Irreversibility, steady state, and non-equilibrium physics in relativistic heavy ion collisions

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Heavy ion collisions at ultrarelativistic energies offer the opportunity to study the irreversibility of multiparticle processes. Together with the many-body decays of resonances, the multiparticle processes cause the system to evolve according to Prigogine's steady states rather than towards statistical equilibrium. These results are general and can be easily checked by any microscopic string-, transport-, or cascade model for heavy ion collisions. The absence of pure equilibrium states sheds light on the difficulties of thermal models in describing the yields and spectra of hadrons, especially mesons, in heavy ion collisions at bombarding energies above 10 GeV/nucleon.

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The hypothesis that local equilibrium (LE) is attained by the system of two heavy ions colliding at ultrarelativistic energies is a basic assumption of macroscopic thermal- and hydrodynamical models of heavy ion collisions. The idea was pushed by Fermi [1] and Landau [2,3] almost 50 years ago for hadron-hadron collisions. Despite the long history of theoretical and experimental attempts there is no rigorous proof of LE yet. The present paper shows that the irreversibility of multiparticle processes, proceeding e.g. via string decays, causes these systems to evolve according to Prigogine's [4] steady state solution, rather than towards statistical equilibrium.

Using Bogolyubov's hierarchy of relaxation times in non-equilibrium statistical mechanics [5] one usually considers the following scheme: Suppose that in the initial stage the system is far from equilibrium. To describe it one has to introduce a set of various many-particle distribution functions rapidly varying in time. Then, due to interactions between the particles, correlations of the distribution functions occur within very short time intervals which are typically on the order of the collision time.

This is the kinetic stage – all many-particle distribution functions may be derived from the single one-particle distribution function. For times significantly larger than the collision time the number of parameters characterizing the system is reduced further to very few average values, namely the number of particles, their energy and velocity, i.e. to the moments of the distribution function. At this stage the system behavior is governed by hydrodynamics.

Unlike in non-relativistic mechanics, in relativistic heavy ion collisions the relaxation picture is more complex because of multiparticle processes. Here the number of particles and their composition are not conserved. Newly produced particles are not thermalized (even if they appear to be, see [6,7]) and this circumstance causes a delay in achieving equilibration. The equilibration time may appear too long as compared to the typical lifetime of the expanding system. Due to the lack of a rigorous first-principles theory of nuclear reactions at relativistic energies, the approach to LE is investigated mainly by virtue of dynamical calculations provided by microscopic semiclassical Monte Carlo models [8–15] which have been intensively studied during the last 15 years.

The analysis of the space time evolution picture obtained in these models reveals that the whole system of colliding nuclei never attains a global equilibrium state after the initial non-equilibrium stage. Still, there is, in principle, a possibility of the occurrence of local equilibrium (e.g. in the central cell), because the approach to LE does not depend on the assumptions of the presence of a heat reservoir, of Gibbs ensembles, etc.

Our study has been inspired by the finding that quasistable states are present in partonic and hadronic matter, as observed independently in dynamical simulations [8,16,17]. On the partonic level an analysis of the thermalization of partons has been performed by the late Klaus Kinder-Geiger [8]. Equilibration of hadronic matter has been studied, e.g., in the Quantum Molecular Dynamics models [9,10,13–15,18]. These simulations have shown that at high energies neither the global system nor its central part seem to reach the state of chemical equilibrium (in the sense of statistical mechanics) [8,14]. This observed feature is not solely restricted to microscopic models. To describe, for instance, the experimental data on yields of strange particles in heavy ion collisions at 200 AGeV [19] or hadron multiplicities at 158 AGeV [20] the standard statistical model of the ideal hadron gas has been modified to invoke the hypothesis of chemical non-equilibrium [21,22] as well.

Does this simply imply that the hadronization time is shorter than the equilibration time? - Not necessarily! In the present paper we show that dissipative processes, such as multiparticle production via strings and many-body ($N \geq 3$) decays of resonances, dominating at high energies, can lead to the creation of a stationary state called steady-state. This steady-state does **not** coincide

with a pure "conventional" equilibrium state, as assumed in the statistical models.

Consider first the necessary and sufficient criteria of LE in the central zone of nuclear reactions, which is usually analyzed in microscopic calculations:
Necessary conditions: **(i)** absence of significant flow effect in the central cell; isotropy of the velocity distributions, and **(ii)** isotropy of the diagonal components of the pressure tensor,

$$P_x = P_y = P_z = \frac{1}{3V} \sum_i \frac{p_{i\{x,y,z\}}^2}{(m_i^2 + p_i^2)^{1/2}} \ . \qquad (1)$$

Here V is the volume of the cell and m_i, p_i are the mass of the i-th hadron and its momentum, respectively.
Sufficient conditions: **(iii)** thermal equilibration which manifests itself in the time independence of the hadronic spectra after a certain period, and **(iv)** chemical equilibration, i.e. the time independence of different hadronic yields.

The necessary conditions look quite simple and evident: Local equilibrium may not be reached in symmetric nuclear collisions earlier than for the time $t^{pass} = 2R/(\gamma_{cm} v_{cm})$, during which noninteracting Lorentz contracted nuclei of radius R/γ_{cm}, which stream freely with the velocity v_{cm}, would have passed through each other. Apparently, early in the collision this is the origin of a substantial initial longitudinal collective flow of hadrons in the cell, which distorts the equilibration picture at the very beginning of the reaction. After t^{pass} this non-equilibrium component rapidly drops [14]. In [15] it has been reported that a stage of *kinetic* equilibrium is attained in heavy ion collisions in a central cell of volume $V = 5 \times 5 \times 5 = 125$ fm^3 at about $t \cong 10$ fm/c, irrespective of the energy of the colliding nuclei from 10.7 AGeV (AGS) to 160 AGeV (SPS). Isotropy of both the pressure and the velocity distributions of hadrons characterizes, **without** the sufficient conditions **(iii)** and **(iv)**, however, a pre-equilibrium stage of the reaction rather than an equilibrium one! In a fully equilibrated system conditions **(iii)-(iv)** must be satisfied as well.

This is the crucial point in our discussion: The statistical thermodynamics of many-particle systems [23,24] determines the thermal equilibrium as the state with maximum entropy. Once thermal equilibrium is attained, the velocity distributions of different particles must be isotropic. If the total number of particles is conserved, *kinetic* equilibrium is equivalent to *thermal* equilibrium [24]. But: this equivalence is broken, both in chemical reactions and in high energy physics.

Indeed, if the mixture of reacting substances is in the "true" equilibrium, then the rates of each chemical reaction must be the same for the direct and inverse processes [4,23]. However, in a cyclic process, in which the concentrations of the reacting substances are time independent, but the partial reaction rates $\omega_j = \omega_j^{\rm dir} - \omega_j^{\rm inv}$ are non-zero, the system is in a stationary state, which may be far from the equilibrium [4].

Consider now an ideal thermostat which contains a few thousand protons with an energy $E \gg m_p c^2$, where m_p is the mass of proton and c is the light velocity. For the sake of simplicity we exclude the (slow) weak processes from this scenario, focusing on strong interactions only. Then, even if the initial momentum distribution of the protons is Maxwellian, thermal equilibrium (in the sense of a state of maximum entropy) is not reached yet. Many new particles, mostly pions, will be produced as a result of initial proton-proton and, later, proton-pion, etc. collisions. When the system will finally reach equilibrium, it will consist of a large number of pions (and heavier mesons) with an admixture of baryons (and antibaryons) whose net number is conserved. The final temperature must, of course, be much lower than the initial one – kinetic energy has been transformed into mass (of produced particles). But: will the particle abundances be the same as those given by the statistical mechanics of an ideal hadron gas? In other words, will the final state be the state of thermal and chemical equilibrium, in which any direct and inverse hadronic processes will be taking place on average at the same rate?

This problem is closely related to the principle of detailed balance and to the irreversibility of multiparticle processes. To avoid ambiguities, we would like to stress that the definition of detailed balance in quantum mechanics (DB^{QM}) does not coincide with the definition of detailed balance in statistical physics (DB^{SP}) and chemistry. Detailed balance in the sense of quantum mechanical invariance under time reversal implies that the transition amplitudes of the direct and the inverse processes must be of the same magnitude,

$$|M_{a \to b}| = |M_{a \leftarrow b}| \quad . \qquad (2)$$

In statistical physics and chemistry, the principle of detailed balance requires that (in thermostatic equilibrium of a system) every separate reaction between its components is in itself in equilibrium, i.e. the rates of the direct and inverse processes are the same. To clarify the difference between DB^{QM} and DB^{SP}, consider the process of multiparticle production, e.g. in string excitation:

$$a + b \longrightarrow x_1 + x_2 + \ldots + x_n, \quad n \gg 1 \qquad (3)$$

According to Fermi's Golden Rule, the probability of n particle production reads

$$\mathrm{d}R_n = \frac{2\pi}{\hbar} |M_{a+b \to n}|^2 \prod_{i=1}^{n} \mathrm{d}^4 p_i \, \delta^4 \left(p - \sum_{i=1}^{n} p_i \right) \quad , \quad (4)$$

where p is the total four-momentum, p_i is the four-momentum of i-th particle, and $|M|$ is the amplitude of the process. The last factor is the space factor, which is fully determined by the kinematics of the reaction [25].

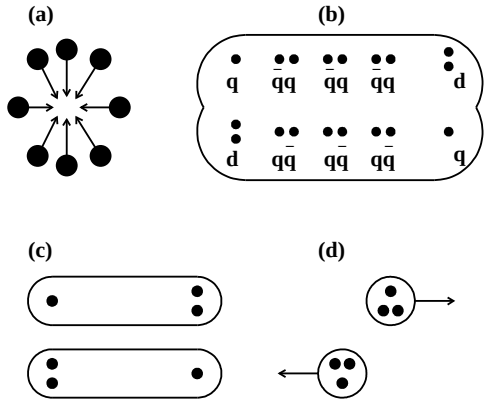

FIG. 1. Irreversible process: recombination of several hadrons into two baryons. Schematic diagram of (a) many-particle collision of hadrons; (b) rearrangement of their quarks and subsequent annihilation of $q\bar{q}$-pairs; (c) production of two quark-diquark strings, which shrink to (d) two baryons of high energy.

Although $|M_{a+b\to n}| = |M_{a+b\leftarrow n}|$ and, therefore, DB^{QM} is satisfied, the rates of the direct and the inverse processes, $R_{a+b\to n}$ and $R_{a+b\leftarrow n}$, are different, due to different space factors. This means that DB^{SP} is not fulfilled. Note that the principle of detailed balance in particle physics has been verified for the reactions

$$a + b \longleftrightarrow c + d \quad , \tag{5}$$

$$a + b \longleftrightarrow c \quad , \tag{6}$$

where a, b, c, d denote hadrons and their resonances. These processes are time reversible, because the space factors (or the densities of states) of the initial and final states are essentially the same.

The space factors are rapidly varying functions of n. Therefore, the matrix elements $|M|$ may be replaced by average values, and a situation typical for statistical mechanics is obtained: the probability of a state is proportional to the volume of the accessible phase space. In other words, multiparticle processes are irreversible in time, because they increase the local entropy of the system. Consequently, the processes of recombination of many hadrons into one string, and two strings colliding to form a couple of ground state hadrons (Fig. 1) of high energy are strongly suppressed, because they violate (locally) the Boltzmann H-theorem.

The irreversibility of multiparticle processes (e.g., strings which provide a steady source of new particles) first drives and then keeps the hadronic system out of the total chemical equilibrium, i.e. out of the full detailed balance in the sense of statistical mechanics. On the other hand, the DB^{SP} principle is the basic assumption of the statistical model (SM) of the ideal hadron gas [26–29] and variations like the statistical bootstrap model (SBM) [6]. Therefore, simply extracting the energy density ε, the baryon density ρ_{B} and the strangeness density ρ_{S} of the system at a given time and inserting these values as an input into the statistical model will be giving misleading results until all multiparticle processes in the system will have ceased.

Still, the conditions **(iii)-(iv)** may be fulfilled, even if the full detailed balance is not reached yet. Such states, which may be stable or not, but are out of local equilibrium, have been dubbed steady states of the system [4]. To decide whether or not a steady state is attained in a microscopic model of heavy-ion collision, the system must be compared with the quasi-equilibrated (in the sense of the criteria **(i)-(iv)**) infinite matter, as simulated within the same microscopic model. In [15] it was shown that the yields and energy spectra of hadrons in a central cell are – after $t \cong 10\,\mathrm{fm}/c$ – very close to those values calculated for infinite hadron matter [16] with the same ε, ρ_{B} and ρ_{S}. This is a strong indication on the occurrence of a steady state.

In conclusion, we have discussed the relaxation of hadronic matter produced in the central zone of heavy ion collisions in the energy range spanning from AGS to RHIC. Apparently, dissipative N-body ($N \gg 1$) decays of strings and resonances, i.e. multiparticle processes, are irreversible in time: the probability of N particles
1) to collide simultaneously in a small volume and
2) to transform into a final state, which consists only of two particles of higher energies,
drops extremely rapidly with rising N.

Therefore, these processes drive the system towards a steady state. Due to the broken symmetry between the rates of direct and inverse processes, this steady state does not coincide with a pure equilibrium state.

The conditions **(iii)-(iv)**, often applied for the analysis of local equilibrium, are generally weaker than the requirement of full local equilibrium usually imposed in the macroscopic models. At low energy densities, when multiparticle processes are rare, the steady state coincides practically with the equilibrium one. At higher energy densities, the difference between the states becomes more and more significant.

One characteristic feature of the steady state would be a strong enhancement of pions, accompanied by a suppression of (many-body decaying) resonances. This is due to the absence of an effective feeding mechanism. This feature of the steady state can explain the fact why conventional thermal models considerably underestimate yields of pions at energies of $E > 10$ AGeV. These results are typical for a large family of microscopic (cascade-, transport-, string-) models, which describe hadronic and nuclear interactions without invoking the hypothesis of

quark-gluon plasma (QGP) creation.

Non-equilibrium thermodynamics of irreversible processes [4] finally comes to high energy physics, where the conservation of mass and particle number, conventional in statistical physics, is obviously violated. The number of possible reaction channels is three order of magnitude higher than in simple chemical reactions (see, however, the role of the equilibrium concept in biochemical/biophysical processes). Therefore, it is a hopeless task to solve the rate equations analytically. On the other hand, microscopic models for hadronic and nuclear collisions provide a very useful tool to probe these fundamental features of nature at very small space and time scale. The non-equilibrium aspects of heavy ion collisions are interesting and require further investigations.

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[1] E. Fermi, Prog. Theor. Phys. **5**, 570 (1950); Phys. Rev. **81**, 683 (1951).
[2] L.D. Landau, Izv.Akad.Nauk SSSR **17**, 51 (1953).
[3] S.Z. Belenkij and L.D. Landau, Suppl. Nuovo Cimento **3**, 15 (1956).
[4] P. Glansdorff and I. Prigogine, *Thermodynamic theory of structure, stability and fluctuations*, (John Wiley & Sons, London, 1971).
[5] N.N. Bogolyubov, J. Phys. (USSR) **10**, 256 (1946); in *Studies in Statistical Mechanics*, edited by D. de Boer and G. E. Uhlenbeck (North-Holland, Amsterdam, 1962), Vol. 1.
[6] R. Hagedorn, Suppl. Nuovo Cimento **3**, 147 (1965); R. Hagedorn and J. Rafelski, Phys. Lett. B **97**, 136 (1980).
[7] F. Becattini, Z. Phys. C **69**, 485 (1996); F. Becattini and U. Heinz, Z. Phys. C **76**, 269 (1997).
[8] K. Geiger, Phys. Rep. **258**, 237 (1995); Phys. Rev. D**46**, 4965 (1992); *ibid.* 4986 (1992); K. Geiger and J.I. Kapusta, Phys. Rev D**47**, 4905 (1994).
[9] M. Berenguer, C. Hartnack, G. Peilert, H. Stöcker, W. Greiner, J. Aichelin, A. Rosenhauer, J. Phys. G **18**, 655 (1992).
[10] K. Puri, E. Lehmann, A. Faessler, S.W. Huang, J. Phys. G **20**, 1817 (1994).
[11] L.V. Bravina, I.N. Mishustin, A.N. Amelin, J. Bondorf, L.P. Csernai, Phys. Lett. B **354**, 196 (1995).
[12] H. Sorge, Phys. Lett. B **373**, 16 (1996).
[13] S.A. Bass, M. Belkacem, M. Bleicher, M. Brandstetter, L. Bravina, C. Ernst, L. Gerland, M. Hofmann, S. Hofmann, J. Konopka, G. Mao, L. Neise, S. Soff, C. Spieles, H. Weber, L.A. Winckelmann, H. Stöcker, W. Greiner, Ch. Hartnack, J. Aichelin, N. Amelin, Prog. Part. Nucl. Phys. **41**, 225 (1998).
[14] L.V. Bravina, M.I. Gorenstein, M. Belkacem, S.A. Bass, M. Bleicher, M. Brandstetter, M. Hofmann, S. Soff, C. Spieles, H. Weber, H. Stöcker, W. Greiner, Phys. Lett. B **434**, 379 (1998).
[15] L.V. Bravina et al, nucl-th/9810036, J. Phys. G. **25** (in press).
[16] M. Belkacem, M. Brandstetter, S.A. Bass, M. Bleicher, L. Bravina, M.I. Gorenstein, J. Konopka, L. Neise, C. Spieles, S. Soff, H. Weber, H. Stöcker, W. Greiner, Phys. Rev. C **58**, 1727 (1998).
[17] M. Brandstetter et al, (in progress).
[18] J. Sollfrank, U. Heinz, H. Sorge, N. Xu, nucl-th/9811011.
[19] S. Abatzis et al., (WA85 Collab.), Heavy Ion Phys. **4**, 79 (1996); Phys. Lett. B **376**, 251 (1996); Phys. Lett. B **347**, 158 (1995).
[20] S.V. Afanasjev et al., (NA49 Collab.), Nucl. Phys. **A610**, 188c (1996); C. Borman et al., (NA49 Collab.), J. Phys. G. **23**, 1817 (1997).
[21] J. Letessier and J. Rafelski, hep-ph/9806386 v4.
[22] G.D. Yen and M.I. Gorenstein, nucl-th/9808012 v2.
[23] L.D. Landau, E.M. Lifshitz, and L.P. Pitaevskii, *Statistical Physics* (Pergamon, New York, 1980), Part 1.
[24] S. de Groot, W.A. van Leuwen, and C.G. van Weert, *Relativistic Kinetic Theory* (North Holland, Amsterdam, 1980).
[25] E. Byckling and K. Kajantie, *Particle Kinematics* (John Wiley & Sons, London, 1973).
[26] H. Stöcker, A.A. Ogloblin, and W. Greiner, Z. Phys. A **303**, 259 (1981).
[27] M.I. Gorenstein, H.G. Miller, R.M. Quick, S.N. Yang, Phys. Rev. C **50**, 2232 (1994).
[28] P. Braun-Munzinger, J. Stachel, J.P. Wessels, N. Xu, Phys. Lett. B **365**, 1 (1996).
[29] J. Cleymans, D. Elliot, H. Satz, R.L. Thews, Z. Phys. C **74**, 319 (1997).