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LETTER TO THE EDITOR

A self-consistent equation of state for nuclear matter

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Abstract. We formulate a phenomenological extension of the mean-field theory approach and define a class of thermodynamically self-consistent equations of state for nuclear matter. A new equation of state of this class is suggested and examined in detail.

One of the foremost goals in tOdays heavy-ion physics is the determination of the nuclear matter equation of state (EOS) (see, for example, [1]). Up to now our knowledge of the the nuclear EOS is restricted to one point in the plane of the independent thermodynamical variables temperature T and net baryon density n. This point is the so-called ground state of nuclear matter: at $T=0$ nuclear matter saturates (i.e. the pressure $p=p_0=0$) at a denisty of about $n_0 \cong 0.16\,\mathrm{fm}^{-3}$. From nuclear physics data one derives the following value for the energy per particle $W(n) \equiv (\epsilon/n)_{T=0} - M$ (ϵ is the energy density, M is the nucleon mass) of infinite nuclear matter:

$$W(n=n_0) \equiv W_0 \cong -16\,\mathrm{MeV}.$$

In [2] a comprehensive analysis of the incompressibility

$$K_0 = 9\left(\frac{\partial p}{\partial n}\right)_{T=0,n=n_0} = 9n_0^2\left(\frac{\mathrm{d}W}{\mathrm{d}n}\right)_{n=n_0}$$

has been performed. It is found that a large value of K_0 ($\simeq 300\,\mathrm{MeV}$, with considerable error) may be more compatible with the data than the previously reported low one, $K_0 = 180 \div 240\,\mathrm{MeV}$ [3, 4]. The estimations of the effective nucleon mass M^* at $T=0$, $n=n_0$ which can be found in the literature are $M_0^* = (0.7 \pm 0.15)\,M$.

Any reasonable model for the nuclear matter EOS must be thermodynamically self-consistent and reproduce the above quantities n_0, W_0, K_0 and M_0^*. Its behaviour in other regions of the n–T plane can be then probed via heavy-ion collisions. The

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aim of the present work is to formulate a phenomenological generalization of the mean-field theory approach and obtain a class of nuclear matter equations of state. We present a new EOS of this class which we investigate in detail and compare with the non-relativistic many-body theory of [5] and various relativistic equations of state.

Following early theoretical suggestions [6], experiments which measure the π-meson multiplicity in heavy-ion collisions [7] have been performed in order to extract the nuclear EOS directly from data. For this purpose the following decomposition for the energy per baryon

$$\epsilon(n,T)/n = M + W_{\mathrm{th}} + W_{\mathrm{c}}$$

was introduced with some phenomenological ansätzen for the thermal energy W_{th} and the compression energy W_{c}. The whole construction was, however, not physically self-consistent: for the calculation of W_{th} the momentum distribution of an ideal nucleon gas was used, but when introducing, in addition, the 'compression energy' one took into account the interaction between nucleons. The interaction, however, modifies the ideal-gas momentum distribution, and one faces the problem of adjusting the relativistic Fermi distribution of the nucleons to the functional form of the 'compression energy'.

To find a solution of this problem we remember that in the relativistic mean-field theory of Walecka [8] (see also [9–10]) the interaction is described by scalar φ and vector U^{μ} mesonic fields with baryon–meson interaction terms in the Lagrangian: $g_{\mathrm{s}}\bar{\psi}\psi\varphi$ and $g_{\mathrm{v}}\bar{\psi}\gamma^{\mu}\psi U_{\mu}$. For nuclear matter in thermodynamical equilibrium these meson fields are considered to be constant classical quantities. The scalar field describes the attraction between nucleons and changes the nucleon mass $M \mapsto M^* = M - g_{\mathrm{s}}\langle\varphi\rangle$. The nucleon repulsion is described by the vector field which adds $U(n) = C_{\mathrm{v}}^2\, n$ ($C_{\mathrm{v}}^2 = \mathrm{const}$) to the nucleon energy ($-U(n)$ for the antinucleon).

Following [12] we now formulate a generalized nuclear matter EOS which includes the mean-field theory and pure phenomenological models as special cases. Restricting ourself at the moment to nucleonic degrees of freedom we suggest the following general form for the nuclear EOS.

$$p(T,\mu) = \frac{\gamma_N}{3}\int \frac{\mathrm{d}^3k}{(2\pi)^3}\frac{k^2}{\sqrt{k^2+M^{*2}}}(f_{\mathrm{N}}+f_{\bar{\mathrm{N}}}) + nU(n) - \int_0^n \mathrm{d}n'\, U(n') + P(M^*) \tag{1}$$

where f_{N} and $f_{\bar{\mathrm{N}}}$ are the distribution functions of nucleons and antinucleons

$$f_{\mathrm{N}(\bar{\mathrm{N}})} = \left[\exp\left(\frac{\sqrt{k^2+M^{*2}} \mp \mu \pm U(n)}{T}\right) + 1\right]^{-1} \tag{2}$$

μ is the baryonic chemical potential and γ_{N} is the number of spin–isospin nucleon states, which equals four for symmetric nuclear matter. The dependence of the effective nucleon mass M^* on T and μ is defined by extremizing the thermodynamical potential (maximum of the pressure):

$$\left(\frac{\delta p}{\delta M^*}\right)_{T,\mu} \equiv \frac{\mathrm{d}P(M^*)}{\mathrm{d}M^*} - \gamma_{\mathrm{N}}\int \frac{\mathrm{d}^3k}{(2\pi)^3}\frac{M^*}{\sqrt{k^2+M^{*2}}}(f_{\mathrm{N}}+f_{\bar{\mathrm{N}}}) = 0. \tag{3}$$

the baryonic number density and energy density can be found from (1)–(3) using the well-known thermodynamical relations:

$$n(T,\mu)\equiv\left(\frac{\partial p}{\partial\mu}\right)_T=\gamma_{\rm T}\int\frac{{\rm d}^3k}{(2\pi)^3}(f_{\rm N}-f_{\bar{\rm N}}) \tag{4}$$

$$\begin{aligned}\epsilon(T,\mu)&\equiv T\left(\frac{\partial p}{\partial T}\right)_\mu+\mu\left(\frac{\partial p}{\partial\mu}\right)_T-p\\&=\gamma_{\rm N}\int\frac{{\rm d}^3k}{(2\pi)^3}\sqrt{k^2+M^{*2}}(f_{\rm N}+f_{\bar{\rm N}})+\int_0^n{\rm d}n'\,U(n')-P(M^*).\end{aligned} \tag{5}$$

It follows from (2) that the nucleon (antinucleon) momentum distribution has the form of the ideal Fermi distribution in 'external fields': the scalar field changes the nucleon (antinucleon) mass M to the effective mass M^* and the vector field adds the energy $U(n)$ ($-U(n)$ for the antinucleon). It is important, however, that additional terms in (2) and (5) appear and represent thermodynamically self-consistent 'field' contributions to the pressure and the energy density. The form of these additional contributions to the pressure (1) is adjusted to the Fermi distributions (2) through the general thermodynamical relation (4)!

Formulae (1)–(5) define, therefore, a special class of thermodynamically self-consistent equations of state for nuclear matter. It is a phenomenological extension of the mean-field theory. Models of this class are fixed by specifying the two functions $U(n)$ and $P(M^*)$. General physical restrictions on these functions have the form:

$$\begin{aligned}&U(-n)=-U(n)\qquad U(n)_{n\to\infty}\sim n^a\qquad 0\leqslant a\leqslant 1\\&\qquad\qquad\qquad\qquad\;\; U(n)_{n\to 0}\sim n^b\qquad b\geqslant 0\\&P(M^*)=\sum_{k\geqslant 2}a_k(M-M^*)^k\qquad \alpha_2<0.\end{aligned} \tag{6}$$

Particular choices of $U(n)$ and $P(M^*)$ satisfying (6) reproduce a great variety of nuclear EOS models known from the literature. The models of [13, 14] correspond to $M^*\equiv M$ ($P(M^*)\equiv O$) in (1) and special forms of $U(n)$. For the mean-field theory models [8–11] $U(n)=C_{\rm v}^2n$. The choice $\alpha_2\equiv-1/2\,C_{\rm s}^2$, $\alpha_k=0$ ($k\geqslant 3$) corresponds to the linear mean-field theory (in the following also referred to as 'Walecka model') [8], while considering $\alpha_2\equiv-1/2\ C_{\rm s}^2$, α_3, $\alpha_4\neq 0$, $\alpha_k=0$ ($\geqslant 5$) we reproduce the non-linear mean-field theory [10, 11].

At $T=0$ we find the general relation for the models (1)–(5):

$$U(n)+\sqrt{\left(\frac{3\pi^2}{2}n\right)^{2/3}+M^{*2}}=M+W(n)+n\frac{{\rm d}W}{{\rm d}n}$$

which corresponds to the Hugenholtz–Van Hove theorem [13] for an interacting Fermi gas at zero temperature. As $({\rm d}W/{\rm d}n)_{n=n_0}=0$, at saturation density we obtain the original Weisskopf relation [14] between Fermi energy and the energy per particle

$$U(n_0)+\sqrt{\left(\frac{3\pi^2}{2}n_0\right)^{2/3}+M_0^{*2}}=M+W_0\cong 922\,{\rm MeV}. \tag{7}$$

Equation (7) gives us the relation between $U(n_0)$ and M_0^*, therefore only one of these quantities (e.g. M_0^*) is free.

We consider now one new example for the nuclear matter EOS from the generalized mean-field theory class (1–5). We choose

$$P(M^*) = -\tfrac{1}{2} C_s^2 (M - M^*)^2 \qquad U(n) = C_v^2 n - C_d^2 n^{1/3}. \tag{8}$$

Thus, the Walecka model is extended by an attractive term in the potential $U(n)$. Such a modification of $U(n)$ is to some extent similar to the approach of the models [15, 16]. However, unlike to these models we now account for the fact that $M^* \neq M$ in the nuclear medium. The introduction to the third parameter C_d^2 allows us to choose M_0^* freely in addition to the required values of n_0 and $W(n_0)$. We stress that, if we require that our 'coupling constant' C_d^2 has the dimension of an integer power of the fundamental units, only the power $\frac{1}{3}$ of n in the new attractive term of $U(n)$ has the property to satisfy the constraints (6).

The additional term in the potential $U(n)$ could be derived (in mean-field approximation) from a Lagrangian containing an additional nucleon–nucleon self-interaction term of the form

$$\tfrac{3}{4} C_d^2 (\bar{\psi}\gamma^\mu \psi \bar{\psi}\gamma_\mu \psi)^{2/3}.$$

Of course, there is no immediate physical motivation for such a term on field-theoretical grounds. However, such a motivation is certainly not strictly required for a phenomenological equation of state. As justification, it is sufficient that the equation of state has physically resonable properties. This will be shown in the following.

Although the incompressibility K_0 cannot be chosen independently from M_0^*, we find that reasonable values of M_0^* lead to values of K_0 which lie in the experimentally found range (see table 1). The first line in table 1 corresponds to the Walecka model for which we have a too small value of M_0^* and a too large value of K_0.

The surprisingly good correlation between M_0^* and K_0 implies that the above model accounts for four ground-state properties of nuclear matter with only three independent parameters. The energy density in our model has the form

$$\epsilon(T, \mu) = \gamma_N \int \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + M^{*2}}\,(f_N + f_{\bar{N}}) + \tfrac{1}{2} C_v^2 n^2 - \tfrac{3}{4} C_d^2 n^{4/3} + \frac{1}{2C_s^2}(M - M^*)^2. \tag{9}$$

For further analysis we fix $K_0 = 300$ MeV (cf [2]). The nuclear EOS at $T = 0$ (i.e.

Table 1. Different sets of 'coupling constants' for fixed $n_0 = 0.15891\ \mathrm{fm}^{-3}$ [5] and $W_0 = -16$ MeV.

M_0^*/M	$C_v^2(\mathrm{GeV}^{-2})$	$C_s^2(\mathrm{GeV}^{-2})$	C_d^2	K_0 (MeV)
0.543	285.90	377.56	0	553
0.600	257.40	326.40	0.124	380
0.635	238.08	296.05	0.183	300
0.688	206.79	251.14	0.254	210
0.720	186.94	244.52	0.288	170

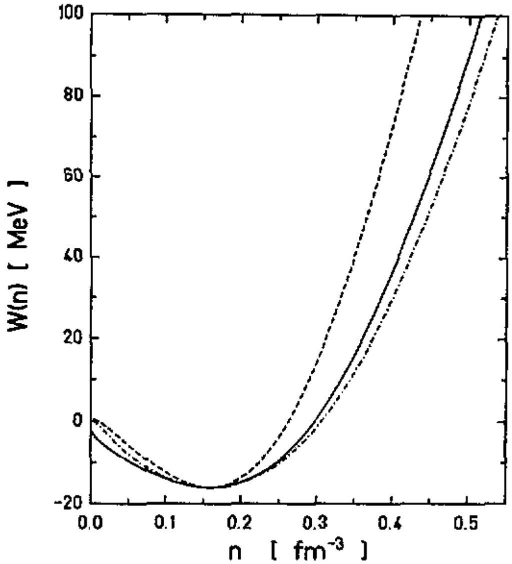

Figure 1. The energy per baryon at zero temperature is shown as a function of n for the new EOS with $M_0^* = 0.635\,M$, $K_0 = 300\,\text{MeV}$ (full line), for the Walecka model (dashed line) and for the non-linear mean-field theory (dashed-dotted line) with the same values of M_0^* and K_0 as for the new EOS.

the function $W(n)$) is shown in figure 1 in comparison to the Walecka model and the non-linear mean-field theory. We note that for this value of K_0 the effective mass M_0^* is in very good agreement with that obtained in the non-relativistic many-body calculations of Friedman and Pandharipande [5]. In figure 2 we compare the free energy per baryon in their calculations [5] with that of our model. We find rather good agreement at small n and systematic deviations from the non-relativistic results at large n. This happens at $n \geqslant 3n_0$ where the nucleon Fermi momentum is large, $k_F \geqslant M$ and therefore relativistic effects become important.

At low temperatures and densities our EOS exhibits a 'liquid–vapour' phase

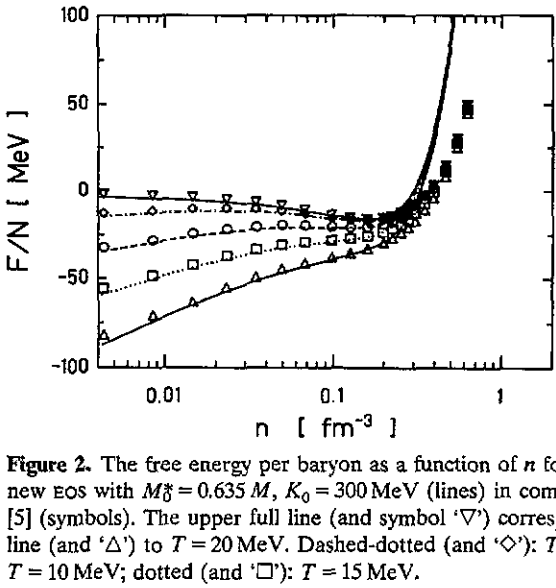

Figure 2. The free energy per baryon as a function of n for constant T is shown for the new EOS with $M_0^* = 0.635\,M$, $K_0 = 300\,\text{MeV}$ (lines) in comparison to the calculations of [5] (symbols). The upper full line (and symbol '∇') corresponds to $T = 0$, the lower full line (and 'Δ') to $T = 20\,\text{MeV}$. Dashed-dotted (and '$\Diamond$'): $T = 5\,\text{MeV}$; dashed (and '$\perp$'): $T = 10\,\text{MeV}$; dotted (and '$\Box$'): $T = 15\,\text{MeV}$.

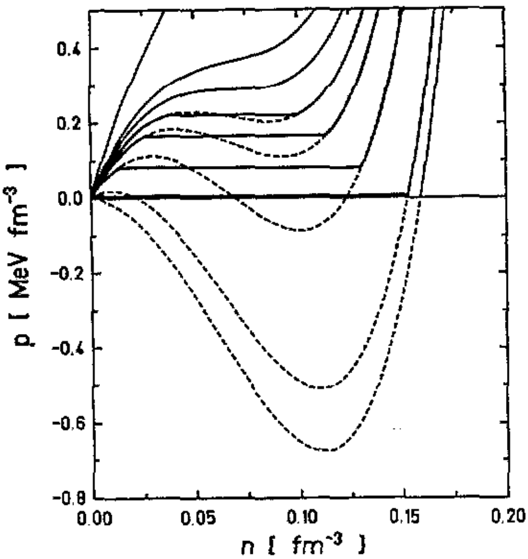

Figure 3. p–n diagram showing the liquid–vapour phase transition in the new model. The curves are isotherms for $T = 0, 5, 10, 12, 13, 14, 15, 20$ MeV (from below to above). Dashed lines mark the unstable part of the isotherms in the phase coexistence region, which are replaced by the full horizontal lines according to Maxwell's construction.

transition, as shown in figure 3. The critical temperature beyond which there is no two-phase equilibrium is ~14 MeV. We point out however, that a realistic description in the region $n \leqslant 0.03\,\mathrm{fm}^{-3}$ and $T \leqslant 5$ MeV requires that one takes into account clustering effects (e.g. deutrons and α-particles).

The non-linear mean-field theory [10] with four parameters C_v^2, C_s^2, α_3, α_4 allows one to choose M_0^* and K_0 at will. However, the value of α_4 is positive for experimentally reasonable sets of M_0^* and K_0 (see [11]). This means that the energy density of the system is not bounded from below (because of the term $-P(M^*)$) with respect to variations of M^*: such a theory is unstable, since its energetic minimum is $-\infty$ for φ (the scalar field) $\rightarrow \pm\infty$.

In conclusion we present the generalized mean-field theory approach to the nuclear matter EOS. It gave us the rules (1)–(5) to construct a class of thermodynamically self-consistent phenomenological models. As an example, we have suggested and investigated a simple version of the nuclear EOS from this class. It allows for a reasonable value of the nucleon effective mass M_0^* and simultaneously allows from an incompressibility K_0 in the range of the experimental values. The non-relativistic many-body calculations of [5] coincide with this EOS up to a few percent in the low-density and low-temperature region. Its agreement with known nuclear matter properties is better than for either the phenomenological models with $M^* \equiv M$, or the Walecka model as well as stable versions ($\alpha_4 \leqslant 0$) of the non-linear mean-field theory.

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