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**Modelling the role of credit rating agencies-
Do they spark off a virtuous circle?**

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Abstract

In this paper, we propose a model of credit rating agencies using the global games framework to incorporate information and coordination problems. We introduce a refined utility function of a credit rating agency that, additional to reputation maximization, also embeds aspects of competition and feedback effects of the rating on the rated firms. Apart from hinting at explanations for several hypotheses with regard to agencies' optimal rating assessments, our model suggests that the existence of rating agencies may decrease the incidence of multiple equilibria. If investors have discretionary power over the precision of their private information, we can prove that public rating announcements and private information collection are complements rather than substitutes in order to secure uniqueness of equilibrium. In this respect, rating agencies may spark off a virtuous circle that increases the efficiency of the market outcome.

Keywords: information production, rating agencies, coordination problems, global game

JEL-Classification: D82, G14, G33

1 Introduction

Despite a lot of recent research effort, the role of credit rating agencies in the market's investment process is still not very well understood. On the one hand, due to the secrecy of rating agencies the rating process itself remains unclear, on the other hand, observers still struggle to understand how the market perceives this process and how the rating influences the rated firms and their outstanding debt. Especially the lack of a solid theoretical basis in grasping these issues is one of the main shortcomings of the academic research to date.[1] The current paper tries to fill this gap. As one of the first approaches in this respect, we introduce a refined utility function of a credit rating agency along which the optimal rating is derived. In accordance with empirical findings and anecdotal evidence, we consider three different utility components. Additional to maximizing her reputation, we presume that a rating agency also has to take into account competitive pressures from other agencies or from institutions that sell similar products.[2] Furthermore, we suppose that each agency has reason to account for potential feedback effects that her rating may have on the rated firms in order to secure future business with these firms.[3] Based on the optimally derived rating, we then analyze the influence of the rating on investment decisions. Additionally, we discuss the way in which the existence of a rating agency can help to eliminate the uncertainty resulting from multiple equilibria on the market and examine the difference between solicited and unsolicited ratings. Even though we do not tackle the process of the watch-list, we take into account regulatory issues that influence the investment decisions of institutional investors in particular.

Contrary to what has been stated in an earlier paper by Carlson and Hale (2005), we find that the existence of rating agencies does *not* necessarily increase the incidence of multiple equilibria. Rather, as long as the rating agency pursues other aims besides maximizing her reputation, a unique equilibrium may be restored - even though the agency, by announcing her rating publicly and as such increasing the precision of public information in the market, makes coordination among investors easier, thereby facilitating the emergence of multiple equilibria. If investors have discretionary power over the precision of their private information, we can prove that public rating announcements and private information collection are complements rather than substitutes in order to secure the prevalence of a unique equilibrium. In this respect, rating agencies may spark off a virtuous circle: The more accurate the announced ratings are, the higher is investors' incentive to increase the precision of their private information as well, which raises the efficiency of the market outcome.

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[1] This is also one of the main conclusions by Cantor (2004) in the editorial of the November 2004 Special Issue on Credit Ratings of the *Journal of Banking and Finance*.

[2] The view that the credit rating industry is competitive and reputation-driven is the dominant one held by scholars. See for instance Cantor and Packer (1994) and Smith and Walter (2001) as advocates of this view but also Partnoy (2001) for an alternative position.

[3] This point is emphasized in the Special Report Credit-rating agencies of *The Economist* (March 26th 2005).

With regard to the difference between solicited and unsolicited ratings, we find that it is indeed contingent on the gap between the rating agency's private information about the firm's credit quality and the quality a priori expected by the market. Hence, unsolicited rating are seriously downward-biased as compared to solicited ratings for firms who can disclose very optimistic private information about their credit quality to the agency while the market a priori expects a much lower quality. However, solicited ratings are strongly influenced by the different components of the agency's utility function. As such, we find that for sufficiently good private information, a solicited rating will be the higher the more emphasis is put on the reputational aim and the less weight is attached to competitive and feedback concerns. Again, the opposite holds for sufficiently bad private information obtained by the rating agency. Interestingly, if we look at the market effects of ratings, we find that in particular those firms have a high incentive to request a solicited rating that feel to be treated unfairly by the market, i.e. that believe to be able to disclose much more optimistic information to the rating agency than what has commonly been expected by the market. For them, the higher solicited rating would strongly reduce their probability of default. Our model therefore relates the observed downward-bias of unsolicited ratings to an adverse selection problem between firms that believe to be of high quality and those that believe to be of lower quality.

Regarding the rating agency's "ability" to assess the relative risk of corporate bonds, we prove anecdotal evidence to be true that agencies of higher ability tend to announce more extreme ratings than do agencies of lower ability. This requires, however, that they indeed do take into account additional aims than simply maximizing their reputation. If this condition is satisfied, we find that a rating agency of higher ability will announce a higher rating after observing good private information than an agency of lower ability. The opposite holds for the case of bad information.

Finally, we show that the typical bond market segregation between institutionalized and non-institutionalized investors increases the probability of default for lowly-rated firms. This result is due to the fact that institutionalized investors are usually required to invest only in investment-grade firms. For highly-rated firms, the existence of institutional investors is beneficial as long as the offered repayment of the bonds is sufficiently high.

The explicit introduction of a rating agency's utility function notwithstanding, our model does not take into account aspects such as the optimal "level of rating activity" or more generally the optimal collection of private information because we do not allow for additional costs from collecting and processing information. Additionally, we assume the weights attached to the different utility components to be exogenous rather than deriving them optimally and as such assigning a strategic role to the credit rating agency. We leave these further refinements for future research.

The paper is related to various strands of the literature. The first is the literature on reputational concerns and reliability of information provided by certification intermediaries such as rating agencies. Mariano (2005) develops a model which analyzes this relationship in the context of both a monopolistic and a duopolistic certification industry.[4] Because of a different modelling setup, her paper has to abstract from a potential coordination function of the credit rating agency and does not consider multiplicity of equilibria.

The second strand of literature builds on signalling models in the tradition of Spence (1973) and examines how different types of firms can signal their quality to the market when there is asymmetric information between firm insiders and market participants. Byoun and Shin (2002) model signalling behavior of firms in an environment in which firms can choose a rating agency as an information specialist who is able to obtain and convey information at lowest costs. They develop conditions for a separating equilibrium where only good firms signal their quality through the rating agency. Their paper differs from ours in many respects. Most importantly, it does not consider the interplay between private and public information and gives no active role to the credit rating agency.

Most closely related to our work are the papers by Boot et al. (2005) and Carlson and Hale (2005). Boot et al. (2005) show that credit ratings can serve as a coordinating mechanism in situations of multiple equilibria. In addition, they explore in detail how a monitoring role vis-à-vis a firm is put in place through the credit watch procedure used by credit rating agencies. In their model, the practice of institutional investors to condition their investment behavior on the rating enforces an implicit contract between the firm and the credit rating agency where the former promises not to misbehave because this may endanger her credit standing. While we share the focus on multiple equilibria with Boot et al. (2005), the methodological contribution of our paper is quite different. We apply the global games theory to examine under which conditions uniqueness of equilibrium can be achieved. This allows us to investigate in detail the role of a credit rating agency in providing information to the market rather than performing a monitoring role, when the rating agency simultaneously fulfills a coordinating function. Carlson and Hale (2005) also propose a model of rating agencies as an application of the global games methodology. However, they do not introduce an explicit utility function for the rating agency. Further differences between the two papers will be discussed at several occasions during the following sections.

The remainder of the paper is organized as follows. Section 2 describes the basic setup of our model. Section 3 derives the agency's optimal rating. Section 4 discusses the condition for a unique equilibrium. Sections 5 to 7 analyze the difference between solicited and unsolicited ratings and their impact on investors' actions. Section 8 concludes.

[4]Also Benabou and Laroque (1992) and Morris (2001) are important examples of this literature. Both papers develop repeated cheap talk models of a sender of information whose honesty is unknown to the information receivers.

2 The model

In a very simple model we capture the interaction between a firm, a continuum of investors[5] and a credit rating agency (CRA).[6] The firm has outstanding debt from ongoing business projects that has to be repaid at maturity. Based on private and public information about the firm's quality (respectively the projects' quality) and a published credit rating, investors have to decide whether or not to prolong credit in the intermediate period.

We assume that the firm's quality is a random variable θ, normally distributed with mean y and variance $1/a$. The distribution of θ is common knowledge in the market and may as such be referred to as public information. The lower a, the higher is the firm's fundamental risk, since firm quality θ may deviate strongly from the ex-ante expected value y. Each market participant additionally observes a private signal about firm quality: $x_i|\theta \sim N(\theta, 1/b)$. These private signals may simply be characterized as investors' individual interpretations of the publicly available information about firm quality. The higher b, the more closely are investors' private signals distributed around the unknown firm quality θ. In this respect, b denotes the precision of investors' private information. Similarly, the rating agency collects information about the firm that results in a private signal of $x_A|\theta \sim N(\theta, 1/c)$. Note that private signals are assumed to be independent of each other.

The time line of the model is as follows:

- In $t = 0$, the firm has outstanding debt that has to be repaid at a rate of R per unit of debt at maturity ($t = 2$). Firm quality θ is represented by a random variable taken from $N(y, 1/a)$. This distribution is common knowledge in the market.

- In $t = 1$, investors observe individual private signals x_i about firm quality. The rating agency observes a private signal x_A. Based on her information, the rating agency publicly announces the rating z. Investors update their beliefs and decide on whether to prolong credit or withdraw early. Early withdrawal is not connected to any premium or punishment and hence delivers a payment of 1 per unit of capital.

- The firm's projects can mature successfully in $t = 2$, if a proportion of less than θ of outstanding debt has been withdrawn prematurely. The firm then repays debt out of the realized project payoff, which is equal to V. Otherwise the firm defaults.

[5]The assumption of atomistic investors is made in order to keep the analysis as simple as possible. It is not critical for the derivation of results as has been shown by Morris and Shin (2003).

[6]Note that even though we consider only one rating agency explicitly, this agency has to assess credit quality under the threat of potential competition from other agencies. However, competition between agencies is not modelled directly in our setup.

In this simple formulation, V is a constant larger than R. Fundamental variable θ measures the firm's ability to meet short term claims from creditors. This very simple payoff function implies that the recovery rate conditional on default is independent of θ (Morris and Shin, 2004).

In order to derive a benchmark result, let us first consider the case *without* a rating agency. Equilibrium values are denoted by subindices "W". In this case, investors have to base their decisions of whether or not to prolong credit solely on the common prior about θ and on their private signals x_i.

Essentially, the depicted model presents a global game in the sense of Carlsson and van Damme (1993), where each player noisily observes the game's payoff structure, which itself is determined by a random draw from a given class of games. Following the solution method of Morris and Shin (2003, 2004) we can derive a unique equilibrium, provided that private information is sufficiently precise. The equilibrium is then characterized by trigger strategies, so that each investor extends his loan whenever he obtains a private signal x_i higher than a trigger value x^*_W and withdraws credit otherwise. Similarly, the firm defaults if a quality value lower than θ^*_W is realized.

Each investor has to choose either to withdraw his money early, i.e. in $t = 1$, in which case each unit of investment will be safely repaid, or to prolong credit. This risky choice delivers a repayment of R in $t = 2$, if the project matures successfully and zero otherwise. The project will be successful only if the firm's fundamental value θ is sufficiently high, i.e. larger than θ^*_W. The marginal investor will be indifferent between foreclosing and extending credit if both actions deliver the same expected payoff:

$$1 \quad = \quad R \cdot \text{prob}(\theta \geq \theta^*_W | x_i, z) \ . \tag{1}$$

If there is no credit rating agency on the market, investors' posterior beliefs about θ are given by:

$$\theta | x_i \sim N\left(\frac{ay + bx_i}{a+b}, \quad \frac{1}{a+b}\right) . \tag{2}$$

Plugging this in (1) delivers the indifference condition for the individual investor:

$$x^*_W = \frac{a+b}{b}\theta^*_W - \frac{a}{b}y - \frac{\sqrt{a+b}}{b}\Phi^{-1}\Big(\frac{R-1}{R}\Big) . \tag{3}$$

The firm's projects, however, need a critical mass of investment in order to proceed successfully. This condition may also be interpreted as the firm being able to refinance internally a certain amount of withdrawn debt. The amount of refinancing that the firm can master depends on the firm's financial strength or quality. For simplicity it is assumed that the firm has to default whenever the proportion of withdrawn debt, denoted by l, is higher than firm quality θ. The firm will therefore be on the brink of default if:

$$\begin{aligned} \theta \quad &= \quad l = \text{prob}(x \leq x^*_W | \theta) \\ \theta \quad &= \quad \Phi(\sqrt{b}(x_i - \theta)) \ . \end{aligned} \tag{4}$$

Note that due to the assumed independence of private signals, the proportion of investors withdrawing their loans prematurely (after observing sufficiently low private signals) is equivalent to the probability with which an individual investor obtains private information lower than x^*_W.

Combining indifference conditions (3) and (4) yields the equilibrium threshold value θ^*_W below which the firm's projects will optimally be abandoned, since the proportion of withdrawn capital is too high to warrant any further internal refinancing on the part of the firm. For quality values of θ above θ^*_W, however, the projects will be continued with certainty, even though there might still be some credit foreclosure. This withdrawal of capital is yet sufficiently small as not to force the firm into default. Equilibrium value θ^*_W is given by:

$$\theta^*_W = \Phi\Big(\frac{a}{\sqrt{b}}(\theta^*_W - y) - \sqrt{\frac{a+b}{b}}\Phi^{-1}\Big(\frac{R-1}{R}\Big)\Big) . \tag{5}$$

Given θ^*_W, the equilibrium threshold value x^*_W for investors' private signals is given by (3). Hence, an investor will foreclose his loan early whenever he observes a private signal about firm quality lower than x^*_W and rolls over otherwise.

As a sufficient condition for uniqueness of equilibrium, consider that the two indifference conditions as represented by (3) and (4) must not cross more than once. Solving (4) for x_i and deriving both functions with respect to θ^*, it can easily be seen that a sufficient condition for a unique equilibrium is given by $b > a^2/(2\pi)$. It requires that *behavioral uncertainty*, introduced via the variance in traders' individual private signals, $1/b$, does not become too strong as compared to *fundamental uncertainty* as represented by the variance $1/a$ of the firm's quality value θ.[7] Stated differently, private information on the part of investors must be sufficiently precise relative to the precision a of public information about θ.

Note that if the exact value of θ were common knowledge in the market, multiple equilibria would be obtained for $\theta \in [0, 1]$. For these intermediate values of θ it would therefore not be possible to predict whether the firm's debt would default or not as both outcomes are fully rational. Whenever an investor believed that sufficiently many other investors might withdraw their loans early, it would be optimal for him to withdraw as well, thereby vindicating any investor's decision to foreclose the loan, so that eventually they will all do so. If, in contrast, he expected others to extend their loans, it would be profit-maximizing for him to extend as well, leading to coordination on the efficient equilibrium. The complete lack of behavioral uncertainty hence gives rise to multiple equilibria based on self-fulfilling expectations. Even if the public knowledge about the firm's quality were faulty, so that fundamental uncertainty prevailed, multiplicity of equilibria would not vanish because investors still held homogeneous, yet erroneous, beliefs (Sbracia and Zaghini, 2001), which guarantees behavioral certainty. Multiple equilibria may, however, be eliminated if investors form heterogeneous beliefs about θ due to individual private information. As long as behavioral uncertainty arising from

[7] What we call behavioral uncertainty corresponds to the notion of *strategic uncertainty* in Morris and Shin (2002, 2003, 2004). Both terms will be used interchangeably.

imprecise private signals is not too strong, a unique equilibrium is obtained that allows to predict the aggregate market solution for each quality value θ.

Now consider the case of a credit rating agency publicly announcing her rating in $t = 1$. For the time being, in order to simplify the presentation of equilibrium we assume the rating z to be exogenously given and normally distributed with a variance of $1/d$. The optimal rating will endogenously be derived in section 3.

The announcement of rating z brings additional public information to the market and leads investors to update their beliefs to:

$$\theta | x_i, z \sim N\Big(\frac{ay + bx_i + dz}{a+b+d}, \frac{1}{a+b+d}\Big) .$$

Based on the same analysis as above, the unique equilibrium value for firm quality with a rating agency being present, θ^*, is then derived as:

$$\theta^* = \Phi\left(\frac{1}{\sqrt{b}}\Big(a(\theta^* - y) + d(\theta^* - z) - \sqrt{a+b+d}\,\Phi^{-1}\Big(\frac{R-1}{R}\Big)\Big)\right) , \qquad (6)$$

with the equilibrium value for private signals given by:

$$x^* = \frac{a+b+d}{b}\theta^* - \frac{a}{b}y - \frac{d}{b}z - \frac{\sqrt{a+b+d}}{b}\Phi^{-1}\Big(\frac{R-1}{R}\Big) . \qquad (7)$$

For quality values higher than θ^*, the firm will never default since a sufficiently large number of investors will optimally decide to roll over debt. A firm quality less than θ^*, in contrast, will always lead to a default by the firm. Note, however, that for $0 < \theta \leq \theta^*$ a default is always inefficient since it has not been triggered by a sufficiently bad firm quality but simply by the number of investors that decided to withdraw their money early.[8] Uniqueness of equilibrium hence does not eliminate inefficiencies. Default occurring in the range of $\theta \in [0, \theta^*]$ represents a coordination failure on the part of investors.

Uniqueness of equilibrium in the case where a credit rating agency is present requires that:

$$b > \frac{(a+d)^2}{2\pi} .$$

Note that in this general case with exogenous rating, the uniqueness condition is stricter than in the absence of a CRA as long as the precision of her rating, d, is perceived to be larger than zero. This is due to the fact that the announced rating increases the precision of information that is public in the market. This result has also been emphasized by Carlson and Hale (2005). However, the condition is sufficient but not necessary for uniqueness of equilibrium. We will return to the question of the CRA's effect on uniqueness versus multiplicity of equilibria and the desirability of each from an investor's point of view in section 4.

[8] Default is efficient only for $\theta < 0$, since in that case, even if all lenders decided to roll over, termination of the project would still be profit maximizing for the firm. The same is true in the case without a rating agency.

3 Optimal Rating

Whereas the preceding section treated the announced rating as exogenous, we will now derive the optimal rating from the CRA's utility function and her information about the firm's quality θ.

Based on empirical and anecdotal evidence, we consider the following utility function to hold for the rating agency. In contrast to earlier studies, we assume that a credit rating agency i is not only concerned with the informational content of her rating, z_i, but also has to take into account the potential effects of competition, i.e. her competitors' ratings z_j, and of her own actions on her future business. These aspects are mirrored in the following utility function:

$$u_i(z_i, \theta) = -(1 - r_1 - r_2)(z_i - \theta)^2 - r_1(L_i - \bar{L}) - r_2\, q\, \text{prob}(\text{default}|x_A) \ , \qquad (8)$$

with

$$L_i = \int_0^1 (z_j - z_i)^2 dj$$

and

$$\bar{L} = \int_0^1 L_j dj \ .$$

The first part in the CRA's utility function displays her aim to increase reputation by disclosing a rating z_i that is as close as possible to the unknown firm quality θ. This *reputational objective* enters the utility function with a weight of $(1-r_1-r_2)$.

Based on the idea of the "island economy" by Lucas (1972, 1973) and Phelps (1970) that formalizes the coordination motive of agents as examined by Morris and Shin (2002), the CRA's utility function also includes an additional "beauty-contest" term as the second part. Since a rating agency will only be able to successfully place her ratings on the market, if firms are willing to buy her expertise and if investors are willing to base their investment decisions on the announced rating, it is reasonable to assume that the market structure forces rating agencies to try to generate ratings that are close to the potential average rating. This *competitive aim* induces the CRA to reduce her risk of either losing the firm as a future customer by announcing a below-average rating or of losing investors as users of her information-provision by announcing inflated ratings that might increase their portfolio risk substantially. This aim essentially arises from a CRA's role as an intermediary between firms and investors. We assume that the CRA attaches a weight of r_1 to this argument of her utility function.

Finally, we presume that a rating agency also accounts for the potential *feedback-effect* that her rating might have on the firm's probability of default. In this respect the CRA should try not to contribute to an (inefficient) firm default via the announcement of her rating. If she succeeds in helping the firm to stay as a going-concern, she might be able to extract additional fees, q, from future business with the firm. This feedback-argument enters the CRA's utility function with a weight of r_2.

There has been plenty of empirical and anecdotal evidence supporting the different objectives in the CRA's utility function. Schwarcz (2002), for instance, argues that simple market forces lead rating agencies to guard their reputation. Otherwise investors will no longer pay attention to their ratings so that issuers will not pay their fees any more. Generally, we may see this argument to hold particularly in a long-run perspective.

Recent years have seen the expansion of a large number of potential competitors to the established credit rating agencies (The Economist, 2005). Apart from smaller, country-specific rating agencies, there is also a growing importance of other predictors of default, for instance via financial derivatives such as credit default swaps. Yet, competition between established CRAs remains tough. Still, Hill (2004) remarks that the market perception of potential rating biases curbs excessive rating competition and instead promotes dissemination of average ratings. Certainly, short-run considerations give bite to the competitive argument in the CRA's utility function that induces her to reduce risk by not deviating too strongly from her competitors' ratings.

Additional proof of the beauty-contest argument and even more detailed hypotheses come from a recent German experience. On June 19, 2005, the so-called "Gewährträgerhaftung" and "Anstaltslast", a maintenance obligation and implied liability of the German public sector for the state banks' public debt, was abolished by the German government because of pressure on the part of the EU competition commission. Contrary to what has been expected, however, the state banks' ratings did not deteriorate after the event. Rather, the required two ratings for each bank remained at a very high level. It was found that particularly those rating agencies that were notorious for being very strict in their rating assessment, were not appointed by the banks to generate a rating. Rather, more generous ratings were announced that complied with the average rating of the German banking landscape. Rating counsellors publicly stated at the time that competition between rating agencies leads agencies to reduce their risk by announcing average ratings, while only agencies of very high ability and long-standing experience dare to disclose "extreme" ratings (Von Heusinger, 2005). This hypothesis will be taken up again and tested in section 5.

Finally, plenty anecdotal evidence suggests support for the feedback-argument in a rating agency's utility function. In particular the existence of rating triggers forces rating agencies to be cautious in their rating process (Hill, 2004). With respect to the Enron case, for instance, it has been mentioned that one reason why the rating agencies acted very late (arguably too late) was that they did not want to begin a downgrading spiral started off by a first downgrading followed by an increase in credit costs due to rating triggers, which would again have increased Enron's probability of default, leading to an even lower rating etc (The Economist, 2005).

Derivation of the optimal rating requires the following steps. A full description is given in appendix A. The optimal rating z_i will maximize the CRA's expected utility function and is, thus, given by:

$$z_i = \frac{1-r_1-r_2}{1-r_2}E(\theta|x_A) + \frac{r_1}{1-r_2}E(\bar{z}|x_A) - \frac{r_2}{1-r_2}\frac{1}{2}q\phi(\cdot)\sqrt{a+c}\frac{\partial\theta^*}{\partial z_i}\ , \tag{9}$$

where $\bar{z}$ denotes the average rating. If we assume a linear strategy as optimal for a rating agency, we may average over all potential ratings. Plugging this in the optimal rating function (9) and comparing coefficients delivers the following optimal rating for a CRA:

$$
\begin{aligned}
z_i &= \frac{(1-r_1-r_2)c}{(1-r_2)a+(1-r_1-r_2)c}x_A + \frac{(1-r_2)a}{(1-r_2)a+(1-r_1-r_2)c}y \\
&\quad -\frac{r_2}{1-r_1-r_2}\,\frac{1}{2}\,q\,\phi(\cdot)\,\sqrt{a+c}\,\frac{\partial\theta^*}{\partial z_i}\,. \qquad (10)
\end{aligned}
$$

As can be seen, the optimal rating is influenced by the CRA's private information about the firm, x_A, the prior expected firm quality, y, and - due to the feedback-argument - also by the impact of the rating itself on the equilibrium threshold value separating defaulting from non-defaulting firms, θ^*. Note that $\partial\theta^*/\partial z_i < 0$. Therefore, the fact that the rating agency takes into account the influence of her rating on the firm's default risk leads her to increase her rating.

It is furthermore interesting to note that r_2, the weight attached to the feedback argument, decreases the impact of both the CRA's private information x_A and of the prior mean y on the optimal rating, while r_1, the weighting factor put on the competitive aim, decreases the effect of x_A but increase the effect of y. Hence, the more importance the CRA attaches to the competition argument, the stronger will the optimal rating be influenced by the prior mean y and the less will it be affected by the CRA's private information. If more weight is put on the feedback component, however, the impact of y and x_A is decreased. Placing more emphasis on the reputation aim, eventually increases the impact of private information x_A on the optimal rating.

4 Uniqueness of Equilibrium

In order to ensure uniqueness of equilibrium, we know that the precision of investors' private information must not become too low compared to the precision of information that is publicly available on the market. Since the announcement of a rating increases the precision of public information above that of the prior distribution of firm quality θ, the sufficient condition for a unique equilibrium becomes stricter after introducing a rating agency into the market. From the optimal rating strategy (10), we know that z displays a variance of $1/[(1-r_2)a+(1-r_1-r_2)c]$, so that the uniqueness condition can be rewritten as:

$$
b > \frac{((2-r_2)a+(1-r_1-r_2)c)^2}{2\pi}\,. \qquad (11)
$$

For a unique equilibrium to hold, investors' private information has to be sufficiently precise, not only relative to the precision of public information, a, but also relative to the precision of the CRA's private information, c. Rearranging the uniqueness condition allows a slightly different view:[9]

$$1 - r_1 - r_2 < \frac{\sqrt{2\pi b} - a(2 - r_2)}{c} .$$

Hence, a unique equilibrium prevails as long as the CRA does not attach a too high weight to her reputational aim. If this is the case, we know from (10) that the CRA's private signal x_A does not dominate the rating z too strongly, so that the "amount" of new information that becomes public via the rating is limited.

Note that the higher the firm's fundamental risk, $1/a$, the larger may the factor $(1 - r_1 - r_2)$ be for the uniqueness condition to still be satisfied. As an intuition for this result consider that the higher a firm's initial fundamental uncertainty, the more valuable is the rating for investors' assessment of the unknown firm quality. Hence, for firms with high fundamental uncertainty, the CRA's private information may dominate the rating to a large degree in order to compensate for the high variance in firm quality and the resulting fundamental uncertainty, before the rating starts to fulfill other, not necessarily informational purposes. For firms with a low variance of firm quality, $1/a$, in contrast, the rating is hardly needed to dispense with the remaining fundamental uncertainty and therefore may easily fulfill a coordinating role as put forward in Boot et al. (2005). Hence, in order to reduce the emergence of multiple equilibria, it may be useful to induce credit rating agencies to pursue several objectives. By doing so, the importance that the CRAs attach to their reputational aim decreases so that multiplicity of equilibria may be prevented. The emergence of multiple equilibria vice versa a unique equilibrium is therefore very susceptible to the CRA's combined role of both an information intermediary and a coordinator of investors' behavior that is at the heart of our analysis.

However, even if the rating agency derives her optimal rating from a complex utility function as given in (8), there is still a considerable chance of multiple equilibria arising after the public announcement of a rating - even if a unique equilibrium prevailed before. If private information on the part of investors is not sufficiently precise, i.e. for $a^2/(2\pi) < b < ((2 - r_2)a + (1 - r_1 - r_2)c)^2/(2\pi)$, introducing a CRA leads to a switch from a unique equilibrium to multiple equilibria because the rating announcement increases the precision of public information on the market and thereby reduces the heterogeneity of investors' posterior beliefs about firm quality.

Yet, the existence of a CRA may still be beneficial. Since in the case of multiple equilibria investors' beliefs are self-fulfilling, the result of each investors' optimality considerations is very susceptible to sunspots, i.e. exogenous incidences that help to coordinate investors' behavior. As for the indicated range of precision values b the market was governed by a unique equilibrium before the introduction of the CRA, it is reasonable to believe that the formerly optimal behavior might affect the new situation. For further explanation see figure 1. It represents equilibrium conditions (6) and (5) for the cases with and without a rating agency. In the case

[9]Since the weights in the CRA's utility function are bounded above by 1, we neglect the remaining condition of $1 - r_1 - r_2 > \frac{a(2-r_2)-\sqrt{2\pi b}}{c}$.

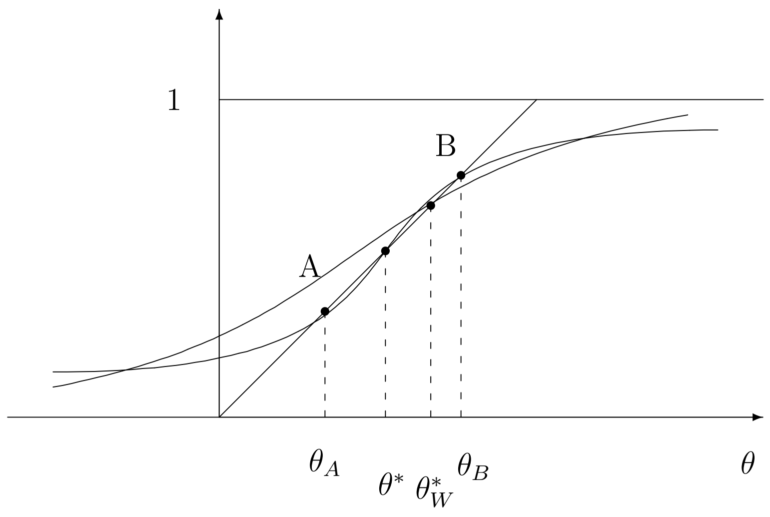

Figure 1: Unique versus multiple equilibria

without a CRA, the unique equilibrium is given by θ_W^*. Let us assume that after the introduction of a CRA, investors' private information is no longer sufficiently precise to prevent multiplicity of equilibria. Therefore two stable equilibria, A and B, and one unstable equilibrium in mixed strategies in θ^* emerge. As long as θ_W^* does not coincide with θ^*, the market may both coordinate on the more efficient equilibrium A, in which all investors roll over their debt for $\theta > \theta_A$, or on the rather inefficient equilibrium B, in which all investors roll over their debt only for $\theta > \theta_B$, with $\theta_B > \theta_A$. Before the presence of the CRA, in contrast, investors rolled over their debt for $E(\theta|x_i) > \theta_W^*$. If $\theta_W^* > \theta^*$, as depicted in the figure, investors withdrew their money for a relatively large range of values θ in the unique equilibrium case. As such, after the introduction of the CRA, it may appear reasonable for them to follow a similar strategy and to withdraw their money rather than to roll over their loans for all intermediate values $\theta \in [\theta_A, \theta_B]$. In this case, the market would move to the inefficient equilibrium B.

In contrast, if $\theta_W^* < \theta^*$, it is very likely that in the multiple equilibria setting, the market will now coordinate on the efficient action so that the firm will not be forced into default as easily and equilibrium A is reached. This is due to the fact that in the unique equilibrium case without the rating agency, investors optimally withdrew their money only for very low project qualities, i.e. for $E(\theta|x_i) \leq \theta_W^*$. The switch to a multiple equilibria setting will then induce investors to rely on a similar strategy, so that they will foreclose their credit for $\theta \leq \theta_A$ and extend otherwise.

Hence, whenever investors' private information is not sufficiently precise and $\theta_W^* > \theta^*$, introducing a CRA to the market should lead to the rather inefficient equilibrium B, whereas for $\theta_W^* < \theta^*$, equilibrium should converge to A, the more efficient one. However, uniqueness of equilibrium can be reinstored if market participants are willing to increase the precision of their private information. If it is possible for them to invest in (costly) information collection and procession, thereby increasing the precision of their private information, they are in essence able to choose between unique or multiple equilibria after the CRA has emerged. Since debt-holders suffer from the downside-risk of losing all the money that they borrowed to the firm while not holding an upside chance, we may reasonably assume that both investors and the firm prefer to reduce the incidence of default. Therefore, it would be reasonable for market participants to invest in the precision of their private information whenever $\theta^*_W > \theta^*$, in order to achieve a unique equilibrium that would reduce the probability of default to prob(default)= $\Phi(\sqrt{a}(\theta^* - y))$. Otherwise, the existence of a CRA would induce the inefficient equilibrium B in which the firm's debt will default unless very high quality values $\theta > \theta_B$ are realized. For $\theta^*_W < \theta^*$, however, investors and the firm would prefer multiple equilibria to the unique equilibrium outcome since this is likely to reduce the firm's probability of default to prob(default)= $\Phi(\sqrt{a}(\theta_A - y))$. Investors' incentive to increase the precision of private information is therefore highest if $\theta^*_W > \theta*$. As is shown in appendix B, this is the case for a sufficiently high rating z. Hence, for sufficiently high ratings z investors prefer a unique equilibrium vis-á-vis multiple equilibria. For sufficiently low ratings, in contrast, it holds that $\theta^* > \theta^*_W$, so that investors prefer multiple equilibria. Proposition 1 summarizes the results obtained so far:

Proposition 1 *For $a^2/(2\pi) < b < ((2 - r_2)a + (1 - r_1 - r_2)c)^2/(2\pi)$, the introduction of a CRA increases the number of equilibria. If the CRA announces a sufficiently high rating, investors have a high incentive to invest in the precision of their private information in order to eliminate multiple equilibria, thereby increasing the efficiency of the market outcome. Whenever a low rating is announced, however, the market is likely to coordinate on the most efficient among multiple equilibria without any investment in private information collection.*

Interestingly, this difference in preferences for equilibrium outcomes coincides with regulatory requirements as far as institutional investors are concerned. In almost any country, institutional investors are allowed to invest only in bonds with high ratings (investment-grade, "mündelsicher", etc.). At the same time, institutional investors usually dispose of research and analysis departments that supply them with additional private information. This has often been taken as evidence for a too low precision of credit ratings, since otherwise it would not have been worthwhile for investors to look for additional sources of information (Partnoy, 1999). According to our analysis, however, it is not the lack of precision in credit ratings but rather the opposite: rating announcements increase the precision of public information too strongly and hence destroy uniqueness of equilibrium. For firms with high bond ratings, however, investors prefer a unique equilibrium as this minimizes the probability of default. Hence, agents' incentive to invest in information is highest when investment is in firms with high-graded bonds, which is exactly what institutional investors are confined to.

Note that in contrast to Carlson and Hale (2005), the CRA in our setup does not disclose her private information to the market but a combination of her posterior information and - indirectly - of her expectation about the firm's default probability. If investors knew the CRA's specific utility function and if equilibrium were unique, they would be able to deduce the expected probability of default. With multiple equilibria, this is no longer true. In this case, the rating agency in essence has a strategic choice of whether to announce a default probability that corresponds to the efficient or to the inefficient equilibrium. But still, investors could choose to reduce the uncertainty emerging from the multiplicity of equilibria by investing in the precision of their private information.

The difference in efficiency-loss between equilibrium B and A furthermore increases in the precision of information that is public on the market. This is due to the fact that the cumulative normal distribution function gets steeper the more precise public information is relative to private information (Morris and Shin, 1999) as can be seen from figure 2. By increasing the precision of the rating,

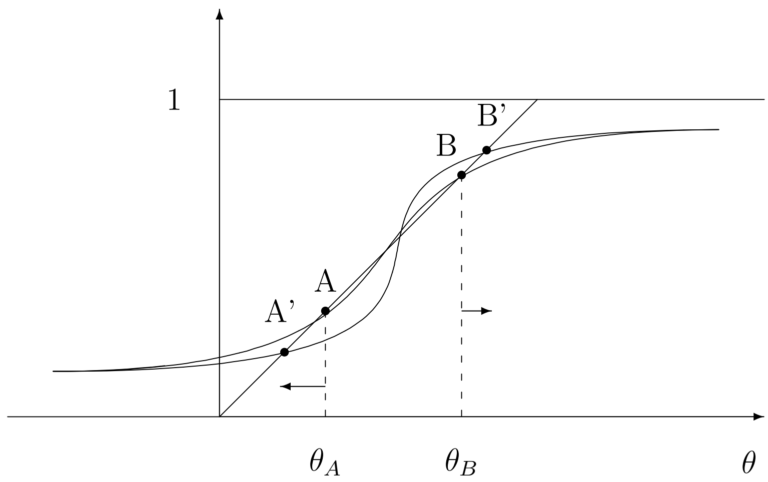

Figure 2: Effects of an increase in the relative precision of public information

equilibrium B becomes even less efficient, as the equilibrium moves to B' to the right, while equilibrium A becomes even more efficient by moving to the left to A'. If we assume that a larger difference in efficiency levels between the two stable equilibria in the multiple equilibria case raises the uncertainty in the market, then a higher precision of public information due to a more precise rating will induce investors to increase the precision of their private information as well. In this respect, the existence of credit rating agencies sparks off a virtuous circle: the more precise an agency's rating assessment is, the higher is the incentive for investors to collect more precise private information themselves in order to restore a unique equilibrium and eliminate the uncertainty stemming from the unpredictability of multiple self-fulfilling equilibria. This result delivers proposition 2:

Proposition 2 *The more precise the announced rating is, the larger is the difference in efficiency between the different multiple equilibria. This raises investors'* *incentive to invest in the precision of their private information to reduce uncertainty, thereby complementing the increase in the rating's precision.*

5 Solicited Versus Unsolicited Ratings

In recent years, rating agencies have started to issue ratings that are not requested by the target firm and that will not be paid for. These "unsolicited" ratings rely only on public information. Empirical studies on unsolicited ratings have raised several questions regarding both the purpose and the informational content of these rating assessments. In our setup unsolicited ratings may be characterized by $c = 0$, since no private information enters the rating process. The unsolicited rating z_U is then given as:

$$z_U = y - \frac{r_2}{1-r_1-r_2}\frac{1}{2}q\phi(\cdot)\sqrt{a}\frac{\partial\theta^*}{\partial z_U}\ . \tag{12}$$

Due to the CRA's objective of preventing inefficient default, even the unsolicited rating is higher than the ex-ante expected firm quality, y. For $q \to 0$, i.e. for decreasing importance of the feedback-argument, however, the unsolicited rating converges to y. Generally, the unsolicited rating increases in y, q, r_1 and r_2 and decreases in $1-r_1-r_2$. It will therefore be higher the better the ex-ante expected firm quality y is, the higher the future fees q are, the more weight the CRA attaches to competitive and feedback arguments and the less importance is given to the reputational concern.

If we assume a solicited rating z_S to be characterized by $c > 0$, it stands to reason whether z_S turns out to be higher or lower than z_U if the firm discloses confidential information to the agency. For the difference between z_S and z_U, we find:

$$z_S - z_U = \frac{(1-r_1-r_2)c}{(1-r_2)a+(1-r_1-r_2)c}(x_A - y) + \frac{r_2}{1-r_1-r_2}\,\frac{1}{2}\,q\,\phi(\cdot)[\sqrt{a}\frac{\partial\theta^*}{\partial z_U} - \sqrt{a+c}\frac{\partial\theta^*}{\partial z_S}]$$

Thus, provided that the CRA's private information, x_A, turns out to be sufficiently higher (lower) than the ex-ante expected firm quality, y, the solicited rating will be higher (lower) than the unsolicited assessment.

Corollary 1 *A solicited rating will only deviate from an unsolicited one if the CRAs private information, x_A, and ex-ante expected firm quality, y, deviate sufficiently strongly. For sufficiently high (low) private information compared to y, the solicited rating will be higher than an unsolicited rating.*

Nevertheless, the provision of private information does not necessarily increase any agency's private information precision c by the same amount. Rather, we may interpret a specific value of $c > 0$ as the "ability" of the CRA to gather and process the information privately provided by the firm.[10]

[10] In this respect, the "ability" of a rating agency would require a solicited rating as a necessary condition.

The solicited rating z_S as given by (10) behaves in the same way as the unsolicited with regard to y and q. It is furthermore intuitive to see that it increases in the face value of the rating agency's private information, x_A. For the influence of the agency's private information precision c, however, we find the following:

$$\frac{\partial z_S}{\partial c} = \frac{(1-r_2)(1-r_1-r_2)a}{[(1-r_2)a+(1-r_1-r_2)c]^2}(x_A - y) - \frac{r_2}{1-r_1-r_2}\frac{1}{2}\, q \cdot \left[\frac{1}{2\sqrt{a+c}}\phi(\cdot)\frac{\partial\theta^*}{\partial z_S} + \sqrt{a+c}\left(\frac{\partial\phi(\cdot)}{\partial c} + \frac{\partial\frac{\partial\theta^*}{\partial z_S}}{\partial c}\right)\right] . \quad (13)$$

Interestingly, the effect of c on the optimal solicited rating depends on the difference between the CRA's private information and the prior information about firm quality, i.e. on $x_A - y$. Obviously, for sufficiently high x_A as compared to the ex-ante expected firm quality y, this partial derivative will be positive. The opposite holds for very low x_A.

What further implications can be derived with regard to the effect of the CRA's ability on the solicited rating? Based on the influence of c as given in (13), we find the following. For very small future fees, i.e. $q \to 0$, so that the feedback argument will hardly influence the optimal rating, the influence of c on the solicited rating depends solely on the factor:

$$\frac{(1-r_2)(1-r_1-r_2)a}{[(1-r_2)a+(1-r_1-r_2)c]^2}\,(x_A - y) .$$

Differentiating the first factor with respect to the weights in the CRA's utility function shows that for sufficiently high precision of the CRA's private information this factor increases in r_1 and r_2 and decreases in $1 - r_1 - r_2$.[11] Hence, a rating agency of high ability, i.e. that disposes of very precise private information, will give a higher solicited rating for sufficiently good private information (i.e. high x_A) as compared to a CRA of lower ability, if she attaches lower weight to her reputational aim and puts more emphasis on competitive and feedback concerns. For sufficiently bad private information, i.e. low x_A, in contrast, she will give a lower solicited rating as a CRA of lower ability. Therefore, the statement that only rating agencies of high ability can afford to deviate strongly from the average rating holds, provided that these agencies do not put too much weight on their reputational objective. In the case of a unique equilibrium we may reasonably assume this condition to be satisfied.

Corollary 2 *CRAs of sufficiently high ability will announce more extreme solicited ratings than agencies of lower ability, if they attach relatively low weight to their reputation objective.*

Analyzing the influence that fundamental uncertainty $1/a$ has on the optimal

[11] For proof, see appendix C.

solicited rating, delivers:

$$\frac{\partial z_S}{\partial a} = \frac{(1-r_1-r_2)r_2 c x_A + [(1-r_2)^2 a + (1-r_1-r_2)(1-r_2)c + (1-r_2)r_2 a]y}{[(1-r_2)a + (1-r_1-r_2)c]^2} - \frac{r_2}{1-r_1-r_2}\frac{1}{2}q\left[\frac{1}{2\sqrt{a+c}}\phi(\cdot)\frac{\partial\theta^*}{\partial z_S} + \sqrt{a+c}\left(\frac{\partial\phi(\cdot)}{\partial a} + \frac{\partial\frac{\partial\theta^*}{\partial z_S}}{\partial a}\right)\right].$$

In order to interpret this result, assume again that $q \to 0$, so that the feedback argument becomes negligible. We then find that for positive private information, x_A, and ex-ante expected firm quality, y, the solicited rating decreases in fundamental uncertainty. For negative x_A and y, the opposite holds. Both are sufficient conditions. As an intuition for this result, consider that higher fundamental uncertainty implies that the unknown firm quality θ might lie far apart from the ex-ante expected value y. For high values of y, there is a considerable likelihood that θ will be much lower, while the opposite is true for low values of y.

Finally, how does the structure of the CRA's utility function, i.e. weights r_1 and r_2, influence the solicited rating? By looking at the derivatives of z_S with respect to r_1 and r_2, we find that both are positive (negative) for sufficiently low (high) private signals, x_A, as compared to ex-ante expected firm quality, y. The proof can be obtained from appendix D. Hence, we find that for sufficiently "bad" private information obtained by the CRA, i.e. for sufficiently low x_A, the optimal solicited rating z_S increases in weights r_1 and r_2 but decreases in $(1-r_1-r_2)$. The opposite holds for sufficiently "good" private information x_A. Stated differently, the more weight a rating agency places on her reputation (as compared to competitive and feedback concerns), the lower will her rating be in case of bad private information about the firm and the higher will it be in case of good private information about θ. The opposite holds if she puts less emphasis on her reputation. The results are summed up in the following corollary:

Corollary 3 *For sufficiently high private information, the optimal solicited rating will* (i) *decrease in fundamental uncertainty,* $1/a$, (ii) *decrease in the weight attached to the competitive argument,* r_1, (iii) *decrease in the weight put to the feedback argument,* r_2 *and* (iv) *increase in the emphasis attached to the repuational aim. The opposite holds for sufficiently low private information.*

Before analyzing the effect that unsolicited and solicited ratings may have on the market outcome, let us consider their role with regard to the uniqueness of equilibrium. Even though the condition for a unique equilibrium is less strict if the rating agency announces only unsolicited ratings, as public information on the market in this case comprises only the common knowledge about the firm's quality distribution, and hence complies with the uniqueness condition in the absence of a rating agency, the existence of a CRA may not induce the mentioned virtuous circle, either. The practice of announcing unsolicited ratings that are not requested by the firms may therefore endanger the process of information aggregation on the market. If investors' private information is not sufficiently precise so that multiple equilibria arise, the difference in efficiency between the inefficient and the efficient equilibrium may not be large enough to induce market participants to invest in the precision of their information. Hence, the practice of generating unsolicited ratings endangers the virtuous circle originally triggered by the announcement of very precise rating assessments.

Are our results in line with the empirical results on the difference between solicited and unsolicited ratings? There is more or less consensus in the empirical literature that unsolicited ratings are lower than solicited ones.[12] However, there are two interpretations that are consistent with this difference that is also known in the literature as the "downward bias". The first is the so-called "punishment hypothesis". Under the punishment hypothesis, a rating agency that is compensated by firms, whose securities are being rated by the agency, has an incentive to assign higher ratings to firms who pay for the service than to issuers who do not. Thus, rating agencies announce unsolicited ratings as a means to blackmail issuers. The second hypothesis may be denoted as the "private information hypothesis". It states that the observed lower unsolicited ratings are the result of self selection based on private information. Of course, this interpretation is more in line with our results. Interestingly, however, our model indicates that it is not the firm quality per se that influences the solicited rating but that it is the difference between the CRA's perception of firm quality, i.e. private signal x_A, and the ex-ante commonly expected firm quality, y. Notice that the power of a rating agency to select the equilibrium in case of multiple equilibria also contains an element of blackmailing. Interestingly, recent empirical studies confirm mainly the private information story (Byoun and Shin ,2003, and Gan, 2004). In addition, studies that investigate the stock market reaction of rating changes from unsolicited to solicited ratings find a significant negative stock market reaction to rating downgrades (Güttler and Behr, 2005). This is also consistent with our theory, as will be shown in the following section.

6 Market Effects

In the following, we assume that the condition for a unique equilibrium holds, i.e. we assume that investors obtain private information of sufficient precision, respectively that the rating agency attaches a sufficiently low weight to her reputation concern. From the analysis in section 2 we know that the ex-ante probability of default is given by

$$\text{prob (default)} = \text{prob}(\theta \leq \theta^*) = \Phi(\sqrt{a}(\theta^* - y)) \ .$$

Obviously, the likelihood of default increases in equilibrium value θ^*, so that all model parameters that reduce θ^* will automatically decrease the probability of default as well.

With rational expectations, investors will learn that the rating's precision d is given by $(1 - r_2)a + (1 - r_1 - r_2)c$. Plugging this into the equilibrium equation

[12] See Byoun and Smith (2002), Poon (2003), Poon and Firth (2004), Gan (2004), and Güttler and Behr (2005) for evidence in this regard.

for θ delivers

$$\theta^* = \Phi\Big(\frac{1}{\sqrt{b}}\Big(a\big((2-r_2)\theta^* - y - (1-r_1)z\big) + (1-r_1-r_2)c(\theta^* - z) - \sqrt{(2-r_2)a + b + (1-r_1-r_2)c}\,\Phi^{-1}\Big(\frac{R-1}{R}\Big)\Big)\Big)\,. \quad (14)$$

As can easily be seen, the probability of default decreases in the ex-ante expected firm quality, y, in the announced rating, z, and in the offered repayment rate, R. But how does θ^* compare to the equilibrium value in the absence of a credit rating agency, θ^*_W? From the following comparison:

$$\begin{aligned} \theta^*_W &> \theta^* \quad \Leftrightarrow \\ \theta^*_W - \theta^* &> \Big[1 - r_1 + (1-r_1-r_2)\frac{c}{a}\Big](\theta^* - z) + \frac{1}{a}\Phi^{-1}\Big(\frac{R-1}{R}\Big)\Big[\sqrt{a+b} \\ &\quad - \sqrt{(2-r_2)a + b + (1-r_1-r_2)c}\Big] \end{aligned} \quad (15)$$

we find that the introduction of a rating agency reduces the probability of default (by reducing the interval in which default occurs with certainty from $[0, \theta^*_W]$ to $[0, \theta^*]$) as long as θ^* lies sufficiently below z, i.e. as long as the rating agency announces a sufficiently high rating. In this case, the l.h.s. of inequality (15) will be positive by assumption, while the r.h.s. is negative, so that the inequality is satisfied. From the preceding section we know that a (solicited) rating increases, among other factors, in the prior expected firm quality, y, in the CRA's private information x_A and in future fees q. Furthermore, the difference between the solicited and the unsolicited rating raises along with the precision of the CRA's private information if the face value of the CRA's private signal deviates positively from y. Hence, for firms that are able to confide sufficiently optimistic information about business prospects to the CRA despite a pessimistic prior expected firm quality, the probability of default will decrease after the announcement of a rating. This contributes to the mentioned "private information hypothesis", that relates the difference between solicited and unsolicited ratings to an adverse selection problem. Our model is even more precise in showing that firms have a high incentive to request a solicited rating in order to reduce their probability of default if they believe to be treated unfair by the market, i.e. whenever they believe to be able to disclose much more optimistic private information to the CRA than what has a priori been expected.

Proposition 3 *In order to reduce the probability of default, firms will request a (solicited) rating if they believe that they are able to disclose more optimistic information to the credit rating agency than what has a priori been expected by the market, i.e. for* $x_A \gg y$.

Since equilibrium value θ^* decreases in the announced rating, it follows naturally from corollary 3 that for sufficiently high x_A, the probability of default will increase in fundamental uncertainty, $1/a$ and in weights r_1 and r_2. For sufficiently low private information x_A, the opposite results are obtained. Hence, the introduction of a rating agency is not necessarily beneficial only for firms of high quality, as has been stated by Carlson and Hale (2005). Rather the benefits of a rating agency are contingent on a very complex system of parameter variations.

7 Institutional Investors

Investors on bond markets can usually be categorized into two different groups. They are either small, individual investors, or large and institutionalized market participants. As institutional investors typically hold their own research departments, they are also presumably much better informed about firm quality than small investors. In order to bring this structure into our model, we assume in this section that the market consists of proportion λ of institutional investors that observe private information about the firm with precision $\bar{b}$ and proportion $(1-\lambda)$ of small investors with information of precision b with $\bar{b} \geq b$. In many countries, institutional investors are furthermore restricted with regard to their investment choice. We build this restriction in our model by assuming that institutional investors are only allowed to invest in bonds with a rating at least as high as $\tilde{z}$. In contrast to Boot et al. (2005), we do not, however, rely on the simplifying assumption that whenever a bond obtains a sufficiently high (usually investment-grade) rating all institutional investors always invest. Even though the market is sometimes so tight that they have to rely on this strategy, we rather consider the case where institutional market participants may but not necessarily have to invest in investment-grade firms. Hence, in the following we assume that institutional investors have to withdraw for $z < \tilde{z}$, but do not necessarily have to roll over for $z \geq \tilde{z}$.

What does this segregation of the buy-side in the bond market imply for the market equilibrium? Whenever the CRA announces a rating $z < \tilde{z}$, we know that institutional investors never roll over their debt but always withdraw their money. Hence the condition for imminent default of the firm is changed to:

$$\theta = \lambda + (1-\lambda)\Phi(\sqrt{b}(x_i - \theta)) \ .$$

This delivers a unique equilibrium value of θ_1^*:

$$\begin{aligned} \theta_1^* &= \lambda + (1-\lambda)\Phi\Big(\frac{1}{\sqrt{b}}(a((2-r_2)\theta_1^* - y - (1-r_1)z) + (1-r_1-r_2)c(\theta_1^* - z) \\ &\quad - \sqrt{(2-r_2)a + b + (1-r_1-r_2)c}\,\Phi^{-1}(\frac{R-1}{R}))\Big) \ . \end{aligned}$$

It is obvious to see that $\theta_1^* \geq \theta^*$, so that for sufficiently low ratings $z < \tilde{z}$, the probability of default is increased by the existence of institutional investors.

If the CRA instead announces a rating of $z \geq \tilde{z}$, whether or not institutional investors roll over their loans depends on their posterior beliefs about θ, so that equilibrium value θ_2^* is given by:

$$\begin{aligned}\theta_2^* &= \lambda\Phi\Big(\frac{1}{\sqrt{\bar{b}}}\Big(a((2-r_2)\theta_2^* - y - (1-r_1)z) + (1-r_1-r_2)c(\theta_2^* - z) \\ &\quad -\sqrt{(2-r_2)a + \bar{b} + (1-r_1-r_2)c}\Phi^{-1}\big(\frac{R-1}{R}\big)\Big)\Big) \\ &\quad +(1-\lambda)\Phi\Big(\frac{1}{\sqrt{b}}\Big(a((2-r_2)\theta_2^* - y - (1-r_1)z) + (1-r_1-r_2)c(\theta_2^* - z) \\ &\quad -\sqrt{(2-r_2)a + b + (1-r_1-r_2)c}\Phi^{-1}\big(\frac{R-1}{R}\big)\Big)\Big)\ .\end{aligned}$$

While it always holds that $\theta_2^* \leq \theta_1^*$, we find that $\theta_2^* \leq \theta^*$ whenever repayment $R > 2$ (as a sufficient condition), while $\theta_2^* > \theta^*$ for $R < 2$, as:

$$\theta_2^* \leq \theta^* \Leftrightarrow \theta_2^* - \theta^* < \frac{\Phi^{-1}(\frac{R-1}{R})[\sqrt{(2-r_2)a + \bar{b} + (1-r_1-r_2)c} - \sqrt{(2-r_2)a + b + (1-r_1-r_2)c}]}{a(2-r_2) + (1-r_1-r_2)c}$$

For $R > 2$, this inequality is satisfied because the l.h.s. is negative by assumption while the r.h.s. is positive. The following corollary sums up the results:

Corollary 4 *The existence of institutional investors reduces the probability of default for firms rated investment-grade, whenever sufficiently high repayment values R are offered. For ratings below investment-grade, however, their existence raises the risk of inefficient firm default due to regulatory reasons.*

8 Conclusion

Since the proposal of the Basel 2 accord, that requires inclusion of borrowers' credit ratings in assessing banks' capital adequacy, general interest in the credit rating industry has greatly increased. Lacking a convincing theoretical basis, empirical and descriptive studies have often come to the conclusion that the credit-rating agency is "curiously devoid of competition and oversight", of which it desperately "needs more" (The Economist, 2005). Moody's Bond Rating Service, a major rating institution, has even been declared as one of the "two superpowers in the world today" (Partnoy, 1999) - together with the United States-, and has been accused of being able to make a "grown man cry" (Euromoney, 1998).

Focussing on a rigorous theoretical analysis and accounting for a complex utility function, we find that credit rating agencies may nevertheless be a benefactor to financial markets. In particular, they may spark off a virtuous circle that supports information aggregation, thereby increasing market efficiency. This result is due to the fact that rating announcements and private information collection prove to be complements rather than substitutes. Hence, the release of precise credit ratings induces market participants to invest in the precision of their private information as well. On the downside, however, the recent practice by the agencies of announcing unsolicited ratings destroys this virtuous circle. Since unsolicited ratings are based only on publicly available information, they cannot be as accurate as ratings that are actually requested by the firms and that comprise also facts that are confidentially disclosed to the rating agencies. Anticipating the lower precision, bond holders reduce their investment in private information precision, which undermines the benevolent effect of credit ratings on information aggregation.

With regard to explaining the observed difference between solicited and unsolicited ratings, our model recommends the “private information hypothesis” that has also been supported by recent empirical work. Following from our model only those firms will request a solicited rating that believe to be treated unfairly by the market, i.e. that expect to be able to disclose more optimistic information about firm quality to the credit rating agency than has a priori been expected. This argument leads to a natural adverse selection between firms that request a solicited rating and those who do not, and contributes to explaining the observed downward-bias in unsolicited ratings.

Appendix

Appendix A

From the analysis in section 2 we know that the probability of firm default is given by prob$(\theta \leq \theta^*)$, while the CRA holds a posterior belief about the firm's quality of:

$$\theta|x_A \sim N\Big(\frac{ay + cx_A}{a+c}, \frac{1}{a+c}\Big) \; .$$

The CRA's expected utility function can therefore be derived as:

$$\begin{aligned} Eu_i(z_i, \theta) &= -(1-r_1-r_2)(z_i - E(\theta|x_A))^2 - r_1 E(L_i - \bar{L}|x_A) - r_2 \, q \, \text{prob}(\theta \leq \theta^*|x_A) \\ &= -(1-r_1-r_2)(z_i - E(\theta|x_A))^2 - r_1 E(z_i^2 - 2z_i\bar{z} + \bar{z}^2 - \int z_j^2 - 2z_j\bar{z} + \bar{z}^2 dj|x_A) \\ & \quad -r_2 \, q \, \Phi(\sqrt{a+c}(\theta^* - \frac{ay + cx_A}{a+c})) \; . \end{aligned} \tag{16}$$

Derivation with respect to z_i and setting equal to zero delivers the optimal rating as given in (9)

Following Morris and Shin (2002), we assume a linear strategy as optimal for a rating agency:

$$z_i = k_1 x_A + k_2 y + k_3 \frac{1}{2} q \phi(\cdot) \sqrt{a+c} \frac{\partial \theta^*}{\partial z_i} \; . \tag{17}$$

This allows to average over all potential ratings:

$$E(\bar{z}|x_A) = k_1 \frac{ay + cx_A}{a+c} + k_2 y + k_3 \frac{1}{2} q \phi(\cdot) \sqrt{a+c} \frac{\partial \theta^*}{\partial z_i} \; , \tag{18}$$

since $E(x_{A_j}|x_{A_i}) = E(\theta|x_{A_i})$. Plugging this into the optimal rating function (9) yields:

$$\begin{aligned} z_i &= \frac{(1-r_1-r_2)c + r_1 k_1 c}{(1-r_2)(a+c)} x_A + \frac{(1-r_1-r_2)a + r_1 k_1 a + r_1 k_2 (a+c)}{(1-r_2)(a+c)} y \\ & \quad + \frac{r_1 k_3 - r_2}{1-r_2} \frac{1}{2} q \phi(\cdot) \sqrt{a+c} \frac{\partial \theta^*}{\partial z_i} \; . \end{aligned}$$

Comparing coefficients to (17) gives us the weights attached to the three arguments of the optimal rating strategy:

$$k_1 = \frac{(1-r_1-r_2)c}{(1-r_2)a - (1-r_1-r_2)c} \; ,$$

$$k_2 = \frac{(1-r_2)a}{(1-r_2)a + (1-r_1-r_2)c} \; ,$$

and

$$k_3 = -\frac{r_2}{1-r_1-r_2} \; .$$

Appendix B

It is optimal for investors to invest in the precision of their private information, if $\theta_W^* > \theta^*$, since otherwise multiple equilibria will lead to the inefficient equilibrium. This condition is satisfied, if:

$$\begin{aligned}
\theta_W^* &> \theta^* \\
\frac{a}{\sqrt{b}}(\theta_W^* - y) - \sqrt{\frac{a+b}{b}}\Phi^{-1}\Big(\frac{R-1}{R}\Big) &> \frac{1}{\sqrt{b}}\Big(a((2-r_2)\theta^* - y - (1-r_1)z) + (1-r_1-r_2)c(\theta^* - z) \\
&\quad -\sqrt{(2-r_2)a + b + (1-r_1-r_2)c}\Phi^{-1}\Big(\frac{R-1}{R}\Big)\Big) \\
\theta_W^* - \theta^* &> [1 - r_1 + (1-r_1-r_2)\frac{c}{a}](\theta^* - z) \\
&\quad +\frac{1}{a}\Phi^{-1}\Big(\frac{R-1}{R}\Big)\cdot[\sqrt{a+b} - \sqrt{(2-r_2)a + b + (1-r_1-r_2)c}
\end{aligned}$$

Since the l.h.s. of the inequality will be positive, if $\theta_W^* > \theta^*$, the inequality will be satisfied for sufficiently high ratings, i.e. for $z > \theta^*$ and $R < 2$, because the r.h.s. will then be negative. It also holds for $z \gg \theta^*$.

Appendix C

With respect to the two weights, r_1 and r_2, we find:

$$\frac{\partial \frac{(1-r_2)(1-r_1-r_2)a}{[(1-r_2)a+(1-r_1-r_2)c]^2}}{\partial r_1} = \frac{(1-r_2)a[(1-r_1-r_2)c - (1-r_2)a]}{[(1-r_2)a + (1-r_1-r_2)c]^3}$$

and

$$\frac{\partial \frac{(1-r_2)(1-r_1-r_2)a}{[(1-r_2)a+(1-r_1-r_2)c]^2}}{\partial r_2} = \frac{ar_1[(1-r_1-r_2)c - (1-r_2)a]}{[(1-r_2)a + (1-r_1-r_2)c]^3},$$

which are both positive if $c > \frac{1-r_2}{1-r_1-r_2}a$ and negative otherwise. For the influence of $(1-r_1-r_2)$ exactly the opposite result is obtained.

Appendix D

$$\frac{\partial z_S}{\partial r_1} = \frac{(1-r_2)ca(y-x_A)}{[(1-r_2)a + (1-r_1-r_2)c]^2} - \frac{r_2}{1-r_1-r_2}\,\frac{1}{2}\,q\,\phi(\cdot)\,\sqrt{a+c}\Big[\frac{\frac{\partial\theta^*}{\partial z_S}}{1-r_1-r_2} + \frac{\partial\frac{\partial\theta^*}{\partial z_S}}{\partial r_1}\Big]$$

This partial derivative is positive for either $y \gg x_A$ or for $y > x_A$ and $q \to 0$ and negative for $y \ll x_A$ or for $y < x_A$ and $q \to 0$.

$$\frac{\partial z_S}{\partial r_2} = \frac{acr_1(y-x_A)}{[(1-r_2)a + (1-r_1-r_2)c]^2} - \frac{1}{1-r_1-r_2}\,\frac{1}{2}\,q\,\phi(\cdot)\,\sqrt{a+c}\Big[(1-r_1)\frac{\partial\theta^*}{\partial z_S} + r_2\frac{\partial\frac{\partial\theta^*}{\partial z_S}}{\partial r_2}\Big]$$

This partial derivative is positive for $y \gg x_A$ and negative for $y \ll x_A$.

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