Nuclear transport models can reproduce charged-particle-inclusive measurements but are not strongly constrained by them

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Nuclear transport models are important tools for interpretation of many heavy-ion experiments and are essential in efforts to probe the nuclear equation of state. In order to fulfill these roles, the model predictions should at least agree with observed single-particle-inclusive momentum spectra; however, this agreement has recently been questioned. The present work compares the Vlasov-Uehling-Uhlenbeck model to data for mass-symmetric systems ranging from $^{12}C+^{12}C$ to $^{139}La+^{139}La$, and we find good agreement within experimental uncertainties at 0.4*A* and 0.8*A* GeV. For currently available data, these uncertainties are too large to permit effective nucleon-nucleon scattering cross sections in the nuclear medium to be extracted at a useful level of precision.

Much of the intermediate-energy heavy-ion research effort over the past decade has been directed towards understanding the nuclear equation of state (EOS) in the hadron gas phase. A determination of whether the evidence from nuclear collisions favors the so-called "stiff" or "soft" EOS (Ref. 1) has been a specific goal of many comparisons between nuclear transport models and 4π measurements, but a consensus on even this preliminary objective has yet to emerge. Several factors have limited the pace of progress. On the experimental side, a 4π detector is typically used to measure EOS-related phenomena such as collective flow, but it is difficult to understand and simulate the complex acceptance normally associated with this type of detector. On the theoretical side, elaborate event-simulating models are used to relate EOS properties to the experimental observables, and a number of the assumptions underlying these models have been the subject of debate in recent years. This paper tests some of these assumptions by comparing the Vlasov-Uehling-Uhlenbeck[2,3] (VUU) model with measured charged-particle-inclusive momentum spectra for a range of polar angles and for various combinations of projectile mass and energy. Because these double-differential cross sections are averaged over all reaction planes and impact parameters, they can be measured with a high and uniform efficiency using a detector with a relatively small solid angle acceptance. It is appropriate to require the models to reproduce this type of data before inferring EOS properties from higher-order differentials of the cross section.

A crucial and common feature of microscopic transport theories,[2–8] like the Boltzmann-Uehling-Uhlenbeck[4] (BUU), VUU,[2,3] and relativistic Vlasov-Uehling-Uhlenbeck[5] (RVU) models, and the more recent quantum molecular-dynamics[6,7] (QMD) approach, is that part of the nucleus-nucleus collision process is described by a sequence of two-body collisions according to $\sigma_{NN}^{\rm UU}$—the experimental scattering cross sections for free particles, corrected for Pauli blocking of occupied final states as per the Uehling-Uhlenbeck prescription. This assumption largely determines the predictions for reaction-plane-averaged momentum spectra. Aichelin *et al.*[9] have demonstrated that the four independent transport models[2–7] mentioned above agree rather well among themselves in predicting such spectra. The differences between transport models pertain mostly to their handling of the EOS: however, reaction-plane-averaged observables show no significant sensitivity to changes in the assumed EOS.

Ter Haar and Malfliet[10] have argued that in-medium effects beyond Pauli blocking of occupied final states should be incorporated in transport models. Their calculations are generally taken to indicate an effective in-medium nucleon-nucleon scattering cross section $\sigma_{NN}^{\rm eff}=\alpha\sigma_{NN}^{\rm UU}$, where $\alpha\approx0.7$. Cugnon *et al.*[11] have argued that α might be strongly dependent on momentum, density, and temperature. One of the goals of the present work is to address these theoretical uncertainties with the help of experimental data. In order to test the assumption that a constant value of α can be used, the widest possible range of experimental conditions should be studied. It is desirable to constrain α within about ±0.1 or better; this requirement originates from the fact that predicted values of EOS-related observables like flow are influenced by α as well as by the assumed EOS, and an $\sim10\%$ change in α has a significantly smaller effect than changing between a soft and a stiff EOS.[12–14,7] Thus, as long as we must rely on transport models to relate experimental observables to the nuclear EOS, we can expect that progress in this endeavor is dependent on improving constraints on α.

Aichelin et al.[9] have recently compared predictions of several models to inclusive p-like momentum spectra at polar angles $\theta_{lab}=20°$, 40°, and 60° for 0.8A GeV La+La, as measured at the Bevalac B30 magnetic spectrometer by Hayashi et al.[15] The term p-like signifies that all detected protons contribute equally to the cross section, be they free or bound in fragments (up to ^{4}He in this case); models that do not incorporate a final-state clustering prescription [such as standard VUU (Ref. 2)] can readily predict this type of cross section. Aichelin et al.[9] reported that all four transport codes[2-7] mentioned above overpredicted the invariant cross section

$$\sigma_{inv}=(E/p^2)d^2\sigma/dp\,d\Omega$$

at 20° by a factor of 1.5–2 for $p_{lab}>1$ GeV/c; the dotted line in Fig. 1 shows the reported VUU calculation. The error bars in Fig. 1 show the systematic uncertainty for the experimental results; the statistical errors are much smaller. This discrepancy was interpreted in terms of a possible failure of the models to adequately incorporate all the effects of the nuclear medium, and it was argued that there is no constant $\alpha\neq1$ that improves the overall agreement relative to $\alpha=1$ at all three polar angles.

In 1985, Kruse et al.[3] compared VUU calculations to inclusive proton spectra for Ar+Ca at 42A, 92A, and 137A MeV,[16] and in the absence of the experimental data needed to construct p-like spectra, used a six-dimensional coalescence model to calculate the contribution from free protons. The overall agreement was reasonably good, but discrepancies ranged up to a factor of ~3 in regions where we expect the difference between proton and p-like spectra to be large. More recently, Madey et al.[17] have compared transport models with inclusive neutron cross sections for Nb+Nb and Au+Au at 0.8A GeV, and they report significant discrepancies at smaller polar angles. Because that study dealt with neutron rather than neutronlike spectra, composite fragments again complicate the interpretation of the transport model comparisons. The advantage of comparing models to p-like data is that shortcomings in clustering calculations are eliminated as a possible source of discrepancies. The present work demonstrates that, for a variety of mass-symmetric systems, an interpretation of p-like data in terms of $\alpha\approx1$ (but with large uncertainties) is tenable.

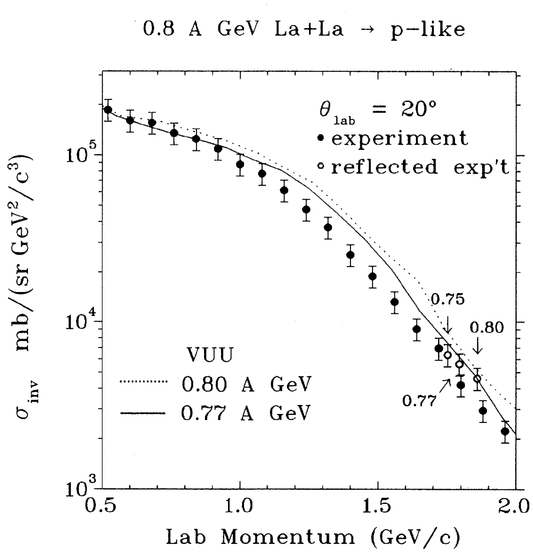

FIG. 1. Inclusive p-like cross section at a laboratory polar angle of 20°, in 0.8A GeV La+La collisions. The experimental data of Hayashi et al. are compared with the Vlasov-Uehling-Uhlenbeck transport model. Statistical errors on the experimental data are negligible, and error bars show systematic uncertainties. The open circles are derived from data at $\theta_{lab}=89°$, and the labels denote the assumed beam energy in A GeV (see text).

In the experiment of Hayashi et al., the kinetic energy of the partially stripped beam ions in the Bevatron was 800A MeV. Based on what is known to have been in the path of the beam, we calculate an energy loss of 31A MeV up to the mean interaction point in the target, and there is a possible additional energy loss that would bring the total to roughly 40A MeV. Projectile-target symmetry provides a means of independently approaching the same question; each phase-space point (θ_{lab},p_{lab}) can be mapped to the corresponding $(p_\perp,y_{c.m.})$, where $y_{c.m.}$ is rapidity measured in the center-of-mass frame, and symmetry requires

$$\sigma_{inv}(p_\perp,y_{c.m.})=\sigma_{inv}(p_\perp,-y_{c.m.})\ .$$

We find that the spectra in the middle of the θ_{lab} range offer relatively poor beam-energy discrimination, and the most significant test comes from reflecting the spectrum at the largest measured angle, $\theta_{lab}=89°$, and comparing with the cross section at the smallest measured angle, $\theta_{lab}=20°$. For a given pair of θ_{lab} values, there is a unique pair of p_{lab} values that satisfies the requirement for both $p_\perp$ values to be the same. The three open circle symbols in Fig. 1 show the reflected 89° results, assuming (from right to left) a beam energy of 0.80A, 0.77A, and 0.75A GeV. While the data satisfy the expected symmetry within the nominal systematic uncertainties for all three assumptions about the beam energy, the lower energies are favored. Figure 1 also shows VUU calculations for beam energies of 0.80A and 0.77A GeV, and it can be seen that a 30A MeV beam-energy loss accounts for about half the difference between the original 0.80A GeV calculation and the experimental data. It should be kept in mind that the plotted error bars represent systematic uncertainties, and so it cannot be assumed that the uncertainties on the various points are independent, or that a given discrepancy becomes more significant if it extends over many points.

We emphasize that the beam-energy loss is of diminishing importance at the larger polar angles—for instance, at $\theta_{lab}=60°$, the difference in predicted cross section between beam energies of 0.80A and 0.77A GeV is about half that shown in Fig. 1, and drops to within statistical uncertainties for our VUU simulations at $\theta_{lab}=89°$. In light of the duplication of information in a mass-symmetric system, and the reduced systematic uncertainty associated with comparisons in the backward c.m. hemisphere, it is more appropriate to rely on this kinematic region for detailed comparisons between transport models and experiment.

Figure 2 shows the data of Hayashi *et al.* at three angles spanning rapidities $y_{\rm lab}=0$ through $\sim\frac{2}{3}y_{\rm beam}$, along with VUU calculations for two values of α. The VUU predictions for $\alpha=1$ are in excellent overall agreement with experiment, and $\alpha=1$ is favored over $\alpha=2$. However, it is clear that the sensitivity to α and the systematic uncertainties in the data are such that the previously discussed goal of constraining α to within ± 0.1 is out of reach in this case.

Other studies aimed at determining α have focused on rapidity spectra, dN/dy. For example, for high multiplicity $1.2A$ GeV Ar+BaI_2 events in the Bevalac streamer chamber, dN/dy favors VUU predictions with $\alpha\approx 1$.[18] Experimental rapidity spectra are strongly dependent on multiplicity and predicted spectra are likewise very sensitive to impact parameter; consequently, particular attention was devoted to simulating the experimental multiplicity selection as realistically as possible. Nevertheless, systematic uncertainties associated with the multiplicity simulation remain and provide a motivation for comparisons such as the present work using inclusive data. dN/dy distributions are, of course, just a subset of the information contained in a comprehensive set of momentum spectra, and VUU dN/dy calculations do not show evidence of improved sensitivity to α if central collisions are selected. Because Fig. 2 uses a logarithmic scale for σ and includes systematic error bars for the experimental data, the present comparison may appear to yield a poorer sensitivity to α than previous studies.[18,7,19]

While many of the assumptions underlying nuclear transport models relate to the incorporation of the EOS and have negligible impact on reaction-plane-averaged observables, the assumption that the precollision nuclei have sharp surfaces with radii given by $r_0A^{1/3}$, $r_0=1.12$ fm, does have some effect on the inclusive momentum spectra. In other contexts, a frequently used value is $r_0=1.18$ fm.[20] Decreasing r_0 increases the equilibrium density, and therefore increases the frequency of hard nucleon-nucleon scattering in the early stages of the nucleus-nucleus collision. Unlike an increase in α, which has a relatively uniform effect on $\sigma_{\rm inv}$ as shown in Fig. 2, we find that a decrease in r_0 has a stronger effect near midrapidity angles (40°–60°) and tends to increase $\sigma_{\rm inv}$ above $p_{\rm lab}\sim 1$ GeV/c while decreasing $\sigma_{\rm inv}$ at lower momenta. The default radius parameter in the VUU code, $r_0=1.12$ fm, yields close to optimum agreement with the data of Hayashi *et al.*, and these data exclude r_0 values that differ by about 10% or more from 1.12 fm. Future data capable of constraining α with improved accuracy may require either a simultaneous optimization of r_0 or a diffuse-surface description of the initial nuclei.

Figure 3 shows data for Ar+KCl at $0.8A$ GeV, also measured at the Bevalac B30 spectrometer by Nagamiya *et al.*[21] The VUU calculations are again in excellent agreement with experiment and still are consistent with $\alpha\approx 1$, albeit at no better sensitivity than is demonstrated in Fig. 2. The effects of beam-energy loss are negligible at $\theta_{\rm lab}\geq 30°$ in the case of a $Z=18$ projectile.

The same paper by Nagamiya *et al.* also presents inclusive spectra for other projectile-target and beam-energy combinations. *p*-like data for these additional systems have been reported in the form of center-of-mass kinetic-energy spectra at $\theta_{\rm c.m.}=90°$. Figure 4 presents data and VUU predictions for three mass-symmetric systems at a beam energy of $0.8A$ GeV. This type of spectrum leads to much the same level of sensitivity to α as

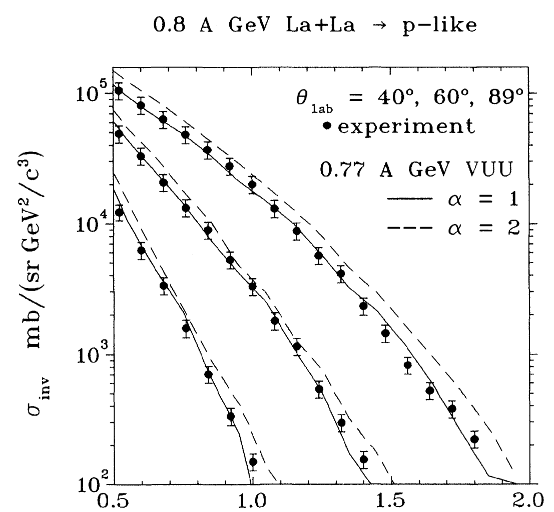

FIG. 2. As Fig, 1, except at three angles covering midrapidity to target rapidity. The two curves show the sensitivity to a doubling of the in-medium *NN* scattering cross sections in VUU.

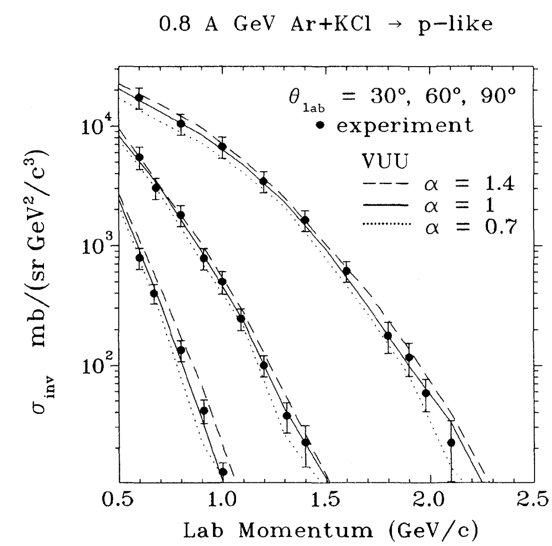

FIG. 3. Inclusive *p*-like cross sections for a lighter mass-symmetric system, Ar+KCl.

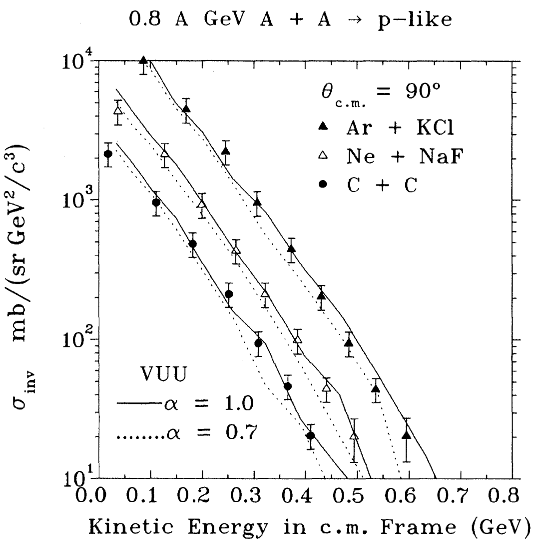

FIG. 4. Inclusive *p*-like cross sections as a function of kinetic energy at a polar angle of 90° in the center-of-mass frame.

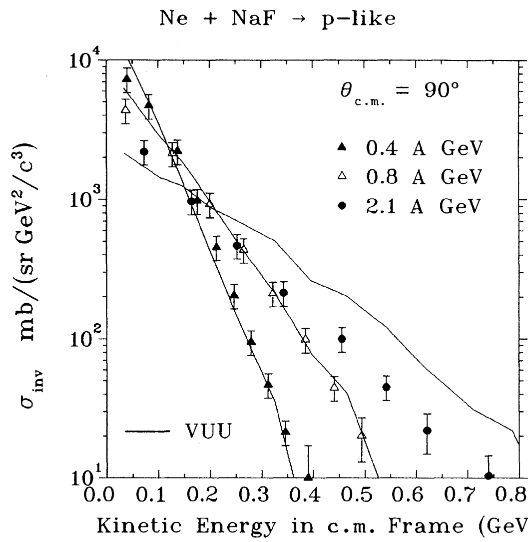

FIG. 5. As Fig. 4, but now showing beam-energy dependence for Ne+NaF.

the spectra in the previous figures and, once again, $\alpha \approx 1$ yields good agreement between VUU and experiment.

Finally, Fig. 5 presents the same type of comparison as Fig. 4, except that the beam-energy dependence between $0.4A$ and $2.1A$ GeV is shown for Ne+NaF. In the case of the highest beam energy, the slope predicted by VUU is significantly "hotter" than experiment. Pion production increases substantially between $0.8A$ and $2.1A$ GeV, raising the possibility that previous findings about the insensitivity of inclusive spectra to the assumed EOS might no longer apply. Our calculations indicate that the discrepancy at $2.1A$ GeV is not affected within statistics by changing the EOS.[22] Although we cannot offer any definitive explanation for this discrepancy, it should be kept in mind that VUU incorporates only the lightest inelastic nucleon resonance [the Δ(1232)] and is a nonrelativistic model; therefore, a divergence from experiment with increasing beam energy is not unexpected.

It should be kept in mind that the dominant uncertainties associated with α are systematic, and therefore a statistical procedure to extract a more precise estimate of this parameter is not appropriate. Only values outside the range $\frac{1}{2} < \alpha < 2$ can be excluded with confidence. These considerations, in conjunction with a similar conclusion for the high multiplicity $Ar+BaI_2$ streamer chamber data of Ref. 18, set limits to the possible dependence of α on details of the medium. To illustrate this point, we have calculated the density in a spherical volume of radius 2 fm surrounding each nucleon-nucleon collision point, averaged over all *NN* collisions at all times in all events. This quantity reflects the average density of the nuclear medium when an *NN* collision occurs. The results for the seven systems of interest are shown in Table I. It can be seen that the average density may increase by as much as ~42% between inclusive $0.4A$ GeV Ne+NaF and moderately central $1.2A$ GeV $Ar+BaI_2$ events. The two colliding nucleons contribute $0.35\rho_0$ to each density given in Table I, with the remainder coming from adjacent nucleons in the medium. If the former contribution is subtracted, the percentage spread in mean densities increases even more.

In summary, we conclude that the VUU model can

TABLE I. Density (in units of equilibrium density ρ_0) in a spherical volume of radius 2 fm centered on each *NN* collision, averaged over collisions at all times in all events, based on a VUU simulation.

	E_{beam} (A GeV)	Stiff EOS	Soft EOS
C+C (all b)	0.8	1.22	1.25
Ne+NaF (all b)	0.4	1.22	1.22
Ne+NaF (all b)	0.8	1.29	1.35
Ne+NaF (all b)	2.1	1.40	1.53
Ar+KCl (all b)	0.8	1.30	1.48
La+La (all b)	0.8	1.43	1.53
$Ar+BaI_2$ (b < 6 fm)	1.2	1.60	1.73

reproduce charged-particle-inclusive spectra within current experimental uncertainties for systems of total mass number ranging from 24 to 278, and for bombarding energies between $0.4A$ and $1.2A$ GeV. Previous work has shown that predictions of inclusive data by different nuclear transport models agree closely, and so the present findings should be valid for all models of this type. The systematic uncertainties associated with our comparisons are generally too large to constrain the cross section for binary collisions in the nuclear medium σ_{NN}^{eff} at the level of precision needed in order for transport models to be used with confidence to infer properties of the nuclear equation of state. If σ_{NN}^{eff} depends on in-medium effects that are neglected in the current transport models, the available evidence suggests that the additional correction factor is within the range $\frac{1}{2}-2$ and cannot be strongly dependent on details of the medium, such as density. Finally, our findings reinforce the need for new, more precise measurements of absolute cross sections for the various fragment species for a wide range of projectile-target masses and bombarding energies, preferably in conjunction with information about the event reaction plane and multiplicity.

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