[9]Using an iterative procedure for obtaining J_a (subtracting from $\Delta E_{0,-1}$ contributions due to spin deviations before assuming the molecular field model), we improve the results, $\delta E = 11.3$ for $RbMnF_3$ and 12.2 for $KMnF_3$. The value for MnF_2 decreases by less than 0.1 cm^{-1}.

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3^- CONTINUUM STATES OF O^{16} IN THE EIGENCHANNEL REACTION THEORY*

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The complete 3^- part of the S matrix for O^{16} has been computed in the one-particle, one-hole approximation. In the continuum states the isospin invariance is totally broken; analogous partial cross sections for protons and neutrons show large differences.

In a recent publication[1] a method has been described by which the eigenstates of the S matrix, i.e., the eigenchannels, can be directly computed. We have tried out this method in the case of the 3^- states of O^{16} in the one-particle, one-hole approximation. In this Letter we report briefly the results of this calculation. The details will be given elsewhere.

The essential points of the method are as follows: The eigenstates of the S matrix are standing waves in all experimental channels with a common phase shift, say $\delta^{(\beta)}$. There are as many eigenstates as there are open channels at this energy. We denote the amplitudes of the standing waves of an eigenstate of the S matrix in the experimental channel c by $V_c^{(\beta)}$. In terms of these quantities the S matrix is given by

$$S_{cc'} = \sum_\beta V_c^{(\beta)} \exp(2i\delta^{(\beta)}) V_{c'}^{(\beta)*}. \qquad (1)$$

A knowledge of the $V^{(\beta)}$ and $\delta^{(\beta)}$ as functions of the energy thus allows the complete description of all one-particle reactions. For example, the total cross section then is (I = spin of target nucleus, s = spin of incident nucleon)

$$\sigma_{tot} = \frac{2\pi\lambda^2}{(2I+1)(2S+1)} \times \sum_J (2J+1) \sum_c [1 - \mathrm{Re} S_{cc}^{[J]}], \qquad (2)$$

where the summation over c is restricted to those channels which contain only the ground state of the target nucleus. We compute here only the term with $J = 3$. The form of the eigenchannel wave function in the asymptotic region, i.e., for $r_c \geq a$, is

$$\Psi^{(\beta)} = \sum_c V_c^{(\beta)} [\cos\delta^{(\beta)} F_c(k_c r_c) - \sin\delta^{(\beta)} G_c(k_c r_c)] \tilde{\psi}_c, \qquad (3)$$

where the F and G are the regular and irregular radial functions of the continuum particle; for a neutron they are simply $j_l(k_c r_c)$ and $n_l(k_c r_c)$, respectively. The channel wave functions $\tilde{\psi}_c$ contain in addition to the wave function of the daughter nucleus (i.e., the hole state) the angular momentum part of the continuum particle.

The computation of the eigenchannels was done as follows: At a given energy, say E, the wave numbers k_c are known for all open channels from the binding energy and the spectrum of the bound states of the daughter nucleus. Assuming a phase shift, say δ, the logarithmic derivatives of the radial wave functions in all open channels are computed from (3) at $r_c = a$. Sets of single-particle wave functions for the different channels are now obtained for a real Saxon-Woods potential[2] using these logarithmic derivatives as the boundary conditions. Arbitrary boundary conditions can be used for the states appearing only in closed channels. An orthonormal set of particle-hole states is now constructed with these single-particle wave functions and the Hamiltonian is diagonalized in the space of these one-particle, one-hole (1p-1h) states. A zero-range force[3] with exchange was employed. The eigenvalues obtained are plotted as a function of δ in Fig. 1 for the case $E = 20$ MeV. The eigen-

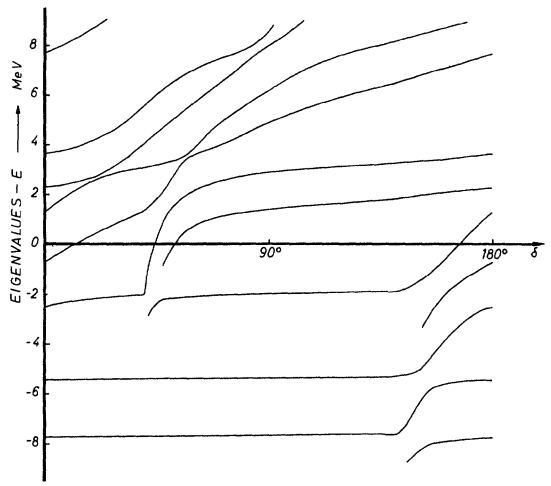

FIG. 1. The eigenvalues of the 3^- compound system as a function of the common phase shift δ of the single-particle continuum states. The boundary conditions applied in this case are those for $E = 20$ MeV, where four channels are open.

phases are found as the crossing points of an eigenvalue curve with the line E, i.e., as the roots of the equations $E_\nu(\delta) - E = 0$. The eigenvectors $V^{(\beta)}$ of the eigenchannels then are given, except for normalization by the eigenvector associated with the eigenvalue $E_\nu(\delta^{(\beta)})$. The eigenphases are plotted in Fig. 2 as functions of the energy. In the region between 12.2 and 15.7 MeV only one channel is open, viz., the $(d_{5/2}p_{1/2}^{-1})$ proton channel. At 15.7 MeV the equivalent neutron channel opens. By the time the first eigenphase goes through 90° at about 17.8 MeV two more channels have appeared, viz., the $(d_{3/2}p_{3/2}^{-1})$ and $(d_{5/2}p_{3/2}^{-1})$ proton channels. At 18 MeV the four open channels of the resonating eigenchannel have the following amplitudes (the subscripts n or p stand for neutron or proton particle-hole pair): $0.95\ (d_{5/2}p_{1/2}^{-1})_p$; $-0.29\ (d_{5/2}p_{1/2}^{-1})_n$; $0.07\ (d_{3/2}p_{3/2}^{-1})_p$; $-0.07\ (d_{5/2}p_{3/2}^{-1})_p$. It is interesting that the energy interval over which the eigenphase rises, i.e., the width of the resonance, is of the order of only 0.5 MeV despite the high kinetic energy of the proton in the channel $(d_{5/2}p_{1/2}^{-1})_p$ which is about 2 MeV above the maximum of the combined Coulomb and angular-momentum barrier. This results from the fact that in the nuclear wave function 94% of the intensity is associated with the bound configurations and only 6% is associated with all the open channels. The width of the peak thus is cut down by a factor of the

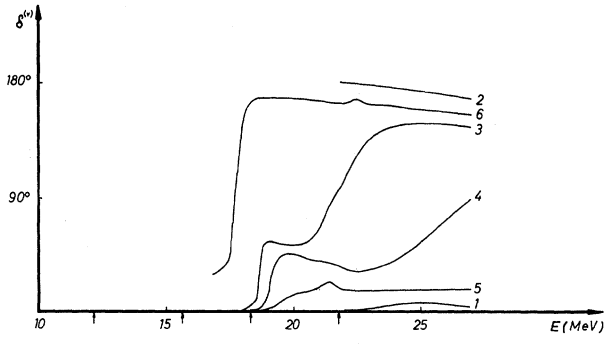

FIG. 2. Eigenphases for the 3^- states as a function of the excitation energy of the compound system. The numbers on the different curves label the channels presumably predominant in the neighbourhood of the corresponding threshold. The sequence is (1) $(d_{3/2}p_{3/2}^{-1})_n$, (2) $(d_{5/2}p_{3/2}^{-1})_n$, (3) $(d_{5/2}p_{1/2}^{-1})_n$, (4) $(d_{3/2}p_{3/2}^{-1})_p$, (5) $(d_{5/2}p_{3/2}^{-1})_p$, (6) $(d_{5/2}p_{1/2}^{-1})_p$. The arrows indicate the various thresholds.

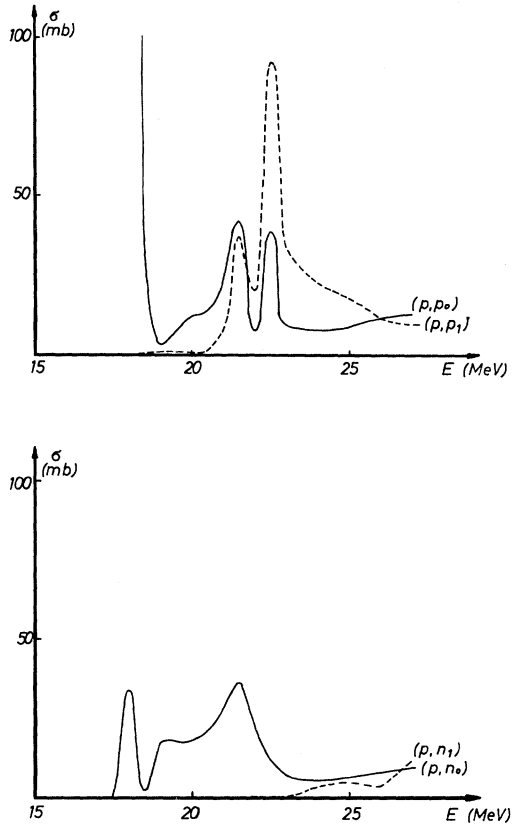

FIG. 3. Contribution of the 3^- states of the O^{16} compound system to proton-induced cross sections. The calculation has been done without inclusion of an imaginary part in the optical potential. Note that the (p,p_0) cross section has a very strong resonance at about 18 MeV.

order 10 as compared to a corresponding peak in potential scattering.

At about 22 MeV anomalies seem to appear in the trends of the eigenphases of the open channels. They could be related to the threshold of the two channels at 21.8 MeV, viz., $(d_{3/2}p_{3/2}^{-1})_n$ and $(d_{5/2}p_{3/2}^{-1})_n$. The rather sudden jump of eigenphase 3 at about 18.7 MeV could also be associated with the threshold at 18.35 MeV.

In Fig. 3 we have plotted the cross sections for several reactions induced by protons incident on N^{15}. It is remarkable that the "analog" reactions, viz., (p,p_0) and (p,n_0) as well as (p,p_1) and (p,n_1), are as different as shown. The reason for this evidently is the large isospin impurity associated with the large difference in the boundary conditions for protons and neutrons. This difference does not appear in a calculation which ignores the continuum character of the unbound states.

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[2]The parameters of the Saxon-Woods potential $V_C \times\{\rho(r)-2(\hbar/2M_C)^2(\vec{l}\cdot\vec{\sigma})\,d\rho(r)/dr\}$ with $\rho(r)=\{1+\exp[(r-R_0)/b]\}^{-1}$ are $V_C=-50$ MeV, $\alpha=35$, $R_0=3.15$ F, $b=0.65$ F; M is the nucleon mass.

[3]The parameters of the force $V_0\delta(r_1-r_2)[a_0+a_\sigma(\vec{\sigma}_1\cdot\vec{\sigma}_2)]$ are $V_0=-1000$ MeV F^3; $a_0=0.865$, $a_\sigma=0.135$.

PERTURBATION THEORY OF MULTIPHOTON PROCESSES

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In the theory of multiphoton processes there would appear to be a considerable number of conflicting views[1-4] as to the importance of the various terms in the perturbation potential given by the nonrelativistic limit of the Dirac equation

$$V=\sum_i (e/mc)\vec{P}_i\cdot\vec{A}(r_i)+(e^2/2mc^2)\vec{A}(r_i)\cdot\vec{A}(r_i). \quad (1)$$

It is possible to remove existing confusion and to derive a simpler formulation of the theory of multiphoton processes by employing an alternative and completely equivalent formulation of the interaction potential due to Richards[5] and developed by Power and Zienau[6]:

$$V=\sum_i (e/c)\dot{\vec{r}}_i\cdot(\partial/\partial t)\vec{A}(r_i)+(e/mc)\vec{P}_i\cdot[\nabla\vec{A}(r_i)]\cdot\vec{r}_i + (e^2/2mc^2)\{[\nabla\vec{A}(\vec{r}_i)]\cdot\vec{r}_i\}^2. \quad (2)$$

The simplifications brought about by using (2) rather than (1) are most easily seen by resolving the perturbation into component perturbations corresponding to the various Fourier components of the electromagnetic field. Writing

$$\vec{A}(\vec{r}_i)=\sum_{k^\pm}[\vec{A}(k^+)+\vec{A}(k^-)],$$

where

$$\vec{A}(k^+)=b_k\vec{\epsilon}_k{}^*\exp[-i(\vec{k}\cdot r-\omega_k t)]a_k{}^+,$$

$$\vec{A}(k^-)=b_k\vec{\epsilon}_k\exp[i(\vec{k}\cdot r-\omega_k t)]a_k,$$

and

$$b_k=(2\pi c^2\hbar/V\omega_k)^{1/2},$$

the perturbation potential V may be expressed as

$$V=\sum_{k^\pm}V(k^\pm)+\sum_{k_1{}^\pm k_2{}^\pm}V(k_1{}^\pm,k_2{}^\pm). \quad (3)$$

In terms of (2), $V(k^\pm)$ and $V(k_1{}^\pm,k_2{}^\pm)$ may be expressed as

$$V(k^\pm)=-\vec{D}\cdot\vec{E}(k^\pm)+\vec{M}\cdot\vec{H}(k^\pm) + (e/2)(\vec{k}\cdot\vec{r})[\vec{r}\cdot\vec{E}(k^\pm)], \quad (4)$$

$$V(k_1{}^\pm,k_2{}^\pm)=(\tfrac{1}{2}mc^2)[\vec{D}\cdot\vec{E}(k_1{}^\pm)][\vec{D}\cdot\vec{E}(k_2{}^\pm)](\vec{n}_1\cdot\vec{n}_2), \quad (5)$$

where $\vec{D}=\sum_i e\vec{r}_i$, $\vec{M}=\sum_i (e/2mc)\vec{L}_i$, and $\vec{n}$ is a unit vector in the direction of propagation of component k of the field. When (4) and (5) are employed to calculate transition amplitudes, it may be shown that amplitudes involving $V(k_1{}^\pm,$