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Gapless phases of color-superconducting matter

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Abstract.   We discuss gapless color superconductivity for neutral quark matter in
β equilibrium at zero as well as at nonzero temperature. Basic properties of gapless
superconductors are reviewed. The current progress and the remaining problems in
the understanding of the phase diagram of strange quark matter are discussed.

1. Introduction

The estimated central densities of compact stars could be sufficiently large to support
the existence of deconfined quark matter. Such matter should develop a Cooper
instability with respect to diquark condensation, and become color superconducting
[1, 2, 3, 4]. Note that typical temperatures inside compact stars are so low that the
diquark condensate, if produced, would not melt.

Matter in the bulk of a compact star should be neutral (at least, on average)
with respect to electric as well as color charges. Otherwise, the star would not be
bound by gravity which is much weaker than electromagnetism. Matter should also
remain in β equilibrium. The latter requires that the rate of the β-decay processes (i.e.,
$d \to u + e^- + \bar{\nu}_e$ and $s \to u + e^- + \bar{\nu}_e$) should be equal to the rate of the corresponding
electron capture processes (i.e., $u + e^- \to d + \nu_e$ and $u + e^- \to s + \nu_e$).

After the charge neutrality and the β-equilibrium conditions are enforced, the
chemical potentials of different quarks satisfy relations that may interfere with the
dynamics of Cooper pairing. If this happens, some color-superconducting phases may
become less favored than others. For example, in Ref. [5], it was argued that a mixture

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of the normal phase, made of strange quarks, and the two-flavor color superconducting (2SC) phase, made of up and down quarks, is less favorable than the color-flavor locked (CFL) phase after the charge neutrality condition is enforced.

Assuming that the constituent medium-modified mass of the strange quark is large (i.e., larger than the corresponding strange quark chemical potential), in Ref. [6] it was shown that neutral two-flavor quark matter in β equilibrium can have another rather unusual ground state called the gapless two-flavor color superconductor (g2SC). The appearance of this phase is directly connected with enforcing the charge neutrality in the system. While the symmetry in the g2SC ground state is the same as that in the conventional 2SC phase, the spectrum of the fermionic quasiparticles is different. The order parameter of the g2SC phase is given by the difference of the number densities of quarks participating in pairing (e.g., the number density of green up quarks and the number density of red down quarks).

The existence of the gapless two-flavor color superconducting phase was confirmed in Refs. [7, 8, 9], and generalized to nonzero temperatures in Refs. [10, 11]. Later it was shown, however, that a chromomagnetic instability develops in such a phase [12]. In view of this, the true ground state remains unknown. If the surface tension between different quark phases is sufficiently small, as suggested in Ref. [13], a mixed phase composed of the 2SC phase and the normal quark phase [14] may be more favored than the gapless phases. If this conclusion holds after the screening effects in the mixed phase are properly taken into account, the mixed phase is likely to be the ground state.

It was also shown that a gapless CFL (gCFL) phase could appear in neutral strange quark matter when the strange quark mass is not very small [15, 16]. At nonzero temperature, the (g)CFL phase and several other new phases (e.g., the so-called dSC and uSC phases) were studied in Refs. [17, 18, 19]. Recently, however, it was claimed that the gCFL phase also has a chromomagnetic instability [20].

2. Gapless color-flavor locked phase

At very large densities, the most favorable phase of quark matter is the CFL phase in which up, down and strange quarks participate in Cooper pairing on almost equal footing [3]. However, at the highest baryon densities existing in stars (which are less than about $10\rho_0$, where $\rho_0 \approx 0.15$ fm^{-3} is the nuclear saturation density), the CFL phase may be replaced by a less symmetric phase. This is because the strange quark is considerably heavier than the up and down quarks, and the ideal strange-nonstrange cross-flavor diquark pairing could be distorted. Indeed, it is most likely that the actual value of the strange quark mass m_s in a dense medium is in the range between about 100 MeV and 500 MeV. This is not negligible compared to the quark chemical potential μ which is of the order of 500 MeV in the center of compact stars.

Here, we consider a Nambu-Jona-Lasinio (NJL) type model for three-flavor quark

matter with a local current-current interaction,

$$\mathcal{L} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} + \gamma^{0}\hat{\mu} - \hat{m}\right)\psi + \frac{g^{2}}{2\Lambda^{2}}\left(\bar{\psi}\gamma^{\mu}\frac{\lambda^{A}}{2}\psi\right)^{2}. \tag{1}$$

where the color-flavor structure of the chemical potential and the mass matrices are given by

$$\hat{\mu} = \mu + \mu_{Q}Q + \mu_{3}T_{3} + \mu_{8}T_{8} \tag{2}$$

and $\hat{m} = \mathrm{diag}_{\mathrm{flavor}}(0, 0, m_{s})$, respectively. The matrices Q, T_{3} and T_{8} are the generators of mutually commuting electric and two color charges.

In the Cooper pairing dynamics responsible for color superconductivity, the main effect of a non-vanishing strange quark mass is a reduction of the strange quark Fermi momentum,

$$k_{F}^{(s)} = \sqrt{\mu^{2} - m_{s}^{2}} \simeq \mu - \frac{m_{s}^{2}}{2\mu}, \quad \text{for} \quad m_{s} \ll \mu. \tag{3}$$

The magnitude of the reduction is approximately given by the value of $m_{s}^{2}/2\mu$. This quantity plays the role of a mismatch parameter in three-flavor quark matter, which is similar to $\delta\mu \equiv \mu_{e}/2$ in two-flavor quark matter [6]. This mismatch interferes with Cooper pairing between strange and non-strange quarks [15].

The simplest way to take into account the effect of the strange quark mass is to replace the chemical potential of the strange quark by its effective shifted value in Eq. (3). This was the approach of Refs. [15, 18]. In this paper, as in Ref. [19], we do not use such an approximation. The strange quark mass is properly taken into account.

Because of a nonzero strange quark mass, the up-down, the up-strange and the down-strange diquark condensates are not equal in the ground state [15, 21],

$$\left\langle \left(\bar{\psi}^{C}\right)_{i}^{a} \gamma^{5}\psi_{j}^{b}\right\rangle \sim \phi_{1}\varepsilon_{ij1}\epsilon^{ab1} + \phi_{2}\varepsilon_{ij2}\epsilon^{ab2} + \phi_{3}\varepsilon_{ij3}\epsilon^{ab3} + \cdots, \tag{4}$$

where the ellipsis denote the terms symmetric in color and flavor. Although the symmetric terms are small and not crucial for the qualitative understanding of strange quark matter, we retain them in our analysis [18].

A nonzero value of the strange quark mass interferes most prominently with the pairing between the strange and the non-strange quarks, i.e., with the pairing described by the gap parameters ϕ_{1} and ϕ_{2}. Because of color-flavor locking, preserved in the diquark condensate (4), this translates into a special role played by the blue color in the ground state (in QCD this is meaningful, provided a specific gauge fixing is done).

Starting from the massless limit ($m_{s} = 0$) and gradually increasing the value of the strange quark mass, one finds that the CFL phase stays robust until a critical value of the control parameter $m_{s}^{2}/2\mu \simeq \Delta$ is reached [15]. Here, Δ is the value of the gap parameter ϕ_{1}. (Note that $\Delta \equiv \phi_{1} = \phi_{2} \approx \phi_{3}$ in the CFL phase, see left panel in Fig. 1 below.) Above the critical value, the charge neutrality exerts too much stress on the CFL phase, and a transition to a new (gapless) phase happens [15].

A nice feature of the CFL phase is that it stays almost automatically electrically neutral [22]. The reason is that Cooper pairing in the CFL phase helps to enforce equal

number densities of all three quark flavors, $n_u = n_d = n_s$. Since the sum of the charges of up, down and strange quarks add up to zero, this insures that the electric charge density is vanishing, $n_Q = \frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s = 0$. This is exactly what happens in the CFL phase even at nonzero, but sub-critical values of the strange quark mass.

In contrast to the g2SC case, it is the color rather than the electric charge neutrality that plays the key role in destabilizing the CFL phase of three-flavor quark matter with increasing the value of $m_s^2/2\mu$. The actual mechanism is directly related to color-flavor locking in the CFL ground state. Because of such a locking, the blue quarks have a special status in the Cooper pairing dynamics. In order to avoid the violation of the color neutrality by these quarks, a nonzero value of the color chemical potential $\mu_8 \propto -m_s^2/2\mu$ is required [5]. Note that the value of μ_8 is monotonically increasing with the strange quark mass. After the stress in the quark system becomes too strong, the CFL phase turns into the gapless CFL phase. As was shown in Ref. [15], this happens when $m_s^2/2\mu \approx \phi_1$. In essence, the mechanism is the same as in two-flavor quark matter studied in Ref. [6].

3. Phase diagram

In this section, we present the phase diagram of dense neutral three-flavor quark matter in the plane of temperature and m_s^2/μ. The first version of such a phase diagram was presented in Ref. [18]. In Ref. [18], however, the effect of the strange quark mass was incorporated only through a shift of the chemical potential of strange quarks, $\mu_s^a \to \mu_s^a - m_s^2/(2\mu)$ (here $a = 1, 2, 3$ is the color index). Such an approach is certainly reliable at small values of the strange quark mass. One should check, however, whether the results are reliable at least qualitatively also at not very small values of the strange quark mass.

The study of the phase diagram [18] was further developed in Ref. [19] where the strange quark mass was properly taken into account. Here, we perform a similar study using our original set of model parameters [18]. As we shall see, the results do not differ very much from those in Ref. [18] even at rather large values of the strange quark mass.

Let us start with the discussion of the effect of a nonzero strange quark mass on the gap parameters. The zero-temperature results for the gaps as functions of m_s^2/μ are shown in the left panel of Fig. 1. At small strange quark mass, the ground state corresponds to the CFL phase. Here, the following relation between the three gaps holds: $\phi_1 = \phi_2 \approx \phi_3$ [19]. At large strange quark mass, on the other hand, the three gap parameters are very different. As we can see from the figure, the qualitative change of the gaps as functions of m_s^2/μ happens at $m_s^2/\mu \approx 2\phi_1$. This is a consequence of the phase transition between the CFL and the gCFL phase [15].

As one can check, the transition point at $m_s^2/\mu \approx 2\phi_1$ corresponds to the appearance of additional gapless modes in the quasiparticle spectrum, justifying the name of the phase. As in the case of the g2SC phase, the order parameter in the gCFL phase could be identified with a difference of number densities of some quarks participating in pairing

[6]. In particular, this is the difference of the number densities of blue down and green strange quarks [18]. In Ref. [15], however, it was suggested to use the number density of electrons as an alternative order parameter. While there are no electrons in the CFL phase [22], there is a non-vanishing density of them in the gCFL phase. Thus, the corresponding transition was called an insulator-metal transition. Note that, at nonzero temperatures, the corresponding transition becomes a smooth crossover [18, 19].

The physical properties of the gCFL phase are very different from those of the CFL phase. The presence of gapless quasiparticle modes has a large effect on the thermodynamics as well as on the transport properties. In contrast to the CFL phase which is an insulator, the gCFL phase is a metal with a nonzero number density of electrons [15, 16, 18, 19]. Therefore, the electrical conductivities of the two phases are very different. (Note that, at low temperature, the electrical conductivity in the CFL phase is dominated by thermally excited electron-positron pairs [23].) Also, the neutrino emissivity from the gCFL phase should be rather high. It is dominated by the β processes involving the gapless modes. In contrast, the corresponding emissivity from the CFL phase is strongly suppressed.

Now, let us discuss how three-flavor neutral quark matter responds to a nonzero temperature. In general, as in Ref. [18], if one starts from the (g)CFL phase and gradually increases the temperature, three consecutive phase transitions occur in the system (see right panel of Fig. 1):

(i) the transition from the (g)CFL phase to the so-called uSC phase;

(ii) the transition from the uSC phase to the 2SC phase;

(iii) the transition from the 2SC phase to the normal quark phase.

Here, the notation uSC (dSC) stands for superconducting phases in which there are only *up*-down and *up*-strange (or up-*down* and *down*-strange) condensates, and there is no down-strange (or up-strange, respectively) condensate [17]. From Eq. (4) one can check that ϕ_1 vanishes in the uSC phase, while ϕ_2 vanishes in the dSC phase.

Our results differ from those of Ref. [17] in that the dSC phase is replaced by the uSC phase in the near-critical region. This was also the case in our previous study [18]. However, Ref. [19] revealed a small region of the dSC phase at temperatures close to the critical temperature and at small values of the strange quark mass. We checked that our numerical calculations produce qualitatively the same results when we use a set of parameters close to that of Ref. [19]. In fact, the main difference between the two studies is the value of the cut-off parameter in the NJL model. From this we conclude that the size of the dSC region in the phase diagram is particularly sensitive to the choice of the cut-off in the NJL model.

Now, let us briefly discuss the main features of the phase diagram of dense neutral three-flavor quark matter in the plane of temperature and m_s^2/μ. This is shown in the right panel of Fig. 1. Here, the results are plotted for a fixed value of the quark chemical potential, $\mu = 500$ MeV. The three solid lines denote the three phase transitions discussed earlier. In the mean-field approximation used in this study, all
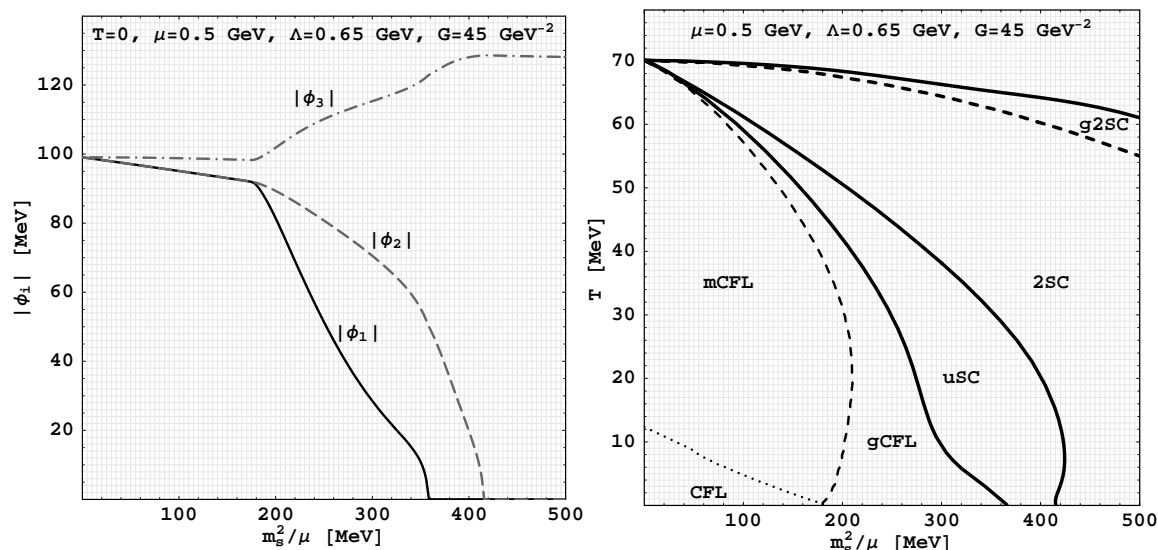
Figure 1. The absolute value of the gap parameters ϕ_i ($i = 1, 2, 3$) as functions of m_s^2/μ at zero temperature (left panel); and the phase diagram of neutral three-flavor quark matter in the plane of temperature and m_s^2/μ at a fixed value of the quark chemical potential, $\mu = 500$ MeV, (right panel). By definition, $G \equiv g^2/\Lambda^2$.

three transitions are second order phase transitions. After taking into account various types of fluctuations, the nature of some of them may change [24]. A detailed study of this issue is, however, outside the scope of this paper. The two dashed lines mark the appearance of gapless modes in the metallic CFL (mCFL) and 2SC phases (see Ref. [18] for the detailed definitions). In addition, there is also an insulator-metal crossover transition between the CFL and mCFL phase. This is marked by the dotted line on the phase diagram in Fig. 1.

By comparing the results in the right panel of Fig. 1 with the corresponding phase diagram in Ref. [18], we find that the results are qualitatively the same and even quantitatively very similar. Thus, with our set of parameters, even a simplified treatment of the strange quark mass reproduces the overall structure of the phase diagram. It should be admitted, however, that this may not always be the case. For example, the simplified method with an effective shift of the strange quark chemical potential is unlikely to capture the appearance of the first order phase transition at small temperatures and large m_s in the phase diagram shown in Figs. 1 and 16 of Ref. [19].

4. Conclusion

In this paper, we studied neutral three-flavor quark matter at large baryon densities. We obtained the phase diagram of dense neutral three-flavor quark matter in the plane

of temperature and m_s^2/μ. In contrast to the approximate treatment of the strange quark mass of Ref. [18], here the mass is properly taken into account. The final results are very similar to those of Ref. [18].

If we ignore the possibility of the chromomagnetic instability [12, 20] for a moment, there are two main possibilities for the strange quark matter ground state at $T = 0$: the CFL and the gCFL phases in the case of small and large strange quark mass, respectively. The transition from the CFL to the gCFL phase is driven by a gradual build-up of the stress in the quark system due to the color neutrality condition. This stress grows with increasing the strange quark mass. The mechanism is directly related to color-flavor locking in the CFL ground state. Turning on the strange quark mass tends to induce an imbalance of the blue color in the system. This imbalance is removed by a nonzero value of the color chemical potential μ_8 in the CFL phase. After reaching a critical value of the strange quark mass, $m_s^2/\mu \approx 2\phi_1$, the CFL turns into the gCFL phase [15]. This is similar to a transition between the 2SC and the g2SC phases which, however, is driven by the electron chemical potential, needed to preserve electric charge neutrality in two-flavor quark matter [6].

In this study we confirm the results of Ref. [18] regarding the existence of several different phases of neutral three-flavor quark matter at nonzero temperature. We also confirm the order in which they appear. In particular, we observe the appearance of the uSC phase as an intermediate state in melting of the (g)CFL phase into the 2SC phase. Formally, this is different from the prediction of Ref. [17]. We find, however, that the difference is connected with the choice of the model parameters. In the NJL model with a cut-off parameter $\Lambda = 800$ MeV used in Ref. [19], there is a non-vanishing (although rather small) region of the dSC phase. On the other hand, in the NJL model with a relatively small value of the cut-off parameter (we have $\Lambda = 653$ MeV) used in this paper, no sizeable window of the dSC phase is found.

Now, if one takes the chromomagnetic instability [12, 20] into account seriously, there is a fundamental problem in the present understanding of the phase diagram of neutral dense quark matter. Indeed, some regions of the phase diagram (see right panel of Fig. 1) correspond to phases that are unstable, and there exist no unambiguous alternatives (two of such alternatives were proposed in Ref. [25]). Of course, it is of prime importance to resolve this crisis.

Acknowledgements

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