# Comment on "Exactly central heavy-ion collisions by nuclear hydrodynamics"

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Problems arising in viscous nuclear fluid dynamical models of high-energy heavy-ion collisions are discussed. The importance of an accurate treatment of the transport properties of the hot and dense nuclear matter is pointed out.

[NUCLEAR REACTIONS Heavy-ion reaction, nuclear hydrodynamics, viscosity.]

In a recent publication,[1] a detailed analysis of central heavy-ion collisions in terms of a fluid dynamical model was presented. An interesting feature of the calculations was the inclusion of the transport processes of the fluid into a two-dimensional model after the previous one-dimensional approaches.[2–4] However, the results and conclusions presented in Ref. 1 are questionable because the following problems were present in the calculations[1]:

(1) Equations (3.11), (3.13), and (3.14) of Ref. 1 are incorrect. The shear viscosity coefficient $\eta$ must enter into these equations with a factor $+\frac{4}{3}$ [see e.g., Ref. 5, Eq. (77.6)] instead of the factor $-\frac{2}{3}$ used in Ref. 1. Consequently, the fitted value for the bulk viscosity coefficient, Eq. (3.15), is incorrect. Also the relative magnitude of the shear and the bulk viscosity, $\eta=0.75$ MeV/(fm$^2c$) $<<\xi=18.76$ MeV/(fm$^2c$), is questionable. If one assumes that the matter consists of pointlike particles without internal degrees of freedom, the bulk viscosity coefficient will vanish, i.e., $\eta>>\xi$.[6] Hence, the damping of the giant monopole resonance may be caused exclusively by the shear viscosity $\eta$, in contrast to the assumptions made in Ref. 1. There the shear viscosity even acts against the damping.

(2) The particular volume and temperature dependence of the shear viscosity $\eta$ chosen in Ref. 1 is questionable. The fixed value of $\eta=0.75$ MeV/(fm$^2c$) corresponds to a temperature $T=30$ MeV, while it is used later to obtain the value of the bulk viscosity from a process at $T\approx0$, where the applied expression diverges. Since this equation describes the low-temperature behavior of the matter, where the particle-particle interactions are important but the kinetic energies are small, its applicability to relativistic heavy ion collisions might be limited. Considerations based on kinetic theory[6] yield a value of $\eta$ that is about ten times higher. Other considerations[7–9] based on fission studies also result in essentially larger viscosity values [$\eta=3-15$ MeV/(fm$^2c$)]. The temperature dependence $\eta\sim T^{-2}$ described by Eq. (3.7) of Ref. 1 is valid only for a Fermi liquid at moderate temperatures.

The constant, small viscosity coefficient used in Ref. 1 results in shock front thicknesses equal to the computational mesh size (0.6−1.2 fm, see Sec. V A of Ref. 1), which causes severe numerical problems (see below). An increase of the viscosity with temperature is necessary[10] to obtain a realistic shock front width.

(3) In the solution of fluid dynamical equations using Eulerian methods, a "numerical viscosity" arises,[11] which should be smaller than the physical viscosity investigated. However, this condition seems not to be fulfilled in Ref. 1. Although we can not evaluate accurately the value of the numerical viscosity present in Ref. 1, since detailed results of the numerical calculations are not available to us, one may estimate the numerical viscosity in the following way: The thickness of the shock front is proportional to the viscosity.[5,10] If the viscosity tends to zero, the shock front becomes infinitely sharp. Then the total viscosity (i.e., the sum of the physical and "numerical" viscosities) can be measured directly by the thickness of the shock front.

 

TABLE I. Total number of nucleons obtained from the cross section tables in Ref. 1.

| Number of table in Ref. 1 | Input nucleon number | Observed outgoing nucleon number | Number of missing nucleons |
|---|---|---|---|
| Ia | 217 | 208.1 | −8.9 (−4%) |
| Ib | 217 | 215.9 | −1.1 (−0.5%) |
| IIa | 416[a] | 56.1 | −359.9 (−86.5%) |
| IIb | 416[a] | 84.3 | −331.7 (−79.7%) |
| IIIa | 217 | 200.4 | −16.6 (−7.7%) |
| IIIb | 217 | 220.9 | + 3.9 (+1.8%) |

[a]Apparently there is a misprint in the table caption of Table II in Ref. 1: "Ne+Au" should be replaced by "Pb+Pb."

This value never tends to zero because of the finite mesh size used in the Eulerian method. This implies that the total viscosity in Ref. 1 has been only slightly larger than the numerical viscosity, since the shock-front thickness is close to the mesh size (Sec. V A of Ref. 1).

On the basis of a similar numerical method,[12] we can estimate the numerical viscosity in the calculation of Ref. 1 to be about 5–25 MeV/($fm^2c$) depending on the mesh size and the details of the numerical procedure. Hence, the use of the terms "small" and "normal" viscosity in Ref. 1 is misleading, since, in fact, both the normal and, in particular, the small viscosity values are negligible in comparison to the numerical viscosity. To actually investigate small viscosity values, a much smaller mesh size must be used.

(4) From the tabluated cross sections, Tables I–III in Ref. 1, the total number of emitted nucleons can be calculated (see Table I). For the case $^{208}$Pb+$^{208}$Pb (Tables IIa and IIb in Ref. 1), we observe that *more than 80%* of the particles do not appear in the calculated cross sections.

(5) The same tables in Ref. 1 give the opportunity to calculate the energy of the outgoing particles in the laboratory system (see Table II). For all cases we find that more than 90% of the incident bombarding energy has disappeared after the collision. Even when we assume that the whole projectile and target breaks up into single nucleons, so that the total binding energy [$\sim 8(A_T+A_P)$ MeV] is subtracted from the incoming projectile energy, the outgoing particles still carry only a small part of the then available energy.

This colossal energy loss might be caused by the numerical method: In Ref. 1, thermal energy can only be produced via the viscous effects taken into account in Eq. (2.3) of Ref. 1. The much larger energy dissipation caused by the sharp shock fronts ("shock heating") has not been included in Ref. 1: Although the physical viscosity and the numerical viscosity both absorbed the incident kinetic energy, only little thermal excitation energy has been produced [a total of 1400 MeV in the "viscous" case (see Sec. V A in Ref. 1)]. These problems are usually[3,11–17] avoided by integrating the total (instead of the thermal) energy conservation equation, which takes the shock heating into account appropriately. The so-called "nonviscous" calculations in Ref. 1 did not include any mechanism for heat production

TABLE II. Total energies of emitted nucleons obtained from the cross section tables in Ref. 1.

| Number of table in Ref. 1 | Input energy (MeV) | Observed outgoing energy (MeV) | Max. binding energy (MeV) | Missing energy (MeV) |
|---|---|---|---|---|
| Ia | 5000 | 362 | 1736 | > −2902 (−58%) |
| Ib | 5000 | 380 | 1736 | > −2885 (−58%) |
| IIa | 20 800 | 1800 | 3328 | >−15672 (−75%) |
| IIb | 20 800 | 2574 | 3328 | >−14898 (−72%) |
| IIIa | 8000 | 443 | 1736 | > −5821 (−73%) |
| IIIb | 8000 | 579 | 1736 | > −5685 (−71%) |

TABLE III. Relative velocities between target and projectile obtained graphically from the density contour plots in Ref. 1.

| Number of figure in Ref. 1 | Input laboratory bombarding energy [MeV/nucleon] | Corresponding relative velocity[a] [$c$] | Observed relative velocity[b] [$c$] | Corresponding "observed" laboratory bombarding energy [MeV/nucleon] |
|---|---|---|---|---|
| 2 | 250 | 0.733 | 0.356 | 59.0 |
| 4 | 250 | 0.733 | 0.366 | 62.4 |
| 6 | 100 | 0.462 | 0.193 | 17.3 |
| 8 | 100 | 0.463 | 0.204 | 19.4 |
| 16 | 50 | 0.328 | 0.143 | 9.5 |

[a]As the calculations in Ref. 1 are nonrelativistic, $v/c=(2E/m_n c^2)^{1/2}$, with $m_n c^2=931$ MeV.
[b]Typical error in the graphical determination of the relative velocity is $\Delta v/c \approx \pm 0.02$.

[see also Sec. V A of Ref. 1: In the small viscosity case, the thermal energy remains negligible (about 1 MeV at most) throughout the entire interaction]. However, a nonviscous, i.e., perfect fluid is also subject to strong energy dissipation and shock heating once sharp discontinuities develop in supersonic flow. The predicted strong dependence of the calculated maximum density on the transport coefficients (see Fig. 15 of Ref. 1) is in contradiction to previous calculations,[3,4,10,12] where the viscosity lowered the density only by a few percent compared to the corresponding exact calculation for a perfect fluid.[3,10,18] In contrast to Ref. 13, the angular distributions shown in Figs. 3, 5, 7, and 9–14 of Ref. 1 exhibit *sharp sidewards peaks* with a small peak width ($\Delta\theta \sim 10°-20°$), in agreement with the early data and predictions based on the Mach shock wave model[15] and fully three-dimensional fluid dynamical calculations.[14] However, the sharp forward ($\theta_{lab}=0°$) and backward ($\theta_{lab}=180°$) peaks,[1] which contain most of the matter in the system (see Tables I and III of Ref. 1), have not been observed in other calculations.[14,16,17] We also do not agree with the authors of Ref. 1 that the inclusion of the thermal velocities into the calculation of the cross sections would cause only negligible changes (see Sec. V A of Ref. 1): If the heat production is properly included, the thermal smearing[4] in the late "breakup" stages of the reaction is very important[12,17]: It changes not only the peak width in the angular distributions dramatically, but also the peak heights and their positions.

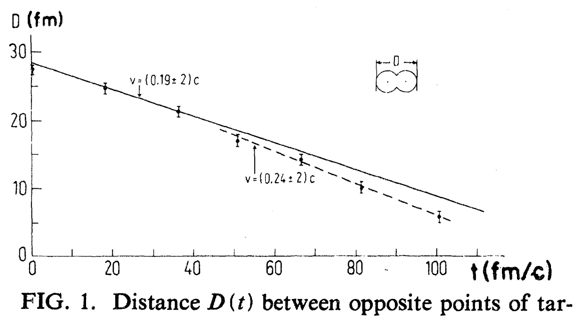


FIG. 1. Distance $D(t)$ between opposite points of target and projectile (see insert in figure) at different time moments for the reaction Pb (100 MeV/nucleon) + Pb shown in Fig. 6 of Ref. 1. The error bars represent the uncertainty in the graphical determination of $D(t)$.

(6) The durations of the collisions depicted in the density contour plots and the interaction times quoted in Sec. II of Ref. 1 seem to be much longer than those usually expected for collisions at these high energies. This point can be checked by reading the approximate values of the relative velocity of projectile and target off the density contour plots (Figs. 2, 4, 6, 8, and 16) shown in Ref. 1 using the time moments and length scales in the figures. The resulting relative velocities (see our Table III and Fig. 1) are actually less than half of the correct values, i.e., the calculations in Ref. 1 correspond to about $\frac{1}{4}$ of the claimed bombarding energies. When this lowered energy scale is considered, the compression in the nonviscous case is tremendous: For Pb+Pb, the calculations in Ref. 1 predict a compression $n/n_0 \approx 4$ at $E_{lab} \sim 90$ MeV/nucleon.

(7) From the time dependence of the distance $D(t)$ between the projectile and target nuclei plotted in Fig. 1 for the "inviscid" reaction Pb(100 MeV/nucleon)+Pb, one observes that the projectile and the target are accelerated towards each other during the collision. This acceleration may be responsible for the increased compression rates in the inviscid case, but it seems to be unphysical and, in

our opinion, requires explanation.

Most of the above listed problems seem to be of computational origin. However, since the transport coefficients of nuclear matter are not small, an accurate treatment of these effects is necessary. Careful fluid dynamical calculations with a realistic treatment of viscosity and thermoconductivity are necessary to investigate other properties of hot dense nuclear matter in high energy heavy ion collisions: In particular, the influence of the transport properties on the reaction dynamics is of similar importance as the influence of the nuclear equation of state,[18] which in turn is one of the most important motivations for doing high energy heavy ion physics.

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[1]H. H. K. Tang and C. Y. Wong, Phys. Rev. C 21, 1846 (1980).

[2]C. Y. Wong, J. A. Maruhn, and T. Welton, Phys. Lett. 66B, 19 (1977).

[3]H. Stöcker, J. A. Maruhn, and W. Greiner, Phys. Lett. 81B, 303 (1979); H. Stöcker, J. Hofmann, J. A. Maruhn, and W. Greiner, Prog. Part. Nucl. Phys. 4, 133 (1980); the viscosity coefficient $\eta$ used in these calculations has been $\eta = 10$ MeV/(fm$^2c$) and not $10\hbar$ fm$^{-3}$ as given in the text. We are grateful to J. R. Nix for bringing this misprint to our attention.

[4]L. P. Csernai, B. Lukacs and J. Zimanyi, Nuovo Cimento Lett. 27, 111 (1980); L. P. Csernai and H. W. Barz; Z. Phys. A 296, 173 (1980).

[5]L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Pergamon, New York, 1959).

[6]K. Huang, *Statistical Mechanics* (Wiley, New York, 1963); V. P. Silin, *Introduction to Kinetic Theory of Gases* (Nauka, Moscow, 1971); D. N. Zubarev, *Nonequilibrium Statistical Thermodynamics* (Nauka, Moscow, 1971); V. M. Galitsky, Yu. B. Ivanov and V. A. Khagulian, Proceedings of the International Conference on Extreme States in Nuclear Systems, Dresden, 1980, see Ref. 4, p. 5.

[7]R. Wieczorek, R. Hasse, and R. W. Süssmann, in *Physics and Chemistry of Fission* (IAEA, Vienna, 1974) Vol. I, p. 523; $\eta = 3-15$ MeV/(fm$^2c$).

[8]R. Wieczorek, R. Hasse, and R. W. Süssmann, in *Proceedings of the International Conference on Nuclear Physics, München, 1973* (North-Holland, Amsterdam, 1973), Vol. I, p. 585; $\eta = 3$ MeV/(fm$^2c$).

[9]J. R. Nix and A. J. Sierk, Los Alamos Report LA-UR-79-1632, 1979; $\eta = 5.6$ MeV/(fm$^2c$).

[10]L. P. Csernai, H. W. Barz, B. Lukacs, and J. Zimanyi, *Proceedings of the EPS Topical Conference on Large Amplitude Collective Nuclear Motion, Keszthely, 1979* (KFKI, Budapest, 1979), Vol. II, p. 533.

[11]F. H. Harlow, A. A. Amsden, and J. R. Nix, J. Comp. Phys. 20, 119 (1976).

[12]G. Buchwald, L. P. Csernai, J. A. Maruhn, W. Greiner, and H. Stöcker, Phys. Rev. C 24, 135 (1981).

[13]J. R. Nix, Prog. Part. Nucl. Phys. 2, 237 (1979); A. A. Amsden, F. H. Harlow and J. R. Nix, Phys. Rev. C 15, 2059 (1977); A. A. Amsden, G. F. Bertsch, F. H. Harlow, and J. R. Nix, Phys. Rev. Lett. 35, 905 (1975).

[14]H. Stöcker, J. A. Maruhn, and W. Greiner, Phys. Rev. Lett. 44, 725 (1980); H. Stöcker, J. A. Maruhn, and W. Greiner, Z. Phys. A 293, 173 (1979); H. Stöcker, R. Y. Cusson, J. A. Maruhn, and W. Greiner, *ibid.* 294, 125 (1980); H. Stöcker, J. A. Maruhn, and W. Greiner, *ibid.* 290, 297 (1979).

[15]H. G. Baumgardt, J. U. Schott, Y. Sakamoto, E. Schopper, H. Stöcker, J. Hoffmann, W. Scheid, and W. Greiner, Z. Phys. A 237, 359 (1975); J. Hofmann, H. Stöcker, U. Heinz, W. Scheid, and W. Greiner, Phys. Rev. Lett. 36, 88 (1976).

[16]J. R. Nix and D. Strottmann, Phys. Rev. C 23, 2548 (1981).

[17]H. Stöcker, L. P. Csernai, G. Graebner, G. Buchwald, H. Kruse, R. Y. Cusson, J. A. Maruhn, and W. Greiner, Phys. Rev. C 25, 1873 (1982); Phys. Rev. C (to be published); H. Stöcker *et al.*, Phys. Rev. Lett. 47, 1807 (1981).

[18]H. Stöcker, M. Gyulassy, and J. Boguta, Phys. Lett. 103B, 269 (1981).