QUASIMOLECULAR STRUCTURE IN ELASTIC $O^{16}+O^{16}$ SCATTERING*

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It is suggested that the experimentally observed intermediate structure in the cross section of elastic $O^{16}+O^{16}$ scattering is due to quasibound molecular states of the ion-ion system while the gross structure originates from virtually bound molecular states.

The excitation function for the elastic scattering of $O^{16}+O^{16}$ as measured by the Yale group[1] exhibits three distinguishable types of structure in the energy range 17 to 35 MeV (c.m.): the gross structure, the intermediate, and the fine structure ("grass"). The gross structure with a large width of about 4 MeV has been successfully analyzed by an optical model.[1-4] The intermediate and fine structures are due to the combined effects of direct nuclear excitations, compound elastic scattering, and transfer reactions. Comparing the elastic cross sections of the $C^{12}+C^{12}$ and the $O^{16}+O^{16}$ reactions[5] it is suggested that the stronger intermediate structure in the $C^{12}+C^{12}$ case arises from the lower stability of the C^{12} shells against deformation. To form the compound nucleus the nuclear deformation has to change during the collision, so that collective states are mainly excited. Hence, inelastic excitations of collective O^{16} states should be strongly coupled both to the elastic entrance and to the compound-nucleus channels. In the following we propose to describe the intermediate structure by inelastic excitations of low-energy states in the individual nuclei. A similar coupling has been mentioned first by Block and Malik.[6] For a survey of statistical interpretations of the intermediate structure we refer to Ref. 1.

The starting point of our considerations is the molecular-type potential shown in Fig. 1 which

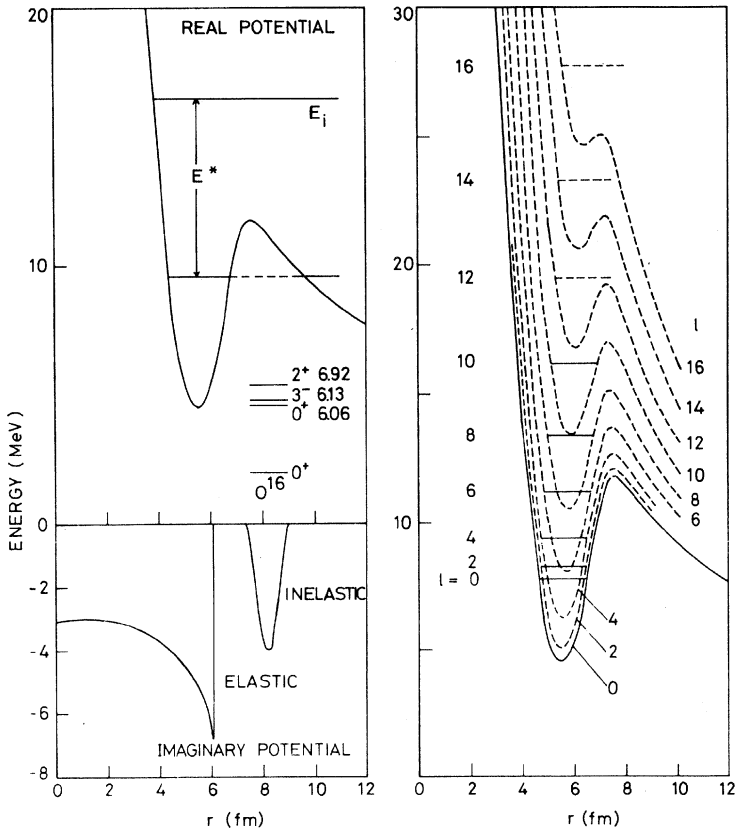

FIG. 1. Real and imaginary parts of the ion-ion potential. The left-hand side shows an elastic channel at energy E_i that de-excites by energy E^* into a quasibound state. The positions of the virtual and quasibound states are found numerically by a phase-shift analysis. They are shown on the right. The de-excitation E^* is compensated for by possible excitations of the O^{16} core as shown in the left part of the figure.

is derived on the basis of a Thomas-Fermi model of the nucleus and used in the elastic $O^{16}+O^{16}$ scattering.[3] As can be seen this potential exhibits quasibound states up to a certain angular momentum and energy ($l=10, E=16$ MeV; all energies in the c.m. system). The positions of these states in Fig. 1 are computed quantum mechanically by a phase-shift analysis. In these states the nuclei can form a quasimolecule[7] which then can decay through the Coulomb and centrifugal barriers as well as into the compound nucleus. Because of the potential barrier the quasimolecular states are not directly reached in the elastic scattering. However, indirect excitation of these quasibound states is possible according to the following mechanism: After the ions have crossed the potential barrier they lose kinetic energy by inelastic excitations of low energy levels of one or both of the ions. Thereby they drop into the potential and are able to form a quasimolecule, if their relative energy E_i-E^* coincides with the energy of a quasibound state (see Fig. 1). This idea is supported by the calculations of Imanishi[8] who found that the resonances seen in the $C^{12}+C^{12}$ reaction below the Coulomb barrier[7] can be interpreted as molecule-like compound states of two C^{12} nuclei, one being in the ground state and the other one in the first excited state.

The potential of Fig. 1 has virtual states for angular momenta exceeding $l=10$ that produce the wide gross structure of the elastic cross section. Each peak of the gross structure is generated by essentially one partial wave in resonance in a virtual state. Since experimentally the intermediate structure is closely associated with the main structure we conclude that the quasibound states are fed through the virtual states as doorway states. Compared with all other ingoing partial waves only that partial wave resonating in a virtual state has a large amplitude to excite quasibound states. Therefore the intermediate structure is generated by a double-resonance event: First an elastic partial wave resonates with its corresponding virtual state. Then a quasibound state is excited by this wave. For this to happen it is necessary that the coupled low-energy excitation accounts for the difference in energy and angular momentum of the virtual and quasibound state. So entrance-channel enhancement of the intermediate structure occurs.

As an example we consider the peak around 20.5 MeV in the 90° differential $O^{16}+O^{16}$ cross section. Its main structure originates from the $l=12$ partial wave according to a phase analysis using the potential of Ref. 3. In Fig. 2 all those low-energy excitations are summarized which connect quasibound states of different angular momenta with the $l=12$ partial wave within its resonance width (from 19 to 23 MeV). There are obviously only a small number of allowed inelastic excitations with the correct angular momentum and energy, in good agreement with the number of peaks observed as intermediate structure in this particular energy range. The description of the gross structure by a particular resonating partial wave is very sensitive to the form of the real part of the elastic-scattering potential. So Rickertsen et al.[4] using somewhat different and deeper potentials attribute an $l=14$ and $l=16$ par-

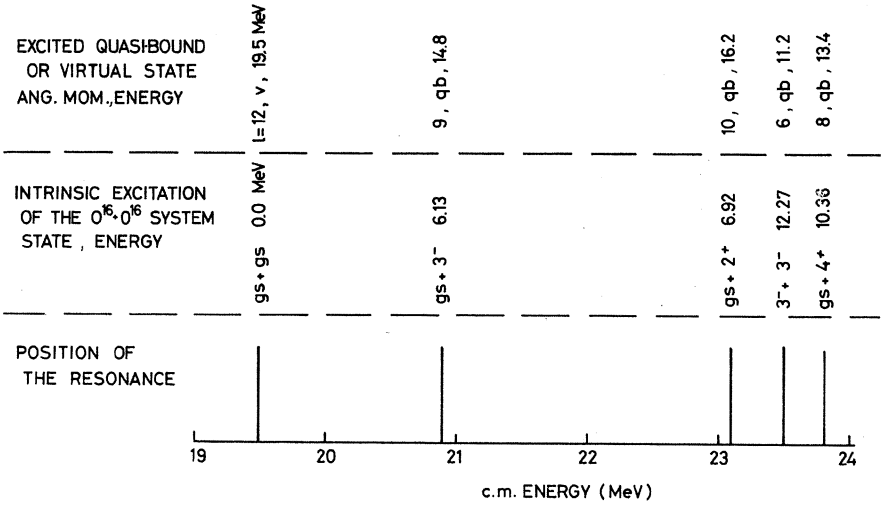

FIG. 2. Composition of the quasibound states that can be excited by the $l=12$ virtual state within its widths (v denotes virtual, qb denotes quasibound, and gs denotes ground state of O^{16}).

tial wave to the peak at 20.5 MeV. This problem is relevant for a spectroscopic analysis of the cross section, but does not affect the interpretation of the intermediate structure.

The Hamiltonian of the system is composed of the kinetic energy of the relative motion, the intrinsic Hamiltonians of two noninteracting O^{16} nuclei, and the interaction between the relative and the internal motion:

$$H = T(\boldsymbol{r}) + H_{O^{16}}(1) + H_{O^{16}}(2) + W_{\mathrm{int}}(\vec{r}, 1, 2). \quad (1)$$

The sets of internal coordinates of both O^{16} nuclei are abbreviated by 1 and 2. We expand the solutions of H with respect to a complete set of states defined by

$$H_{O^{16}}\chi_\alpha = \epsilon_\alpha \chi_\alpha, \quad \alpha = (I, n). \quad (2)$$

The wave function ψ of the composite $O^{16} + O^{16}$ system is symmetrized with respect to the simultanous exchange of all nucleons, thus neglecting exchange of individual nucleons. This symmetrization is suggested because in the entrance channel the two unexcited nuclei act as bosons and the symmetry is conserved by the Hamiltonian (1). Thus[9]

$$\psi = \sum_{l,J,\lambda,I} R_{lJ\lambda I}(\boldsymbol{r})[i^l Y_l \otimes \varphi_{J\lambda}(1, 2)]_{IM} \quad (3)$$

with

$$\varphi_{J\lambda M} = [2(1+\delta_{\alpha_1\alpha_2})]^{-1/2}[\chi_{\alpha_1}(1)\otimes\chi_{\alpha_2}(2) + (-)^I\chi_{\alpha_1}(2)\otimes\chi_{\alpha_2}(1)]_{J,M}, \quad \lambda = (\alpha_1, \alpha_2). \quad (4)$$

If both O^{16} nuclei are in the same state ($\alpha_1 = \alpha_2$) the sum of the angular momenta $l + J$ has to be an even number.

We divide the interaction energy into the optical potential for the elastic channel describing the gross structure of the elastic scattering of two O^{16} nuclei and the residual interaction which couples excited O^{16} states to the entrance channel:

$$W_{\mathrm{int}} = U(\boldsymbol{r}) + \sum_{L,M}(-)^M Q_{LM}{}^{(r,1,2)} Y_{L-M}{}^{(\theta,\varphi)}. \quad (5)$$

Using (3) and (5) we find the coupled system of differential equations for the radial function $R(\boldsymbol{r})$ of the relative motion,

$$\left[-\frac{\hbar^2}{2\mu}\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr} + U(r) + \frac{l(l+1)\hbar^2}{2\mu r^2} + \epsilon_{\alpha_1} + \epsilon_{\alpha_2} - E\right]R_{lJ\lambda I}$$
$$= \sum_{l',J',\lambda',L} i^{l'-l}(-)^{l'+I}\begin{Bmatrix} I & J & l \\ L & l' & J' \end{Bmatrix}\langle l\|Y_L\|l'\rangle\langle J\lambda\|Q_L\|J'\lambda'\rangle R_{l'J'\lambda'I}. \quad (6)$$

The reduced matrix elements of the residual interaction are functions of the relative distance between the O^{16} nuclei. In general nonvanishing diagonal elements occur. They produce a state-dependent contribution to the potential $U(\boldsymbol{r})$ in the excited states. If for simplicity the O^{16} excited states are assumed as vibrational states described by collective surface coordinates the transition potential $Q_{LM}(r, 1, 2)$ of Eq. (5) can be obtained in a similar fashion as used in the calculation of the real part of $U(r)$.[3,9] Introducing the S matrix the wave function ψ, Eq. (3), has the asymptotic form

$$\psi = \sum_{cc'} A_c(J_{c'}\delta_{cc'} - S_{c'c}O_{c'})(i^{l'}Y_{l'}\otimes\varphi_{J'\lambda'})_{I'}, \quad (7)$$

where J_c, $O_c = [1/(v_c{}^{1/2}r)]e^{\pm i\eta_l}(G_l \mp iF_l)$ are Coulomb functions. For the transition where an O^{16} nucleus in the state $\mu_1 = (I_1, M_1, n_1)$ hits an O^{16} target in the state μ_2 and one of the scattered O^{16} nuclei is measured in the direction θ, φ (c.m.) in the state μ_1', the other in the opposite direction in the state μ_2', the symmetrized differential cross section is given by

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mu_1,\mu_2\to\mu_1',\mu_2'} = \Big|f_C(\theta)\delta_{\mu_1\mu_1'}\delta_{\mu_2\mu_2'} + f_C(\pi-\theta)\delta_{\mu_1\mu_2'}\delta_{\mu_2\mu_1'}$$
$$+ \sum_{l,l',m',J,M_J,J',M_J'} \frac{i[\pi(2l+1)]^{1/2}}{k}(I_1M_1I_2M_2|JM_J)(I_1'M_1'I_2'M_2'|J'M_J')(lOJM_J|IM)$$
$$\times(l'm'J'm_J'|IM)[(1+\delta_{\alpha_1\alpha_2})(1+\delta_{\alpha_1'\alpha_2'})]^{1/2}(e^{2i\eta_l}\delta_{ll'}\delta_{JJ'}\delta_{\lambda\lambda'} - S_{l'J'\lambda',lJ\lambda}{}^I)Y_{l'm'}\Big|^2. \quad (8)$$

Here the Coulomb scattering amplitude is denoted by f_C. Since usually the magnetic quantum numbers are not measured, one has to average over the initial states and to sum over the final states.

To test the model some illustrative calculations have been carried out with the following choice of parameters:

(a) The real part of $U(r)$ is taken from Ref. 3 and shown in Fig. 1.

(b) Only the 3^- state at 6.13 MeV and the 2^+ state at 6.92 MeV are assumed to be excited in one of the O^{16} nuclei. Eight channels, including the elastic channel, are coupled for every total angular momentum I. The following orbital angular momenta are involved: $l=I$ (elastic); $I-2$, $I, I+2$ (2^+ excited); and $I-3, I-1, I+1, I+3$ (3^- excited).

(c) The functional form of the reduced matrix elements ($J=2$, $2^+\|Q_2\|0$) and ($J=3$, $3^-\|Q_3\|0$) are chosen as constants of 3 and 2 MeV, respectively, with a radius of $R=7$ F, whereas the imaginary potential in the inelastic channels extends up to about 9 F. All other matrix elements are set equal to zero.

(d) The form of the absorptive potentials in the elastic and inelastic channels is a difficult problem that deserves further study. In this calculation we have made the following choice: As absorptive potential the imaginary potential of Ref. 3 is used in the elastic channel, and a surface-peaked absorption in the inelastic channels (see Fig. 1). Such an absorptive surface potential may arise from nucleon-transfer and cluster-transfer reactions. An optical-model analysis of different heavy-ion reactions supports this assumption.[10]

The calculated excitation functions for elastic scattering are shown in Fig. 3 for the energy range from 19 to 22.5 MeV and compared with the experimental data. The minimum of the 90° curve at 20.3 MeV results from the excitation of the 2^+ state through the elastic partial waves with $l=6, 8, 10$ which resonate with the $l=8$ quasibound state at 13.4 MeV. Correspondingly, in the minimum at 21.0 MeV the $l=9$ quasibound state at 14.8 MeV is excited by the elastic partial wave with $l=12$ through the excitation of the 3^- state. In this particular energy range the excited 2^+ and 3^- states of O^{16} seem to account quite quantitatively for the intermediate structure. The lower part of Fig. 3 gives the calculated inelastic cross section to the 3^- state. It is compared with an experimental cross section[1] which includes also unresolved contributions of the inelastic excitation to the 0^+ state at 6.06 MeV. For a complete analysis of the intermediate structure, higher excited states in the individual ions as well as the simultaneous excitation of both ions would have to be considered. Also the theoretical foundation of the absorptive part needs further investigation. Such calculations are in progress.

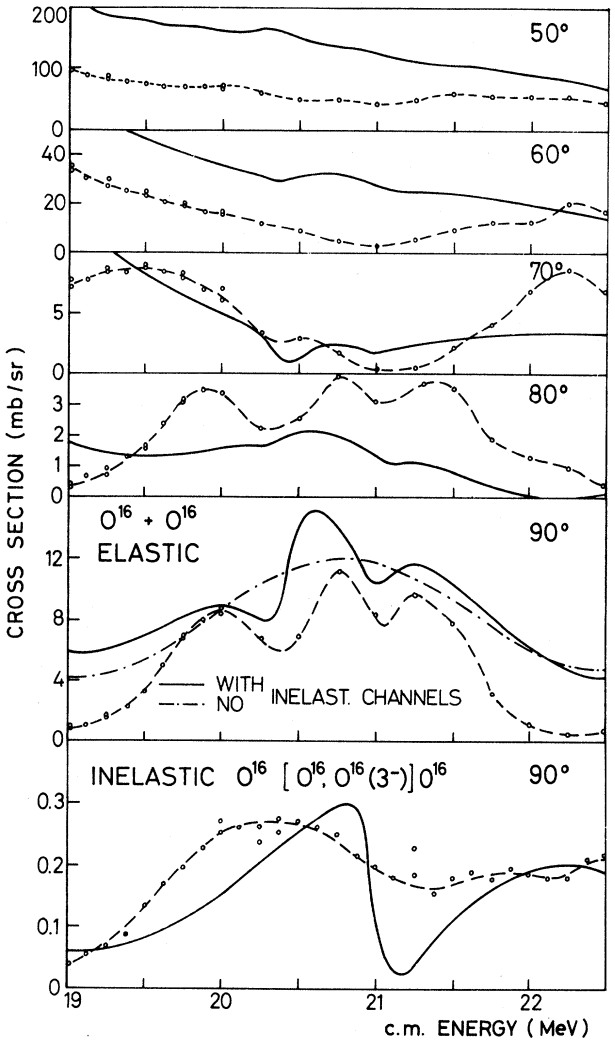

FIG. 3. Elastic and inelastic excitation curves for $O^{16}+O^{16}$ scattering in the energy range between 19 and 22.5 MeV. The experimental points are taken from Ref. 1. The upper part of the figure compares the experimental elastic excitation curves at 49.3°, 60°, 69.8°, 80.3°, and 90° with the theoretical calculations at 50°, 60°, 70°, 80°, and 90°. The first and second minima of the 90° excitation function correspond respectively to the excitation of the 2^+ and 3^- states in O^{16}. The lower part of the figure compares the experimental and calculated cross sections at 90° for exciting the 3^- state.

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ANISOTROPY AND POLARIZATION IN THE GRAVITATIONAL-RADIATION EXPERIMENTS*

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Large anisotropy is observed for gravitational-radiation–detector intensity as a function of sidereal time, with peaks in the direction of the galactic center and in the opposite direction consistent with 12-h antenna symmetry. The 12-h sidereal-time histograms exhibit anisotropy exceeding 6 standard deviations from the mean.

Earlier papers[1,2] have discussed the conception, analysis, and development of the gravitational-radiation antenna. A coincidence experiment involving detectors at Argonne National Laboratory and the University of Maryland was carried out[3,4] in 1969, and gave positive results. Evidence has been presented to support conclusions that the coincidences are not accidental and are not caused by seismic or electromagnetic effects. Results of a study of cosmic-ray interactions, at the Maryland site, have been published.[5]

The antennas are mass quadrupoles and have some directivity. Their axes are horizontal and point east-west at each site. The earth rotates the "antenna beam" among the stars. The time of each coincidence is therefore some measure of the direction of the source. Observation of coincidences over a period of months would be expected to show anisotropy and give information concerning possible sources.

Intensity measurements.—It will be proved that for the detectors (which are harmonic oscillators) the effect of sudden gravitational-radiation excitation is to add the gravitational induced velocity to the initial velocity associated with the thermal fluctuations. The sudden velocity increase decays with the relaxation time of the detector cylinder. Consider a harmonic oscillator excited by a delta function at $t=0$. The equation of motion is

$$m\ddot{x}+r\dot{x}+kx=A\delta(t). \tag{1}$$

The final velocity $\dot{x}_F$ just after $t=0$ is given in terms of the initial velocity $\dot{x}_I$ just before $t=0$, by integration of (1), as

$$\dot{x}_F=(A+m\dot{x}_I)/m. \tag{2}$$

$\dot{x}_I$ will vary over a wide range in consequence of the thermal fluctuations. For small signal-to-noise ratio the final velocity $\dot{x}_F$ and the energy added in consequence of external excitation will therefore also vary over a considerable range.

A procedure will now be described for measuring, to a good approximation, the square root of the source intensity by observing the detector output, which is mainly noise. Each gravitational-radiation detector employs a chain of linear amplifiers followed by a synchronous square-law detector. All voltages referred to are outputs of the linear amplifiers just before the square-law detector. Let v_{JA} be the voltage caused by the Jth coincidence if no noise were present, and let V_{JA} be the noise voltage just before excitation, for channel A. Let v_{JB} and V_{JB} be the corresponding quantities for channel B. Let $\langle V_{JA}{}^2\rangle$ and $\langle V_{JB}{}^2\rangle$ be mean-squared values with averages taken only over a 3-sec interval just pre-