Two-dimensional nuclear inertia: Analytical relationships

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(Received 24 May 1995)

The components of the nuclear inertia tensor, functions of the separation distance R and of the radius of the light fragment R_2, $B_{RR}(R,R_2)$, $B_{RR_2}(R,R_2)$, and $B_{R_2R_2}(R,R_2)$ are calculated within the Werner-Wheeler approximation, by using the parametrization of two intersected symmetric or asymmetric spheres. Analytical relationships are derived. When projected to a path $R_2=R_2(R)$, the reduced mass is obtained at the touching point. The two one-dimensional parametrizations with $R_2=$const, and the volume $V_2=$const previously studied, are found to be particular cases of the present more general approach. Illustrations for the cold fission, cluster radioactivity, and α decay of ^{252}Cf are given.

PACS number(s): 21.60.Ev, 23.70.+j, 25.85.Ca, 27.90.+b

I. INTRODUCTION

One group of methods frequently used to solve quantum dynamical problems in many branches of physics and chemistry (e.g., tunneling phenomena in solid-state and nuclear physics, mass transfer in nuclear reactions, mass distributions in fission, scattering reactions, molecular collisions, chemical reaction-rate theory, etc.) [1–3], relies on quasiclassical approximation, in which an important quantity is the inertia tensor [4]. The components of this tensor are strongly dependent on the arbitrarily chosen set of n generalized coordinates $\{q_1,q_2,\ldots,q_n\}$.

In the present paper we have in mind possible applications for quantum mechanical tunneling in nuclear decay processes like cold fission, cluster radioactivities, and α emission [5], for which it was repeatedly stressed that a number of collective degrees of freedom as low as possible should be chosen in order to represent on an axis or in a plane the main physical quantities determining the basic features of these phenomena.

Glas and Mosel [6] have used the distance R and the angle ϕ to express the kinetic energy in heavy ion collisions. Within fragmentation theory of binary systems [7] the best suited deformation coordinates are the fragment separation distance $q_1=R$ and the mass asymmetry parameter $q_2=\eta=(V_1-V_2)/(V_1+V_2)$, where V_i $(i=1,\ 2)$ are the volumes of the fragments A_1Z_1 (which finally will be the daughter nucleus A_dZ_d) and A_2Z_2 (which becomes the emitted cluster A_eZ_e at the touching point configuration). Such pairs of collective variables have been used to calculate the nuclear inertia for the mass transfer [8–10] in heavy ion collisions, by using either the cranking approximation [3] or the hydrodynamical approach [10].

As a consequence of the assumed incompressibility of nuclear matter, the total volume of the fragments is conserved during the deformation. Also, we are interested to study a given exit channel, hence the final value of the mass asymmetry, $\eta_f=(A_d-A_e)/A$, is known. The parametrization of two intersected spheres with radii R_1 and R_2 has been adopted [11] to generate two different sequences of such shapes for a given mass asymmetry, with an additional constraint of $R_2=R_e=$ const ["clusterlike"(CL) shapes] or $V_2=V_e=$const ["more compact"(MC) shapes]. In this way, by taking into account the total volume conservation and the matching condition in the separation plane, it was possible to arrive at a single independent shape variable which could be either the separation distance, R, of the geometrical centers of the spheres or, z_m—the distance between their centers of mass. The Werner-Wheeler approximation [12,13] allowed us to obtain analytical relationships for the nuclear inertia (a scalar in this case) in a wide range of mass asymmetry.

Our aim at present is to relax the above-mentioned restrictions, leaving both $q_1=R$ and $q_2=R_2$ to vary. We shall show that from the general expressions obtained in this way one can get the preceding ones by giving the corresponding law of variation $R_2=R_2(R)$ in the overlapping region of the two fragments. Also, another local test is provided by the fact that the inertia scalar $B(R)$, which is the result of projection on a general trajectory $R_2=R_2(R)$, is equal to the reduced mass, $B(R_t)=\mu$, at the touching point $R_t=R_d+R_e$.

II. MULTIDIMENSIONAL APPROACH

During the decay process, leading from one parent nucleus, AZ, to two different fragments (A_dZ_d—the daughter or heavy fragment; A_eZ_e—the emitted ion or light fragment), the shape of the system changes continuously. The potential energy surface in a multidimensional hyperspace of deformation parameters, $\{q\}=q_1,q_2,\ldots,q_n$, gives the generalized forces acting on the nucleus. Information concerning how the system reacts to these forces is contained in the inertia tensor $\{B_{ij}\}$. The contribution of a shape change to the kinetic energy of the system at any time, t, is expressed by

$$E_k=\frac{1}{2}\sum_{i,j=1}^{n} B_{ij}(q)\frac{dq_i}{dt}\frac{dq_j}{dt}. \tag{1}$$

The inertia tensor with components $B_{ij}=B_{ij}(q)$ corresponds to the variation in time of the nuclear shape. Their values depend on the particular choice of deformation coordinates. On the other hand, for a system with axial symmetry relative to the z axis, the kinetic energy of a nonviscous fluid is given by

$$E_k=\frac{\sigma}{2}\left[\int_V \mathbf{v}^2 d^3r-\left(\int_V \dot{z}d^3r\right)^2 \Big/ \int_V d^3r\right]. \quad (2)$$

Here V is the volume considered to be conserved, $\sigma=3m/(4\pi r_0^3)$ is the mass density, m is the nucleon mass, $\mathbf{v}$ is the velocity, and $r_0 = 1.16$ fm is the nuclear radius constant.

By assuming irrotational motion ($\nabla\times\mathbf{v}=\text{rot }\mathbf{v}=0$), the velocity field may be derived from a scalar velocity potential φ, i.e., $\mathbf{v}=\nabla\varphi$. From the continuity equation of an incompressible fluid it follows that the Laplace equation, $\nabla^2\varphi=\Delta\varphi=0$, should be satisfied with kinematical boundary conditions,

$$\frac{\partial F}{\partial t}=\mathbf{v}\nabla F+\frac{\partial F}{\partial t}=0, \quad (3)$$

expressing the need of a vanishing normal component of the velocity at the surface. In this way there is no flow of matter through the surface. The surface equation for axially symmetric shapes in cylindrical coordinates (ρ,φ,z) is written as $F(r,t,q)=\rho-\rho_s(z,t,q)=0$ in which ρ_s is the value of ρ on the surface. The velocity components, $\dot{z}=\partial\varphi/\partial z$ and $\dot{\rho}=\partial\varphi/\partial\rho$, are both functions of z and ρ.

As an approximation to the incompressible irrotational flow, one can use the Werner-Wheeler assumption. In this approximation the flow is considered to be a motion of circular layers of fluid, $\dot{z}$ is independent of ρ, and $\dot{\rho}$ is linear in ρ:

$$\dot{z}=\sum_i X_i(z,q)\dot{q}_i; \quad \dot{\rho}=(\rho/\rho_s)\sum_i Y_i(z,q)\dot{q}_i. \quad (4)$$

The quantities X_i are calculated separately for the left- and right-hand side of the body by requiring a vanishing total (convective) time derivative of the fluid volume to the left-hand side, or right-hand side of an arbitrary plane normal to the z axis:

$$X_{il}=-\rho_{sl}^{-2}\frac{\partial}{\partial q_i}\int_{z_{\min}}^{z}\rho_{sl}^2 dz, \quad X_{ir}=\rho_{sr}^{-2}\frac{\partial}{\partial q_i}\int_{z}^{z_{\max}}\rho_{sr}^2 dz. \quad (5)$$

In order to get a vanishing normal component of the velocity at the surface, one needs

$$Y_{ir(l)}=-\frac{\rho_{sr(l)}}{2}\frac{\partial}{\partial z}X_{ir(l)}. \quad (6)$$

The functions X_i and Y_i are found as a sum of two terms for the left- (l) and right- (r) hand side of the shape.

After substitution in the relationship for the kinetic energy and comparison with the initial equation for E_k we find the following equations for the components of the inertia tensor:

$$B_{ij}(q)=\pi\sigma\int_{z_{\min}}^{z_{\max}}\rho_s^2(X_iX_j+\tfrac{1}{2}Y_iY_j)dz+B_{ij}^c(q), \quad (7)$$

where the correction term $B_{ij}^c(q)$ due to the center of mass motion [11]

$$B_{ij}^c(q)=-(\pi^2\sigma/V)\int_{z_{\min}}^{z_{\max}}\rho_s^2X_i dz\int_{z_{\min}}^{z_{\max}}\rho_s^2X_j dz \quad (8)$$

is different from zero if the origin of z is not placed in the center of mass. Here $\rho_s=\rho_s(z)$ is the nuclear surface equation in cylindrical coordinates, with $z_{\min},z_{\max}$ intercepts on the z axis.

For another set of deformation parameters $\{\alpha\}$ describing the same shape,

$$B_{kl}(\alpha)=\sum_{ij}B_{ij}(q)\frac{\partial q_i}{\partial\alpha_k}\frac{\partial q_j}{\partial\alpha_l}. \quad (9)$$

Also, one can define a nuclear inertia scalar $B(s)$ along a trajectory, given parametrically by the equations $q_i=q_i(s)$, $(i=1,2,\ldots,n)$:

$$B(s)=\sum B_{ij}(q_1,q_2,\ldots,q_n)\frac{dq_i}{ds}\frac{dq_j}{ds}. \quad (10)$$

In this way the multidimensional tunneling penetrability can be reduced to a one-dimensional problem. When $s=R$, or $s=z_m$, a good test of accuracy of the computations is obtained at the touching point configurations, where one should obtain the reduced mass $B(R_t)=\mu$.

III. TWO-DIMENSIONAL PARAMETRIZATION OF INTERSECTED SPHERES

The surface equation, assuming a shape of two intersected spheres, can be written as

$$\rho_s^2=\begin{cases}\rho_{sl}^2=R_1^2-z^2, & -R_1\leq z\leq z_s\\ \rho_{sr}^2=R_2^2-(z-R)^2, & z_s\leq z\leq R+R_2,\end{cases} \quad (11)$$

where R_1, R_2 are the radii of the two overlapping fragments, z_s is the position of the separation (intersection) plane, and R is the distance between the two centers.

By taking $q_1=R$ and $q_2=R_2$ as independent deformation parameters and placing the origin of coordinates in the center of the left-hand side fragment, the limits of integration are $z_{\min}=-R_1$ and $z_{\max}=R+R_2$, as can be seen in Eq. (11). The radius of the heavy fragment will be considered as a function of these variables, $R_1=R_1(R,R_2)$. The X_{il} quantities defined above for the left-hand side fragment are

$$X_{lR}=-\frac{1}{\rho_{sl}^2}\frac{\partial}{\partial R}\int_{-R_1}^{z_s}\rho_{sl}^2(x)dx=-\frac{2R_1}{R_1-z}\frac{\partial R_1}{\partial R}, \quad (12)$$

$$X_{lR_2}=-\frac{1}{\rho_{sl}^2}\frac{\partial}{\partial R_2}\int_{-R_1}^{z_s}\rho_{sl}^2(x)dx=-\frac{2R_1}{R_1-z}\frac{\partial R_1}{\partial R_2} \quad (13)$$

and the corresponding Y_{li}

$$Y_{lR}=-\frac{\rho_{sl}}{2}\frac{\partial X_{lR}}{\partial z}=\left[\frac{R_1+z}{(R_1-z)^3}\right]^{\frac{1}{2}}\left(R_1\frac{\partial R_1}{\partial R}\right), \quad (14)$$

$$Y_{lR_2}=-\frac{\rho_{sl}}{2}\frac{\partial X_{lR_2}}{\partial z}=\left[\frac{R_1+z}{(R_1-z)^3}\right]^{\frac{1}{2}}\left(R_1\frac{\partial R_1}{\partial R_2}\right). \quad (15)$$

Similarly, for the right-hand side fragment we have

$$X_{rR}=\frac{1}{\rho_{sr}^2}\frac{\partial}{\partial R}\int_z^{R+R_2}\rho_{sr}^2(x)dx=1, \tag{16}$$

$$X_{rR_2}=\frac{1}{\rho_{sr}^2}\frac{\partial}{\partial R_2}\int_z^{R+R_2}\rho_{sr}^2(x)dx=\frac{2R_2}{R_2+z-R} \tag{17}$$

and the corresponding Y_{ri}

$$Y_{rR}=-\frac{\rho_{sr}}{2}\frac{\partial X_{rR}}{\partial z}=0, \tag{18}$$

$$Y_{rR_2}=-\frac{\rho_{sr}}{2}\frac{\partial X_{rR_2}}{\partial z}=\left[\frac{R_2+R-z}{(R_2+z-R)^3}\right]^{\frac{1}{2}}R_2. \tag{19}$$

The tensor of inertia for two independent variables

$$\begin{pmatrix} B_{RR}(R,R_2) & B_{RR_2}(R,R_2) \\ B_{R_2R}(R,R_2) & B_{R_2R_2}(R,R_2) \end{pmatrix} \tag{20}$$

has the nondiagonal components equal to each other, $B_{R_2R}=B_{RR_2}$, due to the symmetry property. By taking into account Eqs. (7),(8), the inertia components have the following form:

$$\begin{aligned} B_{RR}(R,R_2)&=B_{RR}^l+B_{RR}^r+B_{RR}^c \\ &=\pi\sigma\int_{-R_1}^{z_s}\rho_{sl}^2(X_{lR}^2+\tfrac{1}{2}Y_{lR}^2)dz \\ &\quad+\pi\sigma\int_{z_s}^{R+R_2}\rho_{sr}^2(X_{rR}^2+\tfrac{1}{2}Y_{rR}^2)dz+B_{RR}^c, \end{aligned} \tag{21}$$

where

$$B_{RR}^c(R,R_2)=-\frac{\pi^2\sigma}{V}C_{RR}(R,R_2), \tag{22}$$

$$C_{RR}(R,R_2)=\left(\int_{-R_1}^{z_s}\rho_{sl}^2X_{lR}dz+\int_{z_s}^{R+R_2}\rho_{sr}^2X_{rR}dz\right)^2, \tag{23}$$

$$\begin{aligned} B_{RR_2}(R,R_2)&=B_{RR_2}^l+B_{RR_2}^r+B_{RR_2}^c \\ &=\pi\sigma\int_{-R_1}^{z_s}\rho_{sl}^2(X_{lR}X_{lR_2}+\tfrac{1}{2}Y_{lR}Y_{lR_2})dz+\pi\sigma\int_{z_s}^{R+R_2}\rho_{sr}^2(X_{rR}X_{rR_2}+\tfrac{1}{2}Y_{rR}Y_{rR_2})dz+B_{RR_2}^c, \end{aligned} \tag{24}$$

in which

$$B_{RR_2}^c(R,R_2)=-\frac{\pi^2\sigma}{V}C_{RR_2}(R,R_2), \tag{25}$$

$$C_{RR_2}(R,R_2)=\left(\int_{-R_1}^{z_s}\rho_{sl}^2X_{lR}dz+\int_{z_s}^{R+R_2}\rho_{sr}^2X_{rR}dz\right)\left(\int_{-R_1}^{z_s}\rho_{sl}^2X_{lR_2}dz+\int_{z_s}^{R+R_2}\rho_{sr}^2X_{rR_2}dz\right), \tag{26}$$

and

$$\begin{aligned} B_{R_2R_2}(R,R_2)&=B_{R_2R_2}^l+B_{R_2R_2}^r+B_{R_2R_2}^c \\ &=\pi\sigma\int_{-R_1}^{z_s}\rho_{sl}^2(X_{lR_2}^2+\tfrac{1}{2}Y_{lR_2}^2)dz+\pi\sigma\int_{z_s}^{R+R_2}\rho_{sr}^2(X_{rR_2}^2+\tfrac{1}{2}Y_{rR_2}^2)dz+B_{R_2R_2}^c, \end{aligned} \tag{27}$$

where

$$B_{R_2R_2}^c(R,R_2)=-\frac{\pi^2\sigma}{V}C_{R_2R_2}(R,R_2), \tag{28}$$

$$C_{R_2R_2}(R,R_2)=\left(\int_{-R_1}^{z_s}\rho_{sl}^2X_{lR_2}dz+\int_{z_s}^{R+R_2}\rho_{sr}^2X_{rR_2}dz\right)^2. \tag{29}$$

The following geometrical quantities can be defined: $D_1=z_s$; $H_1=R_1-z_s$; $D_2=R-z_s$; $H_2=R_2-D_2$. By substituting Eqs. (12)–(19) into Eqs. (21)–(29), and performing the integrations we obtain for the first component, B_{RR}, the contribution of the left-hand side fragment:

$$B^l_{RR}=\pi\sigma\left(-3.5D_1-4.5R_1+\frac{2R_1^2}{H_1}+6R_1\ln\frac{2R_1}{H_1}\right)\left(R_1\frac{\partial R_1}{\partial R}\right)^2. \tag{30}$$

For the right-hand side fragment

$$B^r_{RR}=\frac{\pi\sigma}{3}(R_2+D_2)^2(R_2+H_2) \tag{31}$$

and for the correction term

$$B^c_{RR}=-\frac{\pi^2\sigma}{V}\left[-(R_1+D_1)^2R_1\frac{\partial R_1}{\partial R}+\tfrac{1}{3}(R_2+D_2)^2(R_2+H_2)\right]^2. \tag{32}$$

Similar calculations for the mixing component, B_{RR_2}, lead to

$$B^l_{RR_2}=\pi\sigma\left(-3.5D_1-4.5R_1+\frac{2R_1^2}{H_1}+6R_1\ln\frac{2R_1}{H_1}\right)R_1^2\frac{\partial R_1}{\partial R}\frac{\partial R_1}{\partial R_2} \tag{33}$$

for the heavy fragment contribution, and

$$B^r_{RR_2}=\pi\sigma R_2(R_2+D_2)^2 \tag{34}$$

for the light fragment contribution. The correction term is expressed as

$$B^c_{RR_2}=-\frac{\pi^2\sigma}{V}\left[-(R_1+D_1)^2R_1\frac{\partial R_1}{\partial R}+\frac{1}{3}(R_2+D_2)^2(R_2+H_2)^2\right]\left[-(R_1+D_1)^2R_1\frac{\partial R_1}{\partial R_2}+R_2(R_2+D_2)^2\right]. \tag{35}$$

For the last diagonal component of inertia, $B_{R_2R_2}$, the corresponding terms are found to be

$$B^l_{R_2R_2}=\pi\sigma\left(-3.5D_1-4.5R_1+\frac{2R_1^2}{H_1}+6R_1\ln\frac{2R_1}{H_1}\right)\times\left(R_1\frac{\partial R_1}{\partial R_2}\right)^2, \tag{36}$$

$$B^r_{R_2R_2}=\pi\sigma\left(-3.5D_2-4.5R_2+\frac{2R_2^2}{H_2}+6R_2\ln\frac{2R_2}{H_2}\right)R_2^2, \tag{37}$$

$$B^c_{R_2R_2}=-\frac{\pi^2\sigma}{V}\left[-(R_1+D_1)^2R_1\frac{\partial R_1}{\partial R_2}+R_2(R_2+D_2)\right]^2. \tag{38}$$

Finally, by summing up the contribution of the two fragments and the correction term for every of the three tensor components, and by taking into account that all lengths are expressed in units of the radius of the parent nucleus, $R_0=r_0A^{1/3}$, we obtain the three components of the inertia tensor:

$$\frac{1}{m}B_{RR}(R,R_2)=0.25A(R_2+D_2)^2(R_2+H_2)-A\left[0.25(R_2+D_2)^2(R_2+H_2)-0.75(R_1+D_1)^2R_1\frac{\partial R_1}{\partial R}\right]^2+A\left(R_1\frac{\partial R_1}{\partial R}\right)^2\left[R_1\left(1.5\frac{R_1}{H_1}-3.375+4.5\ln\frac{2R_1}{H_1}\right)-2.625D_1\right], \tag{39}$$

$$\frac{1}{m}B_{RR_2}(R,R_2)=A\left(R_1\frac{\partial R_1}{\partial R}\right)\left(\frac{\partial R_1}{\partial R_2}\right)\left[R_1\left(1.5\frac{R_1}{H_1}-3.375+4.5\ln\frac{2R_1}{H_1}\right)-2.625D_1\right]+0.75A(R_2+D_2)^2R_2+0.1875A\left[3(R_1+D_1)^2\left(R_1\frac{\partial R_1}{\partial R}\right)-(R_2+D_2)^2(R_2+H_2)\right]\left[(R_2+D_2)^2R_2-(R_1+D_1)^2R_1\frac{\partial R_1}{\partial R_2}\right], \tag{40}$$

$$\frac{1}{m}B_{R_2R_2}(R,R_2)=A\left(R_1\frac{\partial R_1}{\partial R_2}\right)^2\left[R_1\left(1.5\frac{R_1}{H_1}-3.375+4.5\ln\frac{2R_1}{H_1}\right)-2.625D_1\right]+AR_2^2\left[R_2\left(1.5\frac{R_2}{H_2}-3.375+4.5\ln\frac{2R_2}{H_2}\right)-2.625D_2\right]+0.5625A\left[-R_2(R_2+D_2)^2+R_1(R_1+D_1)^2\frac{\partial R_1}{\partial R_2}\right]^2. \tag{41}$$

The volumes of the two fragments are given by the following relationships:

$$V_1=\frac{\pi}{3}(R_1+D_1)^2(R_1+H_1);$$

$$V_2=\frac{\pi}{3}(R_2+D_2)^2(R_2+H_2). \tag{42}$$

By using the total volume conservation, $V_1+V_2=V=$ const (where $V=4\pi R_0^3/3$, R_0 is the radius of the parent nucleus), and the matching condition in the intersection plane

$$\rho_n^2=R_1^2-D_1^2=R_2^2-D_2^2, \tag{43}$$

we will calculate analytically the two partial derivatives of $R_1(R,R_2)$, according to the theory of implicit functions:

$$\frac{\partial R_1}{\partial R}=-\frac{\partial V/\partial R}{\partial V/\partial R_1};$$

$$\frac{\partial R_1}{\partial R_2}=-\frac{\partial V/\partial R_2}{\partial V/\partial R_1}. \tag{44}$$

The involved quantities are

$$\frac{1}{\pi}\frac{\partial V}{\partial R}=\rho_n^2, \tag{45}$$

$$\frac{1}{\pi}\frac{\partial V}{\partial R_1}=2R_1(R_1+D_1), \tag{46}$$

$$\frac{1}{\pi}\frac{\partial V}{\partial R_2}=2R_2(R_2+D_2). \tag{47}$$

After performing the calculations, we obtain

$$\frac{\partial R_1}{\partial R}=-\frac{H_1}{2R_1}\ ; \qquad \frac{\partial R_1}{\partial R_2}=-\frac{R_2(R_2+D_2)}{R_1(R_1+D_1)}. \tag{48}$$

By choosing a trajectory in the plane of the two independent coordinates, R and R_2 given in a parametric form: $R_2=R_2(s)$; $R=R(s)$, and by taking $s=R$, we can write according to Eq. (10)

$$B(R)=B_{RR}(R,R_2)+2B_{RR_2}(R,R_2)\left(\frac{dR_2}{dR}\right)+B_{R_2R_2}(R,R_2)\left(\frac{dR_2}{dR}\right)^2, \tag{49}$$

expressing an inertia scalar which is used to calculate the tunneling penetrability along this path. In the limit $R\to R_t$, when $H_1\to 0$ and $H_2\to 0$, the diagonal component, $B_{R_2R_2}$, becomes infinitely large. Consequently, for a finite inertia scalar $B(R_t)$, one has to choose a path $R_2(R)$ fulfilling the condition of a vanishing derivative, $R_2'(R_t)=0$, at the touching point.

IV. TWO ONE-DIMENSIONAL SEQUENCE OF SHAPES AS PARTICULAR CASES

We have the possibility to check the validity of the general relationships (39)–(41), by comparing, in the whole range of $R\in(R_i,R_t)$, where $R_i=R_0-R_{2i}$, and $R_{2i}=R_2(R_i)$ is the initial value of R_2, the results for two particular one-dimensional configurations mentioned in the introduction (MC and CL) with the similar equations previously published [11] assuming only one independent variable. Also, at the touching point, $R=R_t$, the inertia scalar $B(R)$ equates the reduced mass.

For the particular parametrization with compact shapes, there is a second restriction besides $V=$const, namely $V_2=$const. We have to take into account the fact that $R_1=R_1(R,R_2)$, in the general case when only the total volume is conserved without any other constraint, so that

$$V_2=V_2[R,R_2,R_1(R,R_2)]. \tag{50}$$

The derivative of $R_2(R)$ with respect to R, is calculated following the same prescription as above,

$$\frac{dR_2}{dR}=-\frac{(\partial V_2/\partial R)+(\partial V_2/\partial R_1)(\partial R_1/\partial R)}{(\partial V_2/\partial R_2)+(\partial V_2/\partial R_1)(\partial R_1/\partial R_2)}. \tag{51}$$

The two partial derivatives of R_1 with respect to R and R_2 are given by Eq. (48). We find for the other terms in the above formula

$$\frac{\partial V_2}{\partial R}=3H_2\frac{D_1}{R}(R_2+D_2), \tag{52}$$

$$\frac{\partial V_2}{\partial R_2}=3\frac{R_2}{R}(R_2+D_2)(2R+H_2), \tag{53}$$

$$\frac{\partial V_2}{R_1}=-3H_2\frac{R_1}{R}(R_2+D_2). \tag{54}$$

After replacing these equations in (51) and performing calculations, we obtain the derivative of R_2 with respect to R for the compact shape parametrization:

$$\frac{dR_2}{dR}=-\frac{H_2(R_1+D_1)}{2R_2(R+R_1+R_2)}, \tag{55}$$

which is identical to the corresponding Eq. (21) from [11].

By substituting (55) in (39)–(41), the three components of the inertia tensor may be written:

$$\frac{1}{m}B_{RR}=A\left\{\frac{H_1^2}{4}K_1+\frac{3}{4\pi}V_2-\frac{9}{16}\left[\frac{H_1}{2}(R_1+D_1)^2+\frac{V_2}{\pi}\right]^2\right\}, \tag{56}$$

$$\frac{1}{m}B_{RR_2}=AR_2(R_2+D_2)\left\{\frac{H_1}{2(R_1+D_1)}K_1+\frac{3}{4}(R_2+D_2)-\frac{9}{16}\left[\frac{H_1}{2}(R_1+D_1)^2+\frac{V_2}{\pi}\right](R+R_1+R_2)\right\}, \tag{57}$$

$$\frac{1}{m}B_{R_2R_2}=AR_2^2\left[\left(\frac{R_2+D_2}{R_1+D_1}\right)^2K_1+K_2-\frac{9}{16}(R_2+D_2)^2(R+R_1+R_2)^2\right], \quad (58)$$

where

$$K_1=-2.625D_1-3.375R_1+1.5\frac{R_1^2}{H_1}+4.5R_1\ln\frac{2R_1}{H_1}, \quad (59)$$

$$K_2=-2.625D_2-3.375R_2+1.5\frac{R_2^2}{H_2}+4.5R_2\ln\frac{2R_2}{H_2}, \quad (60)$$

and V_2 is the volume of the emitted fragment [see Eq. (42)].

Now, in the general expression of the total inertia scalar (49) we introduce the derivative of R_2 with respect to R, (55), obtained for the compact shape parametrization. We get in this way the inertia for $V_2=\text{const}$:

$$\left.\frac{B(R)}{m}\right|_{V_2=\text{const}}=A\left[\left(\frac{H_1}{2}\frac{R_2+D_2}{R+R_1+R_2}\right)^2K_1+\left(\frac{H_2}{2}\frac{R_1+D_1}{R+R_1+R_2}\right)^2K_2-\frac{3}{4}\frac{H_2(R_1+D_1)(R_2+D_2)^2}{R+R_1+R_2}+\frac{3}{4\pi}V_2\right]-\frac{9}{16}A\frac{V_2^2}{\pi^2}. \quad (61)$$

The last term is the correction due to the center of mass motion. One can observe that, during the shape evolution, this correction remains constant $-9AV_2^2/(4\pi)^2=-A_e^2/A$. The final result for the compact shape is

$$\left.\frac{B(R)}{m}\right|_{V_2=\text{const}}=A\left[\left(\frac{H_1}{2}\frac{R_2+D_2}{R+R_1+R_2}\right)^2K_1+\left(\frac{H_2}{2}\frac{R_1+D_1}{R+R_1+R_2}\right)^2K_2-\frac{3}{4}\frac{H_2(R_1+D_1)(R_2+D_2)^2}{R+R_1+R_2}\right]+\mu_A, \quad (62)$$

where $\mu_A=A_eA_d/A$ is the reduced mass number. This term is derived from $3AV_2/(4\pi)-9AV_2^2/(4\pi)^2=A_e-A_e^2/A=\mu_A$. The equation (62) reproduces the formula of inertia obtained for MC shapes [11].

Despite the differences between B and B_{RR} shown in Figs. 1–3 for cold fission with ^{122}Cd light fragment, ^{46}Ar cluster radioactivity, and α decay of ^{252}Cf, the final result of the projection on the $R_2(R)$ path coincides with the one-dimensional $B(R)$, the mixing term, $2B_{RR_2}R_2'$ being negative, due to the sign of the derivative R_2'. Also, by multiplication with R_2' and $(R_2')^2$, the last two terms give no contribution at the touching point, in spite of the general trend of $B_{R_2R_2}$ toward an infinite value when $R\to R_t$.

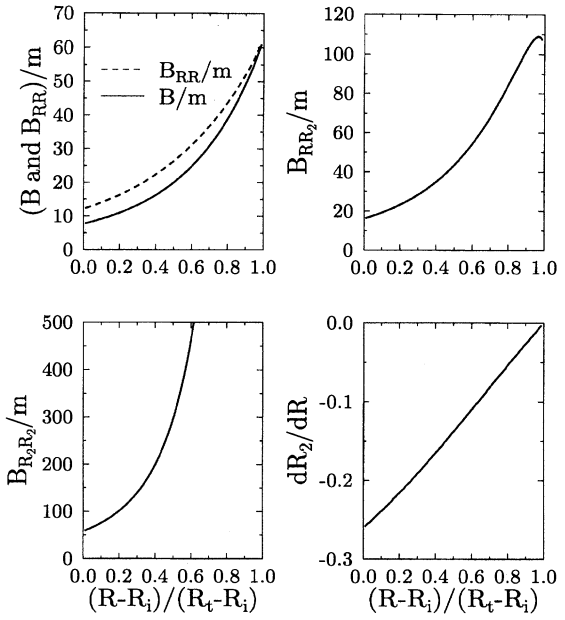

FIG. 1. The components of the nuclear inertia tensor B_{RR} (top left), B_{RR_2} (top right), and $B_{R_2R_2}$ (bottom left), leading to a scalar $B(R)$ along a path $R_2(R)$ with a negative derivative (bottom right), for the cold fission of ^{252}Cf with ^{122}Cd light fragment in the parametrization of two intersected spheres with $V_2=\text{const}$. The one-dimensional inertia B (top left), calculated with the Eq. (49), is exactly reproduced.

For clusterlike shapes, $R_2=\text{const}$, $dR_2/dR=0$. Unlike the preceding case, where all three components of inertia tensor contributed to $B(R)$, now

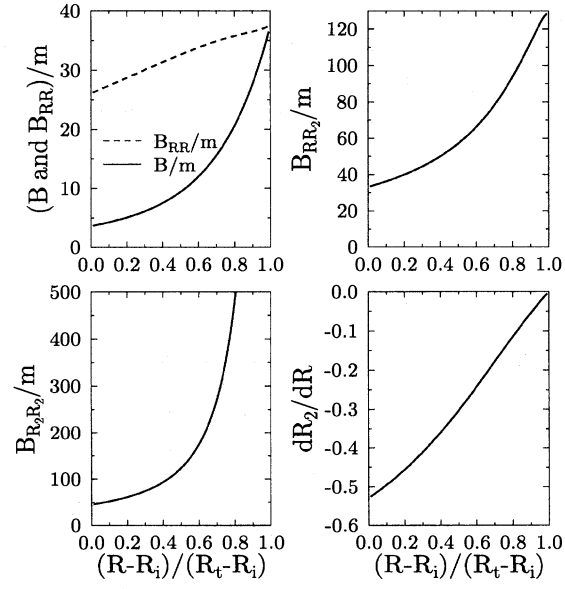

FIG. 2. Same quantities as in Fig. 1, for the ^{46}Ar cluster emission from ^{252}Cf.

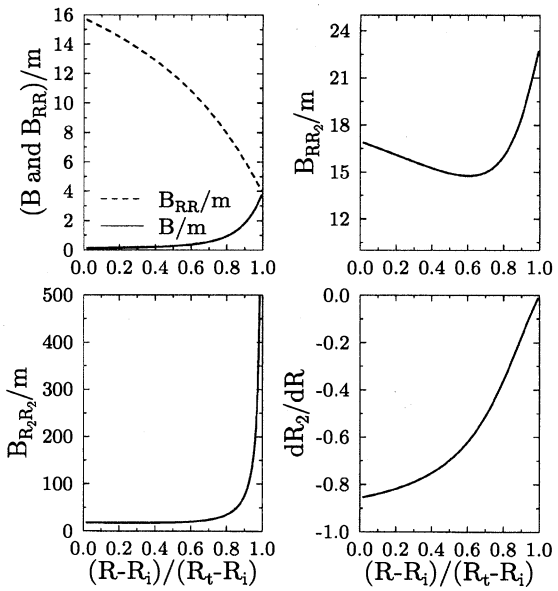

FIG. 3. Same quantities as in Fig. 1, for the α decay of ^{252}Cf.

$$\frac{B(R)}{m}=\frac{B_{RR}}{m}\bigg|_{R_2=\text{const}}=\frac{A}{4}\left\{H_1^2K_1+\frac{3V_2}{\pi}-\frac{9A}{4}\left[\frac{H_1}{2}(R_1+D_1)^2+\frac{V_2}{\pi}\right]^2\right\}, \tag{63}$$

which is again the same as in Ref. [11].

The variations with R of B_{RR} equating $B(R)$, and of B_{RR_2} for CL shapes, and the same decay modes like in Figs. 1–3 are plotted in Fig. 4. The other component $B_{R_2R_2}$ shows a similar behavior with that of MC shapes.

By analyzing the formula of the total inertia $B(R)$, and that of the derivative of R_2 with respect to R, we can see that at the touching point where $H_2\rightarrow 0$, we have from Eq. (55) $(dR_2)/(dR)=0$ when $R\rightarrow R_t$. In this way the two last terms of $B(R)$, namely $2B_{RR_2}(R,R_2)(dR_2)/(dR)$ and $B_{R_2R_2}(R,R_2)[(dR_2)/(dR)]^2$ vanish at $R=R_t$.

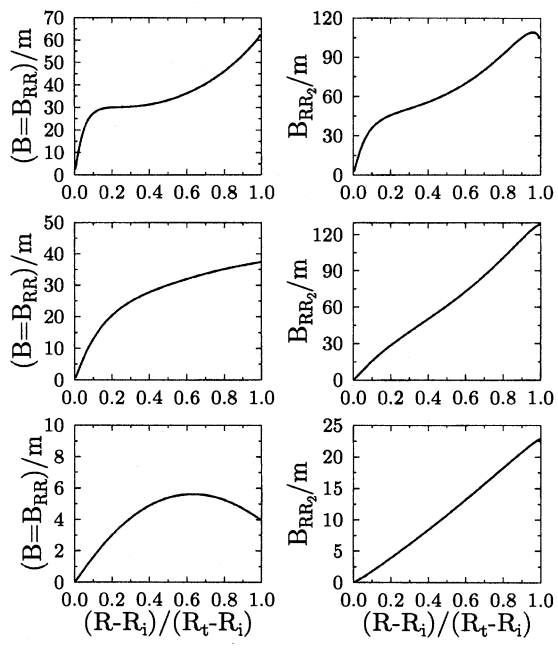

FIG. 4. The components of the nuclear inertia tensor B_{RR} equating exactly the one-dimensional B (left-hand side) and B_{RR_2} [right-hand side, giving no contribution to the scalar $B(R)$ owing to the vanishing derivative $R_2'=0$], for two-intersected spheres with $R_2=$const. The plots refer to the cold fission with ^{122}Cd light fragment (top), the ^{46}Ar cluster radioactivity (middle), and α decay (bottom) of ^{252}Cf.

ACKNOWLEDGMENTS

This work was supported by the Bundesministerium für Forschung und Technologie, Bonn, and the Institute of Atomic Physics, Bucharest. D.N.P. received a donation of computer and copying equipment from the Soros Foundation for an Open Society. R.A.G. is grateful to the Konferenz der Deutschen Akademien der Wissenschaften, Mainz, for a grant of a Volkswagenstfiftung fellowship.

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