four is an even number, and odd numbers are not divisible by even numbers.

b) The number 100 is divisible by 4 (and also 1000, 10,000, etc.).

c) Every other even number is divisible by 4, for example, 24 and 28 but not 26.

d) It only works when the sum of the last two digits is an even number.

5. Mrs. Johnson thinks it is important to vary the whole when she teaches fractions. For example, she might use five dollars to be the whole, or ten students, or a single rectangle. On one particular day, she uses as the whole a picture of two pizzas. What fraction of the two pizzas is she illustrating below? (Mark ONE answer.)



a) 5/4

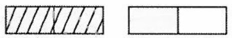
b) 5/3

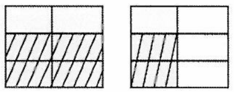
c) 5/8

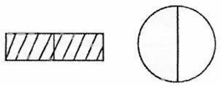
d) 1/4

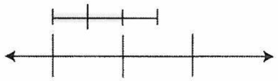
6. At a professional development workshop, teachers were learning about different ways to represent multiplication of fractions problems. The leader also helped them to become aware of examples that do not represent multiplication of fractions appropriately.

Which model below cannot be used to show that $1\frac{1}{2} \times \frac{2}{3} = 1$? (Mark ONE answer.)

A) 

B) 

C) 

D) 

7. Which of the following story problems could be used to illustrate $1\frac{1}{4}$ divided by $\frac{1}{2}$? (Mark YES, NO, or I'M NOT SURE for each possibility.)

	Yes	No	I'm not sure
a) You want to split $1\frac{1}{4}$ pies evenly between two families. How much should each family get?	1	2	3
b) You have $1.25 and may soon double your money. How much money would you end up with?	1	2	3
c) You are making some homemade taffy and the recipe calls for $1\frac{1}{4}$ cups of butter. How many sticks of butter (each stick = $\frac{1}{2}$ cup) will you need?	1	2	3

8. As Mr. Callahan was reviewing his students' work from the day's lesson on multiplication, he noticed that Todd had invented an algorithm that was different from the one taught in class. Todd's work looked like this:

```
  983
x   6
  488
+5410
 5898
```

What is Todd doing here? (Mark ONE answer.)

a) Todd is regrouping ("carrying") tens and ones, but his work does not record the regrouping.

b) Todd is using the traditional multiplication algorithm but working from left to right.

c) Todd has developed a method for keeping track of place value in the answer that is different from the conventional algorithm.

d) Todd is not doing anything systematic. He just got lucky – what he has done here will not work in most cases.

9. Ms. James' class was investigating patterns in whole-number addition. Her students noticed that whenever they added an even number and an odd number the sum was an odd number. Ms. James asked her students to explain why this claim is true for all whole numbers.

After giving the class time to work, she asked Susan to present her explanation:

> I can split the even number into two equal groups, and I can split the odd number into two equal groups with one left over. When I add them together I get an odd number, which means I can split the sum into two equal groups with one left over.

Which of the following best characterizes Susan's explanation? (Circle ONE answer.)

a) It provides a general and efficient basis for the claim.

b) It is correct, but it would be more efficient to examine the units digit of the sum to see if it is 1, 3, 5, 7, or 9.

c) It only shows that the claim is true for one example, rather than establishing that it is true in general.

d) It assumes what it is trying to show, rather than establishing why the sum is odd.

KNOWLEDGE OF CONTENT AND STUDENTS ITEMS

10. Mr. Garrett's students were working on strategies for finding the answers to multiplication problems. Which of the following strategies would you expect to see some elementary school students using to find the answer to 8 x 8? (Mark YES, NO, or I'M NOT SURE for each strategy.)

	Yes	No	I'm not sure
a) They might multiply 8 x 4 = 32 and then double that by doing 32 x 2 = 64.	1	2	3
b) They might multiply 10 x 10 = 100 and then subtract 36 to get 64.	1	2	3
c) They might multiply 8 x 10 = 80 and then subtract 8 x 2 from 80: 80 – 16 = 64.	1	2	3
d) They might multiply 8 x 5 = 40 and then count up by 8's: 48, 56, 64.	1	2	3

11. Students in Mr. Hayes' class have been working on putting decimals in order. Three students — Andy, Clara, and Keisha — presented 1.1, 12, 48, 102, 31.3, .676 as decimals ordered from least to greatest. What error are these students making? (Mark ONE answer.)

a) They are ignoring place value.

b) They are ignoring the decimal point.

c) They are guessing.

d) They have forgotten their numbers between 0 and 1.

e) They are making all of the above errors.

12. You are working individually with Bonny, and you ask her to count out 23 checkers, which she does successfully. You then ask her to show you how many checkers are represented by the 3 in 23, and she counts out 3 checkers. Then you ask her to show you how many checkers are represented by the 2 in 23, and she counts out 2 checkers. What problem is Bonny having here? (Mark ONE answer.)

a) Bonny doesn't know how large 23 is.

b) Bonny thinks that 2 and 20 are the same.

c) Bonny doesn't understand the meaning of the places in the numeral 23.

d) All of the above.

13. Mrs. Jackson is getting ready for the state assessment, and is planning mini-lessons for students focused on particular difficulties that they are having with adding columns of numbers. To target her instruction more effectively, she wants to work with groups of students who are making the same kind of error, so she looks at a recent quiz to see what they tend to do. She sees the following three student mistakes:

I)	II)	III)
1	1	1
38	45	32
49	37	14
+ 65	+ 29	+ 19
142	101	64

Which have the same kind of error? (Mark ONE answer.)

a) I and II

b) I and III

c) II and III

d) I, II, and III

14. Ms. Walker's class was working on finding patterns on the 100's chart. A student, LaShantee, noticed an interesting pattern. She said that if you draw a plus sign like the one shown below, the sum of the numbers in the vertical line of the plus sign equals the sum of the numbers in the horizontal line of the plus sign (i.e., 22 + 32 + 42 = 31 + 32 + 33). Which of the following student explanations shows sufficient understanding of why this is true for all similar plus signs? (Mark YES, NO or I'M NOT SURE for each one.)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

	Yes	No	I'm not sure
a) The average of the three vertical numbers equals the average of the three horizontal numbers.	1	2	3
b) Both pieces of the plus sign add up to 96.	1	2	3
c) No matter where the plus sign is, both pieces of the plus sign add up to three times the middle number.	1	2	3
d) The vertical numbers are 10 less and 10 more than the middle number.	1	2	3

15. Mrs. Jackson is getting ready for the state assessment, and is planning mini-lessons for students around particular difficulties that they are having with subtracting from large whole numbers. To target her instruction more effectively, she wants to work with groups of students who are making the same kind of error, so she looks at a recent quiz to see what they tend to do. She sees the following three student mistakes:

I	II	III
4 12	4 15	6 9 8 15
502	35005	7005
− 6	− 6	− 7
406	34009	6988

Which have the same kind of error? (Mark ONE answer.)

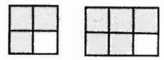
a) I and II

b) I and III

c) II and III

d) I, II, and III

16. Takeem's teacher asks him to make a drawing to compare $\frac{3}{4}$ and $\frac{5}{6}$. He draws the following:



and claims that $\frac{3}{4}$ and $\frac{5}{6}$ are the same amount. What is the most likely explanation for Takeem's answer? (Mark ONE answer.)

a) Takeem is noticing that each figure leaves one square unshaded.

b) Takeem has not yet learned the procedure for finding common denominators.

c) Takeem is adding 2 to both the numerator and denominator of $\frac{3}{4}$, and he sees that that equals $\frac{5}{6}$.

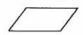
d) All of the above are equally likely.

17. A number is called "abundant" if the sum of its proper factors exceeds the number. For example, 12 is abundant because 1 + 2 + 3 + 4 + 6 > 12. On a homework assignment, a student incorrectly recorded that the numbers 9 and 25 were abundant. What are the most likely reason(s) for this student's confusion? (Mark YES, NO or I'M NOT SURE for each.)

	Yes	No	I'm not sure
a) The student may be adding incorrectly.	1	2	3
b) The student may be reversing the definition, thinking that a number is "abundant" if the number exceeds the sum of its proper factors.	1	2	3
c) The student may be including the number itself in the list of factors, confusing proper factors with factors.	1	2	3
d) The student may think that "abundant" is another name for square numbers.	1	2	3

18. At the close of a lesson on reflection symmetry in polygons, Ms. White gave her students several problems to do. She collected their answers and read through them after class. For the problem below, several of her students answered that the figure has two lines of symmetry and several answered that it has four.

How many lines of symmetry does this figure have?



Which of the following is the most likely reason for these incorrect answers? (Circle ONE answer.)

a) Students were not taught the definition of reflection symmetry.

b) Students were not taught the definition of a parallelogram.

c) Students confused lines of symmetry with edges of the polygon.

d) Students confused lines of symmetry with rotating half the figure onto the other half.

19. Ms. Abdul is preparing a unit to introduce her students to proportional reasoning. She is considering three versions of a problem that are the same except for the numbers used. Which version of the Mr. Short and Mr. Tall problem below is likely to be the most challenging for students? (Circle ONE answer.)

a) A picture depicts Mr. Short's height as 4 paper clips and as 6 buttons. The height of Mr. Tall (not shown) is given as 6 paper clips. How many buttons in height is Mr. Tall?

b) A picture depicts Mr. Short's height as 4 paper clips and as 7 buttons. The height of Mr. Tall (not shown) is given as 5 paper clips. How many buttons in height is Mr. Tall?

c) A picture depicts Mr. Short's height as 2 paper clips and as 9 buttons. The height of Mr. Tall (not shown) is given as 5 paper clips. How many buttons in height is Mr. Tall?

d) All three of the problems are equally challenging.

ELEMENTARY AND MIDDLE SCHOOL KNOWLEDGE OF CONTENT AND TEACHING ITEMS

20. To introduce the idea of grouping by tens and ones with young learners, which of the following materials or tools would be most appropriate? (Circle ONE answer.)

a) A number line

b) Plastic counting chips

c) Pennies and dimes

d) Straws and rubber bands

e) Any of these would be equally appropriate for introducing the idea of grouping by tens and ones.

21. Mr. Foster's class is learning to compare and order fractions. While his students know how to compare fractions using common denominators, Mr. Foster also wants them to develop a variety of other intuitive methods.

Which of the following lists of fractions would be best for helping students learn to develop <u>several</u> different strategies for comparing fractions? (Circle ONE answer.)

a) $\frac{1}{4}$ $\frac{1}{20}$ $\frac{1}{19}$ $\frac{1}{2}$ $\frac{1}{10}$

b) $\frac{4}{13}$ $\frac{3}{11}$ $\frac{6}{20}$ $\frac{1}{3}$ $\frac{2}{5}$

c) $\frac{5}{6}$ $\frac{3}{8}$ $\frac{2}{3}$ $\frac{3}{7}$ $\frac{1}{12}$

d) Any of these would work equally well for this purpose.

22. Ms. Brockton assigned the following problem to her students:

How many 4s are there in 3?

When her students struggled to find a solution, she decided to use a sequence of examples to help them understand how to solve this problem. Which of the following sequences of examples would be <u>best</u> to use to help her students understand how to solve the original problem? (Circle ONE answer.)

a) How many:
4s in 6?
4s in 5?
4s in 4?
4s in 3?

b) How many:
4s in 8?
4s in 6?
4s in 1?
4s in 3?

c) How many:
4s in 1?
4s in 2?
4s in 4?
4s in 3?

d) How many:
4s in 12?
4s in 8?
4s in 4?
4s in 3?

23. Ms. Williams plans to give the following problem to her class:

> Baker Joe is making apple tarts. If he uses $\frac{3}{4}$ of an apple for each tart, how many tarts can he make with 15 apples?

Because it has been a while since the class has worked with fractions, she decides to prepare her students by first giving them a simpler version of this same type of problem. Which of the following would be most useful for preparing the class to work on this problem? (Circle ONE answer.)

I. Baker Ted is making pumpkin pies. He has 8 pumpkins in his basket. If he uses $\frac{1}{4}$ of his pumpkins per pie, how many pumpkins does he use in each pie?

II. Baker Ted is making pumpkin pies. If he uses $\frac{1}{4}$ of a pumpkin for each pie, how many pies can he make with 9 pumpkins?

III. Baker Ted is making pumpkin pies. If he uses $\frac{3}{4}$ of a pumpkin for each pie, how many pies can he make with 10 pumpkins?

a) I only

b) II only

c) III only

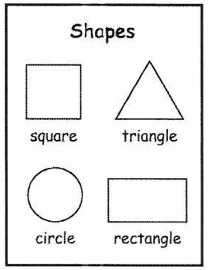
d) II and III only

e) I, II, and III

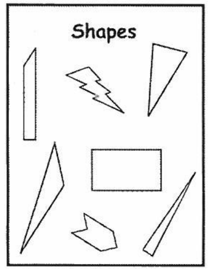
24. Ms. Miller wants her students to write or find a definition for triangle, and then improve their definition by testing it on different shapes. To help them, she wants to give them some shapes they can use to test their definition.

She goes to the store to look for a visual aid to help with this lesson. Which of the following is <u>most</u> likely to help students improve their definitions? (Circle ONE answer.)

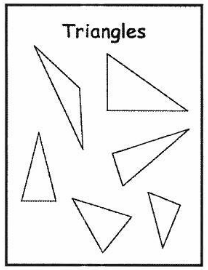
a)



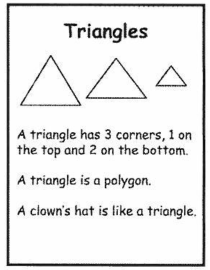
b)



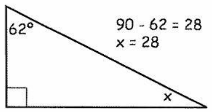
c)



d)



25. Ms. Donaldson's class was working on an assignment where they had to find the measures of unknown angles in triangles. One student consistently found the measures of unknown angles in right triangles by subtracting the known angle from 90. For example:



Ms. Donaldson was concerned that this student might run into difficulty when trying to find the measures of unknown angles in more general triangles. Which of the following questions would be <u>best</u> to ask the student in order to help clarify this issue? (Circle ONE answer.)

a) "What do you get when you add 90 + 62 + 28?"

b) "Why does subtracting 62 from 90 give you the measure of the unknown angle?"

c) "How could you find the missing angle in an isosceles triangle?"

d) "How did you know that this was a right triangle?"

e) "What if this angle measured 17° instead of 62°?"

26. As an early introduction to mathematical proof, Ms. Cobb wants to engage her students in deductive reasoning. She wants to use an activity about the sum of the angles of a triangle, but her students have not yet learned the alternate interior angle theorem. They do, however, know that a right angle is 90 degrees and that a point is surrounded by 360 degrees. Which of the following activities would <u>best</u> fit her purpose? (Circle ONE answer.)

a) Have students draw a triangle and a line parallel to its base through the opposite vertex. From there, have them reason about the angles of the triangle and the angles the triangle makes with the parallel line.

b) Have the students use rectangles with diagonals to reason about the sum of the acute angles in a right triangle.

c) Have students use protractors to measure the angles in several different triangles and from there reason about the sum of the angles of a triangle.

d) Have students cut out a triangle then tear off the three corners and assemble them, and from there reason about the sum of the angles of a triangle.

27. Mrs. Davies' class has learned how to tessellate the plane with any triangle. She knows that students often have a hard time seeing that any quadrilateral can tessellate the plane as well. She wants to plan a lesson that will help her students develop intuitions for how to tessellate the plane with any quadrilateral.

Which of the following activities would <u>best</u> serve her purpose? (Circle ONE answer.)

a) Have students cut along the diagonal of various quadrilaterals to show that each can be broken into two triangles, which students know will tessellate.

b) Provide students with multiple copies of a non-convex kite and have them explore which transformations lead to a tessellation of the plane.

c) Provide students with pattern blocks so that they can explore which of the pattern block shapes tessellate the plane.

d) These activities would serve her purpose equally well.

28. Mr. Shephard is using his textbook to plan a lesson on converting fractions to decimals by finding an equivalent fraction. The textbook provides the following two examples:

Convert $\frac{2}{5}$ to a decimal: $\frac{2}{5}=\frac{4}{10}=0.4$

Convert $\frac{23}{50}$ to a decimal: $\frac{23}{50}=\frac{46}{100}=0.46$

Mr. Shephard wants to have some other examples ready in case his students need additional practice in using this method. Which of the following lists of examples would be best to use for this purpose? (Circle ONE answer.)

a) $\frac{1}{4}$ $\frac{8}{16}$ $\frac{8}{20}$ $\frac{4}{5}$ $\frac{1}{2}$

b) $\frac{1}{20}$ $\frac{7}{8}$ $\frac{12}{15}$ $\frac{3}{40}$ $\frac{5}{16}$

c) $\frac{3}{4}$ $\frac{2}{3}$ $\frac{7}{20}$ $\frac{2}{7}$ $\frac{11}{30}$

d) All of the lists would work equally well.

MIDDLE SCHOOL CONTENT KNOWLEDGE ITEMS

29. Students sometimes remember only part of a rule. They might say, for instance, "two negatives make a positive." For each operation listed, decide whether the statement "two negatives make a positive" sometimes works, always works, or never works. (Mark SOMETIMES, ALWAYS, NEVER, or I'M NOT SURE)

	Sometimes works	Always works	Never works	I'm not sure
a) Addition	1	2	3	4
b) Subtraction	1	2	3	4
c) Multiplication	1	2	3	4
d) Division	1	2	3	4

30. Mr. Garrison's students were comparing different rectangles and decided to find the ratio of height to width. They wondered, though, if it would matter whether they measured the rectangles using inches or measured the rectangles using centimeters.

As the class discussed the issue, Mr. Garrison decided to give them other examples to consider. For each situation below, decide whether it is an example for which different ways of measuring produce the same ratio or a different ratio. (Circle PRODUCES SAME RATIO, PRODUCES DIFFERENT RATIO, or I'M NOT SURE for each.)

	Produces same ratio	Produces different ratio	I'm not sure
a) The ratio of two people's heights, measured in (1) feet, or (2) meters.	1	2	3
b) The noontime temperatures yesterday and today, measured in (1) Fahrenheit, or (2) Centigrade.	1	2	3
c) The speeds of two airplanes, measured in (1) feet per second, or (2) miles per hour.	1	2	3
d) The growths of two bank accounts, measured in (1) annual percentage increase, or (2) end-of-year balance minus beginning-of-year balance.	1	2	3

31. Mrs. Smith is looking through her textbook for problems and solution methods that draw on the distributive property as their primary justification. Which of these familiar situations could she use to demonstrate the distributive property of multiplication over addition [i.e., a (b + c) = ab + ac]? (Mark APPLIES, DOES NOT APPLY, or I'M NOT SURE for each.)

	Applies	Does not apply	I'm not sure
a) Adding $\frac{3}{4} + \frac{5}{4}$	1	2	3
b) Solving 2x – 5 = 8 for x	1	2	3
c) Combining like terms in the expression $3x^2 + 4y + 2x^2 - 6y$	1	2	3
d) Adding 34 + 25 using this method: $\begin{array}{r} 34 \\ \underline{+25} \\ 59 \end{array}$	1	2	3

32. Students in Mr. Carson's class were learning to verify the equivalence of expressions. He asked his class to explain why the expressions a – (b + c) and a – b – c are equivalent. Some of the answers given by students are listed below.

Which of the following statements comes <u>closest</u> to explaining why a – (b + c) and a – b – c are equivalent? (Mark ONE answer.)

a) They're the same because we know that a – (b + c) doesn't equal a – b + c, so it must equal a – b – c.

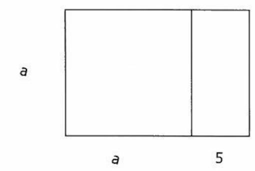
b) They're equivalent because if you substitute in numbers, like a=10, b=2, and c=5, then you get 3 for both expressions.

c) They're equal because of the associative property. We know that a – (b + c) equals (a – b) – c which equals a – b – c.

d) They're equivalent because what you do to one side you must always do to the other.

e) They're the same because of the distributive property. Multiplying (b + c) by –1 produces –b – c.

33. Ms. Whitley was surprised when her students wrote many different expressions to represent the area of the figure below. She wanted to make sure that she did not mark as incorrect any that were actually right. For each of the following expressions, decide whether the expression <u>correctly represents</u> or <u>does not correctly represent</u> the area of the figure. (Mark REPRESENTS, DOES NOT REPRESENT, or I'M NOT SURE for each.)



	Correctly represents	Does not correctly represent	I'm not sure
a) $a^2 + 5$	1	2	3
b) $(a + 5)^2$	1	2	3
c) $a^2 + 5a$	1	2	3
d) $(a + 5)a$	1	2	3
e) $2a + 5$	1	2	3
f) $4a + 10$	1	2	3

34. Ms. Hurlburt was teaching a lesson on solving problems with an inequality in them. She assigned the following problem.

$$-x < 9$$

Marcie solved this problem by reversing the inequality sign when dividing by − 1, so that x > − 9. Another student asked why one reverses the inequality when dividing by a negative number; Ms. Hurlburt asked the other students to explain. Which student gave the <u>best explanation</u> of why this method works? (Mark ONE answer.)

a) Because the opposite of x is less than 9.

b) Because to solve this, you add a positive x to both sides of the inequality.

c) Because −x < 9 cannot be graphed on a number line, we divide by the negative sign and reverse the inequality.

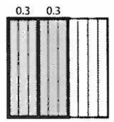
d) Because this method is a shortcut for moving both the x and 9 across the inequality. This gives the same answer as Marcie's, but in different form: −9 < x.

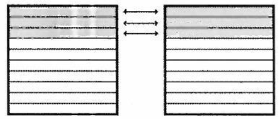
35. Ms. Austen was planning a lesson on decimal multiplication. She wanted to connect multiplication of decimals to her students' understanding of multiplication as repeated addition. She planned on reviewing the following definition with her class:

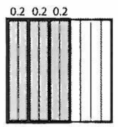
> The repeated addition interpretation of multiplication defines *a* x *b* as *b* added together *a* times, or *a* groups of *b*.

After reviewing this definition of repeated addition, she planned to ask her students to represent the problem 0.3 x 2 using the repeated addition interpretation of multiplication.

Which of the following representations <u>best</u> illustrates the repeated addition definition of 0.3 x 2? (Circle ONE answer.)

a) 

b) 

c) 

d) These representations illustrate the repeated addition definition of 0.3 x 2 equally well.

e) Multiplication of decimals cannot be represented using a repeated addition interpretation of multiplication.

Appendix B: Ethics clearance

Nelson Mandela Metropolitan University

for tomorrow

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• Port Elizabeth • 6031 • South Africa • www.nmmu.ac.za

Chairperson: Research Ethics Committee (Human)
Tel: +27 (0)41 504-2235

Ref: [H15-EDU-CPD-003 /Approval]

Contact person: Mrs U Spies

20 April 2015

Dr L Meiring
Faculty: Education
School for Initial Teacher Education
06-01-31
South Campus

Dear Dr Meiring

THE DEGREE OF ALIGNMENT BETWEEN PRE-SERVICE SECONDARY SCHOOL MATHEMATICS TEACHERS' MATHEMATICAL KNOWLEDGE FOR TEACHING (MKT) AND THEIR SELF-EFFICACY BELIEFS REGARDING THEIR MATHEMATICAL KNOWLEDGE FOR TEACHING

PRP: Dr L Meiring
PI: Ms N van Zyl

Your above-entitled application for ethics approval served at Research Ethics Committee (Human).

We take pleasure in informing you that the application was approved by the Committee.

The ethics clearance reference number is **H15-EDU-CPD-003** and is valid for three years. Please inform the REC-H, via your faculty representative, if any changes (particularly in the methodology) occur during this time. An annual affirmation to the effect that the protocols in use are still those for which approval was granted, will be required from you. You will be reminded timeously of this responsibility, and will receive the necessary documentation well in advance of any deadline.

We wish you well with the project. Please inform your co-investigators of the outcome, and convey our best wishes.

Yours sincerely

CB Cilliers

Prof C Cilliers
Chairperson: Research Ethics Committee (Human)

cc: Department of Research Capacity Development
Faculty Officer: Education

Appendix C: Recruitment letter and Informed consent

Dear Student

Re: Participant permission to take part in a MEd Study entitled:

The degree of alignment between pre-service secondary school teachers' Mathematical Knowledge for Teaching (MKT) and their self-efficacy beliefs regarding their mathematical knowledge for teaching.

I am a lecturer in Mathematics and Mathematics Method for the BEd FET students at the George Campus of the Nelson Mandela Metropolitan University. I am currently conducting research for my Masters' Degree in Education at NMMU. My research project focuses on the alignment between Mathematical Knowledge for Teaching (MKT) and self-efficacy beliefs of pre-service FET teachers.

The results of this research will be absolutely confidential and anonymous. No names, student numbers or any identifying characteristics will be used in any publication resulting from this study and individual participant's results of the MKT questionnaire will not be made public. No comparison will be drawn between the results of any of the participants. Students' names and numbers are only required for the purpose of written consent.

Participation in this project will take about 60 minutes of your time. The questionnaire you will be handed consists of a total of about 30 multiple-choice items regarding the teaching of some topics in the grade 7 to 10 mathematics curriculum for South African schools.

Participation in this research project is completely voluntary, but your participation will be sincerely appreciated, as it might help improve possible gaps in the current curriculum. Please indicate your willingness to participate in this research project by signing in the space provided.

If you have any questions about this research, you may contact Mrs Nicola van Zyl.

Contact details: 071 511 7047 or nicola.vanzyl@nmmu.ac.za

Thank you

Mrs Nicola van Zyl.

Consent letter

I ……………………………..…………………….…...................(name),

NMMU student number ……………..……….…………..……………

agree to participate in the research project entitled:

Investigating the relationship between Mathematical Knowledge for Teaching and self-efficacy of pre-service Mathematical Literacy teachers

by completing the questionnaire on mathematical knowledge for teaching and self-efficacy, conducted by Mrs Nicola van Zyl.

I further agree that the data gathered in this research may be used for possible future research.

……………………………………..   …………………………………

Signature   Date

Appendix D: Numbers of correct responses and ranking

Question	Total number of responses N = 137	Number of correct responses	Percentage correct responses of sample	Average correct responses for 16 items with sub-items	Correct response ranking all 33 items	Correct response ranking 16 items
1	135	18	13.1		3	3
2a	130	78	56.9	68 (49.3%)	26	14
2b	131	70	51.1		24	
2c	131	63	46.0		21	
2d	132	59	43.1		20	
3a	135	66	48.2	57 (41.5%)	22	13
3b	132	46	33.6		15	
3c	133	54	39.4		17	
3d	133	67	48.9		23	
3e	133	51	37.2		16	
4	107	16	11.7		1	1
5	134	32	23.4		9	9
6	134	26	19.0		5	5
7	135	29	21.2		6	6
8	134	17	12.4		2	2
9	136	43	31.4		13	11
10a	108	54	39.4	54 (39.6%)	18	12

10b	107	45	32.8		14	
10c	109	80	58.4		29	
10d	107	38	27.7		10	
11	124	32	23.4		8	8
12	134	41	29.9		11	10
13	131	20	14.6		4	4
14a	129	42	30.7	70 (50.73%)	12	15
14b	129	83	60.6		30	
14c	130	80	58.4		28	
14d	128	73	53.3		25	
15	131	31	22.6		7	7
16a	131	92	67.2	82 (59.85%)	33	16
16b	129	92	67.2		32	
16c	130	79	57.7		27	
16d	130	88	64.2		31	
16e	130	59	43.1		19	

Appendix E: Item difficulty and ranking

Ranking of item difficulty for each item			
Question	Difficulty	Difficulty ranking all 33 items	Difficulty ranking 16 items
1	-0.970	20	13
2a	-1.189	23	12
2b	-1.177	22	
2c	-0.843	19	
2d	-0.549	16	
3a	-1.926	28	11
3b	-1.892	27	
3c	0.574	5	
3d	-1.294	24	
3e	0.520	6	
4	0.733	4	4
5	0.000	10	7
6	-0.311	13	8
7	1.309	2	2
8	1.760	1	1
9	-0.350	15	9
10a	0.382	8	6
10b	-0.313	14	

10c	-0.179	11	
10d	0.288	9	
11	0.510	7	5
12	-1.550	25	15
13	-0.760	18	10
14a	-0.269	12	14
14b	-1.123	21	
14c	-1.863	26	
14d	-0.720	17	
15	1.020	3	3
16a	-2.660	32	16
16b	-2.440	31	
16c	-2.410	30	
16d	-2.161	29	
16e	-3.160	33	

Appendix F: Differences in ranking of number of correct responses and difficulty level

Question number	Difficulty ranking	Correct response ranking all 33 items	Difference in rankings
1	20	3	17
2a	23	26	-3
2b	22	24	-2
2c	19	21	-3
2d	16	20	-4
3a	28	22	6
3b	27	15	12
3c	5	17	-12
3d	24	23	1
3e	6	16	-10
4	4	1	3
5	10	9	1
6	13	5	8
7	2	6	-4
8	1	2	-1
9	15	13	2
10a	8	18	-10
10b	14	14	0
10c	11	29	-18

10d	9	10	-1
11	7	8	-1
12	25	11	14
13	18	4	4
14a	12	12	0
14b	21	30	-9
14c	26	28	-2
14d	17	25	-8
15	3	7	-4
16a	32	33	-1
16b	31	32	-1
16c	30	27	3
16d	29	31	-2
16e	33	19	19