Theory of Fission-Mass Distributions Demonstrated for ^{226}Ra, ^{236}U, ^{258}Fm†

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With the mass asymmetry described by the dynamical collective fragmentation coordinate ξ, and with use of the asymmetric two-center shell model, the fission mass distributions for ^{226}Ra, ^{236}U, and ^{258}Fm (which are typical representatives for triple-, double-, and single-humped distributions) are explained.

The understanding of mass distributions of fissioning nuclei belongs to the most interesting, yet unexplained phenomena in fission physics. Using the concept of mass asymmetry treated as a dynamical collective coordinate, based on the asymmetric two-center shell model (ATCSM), we have calculated, free of parameters, the mass distributions of fissioning ^{226}Ra, ^{236}U, and ^{258}Fm nuclei.

The nuclear shape is defined by five coordinates: λ, the elongation; β_1 and β_2, the fragment deformations; ϵ, the necking-in parameter; and ξ, the mass asymmetry defined as

$$\xi = (A_1 - A_2)/(A_1 + A_2). \tag{1}$$

A_1 and A_2 are the fragment masses obtained from the geometrical size of the fragments. The precise geometrical significance of the parameters is demonstrated in Fig. 1.

For each set of parameters there is an associated single-particle Hamiltonian of the ATCSM,[1,2] consisting essentially of two deformed harmonic-oscillator potentials joined smoothly at the neck, plus $\vec{l}\cdot\vec{s}$ and $\vec{l}^2$ corrections. Single-particle states calculated in this model are used for obtaining shell and pairing corrections. The liquid-drop formula used for renormalization was that of Myers and Swiatecki,[3] with the modification of the surface asymmetry constant κ_s as introduced by Johansson, Nilsson, and Szymanski.[4]

To obtain the behavior of the potential energy as a function of the two parameters interesting in mass asymmetry calculations, we performed a full three-dimensional minimization in ϵ, β_1, and β_2 at each pair of values λ and ξ. Since this is very time consuming, it was feasible only for a small number of points. In particular, for each value of λ we did the full calculation at only five points in ξ.

The collective mass parameters were calculated according to the cranking formula

$$B_{x_i x_j} = 2\hbar \sum_{\mu,\nu} \frac{\langle\mu|\partial/\partial x_i|\nu\rangle\langle\nu|\partial/\partial x_j|\mu\rangle}{\epsilon_\mu + \epsilon_\nu} \times (u_\mu v_\nu + u_\nu v_\mu)^2 \tag{2}$$

in the BCS formulation. The minimization has to be taken into account by taking ϵ, β_1, and β_2 as functions of λ and ξ and substituting

$$\frac{\partial}{\partial\lambda} \rightarrow \frac{\partial}{\partial\lambda} + \frac{\partial\epsilon}{\partial\lambda}\frac{\partial}{\partial\epsilon} + \sum_{i=1}^{2} \frac{\partial\beta_i}{\partial\lambda}\frac{\partial}{\partial\beta_i}, \tag{3}$$

and analogously for $\partial/\partial\xi$, in the cranking formula.

As in the previous paper[3] it is assumed that ξ vibrations are much faster than the relative motion described by λ at the stage of fission considered, i.e., right after the completion of barrier penetration. The potential remains nearly constant in its dependence on asymmetry at later stages (see also the results of Mustafa, Mosel, and Schmitt[5]), so that the main behavior of the distribution should be fixed at this early time. Regarding λ as a parameter, the Schrödinger

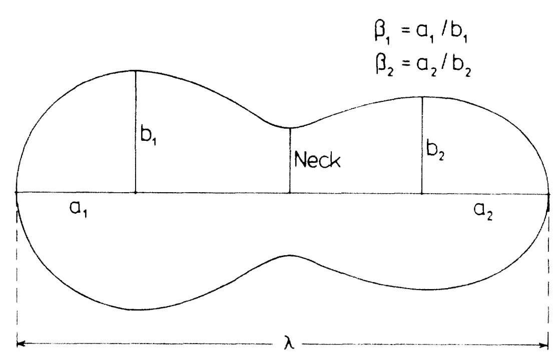

FIG. 1. Explanation of the nuclear shapes occurring in the two-center model and the associated geometrical quantities.

equation in ξ for the νth vibrational state is

$$\left(\frac{-\hbar^2}{2\sqrt{B_{\xi\xi}}}\frac{\partial}{\partial\xi}\frac{1}{\sqrt{B_{\xi\xi}}}\frac{\partial}{\partial\xi}+V(\lambda,\xi)\right)\psi_\lambda^{(\nu)}(\xi)=E_\lambda^{(\nu)}\psi_\lambda^{(\nu)}(\xi). \quad (4)$$

The dependence on λ enters also via the mass parameter $B_{\xi\xi}=B_{\xi\xi}(\lambda,\xi)$.

For complete adiabaticity and starting from the nuclear ground state in spontaneous fission, only the lowest state $\psi_\lambda^{(0)}(\xi)$ should be occupied. However, for fission from excited states, or because of the interaction between the λ and ξ degrees of freedom, higher states in ξ will become excited. We have studied the possible consequences of such excitations by assuming a Boltzmann-like occupation of excited states:

$$|\psi_\lambda|^2=\sum_\nu|\psi_\lambda^{(\nu)}(\xi)|^2\exp[-E_\lambda^{(\nu)}/\theta], \quad (5)$$

with a temperature parameter θ. Calculations were done for three nuclei representative of typical fission types: ^{236}U, ^{226}Ra, and ^{258}Fm, representing asymmetric, triple-humped, and symmetric fission, respectively.

For ^{236}U various values of λ were chosen; we show the results for $\lambda=1.8$ and $\lambda=1.85$, corresponding to a potential energy 2 and 4 MeV below the ground state. The calculated potential energy and mass parameters are exhibited in Figs. 2(a) and 2(b). The potential is rather smooth with the expected minima, whereas the mass parameters oscillate rapidly. Since the curves were obtained by interpolation from only nine calculated points (five plus four reflected under $\xi\to-\xi$), it appears doubtful whether the mass parameters can be trusted to the needed accuracy. However, we did extensive checks on this problem and found that the mass distributions are sensitive only to the overall magnitude of the mass parameters and not to their detailed oscillations; e.g., the distribution changed only very slightly when $B_{\xi\xi}$ was replaced by a constant average value $\bar{B}_{\xi\xi}$. The coupling mass $B_{\lambda\xi}$ is rather small, so that $B_{\lambda\xi}^2\ll B_{\lambda\lambda}B_{\xi\xi}$ holds throughout the range of ξ considered.

The quantity $|\psi_\lambda^{(\nu)}(\xi)|^2$ is interpreted as the probability for finding a certain mass fragmentation ξ at position λ on the fission path. This probability can be scaled to a mass yield Y per mass number A_1 of one fragment as

$$Y(A_1)=|\psi_\lambda(\xi=\xi(A_1))|^2B_{\xi\xi}(A_1)\times 2/\tfrac{1}{2}A. \quad (7)$$

This quantity is plotted in Fig. 2(c) for several temperatures θ. For comparison, experimental

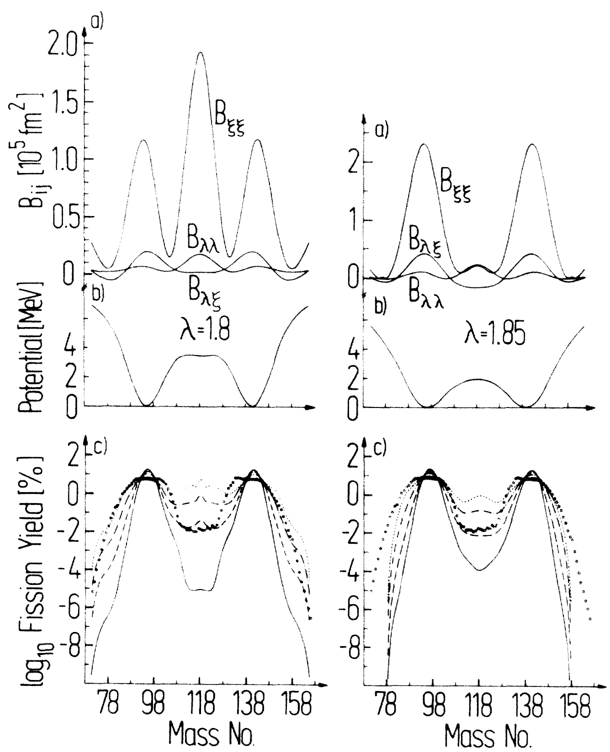

FIG. 2. Results for ^{236}U at two elongations, $\lambda=1.8$ and 1.85. (a) Mass parameters in units of the nucleon mass, (b) potential energy, and (c) theoretical and experimental mass yields. The solid, dashed, dash-dotted, and dotted lines pertain, in that order, to nuclear temperatures of 0, 0.5, 1, and 7 MeV for $\lambda=1.8$ and of 0, 0.25, 0.5, and 1.25 MeV for $\lambda=1.85$.

data[6] are indicated by circles. The experimental data correspond to thermal neutron fission of ^{235}U, and thus to an excitation energy of about 7 MeV. Our calculations show that the shape of the mass distribution is decided several MeV below the saddle point: The asymmetric ξ minima—though they occur roughly already near to the second saddle at ξ values corresponding to the peaks of the final asymmetric mass distribution—are too flat near the saddle to yield the correct spreading of the mass distribution, which occurs only 4–6 MeV below the saddle point during the descent to the scission point.

The calculated distributions show a semiquantitative agreement with experiment. The peaks are somewhat too narrow, and the valley is too low for the ground state ($\theta=0$), but the picture improves as excitation is introduced into the system.[7] Especially the valley rises very rapidly with the temperature and this, as well as the final flattening of the distribution for higher exci-

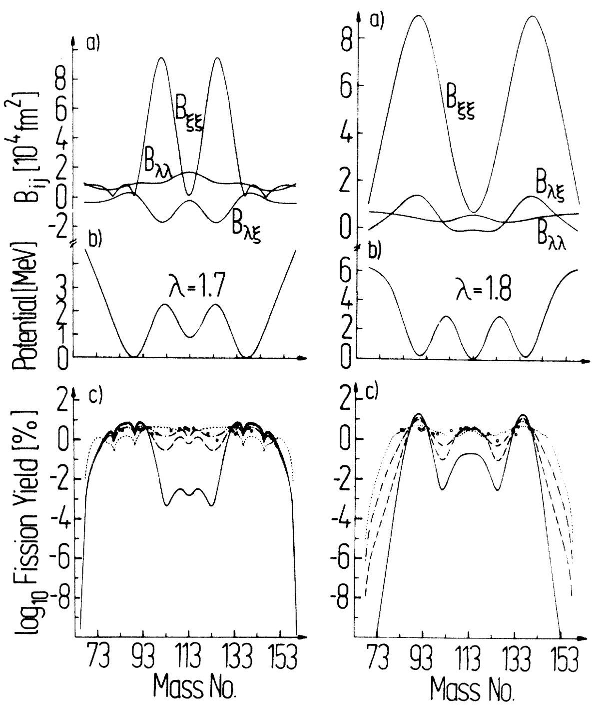

FIG. 3. Same as Fig. 2 but for ^{226}Ra at $\lambda = 1.7$ and 1.8. The temperatures here are 0, 0.5, 1, and 7 MeV in both cases.

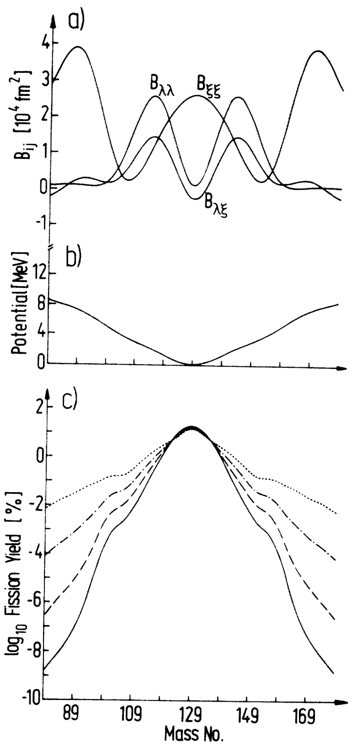

FIG. 4. Same as Fig. 2, but for ^{258}Fm at $\lambda = 1.8$. The temperatures in this case are 0, 0.5, 0.75, and 1.25 MeV.

tations, is in agreement with experiment.

Because of the rather large uncertainty in the determination of the oscillations of the mass parameter, the fine structure in the theoretical curves cannot yet reasonably be compared to experiment, but a more accurate calculation using more points in ξ should shed some light on how much of the experimental fine structure may be due to shell effects in the fissioning nucleus.

For ^{226}Ra, calculations were carried out for $\lambda = 1.7$ and 1.8. The first point is at 6 MeV and the second at 4 MeV above ground-state energy. The results are shown in Fig. 3. These higher-lying points were chosen because data are only available for rather large excitation energies. The experimental data are taken from Jensen and Fairhall[8] and correspond to 11-MeV proton fission of ^{226}Ra. As for ^{236}U, there is semiquantitative agreement with experiment, especially for higher temperatures. It is quite interesting that although the three minima are at approximately the same energy for $\lambda = 1.8$, the mass distribution still has a lower peak in the middle. This effect is due to the rather small $B_{\xi\xi}$ parameter in that region. A sharp minimum in the mass parameter at $\xi = 0$ also causes a dip in the distribution for $\lambda = 1.7$. Though this effect is not entirely certain in the theory itself, there is some hint in the experimental data that there may be such a dip.

Results for ^{258}Fm at $\lambda = 1.8$ are presented in Fig. 4. The distribution is symmetric with two humps appearing at the sides. It seems qualitatively similar to the mass yield observed in thermal neutron fission of ^{257}Fm.[9] Here also rising temperature leads to a rather broad distribution.

The results presented here with their semiquantitative and partly even good agreement with experiment seem to support the interpretation of mass yield distributions as essentially collective vibrations occurring during the descent down the Coulomb slope. It should be kept in mind that not a single parameter has been refitted to yield the experimental distributions. It will be necessary, however, to do a more detailed calculation for one of the cases presented here in order to study the influence of a more accurate determination of the mass parameter $B_{\xi\xi}$.

On the other hand, the conclusions to be drawn from the theoretical results are limited by the apparent dependence of the distributions on λ, which is clearly visible for ^{236}U and ^{226}Ra. Apparently the mass parameters still change considerably with λ, although the potential is rather more constant. This effect will be taken account of in a three-dimensional calculation, where the wave function is set up as

$$\psi = \sum_\nu a_\nu(\lambda)\psi_\lambda^{(\nu)}(\xi)\exp[(-i/\hbar)\int_0^t E_\lambda^{(\nu)}(t)dt] \quad (8)$$

and $\lambda = \lambda(t)$ determined from the classical equation

$$B_{\lambda\lambda}\ddot{\lambda} = -\partial V[\lambda, \xi_0(\lambda)]/\partial\lambda - \gamma\dot{\lambda}. \quad (9)$$

$\xi_0(\lambda)$ can be chosen such as to minimize V for each λ. The introduction of a frictional term with a coefficient γ could yield γ-dependent mass distributions, and a comparison of these with experiment may give information on friction in the fission process.

A second and most important application of the idea of quantized fragmentation dynamics lies in the field of heavy-ion collisions,[10] where it is rather straightforward to calculate fragment distributions after collision, their energy dependence, and their resonance patterns.

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Excitation of a Giant Isoscalar Resonance by α Particles*

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The inelastic scattering of 115-MeV α particles from ^{40}Ca shows an enhancement of the continuum at about 18.25 MeV excitation energy. This observation supports recent electron, proton, and ^{3}He scattering analyses which ascribe similar enhancements at $E_x \approx 63/A^{1/3}$ MeV to an isoscalar giant quadrupole resonance. Although contributions from a monopole excitation cannot be ruled out, attributing all observed yields to a giant quadrupole resonance exhausts only 32% of the energy-weighted sum rule.

Studies of the inelastic scattering of protons,[1-4] electrons,[5-8] and helium ions[9,10] in the region of excitation from 10 to 25 MeV have revealed the existence of a broad resonance. The centroid of the resonance has been found to be consistently 2-3 MeV below the energy at which one expects to find the giant dipole resonance (GDR) as determined by photonuclear experiments. Furthermore, the strength of the resonance is much larger than that predicted by the energy-weighted sum rule for isovector dipole transitions. Although the electron data and some of the proton data[11] are consistent with a monopole assignment, the interpretation of the resonance as an isoscalar quadrupole vibration is generally favored. A monopole excitation is expected to appear at a higher energy because of the incompressibility of nuclear matter. An isovector quadrupole vibration is also expected to be associated with larger field energies. More recent inelastic proton data,[12] as well as the inelastic helium-ion data, tend to favor the quadrupole assignment. A precise determination of the transition strength of the giant quadrupole resonance (GQR) is dif-