Phase transitions in quark-gluon matter

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If the local color symmetry in a quark-gluon matter is broken, the expectation value of the gluon field $\langle A^a_\mu(x)\rangle$ may be different from zero. Such a gluon-condensed phase has been found in mean field approximation. The gluon-condensed phase is characterized by a static, periodic chromomagnetic field, which is coupled to a periodic spin-color density distribution of quarks and antiquarks. Transitions of first and second order type have been found between the gluon-condensed and normal phases, the latter characterized by the vanishing value of the mean gluon field.

I. INTRODUCTION

According to the rather widely accepted view in the hadronic matter at high enough temperature and/or density a transition may take place into a quark-gluon plasma phase.[1,2] In high energy heavy-ion collisions there is a chance to reach that region of physical parameters where this phase transition is possible. How the plasma is produced and what kind of phase transition takes place are questions of extreme interest. A number of approaches has been developed to estimate the temperature and the density of the transition.[3-5] In the quark-gluon plasma phase another phase transition associated with the restoration of the chiral invariance is also expected.[6] In this paper we will assume that the transition from hadronic phase into the quark-gluon plasma is already accomplished and we will search for further possibilities of phase transitions of the plasma. It will be shown that a gluon-condensed phase may exist characterized by the nonvanishing expectation value of the gluon field $\langle A^a_\mu(x)\rangle$, which plays the role of the order parameter. In the gluon-condensed phase a static, periodic chromomagnetic field is present and this induces in a self-consistent way a similar periodic behavior of the spin-color density of the quarks and antiquarks.[7] Recently a somewhat similar approach has been developed by Celenza and Shakin for the treatment of hadronic structure.[8] Gluon condensation characterized by color singlet, scalar order parameters has been investigated in several papers.[9-12]

To have a nonvanishing expectation value of the gluon field $\langle A^a_\mu(x)\rangle$, the local color symmetry must be broken. According to Elitzur's theorem,[13] this symmetry breaking cannot be spontaneous. It was pointed out by Elitzur that it is the introduction of the mean field approximation which produces already a violation of the local gauge symmetry. Consequently, the expectation value of the gluon vector potential may be different from zero; however, it cannot be a good order parameter. Nevertheless, we will use it as an approximate order parameter, since the phase transition found in mean field approximation may survive in the true, symmetric theory.

In Sec. II the mean field approximation of QCD is summarized. In Sec. III the self-consistent set of equations is derived from the field equations. The thermodynamical description of the system is given in Sec. IV. The characteristic features of the gluon-condensed phase are enumerated in Sec. V. Section VI is devoted to the discussion of the phase transitions. Concluding remarks are contained by Sec. VII. The solution of the Dirac equation for SU(2) can be found in Appendix A. The generalization for SU(3) is given in Appendix B. Finally, the equivalence of the self-consistent equations with the necessary conditions of the thermodynamical equilibrium is proved in Appendix C.

II. MEAN FIELD APPROXIMATION

The field equations of the QCD in conventional notations are given by

$$i\gamma^\mu\partial_\mu\psi+gA^a_\mu\gamma^\mu T^a\psi-m\psi=0\ , \quad (1)$$

$$\partial_\nu F^{\nu\mu a}=\mathscr{F}^{\mu a}\ , \quad (2)$$

where the field strength $F^{\mu\nu a}(x)$ and the vector current $\mathscr{F}^{\mu a}(x)$ are defined as follows:

$$F^{\mu\nu a}(x)=\partial^\mu A^{\nu a}-\partial^\nu A^{\mu a}+gf^{abc}A^{\mu b}A^{\nu c}\ , \quad (3)$$

$$\mathscr{F}^{\mu a}(x)=gf^{abc}A^b_\nu F^{\mu\nu c}+g\bar{\psi}\gamma^\mu T^a\psi\ . \quad (4)$$

The generators and the structure constants of the gauge group are denoted by T^a and f^{abc}, respectively. As a consequence of Eq. (2), the divergence of the color vector current vanishes, so that the color charge

$$Q_c=\int\mathscr{F}^{0a=3}(x)d^3x \quad (5)$$

is a conserved quantity. The field equations (1) and (2) are symmetric under the local gauge transformations.

We assume that the field operators carrying both color and Lorentz indices may have nonvanishing expectation values:

$$\langle A^{\mu a}\rangle=\bar{A}^{\mu a}\neq 0\ .$$

Let us introduce the following decomposition:

$$A^{\mu a}(x)=\bar{A}^{\mu a}(x)+\alpha^{\mu a}(x) , \quad (6)$$

where both $A^{\mu a}(x)$ and $\alpha^{\mu a}(x)$ are field operators, while $\bar{A}^{\mu a}(x)$ is a c number and the expectation value of $\alpha^{\mu a}(x)$ vanishes by definition:

$$\langle \alpha^{\mu a}\rangle=0 .$$

Substituting the decomposition (6) into Eq. (2) and taking expectation value on both sides, the following equation is obtained for the mean field $\bar{A}^{\mu a}(x)$:

$$\partial_\nu \bar{F}^{\nu\mu a}=g f^{abc}\bar{A}{}^b_\nu \bar{F}^{\mu\nu c}+\langle g\bar{\psi}\gamma^\mu T^a\psi\rangle - M^2\bar{A}^{\mu a} . \quad (7)$$

Deriving this equation, it is assumed that in addition to $\alpha^{\mu a}(x)$ all of its derivatives have vanishing expectation values and all the expectation values of products of independently fluctuating quantities are also zero. In this way all terms containing $\alpha^{\mu a}(x)$ or its derivatives drop out except for the products of nonindependently fluctuating quantities, which may have nonvanishing expectation values:

$$M^2(\nu)=-g^2\sum_{\substack{\mu\neq\nu\\ c}}\langle \alpha_{\mu c}\alpha^{\mu c}\rangle .$$

The parameter $M^2(\nu)$ formally plays the role of the mass associated with the excitations of the mean gluon field. For the sake of simplicity $M^2(\nu)=M^2$ is assumed. In order to have a logically consistent description, the gluon operators $A^{\mu a}(x)$ in Eq. (1) are substituted by their expectation values $\bar{A}^{\mu a}(x)$ and the coupling constant g by a renormalized one.

It is worthwhile to point out that the field equations obtained by the mean field approximation can be derived from the Lagrangian given by

$$\mathscr{L}=\frac{i}{2}(\bar{\psi}\gamma^\mu\partial_\mu\psi-\partial_\mu\bar{\psi}\gamma^\mu\psi)-g\bar{\psi}\gamma^\mu\bar{A}{}^a_\mu T^a\psi$$
$$-\tfrac{1}{4}\bar{F}{}^a_{\mu\nu}\bar{F}^{\mu\nu a}-m\bar{\psi}\psi+\tfrac{1}{2}M^2\bar{A}^{\mu a}\bar{A}_{\mu a} . \quad (8)$$

Having this Lagrangian we have a unique way to introduce the energy-momentum tensor $\bar{T}^{\mu\nu}$ of the system using the standard definition.

III. SELF-CONSISTENT EQUATIONS

One of the possible strategies for the solution of the coupled, nonlinear set of field equations is to introduce an ansatz for the solution. Substituting this ansatz into the field equations one must check if the resulting equations can be solved for the parameters of the ansatz.

For the mean gluon field we introduce the following ansatz:

$$\bar{A}^{\mu a}(x)=a^\mu\theta^a(kx)/g , \quad (9)$$

which is separable in the Lorentz and color indices and for the case of SU(2) color symmetry the space-time dependence is given by

$$\theta^1(kx)=\sin kx,\ \ \theta^2(kx)=\cos kx,\ \ \theta^3(kx)=0 . \quad (10)$$

The choice of this ansatz is motivated by the remarkable fact that the Dirac-equation can be solved exactly (see Appendix A).[14,15]

Having the fermion single-particle energies, the vector current $\mathscr{J}^{\mu a}$ can be calculated in a straightforward way. Substituting the ansatz (9) into Eq. (7), the following set of self-consistent equations can be derived:

$$\langle j_v^{\mu 1}\rangle=0 , \quad (11)$$

$$\langle j_v^{\mu 2}\rangle=\frac{1}{g}\{M^2a^\mu+[(\mathbf{k}^2-k_0^2)a^\mu+(a_0k_0-\mathbf{ak})k^\mu]\} , \quad (12)$$

$$\langle j_v^{\mu 3}\rangle=-\frac{1}{g}[(a_0^2-\mathbf{a}^2)k^\mu-(a_0k_0-\mathbf{ak})a^\mu] , \quad (13)$$

where

$$\langle j_v^{\mu a}\rangle=\langle g\bar{\psi}_v\gamma^\mu T^a\psi_v\rangle$$

and the quasiparticle field

$$\psi_v(x)=R^+\psi(x) \quad (14)$$

is defined by the help of the operator

$$R=\exp(-ikxT^3) . \quad (15)$$

Our considerations are confined to the study of the case of SU(2), the generalization for SU(3) can be found in Appendix B.

The solution of the self-consistent set of equations provides us the amplitude a^μ and the wave vector k^μ. It is worthwhile to mention that in the mean field approximation the color vector current is conserved only if $a_\mu k^\mu=0$. This condition can be satisfied if $\mathbf{ak}=0$, $a_0=0$, and $k_0=0$. In this case the chromoelectric field vanishes and a static, periodic chromomagnetic field, orthogonal both to $\mathbf{a}$ and $\mathbf{k}$ vectors is present; the color density of the gluons and that of the quarks and antiquarks vanish; the energy eigenvalues for quarks and antiquarks are identical. Without loss of generality the system of coordinates can be chosen in such a way that $k_1=k_2=0$ and $a_2=a_3=0$ are fulfilled. The only nonvanishing parameters are $a=a_1$ and $k=k_3$.

IV. THERMODYNAMICAL EQUILIBRIUM

We assume that the quark-gluon matter under consideration has a volume V and it is in thermodynamical equilibrium with the surrounding world. Neither the energy nor the number of baryons is fixed. The system is characterized by the temperature T and the baryonic chemical potential μ. Thus the thermodynamical properties of the system can be described by the following grand canonical density matrix:

$$\rho=Z^{-1}\exp[-(H-\mu N)/T] , \quad (16)$$

where the partition function Z is given by

$$Z=\mathrm{Tr}\exp[-(H-\mu N)/T] . \quad (17)$$

Here, $H=\int\bar{T}^{00}dV$ is the Hamiltonian and N denotes the number of baryons. We introduce also the thermodynamical potential defined by

$$\Omega=-T\ln Z . \quad (18)$$

In the mean field approximation the quarks behave as independent quasiparticles. In this approximation the Dirac equation can be solved without any further restriction, so one is able to calculate explicitly the thermodynamical potential density

$$\omega=\frac{\Omega}{V}=\epsilon-T\sum_{\lambda}\int\frac{d^3p}{(2\pi)^3}\ln[(1+e^{-(E_\lambda-\mu)/T})\times(1+e^{-(\bar{E}_\lambda+\mu)/T})]\ , \qquad (19)$$

where ϵ is given by

$$\epsilon=-\langle\mathscr{L}\rangle=\frac{1}{2g^2}[(M^2+\mathbf{k}^2-k_0^2)\mathbf{a}^2-(M^2+\mathbf{k}^2)a_0^2-(\mathbf{ak})^2+2a_0k_0\mathbf{ak}]\ . \qquad (20)$$

The energy eigenvalues of the quarks and antiquarks are denoted by E_λ and $\bar{E}_\lambda$, respectively. The 2×2 independent spin and color states are labeled by the index λ.

In addition to the temperature T and chemical potential μ, the generalized thermodynamical potential density is the function also of the parameters a and k:

$$\omega(T,\mu;a,k)\ .$$

The thermodynamical equilibrium is characterized by the minimum of the generalized thermodynamical potential density. This means that at prescribed values of T and μ the conditions

$$\frac{\partial\omega}{\partial a}=0,\quad\frac{\partial\omega}{\partial k}=0 \qquad (21)$$

should be satisfied. It is not difficult to prove, that Eqs. (21), which are the necessary conditions of the thermodynamical equilibrium, are the independent ones among the self-consistent set of Eqs. (12) and (13) (see Appendix C). The solution of Eqs. (21) defines the dependence of the parameters a and k on the temperature and chemical potential in the thermodynamical equilibrium:

$$a(\mu,T),\quad k(\mu,T)\ .$$

Using these functions the generalized thermodynamical potential density for the gluon-condensed phase can be obtained as a function of the temperature and of the baryon chemical potential:

$$\omega_c(\mu,T)=\omega(\mu,T;a(\mu,T),k(\mu,T))\ . \qquad (22)$$

The thermodynamical potential density of the normal phase is given by

$$\omega_n(\mu,T)=\omega(\mu,T;a=0,k=0)\ . \qquad (23)$$

The other thermodynamical characteristics of the system can be easily derived from functions (22) and (23).

In order to examine the order of the phase transitions, we need the first and second derivatives of the thermodynamical potential density. According to their general definitions in thermodynamics, the baryon density ρ_c, the entropy density s_c, and the specific heat C_c of the gluon-condensed system can be given as follows:

$$\rho_c=-\frac{\partial\omega_c}{\partial\mu}=-\frac{\partial\omega(\mu,T;a,k)}{\partial\mu}\ , \qquad (24)$$

$$s_c=-\frac{\partial\omega_c}{\partial T}=-\frac{\partial\omega(\mu,T;a,k)}{\partial T}\ , \qquad (25)$$

$$C_c=T\frac{\partial s_c}{\partial T}=-T\left[\frac{\partial^2\omega}{\partial T^2}+\frac{\partial^2\omega}{\partial T\partial a}\frac{\partial a}{\partial T}+\frac{\partial^2\omega}{\partial T\partial k}\frac{\partial k}{\partial T}\right]\ . \qquad (26)$$

Equations (24)—(26) were obtained by taking into account Eqs. (21). Similarly in the normal phase we have

$$\rho_n=-\frac{\partial\omega_n}{\partial\mu}=-\frac{\partial\omega(\mu,T;a=0,k)}{\partial\mu}\ , \qquad (27)$$

$$s_n=-\frac{\partial\omega_n}{\partial T}=-\frac{\partial\omega(\mu,T;a=0,k)}{\partial T}\ , \qquad (28)$$

$$C_n=T\frac{\partial s_n}{\partial T}=-T\frac{\partial^2\omega(\mu,T;a=0,k)}{\partial T^2}\ . \qquad (29)$$

V. GLUON-CONDENSED PHASE

In the normal phase the mean gluon field is equal to zero and quarks and antiquarks form a noninteracting Fermi gas. In the other phase, gluons are condensed forming a static, periodic chromomagnetic field. The gluon-condensed phase has a layered structure. The 3-vector component of the color vector current is proportional to the vector potential, $\mathscr{F}^{\alpha a}(x)\sim\bar{A}^{\alpha a}(x)$ $(\alpha=1,2,3)$. So in the layers of the thickness π/k, a flow of the color charges appears, the direction of which is opposite in the neighboring layers. The quarks and the antiquarks form a latticelike structure in the periodic mean gluon field, which can be characterized by the spin-color density given as

$$\rho_{\mathrm{SC}}=\langle\bar{\psi}\gamma^0(\tfrac{1}{2}+i\gamma^3\gamma^1T^1)\psi\rangle\ . \qquad (30)$$

In our case the spin-color density has the form

$$\rho_{\mathrm{SC}}=\tfrac{1}{2}\rho_B+\rho_0\cos kx\ , \qquad (31)$$

where

$$\rho_0=\langle\bar{\psi}_v\gamma^0 i\gamma^3\gamma^1T^1\psi_v\rangle\ .$$

The baryon density ρ_B and the amplitude of the oscillations of the spin-color density ρ_0 can be divided into two terms, one coming from quarks and the other from the antiquarks:

$$\rho_B=\rho_B^{(q)}-\rho_B^{(\bar{q})},\quad\rho_0=\rho_0^{(q)}-\rho_0^{(\bar{q})}\ .$$

Let us call the color charges blue and red. Then, according to Eq. (31) the density of the spin-up red and spin-down blue quarks and similarly the density of the spin-up blue and spin-down red quarks vary periodically around the half value of the quark density. The same can be told about the antiquarks. The spatial spin-color density oscillations of quarks and antiquarks have the same phase, but in general are of different amplitudes. This seems to be the analog of the Overhauser effect.[16,17]

VI. PHASE TRANSITIONS

The minimum of the generalized thermodynamical potential density has been found numerically and the thermodynamical state functions as the baryon density ρ_B, the entropy density s and the specific heat C have been computed at the thermodynamical equilibrium.

According to Gibbs's law, in phase equilibrium the temperatures, the pressures $(p=-\omega)$ and the chemical potentials must be equal for the two phases. Typical phase diagrams are shown in Figs. 1(a)–1(c) for the values of the parameters $M^2=0.1$, $g=15.0$, and $m=1.0$. As the field equations (1) and (7) are invariant under the scale transformation

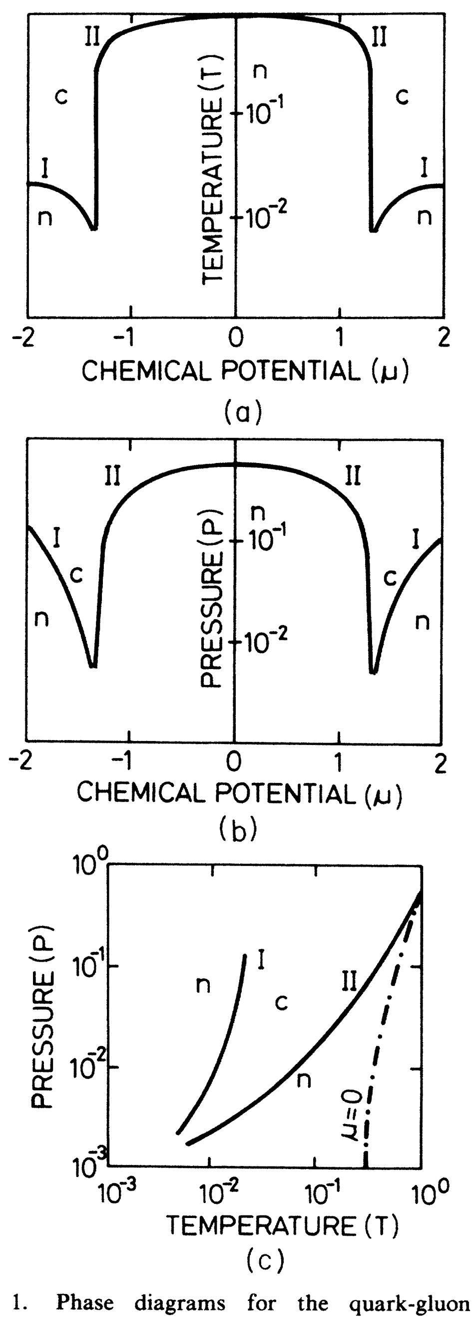

FIG. 1. Phase diagrams for the quark-gluon matter ($M^2=0.1$, $g=15.0$, $m=1.0$).

$$x^\mu \rightarrow \lambda x^\mu,\ k^\mu \rightarrow k^\mu/\lambda,\ m \rightarrow m/\lambda\ ,$$

$$a^\mu \rightarrow a^\mu/\lambda,\ M^2 \rightarrow M^2/\lambda^2,\ g \rightarrow g\ ,$$

where λ is a real constant, the quark mass m was taken as the energy unit. Therefore the physical quantities used have the following units:

$$[a^\mu]=[k^\mu]=m,\ [M^2]=m^2,\ [g]=1\ ,$$

$$[T]=[\mu]=m,\ [\omega]=m^4,\ [\rho_B]=m^3\ .$$

In the phase diagrams shown in Figs. 1(a)–1(c) there is a region limited by the curves labeled by I and II, in which the set of Eqs. (11)–(13) has nontrivial solution. All the states of physical meaning lie on the left-hand side (lhs) of the $\mu=0$ line on the p-T diagram [Fig. 1(c)]. The curve joining the states of phase equilibrium has two branches, branch I in the high density region and branch II in the low density region. Thus, at a given value of the temperature there is a density interval in which the gluon-condensed phase can exist and it is thermodynamically more favored than the normal phase.

The condensed phase is characterized by the nonvanishing mean value of the gluonic vector potential. Therefore in the condensed phase a number of symmetries are broken. Since the normal and the condensed phases have different symmetries, the curve of phase equilibrium cannot be terminated by a single point.

It was established that at the formation of the gluon-condensed phase from the low density normal phase the amplitude of the mean gluon field a increases continuously from zero along an isotherm (Fig. 2). The square of the amplitude a^2 was found proportional to the temperature difference $t=T-T_{\rm cr}$ at a given value of the baryon chemical potential, where $T_{\rm cr}$ denotes the critical temperature at which the phase transition takes place. On the other hand, the wave vector k being different from zero has a finite jump $k_{\rm cr}$ as a result of the phase transition (Fig. 3). At a given value of the chemical potential, the difference $k-k_{\rm cr}$ was found proportional to the temperature difference t too. The only exception is the $M^2=0$ case, when $a=k$, as a consequence of the symmetry of the self-consistent equations. In that case the wave vector has no jump ($k_{\rm cr}=0$) and k is proportional to t.

From Fig. 2 one also can see that along an isotherm the amplitude of the gluon field and the wave vector grow at first with increasing chemical potential. But they have a maximum at which $a \approx k$ holds and then begin to decrease. If the temperature is low enough (as in the case shown on Fig. 2), a phase equilibrium can be reached by increasing the chemical potential. Then the condensed phase disappears by a phase transition of the first order (branch I), accompanied by an absorption of the latent heat $Q=T_{\rm cr}(s_c-s_n)<0$ (given to the system by the surrounding world). At the same time the specific volume of a baryon increases, since the inequality $1/\rho_c<1/\rho_n$ holds.

Branch I of the phase equilibrium curve approaches to branch II in the point μ_0, corresponding to a "condensed" state with $a=k=0$ (see Fig. 3), which is indistinguishable from the normal state. Approaching this point along branch I the latent heat and the difference of the specific volumes go to zero.

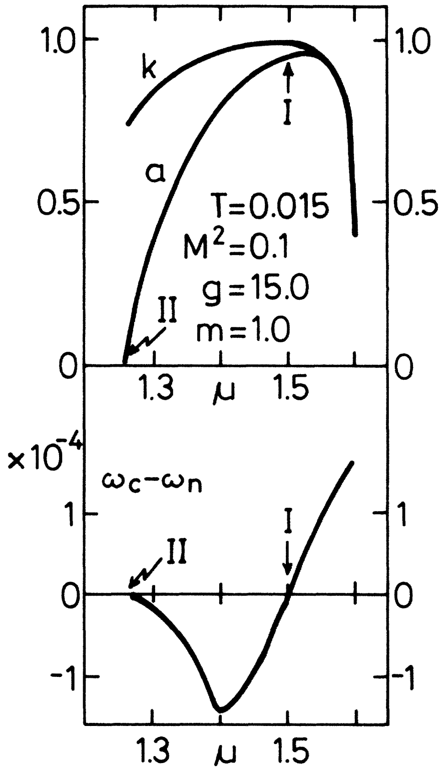

FIG. 2. Amplitude a and the wave vector k of the mean gluon field and the difference of the thermodynamical potential densities of the condensed and the normal phases $\omega_c-\omega_n$ versus the baryon chemical potential along an isotherm.

Through branch II a second order phase transition takes place. Indeed, according to Eqs. (24) and (25) no change in the entropy and in the baryon density (specific volume) appears. Let us now examine the specific heat. According to Eqs. (26) and (29), the difference of the specific heats of the condensed and normal phases can be written as follows:

$$\Delta C=C_c-C_n=-T_{\rm cr}\left[\frac{\partial^2\omega}{\partial T\partial a}\frac{\partial a}{\partial T}+\frac{\partial^2\omega}{\partial T\partial k}\frac{\partial k}{\partial T}\right]. \tag{32}$$

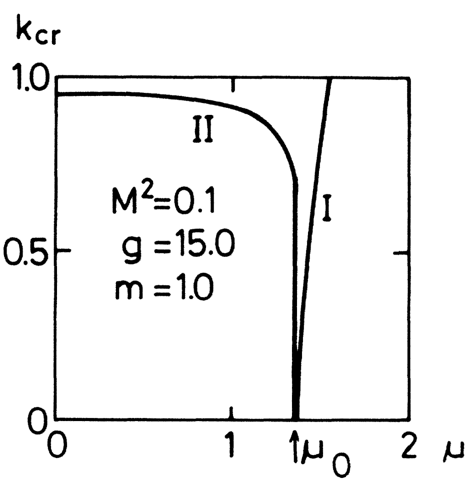

FIG. 3. Variation of the jump of the gluonic wave vector $k_{\rm cr}$ along the phase equilibrium curve versus the chemical potential.

It was established numerically that the second derivatives $\partial^2\omega/\partial T\partial a$ and $\partial^2\omega/\partial T\partial k$ have nonzero values at branch II. As the temperature difference goes to zero $t\to 0$, the wave vector goes also $k\sim t\to 0$, $\partial k/\partial t=\partial k/\partial T$ remaining constant. Since $a^2\sim t$, $\partial a/\partial T\sim t^{-1/2}$ goes to infinity as $t\to 0$. Thus the first term on the right-hand side of Eq. (31) causes an anomaly in the specific heat. In the case of $M^2=0$, the specific heat has only a finite jump, since

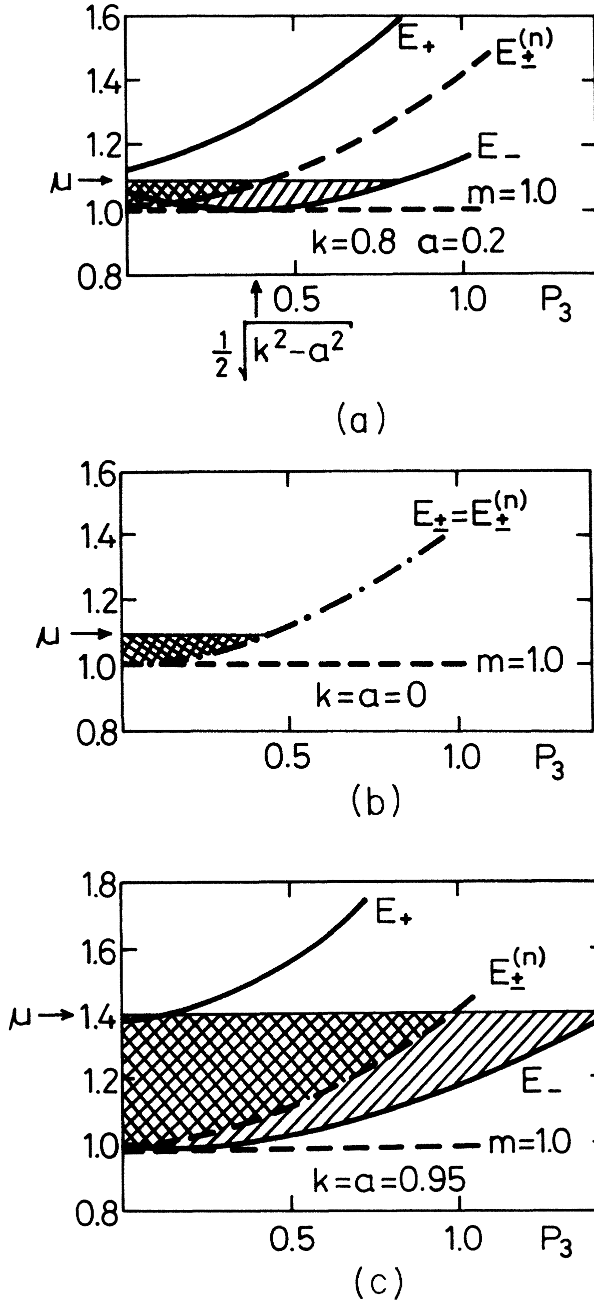

FIG. 4. Fermion single-particle energy levels $E_\pm$ and $E_\pm^{(n)}$ in the condensed and normal phases, respectively, in the case of the second order phase transitions for (a) $M^2\neq 0$, (b) $M^2=0$, and (c) in the case of first order phase transitions. The hatched and the double hatched areas denote the single-particle states occupied in the condensed and the normal phases, respectively.

$a=k\sim t$. According to the general convention, the anomalies of the system can be characterized by the following critical exponents:

$$a\sim t^{\beta},\ C\sim t^{-\alpha},\ \frac{\partial a}{\partial k}\sim t^{-\gamma}\ .$$

In our case they have the following values $\alpha=1/2(0)$, $\beta=1/2(1)$, and $\gamma=1/2(0)$ for $M^2\neq 0$ ($M^2=0$), and satisfy the general relation[18]

$$\alpha+2\beta+\gamma=2\ .$$

Microscopically the phase transition is connected with the redistribution of the fermions among the single-particle energy levels. On Figs. 4(a)–4(c) the single-particle energy levels $E_{\pm}$ and $E_{\pm}^{(n)}$ are shown for the condensed and the normal states, respectively, as functions of the longitudinal momentum p_3 (at $p_1=p_2=0$) in a few typical cases. When a phase transition through branch II takes place, a condensate with $k=k_{cr}\gg a\approx 0$ is formed. In this case the energy level E_- has a minimum at $p_3=\pm\frac{1}{2}(k^2-a^2)^{1/2}\neq 0$ [Fig. 4(a)]. Fermions, originally occupying the twice degenerated levels in the Fermi sphere of the radius μ in the momentum space, can reorder and occupy two spheres of the same radii with centers removed from each other by the "distance" $(k^2-a^2)^{1/2}$. Since this reordering appears at infinitesimally small value of the gluonic amplitude a, no variation of the baryon density takes place. However, when the condensate disappears at the branch I, the equality $k\approx a$ holds. In this case the level E_- has no minimum, but it is smaller than $E_{\pm}^{(n)}$. From Fig. 4(c) one can see that the phase transition takes place as a result of contraction of the Fermi sphere in the momentum space, resulting in the decrease of the baryon density. In the case of $M^2=0$ the condensate with $a=k\approx 0$ is formed without any variation in the momentum space, as the single-particle energy levels for the normal and the condensed states coincide [Fig. 4(b)]. Fermions redistribute among the momentum states only, when increasing the chemical potential a condensate with growing amplitude $a=k$ builds up.

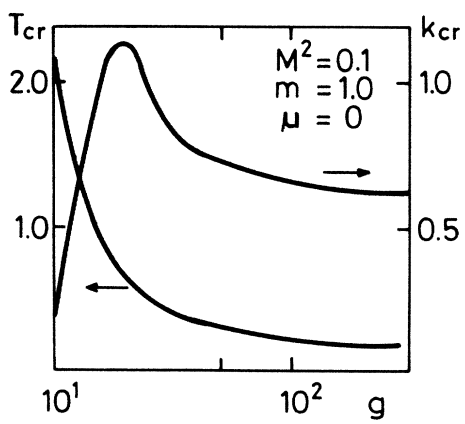

FIG. 5. Temperature T_{cr} at which the second order phase transition takes place at $\mu=0$ and the jump of the wave vector k_{cr} accompanying the transition as a function of the coupling constant g.

The dependence of the phase diagrams on the coupling constant g has also been examined. The phase transition at given value of the chemical potential takes place at higher temperatures if the coupling constant decreases (Fig. 5). The same can be told about the chemical potential at a given value of the temperature. It should be mentioned, that the jump of the wave vector k_{cr} decreases with decreasing coupling constant at its small values (Fig. 5). It has to be noted that the gluon-condensed phase may exist only above some minimal value of g.

VII. CONCLUSIONS

A model of quark-gluon matter is given in relativistic mean field approximation, in which a simple explanation on thermodynamical basis is found of appearing and disappearing a phase with broken local color symmetry, called gluon-condensed phase. Let us suppose, the quark-gluon matter of given volume is cold and dilute enough to form a free quark-antiquark Fermi gas. Putting it into a bath to keep the temperature constant the following happens if the baryon chemical potential is increased, i.e., more and more quarks are pushed into the system. At first, the baryon density reaches a value at which the formation of the gluon-condensed phase becomes energetically favored [branch II on Figs. 1(a)–1(c)]. In the gluon-condensed phase with broken color symmetry, a mean gluon field is continuously building up having a spatial periodicity of the length $2\pi/k$. This phase transition is a second order one, connected with the redistribution of the fermions among the single-particle levels in the momentum space without the variation of the Fermi momentum. With further increase of the chemical potential the amplitude of the mean gluon field grows and a strong gluon field of order of magnitude of the chemical potential builds up. In the meantime the periodicity length $2\pi/k$ has a minimum. In the case of $M^2=0$, the local gauge symmetry is restorted and it can be seen very easily that the ansatz (9) is gauge equivalent with the mean gluon field given by Eq. (4), which is constant in space as well as in time, resulting a constant chromomagnetic field. At low enough temperatures, a phase transition of first order takes place when the chemical potential is further increased [branch I on Figs. 1(a)–1(c)]. Then the fermions redistribute among the single-particle states with sudden decrease of the Fermi momentum and symmetries broken in the condensed phase are restored.

From the phase diagram depicted on Fig. 1 one may expect that the gluon-condensed phase exists at any high enough temperatures. This is the case only if one uses a really constant coupling constant g, as we did. However if one would use the running coupling constant of QCD the gluon-condensed phase ceased to exist at some high enough temperature, since the effective coupling constant would decrease.

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APPENDIX A: SOLUTION OF THE DIRAC EQUATION IN THE CASE OF SU(2)

Substitute ansatz (9) into Eq. (1). First of all, one can get rid of the x dependence of the Hamiltonian performing a rotation around the third axis in the color space given by

$$\psi \to \psi_v = R^{\dagger}\psi, \quad R = \exp(-ikxT^3) . \tag{A1}$$

The solutions of the transformed Dirac equation are plane waves:

$$\psi_v = U(p)e^{-ipx} . \tag{A2}$$

The energy eigenvalues and the eight component spinor $U(p)$ can be obtained by solving the following equation:

$$(\gamma^\mu p_\mu + \gamma^\mu B^a_\mu T^a - m)U(p) = 0 , \tag{A3}$$

where the notation

$$B^{a=1}_\mu = 0, \quad B^{a=2}_\mu = -a_\mu, \quad B^{a=3}_\mu = k_\mu \tag{A4}$$

was introduced. Defining the spinor χ by the formula

$$U = (p_\mu \gamma^\mu + \gamma^\mu B^a_\mu T^a + m)\chi , \tag{A5}$$

we arrive at the equation given by

$$[(p_\mu + B^a_\mu T^a)(p^\mu + B^{\mu b}T^b) + \tfrac{1}{2} f^{abc}\sigma^{\mu\nu}B^a_\mu B^b_\nu T^c - m^2]\chi = 0 . \tag{A6}$$

This equation has a very simple structure which becomes apparent if the $\sigma^{\mu\nu}$ matrices are represented by the Pauli matrices in the conventional manner:

$$\sigma^{0\alpha} = i\begin{bmatrix} 0 & \sigma^\alpha \\ \sigma^\alpha & 0 \end{bmatrix}, \quad \sigma^{\alpha\beta} = \epsilon^{\alpha\beta\gamma}\begin{bmatrix} \sigma^\gamma & 0 \\ 0 & \sigma^\gamma \end{bmatrix} \quad (\alpha,\beta,\gamma = 1,2,3) .$$

Here $\epsilon^{\alpha\beta\gamma}$ is the totally antisymmetric tensor of third rank. Using this representation Eq. (A6) can be written in the following form:

$$\begin{bmatrix} C & D \\ D & C \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = 0 , \tag{A7}$$

where

$$\chi = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

and

$$C = p_\mu p^\mu + \tfrac{1}{4} B^a_\mu B^{\mu a} - m^2 + 2p^\mu B^a_\mu T^a + \epsilon^{\alpha\beta\gamma} B^2_\alpha B^3_\beta \sigma^\gamma T^1 ,$$

$$D = i(B^2_0 B^3_\gamma - B^2_\gamma B^3_0)\sigma^\gamma T^1 .$$

The matrix on the lhs of Eq. (A7) commutes with the Dirac matrix γ_5. Therefore the solutions have the form $u_1 = u_2 = u_+$ and $u_1 = -u_2 = u_-$. $u_\pm$ should satisfy the equations given by

$$(C \pm D)u_\pm = (R^2 + s^a_\pm T^a)u_\pm = 0 , \tag{A8}$$

where

$$R^2 = p_\mu p^\mu + \tfrac{1}{4} B^a_\mu B^{\mu a} - m^2 ,$$

$$s^1_\pm = -\left[\epsilon^{\alpha\beta\gamma} B^3_\alpha B^2_\beta \pm \frac{i}{2}(B^3_0 B^2_\gamma - B^3_\gamma B^2_0)\right]\sigma^\gamma ,$$

$$s^2_\pm = 2p^\mu B^2_\mu, \quad s^3_\pm = 2p^\mu B^3_\mu .$$

Equations (A8) can be diagonalized multiplying by the operator $(R^2 - s^a_\pm T^a)$, making use of the commutation relations

$$[s^a_\pm, s^b_\pm] = 0 \quad (a,b = 1,2,3) .$$

From both of Eqs. (A8) the same secular equation can be obtained in the form of algebraic equation of fourth order:

$$p_0^4 + bp_0^2 + cp_0 + d = 0 , \tag{A9}$$

where

$$b = -2[\mathbf{p}^2 + m^2 + \tfrac{1}{4}(a_0^2 + \mathbf{a}^2) + \tfrac{1}{4}(k_0^2 + \mathbf{k}^2)] ,$$

$$c = 2[a_0(\mathbf{pa}) + k_0(\mathbf{pk})] , \tag{A10}$$

$$d = [\mathbf{p}^2 + m^2 + \tfrac{1}{4}(\mathbf{a}^2 - a_0^2) + \tfrac{1}{4}(\mathbf{k}^2 - k_0^2)]^2 - (\mathbf{pa})^2 - (\mathbf{pk})^2 - \tfrac{1}{4}[k_0^2\mathbf{a}^2 + a_0^2\mathbf{k}^2 + \mathbf{a}^2\mathbf{k}^2 - (\mathbf{ak})^2 - 2a_0k_0\mathbf{ak}] .$$

Thus all fermion energy levels are twice degenerated. From relations (A10) one also can see that in the case of $a_0 = k_0 = 0$, $\mathbf{ak} = 0$ quark and antiquark energy levels are the same and (anti) quarks moving with equal momenta into opposite directions have the same energies.

APPENDIX B: GENERALIZATION FOR SU(3)

In the case of SU(3), the generators of the symmetry group can be expressed by the Gell-Mann matrices λ^a:

$$T^a = \tfrac{1}{2}\lambda^a \quad (a = 1,2,\ldots,8) .$$

The vector potential is defined as

$$\theta^1(kx) = t_1\cos(kx + \alpha_1), \quad \theta^2(kx) = t_1\sin(kx + \alpha_1), \quad \theta^3(kx) = 0, \quad \theta^4(kx) = t_4\cos[\tfrac{1}{2}(1 + 3\vartheta)kx + \alpha_4] ,$$

$$\theta^5(kx) = t_4\sin[\tfrac{1}{2}(1 + 3\vartheta)kx + \alpha_4], \quad \theta^6(kx) = t_6\cos[\tfrac{1}{2}(3\vartheta - 1)kx + \alpha_6], \quad \theta^7(kx) = t_6\sin[\tfrac{1}{2}(3\vartheta - 1)kx + \alpha_6], \quad \theta^8(kx) = 0 ,$$

where ϑ, t_1, t_4, t_6, α_1, α_4, and α_6 are real constants.

Substituting this ansatz into Eq. (7) the following set of self-consistent equations can be derived:

$$\langle j_v^{\mu 1}\rangle = U_0^\mu t_1 \cos\alpha_1 - \frac{3\vartheta}{2} V^\mu t_4 t_6 \cos(\alpha_4-\alpha_6) ,$$

$$\langle j_v^{\mu 2}\rangle = U_0^\mu t_1 \sin\alpha_1 - \frac{3\vartheta}{2} V^\mu t_4 t_6 \sin(\alpha_4-\alpha_6) ,$$

$$\langle j_v^{\mu 3}\rangle = -V^\mu \left[t_1^2 + \frac{1+3\vartheta}{4} t_4^2 + \frac{1-3\vartheta}{4} t_6^2 \right] ,$$

$$\langle j_v^{\mu 4}\rangle = U_+^\mu t_4 \cos\alpha_4 + \frac{3\vartheta-3}{4} V^\mu t_1 t_6 \cos(\alpha_1+\alpha_6) ,$$

$$\langle j_v^{\mu 5}\rangle = U_+^\mu t_4 \sin\alpha_4 + \frac{3\vartheta-3}{4} V^\mu t_1 t_6 \sin(\alpha_1+\alpha_6) ,$$

$$\langle j_v^{\mu 6}\rangle = U_-^\mu t_6 \cos\alpha_6 + \frac{3\vartheta+3}{4} V^\mu t_1 t_4 \cos(\alpha_4-\alpha_1) ,$$

$$\langle j_v^{\mu 7}\rangle = U_-^\mu t_6 \sin\alpha_6 + \frac{3\vartheta+3}{4} V^\mu t_1 t_4 \sin(\alpha_4-\alpha_1) ,$$

$$\langle j_v^{\mu 8}\rangle = -\sqrt{3} V^\mu \left[\frac{1+3\vartheta}{4} t_4^2 + \frac{3\vartheta-1}{4} t_6^2 \right]$$

where the notations

$$U_r^\mu = \frac{1}{g}\{ M^2 a^\mu + \xi_r [(\mathbf{k}^2 - k_0^2) a^\mu + (a_0 k_0 - \mathbf{ak}) k^\mu] \} ,$$

$$V^\mu = -\frac{1}{g}\{ (a_0^2 - \mathbf{a}^2) k^\mu - (a_0 k_0 - \mathbf{ak}) a^\mu \} ,$$

$$\xi_\pm = \tfrac{1}{4}(3\vartheta \pm 1)^2, \quad \xi_0 = 1$$

are used.

The space-time dependence of the Dirac-equation is eliminated by the transformation

$$\psi_v = R^\dagger \psi, \quad R = \exp[-ikx(T^3 + \vartheta\sqrt{3} T^8)] , \tag{B1}$$

where ϑ is a real constant. The matrix of the transformation is diagonal of the form

$$R = \begin{bmatrix} e^{i\varphi_1} & 0 & 0 \\ 0 & e^{i\varphi_2} & 0 \\ 0 & 0 & e^{i\varphi_3} \end{bmatrix} ,$$

where the phases are the following:

$$\varphi_1 = \tfrac{1}{2} kx(\vartheta+1), \quad \varphi_2 = \tfrac{1}{2} kx(\vartheta-1), \quad \varphi_3 = -\vartheta kx .$$

After elementary matrix operations one gets the identity

$$R^\dagger \theta^a(kx)\lambda^a R = \begin{bmatrix} \theta^3 + \frac{1}{\sqrt{3}}\theta^8 & (\theta^1 - i\theta^2) e^{i(\varphi_1-\varphi_2)} & (\theta^4 - i\theta^5) e^{i(\varphi_1-\varphi_3)} \\ (\theta^1 + i\theta^2) e^{-i(\varphi_1-\varphi_2)} & -\theta^3 + \frac{1}{\sqrt{3}}\theta^8 & (\theta^6 - i\theta^7) e^{i(\varphi_2-\varphi_3)} \\ (\theta^4 + i\theta^5) e^{-i(\varphi_1-\varphi_3)} & (\theta^6 + i\theta^7) e^{-i(\varphi_2-\varphi_3)} & -\frac{2}{\sqrt{3}}\theta^8 \end{bmatrix} . \tag{B2}$$

It can be seen, that the space-time dependence of the Dirac equation vanishes, if the functions $\theta^a(kx)$ obey the following set of equations:

$$\begin{aligned} &\theta^1 - i\theta^2 = t_1 e^{-i(kx+\alpha_1)}, \quad \theta^4 - i\theta^5 = t_2 e^{-i(1/2)kx(1+3\vartheta) - i\alpha_4}, \quad \theta^6 - i\theta^7 = t_3 e^{-i(1/2)kx(3\vartheta-1) - i\alpha_6} , \\ &\theta^3 + \frac{1}{\sqrt{3}}\theta^8 = t_4, \quad -\theta^3 + \frac{1}{\sqrt{3}}\theta^8 = t_5, \quad -\frac{2}{\sqrt{3}}\theta^8 = -(t_4+t_5) , \end{aligned} \tag{B3}$$

where $t_1, t_2, \ldots, t_5$ and $\alpha_1, \alpha_4, \alpha_6$ are real constants. A possible solution of Eqs. (B3) is given by

$$\begin{aligned} &\theta_1 = t_1 \cos(kx + \alpha_1), \quad \theta_2 = t_1 \sin(kx + \alpha_1) , \\ &\theta_3 = t_3, \quad \theta_4 = t_4 \cos[\tfrac{1}{2}(1+3\vartheta)kx + \alpha_4] , \\ &\theta_5 = t_4 \sin[\tfrac{1}{2}(1+3\vartheta)kx + \alpha_4] , \\ &\theta_6 = t_6 \cos[\tfrac{1}{2}(3\vartheta-1)kx + \alpha_6], \quad \theta_7 = t_6 \sin[\tfrac{1}{2}(3\vartheta-1)kx + \alpha_6], \quad \theta_8 = t_8 . \end{aligned} \tag{B4}$$

In this case

$$R^\dagger \theta^a(kx)\lambda^a R = \underset{\sim}{t}^a \lambda^a ,$$

where $\underset{\sim}{t}^a$ are real constants:

$$\underset{\sim}{t}^1 = t_1 \cos\alpha_1, \quad \underset{\sim}{t}^2 = t_1 \sin\alpha_1, \quad \underset{\sim}{t}^3 = t_3, \quad \underset{\sim}{t}^4 = t_4 \cos\alpha_4 ,$$

$$\underset{\sim}{t}^5 = t_4 \sin\alpha_4, \quad \underset{\sim}{t}^6 = t_6 \cos\alpha_6, \quad \underset{\sim}{t}^7 = t_6 \sin\alpha_6, \quad \underset{\sim}{t}^8 = t_8 .$$

Choosing $t_3 = t_8 = 0$ in Eqs. (B4), the Dirac equation for the transformed field ψ_v has the form

$$[i\gamma^\mu \partial_\mu + \tfrac{1}{2} k_\mu \gamma^\mu (\lambda^3 + \sqrt{3}\vartheta\lambda^8) - \tfrac{1}{2}\gamma^\mu a_\mu \underset{\sim}{t}^a \lambda^a - m]\psi_v = 0 . \tag{B5}$$

Seeking the solution as a plane wave of the form (A2), and multiplying by $(\gamma^\mu p_\mu + m + \frac{1}{2}\gamma^\mu B_\mu^a \lambda^a)$, the following equation is obtained:

$$[p_\mu p^\mu - m^2 + B_\mu^a p^\mu \lambda^a + \tfrac{1}{4}(B_\mu^a B^{\mu b} - i B_\mu^a B_\nu^b \sigma^{\mu\nu})\lambda^a \lambda^b] U = 0 . \tag{B6}$$

Here the notations

$$B^a_\mu = -a_\mu \underset{\sim}{t}^a/g \quad (a=1,2,4,5,6,7)\ ,$$
$$B^3_\mu = k_\mu,\quad B^8_\mu = \sqrt{3}\vartheta k_\mu$$

have been introduced. Let us seek the solutions of Eq. (B6) as eigenspinors of γ_5, in the form

$$U = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

with $u_1 = u_2 = u_+$ and $u_1 = -u_2 = u_-$. Then for $u_\pm$ one gets the equations

$$(C \pm D)u_\pm = 0\ , \tag{B7}$$

where C and D are given by

$$C = p_\mu p^\mu - m^2 + B^a_\mu p^\mu \lambda^a + \tfrac{1}{4} B^a_\mu B^{\mu b} \lambda^a \lambda^b - \tfrac{1}{4} i\epsilon^{\alpha\beta\gamma}\sigma^\gamma B^a_\alpha B^b_\beta \lambda^a \lambda^b\ ,$$
$$D = \tfrac{1}{4}\sigma^\alpha (B^a_0 B^b_\alpha - B^a_\alpha B^b_0)\lambda^a \lambda^b\ .$$

In the case of $k_0 = 0$, the matrix D vanishes. In the case of a transversal chromomagnetic field $a_\mu = a\delta_{\mu 1}$, $k_\mu = k\delta_{\mu 3}$ we can make use of the commutation relation $[C, \sigma^{\alpha=2}] = 0$, seeking the solution as an eigenspinor of σ^2, i.e., in the form

$$u = \begin{bmatrix} \chi_1 \\ \chi_2 \end{bmatrix}$$

with $\chi_1 = \pm i\chi_2 = \chi_\pm$. Then the equation

$$(R^2 + s^a_\pm \lambda^a)\chi_\pm = 0 \tag{B8}$$

is obtained. Here

$$R^2 = p_\mu p^\mu - m^2 + \tfrac{1}{6} B^a_\mu B^{\mu a}\ ,$$
$$s^a_\pm = B^a_\mu p^\mu + \tfrac{1}{4} d^{abc} B^b_\mu B^{\mu c} \pm \tfrac{1}{4} f^{abc}(B^b_1 B^c_3 - B^b_3 B^c_1)$$

and the structure constants of SU(3) symmetric in all indices are denoted by d^{abc}. Equation (B8) can be solved only if its determinant is equal to zero. Thus an algebraic equation of sixth order can be obtained for the energy eigenvalues, which is of third order in R^2:

$$R^6 - R^2 s^a_\pm s^a_\pm + S_\pm = 0\ , \tag{B9}$$

where

$$S_\pm = 2(s^1_\pm s^4_\pm s^6_\pm - s^2_\pm s^4_\pm s^7_\pm + s^1_\pm s^5_\pm s^7_\pm + s^2_\pm s^5_\pm s^6_\pm) + s^3_\pm[(s^4_\pm)^2 + (s^5_\pm)^2 - (s^6_\pm)^2 - (s^7_\pm)^2]$$
$$+ \frac{2}{\sqrt{3}} s^8_\pm [(s^1_\pm)^2 + (s^2_\pm)^2 + (s^3_\pm)^2 - \tfrac{1}{2}((s^4_\pm)^2 + (s^5_\pm)^2 + (s^6_\pm)^2 + (s^7_\pm)^2) - \tfrac{1}{3}(s^8_\pm)^2]\ .$$

APPENDIX C: SELF-CONSISTENT EQUATIONS AND THERMODYNAMICAL EQUILIBRIUM

In this section the equivalence of the necessary conditions of thermodynamical equilibrium with the independent equations among the self-consistent set of Eqs. (12) and (13) are shown in the SU(2) and SU(3) cases, respectively.

The expectation value of the color vector current of the fermions can be computed exactly:[15]

$$\langle j^{\mu a}_v \rangle = -\sum_\lambda \int \frac{d^3p}{(2\pi)^3} \left\{ \frac{\partial E_\lambda}{\partial B^a_\mu} n_\lambda + \frac{\partial \bar{E}_\lambda}{\partial B^a_\mu} \bar{n}_\lambda \right\} = \frac{\partial(\epsilon - \omega)}{\partial B^a_\mu}\ . \tag{C1}$$

Accounting for Eq. (C1), one can see that not all the equations in sets (12) and (13) are independent from each other. In the case of SU(2), Eq. (C1) contains the derivations

$$\frac{\partial}{\partial B^2_\mu} = -g\frac{\partial}{\partial a_\mu},\quad \frac{\partial}{\partial B^3_\mu} = g\frac{\partial}{\partial k_\mu}$$

and the energy eigenvalues do not depend on $B^1_\mu = 0$. Thus, from the necessary conditions of the minimum of the generalized thermodynamical potential (21) the equations

$$\langle j^{\mu 2}_v \rangle = \frac{\partial \epsilon}{\partial B^2_\mu} = -g\frac{\partial \epsilon}{\partial a_\mu},\quad \langle j^{\mu 3}_v \rangle = g\frac{\partial \epsilon}{\partial k_\mu}$$

and the identity

$$\langle j^{\mu 1}_v \rangle = 0$$

are obtained. So one gets the last two equations of the set (12) as independent ones.

In the case of SU(3) the parameters ϑ, t_1, t_4, and t_6 can be chosen in such a way that conditions (21) coincide with the independent equations from the set (13). In this case (C1) contains the derivations

$$\frac{\partial}{\partial B^3_\mu} = g\frac{\partial}{\partial k_\mu},\quad \frac{\partial}{\partial B^8_\mu} = g\frac{\partial}{\partial k_\mu}\frac{1}{\sqrt{3}\vartheta}\quad (\text{if } \vartheta \neq 0)\ ,$$
$$\frac{\partial}{\partial B^a_\mu} = -g\underset{\sim}{t}^a \frac{\partial}{\partial a_\mu}\quad (a=1,2,4,5,6,7)\ ,$$

and according to the definition of the color vector current the relations

$$\langle j^{\mu a}_v \rangle = -\underset{\sim}{t}^a \langle \underset{\sim}{j}^\mu_v \rangle\ (a=1,2,4,5,6,7)\ ,$$
$$\langle j^{\mu 8}_v \rangle = \begin{cases} \dfrac{1}{\sqrt{3}\vartheta}\langle j^{\mu 3}_v \rangle, & \text{if } \vartheta \neq 0 \\ 0, & \text{if } \vartheta = 0 \end{cases}$$

hold, where $\langle \underset{\sim}{j}^\mu_v \rangle$ is given by

$$\langle \underset{\sim}{j}^\mu_v \rangle = -\sum_\lambda \int \frac{d^3p}{(2\pi)^3} \left\{ \frac{\partial E_\lambda}{\partial a_\mu} n_\lambda + \frac{\partial \bar{E}_\lambda}{\partial a_\mu} \bar{n}_\lambda \right\}\ .$$

We see that only the equations for $\langle j_\nu^{\mu 3}\rangle$ and $\langle \underset{\sim}{j}{}_\nu^{\mu}\rangle$ are independent. From the conditions (12) the equations

$$\langle j_\nu^{\mu 3}\rangle = \frac{\partial\epsilon}{\partial B_\mu^3} = g\frac{\partial\epsilon}{\partial k_\mu} ,$$

$$\langle j_\nu^{\mu a}\rangle = \frac{\partial\epsilon}{\partial B_\mu^a} = -g\underset{\sim}{t}^a\frac{\partial\epsilon}{\partial a_\mu} \quad (a=1,2,4,5,6,7) ,$$

i.e., the equations

$$\langle \underset{\sim}{j}{}_\nu^{\mu}\rangle = g\frac{\partial\epsilon}{\partial a_\mu}$$

can be obtained. These equations are equivalent to the set of Eqs. (13) if

$$\vartheta=0,\quad t_1=0,\quad t_4^2=t_6^2=\tfrac{1}{2} .$$

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