

Chiral Lagrangian for strange hadronic matter

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A generalized Lagrangian for the description of hadronic matter based on the linear $SU(3)_L \times SU(3)_R$ σ -model is proposed. Besides the baryon octet, the spin-0 and spin-1 nonets, a gluon condensate associated with broken scale invariance is incorporated. The observed values for the vacuum masses of the baryons and mesons are reproduced. In mean-field approximation, vector and scalar interactions yield a saturating nuclear equation of state. We discuss the difficulties and possibilities to construct a chiral invariant baryon-meson interaction that leads to a realistic equation of state. It is found that a coupling of the strange condensate to nucleons is needed to describe the hyperon potentials correctly. The effective baryon masses and the appearance of an abnormal phase of nearly massless nucleons at high densities are examined. A nonlinear realization of chiral symmetry is considered, to retain a Yukawa-type baryon-meson interaction and to establish a connection to the Walecka-model.

I. INTRODUCTION

Recently, nuclear physicists have given new attention to the general principles of chiral symmetry and broken scale invariance at finite densities. The underlying theory of strong interactions, QCD, is presently not solvable in the nonperturbative regime of low energies. However, QCD constraints may be imposed on effective theories for nuclear physics through symmetries, which determine largely how the hadrons should interact with each other. In this spirit, models with $SU(2)_L \times SU(2)_R$ symmetry and scale invariance were applied to nuclear matter at zero and finite temperature and to finite nuclei [1–5]. The success of these models established the applicability of this approach to relativistic nuclear many-body physics.

A simultaneous and self-consistent description of strange and nonstrange particles in baryonic matter is of particular interest, since many questions in heavy-ion physics and astrophysics are related to the effect of strangeness in matter: The possible large strangeness content of the nucleon indicates the importance of taking strangeness into account for a deeper understanding of nuclear matter and nuclei [6]. When extrapolating to hadronic systems with a large amount of strangeness, new phenomena as negatively charged multistrange objects may occur [7]. The possible onset of kaon condensation at high baryon densities in heavy-ion collisions and the interior of neutron stars provides another motivation for studying models which include the strange degree of freedom.

Hadrons can be classified in multiplets with (broken) $SU(3)_V$ symmetry, in which they have (almost) degenerate masses [8]. If there is one limit to the strong interactions, in which $SU(3)_V$ is exact, and another one, in which $SU(2)_V \times SU(2)_A$ symmetry holds, then there must be a joint limit in which $SU(3)_V$ is exact *and* all the axial-vector currents are conserved. In this limit the π , K , and η -particles are Goldstone bosons and we are led to a Lagrangian invariant under $SU(3)_L \times SU(3)_R$.

The linear σ -model as a specific realization of $SU(3)_L \times SU(3)_R$ symmetry was extensively studied in free space [9–11]. The spin-0 mass spectrum and meson-nucleon scattering are satisfactorily described within this approach.

In this paper we investigate the applicability of chiral $SU(3)$ symmetry to describe nuclear matter properties by constructing a chiral Lagrangian for hadronic matter including strange particles. To reproduce the binding energy of nuclear matter at saturation density $\rho = 0.15 \text{ fm}^{-3}$ with a reasonable value for the compression modulus, an octet of vector mesons with axial mesons as chiral partners is included.

The work is based on studies of nuclear matter with the $SU(2)_L \times SU(2)_R$ linear σ -model [1–5]. There, a logarithmic potential involving the dilaton field χ introduced in [12] to mimic the trace anomaly of QCD plays an essential role. It eliminates unphysical bifurcations encountered in the linear σ -model when applied to describe nuclear matter properties [13]. A similar concept is adopted here, too. However, there are some important difference to the $SU(2)$ case: the extension to a chiral $SU(3)$ symmetric model is nontrivial, because —in contrast to the nucleon doublet— the baryon octet cannot be assigned to a fundamental representation. Because of this, difficulties in describing the baryon masses and the hyperon potentials simultaneously arise. Furthermore, one needs to reproduce the experimentally well known masses of the baryon octet and the meson nonets. This leads necessarily to the inclusion of cubic and quartic

self-interactions of spin-0 mesons, which were eliminated in [1] to improve their results for nuclear matter and nuclei. The specific form of baryon–meson interaction is crucial for the properties of (hyper–)nuclear matter. The (relativistic) potentials for nucleons and hyperons following from this model depend strongly on the coupling constants of hyperons to vector and scalar mesons. Since these are constrained by chiral symmetry, it is of interest, whether or not the hyperon potentials are described correctly within this approach. Furthermore, the way hyperons are treated has important consequences for the stability of multistrange hypernuclear systems and for the mass of neutron stars. Therefore, different forms of coupling baryons to spin-0 mesons and their influence to the hyperon potentials are examined.

The paper is organized as follows: In the first part we review the chiral transformations of mesons and baryons and their assignment to representations. Then, the Lagrangian in its general form is presented and discussed. The next part is devoted to the approximation scheme used. The results include the investigation of the equation of state for nuclear matter, the hyperon potentials, the effective baryon masses, and a discussion whether a chiral phase transition occurs at high densities. As an outlook, a nonlinear realization of chiral symmetry is examined that is a convenient way to include heavy hadrons.

II. THEORY

The σ -model has been used extensively in exploring the implications of chiral symmetry in low-energy hadron dynamics. Most of these investigations have employed the SU(2) model with mesons and nucleons and the SU(3) σ -model with mesons only. In this section we will discuss the transformation properties of spin-0 and spin-1 mesons as well as those of the baryons as the constituents of our effective theory. This implies the choice of the proper representation under which the particles transform.

A. Representations

The representations of the hadrons result from the direct product of the quark representations, however in the Lagrangian there will be no explicit reference to quarks. For our purpose, they are used as guidance. In the chiral limit, the quarks have to be massless. Therefore, it is sufficient to consider the 2-component spinors

$$\begin{aligned} q_L &= \frac{1}{2}(1 - \gamma_5)q & \sim & (3, 0) \\ q_R &= \frac{1}{2}(1 + \gamma_5)q & \sim & (0, 3). \end{aligned} \quad (1)$$

Since the quarks are massless, the chirality of the spinors is linked to their spin. On the right-hand side, the quark representations are symbolized by the number of flavors, placed left (right) for the left (right) subspace of $SU(3)_L \times SU(3)_R$.

1. Mesons

The mesons as a bound system of a quark and antiquark correspond to the bilinear form $\bar{q}\mathcal{O}q$ where the 12×12 matrix \mathcal{O} is the direct product of the 4×4 Dirac matrices and the 3×3 Gell-Mann matrices ($\mathcal{O} = \Gamma \otimes \lambda$). For the discussion of the representations we will first suppress the explicit reference to the Gell-Mann matrix λ . Consider first the spin-0 mesons. If we suppose that they are s -wave bound states, then the only spinless objects we can form are

$$\bar{q}_R q_L \quad \bar{q}_L q_R. \quad (2)$$

The combinations $\bar{q}_L q_L$ and $\bar{q}_R q_R$ vanish, because the left and right subspaces are orthogonal to each other. The resulting representation is $(3, 3^*)$ and $(3^*, 3)$, respectively (the antiparticles belong to the conjugate representation). We are thus led to consider nonets of pseudoscalar and scalar particles.

For the vector mesons, we have to construct vectorial quantities from q_L and q_R . Again, if we assume that s -wave bound states are involved, the only vectors which can be formed are

$$\bar{q}_L \gamma_\mu q_L \quad \bar{q}_R \gamma_\mu q_R. \quad (3)$$

This suggests assigning the vector and axial vector mesons to the representation $(3 \times 3^*, 0) \oplus (0, 3 \times 3^*) = (8, 1) \oplus (1, 8)$, coinciding with the tensor properties of the currents conserved in the SU(3) \times SU(3) limit [14,15].

2. Baryons

The discussion of baryons differs from that of the mesons in that the construction of baryon multiplets from the basic fields q_L and q_R is not unique. The reason is that a left- or right- handed quark can be added to the spin-0 diquark of one subspace. Consequently, the baryons can be assigned to the representation $(3, 3^*)$ and $(3^*, 3)$ or $(8, 1)$ and $(1, 8)$, respectively. For an explicit construction in terms of quark fields see [16,17].

B. Transformations

Once the chiral-transformation properties of the elementary spinors are known it is straightforward to derive the corresponding transformation properties of the composite fields. An arbitrary element of $SU(3) \times SU(3)$ can be written as

$$\begin{aligned} G(\alpha, \beta) &= e^{-[i\alpha_a Q^a + i\beta_a Q^{5a}]} \\ &= e^{-i(\alpha - \beta) \cdot Q_L} \cdot e^{-i(\alpha + \beta) \cdot Q_R} \equiv L \cdot R. \end{aligned} \quad (4)$$

where α and β are eight-component vectors, and Q^a, Q^{5a} are the vector and axial generators of $SU(3)$, respectively. The spinors q_L of $SU(3)_L$ generated by $Q_L = (Q - Q^5)/2$ and q_R of $SU(3)_R$ generated by $Q_R = (Q + Q^5)/2$ transform as

$$q'_j = L_j^k q_k \quad q'_j = R_j^{\bar{k}} \bar{q}_{\bar{k}}, \quad (5)$$

where we adopted the tensor notation. Here, the (un-)bared indices belong to the (left) right subspace. The complex conjugate spinors transform as

$$q'^{j\bar{k}} = (L_j^k)^* q^{k\bar{k}} \quad q'^{j\bar{k}} = (R_j^{\bar{k}})^* q^{k\bar{k}}. \quad (6)$$

Knowing the representation of the mesonic and baryonic fields, it is straightforward to derive their transformation properties. They are summarized in table I, where we expressed the meson and baryon fields conveniently in a basis of 3×3 Gell-Mann matrices. For example, the spin-0 mesons may be written in the compact form

$$\frac{1}{2} \sum_{a=0}^8 (\bar{q}_R \lambda_a q_L + \bar{q}_R \lambda_a \gamma_5 q_L) \lambda_a \equiv \frac{1}{\sqrt{2}} \sum_{a=0}^8 (\sigma_a + i\pi_a) \lambda_a = \Sigma + i\Pi = M \quad (7)$$

$$\frac{1}{2} \sum_{a=0}^8 (\bar{q}_L \lambda_a q_R + \bar{q}_L \lambda_a \gamma_5 q_R) \lambda_a \equiv \frac{1}{\sqrt{2}} \sum_{a=0}^8 (\sigma_a - i\pi_a) \lambda_a = \Sigma - i\Pi = M^\dagger, \quad (8)$$

where $\sigma_a = \bar{q} \lambda_a q / \sqrt{2}$ (and similar for the π_a -fields) including the diagonal matrix $\lambda_0 = \sqrt{\frac{2}{3}} \mathbb{1}$. The first and second row are connected by the parity transformation, which transforms left-handed quarks to right handed ones. This is achieved in the matrix formulation by taking the adjoint. Therefore, since scalar and pseudoscalar particles have opposite parity, an imaginary unit i is attached to the pseudoscalar matrix Π .

C. Lagrangian formulation

1. Baryon-meson interaction

When generalizing from $SU(2)$ to $SU(3)$, complications arise from the baryon-meson sector, since not only the nucleon mass but the masses of the whole baryon multiplet are generated spontaneously by the vacuum expectation values (VEV) of only *two* meson condensates: of the 18 meson fields σ_a and π_a only the VEV of the components proportional to λ_0 and the hypercharge $Y \sim \lambda_8$ are nonvanishing, and the vacuum expectation value $\langle M \rangle$ reduces to:

$$\langle M \rangle = \frac{1}{\sqrt{2}} (\sigma_0 \lambda_0 + \sigma_8 \lambda_8) \equiv \text{diag} \left(\frac{\sigma}{\sqrt{2}}, \frac{\sigma}{\sqrt{2}}, \zeta \right),$$

in order to preserve parity invariance and assuming, for simplicity, $SU(2)$ symmetry¹ of the vacuum. The quark content of these fields is $\sigma \sim \langle \bar{u}u + \bar{d}d \rangle$ and $\zeta \sim \langle \bar{s}s \rangle$. To see explicitly how these condensates generate the baryon masses, let us consider the simplest ansatz for the baryon-meson interaction, namely the Yukawa-type coupling:

$$\mathcal{L}_{BM}^{(0)} = b_0 \left(\varepsilon_{\bar{a}\bar{b}\bar{c}} \varepsilon^{def} (\bar{\Psi}_L)_{\bar{d}}^{\bar{a}} M_e^{\bar{b}} (\Psi_R)_{\bar{f}}^{\bar{c}} + \varepsilon_{abc} \varepsilon^{\bar{d}\bar{e}\bar{f}} (\bar{\Psi}_R)_{\bar{d}}^{\bar{a}} M_e^{\bar{b}} (\Psi_L)_{\bar{f}}^{\bar{c}} \right). \quad (9)$$

The indices are contracted appropriately to yield a chirally invariant term. Note that the chirally invariant *linear* baryon-meson interaction is only possible in the baryon representation $(3,3^*)$ and $(3^*,3)$ and it is unique (since the product $3 \times 3 \times 3 = 1 + \dots$ leads only to one singlet). Furthermore, the resulting coupling constants are given by the symmetric (d-type) structure constants of $SU(3)$. The reason for this is that three spinors can only be coupled to a singlet by antisymmetrizing them. Since this has to be done in the left and right space, respectively, the resulting coupling will be a symmetric one.

Using the decomposition of the baryon matrix $\Psi = \frac{1}{\sqrt{2}} \sum_{k=0}^8 \psi_k \lambda_k$ by means of the projection operators $(1 \pm \gamma_5)/2$,

$$(\Psi_L)_{\bar{b}}^{\bar{a}} = \frac{1 - \gamma_5}{2} \Psi_b^a, \quad (\Psi_R)_{\bar{b}}^{\bar{a}} = \frac{1 + \gamma_5}{2} \Psi_b^a, \quad (10)$$

one arrives at

$$\mathcal{L}_{BM}^{(0)} = b_0 \varepsilon_{abc} \varepsilon_{def} \bar{\Psi}_{ad} (\Sigma_{be} + i\gamma_5 \Pi_{be}) \Psi_{cf}.$$

After insertion of the vacuum matrix $\langle M \rangle$, one obtains the baryon masses generated by the VEV of the two meson fields. With this kind of coupling, it is not possible to describe the correct baryon mass splitting as the nucleon and the Ξ are degenerate (see table II, first column). To eliminate this flaw, one can either use chirally invariant interaction terms of higher order in the meson fields or break the symmetry explicitly.

Taking the first possibility, one has to compute how the nonlinear terms contribute to the baryon masses. The *quadratic* baryon-meson interaction term reads again for the $(3,3^*)$ and $(3^*,3)$ representation of the baryons²

$$\begin{aligned} \mathcal{L}_{BM}^{(1)} &= b_1 \left((\bar{\Psi}_L)_{\bar{b}}^{\bar{a}} M_c^{\bar{b}} (\Psi_R)_{\bar{d}}^{\bar{c}} M_a^{\bar{d}} + (\bar{\Psi}_R)_{\bar{b}}^{\bar{a}} M_c^{\bar{b}} (\Psi_L)_{\bar{d}}^{\bar{c}} M_a^{\bar{d}} \right) \\ &= b_1 \text{Tr}(\bar{\Psi}_L M \Psi_R M + \bar{\Psi}_R M^\dagger \Psi_L M^\dagger). \end{aligned} \quad (11)$$

But, as can be observed from table II, (second column), this term also fails to remove the nucleon- Ξ -mass degeneracy. Only the inclusion of a *cubic* interaction term of the form

$$\begin{aligned} \mathcal{L}_{BM}^{(2)} &= b_2 \left((\bar{\Psi}_L)_{\bar{b}}^{\bar{a}} M_c^{\bar{b}} (\Psi_R)_{\bar{d}}^{\bar{c}} T_a^{\bar{d}} + (\bar{\Psi}_R)_{\bar{b}}^{\bar{a}} M_c^{\bar{b}} (\Psi_L)_{\bar{d}}^{\bar{c}} T_a^{\bar{d}} \right) \\ &= b_2 \text{Tr}(\bar{\Psi}_L M \Psi_R T + \bar{\Psi}_R M^\dagger \Psi_L T^\dagger). \end{aligned} \quad (12)$$

yields a mass splitting between nucleon and Ξ (table II, third column). Here, the dual tensor is defined as³

$$T_a^{\bar{d}} = \epsilon_{amnp} \epsilon^{\bar{d}\bar{f}\bar{g}} M_{\bar{f}}^m M_{\bar{g}}^n, \quad (13)$$

¹This implies that isopin breaking effects will not occur, i.e., all hadrons of the same isospin multiplet will have identical masses.

²Except for the linear term of equation (9), the quadratic and the cubic interactions are also possible in the $(8,1)$ and $(1,8)$ representation of the baryons. Specifically the quadratic contribution reads $\text{Tr}(\bar{L} M R M^\dagger + \bar{R} M^\dagger L M)$. However, this difference will not play a role for the vacuum masses or in the mean field approximation, since $\langle M \rangle = \langle M^\dagger \rangle = \text{diag}(\frac{\sigma}{\sqrt{2}}, \frac{\sigma}{\sqrt{2}}, \zeta)$.

³The cubic interaction allows for two independent invariants, the other one being analogous to (12) except for exchanging T and M :

$$\begin{aligned} \mathcal{L}_{BM}^{(3)} &= b_3 \left((\bar{\Psi}_L)_{\bar{b}}^{\bar{a}} T_c^{\bar{b}} (\Psi_R)_{\bar{d}}^{\bar{c}} M_a^{\bar{d}} + (\bar{\Psi}_R)_{\bar{b}}^{\bar{a}} T_c^{\bar{b}} (\Psi_L)_{\bar{d}}^{\bar{c}} M_a^{\bar{d}} \right) \\ &= b_3 \text{Tr}(\bar{\Psi}_L T \Psi_R M + \bar{\Psi}_R T^\dagger \Psi_L M^\dagger). \end{aligned}$$

However, this form will not be considered, because it gives poor results for the baryon mass splitting and it does not lead to acceptable nuclear matter fits.

so that it transforms in the same way as the meson matrix M .

The second alternative is to break the symmetry explicitly. However, the transformation properties of the breaking term is restricted due to the necessity to maintain the PCAC relation for the pion (section II C 3). Assuming that the mass differences are entirely due to the quark mass differences, we break the symmetry along the hypercharge Y-direction. This leads to Gell-Mann-Okubo (GMO) mass formulae. We take a term of the type (8,8) [18],

$$\mathcal{L}_{\Delta m} = m_1 \text{Tr}(\bar{\Psi}\Psi - \bar{\Psi}\Psi S) + m_2 \text{Tr}(\bar{\Psi}S\Psi) \quad (14)$$

where $S_b^a = -\frac{1}{3}[\sqrt{3}(\lambda_8)_b^a - \delta_b^a]$ (other types as the (8,1) and (1,8), (3,3*) and (3*, 3) representations lead to similar results and are discussed in [18]).

Since none of the baryon-meson interaction terms alone gives the correct baryon mass splitting, they will be investigated in combination with the explicit symmetry breaking term (14). The baryon masses read:

$$\begin{aligned} m_N &= \frac{b_j}{\sqrt{2}} B_{jN} & m_{\Xi} &= \frac{b_j}{\sqrt{2}} B_{j\Xi} + m_1 + m_2 & j &= 0, 1, 2 \\ m_{\Lambda} &= \frac{b_j}{\sqrt{2}} B_{j\Lambda} + \frac{m_1 + 2m_2}{3} & m_{\Sigma} &= \frac{b_j}{\sqrt{2}} B_{j\Sigma} + m_1, \end{aligned} \quad (15)$$

with the baryon-meson interaction terms B_{ji} ($i = N, \Lambda, \Sigma, \Xi$) of table II. From that one can see that only the strangeness carrying baryon masses are modified (note, that in the case of $m_2 = m_1 \equiv m_s$ the explicit symmetry breaking term corresponds to the strange quark mass in the spirit of the additive quark model). As only the nucleon mass enters the fit to nuclear matter properties, the parameters b_j shall be fixed to reproduce the nucleon mass. The symmetry breaking contributions are then adjusted to the remaining baryon masses.

The interaction terms of baryons with spin-0-mesons, which lead to a saturating nuclear matter equation of state (see table III), are

$$1. \text{ L: } \mathcal{L}_{BM} = \mathcal{L}_{BM}^{(0)} + \mathcal{L}_{\Delta m} \quad \text{Q: } \mathcal{L}_{BM} = \mathcal{L}_{BM}^{(1)} + \mathcal{L}_{\Delta m} \quad \text{C: } \mathcal{L}_{BM} = \mathcal{L}_{BM}^{(2)} + \mathcal{L}_{\Delta m} .$$

Here, L, Q and C stand for the meson fields entering in the baryon-meson interaction terms purely linearly, quadratically and cubic, respectively⁴ (This notation is also used in table III and in figures 1, 2, and 3).

The interaction of the vector meson and axial vector meson nonets

$$V_{\mu} = \frac{1}{\sqrt{2}} \sum_{i=0}^8 v_{\mu}^i \lambda_i \quad A_{\mu} = \frac{1}{\sqrt{2}} \sum_{i=0}^8 a_{\mu}^i \lambda_i \quad (16)$$

with baryons is far less involved. For the baryons belonging to the (3,3*) and (3*,3) representation one has the antisymmetric, f-type coupling to baryons⁵

$$\mathcal{L}_{BV} = g_8^V \text{Tr}(\bar{\Psi}\gamma^{\mu}[V_{\mu}, \Psi] + \bar{\Psi}\gamma^{\mu}\{A_{\mu}\gamma_5, \Psi\}) + g_1^V \text{Tr}(\bar{\Psi}\Psi)\gamma^{\mu}\text{Tr}(V_{\mu} + A_{\mu}). \quad (17)$$

In the mean field treatment, the axial mesons have a zero VEV. The relevant fields in the SU(2) invariant vacuum, v_{μ}^0 and v_{μ}^8 , are taken to have the ideal mixing angle $\sin \theta_v = \frac{1}{\sqrt{3}}$, yielding

$$\begin{aligned} \phi_{\mu} &= v_{\mu}^8 \cos \theta_v - v_{\mu}^0 \sin \theta_v = \frac{1}{\sqrt{3}}(\sqrt{2}v_{\mu}^0 + v_{\mu}^8) \\ \omega_{\mu} &= v_{\mu}^8 \sin \theta_v + v_{\mu}^0 \cos \theta_v = \frac{1}{\sqrt{3}}(v_{\mu}^0 - \sqrt{2}v_{\mu}^8). \end{aligned} \quad (18)$$

For $g_1^V = g_8^V$, the strange vector field $\phi_{\mu} \sim \bar{s}\gamma_{\mu}s$ does not couple to the nucleon. The remaining couplings to the strange baryons are then determined by symmetry relations:

⁴The sum of linear, quadratic and cubic forms leads also to a realistic baryon mass splitting and to a saturating equation of state, even without an explicit symmetry breaking term. However, it only complicates the discussion without significantly improving results. Therefore, we will not consider this option further.

⁵If the baryons are assigned to (8,1) and (1,8), the analogous octet term reads $g_8^V \text{Tr}(\bar{\Psi}\gamma^{\mu}[V_{\mu}, \Psi] + \bar{\Psi}\gamma^{\mu}[A_{\mu}\gamma_5, \Psi])$. Since both representations differ only as to how the axial mesons contribute, there will be no difference in the mean field approximation.

$$g_{\Lambda\omega} = g_{\Sigma\omega} = 2g_{\Xi\omega} = \frac{2}{3}g_{N\omega} = 2g_8^V \quad g_{\Lambda\phi} = g_{\Sigma\phi} = \frac{g_{\Xi\phi}}{2} = \frac{\sqrt{2}}{3}g_{N\omega}, \quad (19)$$

where their relative values are related to the additive quark model. In contrast to the baryon/spin-0-meson interaction, two independent interaction terms of baryons with spin-1 mesons can be constructed. They correspond to the antisymmetric (f-type) and symmetric (d-type) couplings, respectively. However, from the universality principle [19] and the vector meson dominance model the d-type coupling should be small. In mean-field models, large attractive and repulsive contributions from scalar and vector mesons cancel to give the relatively shallow nucleon potential. When extended to the strange sector, a different treatment of the coupling constants disturbs the cancellation and unphysically large hyperon potentials can emerge. We will elaborate on this problem in section III B.

2. Chirally invariant potential

The chirally invariant potential includes the mass terms for mesons, their self-interaction and the dilaton potential for the breaking of scale symmetry. For the spin-0 mesonic potential we take all independent combinations of mesonic self-interaction terms up to fourth order

$$\begin{aligned} \mathcal{L}_0 \equiv -\mathcal{V}_0 = & \frac{1}{2}k_0\chi^2\text{Tr}M^\dagger M - k_1(\text{Tr}M^\dagger M)^2 - k_2\text{Tr}(M^\dagger M)^2 \\ & - k_3\chi(\det M + \det M^\dagger) + k_4\chi^4 + \frac{1}{4}\chi^4 \ln \frac{\chi^4}{\chi_0^4} - \frac{\delta}{3}\chi^4 \ln \frac{\det M + \det M^\dagger}{2\det\langle M \rangle}. \end{aligned} \quad (20)$$

Most of the constants are fixed by the vacuum masses of the pseudoscalar and scalar mesons, respectively (see section III A for details). These are determined by calculating the second derivative of the potential in the ground state. Because of the determinant and the logarithmic terms, mixing between η_8 , η_0 (in the pseudoscalar sector) and σ , ζ , and χ (in the scalar sector) occurs, which makes a diagonalization of the corresponding mass matrices necessary. The quadratic and cubic form of the interaction is made scale invariant by multiplying it with an appropriate power of the dilaton field χ [20]. Originally, the dilaton field was introduced by Schechter in order to mimic the trace anomaly of QCD $\theta_\mu^\mu = \frac{\beta_{QCD}}{2g}\mathcal{G}_{\mu\nu}^a\mathcal{G}_a^{\mu\nu}$ in an effective Lagrangian at tree level [12] ($\mathcal{G}_{\mu\nu}$ is the gluon field strength tensor of QCD). The effect⁶ of the logarithmic term $\sim \chi^4 \ln \chi$ is twofold: First, it breaks the scale invariance and leads to the proportionality $\theta_\mu^\mu \sim \chi^4$ as can be seen from

$$\theta_\mu^\mu = 4\mathcal{L} - \chi \frac{\partial \mathcal{L}}{\partial \chi} - 2\partial_\mu \chi \frac{\partial \mathcal{L}}{\partial(\partial_\mu \chi)} = \chi^4, \quad (21)$$

which is a consequence of the definition of scale transformations [11]. Second, the logarithm leads to a nonvanishing vacuum expectation value for the dilaton field resulting in a spontaneous chiral symmetry breaking. This connection comes from the multiplication of k_0 in Eq. (20) with χ^2 : With the breakdown of scale invariance the resulting mass coefficient becomes negative for positive k_0 and therefore the Nambu–Goldstone mode is entered. The comparison of the trace anomaly of QCD with that of the effective theory allows for the identification of the χ -field with the gluon condensate:

$$\theta_\mu^\mu = \left\langle \frac{\beta_{QCD}}{2g}\mathcal{G}_{\mu\nu}^a\mathcal{G}_a^{\mu\nu} \right\rangle \equiv (1 - \delta)\chi^4. \quad (22)$$

The parameter δ originates from the second logarithmic term with the chiral and parity invariant combination $\det M + \det M^\dagger$. The term is a SU(3)-extension of the logarithmic term proportional to $\chi^4 \ln(\sigma^2 + \pi^2)$ introduced in [1]. An orientation for the value of δ may be taken from β_{QCD} at one loop level, with N_c colors and N_f flavors,

$$\beta_{QCD} = -\frac{11N_c g^3}{48\pi^2} \left(1 - \frac{2N_f}{11N_c} \right) + \mathcal{O}(g^5), \quad (23)$$

⁶According to [12], the argument of the logarithm has to be chirally and parity invariant. This is fulfilled by the dilaton which is a chiral singlet and a scalar.

where the first number in parentheses arises from the (antiscreening) self-interaction of the gluons and the second, proportional to N_f , is the (screening) contribution of quark pairs. Eq. (23) suggests the value $\delta = 6/33$ for three flavors and three colors. This value gives the order of magnitude about which the parameter δ will be varied. For the spin-1 mesons a mass term is needed. The simplest scale invariant form

$$\mathcal{L}_{vec}^{(1)} = \frac{1}{2}m_V^2 \frac{\chi^2}{\chi_0^2} \text{Tr}(V_\mu V^\mu + A_\mu A^\mu) \quad (24)$$

implies a mass degeneracy for the meson nonet. To split the masses one can add the chiral invariant [10,21]

$$\mathcal{L}_{vec}^{(2)} = \frac{1}{8}\mu \text{Tr}[(F_{\mu\nu} + G_{\mu\nu})^2 M^\dagger M + (F_{\mu\nu} - G_{\mu\nu})^2 M^\dagger M], \quad (25)$$

with the vectorial and axial field strength tensors $F_{\mu\nu} = \partial_\nu V_\mu - \partial_\mu V_\nu$ and $G_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$. In combination with the kinetic energy term (see Eq. 31), one obtains for the vector mesons

$$\begin{aligned} & -\frac{1}{4}\left[1 - \mu\frac{\sigma^2}{2}\right](F_\rho^{\mu\nu})^2 - \frac{1}{4}\left[1 - \frac{1}{2}\mu\left(\frac{\sigma^2}{2} + \zeta^2\right)\right](F_{K^*}^{\mu\nu})^2 \\ & -\frac{1}{4}\left[1 - \mu\frac{\sigma^2}{2}\right](F_\omega^{\mu\nu})^2 - \frac{1}{4}\left[1 - \mu\zeta^2\right](F_\phi^{\mu\nu})^2. \end{aligned} \quad (26)$$

Since the coefficients are no longer unity, the vector meson fields have to be renormalized, i.e., the new ω -field reads $\omega_r = Z_\omega^{-1/2}\omega$. The renormalization constants are the coefficients in the square brackets in front of the kinetic energy terms of Eq. (26), i.e., $Z_\omega^{-1} = 1 - \mu\sigma^2/2$. The mass terms of the vector mesons deviate from the mean mass m_V by the renormalization factor⁷, i.e.,

$$m_\omega^2 = m_\rho^2 = Z_\omega m_V^2 \quad ; \quad m_{K^*}^2 = Z_{K^*} m_V^2 \quad ; \quad m_\phi^2 = Z_\phi m_V^2. \quad (27)$$

The constants m_V and μ are fixed to give the correct ω - and ϕ -masses. The other vector meson masses are displayed in table III. The axial vector mesons have a mass around 1 GeV. We refrain from giving their masses explicitly. To treat them appropriately, additional terms are needed [10,22]. This goes beyond the scope of the present paper.

3. Explicit breaking of chiral symmetry

The term

$$\mathcal{L}_{SB} \equiv -\mathcal{V}_{SB} = \frac{\chi^2}{\chi_0^2} \text{Tr}(f\Sigma) = \frac{\chi^2}{\chi_0^2} \left(m_\pi^2 f_\pi \sigma + (\sqrt{2}m_K^2 f_K - \frac{1}{\sqrt{2}}m_\pi^2 f_\pi) \zeta \right) \quad (28)$$

breaks the chiral symmetry explicitly and makes the pseudoscalar mesons massive⁸. It is scaled appropriately to have scale dimension equal to that of the quark mass term $\sim m_q \bar{q}q + m_s \bar{s}s$, which is present in the QCD Lagrangian with massive quarks. This term leads to a nonvanishing divergence of the axial currents. The matrix elements of $f = 1/\sqrt{2}(f_0\lambda_0 + f_8\lambda_8)$ were written as a function of $m_\pi^2 f_\pi$ and $m_K^2 f_K$ to satisfy the (approximately valid) PCAC relations for the π - and K -mesons,

$$\partial_\mu A_\pi^\mu = m_\pi^2 f_\pi \pi, \quad \partial_\mu A_K^\mu = m_K^2 f_K K \quad . \quad (29)$$

Then, by utilizing the equations of motion, the VEV of σ and ζ are fixed in terms of f_π and f_K , i.e.:

$$\sigma_0 = -f_\pi \quad \zeta_0 = \frac{1}{\sqrt{2}}(f_\pi - 2f_K). \quad (30)$$

Since no relation for a partially conserved dilatational current is known, the VEV for the gluon condensate remains undetermined.

⁷One could split the $\rho - \omega$ mass degeneracy by adding a term of the form [10] $(\text{Tr}F_{\mu\nu})^2$ to Eq. (26). Alternatively, one could break the SU(2) symmetry of the vacuum allowing for a nonvanishing vacuum expectation value of the scalar isovector field. However, the $\rho - \omega$ mass splitting is small ($\sim 2\%$), we will not consider this complication.

⁸One may wonder why —besides the explicit symmetry breaking term (14) in the baryon-meson sector— a second chiral noninvariant contribution is needed. This is due to our ignorance as to how to transform the current quark picture into the constituent quark picture.

D. Total Lagrangian

The kinetic energy terms for the fermions and mesons are:

$$\mathcal{L}_{kin} = i\text{Tr}\bar{\Psi}\gamma_\mu\partial^\mu\Psi + \frac{1}{2}\text{Tr}(\partial_\mu M^\dagger\partial^\mu M) + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{4}\text{Tr}(F_{\mu\nu}F^{\mu\nu}) - \frac{1}{4}\text{Tr}(G_{\mu\nu}G^{\mu\nu}). \quad (31)$$

The total general Lagrangian is the sum:

$$\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}_{BM} + \mathcal{L}_{BV} + \mathcal{L}_{vec} + \mathcal{L}_0 + \mathcal{L}_{SB}, \quad (32)$$

with $\mathcal{L}_{vec} = \mathcal{L}_{vec}^{(1)} + \mathcal{L}_{vec}^{(2)}$. For \mathcal{L}_{BM} , we will discuss the effect of various possibilities mentioned in section II C 1 regarding the nuclear matter fits and the hyperon potentials.

E. Mean field Lagrangian

To investigate hadronic matter properties at finite baryon density we adopt the mean-field approximation (see, e.g., [23]). In this approximation scheme, the fluctuations around constant vacuum expectation values of the field operators are neglected:

$$\begin{aligned} \sigma(x) &= \langle\sigma\rangle + \delta\sigma \rightarrow \langle\sigma\rangle \equiv \sigma; & \zeta(x) &= \langle\zeta\rangle + \delta\zeta \rightarrow \langle\zeta\rangle \equiv \zeta \\ \omega_\mu(x) &= \langle\omega\rangle\delta_{0\mu} + \delta\omega_\mu \rightarrow \langle\omega_0\rangle \equiv \omega; & \phi_\mu(x) &= \langle\phi\rangle\delta_{0\mu} + \delta\phi_\mu \rightarrow \langle\phi_0\rangle \equiv \phi. \end{aligned} \quad (33)$$

The fermions are treated as quantum mechanical one-particle operators. The derivative terms can be neglected and only the time-like component of the vector mesons $\omega \equiv \langle\omega_0\rangle$ and $\phi \equiv \langle\phi_0\rangle$ survive if we assume homogeneous and isotropic infinite baryonic matter. Additionally, due to parity conservation we have $\langle\pi_i\rangle = 0$. After performing these approximations, the Lagrangian (32) becomes

$$\begin{aligned} \mathcal{L}_{BM} + \mathcal{L}_{BV} &= -\sum_i \bar{\psi}_i [g_{i\omega}\gamma_0\omega^0 + g_{i\phi}\gamma_0\phi^0 + m_i^*]\psi_i \\ \mathcal{L}_{vec} &= \frac{1}{2}m_\omega^2\frac{\chi^2}{\chi_0^2}\omega^2 + \frac{1}{2}m_\phi^2\frac{\chi^2}{\chi_0^2}\phi^2 \\ \mathcal{V}_0 &= \frac{1}{2}k_0\chi^2(\sigma^2 + \zeta^2) - k_1(\sigma^2 + \zeta^2)^2 - k_2\left(\frac{\sigma^4}{2} + \zeta^4\right) - k_3\chi\sigma^2\zeta \\ &\quad + k_4\chi^4 + \frac{1}{4}\chi^4\ln\frac{\chi^4}{\chi_0^4} - \frac{\delta}{3}\ln\frac{\sigma^2\zeta}{\sigma_0^2\zeta_0} \\ \mathcal{V}_{SB} &= \left(\frac{\chi}{\chi_0}\right)^2 \left[m_\pi^2 f_\pi \sigma + (\sqrt{2}m_K^2 f_K - \frac{1}{\sqrt{2}}m_\pi^2 f_\pi)\zeta \right], \end{aligned}$$

with the effective mass of the baryon i , which is defined according to section II C 1.

1. Grand canonical ensemble

It is straightforward to write down the expression for the thermodynamical potential of the grand canonical ensemble Ω per volume V at a given chemical potential μ and zero temperature:

$$\frac{\Omega}{V} = -\mathcal{L}_{vec} - \mathcal{L}_0 - \mathcal{L}_{SB} - \mathcal{V}_{vac} - \sum_i \frac{\gamma_i}{(2\pi)^3} \int d^3k [E_i^*(k) - \mu_i^*] \quad (34)$$

The vacuum energy \mathcal{V}_{vac} (the potential at $\rho = 0$) has been subtracted in order to get a vanishing vacuum energy. γ_i denote the fermionic spin-isospin degeneracy factors ($\gamma_N = 4$, $\gamma_\Sigma = 6$, $\gamma_\Lambda = 2$, $\gamma_\Xi = 4$). The single particle energies are $E_i^*(k) = \sqrt{k_i^2 + m_i^{*2}}$ and the effective chemical potentials read $\mu_i^* = \mu_i - g_{\omega i}\omega - g_{\phi i}\phi$.

2. Equations of motion

The mesonic fields are determined by extremizing $\frac{\Omega}{V}(\mu, T = 0)$:

$$\frac{\partial(\Omega/V)}{\partial\chi} = -\omega^2 m_\omega^2 \frac{\chi}{\chi_0^2} + k_0 \chi (\sigma^2 + \zeta^2) - k_3 \sigma^2 \zeta + \left(4k_4 + 1 + 4 \ln \frac{\chi}{\chi_0} - 4 \frac{\delta}{3} \ln \frac{\sigma^2 \zeta}{\sigma_0^2 \zeta_0} \right) \chi^3 \quad (35)$$

$$+ 2 \frac{\chi}{\chi_0^2} \left[m_\pi^2 f_\pi \sigma + (\sqrt{2} m_K^2 f_K - \frac{1}{\sqrt{2}} m_\pi^2 f_\pi) \zeta \right] = 0$$

$$\begin{aligned} \frac{\partial(\Omega/V)}{\partial\sigma} &= k_0 \chi^2 \sigma - 4k_1 (\sigma^2 + \zeta^2) \sigma - 2k_2 \sigma^3 - 2k_3 \chi \sigma \zeta - 2 \frac{\delta \chi^4}{\sigma} \\ &+ \left(\frac{\chi}{\chi_0} \right)^2 m_\pi^2 f_\pi + \sum_i \frac{\partial m_i^*}{\partial \sigma} \rho_i^s = 0 \end{aligned} \quad (36)$$

$$\begin{aligned} \frac{\partial(\Omega/V)}{\partial\zeta} &= k_0 \chi^2 \zeta - 4k_1 (\sigma^2 + \zeta^2) \zeta - 4k_2 \zeta^3 - k_3 \chi \sigma^2 - \frac{\delta \chi^4}{\zeta} \\ &+ \left(\frac{\chi}{\chi_0} \right)^2 \left[\sqrt{2} m_K^2 f_K - \frac{1}{\sqrt{2}} m_\pi^2 f_\pi \right] + \sum_i \frac{\partial m_i^*}{\partial \zeta} \rho_i^s = 0. \end{aligned} \quad (37)$$

The vector fields ω and ϕ are determined from $\frac{\partial(\Omega/V)}{\partial\omega} = 0$ and $\frac{\partial(\Omega/V)}{\partial\phi} = 0$, respectively. They may be solved explicitly yielding

$$\omega = \frac{g_{i\omega} \rho_i^s \chi_0^2}{m_\omega^2 \chi^2}, \quad \phi = \frac{g_{i\phi} \rho_i^s \chi_0^2}{m_\phi^2 \chi^2}. \quad (38)$$

The scalar densities ρ_i^s and the vector densities ρ_i can be calculated analytically, yielding

$$\begin{aligned} \rho_i^s &= \gamma_i \int \frac{d^3 k}{(2\pi)^3} \frac{m_i^*}{E_i^*} = \frac{\gamma_i m_i^*}{4\pi^2} \left[k_{Fi} E_{Fi}^* - m_i^{*2} \ln \left(\frac{k_{Fi} + E_{Fi}^*}{m_i^*} \right) \right] \\ \rho_i &= \gamma_i \int_0^{k_{Fi}} \frac{d^3 k}{(2\pi)^3} = \frac{\gamma_i k_{Fi}^3}{6\pi^2}. \end{aligned} \quad (39)$$

The energy density and the pressure follow from the Gibbs–Duhem relation, $\epsilon = \Omega/V + \mu_i \rho_i^s$ and $p = -\Omega/V$. Applying the Hugenholtz–van Hove theorem [24], the Fermi surfaces are given by $E^*(k_{Fi}) = \sqrt{k_{Fi}^2 + m_i^{*2}} = \mu_i^*$.

III. RESULTS

The scope of the present paper is to explore whether it is possible to describe nuclear-matter properties reasonably well within the framework of the $SU(3)_L \times SU(3)_R$ σ -model. Therefore, we discuss only the results for the limit of vanishing *net* strangeness. The case of finite strangeness will be discussed in a forthcoming publication [25]. However, there are strong implications of the Lagrangian for the hyperon potentials and for high densities, which will be elaborated in the following.

A. Fits to nuclear matter and the hadron masses

A salient feature of all chiral models are the strong vacuum constraints. In the present case they fix k_0 , k_2 and k_4 , in order to minimize the thermodynamical potential Ω in vacuum for given values of the fields σ_0 , ζ_0 and χ_0 . Note that these parameters could also be eliminated by adding appropriate chirally invariant terms to ensure that the vacuum energy is minimal for given values of σ_0 , ζ_0 and χ_0 . The parameter k_3 is fixed to the η -mass m_η . There is some freedom to vary parameters, mainly due to the unknown mass of the σ -meson, m_σ , which is determined by k_1 , and due to the uncertainty of the value for the kaon decay constant f_K . While the kaon decay constant is not known precisely, the value for f_π is known very well. Hence, we keep f_π fixed to 93 MeV and vary f_K in the range 115 ± 5 MeV.

In order to reproduce the correct nuclear matter properties, two of the parameters have to be adjusted to the medium.

We choose $g_{N\omega}$ and χ_0 to fit the binding energy of nuclear matter $\epsilon/\rho_B - m_N = -16$ MeV at the saturation density $\rho_0 = 0.15 \text{ fm}^{-3}$. It should be noted that a reasonable nuclear matter fit with acceptable compressibility [26] can be found (row L in table III), where $m_\sigma \approx 500$ MeV. This, in the present approach, allows for an interpretation of the σ -field as the chiral partner of the π -field and as the mediator of the mid-range attractive force between nucleons, though we believe the phenomenon is in reality generated through correlated two-pion exchange [27].

Generally, the fits of table III have an effective nucleon mass of $m_N^* = (0.7-0.75)m_N$ and a compressibility of about 300 MeV. Although these values are reasonable, it might be desirable to fine tune them in order to get acceptable fits to nuclei, too. This could be done by adding a quartic self-interaction of spin-1 mesons, e.g., $\text{Tr} [(V_\mu + A_\mu)^4 + (V_\mu - A_\mu)^4]$.

B. Hyperon potentials

Besides the observables pressure p , energy per baryon ϵ/ρ_B , compressibility K and effective nucleon mass m_N^*/m_N at ground state density ρ_0 , there are some additional important constraints in the medium due to hypernuclear physics: The (relativistic) potential depths U_i of the baryons at ρ_0 , which can serve as input to restrict also the ‘nonstrange’ parameters,

$$U_i = m_i^* - m_i + g_{\omega i} \omega, \quad i = N, \Lambda, \Sigma, \Xi. \quad (40)$$

Experimentally, one finds for the Λ -hyperons a potential of $U_\Lambda = -30 \pm 3$ MeV [28]. For the Σ -potential the situation is unclear, since there is no evidence for bound Σ -hypernuclei. The predictions range from completely unbound Σ 's [29] to $U_\Sigma = -25 \pm 5$ MeV [30]. For Ξ -hyperons, several bound Ξ -hypernuclei candidates have been reported [28]. The potential for the Ξ -hyperon has been extracted to $U_\Xi = -25 \pm 5$ MeV.

The Yukawa-type chirally invariant baryon-meson interaction gives an acceptable mass spectrum of mesons and baryons (row L of table III), and also, the compressibility has a reasonable value ($K \approx 300$ MeV). However, the potential depths of the hyperons are very deep. This is mainly due to the baryon-vector and baryon-scalar meson coupling constants, which determine the strength of the vector and scalar potential, respectively (see Eq. 40). Once $g_{N\sigma}$ and $g_{N\omega}$ are fixed to the nucleon mass and the nuclear potential, the coupling constants of strange baryons to mesons are determined by symmetry relations. As discussed in section II C 1, chiral symmetry restricts the coupling of spin-0 mesons to baryons to a symmetric (d-type) one. This destroys the balance between repulsion and attraction, since the baryon-vector coupling is antisymmetric (f-type), i.e. $g_{\Sigma\sigma} = 0$, whereas $g_{\Sigma\omega} = \frac{2}{3}g_{N\omega}$. To cure this deficiency, nonlinear baryon-meson interaction terms can be introduced, which are also chirally invariant. They lead to coupling constants which differ from the Yukawa-type baryon-meson interaction.

The results of fits to nuclear matter are shown in the rows 2-4 of table III. If quadratic baryon-meson interactions (Q-fit) are used, the hyperon potentials are still too deep. *Cubic* baryon meson interactions (C-fit) allow for a coupling of the strange condensate to the nucleon, such that all baryon potentials are acceptable. This is because the scalar coupling constants approach those for the f-type coupling (Eq. 19). This implies that nonstrange mesons couple according to the OZI rule, i.e. exclusively to the up and down quark, but not to the strange quark. With such a coupling scheme, hypernuclei can be reasonably well described [31]. The potentials of the Σ - and Λ -hyperons are then equal since their density-dependent mass terms are the same (see fourth column of table II). It is remarkable that in this nonlinear scheme a coupling of the strange condensate to nucleons is necessary to yield a Λ -potential of the right magnitude.

Other possibilities than the cubic form of baryon-meson interaction may exist to yield realistic hyperon potentials. The explicit symmetry breaking term in Eq. (14) has no influence on the potential, since it is not medium dependent. Other forms of explicit symmetry breaking, which involve the meson fields (they are listed in [18]), either fail to generate the experimentally known baryon mass spectrum or give unrealistically high/low potentials⁹.

We have also checked the inclusion of a d-type coupling of baryons to spin-1 mesons. Then, the couplings in the baryon-scalar and baryon-vector meson sector can be chosen to be of the same magnitude. Indeed, a pure d-coupling of baryons to spin-1 mesons leads to acceptable hyperon potentials! However, this yields negative couplings of nucleons to the ρ -meson, in contrast to experiment. Correcting this deficiency by adding chiral symmetry breaking terms into baryon-vector sector seems artificial and it does not correct the contradiction to vector meson dominance and to the universality principle [32,19].

The nonlinear (cubic) baryon-meson interaction term that gives reasonable hyperon potentials (row C in table III)

⁹If one includes four instead of two parameters, then it is of course possible to fit both the potentials *and* the baryon masses simultaneously, however for the price of losing the predictive power.

can be considered as an effective description of baryons interacting with multi-quark states. This interpretation is analogous to the common view of the σ -meson in the one-boson-exchange models as a effective parameterization of the correlated two-pion exchange.

C. Equation of state and effective baryon masses

In spite of all the differences in the baryon-meson interaction, all fits of table III lead to almost¹⁰ identical equations of state (see Fig. 1). In contrast, the density dependence of the condensates is characteristic for the specific form of the chirally invariant baryon-spin-0 meson coupling used (Fig. 2).

If the baryons are coupled linearly to spin-0 mesons, the (nonstrange) σ -field decreases linearly at low densities, and then it saturates at nearly 40 % of its VEV (Fig. 2L). This behavior is in contrast to the linear Walecka model where $m_N^* \rightarrow 0$. The strange condensate ζ changes only slightly in the nuclear medium, since it does not couple to the scalar density of nucleons.

This is different for the quadratic (Fig. 2Q) and cubic (Fig. 2C) forms of baryon-meson interaction. There, the strange and nonstrange fields couple either equally to the nucleons (see column Q of table II), or even stronger than σ (column C of table II). Consequently, the medium dependence of the strange condensate becomes stronger, and that of σ weakens with increasing the nonlinearity of the baryon-scalar meson coupling. The dilaton χ changes negligibly in the medium for all kinds of interaction terms, since it corresponds to a heavy (> 1 GeV) particle, and it does not couple to the scalar density of nucleons.

The baryon masses are generated dynamically through the strange and nonstrange condensates. Therefore, they are density dependent, too (Fig. 3). Their medium behavior follows from that of the condensates and from the chirally invariant ‘mass terms’ of table II. At high densities, the masses saturate (or even increase), in contrast to the masses in the Walecka model, which drop dramatically. The main difference between the various fits is the density dependence of the mass of the Ξ -hyperon, which is weakest for the cubic baryon-meson interaction, since there it couples only to σ .

D. Chiral symmetry restoration

Although the Lagrangian with the three different types of baryon-meson interaction is chirally invariant, there is no chiral phase transition at high baryon densities. This is not a deficit of our (purely hadronic) model, since at very high densities the mean-field model with parameters fixed at ρ_0 is most probably out of its range of applicability. Furthermore, it is unclear, whether a chiral symmetry restoration at high densities takes place or not [33].

However, in the chiral $\sigma - \omega$ model, a solution besides the one describing normal nuclear matter can be found, which has the features of a chirally restored phase with, e.g., a vanishing effective nucleon mass. This abnormal solution exists only for a certain range of parameters. As pointed out in [5], the abnormal phase does only exist, if the Lagrangian does not include terms which lead to a contribution in the equation of motion proportional to $1/\sigma$ or higher powers of it. The logarithmic term $\sim \ln \det \sigma^2 \zeta$ is such an example. For the linear baryon-meson interaction, the absence of such a term leads to an unrealistically large nuclear matter compressibility of $K \approx 1400$ MeV [5]. This is not the case for the cubic baryon-meson interaction. There, even with $\delta = 0$, the compressibility is about $K \approx 300$ MeV. Therefore, the nonlinear coupling of baryons to scalar mesons reduces the compressibility as compared to the Yukawa-type of coupling and makes the equations of state softer. However, the abnormal solution following from such a fit is absolutely stable even at ρ_0 . It is possible to shift the abnormal phase to higher energies, so that it becomes metastable, if the term

$$\mathcal{L}_{vec}^2 = g_2 \text{Tr} [(V_\mu + A_\mu) M M^\dagger (V^\mu + A^\mu) + (V_\mu - A_\mu) M^\dagger M (V^\mu - A^\mu)] \quad (41)$$

is included, and the effective ω -meson mass is generated predominantly by σ , e.g.

$$\omega = \frac{g_{i\omega} \rho^i}{m_V^2 \chi^2 / \chi_0^2 + g_2 \sigma^2} \quad (42)$$

¹⁰At higher baryon densities the fits L, Q, and C deviate from each other, since their compressibilities are slightly different.

where m_V and g_2 are fixed to the masses m_ω and m_ϕ (here, the renormalization of the ω -field is neglected by setting $\mu = 0$, see Eq. 25). A fit with $g_2 = 30.0$ and $m_V = 594.7$ MeV, a reasonable compressibility and realistic hyperon potentials is given in table III (C_a -fit, a stands for abnormal¹¹). As shown in Fig. 1, the abnormal phase of nearly massless nucleons has—at zero net strangeness—always a higher energy than the phase describing normal nuclear matter. In contrast to the SU(2)-equation of state [4,5], the abnormal and normal branch do not cross each other, so that no phase transition occurs at high baryon densities. Nevertheless, it is instructive to look at the condensates and the baryonic masses of the abnormal phase: Although the onset of a phase transition is highly parameter dependent, the features of the abnormal or chirally restored phase are not.

In contrast to the nearly vanishing σ field, the strange scalar field ζ has a high value in the abnormal phase (Fig. 4). This is connected to the absence of repulsion in the strange sector: There is no contribution from the ω (since it does not couple to the strange condensate), and the ϕ (which depends on ζ) does not couple to the nucleon density. In the abnormal (chiral) phase not all baryon masses vanish. Their mass difference is due to the explicit symmetry breaking term (14).

A thorough analysis of the parameter dependence and the onset of the chiral phase transition at high densities and nonzero strangeness fraction will be postponed until finite nuclei are described satisfactorily with the cubic baryon-meson interaction [34].

Although the cubic baryon–spin–0 meson interaction term gives reasonable results for infinite nuclear matter, it seems a rather artificial construction. The question still remains as to whether it is possible to keep both the Yukawa-type baryon-meson interaction and at the same time to yield reasonable hyperon potentials in a chiral model. A model which gives a positive answer is proposed in the following section.

IV. A MODEL WITH HIDDEN CHIRAL SYMMETRY

The difficulties encountered when chirally invariant baryon-meson interactions are introduced is presumably related to the large mass of the baryons as compared to the mass of the pion. At this energy scale chiral symmetry is known to be a useful concept. A general framework on how to add ‘heavy particles’ without destroying chiral symmetry was presented in the classic papers of refs. [35–37]. The idea is to go over to a representation where the heavy particles transform equally under left and right rotations. To accomplish this, it is necessary to dress these particles nonlinearly with pseudoscalar mesons. The application of this method to our approach has the following advantages:

- the Yukawa-type baryon/spin–0 meson interaction can be retained,
- the strange baryons have reasonable potential depths,
- the heavy particles transform in the $SU(3)_V$ space, i.e., their interaction terms are not restricted by chiral symmetry, which is expected to hold mostly for light particles,
- baryon masses can be fitted without explicit symmetry breaking terms,
- a connection to the phenomenologically successful Walecka-model exists.

In the following we will outline the argumentation. For a thorough discussion, see [34].

Let the elementary spinors (=quarks) q introduced in section II A transform into ‘new’ quarks \tilde{q} by

$$q_L(x) = U(x)\tilde{q}_L(x) \quad q_R(x) = U^\dagger(x)\tilde{q}_R(x) \quad (43)$$

with the pseudoscalar octet π_a arranged in $U(x) = \exp[-i\pi_a\lambda^a/2]$. Since the algebraic composition of mesons in terms of quarks is known (see section II A 1), it is straightforward to transform from ‘old’ mesons Σ and Π into ‘new’ mesons X and Y :

$$M = \Sigma + i\Pi = U(X + iY)U \quad . \quad (44)$$

Here, the parity even part X is associated with the scalar nonet, whereas Y is taken to be the pseudoscalar singlet [38]. In a similar way, the ‘old’ baryon octet Ψ forming the representation (8,1) and (1,8) is transformed into a ‘new’ baryon octet B :

¹¹For a correct description of the axial vector meson mass splitting, a term of the form $\text{Tr}[(V_\mu + A_\mu)M(V^\mu - A^\mu)M^\dagger]$ should be added.

$$\Psi_L = UB_LU^\dagger \quad \Psi_R = U^\dagger B_RU \quad . \quad (45)$$

The transformations of the exponential U are known [36,37],

$$U' = LUV^\dagger = VUR^\dagger, \quad (46)$$

and with the ‘old’ fields from table I, the ‘new’ baryons B and the ‘new’ scalar mesons X transform as¹²:

$$\begin{aligned} B'_L &= VU^\dagger L^\dagger \cdot L\Psi_L L^\dagger \cdot LUV^\dagger = VB_LV^\dagger \\ B'_R &= VUR^\dagger \cdot R\Psi_R R^\dagger \cdot RU^\dagger V^\dagger = VB_RV^\dagger \\ X' &= \frac{1}{2}(VU^\dagger L^\dagger \cdot LMR^\dagger \cdot RU^\dagger V^\dagger + VUR^\dagger \cdot RM^\dagger L^\dagger \cdot LUV^\dagger) = V XV^\dagger \end{aligned} \quad (47)$$

The pseudoscalars reappear in the transformed model as the parameters of the symmetry transformation. Therefore, chiral invariants (without space-time derivatives) are independent of the Goldstone bosons. Hence, in mean field approximation, the potential (20) does not change its form (see also [39]). Furthermore, the ‘new’ fields allow for invariants which are forbidden for the ‘old’ fields by chiral symmetry: Since the baryons and scalar mesons now transform equally in the left and right subspace, the f-type coupling for the baryon-meson interaction is now allowed. The invariant linear interaction terms of baryons to scalar mesons are

$$\mathcal{L}_{BX} = g_8^S (\alpha [\overline{B}BX]_F + (1 - \alpha) [\overline{B}BX]_D) + g_1^S \text{Tr}(\overline{B}B) \text{Tr}X, \quad (48)$$

with $[\overline{B}BX]_F := \text{Tr}(\overline{B}BX - \overline{B}XB)$ and $[\overline{B}BX]_D := \text{Tr}(\overline{B}BX + \overline{B}XB) - \frac{2}{3}\text{Tr}(\overline{B}B)\text{Tr}M$. In contrast to table II (first column), the baryon masses have an additional dependence on α :

$$\begin{aligned} m_N &= m_0 - \frac{1}{3}g_8^S(4\alpha - 1)(\sqrt{2}\zeta - \sigma) \\ m_\Lambda &= m_0 - \frac{2}{3}g_8^S(\alpha - 1)(\sqrt{2}\zeta - \sigma) \\ m_\Sigma &= m_0 + \frac{2}{3}g_8^S(\alpha - 1)(\sqrt{2}\zeta - \sigma) \\ m_\Xi &= m_0 + \frac{1}{3}g_8^S(2\alpha + 1)(\sqrt{2}\zeta - \sigma) \end{aligned} \quad (49)$$

with $m_0 = g_1^S(\sqrt{2}\sigma + \zeta)/\sqrt{3}$. The three parameters g_1^S , g_8^S and α can be used to fit the baryon masses to their experimental values. Then, no additional explicit symmetry breaking term is needed. For $\alpha = 0$ and $g_1^S = -\sqrt{2/3}g_8^S$, the d-type coupling of table II is recovered, and for $\zeta = \sigma/\sqrt{2}$ (i.e. $f_\pi = f_K$), the masses are degenerate, and the vacuum is $SU(3)_V$ invariant. The potentials following from the fit to nuclear matter are for $\alpha = 1.13$: $U_N = -58.4$ MeV, $U_\Lambda = -39.5$ MeV, $U_\Sigma = -30.0$ MeV, and $U_\Xi = -15.8$ MeV. Note that the sum $U_\Lambda + U_\Sigma$ is independent of the mixing angle α (this can be seen by inserting Eq. (49) in Eq. (40)). As in the cubic fit, a coupling of the strange condensate to the nucleon is necessary to obtain acceptable potential depths.

Since the construction of invariants is only governed by $SU(3)_V$, the form of the ‘new’ Lagrangian is analogous to the one used in RMF-models [23] and allows for equally good results when applied e.g. to finite nuclei [40,34]. In contrast to the Walecka model relations following from chiral symmetry as PCAC and the Goldberger-Treiman relation are incorporated. The model allows also to predict the masses of the meson nonet at zero and finite density [25].

V. SUMMARY AND OUTLOOK

We have presented a chiral $SU(3)_L \times SU(3)_R$ linear σ model for finite baryon density. Besides the meson-meson interaction, which is widely used [9–12], spin-1 mesons and baryons with dynamically generated masses are implemented. In addition, a dilaton field is used to render the Lagrangian scale invariant, except for a scale breaking logarithmic term which simulates the trace anomaly of QCD.

¹²For vector transformations we have L=R=V, whereas for $L \neq R$, V is a complicated nonlinear function of the pseudoscalars $\pi_a(x)$

The parameters are fixed to the hadron masses and to the binding energy of nuclear matter at zero pressure. These parameters can all be related to and are constrained by physical quantities. The equation of state of nuclear matter then has a compressibility constant of about 300 MeV. Nevertheless, the extension to $SU(3)$ is nontrivial, because of the constraints imposed by chiral symmetry on the baryon-meson interaction. The linear form of the interaction leads to coupling constants given by the d_{ijk} -structure constants. Combined with the baryon-vector interaction, which go like f_{ijk} , they generate false hyperon potentials. This problem can be circumvented by using a cubic baryon-meson interaction, whose coupling constants are similar to the f -type ones.

Another possible way out of this dilemma (and maybe more natural) is the nonlinear realization of the σ -model [35,36]. With a nonlinear transformation into ‘new’ scalar fields transforming linearly in $SU(3)_V$ and into ‘new’ pseudoscalar fields transforming nonlinearly, it is possible to construct an f -type baryon–scalar meson interaction. The mixing angle between d and f can then be used to adjust to the known potential of the Λ -hyperon. Furthermore, no additional explicit symmetry breaking mass term for the baryons is needed. The modified form of the Lagrangian can be recast to resemble the nonlinear Boguta–Walecka Lagrangian of ref. [7], which was successfully applied to finite nuclei and hypernuclei. A thorough investigation of this modified model and its connection to the nonchiral mean-field models is presently under way [34].

It is found that both in the cubic form of baryon-scalar meson interaction and in the nonlinear realization of chiral symmetry, the strange condensate needs to be coupled to the nucleon in order to obtain realistic hyperon potentials. This may be viewed as for a large strangeness content of the nucleon [6].

The cubic model (C_a), allows for an abnormal ‘Lee–Wick’ phase with nucleons of nearly vanishing mass. In contrast to $SU(2)$ models involving an abnormal phase [5], here the normal phase, which describes ordinary nuclear matter, has a reasonable compression modulus ($K \approx 300$ MeV). In the abnormal phase, the strange condensate remains—in contrast to the (vanishing) σ field—close to its VEV.

The case of zero net strangeness as well as the effective baryon masses at higher densities were studied here. In a forthcoming publication [34] the extension to finite strangeness and the behavior of the meson masses in matter will be discussed in detail. The application of the model to finite nuclei is currently under investigation.

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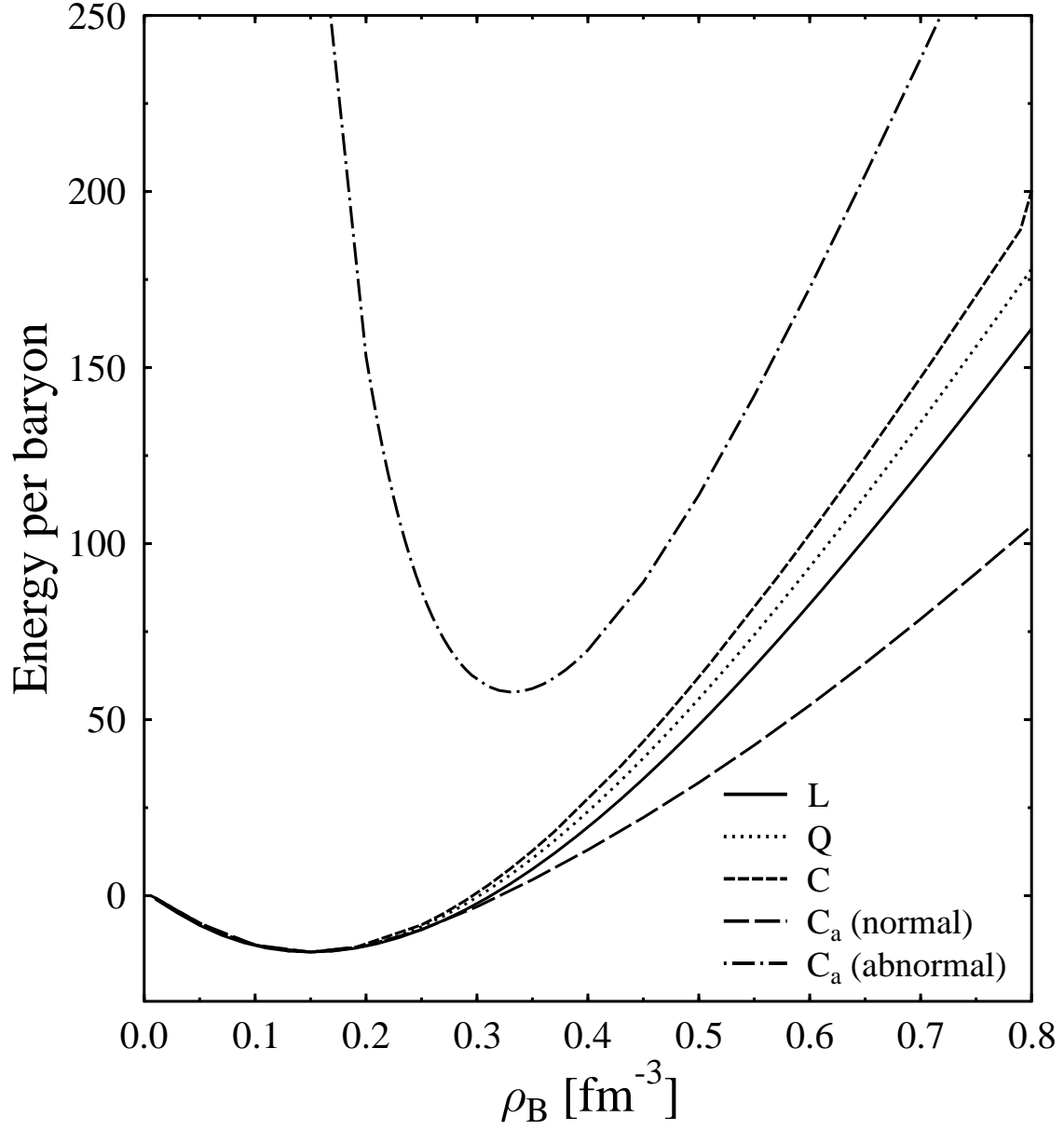


FIG. 1. Binding energy versus baryon density ρ_B for the linear (L), quadratic (Q), and cubic (C) baryon-spin-0 meson interaction (see table III).

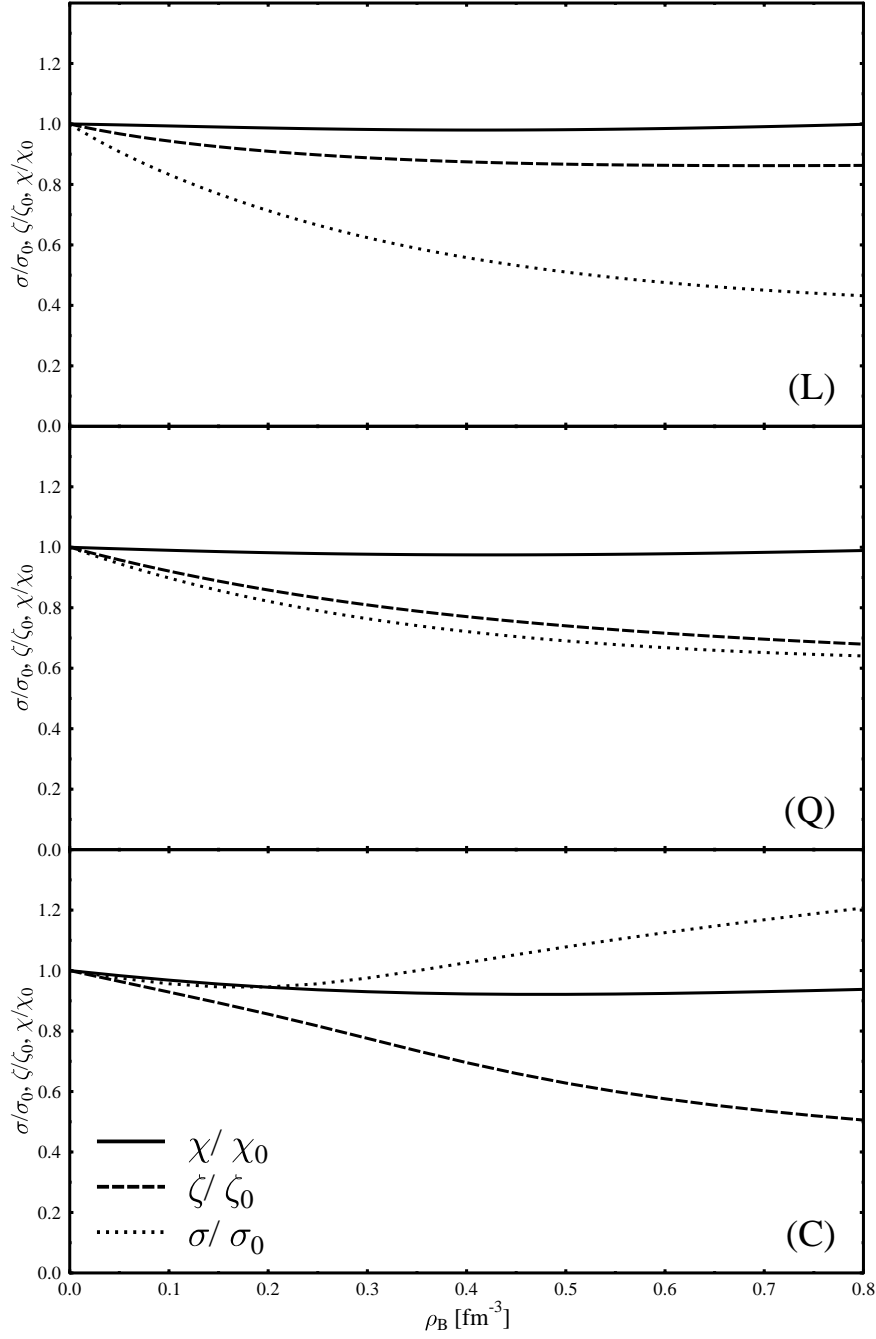


FIG. 2. Nonstrange (σ), strange (ζ) and gluon (χ) S condensates versus baryon density ρ_B for the linear (L), quadratic (Q), and cubic (C) baryon-spin-0 meson interaction (see table III).

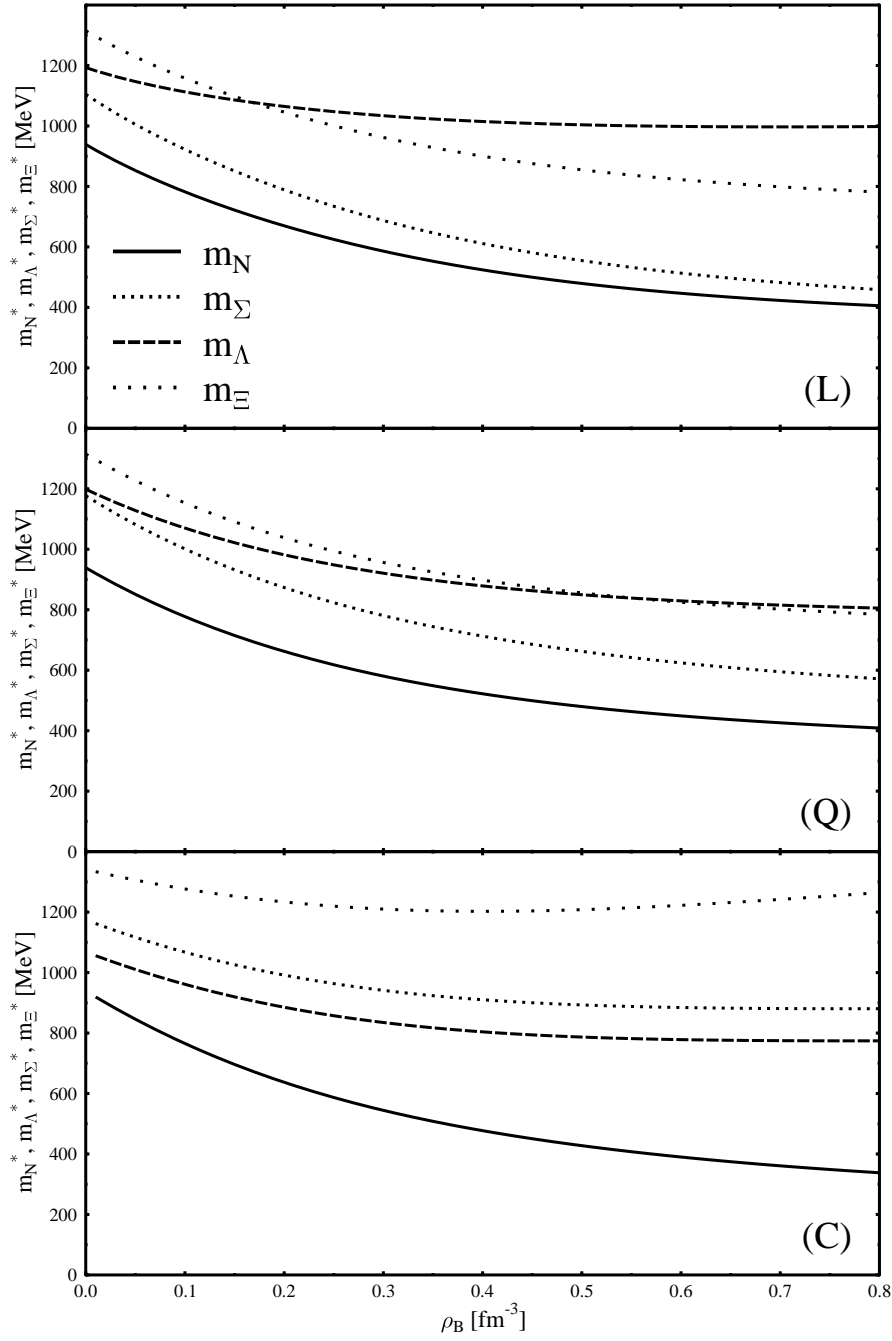


FIG. 3. Baryon masses as a function of the baryon density ρ_B for the linear (L), quadratic (Q), and cubic (C) baryon-spin-0 meson interaction.

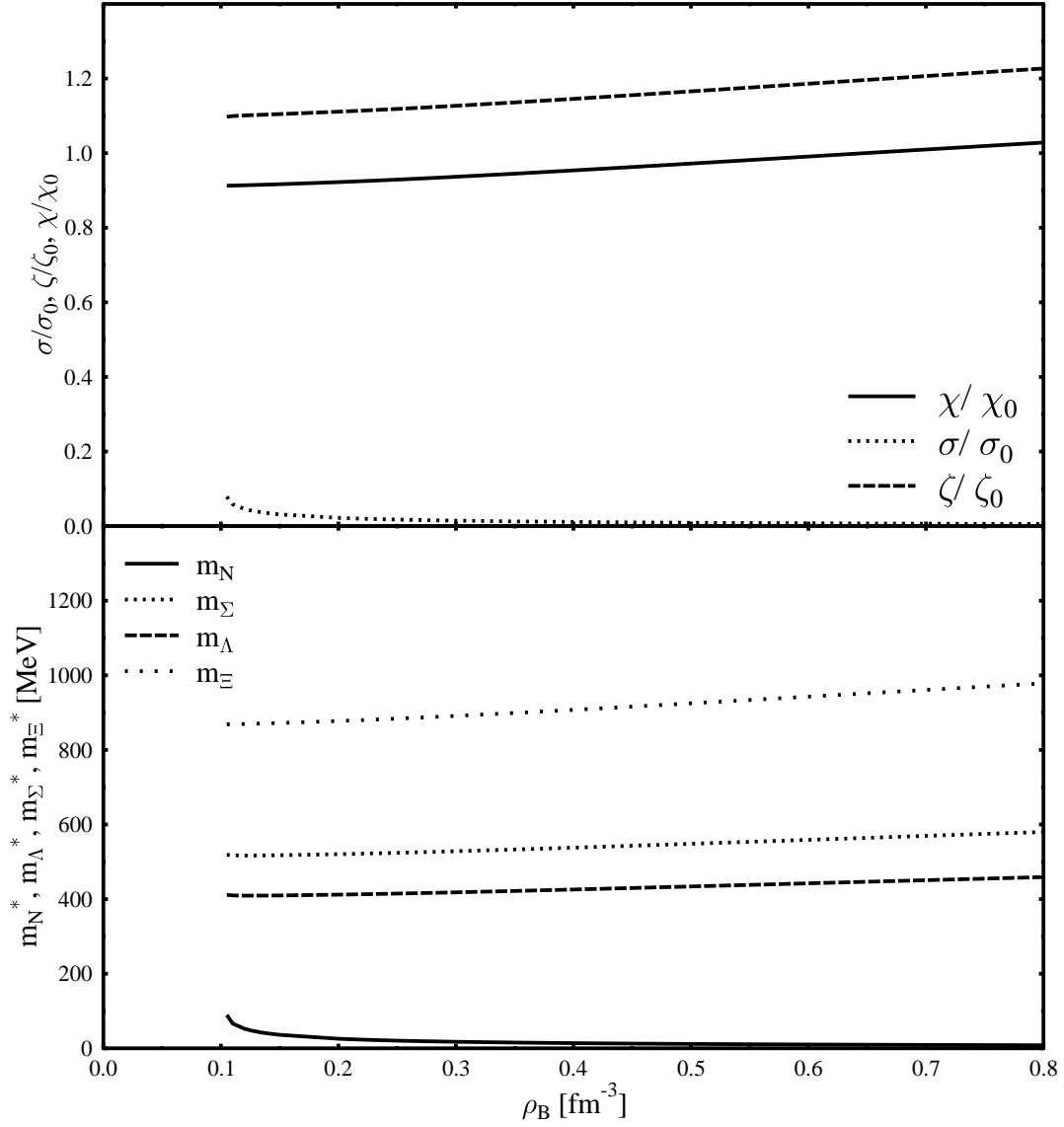


FIG. 4. Nonstrange (σ), strange (ζ) and gluon (χ) condensates (above) and effective baryon masses (below) in the abnormal (chiral) phase.

TABLE I. Chiral transformations of spin-0 mesons ($M = \Sigma + i\Pi$), spin-1 mesons ($V_\mu = l_\mu + r_\mu$ and $A_\mu = l_\mu - r_\mu$) and baryons (see Eq. 10).

Hadrons	J^P	Transformations	
Spin-0 mesons	$0^+, 0^-$	LMR^\dagger	$RM^\dagger L^\dagger$
Spin-1 mesons	$1^-, 1^+$	$Ll_\mu L^\dagger$	$Rr_\mu R^\dagger$
baryons (nonet)	$\frac{1}{2}^+$	$L\Psi_L R^\dagger$	$R\Psi_R L^\dagger$
baryons (octet)	$\frac{1}{2}^+$	$L\Psi_L L^\dagger$	$R\Psi_R R^\dagger$

TABLE II. Mass terms B_{ji} for the baryons (see Eq. 15).

	$j = 0$	$j = 1$	$j = 2$
N	σ	$\sigma\zeta$	$2\sigma\zeta^2$
Ξ	σ	$\sigma\zeta$	σ^3
Λ	$\frac{1}{3}(4\sigma - \sqrt{2}\zeta)$	$\frac{\sqrt{2}}{6}(\sigma^2 + 4\zeta^2)$	$\sqrt{2}\sigma^2\zeta$
Σ	$\sqrt{2}\zeta$	$\frac{1}{\sqrt{2}}\sigma^2$	$\sqrt{2}\sigma^2\zeta$

TABLE III. Parameterizations with linear (L), quadratic (Q), cubic baryon-meson interaction with (C_a) and without (C) abnormal (chiral) phase. All masses are given in MeV.

	Spin-0 particle masses							
	$m_\pi(139)$	$m_K(495)$	$m_\eta(547)$	$m_{\eta'}(958)$	m_{σ_π}	m_{σ_K}	$m_{\sigma_{\eta'}}$	m_{σ_η}
L	139.0	498.0	540.0	931.1	973.3	1065.0	561.0	747.1
Q	139.0	498.0	540.0	972.4	1064.8	1169.3	728.9	993.1
C	139.0	498.0	540.0	954.6	1023.4	1122.8	774.0	1056.6
C_a	139.0	495.0	540.0	946.0	966.8	1041.5	665.5	968.6
	Spin-1 particle masses				Spin- $\frac{1}{2}$ particle masses			
	$m_\omega(783)$	$m_{K^*}(892)$	$m_\rho(770)$	$m_\phi(1020)$	$m_N(939)$	$m_\Lambda(1115)$	$m_\Sigma(1193)$	$m_\Xi(1315)$
L	783.0	878.4	783.0	1020.0	939.0	1104.9	1193.1	1314.9
Q	783.0	878.4	783.0	1020.0	939.0	1177.3	1198.5	1314.9
C	783.0	864.0	783.0	1020.0	939.0	1071.5	1159.5	1339.0
C_a	783.0	867.5	783.0	983.4	939.0	1057.0	1174.7	1348.7
Potential depths [MeV]								
	U_N	U_Λ	U_Σ	U_Ξ	m_1 [MeV]	m_2 [MeV]	$\frac{m_N^*}{m_N}$	K [MeV]
L	-60.6	-149.9	-2.5	-165.5	-230.5	606.4	0.77	279.3
Q	-60.9	-136.4	-68.0	-169.7	531.9	-156.0	0.76	313.6
C	-62.1	-27.9	-27.9	-28.5	512.6	380.7	0.74	329.9
C_a	-61.3	-29.8	-35.0	-47.0	535.3	373.2	0.76	306.3
Parameter								
	33δ	$g_{N\omega}$	k_0	k_1	k_2	k_3	k_4	f_K [MeV]
L	6	9.04	3.77	5.0	-9.25	-0.28	-0.27	117.0
Q	6	9.18	2.63	5.0	-13.57	1.19	-0.26	112.0
C	1.5	9.67	-3.54	-10.0	-11.54	-2.88	-0.07	114.0
C_a	0	9.02	-12.33	-20.0	-7.96	-4.86	0.514	118.0