# Tax Optimized Investment Strategies

*Inaugural-Dissertation*

*zur Erlangung des Doktorgrades*

*des Fachbereichs Wirtschaftswissenschaften*

*der Johann Wolfgang Goethe-Universität*

*Frankfurt am Main*

vorgelegt von Marcel Marekwica

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| *Erstgutachter*: | Prof. Dr. Raimond Maurer |
| *Zweitgutachter*: | Prof. Dr. Christian Schlag |
| *Tag der Promotion*: | 16.01.2008 |

# Danksagung

Mein erster Dank gilt meinem Doktorvater, Herrn Prof. Dr Raimond Maurer, für die Chance an seinem Lehrstuhl zu promovieren. Die vorliegende Arbeit hat von dem regen Gedankenaustausch mit dem Lehrstuhlinhaber und den wissenschaftlichen Mitarbeitern sowie der Möglichkeit, auf die hervorragende technische Ausstattung des Lehrstuhls zugreifen zu können, enorm profitiert.

Der enge Kontakt zu meinen Arbeitskolleginnen und -kollegen Herrn Dr. Ivica Duš, Herrn Dipl.-Kfm. Wolfram Horneff, Herrn Dr. Frank Reiner, Herrn Dipl.-Volksw. Ralph Rogalla, Herrn Prof. Dr. Steffen Sebastian, Frau Dipl.-Kffr. Barbara Somova, Herrn Dr. Michael Stamos sowie Herrn Dipl.-Kfm. Huy Thanh Vo brachte wertvolle Diskussionen und Anregungen für meine Arbeit. Ganz besonders danke ich Herrn Dipl.-Kfm. Alexander Schaefer, mit dem ich mir gerne das Büro geteilt habe. Seine Fähigkeit, sich schnell in meine manchmal sehr plötzlichen Fragestellungen und Ideen hineinzudenken, und seine immerwährende Bereitschaft zur Diskussion haben die vorliegende Arbeit vorangebracht.

Für die hervorragende Doktorandenausbildung im Rahmen seines Kurses "Dynamic Capital Market Theory" sowie seine Gedanken zu meinem weiteren akademischen Weg danke ich meinem Zweitgutachter Herrn Professor Dr. Christian Schlag.

Der Friedrich-Naumann-Stiftung danke ich neben dem Promotionsstipendium aus Mitteln des Bundesministeriums für Bildung und Forschung insbesondere für viele neue Freundschaften und interdisziplinäre Seminare.

Frau Ulrike Witkowski, meiner Oberstufenlehrerin, danke ich dafür, dass sie mich schon vor dem Abitur ermutigt hat, an der Fernuniversität Hagen mein BWL-Studium zu beginnen.

Für die nötige Abwechselung danke ich meinem Freundeskreis, insbesondere Bianca Allendorf, Patrick Berg, Dominik Fiedler, und Conny Uhde.

Mein ganz privater Dank gilt meiner Familie für bedingungsloses Vertrauen, Unterstützung in allen Lebenslagen und so viel mehr als hier Platz hätte.

Frankfurt a.M., den 30. Januar 2008

Marcel Marekwica

Wäre es denn möglich, so würde ich noch meinem Fahrrad

für den klaren Kopf im Fahrtwind danken.

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# List of Symbols

## Symbols used in the first chapter

| symbol | interpretation |
|---|---|
| $\alpha_t$ | total equity exposure in period $t$ |
| $\alpha_{R,i,t}$ | proportion of asset $i$ in period $t$ in retirement account |
| $\alpha_{T,i,t}$ | proportion of asset $i$ in period $t$ in taxable account |
| $\boldsymbol{\alpha}_{R,t}$ | vector of fractions of $n$ assets in period $t$ in retirement account |
| $\boldsymbol{\alpha}_{T,t}$ | vector of fractions of $n$ assets in period $t$ in taxable account |
| $d_{R,i,t}$ | dividend or interest rate for asset $i$ in period $t$ in the retirement account |
| $d_{T,i,t}$ | dividend or interest rate for asset $i$ in period $t$ in the taxable account |
| $\mathbf{d}_{R,t}$ | vector of $n$ dividend rates in period $t$ in retirement account |
| $\mathbf{d}_{T,t}$ | vector of $n$ dividend rates in period $t$ in taxable account |
| $\gamma$ | parameter of risk aversion |
| $g_{R,i,t}$ | capital gains rate for asset $i$ in period $t$ in the retirement account |
| $g_{T,i,t}$ | capital gains rate for asset $i$ in period $t$ in the taxable account |
| $G_T$ | total tax-gift |
| $\mathbf{g}_{R,t}$ | vector of $n$ capital gains rates in period $t$ in retirement account |
| $\mathbf{g}_{T,t}$ | vector of $n$ capital gains rates in period $t$ in taxable account |
| $\mathbf{R}_{R,t}$ | vector of $n$ gross returns in period $t$ for assets in retirement account |
| $\mathbf{R}_{T,t}$ | vector of $n$ gross returns in period $t$ for assets in taxable account |
| $\tau_a$ | effective tax-rate of asset $a$ |
| $\tau_d$ | tax-rate on dividends and interest |
| $\tau_g$ | tax-rate on capital gains |
| $\tau_{p,t}$ | personal income tax-rate in period $t$ |
| $n$ | number of assets in the investment opportunity set |

| | |
|---|---|
| $t$ | time index |
| $T$ | length of investment horizon |
| $T_t$ | relative tax-gift in period $t$ |
| $U(.)$ | utility function |
| $V_t(.)$ | value function at time $t$ |
| $W_t$ | end-of-period $t$ total wealth |
| $W_{T,t}$ | end-of-period $t$ taxable wealth |
| $W_{R,t}$ | end-of-period $t$ retirement wealth |
| $x_a$ | replication cost of asset $a$ |
| $X_t$ | vector of state variables in period $t$ |

# Symbols used in the second chapter

| symbol | interpretation |
|---|---|
| $\alpha_t$ | beginning-of-period wealth allocated to equity after trading |
| $A_H$ | $H$-period annuity factor |
| $A_t$ | age of the investor in period $t$ |
| $\beta$ | annual utility discount factor |
| $b_t$ | number of units of the risky asset with purchase price one, the investor holds during period $t$ |
| $b'_t$ | fraction of beginning-of-period wealth allocated to risk-free bonds after trading |
| $\chi_A$ | characteristic function of $P_t$ which is one if $A$ is true and zero otherwise |
| $c_t$ | consumption-wealth-ratio in period $t$ |
| $C_t$ | consumption during period $t$ |
| $d$ | constant after-tax dividend rate of equity |
| $f$ | fraction of the risky asset the investor has to hold under strategy three, if $L_{t-1} > -M_t$ |
| $f(t)$ | probability of surviving from period $t$ to $t+1$ |
| $F(t)$ | probability that the investor is still alive through period $t$ ($t \leq T$) |
| $\gamma$ | parameter of risk-aversion |
| $g_t$ | capital gain of the stock in period $t$ |
| $G_t$ | net capital gain or loss in period $t$ |
| $H$ | number of periods the investor's beneficiary receives a real payout annuity |
| $i$ | constant inflation rate |
| $J$ | mandatory retirement age |
| $l_t$ | fraction of the investor's loss carryforward to beginning-of-period wealth at the end of period $t$ |
| $L_t$ | loss carryforward that can be carried over from period $t$ to $t+1$ |
| $L_t^{(i)}$ | loss carryforward that can be carried over from period $t$ to $t+1$ when following strategy $i \in \{1,2,3\}$ |
| $\mu$ | annual expected pre-tax capital gain of equity |

| | |
|---|---|
| $m$ | maximum percentage of beginning-of-period wealth qualifying for tax rebates |
| $M$ | maximum amount of losses qualifying for tax rebates |
| $M_t$ | maximum amount of losses qualifying for tax rebates in period $t$ |
| $p_t^*$ | basis-price-ratio in period $t$ after trading |
| $P_t$ | price of the risky asset at the beginning of period $t$ |
| $P_t^*$ | purchase price of the risky asset after trading in period $t$ |
| $P_{t'}$ | price of the risky asset at time $t' \in [t, t+1)$ |
| $q_t$ | number of risky assets the investor holds in period $t$ |
| $r$ | annual risk-free bond return before taxation |
| $r^*$ | after-tax real risk-free return |
| $\rho_t$ | real growth of wealth before capital gains taxes in period $t$ |
| $R$ | annual gross risk-free return after taxation |
| $R_t$ | gross nominal return of the investor's portfolio after trading in period $t$ and payment of taxes on dividends and interest, but before payment of capital gains taxes |
| $\sigma$ | annual standard deviation of the pre-tax capital gains of equity |
| $s_t$ | fraction of the investor's beginning-of-period-$t$ wealth before trading invested in equity |
| $S_t$ | number of units of the risky asset in period $t$ |
| $\tau$ | capital gains tax-rate |
| $t$ | time index |
| $t_t$ | fraction of the investor's beginning-of-period wealth that is taxable at the capital gains tax-rate in period $t$ |
| $T$ | maximum length of the investment horizon |
| $T_t$ | taxable net capital gain in period $t$ |
| $T_t^{(i)}$ | taxable net capital gain in period $t$ when following strategy $i \in \{1, 2, 3\}$ |
| $U(.)$ | utility function |
| $U_t$ | unrealized capital gains or losses at the beginning of period $t$ |
| $v_t(.)$ | normalized value function in period $t$ |
| $V_t(.)$ | value function in period $t$ |
| $W_t$ | beginning-of-period wealth in period $t$ |

| | |
|---|---|
| $W_t^{(i)}$ | beginning-of-period wealth in period $t$ when following strategy $i \in \{1, 2, 3\}$ |
| $x_t$ | vector of normalized state variables in period $t$ |
| $X_t$ | vector of state variables in period $t$ |

# Symbols used in the third chapter

| symbol | interpretation |
|---|---|
| $\alpha_{R,t}$ | fraction of stocks in the tax-deferred account relative to total wealth invested in period $t$ |
| $\alpha_{T,t}$ | fraction of stocks in the taxable account relative to total wealth invested in period $t$ |
| $A_H$ | $H$-period annuity factor |
| $A_t$ | age of the investor in period $t$ |
| $\beta$ | annual utility discount factor |
| $b_t$ | upper bound on contribution to tax-deferred account relative to total beginning-of-period wealth in period $t$ |
| $B_t$ | upper bound on maximum contribution to or withdrawal from tax-deferred account in period $t$ |
| $\chi_A$ | characteristic function which is one if $A$ is true and zero otherwise |
| $c_t$ | consumption-wealth-ratio in period $t$ |
| $C_t$ | consumption during period $t$ |
| $C_{B,u}^{(R)}$ | replication cost in taxable account of one dollar of tax-deferred bonds in tax-system with unlimited capital loss deduction |
| $C_{B,l}^{(R)}$ | replication cost in taxable account of one dollar of tax-deferred bonds in tax-system with limited capital loss deduction |
| $C_{S,u}^{(R)}$ | replication cost in taxable account of one dollar of tax-deferred stocks in tax-system with unlimited capital loss deduction |
| $C_{S,l}^{(R)}$ | replication cost in taxable account of one dollar of tax-deferred stocks in tax-system with limited capital loss deduction |
| $\delta_t$ | capital gain per dollar invested in the taxable account in period $t$ |
| $d$ | constant pre-tax dividend rate of equity |
| $f(t)$ | probability of surviving from period $t$ to $t+1$ |
| $F(t)$ | probability that the investor is still alive through period $t$ ($t \leq T$) |
| $\gamma$ | parameter of risk-aversion |
| $g_t$ | pre-tax capital gains rate of equity in period $t$ |

| | |
|---|---|
| $H$ | number of periods the investor's beneficiary receives a real payout annuity |
| $i$ | annual inflation rate |
| $J$ | mandatory retirement age |
| $\lambda$ | replacement ratio |
| $L(A_t)$ | life expectancy of an investor at age $A_t$ |
| $l_t$ | fraction of the investor's loss carryforward to beginning-of-period wealth at the end of period $t$ |
| $L_t$ | loss carryforward that can be carried over from period $t$ to $t+1$ |
| $\mu$ | annual expected pre-tax capital gain of equity |
| $\mu_{R,t}$ | return per dollar invested in the tax-deferred account in period $t$ |
| $\mu_{T,t}$ | return per dollar invested in the taxable account before capital gains tax payments in period $t$ |
| $m$ | maximum percentage of beginning-of-period wealth qualifying for tax rebates annualy |
| $M$ | maximum amount of losses qualifying for tax rebates annualy |
| $n$ | non-financial income as percentage of beginning-of-period total wealth |
| $N_t$ | non-financial income in period $t$ |
| $p_t^*$ | basis-price-ratio at the end of period $t$ |
| $P_t$ | price of the risky asset at the beginning of period $t$ |
| $P_t^*$ | price of the risky asset at the end of period $t$ |
| $q_{T,t}$ | taxable wealth after transactions relative to beginning-of-period wealth in period $t$ |
| $q_{R,t}$ | tax-deferred wealth after transactions relative to beginning-of-period wealth in period $t$ |
| $Q_{T,t}$ | taxable wealth invested in period $t$ |
| $Q_{R,t}$ | tax-deferred retirement wealth invested in period $t$ |
| $r$ | annual risk-free return before taxation |
| $r^*$ | after-tax real bond return |
| $\rho_t$ | real growth of total wealth before capital gains taxes |
| $R_{T,t}$ | gross return in the taxable account in period $t$ |
| $R_{R,t}$ | gross return of the risky asset in period $t$ |
| $\sigma$ | annual standard deviation of the pre-tax capital gains of equity |

| | |
|---|---|
| $\tau_g$ | capital gains tax-rate |
| $\tau_{g,e}$ | effective capital gains tax-rate |
| $\tau_d$ | tax-rate on dividends and interest |
| $t$ | time index |
| $t_t$ | fraction of the investor's beginning-of-period wealth that is taxable at the capital gains tax-rate in period $t$ |
| $T$ | maximum length of the investment horizon |
| $U(.)$ | utility function |
| $T_t$ | taxable net capital gain in period $t$ |
| $v_t(.)$ | normalized value function in period $t$ |
| $V_t(.)$ | value function in period $t$ |
| $W_{R,t}$ | tax-deferred retirement wealth at the beginning of period $t$ |
| $w_{R,t}$ | fraction of total beginning-of-period wealth in the tax-deferred account in period $t$ |
| $W_{T,t}$ | taxable wealth at the beginning of period $t$ |
| $W_t$ | beginning-of-period wealth in period $t$ |
| $x_t$ | vector of normalized state variables in period $t$ |
| $X_t$ | vector of state variables in period $t$ |
| $z_t$ | contribution-wealth-ratio in period $t$ |
| $Z_t$ | contribution to or withdrawal from tax-deferred account in period $t$ |

# List of Abbreviations

| Abbreviation | Meaning |
|---|---|
| c.p. | ceteris paribus |
| CRRA | constant relative risk aversion |
| e.g. | exempli gratia |
| i.e. | it est |
| RA | retirement account |
| TA | taxable account |
| TDA | tax-deferred account |
| TEA | tax-exempt account |
| US | United States |
| USA | United States of America |

# Einleitung

## Problemstellung

Die Besteuerung von Zinsen, Dividenden und Kursgewinnen privater Anleger hat einen erheblichen Einfluss auf deren Anlageentscheidungen.

Durch die Besteuerung des Wertzuwachses von angelegten Mitteln sinkt der Anreiz, den heutigen Konsum zu Gunsten des zukünftigen Konsums zu reduzieren, da die Prämie für den heutigen Konsumverzicht in Form der Rendite auf die angelegten Mittel geringer ausfällt. Darüber hinaus hat die Besteuerung einen Einfluss auf die optimale Portfoliozusammensetzung privater Anleger, wenn sich die steuerliche Behandlung verschiedener Wertpapiere unterscheidet.

Sowohl in Deutschland als auch in den USA findet eine unterschiedliche steuerliche Behandlung von Erträgen aus Zinsen und Dividenden auf der einen Seite und Erträgen aus Kursgewinnen auf der anderen Seite statt. Insbesondere werden Kursgewinne unter bestimmten Voraussetzungen nach aktuellem Rechtsstand mit einem geringeren Steuersatz belegt als Erträge aus Zinsen und Dividenden. Da verschiedene Anlageklassen, wie etwa festverzinsliche Wertpapiere oder Aktien, zu sehr unterschiedlichen Teilen ihre Erträge aus diesen beiden Quellen beziehen, werden somit auch diese Anlageklassen steuerlich ungleich behandelt.

Die zentralen Kernergebnisse dieser Arbeit werden auf Basis eines Steuersystems erarbeitet, welches in Grundzügen dem US-amerikanischen nachempfunden wurde. Da sich die US-amerikanischen Regelungen und die Regelungen des deutschen Steuerrechts bezüglich der steuerlichen Behandlung von Anlageerfolgen in ihren Grundzügen stark ähneln, lassen sich viele zentrale Kernaussagen auf Deutschland übertragen. Zu der Bedeutung etwaiger Unterschiede wird in den Anhängen zu den Kapiteln 2 und 3 kurz eingegangen.

Nimmt man an, dass die gemeinsame Verteilung der einzelnen Ertragsbestandteile der verschiedenen Wertpapiere bekannt ist und für die beiden Ertragsbestandteile konstante Steuersätze gelten, so lässt sich unter Umständen die optimale Anlageentscheidung unter Berücksichtigung von Steuern in einem Modell ohne Steuern berechnen, indem die gemeinsame Verteilung entsprechend angepasst wird. Jedoch kennt sowohl das deutsche als auch das US-amerikanische Steuerrecht einige besondere Regelungen, die eine Rückführung des Problems mit Steuern auf ein Problem ohne Steuern unmöglich machen und damit eine explizite Modellierung der steuerlichen Regelungen erfordern. Im Rahmen dieser Arbeit wird neben der Berücksichtigung von Steuersätzen unterschiedlicher Höhe für Kursgewinne und Zinsen/Dividenden der Einfluss der folgenden für Privatanleger besonders bedeutenden Regelungen untersucht:

- Kursgewinne werden im Vergleich zu Erträgen aus Dividenden und Zinsen nicht bei Entstehen, sondern erst bei ihrer Realisation versteuert. Dies bietet dem Anleger die Möglichkeit, den Zeitpunkt der Besteuerung durch Wahl des Veräußerungszeitpunktes selbst zu wählen (so genannte tax-timing Option) und bei Nichtveräußerung unter Ausnutzung des Zins- und Zinseszinseffektes noch Erträge auf die noch nicht geleisteten Steuerzahlungen zu erwirtschaften.

- Mittel, die in speziellen Altersvorsorgekonten oder -verträgen, wie etwa den deutschen Riesterverträgen oder den US-amerikanischen Individual Retirement Accounts bespart werden, genießen eine steuerliche Sonderbehandlung. In der Regel kommt es im Rahmen von Altersvorsorgekonten oder -verträgen zu einer so genannten nachgelagerten Besteuerung, die im Wesentlichen darin besteht, dass die Einzahlung aus unversteuertem Einkommen erfolgt, die Entnahme hingegen der Einkommensteuer unterliegt. Weiterhin sind Einzahlungen in solche Konten begrenzt, während Entnahmen nach bestimmten Regelungen erfolgen müssen, vor Eintritt des Rentenalters grundsätzlich steuerlich sanktioniert werden und Erträge, die auf einem solchen Konto erwirtschaftet werden, steuerfrei sind.[1] Durch die Steuerfreiheit der Erträge hat also nicht nur die Wahl des Anlegers, welche Wertpapiere er besparen möchte (asset

[1] Für eine detaillierte Unterscheidung der unterschiedlichen Typen von Altersvorsorgekonten und weitere Einzelheiten der steuerlichen Behandlung von Mitteln in diesen Konten sei auf Kapitel 1 verwiesen.

allocation), sondern auch die Frage, in welchem Kontentyp er welche Wertpapiere besparen möchte (asset location), Einfluss auf die Entwicklung seines Vermögens.

- Realisierte Kursgewinne und Kursverluste werden steuerlich unterschiedlich behandelt. Während in einem Jahr realisierte Kursgewinne grundsätzlich voll steuerpflichtig sind und den Anlageerfolg mindern, werden realisierte Kursverluste nicht unbegrenzt steuerpflichtig. Es findet also keine oder nur eine beschränkte Steuererstattung für erlittene Kursverluste statt.[2] Stattdessen wird ein Verlustvortrag eingeräumt, der in zukünftigen Perioden mit etwaigen realisierten Kursgewinnen verrechnet werden kann und die zukünftige Steuerbelastung mindert.

  Im Gegensatz zu Steuererstattungen weist ein Verlustvortrag zwei wesentliche Nachteile für den Anleger auf. Zum einen bleibt ein solcher Verlustvortrag im Gegensatz zu Steuererstattungen, die wieder angelegt werden können, unverzinst. Zum anderen hat ein Verlustvortrag nur dann für den Anleger einen Wert, wenn er diesen Verlustvortrag einsetzen und zu Geld machen kann, also in zukünftigen Perioden mit realisierten Kursgewinnen verrechnen kann oder in Steuersystemen, die Steuererstattungen für erlittene Kursverluste gewähren, in zukünftigen Perioden als erlittenen Kursverlust deklarieren kann, um Steuerrückerstattungen zu erhalten. Während im deutschen Steuersystem grundsätzlich für Privatpersonen keine Steuererstattungen für erlittene Kursverluste vorgesehen sind, gewährt das US-amerikanische Steuerrecht für Kursverluste, die den Betrag von $ 3.000 im Jahr nicht übersteigen, die Möglichkeit, diese mit anderem Einkommen zu verrechnen.

Während die Bedeutung des tax-timing[3] und die Bedeutung von Altersvorsorgekonten und -verträgen[4] bereits in verschiedenen Arbeiten untersucht worden sind,[5] wurde der Bedeutung der asymmetrischen Behandlung von realisierten Kursgewin-

[2] Durch eine solche Regelung soll sichergestellt werden, dass das Steueraufkommen in Zeiten schlechter Börsenentwicklung nicht zu stark sinkt.

[3] Siehe etwa Constantinides (1983, 1984), Dammon et al. (1989), Dammon und Spatt (1996), Dammon et al. (2001, 2004), Chay et al. (2006) und Gallmeyer et al. (2006).

[4] Siehe etwa Shoven und Sialm (1998, 2003), Dammon et al. (2004), Poterba et al. (2004), Huang (2007), Garlappi und Huang (2006), Gomes et al. (2006) und Zaman (2007).

[5] Die Arbeit von Dammon et al. (2004) vereint sogar beide Ansätze und untersucht simultan den Einfluss von tax-timing und Altersvorsorgekonten.

nen und -verlusten noch keine große Beachtung geschenkt. Deren Untersuchung stellt einen Schwerpunkt der vorliegenden Arbeit dar.

Die vorliegende Dissertationsschrift besteht aus in drei englischsprachigen Aufsätzen. Das Kapitel 1 enthält die Ergebnisse des gemeinsam mit Prof. Dr. Raimond Maurer durchgeführten Forschungsprojekts "How Unobservable Bond Positions in Retirement Accounts Affect Asset Allocation". Der Artikel "Optimal Tax-Timing and Asset Allocation when Tax Rebates on Capital Losses are Limited" bildet das Kapitel 2. Das Kapitel 3 enthält den Aufsatz "Are Bonds Desirable in Tax-Deferred Accounts?". Im nachfolgenden Abschnitt wird auf die wichtigsten Ergebnisse der einzelnen Beiträge eingegangen.

# Einordnung der Ergebnisse

Ziel des Artikels "How unobservable Bond Positions in Retirement Accounts affect Asset Allocation", welches das Kapitel 1 der vorliegenden Arbeit bildet, ist es, die Bedeutung der Möglichkeit in ein steuerbefreites Altersvorsorgekonto anlegen zu können, für die asset allocation eines privaten Anlegers zu analysieren. Im Abschnitt "Tax-Effects in Retirement Accounts", der das Herzstück darstellt, wird gezeigt, dass sich die Entwicklung des zu einem bestimmten Zeitpunkt in einem gewöhnlichen und einem Altersvorsorgekonto angelegten Vermögens in zwei Summanden zerlegen lässt. Der erste Summand beschreibt die Entwicklung des Vermögens, das der Anleger ohne ein steuerbefreites Altersvorsorgekontos erzielt hätte. Der zweite Summand beschreibt den Teil des Endvermögens, der auf die Steuerfreiheit der Erträge im Altersvorsorgekonto zurückzuführen ist.

Der Effekt, der aus der Steuerfreiheit der Erträge resultiert, lässt sich weiter zerlegen in einen direkten und einen indirekten Steuereffekt. Während der direkte Steuereffekt den Effekt der Erhöhung des Gesamtvermögens aus einer Anlageentscheidung misst, der aus der Steuerfreiheit der Erträge in der Entscheidungsperiode resultiert, misst der indirekte Steuereffekt den Einfluss, der durch die Veränderung des Altersvorsorgevermögens entsteht, welches in zukünftigen Perioden die Generierung steuerbefreiter Erträge erlaubt. Insbesondere die Bedeutung des indirekten Steuereffekts nimmt mit zunehmender Länge des verbleibenden Anlagehorizonts aufgrund des Zinseszinseffekts stark zu.

Unter den in der Literatur üblichen Annahmen an das Steuersystem zeigt Huang (2007) mit Hilfe eines Replikationsarguments, dass im Altersvorsorgekonto bevorzugt risikofreie Wertpapiere gehalten werden sollten. Halten private Anleger in ihrem Altersvorsorgekonto bevorzugt risikolose Wertpapiere, so werfen diese aufgrund der Steuerfreiheit der Erträge im Altersvorsorgekonto höhere Erträge ab als sie dies im konventionellen Depot tun würden. Ein risikoloses Wertpapier im Altersvorsorgekonto hat also einen höheren Einfluss auf das zukünftige Vermögen als ein risikoloses Wertpapier im konventionellen Depot. Der Anleger hält also gewissermaßen "unobservable bonds" in Form gegenwärtiger und zukünftiger Überrenditen aus der Steuerfreiheit seiner Erträge der festverzinslichen Wertpapiere im Altersvorsorgekonto. Da der Einfluss der Steuerfreiheit der Erträge auf das Gesamtvermögen mit sinkendem Anlagehorizont abnimmt, muss der Anleger seinen Anteil des risikofreien Wertpapiers im Laufe des Lebenszyklus sukzessive erhöhen und folglich seinen Aktienanteil sukzessive senken, wenn er sein Endvermögen nicht mit steigendem Alter höheren Risiken aussetzen möchte.

Der Artikel "Optimal Tax-Timing and Asset Allocation when Tax Rebates on Capital Losses are Limited" bildet das Kapitel 2 dieser Dissertationsschrift. Im Mittelpunkt der Betrachtung steht hier die Untersuchung optimaler tax-timing Strategien für Anleger, die in Steuersystemen handeln, welche Steuererstattungen für realisierte Kursverluste begrenzen. Eine solche Begrenzung gibt es sowohl im deutschen als auch im US-amerikanischen Steuersystem.

Im analytischen Teil wird in Erweiterung von Constantinides (1983) gezeigt, dass es auch in solchen Steuersystemen optimal bleibt, erlittene Kursverluste sofort zu realisieren, obwohl die Kompensation für die erlittenen Verluste in Form eines sich nicht verzinsenden Verlustvortrags deutlich unattraktiver ausfällt als eine Steuererstattung in einem Steuersystem mit symmetrischer Behandlung von realisierten Kursgewinnen und -verlusten wie bei Constantinides.

Im numerischen Teil wird in Erweiterung der Arbeit von Dammon et al. (2001) gezeigt, dass die asymmetrische Behandlung von Kursgewinnen und -verlusten grundsätzlich zu einer deutlich geringeren Aktienquote führt. Dies liegt daran, dass das Rendite-Risiko-Profil von Aktien deutlich unattraktiver wird, da ein Verlustvortrag einen deutlich geringeren Nutzen mit sich bringt als eine Steuererstattung. Verfügt ein Anleger hingegen bereits über einen Verlustvortrag, so führt dies in Steuersystemen ohne Steuerstattungen für realisierte Kursverluste dazu, dass der Anleger seine Aktienquote wieder geringfügig erhöht, da der Verlustvortrag dem Anleger erlaubt, zukünftige Kursgewinne teilweise steuerfrei zu vereinnahmen, was sein Rendite-Risiko-Profil wiederum verbessert.

Kapitel 3 ist der mit Abstand jüngste Aufsatz der vorliegenden Arbeit und untersucht die Bedeutung der asymmetrischen Behandlung von realisierten Kursgewinnen und -verlusten für die asset location Entscheidung privater Anleger. Während die existierende Literatur fast übereinstimmend feststellt, dass risikofreie festverzinsliche Wertpapiere in Altersvorsorgekonten bespart werden sollten,[6] halten private Anleger in der Realität nicht unerhebliche Aktienquoten in ihren Altersvorsorgekonten. Diese Diskrepanz zwischen den empirisch beobachtbaren Anlagestrategien privater Anleger und den theoretischen Ergebnissen der Literatur wird von Amromin (2003) auch als Asset Location Puzzle bezeichnet.

Kapitel 3 zeigt, dass dieses Puzzle durch die Einführung einer asymmetrischen Behandlung von Kursgewinnen und -verlusten erklärt werden kann. Dies liegt im Wesentlichen daran, dass das Replikationsargument von Huang (2007), auf dem die Dominanz des risikofreien Wertpapiers basiert, aufgrund der asymmetrischen Behandlung von Kursgewinnen und -verlusten nicht mehr anwendbar ist. Wenn man auf positive oder negative Kursgewinne bedingt, also annimmt, dass bekannt ist, ob in der betrachteten Periode Kursgewinne oder -verluste erzielt werden, bleiben risikofreie festverzinsliche Wertpapiere das bevorzugte Wertpapier im Altersvorsorgekonto. Ist das Vorzeichen der Kursgewinne der Periode jedoch nicht zu Beginn dieser Periode messbar, so bleibt dieses Ergebnis nicht gültig, da sich die Replikationsportfolios für positive und negative Kursgewinne voneinander unterscheiden. Somit bleibt ein Replikationsrisiko. Um die Auswirkungen dieses Risikos gering zu halten, kann es sinnvoll sein, Aktien sowohl im konventionellen als auch im Alters-

[6] Arbeiten, die dieser verbreiteten Ansicht nicht folgen, sind die von Shoven (1999), Garlappi und Huang (2006) und Zhou (2007). Die Arbeit von Shoven (1999) basiert auf historischen Renditen und zeigt, dass es historisch für eine Einmalanlage günstiger war, Aktien im Altersvorsorgekonto zu besparen. Allerdings vernachlässigt Shoven in seiner Analyse das höhere Risiko, welches mit einem Aktieninvestment im Altersvorsorgekonto verbunden ist. Garlappi und Huang (2006) zeigen in einem Zweiperiodenmodell mit binomialverteilten Renditen für einen Anleger mit Leerverkaufsrestriktionen und logarithmischer Nutzenfunktion, dass es optimal sein kann, auch im Altersvorsorgekonto einen Aktienanteil zu halten. Zhou (2007) argumentiert, die optimale Allokation von Aktien hänge davon ab, wie oft Gewinne im konventionellen Konto realisiert werden. In ihrer Studie ignoriert sie dabei jedoch Diversifikationsgesichtspunkte, welche, wie etwa Dammon et al. (2001) und die Ergebnisse von Kapitel 2 zeigen, von enormer Bedeutung sind. Zhou betrachtet den Anteil der realisieren Kursgewinne als exogen.

vorsorgekonto zu besparen.

Während für positive Kursgewinne die Rendite von Aktien im Altersvorsorgekonto höher ist und bei entsprechender Höhe den Steuervorteil von risikofreien festverzinslichen Wertpapieren überkompensieren kann, sind Aktien bei negativen Kursgewinnen im konventionellen Konto aufgrund des dort gewährten Verlustvortrags besser aufgehoben. Unter Diversifikationsgesichtspunkten bietet es sich deshalb an, Aktien sowohl im konventionellen als auch im Altersvorsorgekonto zu besparen.

Dieses Ergebnis steht im direkten Gegensatz zur bisherigen asset location Literatur, die in Steuersystemen mit symmetrischer Behandlung von Kursgewinnen und -verlusten arbeitet und grundsätzlich risikofreie festverzinsliche Wertpapiere im Altersvorsorgekonto empfiehlt. Insbesondere lässt sich also mit Hilfe einer unterschiedlichen steuerlichen Behandlung von Kursgewinnen und -verlusten das Asset Location Puzzle erklären.

# 1 How Unobservable Bond Positions in Retirement Accounts Affect Asset Allocation

This paper is the result of a joint project with Raimond Maurer.

First draft: March 2006

This version: June 2007

Until July 2, 2007, this paper has been accepted for presentation at the following conferences:

- 11th international congress on Insurance: Mathematics and Economics (IME), July 10-12, 2007, Piraeus, Greece
- 34th Meeting of the European Finance Association (EFA), August 22-25, 2007, Ljubljana, Slovenia
- Jahrestagung Verein für Socialpolitik, October 9-12, 2007, Munich, Germany,
- Southern Finance Association (SFA) Annual Meeting 2007, November 14-17, 2007, Charleston, South Carolina, USA

Abstract

Many tax-codes around the world allow for special taxable treatment of savings in retirement accounts. In particular, profits in retirement accounts are usually tax exempt which allow investors to increase an asset's return by holding it in such a retirement account. While the existing literature on asset location shows that risk-free bonds are usually the preferred asset to hold in a retirement account, we explain how the tax exemption of profits in retirement accounts affects private investors' asset allocation. We show that total final wealth can be decomposed into what the investor would have earned in a taxable account and what is due to the tax exemption of profits in the retirement account. The tax exemption of profits can thus be considered a tax-gift which is similar to an implicit bond holding. As this tax-gift's impact on total final wealth decreases over time, so does the investor's equity exposure.

**JEL Classification Codes:** G11, H24

**Key Words:** asset location, asset allocation, tax-deferred accounts, tax exempt accounts

## 1.1 Introduction

Profits from dividends, interest and capital gains are usually subject to taxation. In tax-sheltered retirement accounts, however, profits remain untaxed which allows the return of an asset to increase from its after-tax to its pre-tax return by holding it in such a tax-sheltered retirement account. This paper explains optimal asset allocation (i.e. which assets to hold) and asset location (i.e. in which account to hold these assets) for investors having the opportunity to invest in both taxable and retirement accounts. In particular, it explains why the equity exposure in both taxable as well as retirement wealth decreases with a decreasing investment horizon. It is most closely related to the recent literature on optimal asset location decisions including Shoven and Sialm (1998), Shoven and Sialm (2003), Dammon et al. (2004), Poterba et al. (2004), Huang (2007) and Garlappi and Huang (2006). These papers conclude that assets facing a high tax-burden should generally be located in retirement accounts, while assets facing a low tax-burden are better located in conventional taxable accounts. Garlappi and Huang (2006), however, show that this finding does not hold in general.

While, in general, the existing literature shows that risk-free bonds are the preferred asset to hold in the retirement account, we explain how the investor's equity exposure in both the taxable and the retirement account depends on the length of the remaining investment horizon and the fraction of total wealth in the retirement account. Due to the tax exemption of profits in the retirement account, total wealth contains a tax-gift which becomes bigger as the remaining investment horizon increases. As bonds are usually the preferred asset to hold in a retirement account the tax-gift can be considered an implicit bond holding that is not directly observable in security accounts. As this tax-gift decreases with decreasing length of the remaining investment horizon, so does the unobservable bond holding and the observable equity exposure.

The paper proceeds as follows. Section 1.2 reviews the related literature. Section 1.3 introduces the model and discusses the tax-effects of investments into tax-sheltered retirement accounts. Section 1.4 provides numerical evidence. Section 1.5 concludes.

| Taxation of | TA | TEA | TDA |
|---|---|---|---|
| Contribution | X | X | |
| Profits | X | | |
| Withdrawal | | | X |

Table 1.1: Taxable treatment in different accounts

## 1.2 Prior Studies

Dilnot (1995) categorizes retirement accounts according to the taxable treatment of contributions, profits and withdrawals. Even though theoretically, there are many combinations of taxable treatments, in practice there are only three main types of retirement accounts to be found. The so-called taxable accounts (TAs) are accounts in which contributions can only be made from after-income-tax dollars, and where profits are taxable and withdrawals are tax exempt. Tax-deferred accounts (TDAs) such as IRAs are characterized by the opportunity to contribute to them from pre-income-tax dollars, and withdrawals are taxable while profits are tax exempt. In tax exempt accounts (TEAs) like Roth-IRAs, contributions are made from after income tax dollars, profits are tax exempt, and withdrawals are usually tax-exempt. In many countries, different kinds of accounts coexist with different taxable treatment. Whereas TAs can be used for other investment objectives besides saving for retirement, TEAs and TDAs are pure retirement accounts. Early withdrawals from them are subject to a penalty tax and contributions to them are limited by law. As only TAs are flexible enough to be used as saving accounts for other investment objectives as well, only TEAs and TDAs are referred to as retirement accounts (RAs) in this article. Due to the differing taxable treatment of the funds in the various accounts, investing a pre-tax dollar in each of the separate types of accounts usually results in varying risk-return profiles of that dollar and different changes in the investor's wealth. Table 1.1 summarizes the taxable treatment of the three different types of accounts.

Although optimal asset allocation is a topic that has been intensively discussed in economic literature, research on optimal asset location is quite a recent field of research. It goes back to studies of Tepper and Affleck (1974), Black (1980) and Tepper (1981). They analyze optimal investment strategies of companies that run defined-benefit pension plans. If these companies do not face any short-selling restrictions and their gains are fully taxable at the moment of occurrence (i.e. there is no deferral option), they should hold bonds only in their defined-benefit pension plan, where the taxable treatment is similar to that of TEAs. Auerbach and King (1983) point out that this result also applies to investors having the opportunity to invest into a retirement account and a TA.

Shoven and Sialm (1998) and Shoven (1999) introduce the problem of asset location to household investment decisions and point out that each choice of an investment strategy for retirement saving does not only contain a choice about the assets to invest in (the so-called asset allocation problem), but also a choice about which of these assets to locate in a retirement account and which to locate in a taxable account (the so-called asset location problem). In their studies, they analyze asset location and asset allocation decisions for simulated distributions of wealth. They arrive at the conclusion that due to tax-inefficiency of many actively managed mutual funds, these funds have their preferred location in a retirement account, while tax exempt bonds should be held in a taxable account.

Bodie and Crane (1997), Poterba and Samwick (2001) and Barber and Odean (2003) analyze asset location strategies used by private investors in practice. They report that investors do not realize the opportunities that TDAs offer to them. In particular, they often choose suboptimal asset location strategies. Contribution rates have been found to have an especially substantial impact on utility losses (Gomes et al. (2006)). Amromin (2002) and Bergstresser and Poterba (2004) report similar findings and further point out that many investors have considerable amounts of money in both accounts and hold significantly more stocks in their TDAs than in their TAs. Benartzi and Thaler (2001) point out that investors tend to follow naive $\frac{1}{n}$-diversification strategies in their retirement accounts and pension plans and thus the more stock funds offered to them by these plans the more stocks investors tend to hold.

According to Gale and Scholz (1994), the majority of investors in the US that contribute to an individual retirement account are either older than 59 or have substantial funds in a TA in addition to the funds in the IRA. Either of these cases makes the need for an early withdrawal - which is accompanied by a penalty tax - quite unlikely. This is why we put the focus of our analysis on the asset location and asset allocation decisions induced by the tax-effects and ignore early withdrawals.

A dollar invested into a risk-free asset in an RA results in a higher after-tax yield than a dollar invested into the same risk-free asset in a TA. Hence, the dollar in the RA is worth more than the dollar in the TA, a fact which has been pointed out by Dammon et al. (2004) and Poterba (2004). Dammon et al. (2004) call the difference between the value of the dollar in the TEA and the value of the dollar in the TA the "shadow price" of that dollar. This "shadow price" depends on the relative dividend of the assets, the relative capital gains of the assets, the level of tax-rates on these dividends and gains, and the length of the remaining investment horizon. Due to the penalty tax for early withdrawal and the maximum contribution limits for TEAs and TDAs, TAs are often used for retirement saving as well. Hence, private investors saving for retirement usually only locate some part of their retirement savings in a retirement account. Due to the difference in taxable treatment of assets between a taxable account and a retirement account, it is important to make an informed decision as to which asset to locate in the retirement account and which to locate in the taxable account.

The impact of tax-timing strategies on optimal asset location decisions is analyzed in Dammon et al. (2004) and Zaman (2007). In Zaman's numerical study, stocks tend to have their preferred location in the taxable account to use potential benefits from tax-timing and exploit the higher tax-burden of bonds. This result does not differ from that in Dammon et al. (2004), whose analysis contains only one risky asset. This finding suggests that it suffices to consider one risky asset in our numerical analysis.

Furthermore, (long-term) capital gains are often subject to a lower tax-rate than dividends and interest. This is why the returns on stocks that mainly consist of capital gains tend to be taxed at a lower rate than the returns on bonds mainly consisting of interest payments. For this reason, Shoven and Sialm (2003) argue that bonds should have their preferred location in a retirement account and stocks shall only be held there if no bonds are held in the taxable account at all. This argument seems to be very convincing at first sight and has been shown to be correct for investors that are not facing short-selling restrictions (see e.g. Dammon et al. (2004), Shoven and Sialm (2003) or the theoretical paper of Huang (2007)). However, Garlappi and Huang (2006) show that this does not necessarily hold if the investor is short-selling constrained.

If investors are facing high labor income risk and have limited liquid financial resources, some papers, among them Amromin (2003), Dammon et al. (2004) and Amromin (2005), argue that holding stocks in the retirement account and bonds in the taxable account can also be an efficient investment strategy when taking labor income shocks into account. They base their argument on the fact that due to the lower volatility of bonds, holding them in the taxable account reduces the probability of having to withdraw funds from the retirement account and to pay the penalty tax. The higher the probability and the order of magnitude of income shocks and the lower the liquid financial resources, the better the strategy to hold sufficient TA-wealth in bonds. Besides, under current law there seem to be tax arbitrage opportunities between savings in retirement accounts and mortgage payments (Amromin et al. (2007)). The literature on taxation and optimal portfolio choice is surveyed in Poterba (2002b) and Campbell (2006).

While the focus of the existing literature is on the asset location decision, we concentrate on the impact of the tax-effects in retirement accounts for the investor's asset allocation decision.

## 1.3 Tax-Effects in Retirement Accounts

### 1.3.1 Effects of Tax Deferral

In this article it is assumed that dividends, interest, and capital gains in the TA are taxable at occurrence. That is, no matter if realized or not, capital gains are fully taxable, i.e. there is no tax-timing option. Equivalently one could also assume the corresponding tax-rate to be the "effective tax-rate" as described in Constantinides (1983), which reflects the unmodeled optimal tax realization strategy in the risky asset. It is further assumed that the investor cannot go short and there is no need for an early withdrawal from a retirement account. This in particular implies that the investor does not have to pay the penalty tax for early withdrawal at any point in time. A market with $n$ assets is considered. Assets can be traded without incurring any transaction costs.

Let $\alpha_{l,i,t}$ denote the proportion of asset $i$ in period $t$ with location $l \in \{R, T\}$, in which $R$ denotes the location in a retirement account (TDA or TEA) and $T$ denotes a location in the taxable account. Let $\boldsymbol{\alpha}_{l,t} \equiv (\alpha_{l,1,t}, \ldots, \alpha_{l,n,t})^\top$ be the vector of proportions of the $n$ assets in period $t$ with location $l$. $0 \leq \tau_d < 1$ denotes the constant tax-rate on dividends and interest, and $0 \leq \tau_g < 1$ the constant tax-rate on capital gains. In particular, it is assumed that short-term and long-term capital gains are subject to the same tax-rate. $0 \leq d_{R,i,t}$ and $-1 \leq g_{R,i,t}$ are the dividend or interest rate (dividend rate in this article) and the capital gains rate, respectively, for asset $i$ in period $t$ in the retirement account (and thus on a pre-tax basis). $d_{T,i,t} \equiv (1 - \tau_d)d_{R,i,t}$ and $g_{T,i,t} \equiv (1 - \tau_g)g_{R,i,t}$ denote the dividend rate and the capital gains rate of asset $i$ in period $t$ in the TA (and thus on an after-tax basis). $\mathbf{d}_{l,t} \equiv (d_{l,1,t}, \ldots, d_{l,n,t})^\top$ denotes the vector of dividend rates in period $t$ with location $l$ and $\mathbf{g}_{l,t} \equiv (g_{l,1,t}, \ldots, g_{l,n,t})^\top$ denotes the vector of capital gains rates in period $t$ with location $l$. $0 \leq \tau_{p,t} < 1$ denotes an exogenously given personal income tax-rate of the investor in period $t$. $\mathbf{1}$ denotes a column vector of $n$ ones. The vector of gross returns $\mathbf{R}_{l,t}$ in period $t$ for assets with location $l$ is given by $\mathbf{R}_{l,t} \equiv \mathbf{1} + \mathbf{d}_{l,t} + \mathbf{g}_{l,t}$. $W_{T,t}$ denotes the wealth of some investor in a TA and $W_{R,t}$ the wealth of that investor in a RA at the end of period $t$.

Tax-deferred accounts allow deferring income taxation until withdrawal. That means contributions to such tax-deferred accounts are made from pretax-income. This is in contrast to taxable accounts and tax exempt accounts, where contributions can only be made from after-tax dollars. Investors facing a high marginal income tax rate at the time of contribution, but expecting a lower personal income tax-rate at the time of withdrawal can lower their expected relative income tax burden by an investment in a TDA.

Assume the investor is initially endowed with $W_{T,0}$ dollars in a TA and $W_{R,0}$ dollars in a TEA. For simplicity, she is not allowed to shift funds between the two accounts and maximizes utility over a $T$-period investment horizon from total final wealth $W_T \equiv W_{T,T} + W_{R,T}$.

The investment decision of the investor can be decomposed into two parts. On the one hand, the investor has to decide which assets to hold. This problem is known as the asset allocation problem. On the other hand, she has to decide which of the assets to hold in the retirement account and which in the taxable account. This problem is known as the asset location problem. In the remainder of this paper an asset allocation for a $T$-period investment horizon is defined to be a tuple $(\boldsymbol{\alpha}_1, \ldots, \boldsymbol{\alpha}_T)$ of vectors $\boldsymbol{\alpha}_t$ $(t \in \mathbb{N}_T \equiv \{n \in \mathbb{N} | n \leq T\})$, which contain the proportions of the assets relative to total wealth $W_t$. An asset location for a $T$-period investment horizon with investment opportunities in a retirement account and a TA is a tuple $(L_1, \ldots, L_T)$ of tuples $L_t = (\boldsymbol{\alpha}_{R,t}, \boldsymbol{\alpha}_{T,t})$ $(t \in \mathbb{N}_T)$, such that $\boldsymbol{\alpha}_t = \frac{W_{T,t-1}}{W_{t-1}} \boldsymbol{\alpha}_{T,t} + \frac{W_{R,t-1}}{W_{t-1}} \boldsymbol{\alpha}_{R,t}$ $(t \in \mathbb{N}_T)$. When returns are stochastic, $W_{T,t}$, $W_{R,t}$, and $W_t$ are not known before the end of period $t$, which is why $\boldsymbol{\alpha}_{T,t+1}$,$\boldsymbol{\alpha}_{R,t+1}$, and $\boldsymbol{\alpha}_{t+1}$ also cannot be determined before the end of period $t$, that is at the beginning of period $t+1$. The tuple $(I_1, \ldots, I_T)$ of tuples $I_t = (\boldsymbol{\alpha}_t, \boldsymbol{\alpha}_{R,t}, \boldsymbol{\alpha}_{T,t})$ $(t \in \mathbb{N}_T)$ is called an investment strategy. While the asset allocation $(\boldsymbol{\alpha}_1, \ldots, \boldsymbol{\alpha}_T)$ provides information about proportions of the asset relative to total wealth, the asset location $(L_1, \ldots, L_T)$ provides information about the accounts in which these assets are held and with which proportions.

If one pre-income-tax dollar at the end of period 0 is invested in a TDA for a $T$-period investment horizon and $x \cdot y$ denotes the Euclidian scalar product of two $n$-dimensional vectors $x$ and $y$, total final wealth after income tax is given by

$$\prod_{t=1}^{T} \Big( \boldsymbol{\alpha}_{R,t} \cdot (\mathbf{1} + \mathbf{g}_{R,t} + \mathbf{d}_{R,t}) \Big) (1 - \tau_{p,T}) = \prod_{t=1}^{T} (\boldsymbol{\alpha}_{R,t} \cdot \mathbf{R}_{T,t}) (1 - \tau_{p,T}) \, . \tag{1.1}$$

If one invests one such dollar for a $T$-period investment horizon in a TEA, income taxation is already due at the time of contribution and one ends up with a final wealth after income tax of

$$(1 - \tau_{p,0}) \prod_{t=1}^{T} \Big( \boldsymbol{\alpha}_{R,t} \cdot (\mathbf{1} + \mathbf{g}_{R,t} + \mathbf{d}_{R,t}) \Big) = (1 - \tau_{p,0}) \prod_{t=1}^{T} (\boldsymbol{\alpha}_{R,t} \cdot \mathbf{R}_{T,t}) \, . \tag{1.2}$$

The only difference in total final after income tax wealth of an investment in a TDA and a TEA is the point in time at which the investor has to pay the income tax. As $\tau_{p,0}$ might differ from $\tau_{p,T}$, an investment in a TDA might result in different final wealth than an investment in a TEA. An investment in a TDA offers the opportunity to face an income tax-rate at withdrawal $\tau_{p,T}$ that is lower than the income tax-rate at contribution $\tau_{p,0}$, but bears the risk that $\tau_{p,T}$ is higher than $\tau_{p,0}$. If contributions to a TEA are made from income that has already been subject to income taxation, the factor $1 - \tau_{p,0}$ is to be omitted.

If one invests one pre-income-tax dollar in a TA for a $T$-period investment horizon, total final after income tax wealth is given by

$$(1-\tau_{p,0})\prod_{t=1}^{T}\Big(\boldsymbol{\alpha}_{T,t}\cdot\big(\mathbf{1}+(1-\tau_g)\,\mathbf{g}_{R,t}+(1-\tau_d)\,\mathbf{d}_{R,t}\big)\Big)=(1-\tau_{p,0})\prod_{t=1}^{T}(\boldsymbol{\alpha}_{T,t}\cdot\mathbf{R}_{T,t}). \tag{1.3}$$

Total final wealth after income tax of an investment in a TA and a TEA with the same proportions of the assets $\boldsymbol{\alpha}_{R,t}=\boldsymbol{\alpha}_{T,t}\;\forall t\in\mathbb{N}_T$ only differs in the rates of return. In the TA, dividend rates and capital gains rates shrink towards zero by the factor $1-\tau_d$ and $1-\tau_g$, respectively. If one invests in assets whose dividend rates and capital gains rates cannot become negative, an investment in a TEA is at least as good as an investment into a TA. If $\tau_{p,0}\geq\tau_{p,T}$, this also holds for a comparison between an investment in a TA and a TDA. If however $\tau_{p,T}>\tau_{p,0}$, an investment in a TA does not necessarily dominate an investment in a TDA, because for investment horizons of sufficient length, the tax exemption of profits in the TDA can outweigh the tax-burden of the higher income tax-rate.

### 1.3.2 Effects of Tax Exemption of Profits in Retirement Accounts

Saving for retirement is usually a process lasting several decades, where the effect of tax exemption on profits in TDAs and TEAs becomes of increasing importance due to the compounding of interest and the length of the investment horizon. When combined with the effect of shrinking returns on total final wealth, the tax exemption of profits in retirement accounts can be considered a public contribution to private retirement saving. This contribution does not come directly in the form of a payment to the retirement account, but indirectly as what can be called a tax-gift. For an investor who can invest in a TA and a TEA total final wealth is given by

$$W_T=W_{T,0}\prod_{t=1}^{T}(\boldsymbol{\alpha}_{T,t}\cdot\mathbf{R}_{T,t})+W_{R,0}\prod_{t=1}^{T}(\boldsymbol{\alpha}_{R,t}\cdot\mathbf{R}_{R,t})\,. \tag{1.4}$$

For positive returns and $\boldsymbol{\alpha}_{T,t}=\boldsymbol{\alpha}_{R,t}$ $(t\in\mathbb{N}_T)$, growth of wealth in the TEA is higher than in the TA due to the tax exemption of profits. Thus, the longer the investment horizon the stronger the impact of wealth in the TEA on total final wealth. The tax advantage that results from the tax exemption of profits in the TEA in period $t$ is given by

$$T_t \equiv \boldsymbol{\alpha}_{R,t} \cdot (\mathbf{R}_{R,t} - \mathbf{R}_{T,t}) = \boldsymbol{\alpha}_{R,t} \cdot (\tau_d \mathbf{d}_{R,t} + \tau_g \mathbf{g}_{R,t}) \,. \tag{1.5}$$

$T_t$ can be interpreted as a relative tax-gift in period $t$ that is paid for each dollar invested into a TEA. Besides the vector of pre-tax and after-tax returns on the assets and the tax-rates on gains and dividends, the relative tax-gift depends in particular on the choice of the proportions of the assets $\boldsymbol{\alpha}_{R,t}$ in the TEA in period $t$. Equation (1.4) can be rewritten as

$$W_T = (W_{T,0} + W_{R,0}) \prod_{t=1}^{T} (\boldsymbol{\alpha}_t \cdot \mathbf{R}_{T,t}) + \sum_{t=1}^{T} W_{R,0} \prod_{j=1}^{t-1} (\boldsymbol{\alpha}_{R,j} \cdot \mathbf{R}_{R,j}) \, T_t \prod_{j=t+1}^{T} (\boldsymbol{\alpha}_j \cdot \mathbf{R}_{T,j}) \,. \tag{1.6}$$

Total final wealth after income tax $W_T$ at the end of period $T$, thus consists of two components. The first summmand is total final wealth an investor would have attained without having had the opportunity to invest in a TEA and is thus driven to invest her entire initial wealth $W_{T,0} + W_{R,0}$ in a TA. The second

$$G_T \equiv \sum_{t=1}^{T} W_{R,0} \prod_{j=1}^{t-1} (\boldsymbol{\alpha}_{R,j} \cdot \mathbf{R}_{R,j}) \, T_t \prod_{j=t+1}^{T} (\boldsymbol{\alpha}_j \cdot \mathbf{R}_{T,j}) \tag{1.7}$$

is the amount of total final wealth $W_T$ that results from the tax exemption of profits as well as interest and compound interest on the profits in the TEA. It can be interpreted as a public tax-gift for private retirement saving in terms of non-levied taxes on gains and dividends as well as interest and compound interest and will be referred to as total *tax-gift*.

Especially for long investment horizons this tax-gift can be a substantial fraction of the investor's total final wealth. To demonstrate the power of this tax-gift consider an investor who is initially endowed with \$ 5,000 in both her taxable and her retirement account and can only invest into one risk-free asset with a pre-tax return of 6%. If the investor's investment horizon is 40 years and the tax-rate applicable to the return of the asset is 36%, i.e. her after-tax return is 3.84%, her total final wealth is \$ 74,000 and her tax-gift is \$ 28,857, which is about 39% of total final wealth. The tax-gift thus has a tremendous impact on total final wealth.

Equation (1.7) can be further decomposed into

$$\begin{aligned} G_T = & \sum_{\substack{t=1\\t\neq k}}^{T} W_{R,0} \prod_{\substack{j=1\\j\neq k}}^{t-1} (\boldsymbol{\alpha}_{R,j} \cdot \mathbf{R}_{R,j})\, T_t \prod_{j=t+1}^{T} (\boldsymbol{\alpha}_j \cdot \mathbf{R}_{T,j}) \\ & + W_{R,0} \prod_{j=1}^{k-1} (\boldsymbol{\alpha}_{R,j} \cdot \mathbf{R}_{R,j})\, T_k \prod_{j=k+1}^{T} (\boldsymbol{\alpha}_j \cdot \mathbf{R}_{T,j}) \\ & + \sum_{t=k+1}^{T} W_{R,0} \prod_{\substack{j=1\\j\neq k}}^{t-1} (\boldsymbol{\alpha}_{R,j} \cdot \mathbf{R}_{R,j})\, (\boldsymbol{\alpha}_{R,k} \cdot \mathbf{R}_{R,k} - 1)\, T_t \prod_{j=t+1}^{T} (\boldsymbol{\alpha}_j \cdot \mathbf{R}_{T,j})\,. \end{aligned} \tag{1.8}$$

According to Equation (1.8), the total tax-gift contained in total final wealth $W_T$ can be decomposed into three summands. The part of $G_T$ that is independent from the investment decision in period $k$ and only depends on the tax-effects of the other periods is given by the first summand. The absolute change in total final wealth that results from the second summand

$$W_{R,0} \prod_{j=1}^{k-1} (\boldsymbol{\alpha}_{R,j} \cdot \mathbf{R}_{R,j})\, T_k \prod_{j=k+1}^{T} (\boldsymbol{\alpha}_j \cdot \mathbf{R}_{T,j}) \tag{1.9}$$

describes the change in total final wealth that results from the relative tax-gift in period $k$ and is called the *direct tax-effect* of the investment decision in period $k$, or just the direct tax-effect in this article. It depends on growth of TEA-wealth until the end of period $k-1$ and therefore also on the proportions of the assets in the TEA $\boldsymbol{\alpha}_{R,j}$ $(j \in \mathbb{N}_{k-1})$ until period $k-1$, the relative tax-gift $T_k$, and the after-tax growth rate $\prod_{j=k+1}^{T} (\boldsymbol{\alpha}_j \cdot \mathbf{R}_{T,j})$ until the end of the investment horizon. $W_{R,0} \prod_{j=1}^{k-1} (\boldsymbol{\alpha}_{R,j} \cdot \mathbf{R}_{R,j})\, T_k$ describes the *tax-gift* in period $k$ which then grows by the after-tax growth rate $\prod_{j=k+1}^{t} (\boldsymbol{\alpha}_j \cdot \mathbf{R}_{T,j})$ until the end of the investment horizon. The absolute change in total final wealth that results from the third summand

$$\sum_{t=k+1}^{T} W_{R,0} \prod_{\substack{j=1\\j\neq k}}^{t-1} (\boldsymbol{\alpha}_{R,j} \cdot \mathbf{R}_{R,j})\, (\boldsymbol{\alpha}_{R,k} \cdot \mathbf{R}_{R,k} - 1)\, T_t \prod_{j=t+1}^{T} (\boldsymbol{\alpha}_j \cdot \mathbf{R}_{T,j}) \tag{1.10}$$

is the change in total final wealth that results from the change of period $k$ retirement wealth and is called the *indirect tax-effect* of the investment decision in period $k$ or just the indirect tax-effect in the preceding. Each summand in (1.10) can be interpreted as follows: $W_{R,0}\prod_{\substack{j=1\\j\neq k}}^{t-1}\left(\boldsymbol{\alpha}_{R,j}\cdot\mathbf{R}_{R,j}\right)\left(\boldsymbol{\alpha}_{R,k}\cdot\mathbf{R}_{R,k}-1\right)T_t$ is the effect on period $t$ total wealth, which results from the choice of $\boldsymbol{\alpha}_{R,k}$ in period $k$ and the increase of the tax exempt basis in that period. It then grows with after-tax return $\prod_{j=t+1}^{T}\left(\boldsymbol{\alpha}_j\cdot\mathbf{R}_{T,j}\right)$ until the end of period $T$ to the indirect tax-effect.

The direct tax-effect from Equation (1.9) has a singular effect, as it only has an impact on total wealth in one single period and then grows with an after income tax return until the end of the investment horizon. The expression for the indirect tax-effect in Equation (1.10), however, has more than one summand for $k \leq T-2$. Due to the change in the tax exempt basis, it has an impact on all future periods. This is why the impact on total final wealth from the indirect tax-effect in period $k$ can be quantified by the direct tax-effects of all future periods on that part of TEA wealth that results from the investment decision in period $k$.

Both the direct and the indirect tax-effect are not necessarily an advantage to the investor. If the total return on TEA-wealth in period $t$ is negative, the relative tax-gift $T_t$ can become negative. This is because contrary to an investment in a TA, the treasury does not participate in these losses via the taxation of dividends and gains. If $T_t$ becomes negative, so does the direct tax-effect. As with the direct tax-effect, negative gains that outweigh potential interest or dividends can lead to a negative return on TEA wealth, which causes a relative reduction in the tax exempt basis of forthcoming periods by $\left(\boldsymbol{\alpha}_{R,t}\cdot\mathbf{R}_{R,t}-1\right)<0$, and shows a negative indirect tax-effect.

### 1.3.3 Generalization to TDAs

The argument for the case of an investment opportunity set with a TEA and a TA can be generalized to the case of an investment opportunity set with a TDA and a TA as follows: Initial wealth $W_{R,0}$ which has been invested into a TEA at the end of period 0 must have been made from after income tax dollars. As an investment into a TDA is made from pre income-tax dollars, instead of investing $W_{R,0}$ after income tax dollars into a TEA, one can invest $\frac{W_{R,0}}{1-\tau_{p,0}}$ pre income-tax dollars in a TDA. As wealth in a TDA is still subject to income taxation at the end of the investment horizon at rate $\tau_{p,T}$, the investor can only consider $W'_{R,t}\equiv W_{R,t}\frac{1-\tau_{p,T}}{1-\tau_{p,0}}$ her effective wealth, as the remainder falls to the treasury at the end of period $t$. If $\tau_{p,T}<\tau_{p,0}$, then $W'_{R,t}>W_{R,t}$ and an investment in a TDA results in a higher final wealth than an investment into a TEA due to the lower burdening with income tax. As $W'_{R,t}$ and $W_{R,t}$ only differ by the constant factor $\frac{1-\tau_{p,T}}{1-\tau_{p,0}}$, the problem of finding the optimal investment strategy with an investment opportunity set with both a TEA and a TA, is equivalent to the problem of with a TDA and a TA. Hence, it suffices to consider an investor with the opportunity to invest into both a TA and a TEA. As before, $W_{R,t}$ denotes the wealth in the RA if the retirement account is a TEA and the effective wealth in the RA if the retirement account is a TDA.

### 1.3.4 Optimal Asset Location Decisions

According to the seminal work of Huang (2007), investors that do not face short-selling constraints should invest their entire retirement-wealth into the asset $a$ with the highest effective tax-rate

$$\tau_a \equiv \frac{(1+(1-\tau_d)\,d_B)\,(x_a-1)}{(1+(1-\tau_d)\,d_B)\,x_a-1} \tag{1.11}$$

in which $x_a \equiv \frac{1}{1-\tau_g} + \frac{d_a(\tau_d-\tau_g)-\tau_g}{(1-\tau_g)(1+(1-\tau_d)d_B)}$ is the replication cost in the TA of one dollar of asset $a$ in the RA. Furtermore, in this case, the asset location and asset allocation problem are independent from each other, as each dollar in asset $a$ in the RA can be replicated by $\frac{1}{1-\tau_g}$ dollars of that asset and $\frac{d_a(\tau_d-\tau_g)-\tau_g}{(1-\tau_g)(1+(1-\tau_d)d_B)}$ dollars of the risk-free asset in the TA. In particular, in such a setting the asset allocation and asset location decision are independent from each other.

If, however, the investor is not allowed to go short, it is no longer necessarily optimal for him to hold the asset with the highest effective tax-rate in the RA as shown in Garlappi and Huang (2006). This is due to the fact that the investor cannot replicate the tax-deferred portfolio in the taxable account if this required to go short. Garlappi and Huang (2006) have also pointed out that holding stocks in the RA can help smooth the ratio of the relative tax-gift $T_t$ times $W_{R,t}$ to total wealth $W_t$. They call this ratio the tax-subsidy. This tax-subsidy can be interpreted as that part of growth in total wealth that results from the relative tax-gift in period $t$. This growth in total wealth can be smoothed by constructing portfolios in the RA and the TA that have similar weights in the two assets.

Smoothing these extra growth-rates results in less volatile distributions of final wealth, which is desirable for risk-averse investors. However, Garlappi and Huang

(2006) assume the return of the risky asset to be binomially distributed. In this case, it is possible to smooth the tax-subsidy in such a way that it takes the same value, independent from the realization of the return from the risky asset. If, however, the return on the risky asset $S$ has a more complex distribution and can take more than two realizations, it is no longer possible to keep the tax-subsidy at the same level independent from the realization of the return. Nevertheless, in such a case, it is still possible to dampen the impact of the total tax-gift, which, according to Equation (1.7), is ceteris paribus best attained when only holding bonds in the RA. As one can see from Equation (1.6), in addition to the risks due to the volatility of growth of total wealth after-tax, the volatility of the total tax-gift is a second source of risk to which total final wealth is exposed when holding stocks in a retirement account.

For a given asset allocation $\boldsymbol{\alpha}_t$ ($t \in \mathbb{N}_T \backslash \{k\}$), the decision to locate stocks instead of bonds in the retirement account in period $k$ has three effects on the risk-return profile of total final wealth given everything else as equal. First, it has an impact on the direct tax-effect whose sign is depending on the absolute tax-burden of stocks and bonds. Second, due to the higher expected pre-tax return on stocks the expected indirect tax-effect increases. Third, if the pre- and the after-tax returns of $S$ and $B$ are not negatively correlated, the volatility of total final wealth increases as both the direct and the indirect tax-effect are subject to higher volatility. While according to the first and third effect, bonds should be preferably located in the retirement account, the second effect suggests that stocks could be preferably held in the retirement account as well.

However, the changes of the risk-return profiles of stocks and bonds when shifting them from the TA to the RA suggest that bonds should still be the preferred asset to hold in the RA. Shifting bonds from the TA to the RA increases their return. However, shifting stocks from the TA to the RA does not only increase their expected return, but also its volatility. Moreover, bonds usually come with a higher effective tax-rate than stocks.

According to Equation (1.6) total final wealth consists of two components: total final wealth the investor would have attained without the opportunity of investing into a retirement account and a tax-gift that results from the tax exemption of profits in these accounts. As the impact of the tax-gift decreases with a decreasing investment horizon, its impact is stronger, the longer the remaining investment horizon. Due to the fact that investors usually prefer bonds in their retirement account, the properties of the tax-gift are more similar to those of bonds than to those of stocks. The tax-gift can thus be regarded an implicit bond position that is not directly observable in the investor's security accounts. The higher the unobservable implicit bond position, the higher the observable equity position. This is why for longer remaining investment horizons investors will have a higher observable bond position than for shorter remaining investment horizons. Hence, besides decreasing future labor income, decreasing unobservable bond holding in retirement accounts are another reason for decreasing equity exposure over the life cycle.

Due to the risks associated with the indirect tax-effect, the preference for bonds in the retirement account, and the shadow price of one dollar in the retirement account, an investor with a high fraction of her wealth in a retirement account will hold a lower total fraction of stocks than an investor with a high fraction of her wealth in a taxable account.

## 1.4 Numerical Evidence

For the numerical analysis, a short-selling constrained investor who can invest in a market with a risky asset $S$ and a risk-free asset $B$ is considered. The characteristics of $S$ and $B$ are similar to those of stocks and bonds, respectively. The capital gains rate of $S$ is lognormally distributed with an expected gain of $\mu_{g,S} = 0.09$ and a standard deviation of $\sigma_{g,S} = 0.20$. Asset $S$ has a constant dividend rate of $d_S = 0.02$. Asset $B$ has a certain interest-rate of $d_B = 0.06$ and no capital gains. An investment horizon of $T = 40$ years is considered which is a realistic investment horizon for retirement saving. The tax-rates on dividends and gains are assumed to be $\tau_d = 0.36$ and $\tau_g = 0.2$, respectively.[1] This parameter choice follows Dammon et al. (2004). For these parameter values, the effective tax-rates for stocks and bonds are $\tau_S = 0.16$ and $\tau_B = 0.36$, respectively. Thus, investors that are not short-selling constrained in their taxable accounts should hold their entire retirement wealth in bonds. Even though we consider short-selling constraint investors, this result

[1] Since capital gains that are realized short-term are taxable at the investor's personal income tax-rate which corresponds to the tax-rate on dividends, choosing $\tau_g$ to be smaller than $\tau_d$ implies the assumption that the long-term capital gains rate applies to the investor's capital gains.

remains valid.

The investor is assumed to maximize her utility of total final wealth $W_T$ from a CRRA-utility function with a parameter of risk aversion of $\gamma \in [0, \infty)$

$$U(W_T) \equiv \begin{cases} \frac{W_T^{1-\gamma}}{1-\gamma} & \text{for } \gamma \neq 1 \\ \ln(W_T) & \text{for } \gamma = 1. \end{cases} \tag{1.12}$$

Her optimization problem is

$$\max_{\boldsymbol{\alpha}_T, \boldsymbol{\alpha}_R} \mathbb{E}\left[U\left[W_T\right]\right] \tag{1.13}$$

s.t.

$$W_t = W_{R,t} + W_{T,t} \tag{1.14}$$

$$W_{R,t+1} = W_{R,t}\left(\boldsymbol{\alpha}_{R,t+1} \cdot \mathbf{R}_{R,t+1}\right) \tag{1.15}$$

$$W_{T,t+1} = W_{T,t}\left(\boldsymbol{\alpha}_{T,t+1} \cdot \mathbf{R}_{T,t+1}\right) \tag{1.16}$$

$$0 \leq \boldsymbol{\alpha}_{R,t}, \boldsymbol{\alpha}_{T,t} \leq 1 \tag{1.17}$$

in which $\boldsymbol{\alpha}_T \equiv (\boldsymbol{\alpha}_{T,1}, \ldots, \boldsymbol{\alpha}_{T,T})$ and $\boldsymbol{\alpha}_R \equiv (\boldsymbol{\alpha}_{R,1}, \ldots, \boldsymbol{\alpha}_{R,T})$. Normalizing by $W_t$ and taking into account that $U$ is homogeneous in wealth implies that the optimization problem is equivalent to the solution of

$$\max_{\boldsymbol{\alpha}_T, \boldsymbol{\alpha}_R} \mathbb{E}\left[U\left[\frac{W_T}{W_t}\right]\right] \tag{1.18}$$

s.t.

$$1 = \frac{W_{R,t}}{W_t} + \frac{W_{T,t}}{W_t} \tag{1.19}$$

$$\frac{W_{R,t+1}}{W_t} = \frac{W_{R,t}}{W_t}\left(\boldsymbol{\alpha}_{R,t+1} \cdot \mathbf{R}_{R,t+1}\right) \tag{1.20}$$

$$\frac{W_{T,t+1}}{W_t} = \frac{W_{T,t}}{W_t}\left(\boldsymbol{\alpha}_{T,t+1} \cdot \mathbf{R}_{T,t+1}\right) \tag{1.21}$$

$$0 \leq \boldsymbol{\alpha}_{R,t}, \boldsymbol{\alpha}_{T,t} \leq 1. \tag{1.22}$$

If $V_t(X_t)$ denotes the investor's utility as a function of her states variables $X_t$ at time $t$, it holds for an investor with risk-aversion of $\gamma \neq 1$ that

$$V_t(X_t) \equiv \max_{\boldsymbol{\alpha}_T, \boldsymbol{\alpha}_R} \mathbb{E}\left[U\left[\frac{W_{R,t}}{W_t}\right]\right] \tag{1.23}$$

$$= \max_{\boldsymbol{\alpha}_T, \boldsymbol{\alpha}_R} \mathbb{E}\Bigg[V_{t+1}(X_{t+1}) \cdot \Bigg(\frac{W_{R,t}}{W_t}(\boldsymbol{\alpha}_{R,t+1} \cdot \mathbf{R}_{R,t+1}) + \left(1 - \frac{W_{R,t}}{W_t}\right)(\boldsymbol{\alpha}_{T,t+1} \cdot \mathbf{R}_{T,t+1})\Bigg)^{1-\gamma}\Bigg]. \tag{1.24}$$

The solution of this problem can thus be computed numerically using backward induction. For the optimization procedure one state-variable (the percentage of wealth in the RA relative to total wealth $X_t = \frac{W_{R,t}}{W_t}$ at the end of period $t$) is sufficient. The grid is spanned with 101 grid points that are equally distributed on $[0, 1]$. The integral of the expectation in Equation (1.24) is computed using Gaussian quadrature.

If the investor would only have the opportunity to invest into one account, she would hold the same fraction of stocks in all periods according to the classical result of Merton (1969) and Samuelson (1969). All derivations from a constant fraction of stocks thus have to be driven by the different taxable treatment of the assets in the two accounts.

Figure 1.1 shows the optimal equity proportion for an investor with risk-aversion of $\gamma = 3$ as a function of time passed since the initial investment and her fraction of retirement wealth to total wealth. The upper left and the upper right graphs show her optimal fraction of stocks in the TA and the RA, respectively. If her fraction of wealth in the retirement account is zero or one, the investor is in the one-account world, there is no asset location decision, and in line with Merton (1969) and Samuelson (1969) she holds the same fraction of stocks independent from the remaining investment horizon. If, however, the investor is endowed with both taxable and retirement wealth, she has to decide, which assets to hold in the retirement account and which in the taxable account. Confirming the findings of the recent literature on optimal asset location, we find the investor to prefer bonds in the retirement account and stocks in the taxable account.

The lower right graph, which depicts the sum of the investor's optimal fraction of stocks in the TA and the RA, shows that the investor only holds stocks in the retirement account if her taxable wealth is entirely invested in stocks. With an

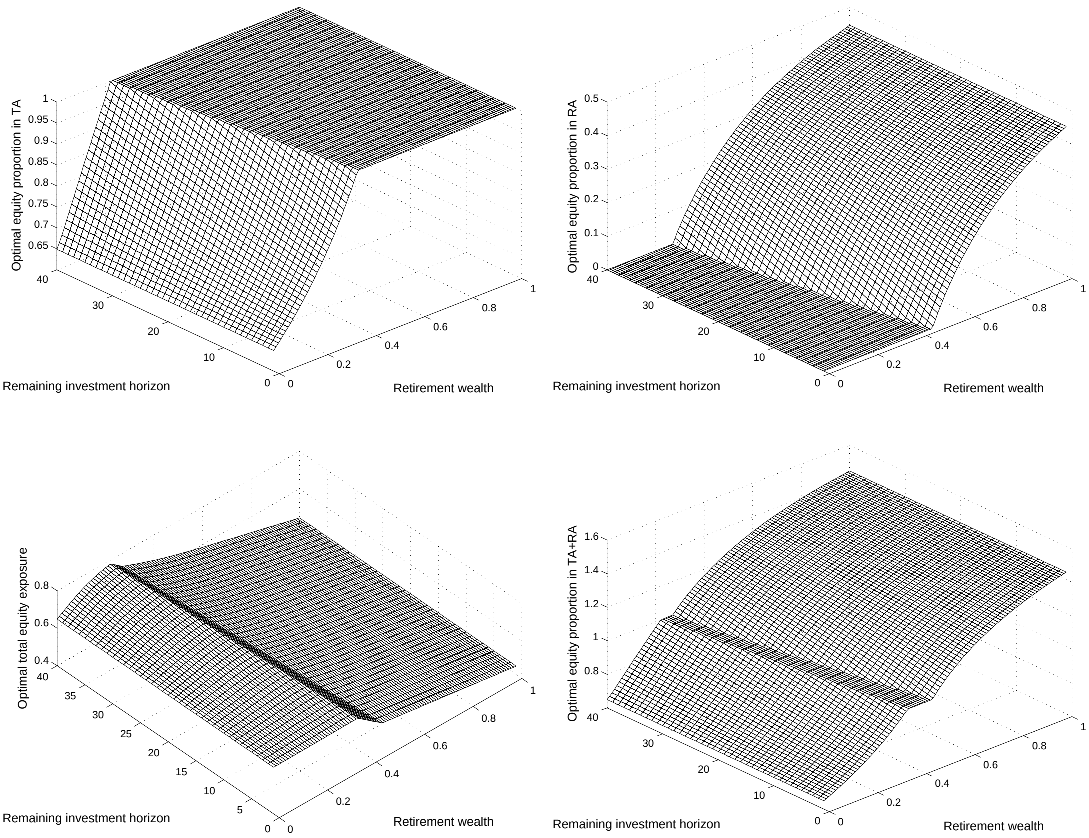


Figure 1.1: This Figure shows the optimal asset location and asset allocation strategy as a function of the investor's fraction of retirement wealth to total wealth (retirement wealth) and her remaining investment horizon. The upper left graph shows her optimal fraction of stocks in the TA, the upper right graph depicts her optimal fraction of stocks in the RA. The lower left graph shows the optimal overall equity exposure, the lower right graph shows the sum of the optimal relative equity exposure in the TA and the RA.

increasing fraction of wealth in the retirement account, the investor first increases her fraction of stocks in the TA until it has attained 100% as can be seen from the upper left graph. However, increasing her fraction of retirement wealth further, does not cause the investor to immediately increase her fraction of stocks in the retirement account - as can be seen from the lower right graph. For small increases in retirement wealth, she keeps her fraction of stocks in the retirement account at 0% (*plateau effect*), which reflects the fact that bonds are preferred in the RA as they come with a more advantageous risk-return profile in the retirement account. Furthermore, if only risk-free bonds are held in the RA, there is no additional source of risk from the tax-effect. The size of the plateau effect increases with a short-term investment horizon. This is due to the fact that for short investment horizons the probability of facing a negative tax-effect when being invested into equities in the retirement account is significantly higher than for long investment horizons.

If the fraction of retirement wealth increases even further, the investor increases the fraction of stocks in the retirement account. This reflects her desire not to get too heavily invested into bonds. However, the increase in the fraction of stocks in the retirement account is slower than in the taxable account. If the investor does not have any retirement wealth, her optimal equity exposure is about 70%, if she does not have any taxable wealth her optimal equity exposure is only about 50%. This is due to the fact that bonds have a more advantageous risk-return profile in the retirement account.

The investor's fraction of stocks in both the taxable account and the retirement account decreases with decreasing length of the remaining investment horizon. This is due to the fact that with decreasing investment horizon, the expected tax-gift decreases. As the investor prefers bonds in the retirement account for lower levels of retirement wealth, the tax-gift can be considered a certain income stream which is similar to a risk-free bond. If the investor has a higher retirement wealth, the tax-gift is still more similar to a risk-free bond than to a stock as the investor's fraction of stocks in the RA remains below 50%, despite the associated risk. This does not necessarily hold for investors with even lower risk-aversion which is why in that case the tax-gift can become more similar to an implicit stock-holding. Due to the absence of this implicit bond holding, the investor increases her explicit bond holding and thus decreases her equity exposure for short investment horizons.

The lower left graph depicts the fraction of stocks relative to total wealth. For low levels of retirement wealth, the stock fraction is more prevalent. This is due to sharp increases in equity exposure in the taxable account. This is explained by the fact that one dollar of bonds in the retirement account has a higher impact on total final wealth than one dollar in the taxable account due to the tax-effect. In particular, the indirect tax-effect has a tremendous impact for long investment horizons. As the tax-effect diminishes with decreasing investment horizon, the increase in the total fraction of stocks decreases with increasing time passed since the initial investment.

For higher levels of retirement wealth, the total fraction of stocks first rapidly decreases due to the plateau effect and the investor's aversion against holding stocks in the retirement account. For even higher levels of retirement wealth, the total fraction of stocks decreases even further, a finding noted by Dammon et al. (2004).

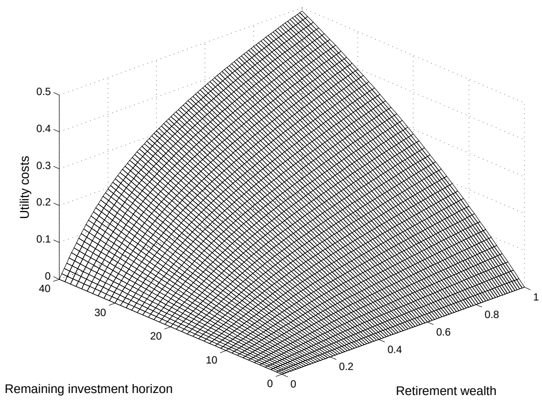


Figure 1.2: This Figure shows the increase in taxable wealth an investor who does not have retirement wealth needs to be compensated with to attain the same level of expected utility as an investor who is endowed with the same total wealth of which some is invested in a retirement account (utility costs) as a function of this investor's fraction of total wealth in the retirement account (retirement wealth) and the remaining investment horizon.

Consequently, one dollar of stocks in the retirement account has a higher impact on total final wealth than one dollar of stocks in the taxable account. However, the slope of the decrease is somewhat lower as the investor starts holding stocks in her retirement account. Nevertheless, as bonds come with a more advantageous risk-profile in the retirement account, this increase is not strong enough to prevent the total fraction of stocks from decreasing as retirement wealth increases. Our obtained results remain valid when varying the investor's risk-aversion parameter $\gamma$.

Since retirement accounts allow for earning pre-tax returns while taxable accounts only allow for earning after-tax returns, one dollar in a retirement account is worth more than one dollar of taxable wealth. We analyze the value of retirement wealth by computing the increase in taxable wealth an investor with some given total wealth which is entirely held in a taxable accounts needs to be compensated with to attain the same level of expected utility as an investor who is holding the same amount of total wealth of which some part of is invested in a retirement account.

Figure 1.2 shows the increase in taxable wealth an investor who is initially endowed with only taxable wealth needs to be compensated with to attain the same level of expected utility as an investor who is endowed with the same total wealth of which some part is held in a retirement account (utility costs). The investor's utility costs are increasing both in her retirement wealth and her remaining investment horizon. Each dollar in the retirement account provides the investor with some tax-gift, which is why the investor's utility costs increase in her retirement wealth. With increasing length of the investment horizon the impact of the tax-gift increases. Consequently, the investor's utility costs increase in the length of her remaining investment horizon.

However, the investor's utility costs are concave in her retirement wealth. This is due to the fact that with increasing retirement wealth the investor increases her fraction of stocks in the retirement account for not getting too heavily invested into bonds. Since bonds are the preferred asset to hold in the retirement account, each dollar of bonds that is held in the retirement account provides the investor with a higher advantage than a dollar of stocks that is held in the retirement account. Consequently, her marginal utility gains decrease when she holds stocks in her retirement account.

## 1.5 Conclusion

In this paper, the field of asset location decisions for retirement savers having the opportunity to invest into both a retirement account and a taxable account is outlined.

Confirming recent findings in economic literature that bonds are the preferred asset to hold in the retirement account, we show that the investor only holds stocks in the retirement account if her taxable wealth is entirely invested into equity. We further show that the investor does not increase her equity exposure in the retirement account immediately with increasing retirement wealth, but prefers to hold only bonds in her retirement account for small increases in retirement wealth which is another indication for her preference for bonds in the retirement account. If, however, the investor's retirement wealth is substantial, she also holds some stocks in her retirement account in order to prevent from investing too heavily in bonds.

While the literature on optimal asset location concludes in (almost) one voice that bonds are the preferred asset to hold in retirement accounts, this paper focuses on the relation of asset location and asset allocation and shows that besides the locational preference of bonds in retirement accounts, the opportunity to invest in a retirement account also has an impact on asset allocation. It is argued that the different taxable treatment of capital gains and dividends in taxable as well as tax-sheltered retirement accounts has an impact on asset allocation. Compared to the benchmark of a constant equity proportion in a one-account problem, the investor's equity proportion depends on both her fraction of total wealth in the retirement account and the length of the remaining investment horizon. The longer the remaining investment horizon, the higher the investor's equity exposure in both taxable and retirement account. This finding is due to the fact that total final wealth can be decomposed into what the investor would have attained in the absence of a tax-deferred investment vehicle and a tax-gift resulting from the tax exemption of profits in retirement accounts. As the properties of the tax-gift are more similar to those of risk-free bonds than to those of stocks and its impact is decreasing with decreasing remaining investment horizon, the investor is endowed with some implicit bond holding that is unobservable in her security account. As the impact of the tax-gift on total final wealth is stronger the longer the remaining investment horizon, the investor's equity exposure is higher the longer the remaining investment horizon.

# 2 Optimal Tax-Timing and Asset Allocation when Tax Rebates on Capital Losses are Limited

First draft: November 2006

This version: July 2007

Until July 2, 2007, this paper has been accepted for presentation (or already been presented) at the following conferences:

- 40th Meeting of the Euro Working Group on Financial Modelling (EWGFM), May 10-12, 2007, Rotterdam, Netherlands
- Advances in Portfolio Decision Making, June 8-10, 2007, Notre Dame, Indiana, USA
- European Financial Management Association (EFMA) Annual Meeting 2007, June 27-30, 2007, Vienna, Austria
- 11th international congress on Insurance: Mathematics and Economics (IME), July 10-12, 2007, Piraeus, Greece
- Southern Finance Association (SFA) Annual Meeting 2007, November 14-17, 2007, Charleston, South Carolina, USA
- American Economic Association (AEA) Annual Meeting 2008, January 4-6, 2008, New Orleans, Louisiana, USA

# Abstract

Since Constantinides (1983), it is well known that it is optimal to realize losses immediately if capital losses qualify for indefinite tax rebates. However, many tax-codes around the world restrict tax rebates on capital losses. This paper shows that Constantinides' result generalizes to markets in which capital loss deduction is limited and that realizing losses immediately remains an optimal tax-timing strategy even though the compensation for the loss comes as a loss carryforward, which is less attractive than compensation in cash. Nevertheless, in such markets, an investor without an initial loss carryforward will hold substantially less risky assets than in markets with unlimited tax rebates, due to the less attractive taxable treatment of capital losses. An investor with a significant loss carryforward, however, increases her equity exposure due to the opportunity of earning capital gains tax-free. The less capital losses qualify for tax rebates, the lower the equity exposure is.



**JEL Classification Codes:** G11, H24

**Key Words:** tax-timing, asset allocation, capital losses, loss carryforward, limits on tax rebates

## 2.1 Introduction

The tax rules private investors are confronted with are an important factor influencing household portfolio structure. Even though pre-tax returns are reported in newspapers, on television, or on the internet, only after-tax returns should have an impact on investment decisions of rational private investors, as only these returns have an impact on potential consumption and bequest.[1]

Many assets generate profits from both capital gains and dividends. The taxable treatment of these two types of profits differs in two ways. First, dividends are taxable the year they are obtained, while capital gains are taxable the year when the asset is sold and the gains are realized. Second, dividends are subject to a higher tax-rate than capital gains that qualify for long-term treatment.

The taxation of capital gains has several impacts on asset allocation decisions of private investors. First, it reduces the expected after-tax return, which might lead some investors to increase their present consumption and decrease their future consumption by investing less. Second, the deferral of capital gains taxation until realization of the capital gains results in compound returns on the postponed taxes and thus decreases the effective capital gains tax-rate (Chay et al. (2006)). Third, investors having high unrealized capital gains in some assets might not want to sell them in order to avoid paying the capital gains tax, and thus get "locked in".[2] The decision whether or not to sell an asset with embedded unrealized capital gains is influenced by two opposing effects. On the one hand, the investor should sell some part of the asset to rebalance the portfolio. On the other hand, the investor should not sell any part of the asset at all to defer paying the capital gains tax and exploit the effect of compound returns.

A capital gains tax an investor has to pay when selling an asset with unrealized capital gains is similar to a transaction cost. However, while the transaction cost in future periods are usually independent from present transaction costs, future unrealized capital gains depend on present trading behavior. Especially for aged

[1] However, when the investor bequests assets with unrealized capital gains, these gains are not taxable for the beneficiary under current US tax-law. Hence, at the end of the investor's life cycle, pre-tax capital gains might matter more than after-tax capital gains.

[2] An investor is said to be "locked in" when she has unrealized capital gains in some asset and a higher fraction of total wealth is invested in that asset to avoid the capital gains tax-payment than she would hold when not facing unrealized capital gains.

investors, the disadvantages of unbalanced portfolios might be outweighed by the forgiveness of capital gains under current US tax-law when bequeathed. Fourth, if tax rebates on realized capital losses are limited, the risk-return profile of the assets becomes worse than they would be if tax-rebates were unlimited. This results in a lower exposure to the risky asset. Under current US tax-law, there is an annual upper bound of $ 3,000 of unrealized capital losses that can be offset against other income and thus qualifies for tax rebates.[3] According to Poterba (1987), this loss-offset constraint applies to about twenty percent of US taxpayers.

On the assumption that the returns of each asset are subject to the same tax-rate, Auerbach and King (1983) show that an optimal portfolio is a weighted average of a market portfolio and a portfolio that is chosen on the basis of tax considerations ignoring risk. Thus, expected changes in the capital gains tax-rate result in vast realizations of capital gains (Auerbach (1988)).

Auerbach et al. (2000) and Ivkovich et al. (2005) show that a large part of the investor public does not engage in tax-minimizing transactions. According to Shefrin and Statman (1985), Odean (1998), Barber and Odean (2000, 2001, 2003), investors tend to hold assets incurring losses too long and tend to sell assets with unrealized capital gains too early which clearly violates efficient tax-planning. The study by Jin (2006), however, finds selling decisions by institutions serving tax-sensitive clients to be sensitive to cumulative capital gains. Confirming this finding, Barclay et al. (1998) show that fund managers manage tax liabilities to attract new investors. Bergstresser and Poterba (2002) find fund flows to be sensitive to tax burdens.

This finding suggests that private investors might be more subject to the disposition effect than institutional investors. Ivkovich et al. (2005) compare the realizations of capital gains in taxable and tax-deferred accounts and find a strong "lock in" effect for capital gains in the former type of accounts. This finding in turn suggests that private investors might ride efficient tax-timing strategies. The empirical evidence in Seyhun and Skinner (1994), however suggests that the $ 3,000 limit on capital losses qualifying for tax rebates under current US-tax code represents an important constraint on tax-reduction strategies, and investors tend to follow very simple tax-timing strategies like realizing losses early and postponing gains. How-

[3]This rule assures that public finances are not affected too strongly when capital markets perform poorly.

ever, in their study, about 90% of the investors seem to follow simple buy-and-hold strategies. Other studies on the relation between taxation and investment behavior include Blouin et al. (2003) and the surveys in Poterba (2002a) and Campbell (2006).

According to the seminal work of Constantinides (1983), it is optimal to realize losses immediately if capital loss deduction is unlimited and if wash sale rules do not apply.[4] Under current US tax law wash sales are permitted and it is not allowed to short a security in which one has a long position to avoid realizing capital gains. Investors realizing such a "shorting-the-box-strategy" are treated as if they had sold the long position and hence their capital gains are taxed. Gallmeyer et al. (2006) address this issue and propose tax-management strategies to circumvent the capital gains tax. Their so-called trading flexibility strategy minimizes future tax-induced trading costs by shorting one of several stocks, even if none of these stocks has an unrealized gain. This strategy is in particular useful if the benefits from holding a well-diversified portfolio are outweighed by the expected future rebalancing costs. This is in particular the case if two assets are highly correlated. Constantinides and Scholes (1980) argue that even when an investor sells an asset with an unrealized capital gain, the realization of that gain can be deferred by hedging. To circumvent the "shorting-the-box-strategy" Stiglitz (1983) suggests selling (or shorting, if necessary) highly correlated assets instead of realizing capital gains. Nevertheless, this shorting-strategy can be subject to significant transaction costs and is subject to the risk that correlations might change in time.

Dammon et al. (2001), on the other hand, show that selling an asset with an unrealized capital gain can be an optimal tax-timing strategy. According to their study, the diversification benefit of reducing a volatile position can significantly outweigh the tax cost of selling the asset with an unrealized capital gain. The results of Dammon et al. (1989) suggest that the value of the option to realize long-term gains in order to regain the opportunity of realizing short-term losses is negatively related to the stocks price volatility.

If short-term capital gains are taxed at a higher tax-rate than long-term capital gains, Constantinides (1984) shows that it can be optimal to sell assets with an

[4] A transaction is termed a wash sale if a stock is sold to realize a capital loss and repurchased immediately. Under current US tax-rules wash sales do not qualify for the capital loss deduction if the same stock is repurchased within thirty days before or after the sale.

unrealized capital gain as soon as they qualify for long-term treatment in order to regain the opportunity of producing short-term losses. Cadenillas and Pliska (1999) show that in a single-security market, even with only one tax-rate, it can be optimal to cut unrealized capital gains short. This surprising result is most likely due to the reduction in volatility.

Dammon and Spatt (1996) extend the approach of Constantinides (1984) by allowing the number of trading periods before a short term position becomes a long term position to be greater than one. In particular, they show that contrary to intuition, it can be optimal to defer small short-term losses even in the absence of transaction costs. This finding is due to the fact that realizing these losses and repurchasing the asset restarts the short-term holding period and thus the time the investor has to wait until potential future gains qualify for long-term treatment. Under plausible parameter values, they find that it can be optimal for private investors to defer realizing short-term capital losses of about 10% in the absence of transaction costs.

The articles closest to ours are Constantinides (1983) and Dammon et al. (2001). Our paper differs from the existing literature on optimal asset allocation by allowing for limitations on tax rebates for capital losses. According to the seminal work of Constantinides (1983), it is optimal to realize losses immediately if capital loss deduction is unlimited and if wash sale rules do not apply. This paper generalizes his result to tax-systems in which capital loss deduction is limited and the remaining losses can be carried over indefinitely as a loss carryforward. It shows that in such tax-systems, Constantinides' result of realizing losses immediately remains an optimal tax-timing strategy even though the investor's compensation for incurred losses does not come in cash as a tax rebate, but as a less desirable loss carryforward that does not pay any interest and potentially remains unused.

It further extends the approach of Dammon et al. (2001) by allowing for limits on tax rebates and shows that such limits decrease the optimal equity exposure of investors that do not have an initial loss carryforward. This is due to the fact that in tax-systems with unlimited tax rebates, the investor effectively receives some tax-refund in cash which can be reinvested to earn some returns on the tax rebate. In contrast, the loss carryforward does not earn any interest. Furthermore, in a tax-system with limited tax rebates, the investor bears the risk that the loss carryforward potentially remains unused if the investor does not live long enough to make use of it.

Limits on tax rebates thus worsen the risk-return-profile of risky assets. This is why in tax-systems with limited tax rebates, investors with no initial loss carryforward tend to hold less risky assets than in tax-systems with unlimited tax rebates.

The remainder of this paper proceeds as follows. Section 2.2 introduces the model and generalizes the finding of Constantinides (1983) that it is optimal to realize capital losses immediately to tax-systems with limits on tax rebates. Section 2.3 discusses the impact of the tax-timing option and limited tax rebates on asset allocation. Section 2.4 presents our numerical results extending the approach of Dammon et al. (2001) to tax-systems with limited tax rebates. Section 2.5 concludes.

## 2.2 Optimal Tax-Timing with Unrealized Capital Losses

The model in this section builds on Constantinides (1983). Following his approach, a market is considered in which investors are price takers and only trade at equilibrium prices. There are no transaction costs and the tax-system allows for wash sales.[5] There is only one capital gains tax-rate, i.e. the tax-system does not distinguish between long-term and short-term capital gains. The investment opportunity set consists of a risk-free asset that pays an after-tax return of $r > 0$ and a risky asset which is infinitely divisible.

The difference between a realized capital gain and a realized capital loss plus an initial loss carryforward $L_{t-1} \leq 0$ in period $t$ is called a net capital gain $G_t$, if $G_t \geq 0$, or a net capital loss, if $G_t < 0$. A net capital gain is taxable at tax-rate $\tau > 0$ when being realized. Unrealized capital gains or losses are not subject to taxation. In line with Constantinides, we assume the same tax-rate $\tau$ to apply for capital gains and losses.[6] In contrast to Constantinides, we allow for limits on tax rebates for capital losses, i.e. net capital losses only qualify for tax rebates for

[5]This assumption assures that unrealized capital losses can be realized without having to decrease the exposure in the asset bearing the unrealized capital loss.

[6]Under current tax-law realized capital losses up to \$ 3,000 can be offset from other income such that the tax-rate which applies to such losses is the investor's personal tax-rate, which can differ from the tax-rate on capital gains.

that part of the absolute value of the net capital loss $G_t$ not exceeding a maximum amount of $M_t \geq 0$ dollars in period $t$. This tax rebate of $\tau$ dollars per dollar of net capital loss is paid when the loss is realized or the investor has a loss carryforward remaining from the previous period. This modeling follows the taxable treatment of net capital losses under current US tax-law. The net capital gain (or loss) $T_t$ in period $t$ that is subject to the capital gains tax-rate is given by

$$T_t \equiv \max\left(G_t + L_{t-1}; -M_t\right). \tag{2.1}$$

Realized net capital losses that do not qualify for tax rebates can be indefinitely carried forward to the following periods. Thus, the loss $L_t < 0$ that can be carried forward from period $t$ to period $t+1$ is given by

$$L_t \equiv G_t - T_t + L_{t-1}. \tag{2.2}$$

In a one-period model, the evolution of an investor's wealth in a tax-system with limited tax rebates is equal to that of an investor in a tax-system with unlimited tax rebates who has sold a put-option at time zero without receiving a premium for that put-option. However, in a tax-system with limited tax rebates, the investor receives the loss carryforward according to Equation (2.2) as a compensation when the put-option is in the money, i.e. if $T_1 < -M_1$ at time of maturity.

### 2.2.1 Wealth, Unrealized Gains and Loss Carryforward

In tax-systems with limited tax rebates, optimal asset allocation depends on total wealth $W_t$ before trading including unrealized capital gains, the initial loss carryforward $L_{t-1}$, unrealized capital gain $U_t$ before trading at time $t$, and the length of the remaining investment horizon. The key to understanding optimal tax-timing in such a tax-system is understanding the relation between $W_t$, $U_t$ and $L_{t-1}$.

A loss carryforward $L_{t-1}$ of one dollar in period $t$ can be used in two ways. First, it can be subtracted from a realized capital gain to reduce capital gains taxes. Second, in the absence of a realized capital gain, the loss carryforward can be claimed a net capital loss that qualifies for tax rebates if $M_t > 0$. Thus, one dollar of loss carryforward can be shifted to $\tau$ dollars of wealth if $M_t \geq \tau$. Shifting the loss carryforward to wealth is a dominating strategy, since one dollar of loss carryforward can reduce future tax burden by not more than $\tau$ dollars. Furthermore, in contrast with the loss carryforward, the $\tau$ dollars of tax rebate can be reinvested and earn profits. By investing them in the risk-free asset, their value is always at least as high as the future tax burden of the unrealized capital gain.

Thus, if two investment strategies result in the same unrealized capital gains at some point in time $t$ before trading, but one of them results in a higher pre-tax wealth $W_t$ before trading and the other in a higher loss carryforward $L_{t-1}$ (in absolute value), the strategy with the higher pre-tax wealth is at least as good as the strategy with the higher loss carryforward, if for every $\tau$ extra dollars of wealth $W_t$ of the first strategy, the second strategy does not have more than one dollar of extra loss carryforward $L_{t-1}$. If $A \succeq B$ denotes "A is at least as good as B", than this finding can also be expressed as

$$\begin{pmatrix} W_t = \tau \\ L_{t-1} = 0 \end{pmatrix} \succeq \begin{pmatrix} W_t = 0 \\ L_{t-1} = -1 \end{pmatrix} \tag{2.3}$$

An investor endowed with one dollar of unrealized capital gains $U_t = 1$ at the beginning of period $t$ before trading and one dollar of loss carryforward $L_{t-1} = -1$ can use the loss carryforward in two ways. It can either be used to realize the capital gain without having to pay the capital gains tax or it can be used to generate a net capital loss at time $t$ and thus to earn a tax rebate of $\tau$ dollars if $M_t \geq \tau$. As argued above, the value of the tax rebate is at least as high as the future tax burden due to the unrealized capital gain when invested in the risk-free asset. Accordingly, realizing the net capital loss to increase $W_t$ but not to realize the capital gains is a dominating tax-timing strategy.

An investor who is neither endowed with that dollar of unrealized capital gain nor that dollar of loss carryforward can be considered an investor who has realized that capital gain and used the loss carryforward to avoid the capital gains tax. The investor then lacks the dominating opportunity of realizing the net capital loss. Hence:

$$\begin{pmatrix} U_t = 1 \\ L_{t-1} = -1 \end{pmatrix} \succeq \begin{pmatrix} U_t = 0 \\ L_{t-1} = 0 \end{pmatrix} \tag{2.4}$$

The unrealized capital gain $U_t$ is the product of the number of units $S_t$ of the risky asset and the unrealized capital gain $P_t - P^*_{t-1}$ per unit of the risky asset where $P_t$ denotes the price of the risky asset at time $t$ and $P^*_t$ is the investor's purchase price of that asset at the end of period $t$ after trading. That is, $P^*_{t-1}$ is the purchase price of that asset at the beginning of period $t$ before trading. Then $U_t$ is given by

$$U_t = S_t \cdot \left(P_t - P^*_{t-1}\right) \tag{2.5}$$

Equation (2.4) only depends on $U_t$. In particular, it is independent from the composition of $U_t$, i.e. whether a given capital gain $U_t$ results from a high equity exposure with a small capital gain or a small equity exposure with a high capital gain.

The relation between wealth and unrealized capital gains is a consequence of the relation between wealth and losses and the relation between unrealized gains and losses. If two investment strategies result in a loss carryforward of zero at some point in time $t$ before trading, but the first of them results in a higher pre-tax wealth and in higher capital gains than the other, the first strategy is at least as good as the second strategy, if for every $\tau$ extra dollars of pre-tax wealth, the unrealized capital gains of the first strategy does not exceed one dollar. This is due to the fact that according to Equations (2.3) and (2.4) it holds that

$$\begin{aligned}
&\begin{pmatrix} W_t = 0 \\ U_t = 1 \\ L_{t-1} = -1 \end{pmatrix} + \begin{pmatrix} W_t = \tau \\ U_t = 0 \\ L_{t-1} = 0 \end{pmatrix} \succeq \begin{pmatrix} W_t = 0 \\ U_t = 0 \\ L_{t-1} = 0 \end{pmatrix} + \begin{pmatrix} W_t = 0 \\ U_t = 0 \\ L_{t-1} = -1 \end{pmatrix} \\
\Leftrightarrow &\begin{pmatrix} W_t = \tau \\ U_t = 1 \\ L_{t-1} = -1 \end{pmatrix} \succeq \begin{pmatrix} W_t = 0 \\ U_t = 0 \\ L_{t-1} = -1 \end{pmatrix} \Leftrightarrow \begin{pmatrix} W_t = \tau \\ U_t = 1 \end{pmatrix} \succeq \begin{pmatrix} W_t = 0 \\ U_t = 0 \end{pmatrix}
\end{aligned} \tag{2.6}$$

The economic reason for this result is that each dollar of unrealized capital gains results in a tax burden of $\tau$ dollars. Whereas when $U_t = 1$ and $W_t = \tau$, the $\tau$ dollars of wealth allow for earning profits on these $\tau$ dollars, the investor with $W_t = 0$ and $U_t = 0$ cannot earn these profits. By investing the $\tau$ dollars in the risk-free asset, its value is always at least as high as the present unrealized capital gain.

### 2.2.2 The Optimal Tax-Timing Strategy

In the following, the investment decision at time $t$ of an investor endowed with an initial loss carryforward of $L_{t-1}$ is considered. We assume that the return of the risky asset consists only of capital gains, i.e. the asset does not to pay any dividend or interest.[7] If the investor does not trade the risky asset at time $t$, the purchase price of the risky asset does not change and $P_t^* = P_{t-1}^*$. If the investor purchases the asset in period $t$ at price $P_t$, the purchase price is given by $P_t^* = P_t$.

Let $P_{t'} \in [\inf_{i\in[t,t+1)} P_i, \sup_{i\in[t,t+1)} P_i]$ be some price of the risky asset at time $t' \in [t, t+1)$. In the case that $P_{t'} < P_{t-1}^*$ the investor could realize a net capital loss in period $t$ by trading at price $P_{t'}$. If that loss does not exceed $M_t$ in absolute value, that is, if $-\left(P_{t'} - P_{t-1}^*\right) \leq M_t$, the classical result of Constantinides (1983) applies and the investor should sell the asset to realize that loss and earn the tax rebate on it.

If, however, the net capital loss exceeds $M_t$, i.e. $P_{t'} - P_{t-1}^* < -M_t$, the preconditions under which the result of Constantinides (1983) holds are no longer full-filled.[8] In the following it is shown that it remains optimal to sell the asset even though the tax rebate is potentially lower than in the tax-system with unlimited tax rebate, a potential loss carryforward is a less attractive compensation than a tax refund, and the purchase price $P_t^*$ is increased from $P_{t-1}^*$ to $P_{t'}$, thereby increasing the risk of getting "locked in" in forthcoming periods.

### 2.2.3 The Case with One Risky Asset

To prove that the optimal tax-timing strategy is to realize losses immediately, we consider three strategies of an investor who is initially endowed with one unit of the risky asset at time $t$ acquired at price $P_{t-1}^*$ and who wants to hold one unit of the risky asset from time $t$ to $t+1$.[9] Since all other strategies are linear combinations

[7] We will show later in this section that the optimal tax-timing strategy is not affected by this assumption and does not differ from the optimal tax-timing strategy with an asset that pays dividend or interest.

[8] For $t' > t$, it is also possible that the investor has already realized capital losses during time $[t, t')$ summing up to a value of $G$. In this case Constantinides' result does not apply any longer if $P_{t'} - P_{t-1}^* < \min(0, -M_t - G)$.

[9] It suffices to consider an investor who does not change the number of risky assets in her portfolio. An investor who wants to increase the number of risky assets in her portfolio faces the same tax-timing decision (with potentially different purchase prices after trading in period $t$) as an investor who does not change the number of risky assets in her portfolio. An investor who decreases the

of these three strategies, it suffices to show that one of these strategies is at least as good as the two other strategies.[10] First, the investor can sell the risky asset to realize the unrealized net capital loss, and immediately repurchase it (strategy one). Second, the investor can hold the asset and do no transactions (strategy two). Third, the investor can sell just enough of the risky asset to realize the maximum loss that can be offset in period $t$, and repurchase the sold amount of the risky asset immediately (strategy three). In case the loss carryforward $L_{t-1}$ exceeds the upper limit qualifying for tax rebates, the investor does not even have to sell any assets to realize the desired capital loss and strategies two and three coincide.

All other tax-timing strategies are linear combinations of these three strategies. Any strategy selling a fraction of the risky asset which is greater than that in strategy three, but less than that in strategy one results in a portfolio and a loss carryforward that is a linear combination of those of strategy one and three. Accordingly, any strategy selling some fraction of the risky asset which is less that that of strategy three, but more that that of strategy two results in a portfolio and a loss carryforward that is a linear combination of those of strategies two and three. To prove that strategy one is an optimal tax-timing strategy, it thus suffices to show that strategy one performs at least as good as strategies two and three.

The three strategies at time $t$ only differ in their purchase price of the risky asset $P_t^*$, the initial loss carryforward $L_t$, the unrealized capital gain $U_{t+1}$, and the investor's wealth $W_{t+1}$ at the beginning of period $t+1$ before trading.

When the investor follows strategy one and sells the risky asset, a net capital loss of $P_{t'} - P_{t-1}^*$ is realized and the purchase price decreases to $P_t^* = P_{t'}$. As $P_{t'} - P_{t-1}^* < -M_t \Rightarrow P_{t'} - P_{t-1}^* + L_{t-1} < -M_t$, the taxable net capital loss is

$$T_t^{(1)} = \max\left(P_{t'} - P_{t-1}^* + L_{t-1}; -M_t\right) = -M_t. \tag{2.7}$$

Thus, the tax refund is $M_t\tau$ dollars. The remaining loss carryforward is given by

$$L_t^{(1)} = P_{t'} - P_{t-1}^* + M_t + L_{t-1}. \tag{2.8}$$

number of risky assets in her portfolio faces a given minimum realized net capital loss which is equivalent to a higher given initial loss carryforward from a technical perspective.

[10] To derive the optimal tax-timing strategy of an investor who additionally holds some risk-free bonds from time $t$ to $t+1$, it suffices to analyze the case of an investor who holds only one unit of the risky asset since the return on the risk-free bonds do not have an impact on optimal tax-timing.

If the investor follows strategy two and does not do any transactions in period $t$, the purchase price remains at $P_t^* = P_{t-1}^*$, the net capital loss is

$$T_t^{(2)} = \max\left(L_{t-1}; -M_t\right). \tag{2.9}$$

Thus, the tax refund is $\max\left(L_{t-1}; -M_t\right)\tau$ and the remaining loss carryforward is

$$L_t^{(2)} = L_{t-1} - \max\left(L_{t-1}; -M_t\right). \tag{2.10}$$

If the investor follows strategy three, an investment strategy is chosen such that the net capital loss is given by

$$T_t^{(3)} = -M_t \tag{2.11}$$

and accordingly, the tax refund under strategy three is $M_t\tau$. The remaining loss carryforward is

$$L_t^{(3)} = 0, \tag{2.12}$$

Let $W_t^{(i)}$ denote the pre-tax wealth in period $t$ of strategy $i$ ($i \in \mathbb{N}_3 \equiv \{n \in \mathbb{N} | n \leq 3\}$) before trading. Then

$$W_{t+1}^{(1)} = P_{t+1} + M_t\tau \exp\Big(r\left(t+1-t'\right)\Big) \tag{2.13}$$

$$W_{t+1}^{(2)} = P_{t+1} - \max\left(L_{t-1}; -M_t\right)\tau \exp\Big(r\left(t+1-t'\right)\Big) \tag{2.14}$$

$$W_{t+1}^{(3)} = P_{t+1} + M_t\tau \exp\Big(r\left(t+1-t'\right)\Big). \tag{2.15}$$

If the investor follows tax-timing strategy three, two cases have to be distinguished concerning the amount of the risky asset to be sold. First, if $\max\left(L_{t-1}; -M_t\right) = -M_t$, then the loss carryforward $L_{t-1}$ from period $t-1$ suffices to realize the desired net capital loss in period $t$. In this case, the investor does not have to do any transactions, and strategies two and three coincide. For case three, it thus suffices to consider the case that $L_{t-1} > -M_t$ in which the investor still has to sell some fraction of the risky assets. The amount of the risky assets the investor has to sell is then equivalent to a fraction $f$ of the risky asset, such that $-M_t = f\left(P_{t'} - P_{t-1}^*\right) + L_{t-1} \Leftrightarrow$

**Comparison of Investment Strategies**

| | strategy one | strategy two | strategy three |
|---|---|---|---|
| $P_t^*$ | $P_{t'}$ | $P_{t-1}^*$ | mixed |
| $W_{t+1}$ | $P_{t+1}+M_t\tau\exp\Big(r(t+1-t')\Big)$ | $P_{t+1}-\max\Big(L_{t-1};-M_t\Big)$ $\cdot\tau\exp\Big(r(t+1-t')\Big)$ | $P_{t+1}+M_t\tau\exp\Big(r(t+1-t')\Big)$ |
| $U_{t+1}$ | $P_{t+1}-P_{t'}$ | $P_{t+1}-P_{t-1}^*$ | $P_{t+1}-P_{t-1}^*+M_t+L_{t-1}$ |
| $L_t$ | $P_{t'}-P_{t-1}^*+M_t+L_{t-1}$ | $L_{t-1}-\max\left(L_{t-1};-M_t\right)$ | 0 |

Table 2.1: This table shows the investor's purchase price $P_t^*$, her total wealth $W_{t+1}$, her unrealized capital gains $U_{t+1}$ and her loss carryforward $L_t$ when following strategy one, two or three.

$f=\frac{-M_t-L_{t-1}}{P_{t'}-P_{t-1}^*}$.

Let $U_t^{(i)}$ denote the unrealized capital gains (or losses) in period $t$ of strategy $i$ ($i\in\mathbb{N}_3$) before trading. Then

$$U_{t+1}^{(1)}=P_{t+1}-P_{t'} \tag{2.16}$$

$$U_{t+1}^{(2)}=P_{t+1}-P_{t-1}^* \tag{2.17}$$

$$\begin{aligned}U_{t+1}^{(3)}&=P_{t+1}-\left(fP_{t'}+(1-f)\,P_{t-1}^*\right)\\&=P_{t+1}-\left(\frac{-L_{t-1}-M_t}{P_{t'}-P_{t-1}^*}P_{t'}+\left(1-\frac{-L_{t-1}-M_t}{P_{t'}-P_{t-1}^*}\right)P_{t-1}^*\right)\\&=P_{t+1}-\left(\frac{-L_{t-1}-M_t}{P_{t'}-P_{t-1}^*}\left(P_{t'}-P_{t-1}^*\right)+P_{t-1}^*\right)\\&=P_{t+1}-P_{t-1}^*+L_{t-1}+M_t.\end{aligned} \tag{2.18}$$

Table 2.1 summarizes the properties of the three tax-timing strategies.

With Equation (2.4), it holds in case that $\max\left(L_{t-1};-M_t\right)=L_{t-1}$ for the relation between strategies one and three that

$$\begin{aligned}\begin{pmatrix}W_{t+1}^{(1)}\\U_{t+1}^{(1)}\\L_t^{(1)}\end{pmatrix}&=\begin{pmatrix}P_{t+1}+M_t\tau\exp\Big(r\left(t+1-t'\right)\Big)\\P_{t+1}-P_{t'}\\P_{t'}-P_{t-1}^*+M_t+L_{t-1}\end{pmatrix}\\&\succeq\begin{pmatrix}P_{t+1}+M_t\tau\exp\Big(r\left(t+1-t'\right)\Big)\\P_{t+1}-P_{t-1}^*+M_t+L_{t-1}\\0\end{pmatrix}=\begin{pmatrix}W_{t+1}^{(3)}\\U_{t+1}^{(3)}\\L_t^{(3)}\end{pmatrix}.\end{aligned} \tag{2.19}$$

Thus, strategy one is at least as good as strategy three if $\max\left(L_{t-1};-M_t\right)=L_{t-1}$. The economic reason for this finding is that the loss carryforward of strategy one can be more easily converted to wealth and earn profits than the lower unrealized capital gain of strategy three. In case that $\max(L_{t-1}; -M_t) = -M_t$ strategies two and three coincide. To show that strategy one is an optimal tax-timing strategy it remains to show that strategy one is at least as good as strategy two.

For the relation between strategies one and two, we distinguish two cases. First, if $M_t + L_{t+1} \leq 0 \Leftrightarrow \max(L_{t-1}; -M_t) = -M_t$, it holds with Equation (2.3) that

$$\begin{aligned}
\begin{pmatrix} W_{t+1}^{(1)} \\ U_{t+1}^{(1)} \\ L_t^{(1)} \end{pmatrix} &= \begin{pmatrix} P_{t+1} + M_t \tau \exp\Big(r\left(t+1-t'\right)\Big) \\ P_{t+1} - P_{t'} \\ P_{t'} - P_{t-1}^* + M_t + L_{t-1} \end{pmatrix} \succeq \begin{pmatrix} P_{t+1} + M_t \tau \exp\Big(r\left(t+1-t'\right)\Big) \\ P_{t+1} - P_{t-1}^* \\ L_{t-1} + M_t \end{pmatrix} \\
&= \begin{pmatrix} P_{t+1} - \max\left(L_{t-1}; -M_t\right) \tau \exp\Big(r\left(t+1-t'\right)\Big) \\ P_{t+1} - P_{t-1}^* \\ L_{t-1} - \max\left(L_{t-1}; -M_t\right) \end{pmatrix} = \begin{pmatrix} W_{t+1}^{(2)} \\ U_{t+1}^{(2)} \\ L_t^{(2)} \end{pmatrix}.
\end{aligned} \tag{2.20}$$

Second, if $M_t + L_{t-1} > 0 \Leftrightarrow \max(L_{t-1}; -M_t) = L_{t-1}$, it holds that

$$\begin{aligned}
\begin{pmatrix} W_{t+1}^{(1)} \\ U_{t+1}^{(1)} \\ L_t^{(1)} \end{pmatrix} &= \begin{pmatrix} P_{t+1} + M_t \tau \exp\Big(r\left(t+1-t'\right)\Big) \\ P_{t+1} - P_{t'} \\ P_{t'} - P_{t-1}^* + M_t + L_{t-1} \end{pmatrix} \\
&= \begin{pmatrix} P_{t+1} + \left(M_t + L_{t-1} - L_{t-1}\right) \tau \exp\Big(r\left(t+1-t'\right)\Big) \\ P_{t+1} - P_{t'} \\ P_{t'} - P_{t-1}^* + M_t + L_{t-1} \end{pmatrix} \\
&\succeq \begin{pmatrix} P_{t+1} - L_{t-1} \tau \exp\Big(r\left(t+1-t'\right)\Big) \\ P_{t+1} - P_{t'} \\ P_{t'} - P_{t-1}^* + M_t + L_{t-1} - \left(M_t + L_{t-1}\right) \end{pmatrix} \succeq \begin{pmatrix} P_{t+1} - L_{t-1} \tau \exp\Big(r\left(t+1-t'\right)\Big) \\ P_{t+1} - P_{t-1}^* \\ L_{t-1} - L_{t-1} \end{pmatrix} \\
&= \begin{pmatrix} P_{t+1} - \max\left(L_{t-1}; -M_t\right) \tau \exp\Big(r\left(t+1-t'\right)\Big) \\ P_{t+1} - P_{t-1}^* \\ L_{t-1} - \max\left(L_{t-1}; -M_t\right) \end{pmatrix} = \begin{pmatrix} W_{t+1}^{(2)} \\ U_{t+1}^{(2)} \\ L_t^{(2)} \end{pmatrix}.
\end{aligned} \tag{2.21}$$

Thus, strategy one is at least as good as strategy two, which shows that independent from the realization of future prices $P_{t+1}$ of the stock and the relation of the maximum loss deduction $M_t$ to the initial loss carryforward $L_{t-1}$, strategy one always does at least as good as strategies two and three. Furthermore, strategy one sometimes results in higher wealth than strategy two by allowing to earn the risk-free interest rate on the tax rebates. Hence, strategy one is an optimal tax-timing strategy and unrealized capital losses should be realized immediately.

Furthermore, if $P_{t'} \neq \inf_{i \in [t,t+1)} P_i$, the investor can still increase the realized loss in period $t$ by trading whenever the price of the asset is below the purchase price. In this case the above results with $P_{t'} = \inf_{i \in [t,t+1)} P_i$ apply.

It has thus far been assumed that the risky asset does not pay any dividend or interest. If, however, the risky asset does pay some dividend or interest, all strategies are affected from these payments in the same way, since under all three strategies, the investor holds one unit of the risky asset and thereby receives the same amount of dividend or interest. Hence, the results derived above also hold for risky assets whose returns consist of both capital gains and dividend or interest payments.

### 2.2.4 The Case with Multiple Risky Assets

If the investor only holds one risky asset, it is optimal for the investor to realize losses at time $t$ immediately in order to earn the interest on the tax rebate if $M_t > 0$. If $M_t = 0$, there is no tax rebate and the investor can only use a loss carryforward to reduce future realized capital gains. Consequently, for the one-asset case when $M_t = 0$, both strategies one and three and any combination of them is an optimal tax-timing strategy, as there are no tax rebates that allow for earning an extra return.

In the one-asset case, the investor never faces the situation in which one of the assets comes with an unrealized capital gain and another is endowed with an unrealized capital loss. That being so, a net capital loss that exceeds the amount of $M_t$ can only be carried forward. If, however, the investor holds more than one risky asset, a loss carryforward $L_{t-1}$ realized in some asset $S_1$ in period $t$ can be used to reduce net capital gains from some other asset $S_2$ realized in some period $k \geq t$ if the investor wants to reallocate the portfolio. As, in contrast to strategy one, strategy three does not allow for this transfer of realized losses of some asset $S_1$ to some other asset $S_2$, strategy one dominates strategy three in the multiple-asset case.

### 2.2.5 Attainable Wealth

As shown in Constantinides (1983) and above, both in tax systems with limited and unlimited tax rebates, it is optimal to realize losses immediately. Let $W_t^{(l)}$ denote the beginning of period $t$ wealth before trading that an investor following the optimal tax-timing strategy can attain in a tax-system with limited tax rebates, i.e. $W_t^{(l)} \equiv W_t^{(1)}$ . Furthermore, let $W_t^{(u)}$ denote the corresponding wealth the investor can attain in a tax-system with unlimited tax rebates following the optimal tax-timing strategy to realize losses in the period they occur. As there are no loss carryforwards in tax-systems with unlimited tax rebates, it is assumed that $L_{t-1} = 0$ in the tax-system with limited tax rebates to make the two tax-systems comparable. In case an investor realizes a capital loss in period $t$ that does not exceed $M_t$, the evolution of the wealth from $t$ to $t+1$ is the same in both tax systems. If, however, the capital loss exceeds $M_t$, i.e. $P_{t'} - P_{t-1}^* < -M_t$, then

$$W_{t+1}^{(l)} = P_{t+1} + M_t \tau \exp\Big(r\left(t+1-t'\right)\Big) \tag{2.22}$$

$$U_{t+1}^{(l)} = P_{t+1} - P_t \tag{2.23}$$

$$L_t^{(l)} = P_{t'} - P_{t-1}^* + M_t \tag{2.24}$$

and

$$W_{t+1}^{(u)} = P_{t+1} - \left(P_{t'} - P_{t-1}^*\right) \tau \exp\Big(r\left(t+1-t'\right)\Big) \tag{2.25}$$

$$U_{t+1}^{(u)} = P_{t+1} - P_{t'} \tag{2.26}$$

$$L_t^{(u)} = 0. \tag{2.27}$$

With Equation (2.3) it holds that

$$\begin{aligned} \begin{pmatrix} W_{t+1}^{(u)} \\ U_{t+1}^{(u)} \\ L_t^{(u)} \end{pmatrix} &= \begin{pmatrix} P_{t+1} + \left(M_t \tau - \left(P_{t'} - P_{t-1}^* + M_t\right)\tau\right) \exp\Big(r\left(t+1-t'\right)\Big) \\ P_{t+1} - P_{t'} \\ 0 \end{pmatrix} \\ &\succeq \begin{pmatrix} P_{t+1} + M_t \tau \exp\Big(r\left(t+1-t'\right)\Big) \\ P_{t+1} - P_{t'} \\ P_{t'} - P_{t-1}^* + M_t \end{pmatrix} = \begin{pmatrix} W_{t+1}^{(l)} \\ U_{t+1}^{(l)} \\ L_t^{(l)} \end{pmatrix}. \end{aligned} \tag{2.28}$$

Thus, not very surprisingly, an investment opportunity set with unlimited tax rebates is preferable to an investment opportunity set with limited tax rebates. The advantage of the investment opportunity set with unlimited tax rebates is the opportunity to get an unlimited tax rebate on capital losses and earn the interest on these losses, as in a tax-system with limited tax rebates, no interest is paid on the loss carryforward. Furthermore, one dollar of cash at hand can be used much more flexibly than one dollar of loss carryforward – especially when limits on maximum losses qualifying for tax rebates $M_1, \ldots, M_T$ are small.

## 2.3 Capital Gains Taxation and Asset Allocation

Having derived the optimal tax-timing strategy for an investor endowed with an initial loss carryforward, we now turn to the impact of the tax-timing option and limits on tax rebates for asset allocation.

In the absence of a tax-timing option and tax rebates on capital losses, an investor maximizing utility from terminal wealth whose wealth does not face exogenous increases or decreases, for instance through non-financial income or consumption, would hold the same fraction of stocks in each period, according to the classical result of Merton (1969) and Samuelson (1969). Thus, in such a setting, each deviation of the investor's asset allocation from this benchmark is due to the tax-timing option and the limits on tax rebates. This section discusses how taxation of capital gains and in particular, how limits on capital loss deduction affect the investor's asset allocation.

The taxation of capital gains differs in four ways from the Merton-Samuelson benchmark. First, capital gains are often taxed at lower tax-rates than dividends and interest. Second, capital gains are only taxable when realized. That is, the investor has a deferral option, which is exercised by not selling the asset. Third, net capital losses only qualify for tax rebates up to a certain limit. Fourth, capital gains are forgiven when being bequeathed.

It is true that the different tax-rates applied to capital gains and dividends and interest can still be analyzed in a model without taxes by applying the after-tax risk-return profiles of the two assets. However, the deferral option, limits on tax rebates, and the forgiveness of capital gains when being bequeathed deserve explicit modeling.

The deferral option allows the investor to earn compound returns on the taxes that have been postponed, which reduces the effective capital gains rate (Chay et al. (2006)). On the other hand, postponing the realization of capital gains can result in unbalanced portfolios. In particular when unrealized capital gains are substantial, rebalancing the portfolio can result in substantial capital gains tax payments, which make it even less attractive to sell an asset with a substantial unrealized capital gain. This is also referred to as the danger of getting "locked in."

In a tax-system with unlimited tax rebates, capital gains and capital losses are treated symmetrically. That is, it does not matter if a net capital gain or a net capital loss is realized; the investor is confronted with the same taxable treatment. In tax-systems with limited tax rebates, however, there is an asymmetric taxation of capital gains and losses. Although capital gains are taxed at the capital gains tax-rate independent of their amount, capital losses qualify for tax rebates only up to a certain limit. The investor thus keeps the fraction $1 - \tau$ of realized capital gains, but bears the entire risk for losses exceeding the maximum loss deduction if there is no initial loss carryforward. The compensation for this risk comes as a loss carryforward. However, in contrast to tax rebates, a loss carryforward does not pay any interest.

Furthermore, a loss carryforward also bears the risk of never being used and thus ending up worthless. This risk is especially important if the remaining investment horizon of the investor is short. Therefore, in comparison with a tax-system with unlimited tax rebates in the absence of an initial loss carryforward, an investment into a risky asset offers the same opportunities to the investor when returns are positive, but bears higher risks when returns are negative. As a result, in a tax-system with limited tax rebates, investors that are not endowed with an initial loss carryforward will hold less risky assets than in a tax-system with unlimited tax rebates. The higher the level of the initial loss carryforward, the lower the investor's advantage of an additional loss carryforward since the risk that this additional loss carryforward remains unused is higher when with the investor already has some loss carryforward. Furthermore, even if it is entirely used, the time passed until its entire usage is longer, which means it will take longer until the investor receives the interest payments on the tax rebates.

The size of the advantage to an investor without a loss carryforward who has the opportunity to invest in a tax-system with unlimited tax rebates instead of a tax-system with limited tax rebates depends on seven factors. First, it depends on $M_1, \ldots, M_T$, the amounts up to which realized losses qualify for tax rebates. The higher these values are, the lower the advantage is of the tax-system with unlimited tax rebates. Second, it depends on the capital gains tax-rate $\tau$. The higher $\tau$ is, the higher the tax rebates are, and thus the more advantageous the tax-system with unlimited tax rebates is. Third, it depends on the evolution of the price of the risky asset, $P_1, \ldots, P_T$. The earlier and the higher capital losses are that exceed $M_1, \ldots, M_T$, the bigger the advantage is in the tax-system with unlimited tax rebates. As one can see, the more volatile the risky asset, the higher the disadvantage of being in a tax-system with limited tax rebates. Hence, in tax-systems with limited capital loss deduction, investors will decrease their holdings in risky assets when those assets become more volatile. Fourth, the lower the risk-aversion of an investor is, the higher the exposure to risky assets is, and consequently, the higher the disadvantage is when being confronted with a tax-system with limited tax rebates. Fifth, the higher the risk-free rate is, the higher the disadvantage is of the loss carryforward not paying any interest, and thus the higher the advantage is of the tax-system with unlimited tax rebates. Sixth, the higher the level of the investor's wealth is, the higher the probability is of exceeding the lower bound on maximum losses qualifying for tax rebates for a given equity proportion. Seventh, the investor's advantage depends on the investor's remaining investment horizon. The longer the investment horizon is, the bigger the advantage is of the tax-system with unlimited tax rebates due to the compounding of interest on tax rebates.

If, however, the investor has an initial loss carryforward, this has a positive impact on the after-tax risk-return profile of the risky asset, as it allows the investor to earn an amount of capital gains not exceeding the loss carryforward tax-free. As a result of this, if the investor has some loss carryforward, there is no longer a dominating relationship between the two tax-systems from the investor's point of view. On the one hand, the investor bears the risk that the treasury does not participate in high losses via tax rebates. On the other hand, the investor has the opportunity of earning some capital gains tax-free due to the initial loss carryforward.

Given the parameters of the tax-system, the investor's initial wealth, the length of the remaining investment horizon, and the distribution of future changes in the price of the risky asset, having sold the assets with unrealized capital losses, the investment decision depends on three factors: first, the loss carryforward, second, the remaining unrealized capital gains, and third the remaining initial equity proportion. Altogether, six different cases can be distinguished.

First, when the investor is endowed with an initial loss carryforward, unrealized capital gains and an equity proportion below the optimal level, the investor can increase her equity proportion to the optimal level. Compared to the case with no initial loss carryforward, the optimal equity proportion will be higher, as the loss carryforward allows earning future profits tax-free. In particular, when the remaining investment horizon is short, this effect is important, as otherwise the loss carryforward does not have any value to the investor.

Second, when the investor has an initial loss carryforward, unrealized capital gains and a high initial equity proportion, the investor can use the loss carryforward to rebalance her portfolio without having to pay the capital gains tax for unrealized gains not exceeding the loss carryforward.

Third, when the investor does not have an initial loss carryforward, but has unrealized capital gains, and the initial equity proportion is above the optimal equity exposure of an investor who is not endowed with unrealized capital gains, the desire to rebalance the portfolio and the desire to defer the realization of capital gains are opposing effects. The higher the initial equity proportion is, and the lower the unrealized capital gain is per unit of the risky asset, the higher the probability is that the desire to rebalance the portfolio outweighs the desire to defer the realization of capital gains and that the investor hence realizes at least part of the unrealized capital gains.

Fourth, when the investor has no initial loss carryforward, but has unrealized capital gains, and the initial equity proportion is low, the investor will increase the equity proportion to the optimal level, as this transaction does not confront her with capital gains taxes.

Fifth, when the investor has an initial loss carryforward, but no unrealized capital gains, the initial equity proportion does not have an impact on the optimal equity proportion, as the investor can trade without facing capital gains tax-payments.

Sixth, when the investor has neither an initial loss carryforward nor an unrealized capital gain, the investor holds a lower equity proportion than in the case with an initial loss carryforward, as there is no opportunity of earning positive future capital gains tax-free.

In total, a limitation of capital loss deduction should result in a lower equity exposure than in a tax-system with unlimited tax rebates. The stricter the limitation, the lower the equity exposure for an investor who does not have an initial loss carryforward.

## 2.4 Optimal Tax-Timing over the Life Cycle

Having derived optimal tax-timing decisions for investors endowed with unrealized capital losses and discussed the impact of the tax-timing option and limits on tax rebates for the investor's asset allocation, we now turn to optimal tax-timing strategies over the life cycle. The model in this section builds on Dammon et al. (2001). We consider an economy consisting of short-selling constrained investors living for at most $T$ years, who can only trade at time $t = 0, 1, \ldots, T$. $F(t)$ denotes the probability that the investor is still alive through period $t$ ($t \leq T$). We assume the investor to derive utility from the consumption $C_t$ of a single good at time $t$ and bequest, and have CRRA-utility with a parameter of risk-aversion of $\gamma \in [0, \infty)$, i.e.

$$U(C_t) \equiv \begin{cases} \frac{C_t^{1-\gamma}}{1-\gamma} & \text{for } \gamma \neq 1 \\ \ln(C_t) & \text{for } \gamma = 1. \end{cases} \tag{2.29}$$

The parameter $\gamma$ represents the investor's willingness to substitute consumption among different states in time. It also represents the elasticity of consumption, which is given by $\frac{1}{\gamma}$. The investor's task is to optimize the discounted expected utility of lifetime consumption and bequest, given an initial endowment, subject to the intertemporal budget constraint. The parameter $\beta$ represents the investor's utility discount factor, and $i$ is a constant annual inflation rate. Following Dammon et al. (2001), we assume that at the time of death, the investor's remaining wealth is used to purchase an $H$-period annuity payable to the investor's beneficiary and that the $H$-period annuity provides the beneficiary with nominal consumption of $A_H W_t (1+i)^{k-t}$ at date $k$ ($t + 1 \leq k \leq t + H$), in which $A_H \equiv \frac{r^*(1+r^*)^H}{(1+r^*)^H - 1}$ is the

$H$-period annuity factor, $r^* = \frac{(1-\tau_d)r-i}{1+i}$ is the after-tax real bond return, $W_t$ is the investor's total beginning-of-period-$t$-wealth, and $i$ is a constant inflation rate. For simplicity, the beneficiary is assumed to have the same preferences as the investor. On this assumption, $H$ can be interpreted as a measure for the investor's bequest motive. The higher $H$ the stronger the bequest motive.

Following Dammon et al. (2001), we consider an investor who is not endowed with non-financial income[11] and assume that the purchase price used to compute capital gains is the average weighted historical purchase price. This assumption assures that stochastic dynamic programming is an efficient method for solving the investor's optimization problem.[12] If $P_t^*$ denotes that purchase price after trading at time $t$, and $q_t$ denotes the number of risky assets the investor holds in period $t$, $P_t^*$ is given by

$$P_t^* = \begin{cases} \frac{q_{t-1}P_{t-1}^* + \max(q_t - q_{t-1}, 0)P_t}{q_{t-1} + \max(q_t - q_{t-1}, 0)} & \text{if } P_{t-1}^* < P_t \\ P_t & \text{if } P_{t-1}^* \geq P_t. \end{cases} \tag{2.30}$$

This specification takes into account that it is optimal to realize capital losses (i.e. $P_{t-1}^* \geq P_t$) immediately, which decreases the average purchase price from $P_{t-1}^*$ to $P_t$. If, however, the investor is endowed with an unrealized capital gain (i.e. $P_{t-1}^* < P_t$) the change in the tax basis depends on the trading in period $t$. If the investor sells some assets (i.e. $q_t < q_{t-1}$), the tax basis remains unchanged. If instead, the investor purchases some assets (i.e. $q_t > q_{t-1}$) the tax basis is a weighted average of the previous tax basis and the purchase price $P_t$ of the new asset. As according to the optimal tax-timing strategy losses shall be realized immediately, the investor's realized net capital gain in period $t$ is given by

$$G_t = \left(\chi_{\{P_{t-1}^* > P_t\}} q_{t-1} + \chi_{\{P_t \geq P_{t-1}^*\}} \max\left(q_{t-1} - q_t, 0\right)\right) \cdot \left(P_t - P_{t-1}^*\right) \tag{2.31}$$

where $\chi_{\{P_{t-1}^* > P_t\}}$ denotes the characteristic function of $P_t$, which is one for $P_{t-1}^* > P_t$

[11] We relax this assumption in section 2.4.5.

[12] In an exact cost basis calculation, it is possible that the investor holds positions in the risky asset with different purchase prices. In particular, the investor should prefer to sell the asset with the highest purchase price first which could allow her to slightly decrease her tax payments. However, DeMiguel and Uppal (2005) show that the certainty equivalent losses from using the average tax basis instead of the exact tax basis is small. The average purchase-price method is e.g. mandatory under current Canadian or Danish tax-law.

and zero otherwise. By $R$ we denote the gross after-tax return of the risk-free asset. $d$ is a constant after-tax dividend of equity, $b_t$ is the number of units of the risk-free asset with purchase price one the investor holds during period $t$.

In a tax-system with unlimited tax rebates, the optimization problem is the same as in Dammon et al. (2001). In a tax-system with limits on tax rebates, the investor's optimizing problem becomes

$$\max_{C_t, q_t} \mathbb{E}\left[\sum_{t=0}^{T} \beta^t \Big( F(t) U\left(\frac{C_t}{(1+i)^t}\right) + (F(t-1) - F(t)) \sum_{k=t+1}^{t+H} \beta^{k-t} U\left(\frac{A_H W_t}{(1+i)^t}\right)\Big)\right] \tag{2.32}$$

s.t.

$$W_t = q_{t-1} \cdot (1+d)\, P_t + b_{t-1} \cdot R, \qquad t = 0, \ldots, T \tag{2.33}$$

$$W_t = \tau T_t + q_t \cdot P_t + b_t + C_t \qquad t = 0, \ldots, T-1 \tag{2.34}$$

$$q_T = 0, b_T = 0 \tag{2.35}$$

$$q_t \geq 0, b_t \geq 0 \qquad t = 0, \ldots, T-1 \tag{2.36}$$

given the initial holding of bonds $b_{-1}$, stocks $q_{-1}$, the initial tax-basis $P^*_{-1}$, the price of one unit of the stock $P_0$, and the initial loss carryforward $L_{-1}$. According to Equation (2.32), the investor maximizes discounted expected utility of lifetime consumption and bequest. Equation (2.33) defines the investor's beginning of period $t$ wealth as the sum of wealth in stocks and wealth in bonds before trading at time $t$, including the after-tax interest and dividend income, but before any capital gains taxes resulting from trading at time $t$. Equation (2.34) is the investor's budget constraint at time $t$. If the investor trades equity, the net capital gain $T_t$ is subject to the capital gains tax-rate.

By letting $X_t$ denote the vector of the investor's state variables, $V_t(.)$ the investor's value function at time $t$, $f(t)$ the probability of surviving from period $t$ to $t+1$ given the investor is alive at the beginning of period $t$, and taking into account that the sum in the last term of the objective function (2.32) can be simplified by taking into account that $\sum_{k=t+1}^{t+H} \beta^{k-t} = \frac{\beta\left(1-\beta^H\right)}{1-\beta}$, the Bellmann equation for the optimization problem can be written as

$$\begin{aligned} V_t(X_t) = \max_{C_t, q_t} \Bigg[ & f(t) U\left(\frac{C_t}{(1+i)^t}\right) + f(t)\beta \mathbb{E}_t\left[V_{t+1}\left(X_{t+1}\right)\right] \\ & + (1-f(t)) \frac{\beta\left(1-\beta^H\right)}{1-\beta} U\left(\frac{A_H W_t}{(1+i)^t}\right)\Bigg] \end{aligned} \tag{2.37}$$

for $t = 0, \ldots, T-1$ subject to Equations (2.1), (2.2), (2.30), (2.31), and (2.33) to (2.36) with terminal condition $V_T\left(X_T\right) = U\left(\frac{A_H W_T}{(1+i)^T}\right)$. The state variables needed to solve the problem at time $t$ are the investor's beginning-of-period-wealth $W_t$ before trading, the initial loss carryforward $L_{t-1}$, the price of the risky asset $P_t$, its tax basis $P^*_{t-1}$, and the number of stocks $q_{t-1}$ the investor is holding at the beginning of period $t$ before trading. Thus, the vector of state variables $X_t$ at time $t$ can be represented as

$$X_t = [P_t, W_t, L_{t-1}, P^*_{t-1}, q_{t-1}]. \tag{2.38}$$

The US tax-code allows a constant amount of \$ 3,000 as the maximum amount that qualifies for tax rebates. Due to this, the impact of this upper bound depends on the investor's level of beginning-of-period wealth. If $M > 0$, investors with higher beginning-of-period-wealth should have a lower fraction invested in equity, since they run a higher risk of facing net capital losses exceeding $M$ than investors with lower beginning-of-period-wealth. In order to reduce the number of state variables and keep our optimization problem numerically tractable, we assume $M_t$ to be a non-positive multiple of $W_t$, i.e. $m \equiv \frac{M_t}{W_t}$ $(t \in \mathbb{N}_T)$ to be some finite non-positive real value.[13]

By the assumption that $M_t$ is a constant multiple of $W_t$, the above optimization problem can be simplified by normalizing with beginning-of-period-wealth $W_t$. Let $s_t \equiv \frac{q_{t-1}P_t}{W_t}$ denote the fraction of the investor's beginning-of-period-wealth before trading invested into equity, $\alpha_t \equiv \frac{q_t P_t}{W_t}$ the investor's fraction of beginning-of-period-wealth allocated to equity after trading, $b'_t \equiv \frac{b_t}{W_t}$ the fraction of the beginning-of-period-wealth allocated to risk-free bonds after trading, $c_t \equiv \frac{C_t}{W_t}$ the consumption-

[13]Except for $m = 0$, this approach does not capture the impact of changes in beginning-of-period wealth $W_t$ on the relation between $M$ and $W_t$. However, the impact of different wealth levels on optimal asset allocation can be analyzed by varying $m$.

wealth-ratio, $p^*_{t-1} \equiv \frac{P^*_{t-1}}{P_t}$ the investors basis-price-ratio, $t_t \equiv \frac{T_t}{W_t}$ the fraction of the investor's beginning-of-period-wealth that is taxable at the capital gains tax-rate, $l_{t-1} \equiv \frac{L_{t-1}}{W_t}$ the fraction of the investor's loss carryforward to beginning-of-period-wealth, $g_t \equiv \frac{P_{t+1}}{P_t} - 1$ the capital gain on the stock in period $t$, and

$$R_t \equiv \frac{\alpha_t (1+d)(1+g_t) + b'_t R}{\alpha_t + b'_t} \tag{2.39}$$

the gross nominal return on the investor's portfolio after trading in period $t$ and payment of taxes on dividends and interest, but before payment of capital gains taxes.

Defining $v_t(x_t) \equiv \frac{V_t(X_t)}{W_t^{1-\gamma}}$ to be the normalized value function, $\rho_t \equiv \frac{W_{t+1}}{W_t(1+i)}$ to be the investor's real growth of wealth before capital gains taxes, the investor's optimization problem can be rewritten as

$$\begin{aligned} v_t(x_t) = \max_{c_t, \alpha_t} \Bigg[ & f(t)U(c_t) + f(t)\beta \mathbb{E}\left[v_{t+1}(x_{t+1})\,\rho_t^{1-\gamma}\right] \\ & + (1-f(t))\frac{\beta\left(1-\beta^H\right)}{1-\beta} U(A_H) \Bigg] \end{aligned} \tag{2.40}$$

s.t.

$$1 = \tau t_t + \alpha_t + b'_t + c_t \qquad t = 0, \ldots, T-1 \tag{2.41}$$

$$\rho_t = (1 - \tau t_t) R_t \qquad t = 0, \ldots, T-1 \tag{2.42}$$

$$\alpha_t, b'_t \geq 0 \qquad t = 0, \ldots, T-1 \tag{2.43}$$

in which $t_t$ is given by

$$t_t = \max(\delta_t + l_{t-1}; -m) \tag{2.44}$$

The fraction of realized gains to beginning-of-period-wealth needed to compute $t_t$ is given by

$$\delta_t \equiv \frac{G_t}{W_t} = \left(\chi_{\{p^*_{t-1} > 1\}} s_t + \chi_{\{p^*_{t-1} \leq 1\}} \max(s_t - \alpha_t, 0)\right) \cdot \left(1 - p^*_{t-1}\right) \tag{2.45}$$

and $p_t^*$ is given by

$$p_t^* = \begin{cases} \frac{s_t p_{t-1}^* + \max(\alpha_t - s_t;0)}{(s_t + \max(\alpha_t - s_t;0))(g_t+1)} & p_{t-1}^* < 1 \\ \frac{1}{g_t+1} & \text{otherwise} \end{cases} \tag{2.46}$$

At time $T$, the investor's value function takes the value

$$v_T = \frac{\beta\left(1-\beta^H\right)}{1-\beta} U\left(A_H\right) \tag{2.47}$$

in all states. Our optimization problem can be solved numerically using backward induction with state variables $x_t = [s_t, p_{t-1}^*, l_{t-1}]$. To do so, a $41 \times 40 \times 41$ grid with equally distributed grid points over [0,1], [0,1.5] and [-1,0] is spanned. For values of $[s_t, p_{t-1}^*, l_{t-1}]$ between the grid, cubic spline interpolation is performed. To expedite computation, the interpolation function for each of the two problems is computed symbolically as a function of $s_t$, $p_{t-1}^*$, and $l_{t-1}$ for each period $t$. The cubic spline interpolation at time $t$ can then be performed by plugging $s_t$, $p_{t-1}^*$, and $l_{t-1}$ into this interpolation function. The integral in the expectation in Equation (2.40) is computed using Gaussian quadrature. The optimization problem with unlimited tax rebates with state variables $x_t = [s_t, p_{t-1}^*]$ is solved accordingly for a $41 \times 40$ grid.

For the numerical analysis, it is assumed, that annual inflation is $i = 3.5\%$, and mandatory retirement age is $J = 66$, indicating that the investor still works at age 65 and retires when attaining age 66. The pre-tax risk-free rate is 6%. The return on equity is lognormally distributed, independent in time, comes with an expected capital gain of $\mu = 7\%$ $(t \in \mathbb{N}_T)$, a standard deviation of $\sigma = 20.7\%$ (which corresponds to a standard deviation of the real return of about 20%) and a constant pre-tax dividend rate of 2% in each period.

The correct choice of the risk-premium for equity has been subject to numerous theoretical and empirical research (see Siegel (2005) for a survey). While the historical risk-premium has been about 6% (Mehra and Prescott (1985)) in the US since 1872, economists doubt whether this will be true in future periods. We follow the current consensus which is about 3% to 4% (see e.g. Dammon et al. (2001, 2004) Cocco et al. (2005), and Gomes and Michaelides (2005)).

The tax-rate on interest, dividends and income is assumed to be $\tau_d = 36\%$. The

**Base-case Parameter Values**

| Description | Parameter | Value |
|---|---|---|
| Risk-aversion | $\gamma$ | 3 |
| Length of investment horizon | $T$ | 80 |
| Number of years annuity beneficiary | $H$ | 60 |
| Mandatory retirement age | $J$ | 66 |
| Discount factor | $\beta$ | 0.96 |
| Dividend rate | $d$ | 2% |
| Expected capital gains rate stock | $\mu$ | 7% |
| Standard deviation stock | $\sigma$ | 20.7% |
| Interest payment of bond | $r$ | 6% |
| Inflation rate | $i$ | 3.5% |
| Tax-rate on dividends, interest and income | $\tau_d$ | 36% |
| Tax-rate on capital gains | $\tau_g$ | 20% |

Table 2.2: This table reports the parameter values used in the base-case.

tax-rate on realized capital gains is assumed to be $\tau_g = 20\%$. Since short-term capital gains are usually subject to the same tax-rate as dividends and interest, this implies the assumption that the investor realizes capital gains long-term, which is can be the case, for example, when the investor sells them after one year and one day. We assume the investor makes decisions annually starting at age 20 ($t = 0$). The maximum age the investor can attain is set to 100 years ($T = 80$). It is also assumed that the relative risk-aversion of the investor is $\gamma = 3$ and the annual subjective utility discount factor is $\beta = 0.96$. We assume the investor finances consumption entirely from her investments and does not have any non-financial income (we relax this assumption in section 2.4.5). This choice of parameters follows Gallmeyer et al. (2006) and is quite similar to that of Dammon et al. (2001). Initially, $H$ is set to $H = 60$ in the bequest function, indicating the investor wishes to provide the beneficiary with an income stream for the next 60 years. The data for the survival probabilities of our investor were set equal to the survival probabilities for female investors according to the 2001 Commissioners Standard Ordinary Mortality Table. Table 2.2 summarizes these parameters for the base-case.

w We first turn to the base-case scenario and analyze optimal consumption and asset allocation in sections 2.4.1 and 2.4.2. In section 2.4.3, we quantify the value of an initial loss carryforward. Section 2.4.4 compares optimal asset allocation in tax-systems with limited capital loss deduction and unlimited capital loss deduction. Section 2.4.5 explores the sensitivity of our results to non-financial income and tax-systems that allow for tax rebates (i.e. $m > 0$).

### 2.4.1 Optimal Consumption Policy

We begin the discussion of our numerical results for the base-case in a life cycle model by presenting the investor's optimal consumption policy depending on her age, her initial fraction of stocks, her unrealized capital gains and losses and her initial loss carryforward. As a benchmark case, we consider an investor confronted with a tax-system that does not pose any limits on tax rebates. The relation between the optimal consumption-wealth-ration depends heavily on the investor's bequest motive. As the investor ages her mortality rate increases such that the impact of her bequest motive on her consumption and investment decision increases. The bequest motive used in the base-case is of such magnitude that the investor increases her consumption slightly over the life cycle. For high initial levels of the equity exposure and unrealized capital gains, the investor is "locked in," and her optimal consumption also depends on the capital gains tax she faces to finance her consumption and rebalance her portfolio.

Figure 2.1 shows the investor's optimal consumption-wealth-ratio. The investor's basis-price-ratio indicates whether she faces unrealized gains or losses. For $p^* < 1$, the investor faces unrealized capital gains that become higher as $p^*$ becomes lower. If $p^* = 1$, the investor neither faces unrealized capital gains nor losses, whereas if $p^* > 1$, the investor is endowed with an unrealized capital loss.

The upper left graph depicts the consumption-wealth-ratio depending on age and initial equity proportion in a tax-system that does not allow for any tax rebates on realized capital losses ($m = 0$) for an investor who is has no initial loss carryforward ($l = 0$), and whose basis-price-ratio is $p^* = 0.5$, indicating that the investor faces substantial unrealized capital gains in her equity. The upper right graph shows the optimal consumption-wealth-ratio for an investor who is endowed with an initial loss carryforward of 10% of her beginning-of-period-wealth ($l = -0.1$).

The upper graphs in Figure 2.1 show that the investor's optimal consumption-wealth-ratio is slightly increasing with her age. This increase is due to the fact that due to increasing mortality rates and the forgiveness of capital gains when being bequeathed the value of the tax-timing option decreases as the investor ages. The decreasing value of the tax-timing option tends to decrease the desire to hold

**Optimal Consumption Policy**

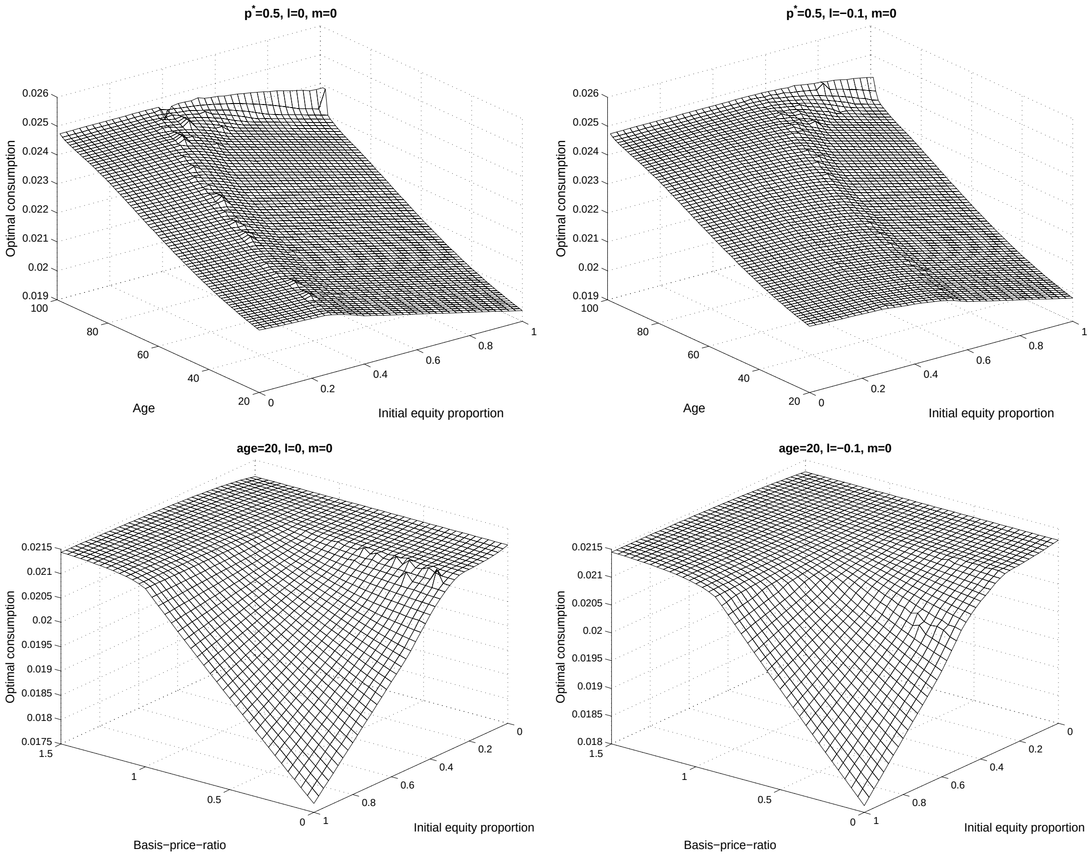


Figure 2.1: This Figure shows the investor's optimal consumption-wealth-ratio (optimal consumption) in a tax-system with no tax rebates on capital losses ($m = 0$). The investor is either endowed with no initial loss carryforward ($l = 0$, left graphs) or an initial loss carryforward of 10% of her beginning-of-period wealth ($l = -0.1$, right graphs). The upper graphs depict the investor's optimal consumption-wealth-ratio depending on her age and her initial equity proportion when the investor is facing substantial unrealized capital gains in her equity ($p^* = 0.5$), the lower graphs depict her optimal consumption-wealth-ratio at age 20 depending on her initial equity proportion and her basis-price-ratio.

equity thereby reducing the expected return on investments which increases present consumption.[14] The upper left graph further shows that the investor decreases her consumption with increasing initial equity proportion. This decline is due to the fact that her basis-price-ratio is smaller than one, and she thus faces unrealized capital gains. If the investor's initial equity proportion is of significant height, she is "locked in". To finance her consumption, she therefore either has to decrease her risk-free bond position, which leaves her with a badly diversified portfolio, or she has to sell some fraction of her equity, which confronts her with capital gains tax payments.

[14]However, due to the forgiveness of capital gains when being bequeathed there is an opposing effect on the investor's equity exposure. This effect will be discussed in more detail in section 2.4.2.

In both cases, she decreases her consumption to either improve diversification or decrease the amount of capital gains tax to pay.

If, however, the investor is endowed with an initial loss carryforward (upper right graph), she uses that loss carryforward to realize capital gains and avoid the capital gains tax. Due to her initial loss carryforward, she can thus avoid either ending up with an unbalanced portfolio or paying the capital gains tax. Consequently, she only decreases her consumption-wealth-ratio if her initial loss carryforward is not sufficient to rebalance her portfolio and withdraw funds to finance her consumption.

The lower left graph in Figure 2.1 shows the optimal consumption-wealth-ratio of an investor at age 20 who has no initial loss carryforward ($l = 0$) in a tax-system that does not allow for any tax rebates for realized capital losses ($m = 0$) depending on her initial equity proportion and her initial basis-price-ratio. The lower right graph shows her optimal consumption-wealth-ratio when she is given an initial loss carryforward of 10% of her beginning-of-period wealth ($l = -0.1$).

When a 20-year old investor is given an initial loss carryforward (left graph), she decreases her consumption if she has to sell equity with unrealized capital gains to lower her capital gains tax payments. The investor faces unrealized capital gains if her basis-price-ratio is smaller than one. If, in addition, her initial equity proportion exceeds the optimal equity proportion she would choose if she was not confronted with the capital gains tax-rate, she is "locked in". To finance her consumption, she either has to decrease the fraction of risk-free bonds in her portfolio, which might leave her with an unbalanced portfolio, or she has to sell equity with unrealized capital gains and pay the capital gains tax-rate. Since both options are quite unattractive, she reduces her consumption when being "locked in". The higher her initial equity proportion and the lower her basis-price-ratio, the higher her unrealized capital gains. Since higher unrealized capital gains confront the investor with higher capital gains tax payments when rebalancing her portfolio, her consumption wealth-ratio is lower when facing a high initial equity proportion and a low basis-price-ratio.

If, however, the investor has an initial loss carryforward (lower right graph), she can avoid the capital gains tax-payment on realized capital gains. Consequently, her consumption is above the level of an investor has no initial loss carryforward. However, her consumption is below the level of an investor who does not have unrealized capital gains. This reflects the fact that the investor wishes to keep a certain loss carryforward for future periods and thus does not use her entire loss carryforward at once.

### 2.4.2 Optimal Investment Policy

Having discussed the optimal consumption policy in the previous section, we now turn to the optimal investment policy in the base-case scenario. This policy depends on the investor's basis-price-ratio, her initial equity exposure, her initial loss carryforward, and the length of her remaining investment horizon. The basis-price-ratio indicates whether the investor faces an unrealized capital gain or loss, and thereby influences potential capital gains tax-payments when trading equity. The initial equity proportion indicates to which extend the investor is affected by the unrealized capital gains or losses. An initial loss carryforward provides the investor with the opportunity of avoiding capital gains tax-payments when rebalancing her portfolio. The length of the remaining investment horizon has an impact on the investor's optimal tax-timing and asset allocation due to the forgiveness of capital gains when being bequeathed. Due to the uncertain death of the investor, the length of the remaining investment horizon is not known in advance. However, its distribution depends on the investor's age since mortality rates are increasing as the investor ages.

Figure 2.2 shows the investor's optimal equity proportion after trading in our base-case setting in which the tax-system does not provide any tax rebates. The upper left graph depicts the investor's optimal equity proportion as a function of age, and the initial equity proportion for an investor who has no initial loss carryforward ($l = 0$), and whose initial basis-price-ratio is $p^* = 0.5$, such that the investor faces substantial unrealized capital gains in her equity. The upper right graph shows the optimal equity proportion for an investor who has an initial loss carryforward of 10% of her beginning-of-period-wealth ($l = -0.1$). Compared to the benchmark case with no tax-timing option (not shown here), the investor's equity exposure increases significantly, which reflects the value of the tax-timing option.

The upper left graph in Figure 2.2 shows that the investor's optimal equity proportion depends on both her initial equity proportion and her age. As the investor gets older, she sharply increases her optimal equity proportion due to the forgiveness of capital gains when being bequeathed. This increase is of tremendous size if she

**Optimal Investment Policy**

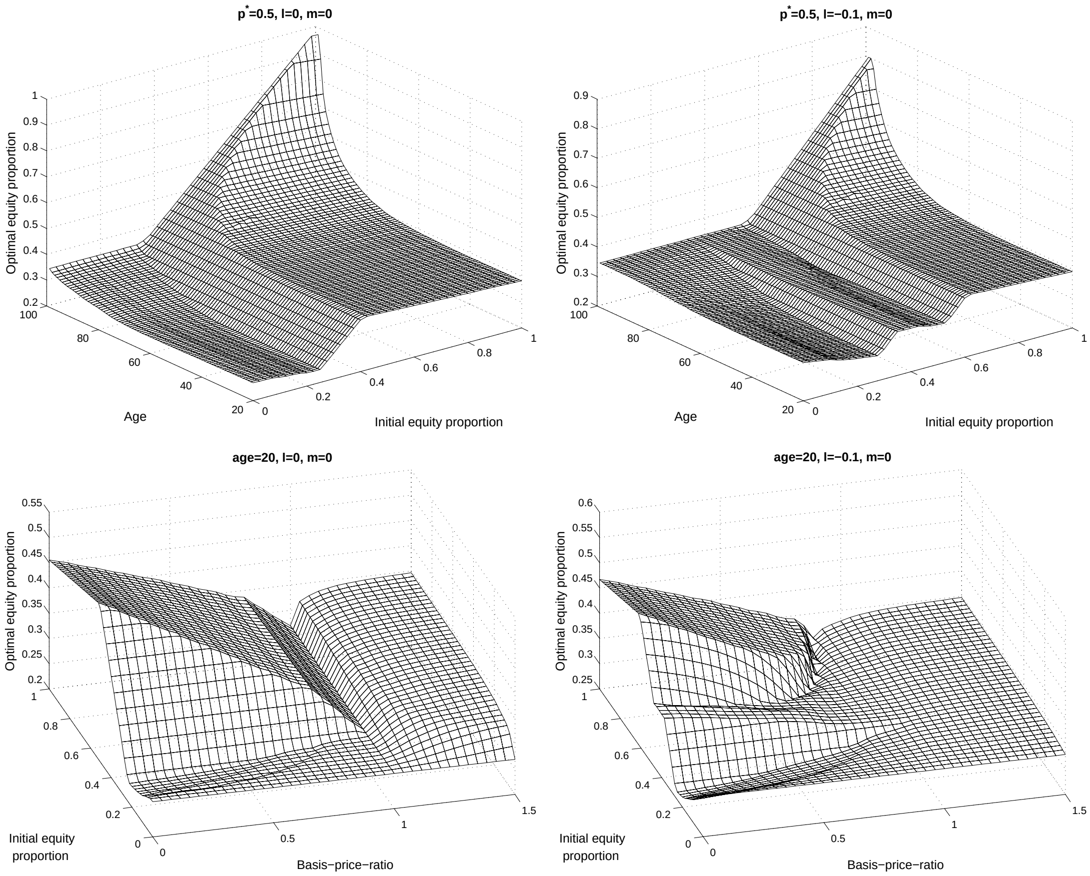


Figure 2.2: This Figure shows the investor's optimal equity exposure in a tax-system with no tax rebates on capital losses ($m = 0$), for an investor who either has no initial loss carryforward ($l = 0$, left graphs) or an initial loss carryforward of 10% of her beginning-of-period wealth ($l = -0.1$, right graphs). The upper graphs depict the investor's optimal equity proportion (money invested into equity relative to beginning-of-period-wealth), depending on her age and her initial equity proportion when facing substantial capital gains in her equity (basis-price-ratio $p^* = 0.5$). The lower graphs depict her optimal equity exposure at age 20 depending on her initial equity proportion and her basis-price-ratio.

is older than 90 years and is "locked in" with a high amount of money, i.e. she faces a high initial equity proportion. However, even if the investor is not "locked in" she increases her equity exposure. This is due to the reason that she can mainly base her optimization problem on after-tax returns that come with a higher expected return which increases the desirability of holding equity.

For young investors being "locked in", however, the rebalancing motive is a lot more important than the forgiveness of capital gains at death, which is why they realize some part of the capital gains and pay the capital gains tax to decrease their equity exposure. Still, even young investors accept a higher equity exposure than they would choose not being "locked in" to avoid paying capital gains taxes. When the initial equity proportion is low, the investor even slightly decreases her optimal equity proportion as her initial equity proportion increases. This finding is due to the assumption that her basis-price-ratio is a weighted average of her purchase prices. Hence, by decreasing her equity proportion she reduces her risk of getting "locked in" in future periods.

The upper right graph in Figure 2.2 differs from the upper left graph because it considers an investor who has an initial loss carryforward of 10% of her beginning-of-period-wealth. When her optimal equity proportion is below her initial equity proportion she uses the loss carryforward to realize capital gains, avoids the capital gains tax-rate and lowers her equity exposure. If, however, her initial equity proportion is only slightly above her optimal equity proportion, she accepts a slightly higher equity proportion since she faces a lower risk of getting locked in because of her initial loss carryforward.

The lower left graph in Figure 2.2 depicts the optimal equity proportion of an investor at age 20 who does not have an initial loss carryforward ($l = 0$), depending on her initial equity proportion and her basis-price-ratio. If the investor's basis-price-ratio is below one, she faces unrealized capital gains. If her basis-price-ratio is above one, she faces unrealized capital losses. According to our results in section 2.2, she should realize these losses immediately, which leaves her with a loss carryforward. This loss carryforward increases the desirability of equity, since it allows the investor to earn some capital gains tax-free. Consequently, her equity exposure is higher when initially facing unrealized capital losses than the equity exposure of an investor who is neither facing unrealized losses nor capital gains, i.e. whose basis-price-ratio is one.

If, however, the investor's basis-price-ratio is smaller than one, indicating that she faces an unrealized capital gain, she has to decide whether to realize that gain or to postpone realizing it. Concerning this decision, there are two opposing effects. On the one hand, postponing the realization of the capital gain allows her to earn compound returns on the tax-payments that have been postponed. On the other hand, postponing the realization of the capital gains can result in unbalanced portfolios. Especially if the risky asset has performed well in the past, its fraction relative to the investor's total wealth has been increasing which might result in an unbalanced portfolio. However, selling this asset to rebalance the portfolio would result in capital gains tax payments. To avoid the capital gains tax payment, the investor accepts a deviation from her optimal equity proportion, which is higher when her basis-price-ratio is lower, i.e. when her unrealized capital gains per unit of the risky asset is higher.

In case the investor is endowed with a low initial equity exposure, the investor's fraction of stocks after trading is decreasing in her initial equity exposure and increasing in her basis-price-ratio $p^*$. Even though her total fraction of stocks after trading is higher than her fraction of stocks before trading, it is not as high as if the investor was not facing a capital gain (i.e. $p^* = 1$). This is due to the fact that next period's basis-price-ratio is a weighted average of the beginning-of-period basis-price-ratio and one. The lower the beginning-of-period basis-price-ratio and the higher the initial equity exposure are, the lower next period's basis-price-ratio and thus the higher the risk of getting "locked in" with a significant amount in future periods will be. To avoid increasing that risk too much, the investor's optimal equity exposure is lower than that of an investor who is not facing unrealized capital gains or losses.

With increasing initial equity exposure, the investor attains an initial equity exposure exceeding her optimal equity exposure. As long as her optimal equity exposure does not deviate too far from the equity exposure, she has to hold to avoid paying the capital gains tax. The rebalancing motive is outweighed by the opportunity to earn compound returns on the unrealized capital gains. If the investor is given an even higher initial equity exposure, the rebalancing motive outweighs the tax-saving motive, and the investor's wealth after trading is reduced by the capital gains tax payment, which is why her fraction of stocks after trading relative to beginning-of-period-wealth slightly decreases in her initial equity exposure. With high capital gains tax payments, the investor decreases her beginning-of-period-wealth after trading. As we defined the optimal equity proportion to be the investor's stocks holdings relative to her beginning-of-period-wealth before trading, her optimal equity exposure slightly decreases when realizing capital gains.

The lower right graph in Figure 2.2 differs from the lower left graph by considering an investor who is endowed with an initial loss carryforward of 10% of her beginning-of-period-wealth. If her basis-price-ratio is above one, indicating that the investor faces an unrealized capital loss, she slightly increases her optimal equity exposure due to the higher initial loss carryforward after trading.

If, in turn, the investor faces a basis-price-ratio below one, indicating that she faces unrealized capital gains, her optimal equity exposure differs significantly depending on the different levels of the initial equity proportion. For low levels of her initial equity proportion, she can increase her equity exposure to the desired level without being confronted with any tax payments. As her initial equity proportion increases, she uses her loss carryforward to rebalance her portfolio without facing the capital gains tax. If, however, her unrealized capital gains exceed her loss carryforward, she accepts about the same equity exposure as if she did not start with an initial loss carryforward.[15]

### 2.4.3 The Value of an Initial Loss Carryforward

In this section, we analyze the value of a loss carryforward by computing the equivalent wealth increase an investor who has no loss carryforward would need to attain the same level of expected utility as an investor who has an initial loss carryforward. Since each dollar allows the investor to decrease tax-payments by not more than $\tau$ dollars, a dollar of loss carryforward is never worth more than these $\tau$ dollars. However, if the investor faces a significant loss carryforward, her loss carryforward might be worth less for two reasons. First, she might not use her entire loss carryforward, implying that the potential value of the loss carryforward never turns into wealth that can be consumed or bequeated. This type of risk is especially important for older investors facing high mortality rates. Second, even if she uses her entire loss carryforward, it might take several periods until her entire loss carryforward is converted to wealth and she can earn profits from it. Since the loss carryforward does not pay any interest, whereas – in contrast – tax rebates can be reinvested, one dollar in a significant loss carryforward can be worth less than one dollar in a small loss carryforward.

Figure 2.3 shows the equivalent wealth increase an investor who has no initial loss carryforward needs to attain the same level of expected utility as an investor

[15] In fact, her optimal equity exposure is slightly above the level when she has no initial loss carryforward, since the loss carryforward allows her to decrease her beginning-of-period-wealth after trading less sharply than in the absence of a loss carryforward. However, the difference is very small and hardly visible in Figure 2.2.

**Value of an Initial Loss Carryforward**

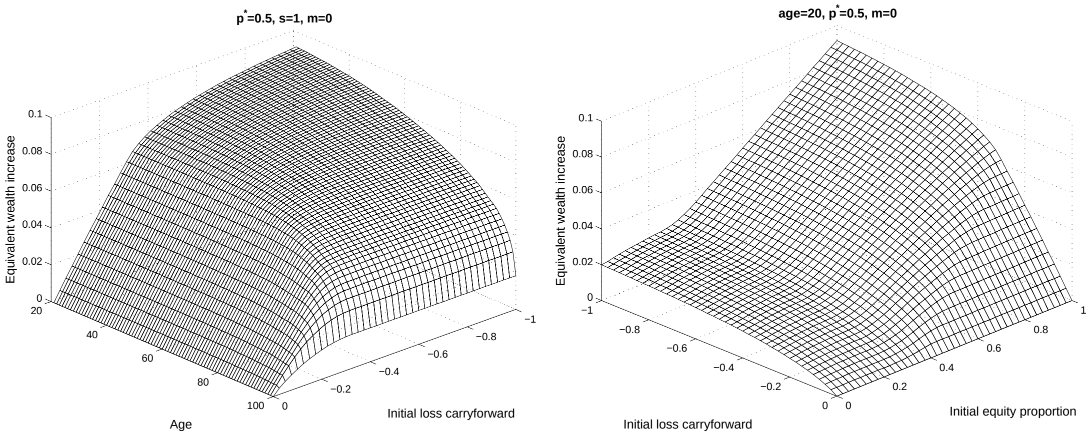


Figure 2.3: This Figure shows the equivalent wealth increase an investor who has no initial loss carryforward needs to be compensated with to attain the same level of expected utility as an investor who has an initial loss carryforward in a tax-system that does not grant any tax rebates on capital losses ($m = 0$). The left graph shows the investor's equivalent wealth increase when the investor's basis price-ratio is $p^* = 0.5$ and her initial equity exposure is $s = 1$ depending on her age and her initial loss carryforward. The right graph shows the equivalent wealth increase of an investor at age 20 whose basis-price-ratio is $p^* = 0.5$ as a function of her initial equity proportion and her initial loss carryforward.

who has an initial loss carryforward. The left graph shows the equivalent wealth increase an investor who has no initial loss carryforward needs to be compensated with to attain the same level of expected utility as an investor who has an initial loss carryforward, given that the investor faces unrealized capital gains ($p^* = 0.5$) and her initial equity exposure is $s = 1$. The graph depicts the equivalent wealth increase depending on the investor's age and her initial loss carryforward. The investor's equivalent wealth increase is rising in the initial loss carryforward, since each dollar of loss carryforward allows the investor to earn capital gains tax-free, which is obviously an advantage. However, there is no linear relationship between the level of the initial loss carryforward and the equivalent wealth increase. In fact, the equivalent wealth increase is concave in the level of the initial loss carryforward. This reflects the fact that a substantial loss carryforward faces a lower probability of being entirely used than a low loss carryforward and that the loss carryforward does not pay any interest, while the equivalent wealth increase can be invested.

The equivalent wealth increase is strictly decreasing as the investor ages. This is because with increasing age of the investor, mortality rates increase such that the probability that the loss carryforward remains unused and is worthless increases.

Consequently, her equivalent wealth increase for the loss carryforward decreases as the investor ages.

The right graph shows the equivalent wealth increase of an investor at age 20 who faces substantial unrealized capital gains in her equity ($p^* = 0.5$) in a tax-system that does not grant any tax rebates for capital losses. The equivalent wealth increase is depicted against the investor's initial loss carryforward and her initial equity proportion. Confirming the findings of the left graph, we find that the value of the loss carryforward is a concave function of the level of the initial loss carryforward, indicating that an additional dollar of loss carryforward provides the investor with a lower utility when the investor's initial loss carryforward is at a high level than when it is at a low level.

Furthermore, the value of the loss carryforward is strictly increasing in the level of the investor's initial equity proportion. While for low levels of the initial equity proportion, the investor can increase her equity proportion to the desired level without facing any tax-payments, for higher levels of the initial loss carryforward the investor can use her loss carryforward to rebalance her portfolio without facing the capital gains tax payments. Even when the investor's initial equity proportion is zero, indicating that the investor does not face any unrealized capital gains, the initial loss carryforward is valuable for her due to the fact that she can carry it forward and use it in forthcoming periods where she might face unrealized capital gains.

### 2.4.4 Limited versus Unlimited Tax Rebates

Having analyzed optimal consumption, investment and equivalent wealth increases for investors facing tax-systems with limited capital loss deduction, we now turn to the impact of limits on tax rebates for the investor's asset allocation decision. Limits on tax rebates lessen the appeal of risky assets, since they confront the investor with the same taxable treatment as in a tax-system with unlimited tax rebates when the investor faces positive capital gains, but leaves the investor with higher wealth decreases when she realizes capital losses. Even though the investor is granted a loss carryforward that can be carried forward indefinitely, she is worse off in a tax-system with limited capital loss deduction, as a tax rebate is more attractive than a loss carryforward.

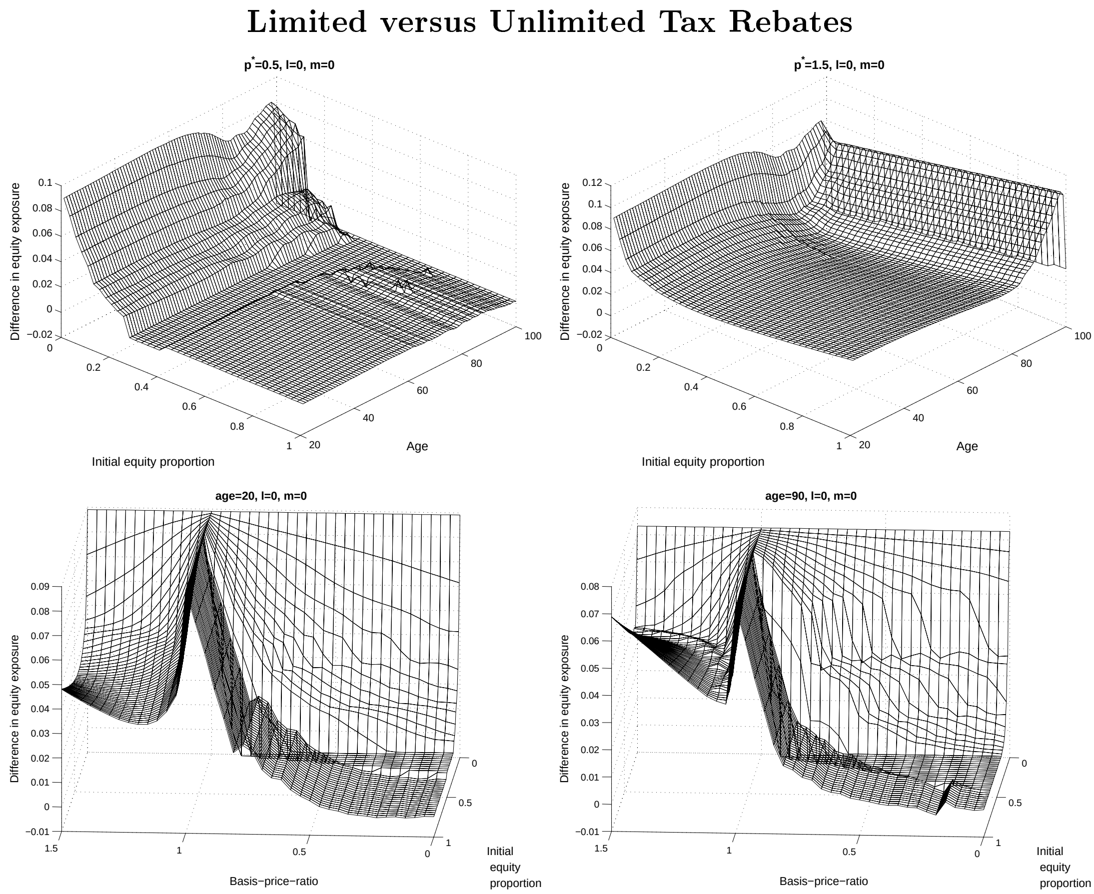


Figure 2.4: This Figure shows the difference between the investor's optimal equity exposure in a tax-system with unlimited tax rebates and a tax-system with no tax rebates (difference in equity exposure). The upper graphs depict the change in equity exposure as a function of age and the initial loss carryforward, given a basis-price-ratio of $p^* = 0.5$ (upper left graph) and $p^* = 1.5$ (upper right graph). The lower graphs show the change in equity exposure as a function of the basis-price-ratio and the initial equity exposure given a 20 year old investor (lower left graph) and a 90 year old investor (lower right graph).

In this section, we analyze the optimal asset allocation of investors that have no initial loss carryforward in tax-systems with unlimited tax rebates in comparison to tax-systems with limited tax rebates.[16] We show that due to limits on tax rebates, the investor significantly decreases her equity exposure – especially when having neither substantial unrealized capital gains nor losses. For an investor in a tax-system with unlimited capital loss deduction, our results confirm the findings of Dammon et al. (2001).

Figure 2.4 shows the difference between the optimal equity exposure (change in

[16] We do not compare asset allocation strategies in tax-systems with unlimited tax rebates to asset allocation strategies in tax-systems with limited tax rebates for investors that have an initial loss carryforward since in tax-systems with unlimited tax rebates, investors can never face a loss carryforward since all their losses qualify for immediate tax rebates.

equity exposure) for an investor trading in a tax-system with unlimited tax rebates for capital losses and a tax-system with no tax rebates for capital losses that only grants a loss carryforward. The upper graphs depict the change in equity exposure as a function of age and the initial loss carryforward, given a basis-price-ratio of $p^* = 0.5$ (upper left graph) and $p^* = 1.5$ (upper right graph). The lower graphs show the change in equity exposure as a function of the basis-price-ratio and the initial equity exposure given a 20 year old investor (lower left graph) and a 90 year old investor (lower right graph).

The upper left graph of Figure 2.4 shows the change in equity exposure of an investor who is endowed with a significant unrealized capital loss in her equity ($p^* = 0.5$). If such an investor is facing an initial equity exposure that is above her desired level, she is "locked in". In both tax-systems, the opportunity to earn compound returns is a strong incentive to postpone the realization of her capital gains. Consequently, her desired level of equity is about the same in both tax-systems. If, however, the investor has a small initial equity exposure below her desired equity exposure, she increases her equity exposure in both tax-systems. However, in the tax-system with unlimited tax rebates, the risk-return profile of the risky asset is more attractive which is why the increase in her equity exposure is of a higher order of magnitude in the tax-system with unlimited tax rebates.

The upper right graph of Figure 2.4 shows the change in equity exposure of an investor who is endowed with a significant unrealized capital loss in her equity ($p^* = 1.5$). As it is optimal to realize that loss immediately in both tax-systems, the investor in the tax-system with unlimited tax rebates ends up with a higher level of wealth, whereas the investor in the tax-system with no tax rebates ends up with a loss carryforward she can use in forthcoming periods. For low levels of the initial equity exposure, the tax-gift or the loss carryforward are quite small and the investor in the tax-system with unlimited tax rebates increases her equity exposure to a higher level than the investor in the tax-system with limited tax rebates. Again, this finding is due to the more attractive risk-return-profile of the risky asset in the former tax-system.[17]

[17] The sharp decrease in the change in equity exposure for an investor at age 99 is due to the assumption that the investor's maximum age is 100. Since next period's capital gains are tax-exempt when being bequeathed, the remaining loss carryforward does not provide the investor with an advantage and provides a strong incentive not to sell equity with embedded unrealized capital gains in both tax-systems.

If the investor is given neither an unrealized capital gain nor loss ($p^* = 1$, not shown here), the investor's optimal equity proportion does not depend on her initial equity exposure in neither the tax-system with limited nor with unlimited capital loss deduction. This is due to the fact that for $p^* = 1$, the investor can rearrange her portfolio without facing capital gains tax-payments or tax rebates.

If the investor has a high initial equity exposure, she increases her equity exposure in both the tax-system with unlimited and limited tax rebates. Even though the initial loss carryforward granted in the tax-system with limited capital loss deduction increases the attractiveness of equity, the increase in wealth in the tax-system with unlimited capital loss deduction outweighs this effect and the change in equity exposure increases as the initial equity exposure increases. The lower increase in the tax-system with limited tax rebates is due to the reason that with increasing loss carryforward, the probability of entirely using that loss carryforward and potentially building up an even higher loss carryforward that has an even lower probability of being used is increasing in the level of the loss carryforward. In contrast, one dollar of tax rebate increases the investor's beginning-of-period-wealth and thus the increase in her equity exposure is increasing in her initial equity exposure.

The lower graphs in Figure 2.4 depict the change in equity exposure of an investor at age 20 (lower left graph) and age 90 (lower right graph), respectively. If the investor has neither an unrealized capital gain nor with an unrealized capital loss (i.e. $p^* = 1$), which is, for example, the case for an investor entering the stock market, her equity exposure at age 20 in a tax-system without capital loss deduction is about 26.9%, which is below the 35.9% in a tax-system with unlimited tax rebates. This is due to the fact that on the one hand in the tax-system with limited tax rebates, the investor bears the entire risk when having net capital losses, as she does not get any tax rebates on them. On the other hand, capital gains are treated the same way as in a tax-system with unlimited tax rebates. Hence, equity is less attractive in the tax-system with limited tax rebates which is why the investor holds a substantially lower fraction of her wealth in equity.

If the investor is facing an unrealized capital loss (i.e. $p^* > 1$), it is optimal to realize that loss immediately. The investor has a loss carryforward, which is higher as the net capital loss $p^*$ and her initial equity exposure increase. As this loss carryforward allows for earning some future capital gains tax-free, the investor chooses a higher equity proportion after trading compared to the case of $p^* = 1$. While in the tax-system with unlimited tax rebates, the investor receives the loss carryforward in cash as a tax rebate, in the tax-system with no tax rebates, it only has value to the investor if it can be subtracted from forthcoming capital gains. This is why in the tax-system with limited tax rebates, the increase in the fraction of stocks is stronger than in the tax-system with unlimited tax rebates, but remains at a certain level with increasing realized losses to avoid ending up with a portfolio that is too heavily invested into equity.

If, however, the investor is facing an unrealized capital gain (i.e. $p^* < 1$) and her fraction of stocks before trading is low, her fraction of stocks after trading will be about the same as without the capital gain and her basis-price-ratio after trading will decline. Compared to the tax-system with unlimited tax rebates, the danger of getting "locked in" with a higher amount of wealth seems to be neglectable. The reason for this is that in the tax-system with limited tax rebates, each capital gain in equity increases her future capital gains tax, while each loss in equity decreases her future capital gains tax. This decrease in the future capital gains tax results in future capital gains and capital losses being treated equally and thus has a higher value for the investor than a loss carryforward, as the loss carryforward carries the risk of potentially remaining unused. The numerical result suggests that the opportunity of reducing the embedded capital gain outweighs the risk of getting locked in.

If the investor is facing an unrealized capital gain, and her fraction of stocks before trading is substantial, the rebalancing motive outweighs the opportunity of earning compound returns on the unrealized capital gains and the results in the tax-system do not differ from those in the tax-system with unlimited tax rebates.

Comparing the lower left and the lower right graphs of Figure 2.4 shows that the change in equity exposure decreases for an investor who does not have an unrealized capital gain or loss ($p^* = 1$). This is due to the fact that with increasing age the investor runs a higher risk of generating a loss carryforward that she cannot use in her remaining lifetime, which is why she reduces her equity exposure. If, however, the investor faces unrealized capital losses she realizes these capital losses, immediately and generates a loss carryforward. Since a 20 year old investor may expect to have a higher remaining life expectancy than a 90 year old investor, the 90 year old investor tries to make use of the loss carryforward as early as possible, which is why she increases her equity exposure more strongly than the 20 year old investor. However, in a tax-system with unlimited tax rebates there is an even stronger increase in equity exposure due to the forgiveness of capital gains when being bequeathed. Consequently, the difference in equity exposure is higher for the 90 year old investor.

### 2.4.5 Comparative Static Analysis

Having analyzed the optimal tax-timing and asset allocation decision of an investor in our base-case scenario, we now turn to a comparative static analysis and analyze the impact of exogenous non-financial income and the upper bound on capital losses qualifying for tax rebates.

Ideally non-financial income should be introduced into our model with its own stochastic process. However, this would require increasing the number of state variables, and significantly complicating our numerical analysis. We therefore follow recent literature (see e.g. Dammon et al. (2001, 2004)) and assume the investor's non-financial income to be a constant proportion of her beginning-of-period wealth. However, since present savings thereby do not only increase future wealth, but also future non-financial income, this assumption overstates the impact of present savings. We consider an investor who has a non-financial income of 15% of her beginning-of-period-wealth before taxation during her working life until age 65, corresponding to a non-financial income after taxes of 9.6%. When retired, we assume the investor's non-financial income to decrease to 70% of her non-financial income during her working life.

In contrast to an investor who has no non-financial income, the investor who is endowed with non-financial income of 15% of her beginning-of-period-wealth decreases her consumption over the life cycle (not shown here). This is due to the fact that some part of her non-financial income is used for consumption while some other part is used to increase her savings and thereby her bequest potential. The younger the investor is, the less important the bequest potential is, and thus the higher the part of her non-financial income is that she uses to increase her consumption.

Figure 2.5 shows the optimal asset location of an investor with non-financial income of 15% of her beginning-of-period-wealth. Compared to the optimal investment strategy for an investor who has no non-financial income (see Figure 2.2), the optimal

**Optimal Investment Policy with Non-Financial Income**

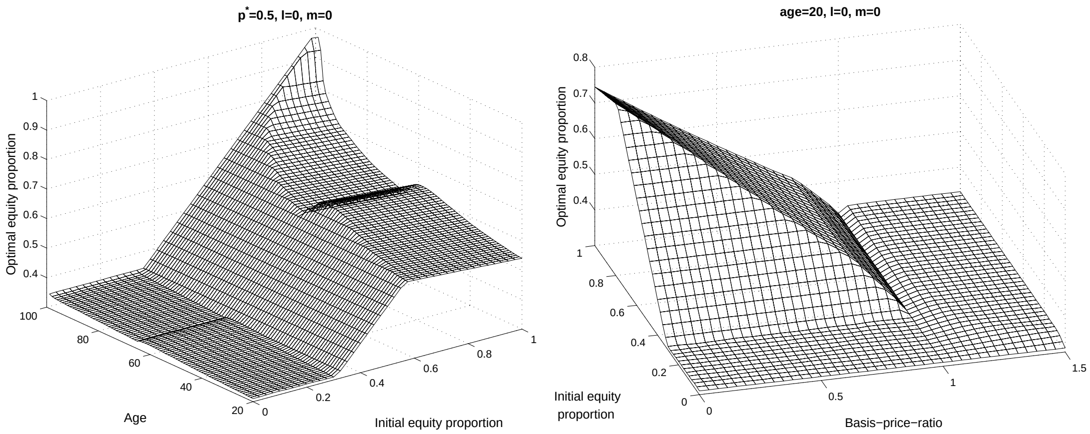


Figure 2.5: This Figure shows the optimal equity exposure in a tax-system with no tax rebates on capital losses ($m = 0$) for an investor who has no initial loss carryforward ($l = 0$) and has non-financial income of 15% of her beginning-of-period-wealth before taxation during working-age, corresponding to an after-tax income of 9.6%. When being retired, non-financial income is cut down by 30%. The left graph depicts her optimal equity exposure depending on her age and her initial equity proportion when she is facing substantial capital gains in her equity ($p^* = 0.5$). The right graph depicts her optimal equity exposure at age 20 when she has no initial loss carryforward ($l = 0$), depending on her initial equity proportion and her basis-price-ratio.

equity proportion is significantly higher. This is due to the fact that non-financial income grants the investor a certain increase in her beginning-of-period-wealth and is thus more similar to the return on the risk-free asset than the return on the risky asset. Consequently, the investor wishes to hold a higher fraction of stocks than an investor who is not endowed with non-financial income.

The left graph depicts her optimal equity exposure over the life cycle, given that she faces a basis-price-ratio of $p^* = 0.5$ and has no initial loss carryforward ($l = 0$). It shows that, in contrast to the base-case without non-financial income, the investor now accepts a higher deviation from her optimal equity exposure when she is locked in. This is due to the fact that future non-financial income allows her to rebalance her portfolio without having to pay the capital gains tax. When the investor approaches retirement age, she consequently accepts a lower deviation from her optimal equity exposure, since her future non-financial income decreases. As her age increases, the accepted deviation increases again. This reflects the fact that capital gains are forgiven when being bequeathed, which is a strong incentive not to realize the capital gains.

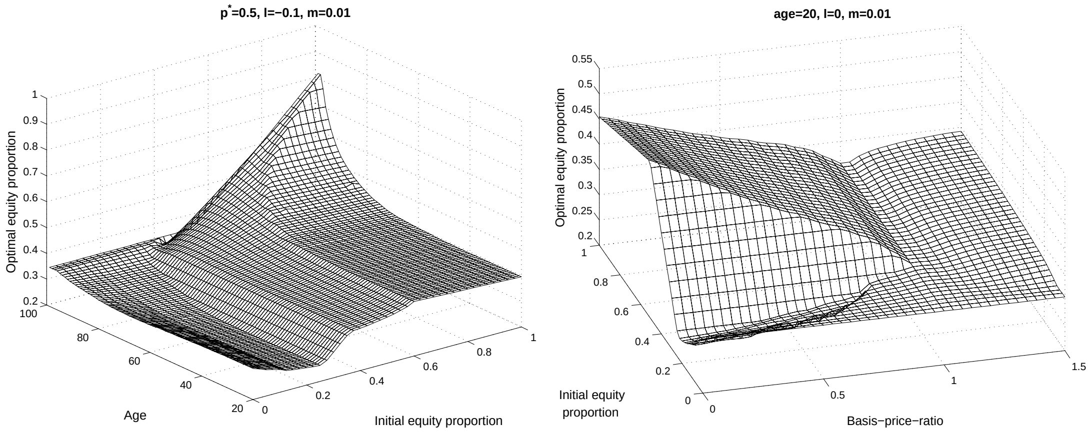


Figure 2.6: This Figure shows the optimal equity exposure in a tax-system with tax rebates on capital losses not exceeding 1% of the investor's beginning-of-period-wealth ($m = 0.01$). The left graph shows the optimal equity proportion of an investor who has substantial capital gains in her equity ($p^* = 0.5$) and an initial loss carryforward of 10% of her beginning-of-period-wealth ($l = -0.1$). The right graph shows the optimal equity exposure of an investor at age 20 depending on her initial equity exposure and her basis-price-ratio.

The right graph depicts the optimal equity exposure when the investor is 20 years old and has no initial loss carryforward ($l = 0$). It shows that, compared to the base-case scenario without non-financial income, the investor's equity exposure is significantly higher. This is due to the fact that future non-financial income guarantees the investor a safe after-tax return of 9.6% of her beginning-of-period-wealth. The impact of non-financial wealth on her return is therefore similar to the return of the risk-free asset. To end up with a well-diversified portfolio, her optimal equity exposure is higher in a setting with non-financial income than in a setting without non-financial income.

While so far, we have analyzed optimal tax-timing and asset allocation in a tax-system that does not grant any tax rebates on capital losses, we now consider the case of a tax-system that allows for small tax rebates not exceeding 1% of the investor's beginning-of-period-wealth ($m = 0.01$) in Figure 2.6. An investor who has no initial loss carryforward and has substantial unrealized capital gains in her equity is in a similar situation as an investor who trades in a tax-system with no tax rebates (not shown here). This is due to the fact that the tax rebates only provide her with an advantage if she has an initial loss carryforward or unrealized capital losses. If her unrealized capital gains are substantial her advantage from trading in a tax-system with tax rebates is negligible, which is why her optimal equity exposure does not deviate significantly from the optimal equity exposure of an investor trading in a tax-system with no tax rebates on capital losses.

If, however, such an investor has an initial loss carryforward, the left graph in Figure 2.6, which considers an investor with an initial loss carryforward of 10% of her beginning-of-period-wealth, shows that the opportunity of getting tax rebates has an impact on the optimal equity exposure. In contrast to the base-case-investor (upper right graph of Figure 2.2) the investor does not use her entire loss carryforward immediately to rebalance her portfolio. This is due to the fact that in the tax-system that allows for tax rebates, she can not only subtract her loss carryforward from realized capital gains, but can also use it to earn tax rebates. In contrast to the tax-system with no tax rebates, she thus has the opportunity of both making use of her loss carryforward and earning compound returns on the unrealized capital gains. Consequently, her decrease in the optimal equity exposure is less severe than in the tax-system that does not allow for tax rebates.

The right graph in Figure 2.6 depicts the optimal equity exposure of an investor at age 20 with no initial loss carryforward ($l = 0$) as a function of her initial equity proportion and her basis-price-ratio. If the investor's basis-price-ratio is above one, indicating that the investor has an unrealized capital loss, her optimal equity exposure increases less compared to an investor has no initial loss carryforward than in a tax-system with no tax rebates (lower left graph in Figure 2.2). This is due to the fact that without tax rebates, the investor can only make use of the loss carryforward when generating capital gains. In the tax-system with tax rebates, however, the investor can also make use of the loss carryforward by realizing it in forthcoming periods and earning the tax rebates granted for it. Consequently, she ends up with a smaller remaining loss carryforward, which is why she does not increase her equity exposure as strongly as an investor in a tax-system without tax rebates, whose remaining loss carryforward is at a higher level.

## 2.5 Conclusion

In his seminal 1983 paper, Constandinides shows that in a tax-system that permits wash sales and provides unlimited tax rebates for incurred net capital losses, and in a market without transaction costs, it is optimal to realize capital losses immediately, even though this trading strategy increases the investor's purchase price. This paper generalizes his finding to tax-systems with limited tax rebates. It further shows, not very surprisingly, that generally in such a tax-system, investors cannot attain the same level of wealth as in a tax-system with unlimited tax rebates. Due to the asymmetric taxable treatment of capital gains and losses in such a tax-system, investors tend to hold substantially less equity than in tax-systems with unlimited tax rebates.

If the investor is endowed with an initial loss carryforward, the optimal equity exposure increases, since the loss carryforward allows the investor to earn capital gains tax-free, which increases the desirability of stocks. The less that capital losses qualify for tax rebates, the lower the equity exposure is. This finding suggests that investors in countries with more generous capital loss deduction should have a higher equity exposure than investors in countries with lower or no capital loss deduction. Whether this is in fact the case or whether the equity exposure is more driven by national risk-attitude, different risk-return profiles of the assets for home-biased investors in different national markets or other factors is an interesting question for empirical research.

Inevitably, this paper neglects many important issues. Instead of considering the impact of an absolute bound on capital loss deductions, it considers a bound relative to current beginning-of-period-wealth and thus does not take the impact of the absolute level of wealth on tax-timing into account. Furthermore, it ignores different tax-rates on long-term and short-term capital gains and losses, transaction costs, potential income shocks and more than one risky asset. When having the opportunity of investing in more than one risky asset, the investor could potentially realize losses in one risky asset to reduce diversification costs when selling another asset with unrealized capital gains that makes up a substantial fraction of the investor's portfolio. Interesting avenues for further research are to include these factors into the model, to compute tax-rates that allow investors to end up with the same level of utility in a tax-system with limited tax rebates for given wealth and tax-rates in a tax-system with unlimited tax rebates, and to analyze optimal tax-timing in a tax-system where wash sale rules apply.

# Appendix:

## Anmerkungen für Deutschland

Im vorangehenden Artikel ist die Bedeutung einer asymmetrischen Besteuerung von Kursgewinnen in einem Steuersystem untersucht worden, welches in wesentlichen Grundzügen dem US-amerikanischen Steuersystem nachempfunden wurde. Im Rahmen dieses Appendix soll kurz auf den Fall einer Anlegerin eingegangen werden, die sich in einem Steuersystem bewegt, welches in den wesentlichen Grundzügen dem deutschen Steuerrecht entspricht. Gegenüber dem US-amerikanischen Steuersystem sieht sich die Anlegerin nach aktuell gültiger Rechtslage dabei insbesondere folgenden Unterschieden ausgesetzt:

- Der Spitzensteuersatz liegt höher als in den USA.
- Kursgewinne sind nach einer Mindesthaltedauer von einem Jahr steuerfrei.

Auf Grund der Steuerfreiheit von Kursgewinnen, die nicht als Spekulationsgewinne gelten, kann die Anlegerin nach Ablauf der Mindesthaltedauer ihr Portfolio umschichten, ohne im Falle des Verkaufs von Wertpapieren mit Kursgewinnen, ihre Kursgewinne versteuern zu müssen. Dadurch besteht für sie langfristige keine Gefahr "locked in" zu werden. Somit sind für deutsche Anlegerinnen im betrachteten Modell Kursgewinne steuerfrei vereinnehmbar. Insbesondere gibt es für sie mangels steuerlicher Belastung ihrer Kursgewinne auch keine tax-timing Option. $\tau_g = 0$ führt also zu einem strukturell einfacherem als dem oben betrachteten Optimierungsproblem.

Durch die geplante Abgeltungssteuer, nach der Kursgewinne grundsätzlich steuerpflichtig werden sollen, könnten tax-timing Strategien jedoch auch für deutsche Privatanlager schon bald Bestandteil effizienter Anlagestrategien werden.

# 3 Are Bonds Desirable in Tax-Deferred Accounts?

First draft: April 2007

This version: June 2007

Until July 2, this paper has been accepted for presentation at the following conferences:

- 11th international congress on Insurance: Mathematics and Economics (IME), July 10-12, 2007, Piraeus, Greece

# Abstract

The literature on optimal asset location concludes almost unanimously that bonds are the preferred asset to hold in tax-deferred accounts. We argue that in tax-systems like the US where tax rebates on capital losses are limited, this is not necessarily true. Depending on either positive or non-positive capital gains, bonds remain the preferred asset to hold in a tax-deferred account. However, if the sign of the capital gains is not known in advance, this result may not be true. This surprising finding is due to the fact that in contrast to tax-systems without limits on tax rebates, the return on an asset in the tax-deferred account can no longer be perfectly replicated in the taxable account since a replication risk is caused by differing replicating portfolios for positive and non-positive capital gains. While for positive capital gains, the better performance of stocks might outweigh the higher yield of bonds in tax-deferred account, for negative capital gains holding stocks in the taxable account is more desirable due to the compensation for incurred losses via potential tax rebates and a loss carryforward. A well diversified portfolio can thus consist of stocks in both the taxable and the tax-deferred account. As one can see, limits on tax rebates for capital losses can explain the asset location puzzle, i.e. why private investors hold stocks in tax-deferred accounts.



**JEL Classification Codes:** G11, H24

**Key Words:** asset location, asset allocation, limited capital loss deduction, tax-deferred accounts, loss carryforward, asset location puzzle

## 3.1 Introduction

The taxable treatment of private investors' profits is a potentially important factor influencing household portfolio structure. While profits in taxable accounts are subject to taxation, profits in tax-deferred accounts remain untaxed.

This paper explains optimal asset allocation (i.e. which assets to hold) and optimal asset location (i.e. in which account to hold these assets) for investors that are confronted with a tax-system in which tax rebates on capital losses are limited – as under current US tax-law. It is most closely related to recent literature on optimal asset location including Shoven and Sialm (1998, 2003), Dammon et al. (2004), Poterba et al. (2004), Gomes et al. (2006), and Huang (2007). These papers conclude that in general, bonds should be preferred to stocks in tax-deferred accounts. Garlappi and Huang (2006), in contrast, provide an example for short-selling constrained investors in which holding a mixed portfolio in the tax-deferred account is optimal. Their result, however, is based on strong assumptions. Shoven (1999) argues that based on historical returns holding stocks in tax-deferred accounts resulted in higher wealth for a buy and hold investor. However, his approach fails to control for risk properly.

This paper differs from the existing literature by taking into account limitations on tax rebates for capital losses. We show that in the presence of limited tax rebates, it can be optimal to hold stocks in both taxable and tax-deferred accounts.

Conditioned on either a negative or a non-negative capital gain on the risky stock tax-deferred bonds have higher replication costs in the taxable account and should therefore be held in the tax-deferred account. However, stocks can become the preferred asset to hold in the tax-deferred account, if the sign of the capital gain is not known in advance. This is due to the fact that the portfolios replicating one dollar of tax-deferred wealth in the taxable account are different for negative and non-negative capital gains, making perfect replication no longer possible. While for positive capital gains, the extra return resulting from the tax-exemption of profits in the tax-deferred account might be higher for stocks than for bonds, stocks should preferentially be held in the taxable account for negative capital gains to provide the investor with the loss carryforward. Since the sign of a future capital gain is not known in advance, it can be optimal to hold mixed portfolios in both accounts for diversification purposes. Another reason for holding stocks in tax-deferred accounts is that limitations on tax rebates for capital losses worsen stocks' risk-return-profile in the taxable account. The risk of generating a substantial loss carryforward which could potentially remain unused in the investor's remaining lifetime is especially important for investors facing high mortality rates.

Limitations on tax rebates for capital losses can thus explain the asset location puzzle (Amromin (2003)), i.e. why private investors hold substantial amounts of stocks in their tax-deferred accounts.

In general, the taxation of profits has several impacts on private investors' investment behavior. Firstly, due to shrinking returns, investors might increase their present consumption and decrease their present investments. Secondly, if different assets are taxed at different tax-rates, taxation has an impact on optimal asset allocation. Historically, dividends and interest payments have been taxed at a higher rate than long-term capital gains (Sialm (2006)). While the impact of different tax-rates on asset allocation can still be analyzed implicitly by adjusting returns, volatilities, and correlations accordingly, the tax-system confronts private investors with additional tax-rules that deserve explicit modeling.

First, in the 1980s tax-deferred accounts like IRAs and 401(k)s with favorable taxable treatment were introduced to stimulate private savings for retirement. In particular, profits in such accounts are not subject to taxation. Furthermore, tax-deferred accounts allow one to defer the taxation of income until withdrawal, which is an advantage to private investors facing lower personal income tax-rates when retired than when employed. However, early withdrawals from tax-deferred accounts are subject to a 10% penalty tax. As a result of the penalty tax and limits on contributions to tax-deferred accounts, taxable accounts are often used as a retirement saving vehicle, as well. Due to the different taxable treatment of assets in taxable and tax-deferred accounts, it is important to make an informed decision as to which assets to locate in each account.

Second, capital losses exceeding the amount of $ 3,000 cannot be offset against other income and thus do not qualify for tax rebates. This tax-rule has an unfavorable impact on the risk-return-profile of volatile assets as it confronts the investor with tax-payments in case of a capital gain, but limits tax rebates for capital losses.[1]

[1]This rule assures that public finances are not affected too strongly when capital markets perform

Instead, for losses that do not qualify for tax rebates, the investor receives a loss carryforward that can be carried forward indefinitely and can be subtracted from future capital gains to reduce future tax-burden or to generate a future capital loss that qualifies for a tax rebate.

Third, capital gains are not only taxed at a lower rate when being realized long-term, they also come with a tax-timing option. That is, capital gains are not taxable the moment they occur, but the moment they are realized. This option provides the investor with an opportunity to defer the realization of capital gains and to exploit the effect of compound returns, resulting in a reduction of the effective capital gains tax-rate (Chay et al. (2006)). A tax-timing option thus increases the equity exposure. However, tax-timing is associated with a risk of ending up with an unbalanced portfolio. An asset that comes with substantial capital gains in some period tends to increase its fraction in the investor's portfolio. If the investor does not sell some units of that asset, the investor's portfolio may become too heavily invested in it. Hence, for assets with an unrealized capital gain, postponing the taxation of the unrealized gain and diversification can be opposing desires. If an asset comes with an unrealized capital loss, and wash-sale rules do not apply, Marekwica (2007) extends the classical result of Constantinides (1983) to tax-systems where tax rebates on capital losses are limited. He shows that it is optimal to realize that loss immediately even though that realization increases the investor's tax-basis. In addition, the investor's compensation potentially comes as a loss carryforward, which is less attractive than a tax rebate, since the loss carryforward does not pay any interest and potentially remains unused.

In this paper, we focus on the impact of tax-deferred investing and limitations on tax rebates for capital losses, and ignore the tax-timing option to concentrate on the impact of limits on tax rebates for asset allocation and to keep our optimization problem numerically tractable.

While optimal asset allocation is intensively discussed in the economic literature, research on optimal asset location is a more recent field of research going back to Tepper and Affleck (1974), Black (1980) and Tepper (1981). They analyze investment strategies of companies running defined-benefit pension plans and conclude that in

poorly. The revenue from capital gains taxation for public finances are thus similar to the payoff of a call option on a fraction of the capital gains with forced realization for losses not exceeding \$ 3,000.

the absence of tax-timing options, limits on capital loss deduction, and short-selling restrictions, only bonds should be held in defined-benefit pension plans. Auerbach and King (1983) point out that this result can also be applied for private investors with a tax-deferred account. Along this line, Shoven (1999) and Shoven and Sialm (1998) introduce the asset location problem to household investment decisions.

Asset location strategies private investors follow in practice are analyzed in Bodie and Crane (1997), Poterba and Samwick (2001) and Barber and Odean (2003). According to these studies, private investors fail to exploit the opportunities tax-deferred accounts offer them. Particularly for long-term investment horizons, contribution rates have an important impact on total final wealth (Gomes et al. (2006)). This is due to the tax-exemption of profits in tax-deferred accounts, which has a tremendous impact on total final wealth for long-investment horizons due to the compounding on the taxes saved. It would seem that if there were no early withdrawal penalty, all individuals would contribute to tax-deferred accounts to the limit.

Amromin (2002) and Bergstresser and Poterba (2004) point out that many investors hold considerable amounts of equity in their tax-deferred accounts. Furthermore, according to the study of Benartzi and Thaler (2001), private investors' equity exposure in tax-deferred accounts and pension plans increases with the number of equity funds the investors can choose from. In particular, many investors seem to follow simple $\frac{1}{n}$-diversification strategies.

If investors have limited taxable wealth and face high labor income risk, some authors, among them Amromin (2003), Dammon et al. (2004) and Amromin (2005) argue that bonds can be the preferred asset to hold in the tax-deferred account to avoid the penalty tax on early withdrawals and to smooth consumption over the life cycle. However, even with substantial income shocks, Dammon et al. (2004) show that it usually still remains optimal to hold stocks in the taxable account.

While the impact of the opportunity to invest in a tax-deferred account[2] and the impact of tax-timing on asset allocation[3] have been intensively discussed in the literature, only limited guidance is available to investors faced with tax-systems where tax rebates on capital losses are limited.

[2]See e.g. Shoven and Sialm (1998, 2003), Dammon et al. (2004), Poterba et al. (2004), Huang (2007), Garlappi and Huang (2006), Gomes et al. (2006) and Zaman (2007).

[3]See e.g. Constantinides (1983, 1984) Dammon et al. (1989), Dammon and Spatt (1996), Dammon et al. (2001, 2004), Chay et al. (2006) and Gallmeyer et al. (2006).

Nearly all of the literature concludes in the absence of limits on tax rebates, risk-free bonds are the preferred asset to hold in a tax-deferred account. We, however, show that with limited capital loss deduction, it can be optimal to hold stocks in tax-deferred accounts. We propose that limits on tax rebates for capital losses can thus solve the asset location puzzle.

The remainder of this paper proceeds as follows. Section 3.2 derives the effects driving optimal asset location decisions when tax rebates on capital losses are limited. Section 3.3 introduces the model and presents our numerical results. Section 3.4 summarizes and gives some hints for further research.

## 3.2 Effects of Limits on Tax Rebates

We consider a market in which the investor has the opportunity to invest into a risk-free bond paying a pre-tax return of $r > 0$ and a risky stock paying a constant pre-tax dividend rate of $d > 0$, and a risky capital gains rate $g_t$ in period $t$. The parameters $\tau_g \in (0, 1)$ and $\tau_d \in (0, 1)$ respectively denote the tax-rate on capital gains and losses and on dividends and interest. $\alpha_{T,t}$ denotes the fraction of stocks in the taxable account relative to taxable wealth invested in period $t$. That is, $\alpha_{T,t}$ is the fraction of stocks in the taxable account relative to beginning-of-period-taxable wealth adjusted for income, consumption, and contributions to or withdrawals from the tax-deferred account. $\alpha_{R,t}$ denotes the fraction of stocks relative to tax-deferred retirement wealth invested in period $t$. In the absence of a limitation on capital loss deduction, the investor's period $t$ gross-return $R_{T,t}$ on a risky asset in a taxable account is given by

$$R_{T,t} = 1 + (1 - \tau_d)\, d + (1 - \tau_g)\, g_t. \tag{3.1}$$

We, however, analyze a tax-system where capital losses exceeding a certain amount $M \geq 0$ do not qualify for tax rebates.[4] If $G_t$ denotes the realized capital gain (or loss) of an investor in period $t$ and $L_{t-1}$ the loss carryforward that has been carried over from period $t-1$ to $t$, the net capital gain (or loss) $T_t$ in period $t$ that is subject to the capital gains tax is given by

$$T_t \equiv \max\left(G_t + L_{t-1}; -M\right). \tag{3.2}$$

[4]Under current US tax-law $M$ is equal to \$ 3,000.

Realized net capital losses that exceed the amount of $M$ can be indefinitely carried forward to the following periods. Thus, the loss carryforward $L_t$ that can be carried over from period $t$ to period $t+1$ is given by

$$L_t \equiv G_t - T_t + L_{t-1}. \tag{3.3}$$

For simplicity, we assume capital losses to be limited by zero in this section, i.e. there are no tax rebate payments at all and $M = 0$ (this assumption is relaxed in section 3.3.5).[5] If $\chi_{g_t>0}$ denotes the characteristic function which is 1 for $g_t > 0$ and 0 for $g_t \leq 0$, the gross-return on the risky asset in the taxable account $R_{T,t}$ in period $t$ is given by

$$R_{T,t} = 1 + (1-\tau_d)\,d + \left(1 - \tau_g \chi_{\{g_t>0\}}\right) g_t = \begin{cases} 1 + (1-\tau_d)\,d + (1-\tau_g)\,g_t & \text{if } g_t > 0 \\ 1 + (1-\tau_d)\,d + g_t & \text{if } g_t \leq 0. \end{cases} \tag{3.4}$$

The limitation of capital loss deduction in the taxable account is an obvious disadvantage to the investor, as although there is no longer a tax rebate when facing capital losses, the investor is still confronted with tax-payments when the capital gain is positive. The gross-return on the risky asset in the tax-deferred account $R_{R,t}$ at time $t$ is not affected by a limitation on capital loss deduction due to the tax-exemption of profits and is given by

$$R_{R,t} = 1 + d + g_t. \tag{3.5}$$

Huang (2007) uses a tax-arbitrage argument to show that the investor should only hold bonds in the tax-deferred account when there is no limit on tax rebates for capital loss deduction and the investor has the opportunity to go short in the taxable account. Her tax-arbitrage argument considers the replication costs of one dollar of bonds $C^{(R)}_{B,u}$ and one dollar of stocks $C^{(R)}_{S,u}$ in the tax-deferred account. Assuming that the tax-rate on coupon payments is higher than on capital gains, that is, $\tau_d > \tau_g$, and the interest rate on the risk-free asset is higher than the dividend rate on the

[5]Such a taxable treatment of capital losses is, for example, implemented in the German tax-code.

stock, i.e. $r > d$, she shows that $C^{(R)}_{B,u}$ and $C^{(R)}_{S,u}$ are given by

$$C^{(R)}_{B,u} = \frac{1+r}{1+(1-\tau_d)\,r} \tag{3.6}$$

$$C^{(R)}_{S,u} = \frac{1}{1-\tau_g} + \frac{1}{1+(1-\tau_d)\,r}\frac{d\,(\tau_d-\tau_g)-\tau_g}{1-\tau_g} \tag{3.7}$$

and that the replication cost in the taxable account for one dollar of tax-deferred bonds is higher than for one dollar of tax-deferred stocks, i.e. $C^{(R)}_{B,u} > C^{(R)}_{S,u}$. One can thus replicate one dollar of bonds in tax-deferred wealth by $C^{(R)}_{B,u}$ dollars of taxable wealth in bonds. One dollar of stocks in tax-deferred wealth can be replicated by $\frac{1}{1-\tau_g}$ dollars of stocks and $\frac{1}{1+(1-\tau_d)r}\left(\frac{d(\tau_d-\tau_g)-\tau_g}{1-\tau_g}\right)$ dollars of bonds in the taxable account. Therefore, bonds are the preferred asset to hold in the tax-deferred account.

Even in the absence of a short-selling opportunity, the recent literature except for Garlappi and Huang (2006) concludes that it usually remains optimal to hold bonds in the tax-deferred account.

If, however, capital loss deduction is limited, a stock in a tax-deferred account in period $t$ can no longer be perfectly replicated in a taxable account as the replicating portfolio depends on whether the capital gain $g_t$ is positive or not. In this case, stocks in the taxable and the tax-deferred account are only close but no longer perfect substitutes in the sense that the return in the tax-deferred account can no longer be perfectly replicated in the taxable account.

For $g_t \leq 0$, the replicating portfolio for one dollar of stocks in the tax-deferred account is one dollar of stocks and $\frac{\tau_d d}{1+(1-\tau_d)r}$ dollars of bonds in the taxable account. Its replication cost $C^{(R)}_{S,l}$ is given by

$$C^{(R)}_{S,l} = 1 + \frac{\tau_d d}{1+(1-\tau_d)\,r} < C^{(R)}_{B,l} \tag{3.8}$$

in which $C^{(R)}_{B,l} = \frac{1+r}{1+(1-\tau_d)r}$ is the replication cost for one dollar of bonds in the tax-deferred account. Since the risk-free asset does not come with any capital gains or losses its replication cost is the same as in a tax-system with unlimited capital loss deduction.

For $g_t > 0$ the replicating portfolio thus is the same as in the case with unlimited tax rebates for capital losses and thus consists of $\frac{1}{1-\tau_g}$ stocks and $\frac{1}{1+(1-\tau_d)r}\left(\frac{d(\tau_d-\tau_g)-\tau_g}{1-\tau_g}\right)$ dollars of bonds in the taxable account. As $C^{(R)}_{S,u} < C^{(R)}_{B,u} = C^{(R)}_{B,l}$, the replicating portfolio for both $g_t > 0$ and $g_t \leq 0$ is more expensive for bonds than for stocks in the tax-deferred account.

As one can see, conditioned on either $g_t \leq 0$ or $g_t > 0$ bonds are more expensive to replicate in the taxable account than stocks. This implies that given $g_t \leq 0$ or $g_t > 0$ bonds have their preferred location in the tax-deferred account if the replication cost of tax-deferred wealth in the taxable account is state independent at each point in time (Huang (2007)). However, this does not imply that bonds are the preferred asset to hold in the tax-deferred account. This is due to the fact that the replicating portfolios for $g_t \leq 0$ and $g_t > 0$ are different. If the investor chooses the replicating portfolio for $g_t \leq 0$, but $g_t$ turns out to be positive, the return of the replicating portfolio is given by

$$1 + d + g_t (1 - \tau_g) < 1 + d + g_t. \tag{3.9}$$

In this case, the return of the replicating portfolio is lower than in the tax-deferred account, due to the taxation of capital gains in the taxable account. If, on the other hand, the investor chooses the replicating portfolio for $g_t > 0$, but $g_t$ turns out to be negative, the return of the replicating portfolio is given by

$$1 + d + \frac{g_t}{1 - \tau_g} < 1 + d + g_t. \tag{3.10}$$

Again, the return of the replicating portfolio is below the return of one unit of stocks in the tax-deferred account. Hence, in addition to the uncertainty about the return of the risky asset, replication itself is another source of risk. In contrast to the setting in Huang (2007) with limits on tax rebates there is no longer a dominating asset location strategy. In the presence of limits on tax rebates for capital losses, it can be optimal to hold stocks in both taxable and tax-deferred accounts for two reasons.

First, whereas so far, the focus of our analysis has been on the impact of asset location decisions for cash flows, the asset location decision also has an impact on a future loss carryforward. An investment strategy that does not invest heavily in stocks in the taxable account in some given period potentially generates a small loss carryforward that has a high probability of being entirely used in future periods In contrast, an investment strategy that does invest heavily in stocks in the taxable account potentially generates a huge loss carryforward that has a lower probability of being entirely used in future periods. It can be optimal to invest in stocks in both the taxable and the tax-deferred account to have the opportunity of generating a small loss carryforward, while avoiding generating a high loss carryforward with a lower probability of being entirely used.

Second, for negative returns, stocks are better held in the taxable account to have the opportunity of being compensated for capital losses with a loss carryforward. For substantial positive returns, however, stocks are better held in the tax-deferred account to benefit from the tax exemption of capital gains in the tax-deferred account. Thus, holding stocks in both taxable and tax-deferred accounts can be part of a diversification strategy and limits on tax rebates for capital losses can explain the asset location puzzle.

Due to the different taxable treatment in tax-deferred and taxable accounts, shifting assets between these two accounts changes their risk-return profiles. While shifting bonds from a taxable to a tax-deferred account increases their return by $\tau_d r$, shifting stocks from the taxable to the tax-deferred account increases their return by $\tau_d d + \tau_g g_t$ in a tax-system with unlimited capital loss deduction and by $\tau_d d + \tau_g \chi_{\{g_t>0\}} g_t$ in a tax-system with limits on tax rebates. Thus, stocks are less attractive in the taxable account in the tax-system without tax rebates on capital losses. This is because the taxable treatment for positive returns remains the same, while for negative returns the investor is not compensated for losses in cash, but instead with a loss carryforward that – in contrast to tax rebates – does not pay any interest and bears a risk of never being converted into cash. The second source of risk is especially important for aged investors with higher mortality rates.

In a tax-system with unlimited tax rebates on capital losses, the tax-deferred account provides an investor with the opportunity of either earning a safe $\tau_d r$ extra dollars per dollar of bonds in the tax-deferred account or $\tau_d d + \tau_g g_t$ extra dollars for each dollar of stocks in the tax-deferred account in period $t$. In particular, as $\tau_g g_t$ is negative for $g_t < 0$, the tax-exemption of profits in the tax-deferred account can be a disadvantage to the investor since there are no tax rebates on capital losses in tax-deferred accounts. In a tax-system without tax rebates on capital losses, the tax-deferred account provides an investor who has no initial loss carryforward with the opportunity of either earning a safe $\tau_d r$ extra dollars and a loss carryforward of

**Effects of Shifting Asset from Taxable to Tax-Deferred Account**

| Asset | Effect of shift on | Unlimited tax rebates | No tax rebates |
|---|---|---|---|
| Bonds | Return | $\tau_d r$ | $\tau_d r$ |
| | Extra loss carryforward | - | - |
| Stocks | Return | $\tau_d d + \tau_g g_t$ | $\tau_d d + \tau_g g_t \chi_{\{g_t \geq 0\}}$ |
| | Extra loss carryforward | - | $\tau_g g_t \chi_{\{g_t < 0\}}$ |

Table 3.1: This table shows the ffects of shifting asset from a taxable to a tax-deferred account in tax-systems with limited and unlimited tax rebates on capital losses.

$\tau_g g_t$ dollars, if $g_t < 0$, per dollar of bonds shifted to the tax-deferred account or a safe $\tau_d d$ extra dollars and an extra $\tau_g g_t$ extra dollars if $g_t > 0$, per dollar of stocks shifted to the tax-deferred account.

While in the tax-system with unlimited capital loss deduction, shifting stocks from the taxable to the tax-deferred account increases the order of magnitude of a capital loss, this does not hold in a tax-system without capital loss deduction. Thus, shifting stocks from the taxable to the tax-deferred account does not increase the downside risk, but increases the upside potential. However, for negative capital gains, stocks have their preferred location in the taxable account due to the loss carryforward granted for incurred losses and the preferential taxable treatment of bonds in the tax-deferred account.

Table 3.1 summarizes the effects of shifting assets from the taxable to the tax-deferred account in tax-systems without tax rebates on capital losses and unlimited tax rebates.

If, however, the investor is endowed with an initial loss carryforward, potential capital gains in the taxable account not exceeding that loss carryforward can be earned tax-free by making use of the loss carryforward. In that case, shifting stocks to the tax-deferred account only allows the investor to earn $\tau_d d$ extra dollars, whereas shifting bonds to the tax-deferred account allows her to earn $\tau_d r > \tau_d d$ extra dollars per dollar invested in the tax-deferred account. Thus, for investors that are endowed with an initial loss carryforward of substantial size, holding bonds in the tax-deferred account increases their total wealth more than holding stocks in the tax-deferred account.

However, due to penalties on early withdrawal, the opportunity to earn profits tax-free and maximum contribution limits in tax-deferred accounts, taxable and tax-deferred wealth are imperfect substitutes. Besides the impact on total wealth, the distribution on future taxable and tax-deferred wealth can have an impact on the optimal investment decision as well.

Furthermore, an investment decision affects the next period's distribution of the loss carryforward. Since a loss carryforward does not pay any interest, a given increase in a loss carryforward has a higher value for an investor who is endowed with a low initial loss carryforward than for an investor who is endowed with a high initial loss carryforward.[6] While the investor who is endowed with an initial loss carryforward performs better in case of a positive capital gain, the investor without an initial loss carryforward generates a higher advantage from the loss carryforward. Due to the higher value of a potential increase in the loss carryforward, for an investor who has no loss carryforward it can even be rational to hold a higher equity proportion in the taxable account than for an investor with a small loss carryforward.

In addition, the advantage of an initial loss carryforward depends on the distribution of the investor's total wealth. Investors that hold a high fraction of their total wealth in a tax-deferred account should c.p. assign a lower value to an initial loss carryforward than investors holding a substantial fraction of total wealth in a taxable account.

While so far it has been assumed that the tax-system does not allow for tax rebates ($M = 0$), we now turn to the case that the tax-system does allow for tax rebates ($M > 0$). In this case, it can be optimal to carry losses forward and receive tax rebates from the loss carryforward in future periods. For not decreasing the loss carryforward, and thereby tax rebate payments, it can be optimal to hold stocks in the tax-deferred account. If, however, an investor only has a small or no loss carryforward, it can be optimal to hold mixed portfolios in both the taxable and the tax-deferred account.

## 3.3 Numerical Evidence

Having analyzed the effects of limits on tax rebates for optimal asset location decisions from a theoretical perspective, we now focus on the optimal asset location and asset allocation over the life cycle in a numerical setting. We consider an econ-

[6]This issue is analyzed in more detail in section 3.3.3.

omy consisting of investors living for at most $T$ years that can only trade at time $t = 0, 1, \ldots, T$. $F(t)$ denotes the probability that the investor is still alive through period $t$ ($t \leq T$). Investors in that economy derive utility from the consumption of a single good and have CRRA-utility with parameter of risk-aversion $\gamma \in [0, \infty)$, i.e.

$$U(C_t) = \begin{cases} \frac{C_t^{1-\gamma}}{1-\gamma} & \text{for } \gamma \neq 1 \\ \ln(C_t) & \text{for } \gamma = 1. \end{cases} \tag{3.11}$$

The parameter $\gamma$ represents the investor's willingness to substitute consumption among different states in time. It also represents the elasticity of consumption, which is given by $\frac{1}{\gamma}$. Given an initial endowment, the investor optimizes the discounted expected utility of lifetime consumption and bequest, subject to the intertemporal budget constraint. Following Dammon et al. (2001), we assume that at the time of death, the investor's remaining wealth is used to purchase an $H$-period annuity payable to the investor's beneficiary and that the $H$-period annuity provides the beneficiary with nominal consumption of $A_H W_t (1+i)^{k-t}$ at date $k$ $(t+1 \leq k \leq t+H)$, in which $A_H \equiv \frac{r^*(1+r^*)^H}{(1+r^*)^H - 1}$ is the $H$-period annuity factor, $W_t$ is the sum of the investor's taxable and after-tax tax-deferred wealth, and $r^* \equiv \frac{(1-\tau_d)r - i}{1+i}$ is the after-tax real bond return. For simplicity, the beneficiary is assumed to have the same preferences as the investor. On this assumption, $H$ can be interpreted as a measure for the strength of the investor's bequest motive. High values of $H$ denote a strong bequest motive and low values denote a weak bequest motive. The parameter $\beta$ represents the investor's utility discount factor and $i$ is a constant annual inflation rate. If $C_t$ denotes the investor's consumption, $Z_t$ the contribution to (or withdrawal from) the tax-deferred account that is limited by some upper bound $B_t$, i.e. $Z_t \leq B_t$, $\tau_p$ is the penalty tax-rate applicable to early withdrawals from tax-deferred accounts, $A_t$ is the investor's age in period $t$, $J$ is the investor's mandatory retirement age, and $N_t$ is the investor's non-financial income in period $t$, the investor's optimization problem over the life cycle can be expressed as

$$\max_{C_t,\alpha_{T,t},\alpha_{R,t},Z_t} \mathbb{E}\left[\sum_{t=0}^{T}\beta^t\left[F(t)U\left(\frac{C_t}{(1+i)^t}\right)+[F(t-1)-F(t)]\sum_{k=t+1}^{t+H}\beta^{k-t}U\left(\frac{W_tA_H}{(1+i)^t}\right)\right]\right] \tag{3.12}$$

subject to

$$W_t = W_{T,t} + (1-\tau_d)\,W_{R,t} \qquad t=1,\ldots,T \tag{3.13}$$

$$\begin{aligned} W_{T,t} = {} & N_t\,(1-\tau_d) + Q_{T,t-1}\Big(\alpha_{T,t-1}\,(1+(1-\tau_d)\,d+g_{t-1}) \\ & +(1-\alpha_{T,t-1})\,(1+(1-\tau_d)\,r)\Big) - \tau_g T_{t-1} \qquad t=1,\ldots,T \end{aligned} \tag{3.14}$$

$$Q_{T,t} = W_{T,t} - C_t - Z_t\left(1-\tau_d-\tau_p\chi_{\{Z_t<0\}\cap\{A_t<J\}}\right) \qquad t=0,\ldots,T-1 \tag{3.15}$$

$$W_{R,t} = Q_{R,t-1}\Big(\alpha_{R,t-1}\,(1+d+g_{t-1}) + (1-\alpha_{R,t-1})\,(1+r)\Big) \qquad t=1,\ldots,T \tag{3.16}$$

$$Q_{R,t} = W_{R,t} + Z_t \qquad t=0,\ldots,T-1 \tag{3.17}$$

$$Q_{T,t}, Q_{R,t}, C_t \geq 0; \quad \alpha_{T,t},\alpha_{R,t}\in[0,1]; \quad Z_t \leq B_t \qquad t=0,\ldots,T-1 \tag{3.18}$$

given the investor's initial taxable wealth $W_{T,0}$, the initial tax-deferred wealth $W_{R,0}$ and the initial loss carryforward $L_{-1}$. $F(-1)$ is set equal to one to indicate that the investor is alive at the end of period 0. $U(.)$ denotes the utility function of the investor and the beneficiary.

According to Equation (3.12), the investor's current expected utility is a weighted sum of utility from consumption and utility from bequest. Equation (3.13) defines the investor's beginning-of-period-wealth $W_t$ at time $t$ to be the sum of taxable wealth $W_{T,t}$ and $W_{R,t}$ that part of tax-deferred retirement wealth that does not fall to the treasury at withdrawal.

Equation (3.14) describes the evolution of wealth in the taxable account. $W_{T,t}$ is the investor's taxable wealth in period $t$ before consumption and investment decisions. $Q_{T,t-1}$ is the investor's taxable wealth invested in period $t-1$ after consumption, contributions or withdrawals, and the tax-payments or tax-rebates resulting from the contribution or withdrawal. Equation (3.15) is the investor's budget constraint. It shows how the investor's reinvestable wealth in the taxable account depends on consumption as well as the contribution or withdrawal from the tax-deferred account.

Equations (3.16) and (3.17) define the evolution of wealth in the tax-deferred account. Equation (3.16) shows the evolution of wealth $W_{R,t}$ in the tax-deferred account. $Q_{R,t-1}$ is the investor's tax-deferred wealth invested from period $t-1$ to period $t$. Equation (3.17) defines $Q_{R,t}$. If the investor dies before retirement age, we assume that the penalty tax on early withdrawal does not apply, which, among other reasons, can be the case if the beneficiary is the investor's spouse.

By letting $X_t$ denote the vector of the investor's state variables, $V_t(.)$ the investor's value function at time $t$, and $f(t)$ the probability of surviving from period $t$ to $t+1$, and taking into account that

$$\sum_{k=t+1}^{k+H} \beta^{k-t} = \frac{\beta\left(1-\beta\right)^H}{1-\beta} \tag{3.19}$$

as shown in Dammon et al. (2001), the optimization problem can also be stated as

$$\begin{aligned} V_t(X_t) = \max_{C_t,\alpha_{T,t},\alpha_{R,t},Z_t} & \Bigg[ f(t)U\left(\tfrac{C_t}{(1+i)^t}\right) + f(t)\beta\mathbb{E}_t\left[V_{t+1}\left(X_{t+1}\right)\right] \\ & + (1-f(t))\,\tfrac{\beta\left(1-\beta^H\right)}{1-\beta} U\left(\tfrac{A_H W_t}{(1+i)^t}\right)\Bigg] \end{aligned} \tag{3.20}$$

for $t=0,\ldots,T-1$ subject to Equations (3.2), (3.3) and (3.13) to (3.18) with vector of state variables

$$X_t = \left[W_{T,t}, W_{R,t}, L_{t-1}\right] \tag{3.21}$$

and terminal condition $V_T(X_T) = U\left(\frac{A_H W_T}{(1+i)^T}\right)$. For the numerical analysis, we assume that $M$ is a constant multiple of $W_t$, i.e. $m \equiv \frac{M}{W_t}$ is a constant. Following Dammon et al. (2004), we assume that non-financial income $N_t$ is a constant multiple of $W_t$, i.e. $n \equiv \frac{N_t}{W_t}$ $(t \in \mathbb{N}_J \equiv \{t \in \mathbb{N} | t \leq J\})$ is a constant during the accumulation phase and a multiple of $n$ that is given by the replacement rate $\lambda$ during retirement. We further assume that $b_t \equiv \frac{B_t}{W_t}$ is an exogenously given constant during the investor's working age.

When the investor is retired, we do not allow any further contributions and require the investor to withdraw at least a fraction of $\frac{1}{\mathbb{E}[L(A_t)]}$ of the remaining tax-deferred wealth when the investor is aged 70.5 and older where $L(A_t)$ is the remaining life-expectancy of an investor at age $A_t$. The optimization problem can then be simplified by normalizing both the objective function and the constraints by $W_t$. We let $w_{R,t} \equiv (1-\tau_d)\frac{W_{R,t}}{W_t}$ denote the fraction of total beginning-of-period-wealth in the tax-deferred account. $q_{T,t} \equiv \frac{Q_{T,t}}{W_t}$ is the investor's taxable wealth after transactions relative to beginning-of-period-wealth and $q_{R,t} \equiv (1-\tau_d)\frac{Q_{R,t}}{W_t}$ is the investor's retirement wealth after transactions relative to beginning-of-period-wealth. The investor's initial loss carryforward beginning-of-period-wealth ratio is $l_t \equiv \frac{L_{t-1}}{W_t}$. $z_t \equiv \frac{Z_t}{W_t}$ is the investor's contribution-wealth-ratio, and $c_t \equiv \frac{C_t}{W_t}$ is the investor's consumption-wealth-ratio. Finally,

$$\delta_t \equiv \frac{G_t}{Q_{T,t}} = \alpha_{T,t} g_t \tag{3.22}$$

is the investor's capital gain per dollar invested in the taxable account from period $t$ to $t+1$. The investor's taxable capital gains wealth ratio $t_t \equiv \frac{T_t}{W_t}$ is then given by

$$t_t = \max\left(\delta_t q_{T,t} + l_{t-1}, -m\right) \tag{3.23}$$

in which $q_{T,t} = 1 - w_{R,t} - c_t - z_t\left(1 - \tau_d - \tau_p \chi_{\{z_t<0\}\cap\{A_t<J\}}\right)$ defines the ratio between $Q_{T,t}$ and $W_t$. If

$$\mu_{T,t} = \alpha_{T,t}\Big(1 + (1-\tau_d)\, d + g_t\Big) + (1-\alpha_{T,t})\Big(1 + (1-\tau_d)\, r\Big) \tag{3.24}$$

and

$$\mu_{R,t} = \alpha_{R,t}\Big(1 + d + g_t\Big) + (1-\alpha_{R,t})\Big(1 + r\Big) \tag{3.25}$$

denote the investor's return per dollar invested in the taxable account and the tax-deferred account before capital gains tax payments, respectively, the evolution of the investor's total wealth is given by

$$\frac{W_{t+1}}{W_t} = \frac{q_{T,t}\mu_{T,t} - \tau_g t_t + q_{R,t}\mu_{R,t}}{1 - n_{t+1}(1-\tau_d)}. \tag{3.26}$$

For the evolution of the investor's tax-deferred retirement wealth, it holds that

$$w_{R,t+1} = q_{R,t}\mu_{R,t}\frac{W_t}{W_{t+1}}, \tag{3.27}$$

and the evolution of the investor's loss carryforward is given by

$$l_t \equiv \frac{L_t}{W_{t+1}} = \frac{L_t}{W_t}\frac{W_t}{W_{t+1}} = \left(\delta_t q_{T,t} - t_t + l_{t-1}\right)\frac{W_t}{W_{t+1}}. \tag{3.28}$$

By defining $v_t(x_t) \equiv \frac{V_t(X_t)}{\left(\frac{W_t}{(1+i)^t}\right)^{1-\gamma}}$ to be the normalized value function, and $\rho_t \equiv \frac{W_{t+1}}{W_t(1+i)}$ to be the real growth rate of total wealth in period $t$, the assumption of CRRA-utility assures that the investor's objective function can be rewritten as

$$\begin{aligned} v_t(x_t) \;=\; \max_{c_t,\alpha_{T,t},\alpha_{R,t},z_t} & \Bigg[ f(t)U(c_t) + f(t)\beta\mathbb{E}\left[v_{t+1}\left(x_{t+1}\right)\rho_t^{1-\gamma}\right] \\ & + (1-f(t))\frac{\beta\left(1-\beta^H\right)}{1-\beta}U\left(A_H\right)\Bigg] \end{aligned} \tag{3.29}$$

with vector of state-variables

$$x_t = \left[w_{R,t}, l_{t-1}\right]. \tag{3.30}$$

The investor's optimization problem can then be rewritten as

$$\max_{c_t,\alpha_{T,t},\alpha_{R,t},z_t} \left[ f(t)U(c_t) + f(t)\beta\mathbb{E}\left[v_{t+1}\left(x_{t+1}\right)\rho_t^{1-\gamma}\right] + (1-f(t))\frac{\beta\left(1-\beta^H\right)}{1-\beta}U\left(A_H\right)\right] \tag{3.31}$$

subject to

$$q_{T,t}, q_{R,t}, c_t \geq 0; \quad \alpha_{T,t}, \alpha_{R,t} \in [0,1]; \quad z_t \leq b_t \tag{3.32}$$

and Equations (3.22) through (3.28).

For the numerical analysis, it is assumed that annual inflation is $i = 3.5\%$, mandatory retirement age is $J = 66$, in that the investor retires when she turns 66. It is further assumed that exogenous income $n_t$ is 0.15 during the accumulation phase and 0.105 during retirement, corresponding to a replacement ratio of $\lambda = 0.7$.[7] The risk-free rate is set to $r = 6\%$, the return on equity is lognormally distributed, independent in time, and comes with an expected capital gain of $\mu = 7\%$ ($t \in \mathbb{N}_T$), a standard deviation of $\sigma = 20.7\%$ (which corresponds to a standard deviation of the

[7] Ideally, non-financial income should be introduced into our model with its own stochastic process. However, this would require increasing the number of state variables and significantly complicate our numerical analysis. We therefore follow recent literature (see e.g. Dammon et al. (2001, 2004)) and assume the investor's non-financial income to be a constant proportion of her beginning-of-period wealth. However, since present savings do not only increase future wealth, but also future non-financial income, this assumption overstates the impact of present savings.

real return of about 20%), and a constant dividend rate of $d = 2\%$ in each period.

The correct choice of the risk-premium for equity has been subject to numerous theoretical and empirical research (see Siegel (2005) for a survey). While the historical risk-premium has been about 6% in the US since 1872 (Mehra and Prescott (1985)), economists doubt whether this will be true in future periods. We follow the current consensus which is about 3% to 4% (see e.g. Dammon et al. (2001, 2004), Cocco et al. (2005) or Gomes and Michaelides (2005)). The tax-rate on interest, dividends and income is assumed to be $\tau_d = 36\%$. The tax-rate on realized capital gains is assumed to be $\tau_g = 20\%$.[8] This choice of parameters follows Gallmeyer et al. (2006) and is quite similar to that of Dammon et al. (2004). For this parameter choice, the asset location problem is the problem of whether the investor prefers a safe extra return of $\tau_d r = 0.0216$ and a loss carryforward of $\tau_g g_t$ for $g_t < 0$ per dollar of bonds in the tax-deferred account in period $t$, or a safe extra return of $\tau_d d = 0.0072$ and an extra return of $\tau_g g_t$ where $g_t \geq 0$ for each dollar of stocks held in the tax-deferred account in period $t$. We consider an investor who makes decisions annually starting at age 20 ($t = 0$). The maximum age the investor can attain is set to 100 years ($T = 80$). It is assumed that the relative risk-aversion of the investor is $\gamma = 3$ and the annual subjective utility discount factor is $\beta = 0.96$. The data for the survival probabilities of our investor were set equal to the survival probabilities for female investors according to the 2001 Commissioners Standard Ordinary Mortality Table.

During the accumulation phase until retirement age $J$, the maximum contribution is set to $b_t = 5\%$ ($t \in \mathbb{N}_{J-1}$). When the investor is retired, we do not allow any further contributions and require an investor at age $A_t$ with remaining life-expectancy of $L(A_t)$ to withdraw at least a fraction of $\frac{1}{\mathbb{E}[L(A_t)]}$ of the remaining tax-deferred wealth when the investor is aged 70.5 and older. Initially, $H$ is set to $H = 10$ in the bequest function, indicating that the investor wished to provide the beneficiary with an income stream for the next 10 years. This set of parameter values is referred to as the base case scenario. Table 3.2 summarizes these base-case parameters.

Our optimization problem can be solved numerically using backward induc-

[8]Since the tax-rate on long-term capital gains is equal to the tax-rate on dividend income, we thereby implicitly assume that the capital gains qualify for long-term treatment. The case with $\tau_g = \tau_d = 0.36$ is analyzed in section 3.3.5.

**Base-case Parameter Values**

| Description | Parameter | Value |
|---|---|---|
| Risk-aversion | $\gamma$ | 3 |
| Length of investment horizon | $T$ | 80 |
| Number of years annuity beneficiary | $H$ | 10 |
| Mandatory retirement age | $J$ | 66 |
| Utility discount factor | $\beta$ | 0.96 |
| Dividend rate | $d$ | 2% |
| Expected return stock | $\mu$ | 7% |
| Standard deviation stock | $\sigma$ | 20.7% |
| Interest payment of bond | $r$ | 6% |
| Inflation rate | $i$ | 3.5% |
| Non-financial income | $n$ | 15% |
| Tax-rate on dividends, interest and income | $\tau_d$ | 36% |
| Tax-rate on capital gains | $\tau_g$ | 20% |

Table 3.2: This table reports the parameter values used in the base-case.

tion. The state-space for the vector of endogenous state-variables $x_t = [w_{R,t}, l_{t-1}]$ is spanned over a $51 \times 51$ grid with equally distributed grid points on $[0, 1]$ and $[-0.5, 0]$, respectively, for each period $t$. For values of $[w_{R,t}, l_{t-1}]$ within the grid, cubic spline interpolation is performed. The integral of the expectation in Equation (3.31) is computed using Gaussian quadrature.

In the following, we present our numerical results for the optimal consumption-investment problem in the presence of taxes, limits on tax rebates for capital losses and short-sale restrictions. The model is first solved for the base-case parameters. To demonstrate the effect of limits on tax rebates for capital losses and tax-deferred investment opportunities, the model is also solved for a setting with unlimited tax rebates for capital losses and a setting without a tax-deferred account. These two settings serve as benchmarks for our forthcoming analysis. The optimal consumption policy is considered in section 3.3.1, and the optimal investment policy is analyzed in section 3.3.2. In section 3.3.3, we compute the value of an initial loss carry-forward. Section 3.3.4 examines the effect of optimal asset location on expected utility by comparing the optimal investment strategy with the strategy of preferring bonds in tax-deferred accounts. Section 3.3.5 examines the sensitivity of the optimal investment and liquidation policies to various model parameters.

### 3.3.1 Optimal Consumption Policy

We begin the discussion of our numerical results by presenting the investor's optimal consumption policy over the life cycle depending on age, the fraction of wealth in the tax-deferred account and an initial loss carryforward the investor is potentially endowed with. As benchmark cases, we consider an investor in a tax-system with unlimited capital loss deduction and an investor who does not have the opportunity of locating funds in a tax-deferred account.

The relation between the optimal consumption-wealth ratio and age depends heavily on the investor's bequest motive. As the investor ages, her mortality rate increases and the impact of the bequest motive on her consumption decision increases. Investors with strong bequest motives will decrease their consumption-wealth ratio; while in contrast, investors with low levels of bequest motive will increase their consumption-wealth ratio. The bequest motive used in the base-case is not so strong that the investor decreases her consumption throughout her entire life nor is it so weak that she increases her consumption throughout her entire life. Due to tax-effects and a replacement ratio below one, the investor decreases her consumption until retirement age and increases it again afterwards.

However, the optimal consumption-wealth ratio does not only depend on age, but also on the investor's initial loss carryforward and the distribution of her wealth to the taxable and the tax-deferred account. The higher the loss carryforward, the more advantageous the risk-return profile of equity in the taxable account, and thus the higher the increase of potential future consumption for each dollar that is not consumed today. The higher the investor's fraction of wealth in the tax-deferred account, the higher the investor's expected return on her total wealth. Thus, to generate the same expected wealth in the forthcoming period, an investor who has a high fraction of tax-deferred wealth can consume more than an investor who has a small fraction of tax-deferred wealth.

Figure 3.1 depicts the optimal consumption-wealth ratios of an investor in our base-case setting who is endowed with either no initial loss carryforward ($l = 0$, left graph), or an initial loss carryforward of 10% of her beginning-of-period wealth ($l = -0.1$, right graph).

The investor's optimal consumption depends on both age and the fraction of

**Optimal consumption over the life cycle in the base-case**

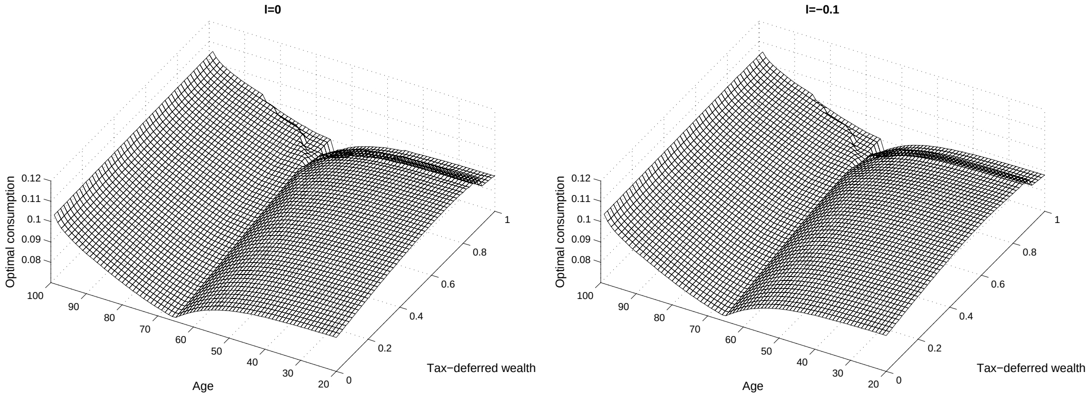


Figure 3.1: Optimal consumption-wealth-ratio (optimal consumption) for a female investor in the base-case scenario who has an initial loss carryforward of zero ($l = 0$) or 10% of her initial wealth ($l = -0.1$) as a function of her age and her initial fraction of tax-deferred wealth.

tax-deferred wealth. The optimal consumption is strictly increasing in tax-deferred wealth except for very high levels of tax-deferred wealth where the investor has to withdraw from her tax-deferred account to finance consumption. This is due to the fact that tax-deferred wealth provides the investor with higher returns than taxable wealth due to the tax-exemption of profits. Due to the tax-exemption of profits in tax-deferred accounts, one dollar of tax-deferred wealth is worth more than one dollar of taxable wealth if the investor can avoid the penalty-tax on early withdrawal (Dammon et al. (2004), Poterba (2004)). Hence, with a higher level of tax-deferred wealth, the investor can attain the same level of next-period-wealth by investing a lower amount of wealth today. Furthermore, the investor can both increase her present consumption by consuming some part of the difference between the amount of wealth needed in the absence of a tax-deferred investment opportunity and the amount needed in the presence of tax-deferred investment opportunities, and investing the remainder. Young investors, however, face penalties on early withdrawals from tax-deferred accounts.

Prior to the retirement age of 66, the investor's consumption gradually decreases. This reflects the fact that at retirement age, the investor's non-financial income sharply decreases. To keep up a certain level of consumption, the investor builds up capital she can use to compensate for the decrease in non-financial income when she reaches retirement age.

Over the life cycle, the investor faces two structural changes in the taxable treatment of her wealth. First, having attained retirement age, the investor is not allowed to make any further contributions to the tax-deferred account, but is allowed to withdraw from it. For an investor with very high levels of tax-deferred wealth, the opportunity to withdraw funds from the tax-deferred account is an advantage since it allows her to consume without being confronted with the tax-penalty on early withdrawal. Hence, having attained retirement age consumption increases significantly.[9] An investors with lower levels of tax-deferred wealth, however, does not consider the opportunity of withdrawing funds from the tax-deferred account a tremendous advantage, since she is endowed with sufficient amounts of taxable wealth. The upper bound on contributions of zero prevents such investors from shifting taxable wealth to tax-deferred wealth for earning higher returns. The investor thus can no longer increase returns by shifting the location of assets, which reduces the intertemporal rate of consumption, and makes current consumption more attractive due to the decreasing premium on consumption deferral. Consequently, the consumption level increases having attained retirement age.

Second, at age 70.5 the investor is subject to the minimum withdrawal rules which force her to withdraw at least a fraction, equal to one divided by her remaining life expectancy, from her tax-deferred account. Due to this forced withdrawal, the investor's return is decreasing since assets in the tax-deferred account are not subject to taxation of profits while assets in the taxable account are. Hence, the appeal of current consumption increases as the premium for a deferral of consumption decreases. This is why starting at age 70.5, the slope of the investor's consumption increases with time even stronger than before.

Compared to the benchmark case with no tax-deferred account (not shown here), the investor's optimal consumption is significantly higher, since the tax-deferred investment opportunity allows the investor to generate substantially higher returns than investments in the taxable account. Compared to the benchmark case with unlimited capital loss deduction, the investor's optimal consumption does not change significantly.

Figure 3.1 further shows that the level of an initial loss carryforward does not have a significant impact on optimal consumption. However, an initial loss car-

[9] Due to the assumption of non-financial income increasing taxable wealth, the base-case investor can never end up in such a state. In section 3.3.5 we consider an investor who does not have non-financial income.

ryforward does have a tremendous impact on the optimal investment policy. In particular, it has a strong impact on optimal asset location.

### 3.3.2 Optimal Investment Policy

Having discussed the optimal consumption policy in the previous section, we now turn to the optimal investment policy in the presence of tax-deferred investment opportunities and limitations on capital loss deductions.

In the absence of tax-deferred investment opportunities (not shown here), there is no longer an asset location problem. In that case, differences in the investor's equity exposure only depend on the remaining investment horizon and the level of the initial loss carryforward. Not very surprisingly, the investor's equity exposure tends to increase with an increasing initial loss carryforward. This is due to the fact that in the presence of a loss carryforward, the risk-return-profile of the asset becomes more advantageous to the investor. Furthermore, for low levels of initial loss carryforward, the investor's equity exposure decreases when the investor ages. This is due to the fact that because by investing into equity, the investor faces a high probability of generating a loss carryforward that potentially cannot be used anymore.

Without limits on capital loss deduction, the tax-arbitrage argument of Huang (2007) suggests that the investor should hold her entire tax-deferred wealth in bonds. Our numerical analysis supports this finding. Due to the unlimited tax rebates on capital losses, the risk-return-profile of stocks in the taxable account is quite attractive, which is why the preferred asset to hold in the tax-deferred account are bonds – a result which is in line with the findings of the existing literature on optimal asset location.

Figure 3.2 depicts the optimal proportion of stocks relative to taxable and tax-deferred wealth after consumption, contributions, and withdrawals, in both the taxable and the tax-deferred retirement wealth for an investor in our base-case setting who is either endowed with no initial loss carryforward ($l = 0$, upper graphs), endowed with an initial loss carryforward of 4% of beginning-of-period wealth ($l = -0.04$, middle graphs), or endowed with an initial loss carryforward of 10% of her beginning-of-period wealth ($l = -0.1$, lower graphs) as a function of age and the fraction of beginning-of-period-wealth in the tax-deferred account.

## Optimal fraction of stocks in taxable and tax-deferred wealth in the base-case

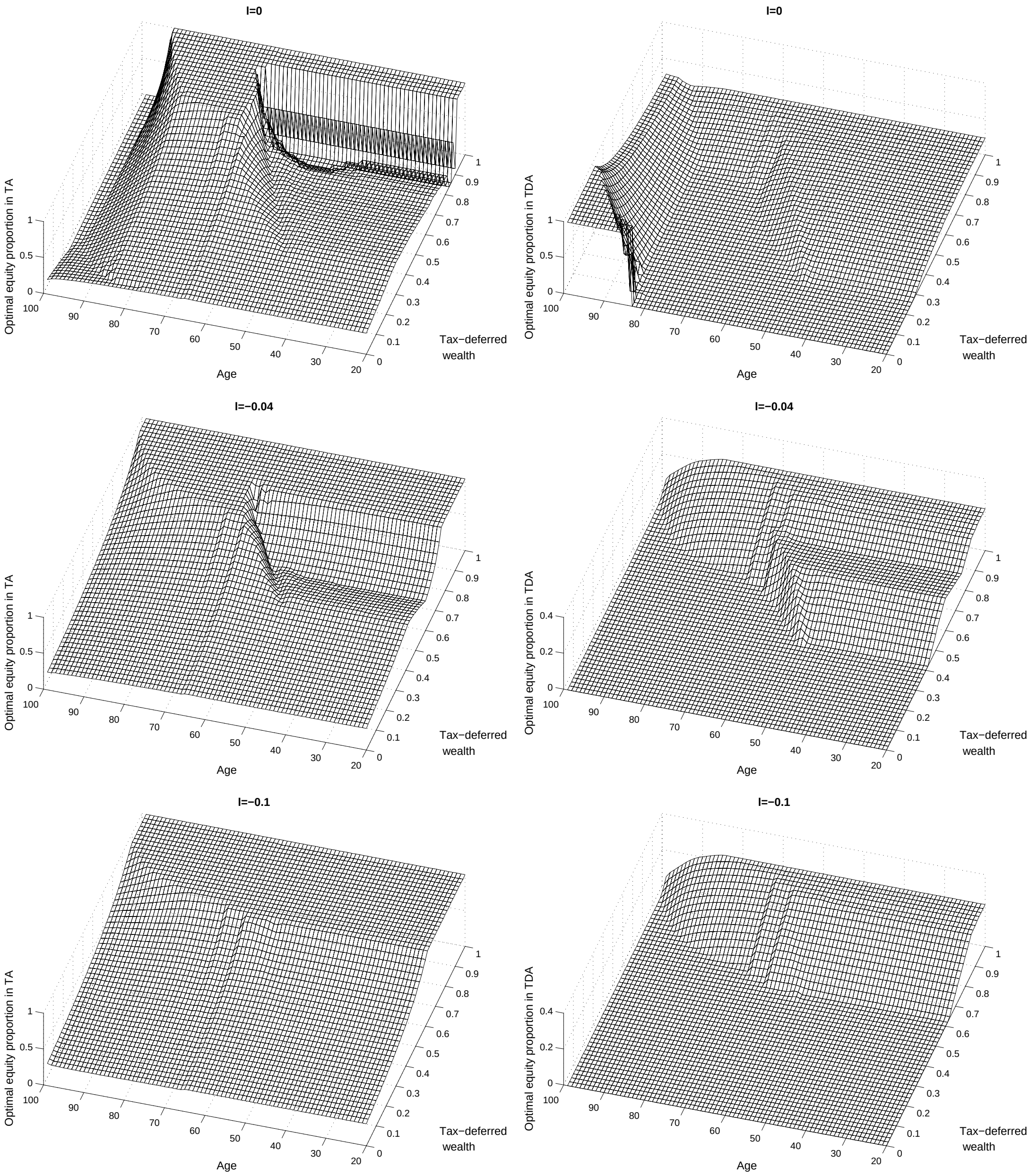


Figure 3.2: Optimal stock holding relative to taxable and tax-deferred wealth after consumption, contributions, and withdrawals for a female investor in the base-case scenario in taxable account (TA, left graphs), and tax-deferred account (TDA, right graphs) who is endowed with an initial loss carryforward of zero ($l = 0$, upper graphs), 4% of her initial wealth ($l = -0.04$, middle graphs) or 10% of her initial wealth ($l = -0.1$, lower graphs) as a function of age and initial fraction of tax-deferred wealth.

The upper left graph shows the optimal equity proportion in the taxable account for an investor who is not endowed with an initial loss carryforward. The upper right graph shows the optimal fraction of stocks in the tax-deferred account. When the investor is young, more stocks are held in the taxable account as the certain tax-gift in the tax-deferred account of $\tau_d r$ extra dollars and the loss carryforward of $\tau_g g_t$ in period $t$ is very attractive when she is young and can expect to make use of the loss carryforward at a significant probability. In contrast, if the investor is old, stocks are preferentially held in the tax-deferred account. This is due to the fact that for an old investor, the probability of using the entire loss carryforward in future periods is much lower than for a young investor. That is, a safe extra return of $\tau_d d$ plus an extra return of $\tau_g g_t$ if $g_t > 0$ in period $t$ tends to be more attractive for a young investor than a safe $\tau_d r$ extra dollars and a loss carryforward of $\tau_g g_t$ dollars if $g_t < 0$. However, to hold a well diversified portfolio, it can be rational to hold stocks in both the taxable and the tax-deferred account.[10]

During the accumulation phase, the investor takes advantage of that diversification strategy and holds stocks both in her tax-deferred and her taxable account. Having attained retirement age, she preferentially locates bonds in the tax-deferred account. This preference is caused by two tax-effects.

First, having passed age 70.5, the investor is subject to the minimum withdrawal rules such that an increase in the return on tax-deferred wealth does not only increase tax-deferred wealth, but also increases the minimum withdrawal in forthcoming periods, which in turn decreases the appeal of tax-deferred investing. However, the results of section 3.3.5 in the absence of minimum withdrawal rules suggest that such rules cannot explain the entire decrease in the equity exposure of retired investors.

Second, having passed retirement age, the investor is no longer allowed to contribute to the tax-deferred account. Since the investor's equity exposure in the taxable account is above the equity exposure in the tax-deferred account, and taxable wealth can no longer be decreased by contributing to the tax-deferred account, the optimal equity exposure in the tax-deferred account is decreasing. Accordingly,

[10]The optimal equity proportion of one in the taxable account for high levels of tax-deferred wealth is a side-effect of the modeling. Whereas tax-deferred wealth is measured as beginning-of-period tax-deferred wealth, the optimal equity proportion in the TA is given as a fraction of the investor's taxable wealth after consumption, contributions to and withdrawals from the taxable account. This modeling allows us to reduce the number of decision variables. Since the investor's optimal consumption is about 10%, she has no taxable wealth left that can be reinvested and thus any equity proportion in the taxable account is optimal.

the minimum distribution rules increase the investor's taxable wealth, which in turn causes her to decrease her taxable equity exposure when she passes age 70.5.

When investors are very old, these effects are outweighed by the high probability of generating a loss carryforward that cannot be used anymore due to the high mortality. Consequently, the loss carryforward has a low value for the investor, which causes her to increase her stock holdings in the tax-deferred account and decrease her equity exposure in the taxable account when approaching her maximum age of 100.

The middle left graph of Figure 3.2 shows the optimal equity exposure in the taxable account of an investor who is endowed with an initial loss carryforward of 4% of her initial total wealth. Compared to the case without a loss carryforward, there are two important differences.

First, the investor's equity exposure in the tax-deferred account at high age decreases while her equity exposure in the taxable account increases in comparison to the case with no initial loss carryforward. If the investor is endowed with an initial loss carryforward, her equity exposure in the taxable account is significantly above her equity exposure in the absence of an in initial loss carryforward. This is due to the fact that the loss carryforward changes the risk-return-profile of equity in the taxable account in an attractive manner. The increase in the investor's equity exposure is such that her loss carryforward suffices to cover potential capital gains at a satisfactory probability. Consequently, the investor decreases her equity exposure in the tax-deferred account to avoid getting too heavily invested into equity.

Second, in the presence of a small loss carryforward, a retired investor holds stocks in the tax-deferred account for lower levels of tax-deferred wealth than in the absence of a small loss carryforward. This is because when endowed with a substantial loss carryforward, the investor holds 100% of stocks in the taxable account for a lower level of initial tax-deferred wealth and thus can only increase her equity exposure by holding additional stocks. To avoid getting too underweight in equity, the investor holds some equity in the tax-deferred account as well. Again, bonds are the preferred asset in the tax-deferred account during retirement.

With increasing length of the investment horizon, i.e. for younger investors, a higher loss carryforward becomes more attractive, as the investor may expect to live longer and thus the probability of making use of the loss carryforward increases.

That is, with decreasing age, the investor increases her equity exposure in the taxable account and decreases her equity exposure in the tax-deferred account.

The lower left graph of Figure 3.2 shows the optimal equity exposure in the taxable account of an investor who is endowed with a substantial initial loss carryforward of 10% of her total wealth ($l = -0.1$). The lower right graph shows her optimal equity exposure in the tax-deferred account. While for a small loss carryforward, the investor holds equity in both the taxable and the tax-deferred account at high age, with substantial initial loss carryforward, equity is strictly preferred in the taxable account over the entire life cycle.

### 3.3.3 Valuing an Initial Loss Carryforward

In section 3.2 we argued that an initial loss carryforward increases the investor's utility since it allows her to earn some capital gains in the taxable account not exceeding that loss carryforward tax-free. In this section, we quantify the value of an initial loss carryforward. We do this by computing the increase in both taxable and tax-deferred an investor who is not endowed with an initial loss carryforward needs to be compensated with to attain the same level of utility as an investor who is endowed with some given initial loss carryforward.

Figure 3.3 shows the increase in both taxable wealth and tax-deferred wealth (equivalent wealth increase) an investor who is not endowed with an initial loss carryforward needs to be compensated with to attain the same level of utility as an investor who is endowed with an initial loss carryforward. The left graph shows the equivalent wealth increase for an investor at age 20 as a function of her tax-deferred wealth and the level of her initial loss carryforward. The right graph shows her equivalent wealth increase over the life cycle given a level of her tax-deferred wealth of 20%. Since one dollar of loss carryforward allows saving $\tau_g$ dollars of tax-payments, one dollar of loss carryforward cannot be worth more than $\tau_g$ dollars. However, since in contrast to present wealth, the loss carryforward does not pay any interest, it can be, and is, worth less as Figure 3.3 shows.

The left graph shows that if the investor is not endowed with an initial loss carryforward, her equivalent wealth increase is strictly increasing in the value of the initial loss carryforward, since a higher loss carryforward allows her to improve her after-tax return in the taxable account, leaving her with a higher after-tax wealth.

**Value of an initial loss carryforward**

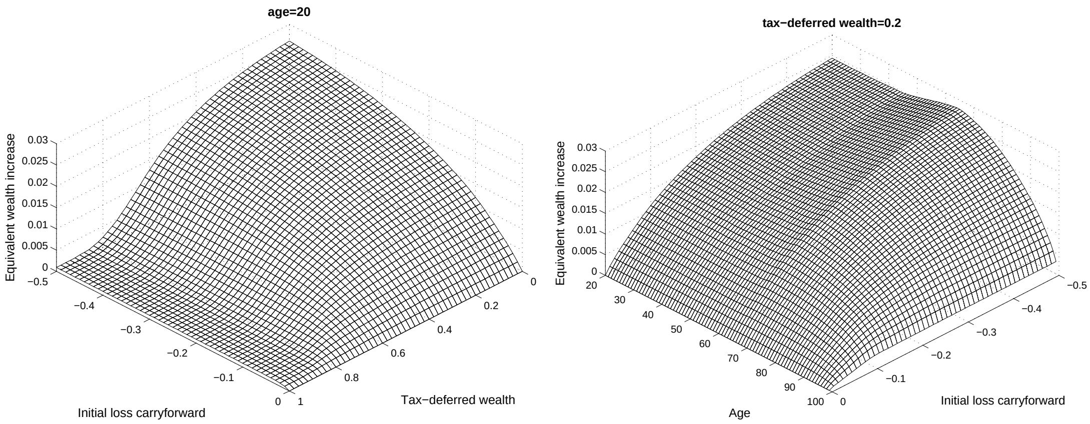


Figure 3.3: This Figure shows the increase in both taxable wealth and tax-deferred wealth (equivalent wealth increase) in the base-case scenario with which an investor who is not endowed with an initial loss carryforward needs to be compensated to attain the same level of utility as an investor who is endowed with an initial loss carryforward. The left graph shows the equivalent wealth increase for an investor at age 20 as a function of her tax-deferred wealth and the level of her initial loss carryforward. The right graph shows her equivalent wealth increase over the life cycle given a level of her tax-deferred wealth of 20%.

Furthermore, the equivalent wealth increase is strictly decreasing in the investor's tax-deferred wealth. This finding is also quite intuitive as a loss carryforward only provides the investor with an advantage in our base-case setting if she can subtract it from future positive capital gains in her taxable account. The higher the level of her tax-deferred wealth, the lower the level of her taxable wealth, and thus the lower the level of potential capital gains in the taxable account relative to total wealth. Consequently, her equivalent wealth increase is strictly decreasing in her tax-deferred wealth.

The right graph of Figure 3.3 shows that the wealth increase of an investor being endowed with a level of tax-deferred wealth of 20% is strictly decreasing with age after the investor has attained retirement age. This is due to the fact that the probability of not making use of the loss carryforward is increasing as she ages. Opposing the age-effect, the minimum distribution rules increase the investor's taxable wealth, which in turn increase the attractiveness of an initial loss carryforward. Figure 3.3 shows that the age-effect outweighs this opposing effect.

Prior to retirement age, the equivalent wealth increase remains at about the same level for a given initial loss carryforward. Approaching retirement age the investor's equivalent wealth increase goes up slightly with age, since at retirement age, the investor is no longer allowed to shift funds from her taxable to her tax-deferred account. Thus, her taxable wealth remains at a higher level than for an investor who can shift money to the tax-deferred account, which is why a loss carryforward has a higher value for her.

Both the left and the right graph in Figure 3.3 show that the equivalent wealth increase is concave in the level of the initial loss carryforward. With increasing initial loss carryforward, an additional loss carryforward results in a decreasing impact on equivalent wealth increase. This is due to the reason that, first, a higher loss carryforward has a lower probability of entirely being used over the investor's life cycle and, second, the loss carryforward does not pay any interest. Even for high levels of the initial loss carryforward, the level of the equivalent wealth increase remains quite low. An investor at age 20 who is endowed with an initial level of tax-deferred wealth and an initial level of her loss carryforward has the same expected utility as an investor who is endowed with no loss carryforward, but an initial wealth level that is about 2.5% above that of the first investor.

### 3.3.4 Welfare Analysis

In the absence of limits on tax rebates for capital losses, recent literature on optimal asset location suggests that locating bonds preferentially in tax-deferred accounts is an optimal asset location strategy. The previous sections examined the effect of limits on tax rebates for capital losses on optimal investment and liquidation policies. In this section, we investigate the welfare costs of preferentially locating bonds in the tax-deferred account and stocks in the taxable account. That is, we assume the investor to hold stocks in her tax-deferred account only if her entire taxable wealth is invested into stocks. We solve numerically for the optimal consumption, contribution, and investment strategies in taxable and tax-deferred accounts using the base-case parameters. The resulting value functions are compared to the value function from the base-case model with optimal asset location over the entire state space. For each point in the state space, the percentage increase is computed in both taxable and tax-deferred wealth that would be needed to bring the value function for the heuristic asset location strategy of preferring bonds in the tax-deferred account up to the level of the value function following the optimal asset location strategy (utility costs).

**Welfare costs of ”pecking-order” asset location strategy**

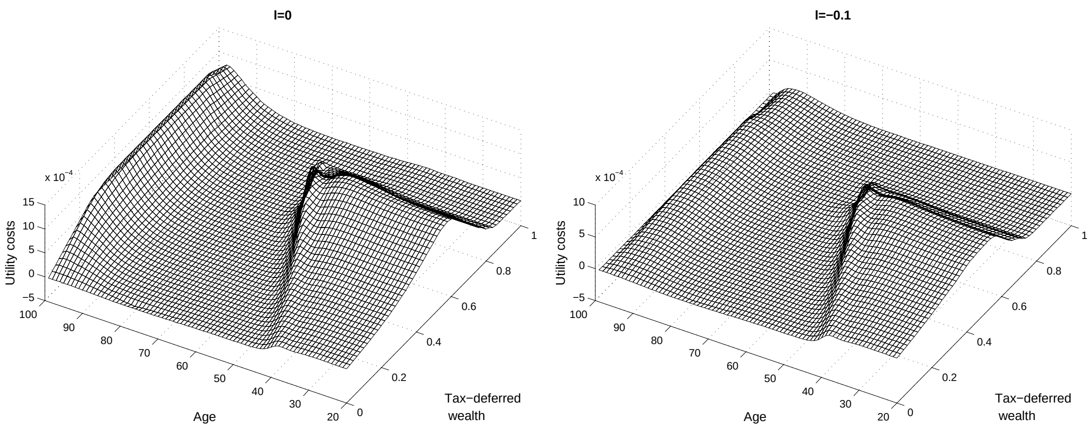


Figure 3.4: Welfare costs measured as wealth increase in both taxable and tax-deferred wealth (utility costs) needed to compensate an investor following the suboptimal asset location strategy as compared to the optimal policy in the base-case scenario. The Figure shows the results for an investor who is initially endowed with no loss carryforward ($l = 0$, left graph), or an initial loss carryforward of 10% of her total wealth ($l = -0.1$, right graph).

Figure 3.4 shows that the utility costs of following a ”pecking-order” rule which locates as many bonds as possible in the tax-deferred account results in a higher initial wealth needed to bring the investor up to the same level of utility as an investor following an optimal asset location strategy. The left graph in Figure 3.4 shows the investor’s utility costs when the investor is endowed with no initial loss carryforward ($l = 0$). The right graph shows the utility costs when the investor is endowed with an initial loss carryforward of 10% of her initial wealth ($l = -0.1$). When the investor is not endowed with an initial loss carryforward, her utility costs are positive if ever her optimal investment strategy differs from the strategy of preferring bonds in the tax-deferred account. This is the case when she is very old and faces high mortality rates, and during her accumulation phase, when she holds mixed portfolios in both the taxable and the tax-deferred account to diversify the different types of risks of stocks in the taxable and the tax-deferred account. The maximum level of the utility costs is about $\frac{1}{10}$% for a 99 year old investor with an initial fraction of tax-deferred wealth of 50% which is a very modest utility gain.

The low level of the utility costs is due to the fact that much of the investor’s utility is derived from present consumption and the investor has non-financial income such that her future level of wealth is only partly driven by her investment decisions. In addition, especially for older investors, those states in which the investor prefers stocks in her tax-deferred account are only attained at a quite low probability explaining why the investor's utility costs at the beginning of retirement are close to zero.

If the investor is endowed with an initial loss carryforward (right graph), her utility costs when preferring bonds in the tax-deferred account and stocks in the taxable account decrease even further. This is because the loss carryforward improves the risk-return profile of stocks in the taxable account, which makes the "pecking-order" asset location strategy approach the optimal investment strategy. Consequently, the utility costs of following that "pecking-order" rule decrease.

### 3.3.5 Comparative Static Analysis

In this section, we provide some comparative static results by: varying the maximum percentage of beginning-of-period-wealth that qualifies for tax rebates, allowing for non-financial income, increasing the tax-rate on capital gains $\tau_g$ to the tax-rate on dividends and interest $\tau_d$, increasing the volatility of equity, considering the mortality table of a male investor, and ignoring minimum withdrawal rules.

If the percentage $m$ of the investor's initial wealth qualifying for tax rebates is greater than zero, the risk-return profile of stocks in the taxable account becomes more attractive, since a potential loss carryforward cannot only be subtracted from future capital gains, but also be claimed a loss carryforward in a forthcoming period qualifying for tax rebates if the investor has no capital gains in that period. If the investor is not endowed with an initial loss carryforward ($l = 0$), and 2% of the investor's beginning-of-period-wealth qualifies for tax rebates ($m = 0.02$), she strictly prefers stocks in her taxable account except when she is very old, due to the high mortality rates at the end of her life that leave her with a high probability of not making use of a potential loss carryforward. If only 1% of the investor's beginning-of-period-wealth qualifies for tax rebates ($m = 0.01$), the investor starts shifting stocks from the taxable to the tax-deferred account at age 95, and prefers stocks to bonds in the taxable account when younger than 95.

If the investor does not receive any non-financial income and has to finance her entire consumption from her investments, her consumption is much lower when she is young and strictly increases to the level of an investor endowed with exogenous income.[11] Her optimal investment strategy remains similar to that of an investor who is endowed with non-financial income.

Increasing the volatility of stocks from $\sigma = 20.7\%$ to $\sigma = 31.05\%$, corresponding to a volatility of real stock returns of about 30%, causes the investor to sharply decrease her equity exposure. Furthermore, with $\sigma = 31.05\%$, the investor prefers stocks in her tax-deferred account when not being endowed with an initial loss carryforward. Again, this is due to the fact that the higher volatility provides the investor with a higher increase in wealth when holding stocks in the tax-deferred account and a higher loss carryforward when holding stocks in the taxable account compared to the base-case. Using the same argument as in the case with a higher tax-rate on capital gains, the investor prefers stocks in the tax-deferred account.

In the base-case scenario, we have considered the mortality table of a female investor. If, instead, one considers the mortality table of a male investor, our results do not change significantly (not shown here). While the optimal consumption policy remains about the same as for a female investor, the optimal asset location at the end of the life cycle slightly differs. Since men are subject to higher mortality rates, male investors start shifting stocks from their taxable to their tax-deferred accounts at a lower age than female investors. While female investors start this shifting around age 88, male investors already start shifting stocks around age 81.

The two most important changes in the taxable treatment of the investor's wealth occur, firstly, when attaining retirement age, when the investor is no longer allowed to contribute to her tax-deferred account but the penalty-tax on early withdrawal no longer applies, and secondly at age 70.5, when minimum distribution rules apply. In a tax-system where early withdrawals from the tax-deferred account are not subject to a penalty-tax, i.e. $\tau_p = 0$, the investor with high level of tax-deferred wealth decreases her consumption less than in the base-case with a penalty-tax on early withdrawal of $\tau_p = 10\%$. However, she still decreases her consumption. This is due to the fact that to finance her consumption, she has to withdraw from her tax-deferred account, which pays a tax-free return, whereas for low levels of tax-deferred wealth, she can consume from her taxable wealth, which only pays an after-tax return.

If minimum distribution rules do not apply and the investor is never forced to

[11] In fact, her optimization problem is the same as for an investor with non-financial income in the very last period.

**Optimal fraction of stocks in taxable and tax-deferred wealth without minimum distribution**

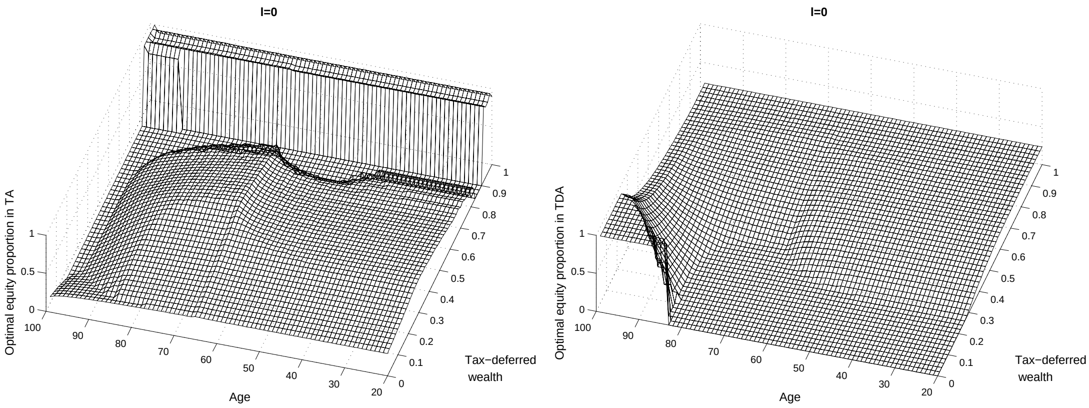


Figure 3.5: Optimal stock holding relative to taxable and tax-deferred wealth after consumption, contributions, and withdrawals for a female investor in taxable account (TA, left graph) and tax-deferred account (TDA, right graph) who is endowed with an initial loss carryforward of zero ($l = 0$), when minimum distribution rules do not apply.

withdraw funds from her tax-deferred account for legal reasons, Figure 3.5 shows that the optimal fraction of stocks in the taxable account is significantly below the level of an investor in a tax-system where minimum distribution rules apply for an investor endowed with some tax-deferred wealth.[12] This is due to the fact that the investor only withdraws funds from her tax-deferred account for consumption, since investments in the tax-deferred account result in higher returns than investments in the taxable account. Consequently, the investor's taxable wealth is primarily used for consumption and thus a low volatility in it is a desirable feature. This is why the equity exposure in the taxable account is quite low compared to the case with a tax-system where minimum distribution rules apply. For not getting too underinvested into equity, the investor increases her equity exposure in the tax-deferred account.

Increasing the investor's bequest motive by increasing $H$ from 10 to 20 causes the investor to decrease her consumption over the entire life cycle, but does not change her investment strategy in either the taxable or the tax-deferred account significantly.

In our analysis, we have so far assumed that capital gains are taxable when they occur. Many tax-systems around the world do not tax capital gains until they are realized, which provides the investor with a tax-timing opportunity. As tax-timing can only be performed in the taxable account, a tax-timing option increases the

[12] As in the base-case setting, the high fraction of stocks in the taxable account for high levels of tax-deferred wealth is a side-effect.

**Optimal fraction of stocks in taxable and tax-deferred wealth for $\tau_g = 14\%$**

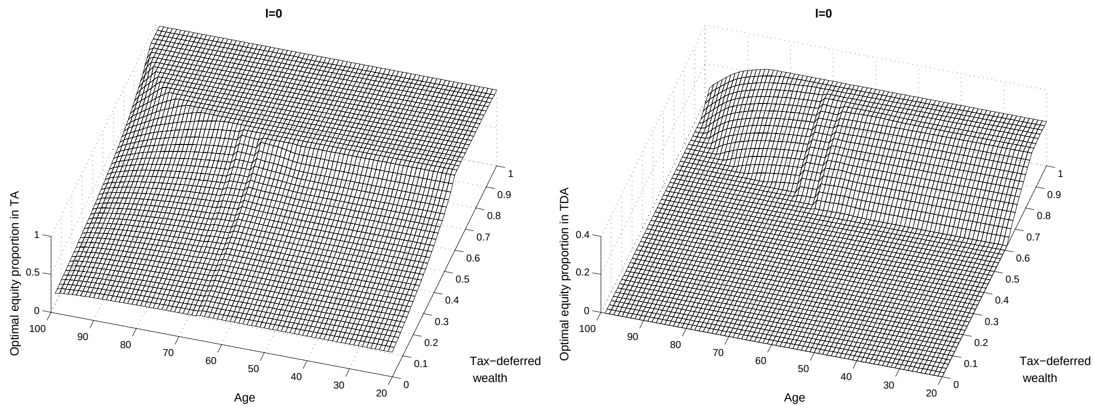


Figure 3.6: Optimal stock holding relative to taxable and tax-deferred wealth after consumption, contributions, and withdrawals for a female investor in taxable account (TA, left graph) and tax-deferred account (TDA, right graph) who is endowed with an initial loss carryforward of zero ($l = 0$), when the tax-rate on capital gains corresponds to the effective capital gains rate of Chay et al. (2006) of $\tau_g = 14\%$.

attractiveness of holding stocks in the taxable account. Zhou (2007) argues that optimal asset location should depend on the frequency of capital gains realizations since this frequency affects the tax burden on stocks. The study of Chay et al. (2006) shows that due to the tax-timing option, the effective capital gains rate $\tau_{g,e}$ that the investor faces is lower than the capital gains tax-rate $\tau_g$. According to their study, the effective capital gains tax-rate of an investor with capital gains tax-rate $\tau_g = 20\%$ is $\tau_{g,e} = 14\%$.

Figure 3.6 shows the optimal asset location and asset allocation of an investor who is confronted with a capital gains tax-rate of $\tau_g = 14\%$, the effective capital gains tax-rate of Chay et al. (2006). An investor who faces such a low capital gains tax-rate strictly prefers bonds in her retirement account, due to the high tax-burden on these assets. These findings suggest that in the presence of a tax-timing option, the asset location puzzle cannot entirely be explained by asymmetric treatment of capital gains and losses in taxable and tax-deferred accounts and other factors like the desire for liquidity might haven an important impact as well. However, to describe the advantages of the tax-timing option in one single value, Chay et al. (2006) require some simplifying assumptions. In particular, they assume that the optimal liquidation price is independent of the purchase price, which ignores the fact that with unrealized capital gains, investors tend to hold a higher fraction of

**Optimal fraction of stocks in taxable and tax-deferred wealth for $\tau_g = 36\%$**

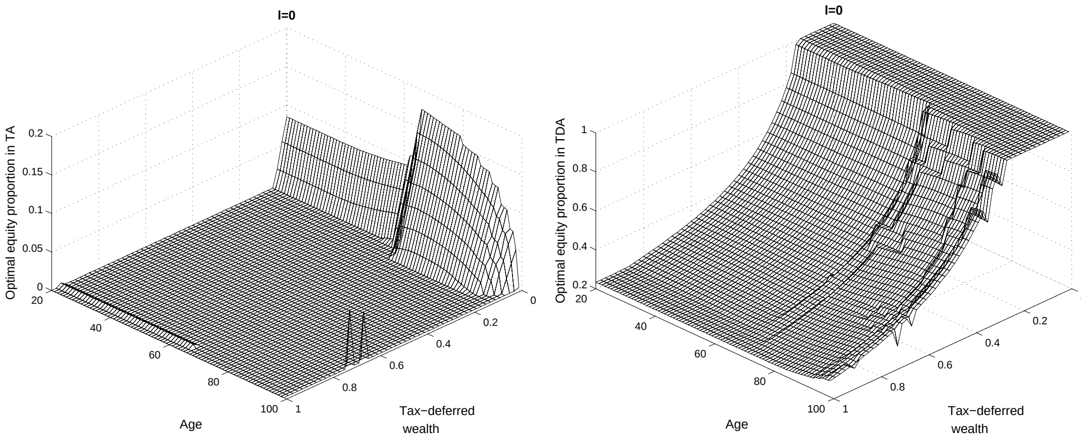


Figure 3.7: Optimal stock holding relative to taxable and tax-deferred wealth after consumption, contributions, and withdrawals for a female investor in taxable account (TA, left graph) and tax-deferred account (TDA, right graph) who is endowed with an initial loss carryforward of zero ($l = 0$), when capital gains are considered short-term gains and $\tau_g = 36\%$.

that asset than without to avoid the capital gains tax payment. This assumption thus casts doubt on whether the tax-timing opportunity causes investors to prefer bonds in their tax-deferred accounts.

In the base-case, it has been assumed that the tax-rate on capital gains $\tau_g$ is $\tau_g = 20\%$, which is below the dividends tax-rate that we set as $\tau_d = 36\%$. The tax-rate on capital gains is only lower than the tax-rate on dividends if capital gains are realized long-term. In case capital gains are realized short-term, the tax-rate on capital gains is equal to that of dividends and $\tau_g = \tau_d$. Considering all capital gains short-term gains, $\tau_g$ increases to $\tau_g = 36\%$. This increase in the tax-rate on capital gains has a tremendous impact on optimal asset location for an investor who is not endowed with an initial loss carryforward, as shown in Figure 3.7.

Figure 3.7 shows that when capital gains are realized short-term, an investor who is not endowed with an initial loss carryforward prefers holding stocks in the tax-deferred account to holding stocks in the taxable account. This result is due to changes in the investor's risk-return profile caused by the increase of the capital gains rate. By increasing $\tau_g$ from 20% to 36%, the asset location decision depends on whether the investor prefers a safe extra return of $\tau_d r = 0.0216$ and a loss carryforward of $\tau_g g$ for $g_t \leq 0$ per dollar of bonds in the tax-deferred account in period $t$, or a safe extra return of $\tau_d d = 0.0072$ and an extra return of $\tau_g g_t$ in case that $g_t > 0$ for each dollar of stocks held in the tax-deferred account. The increase of $\tau_g$ thus increases the potential increase in wealth in the tax-deferred account for positive capital gains, while in the taxable account it only provides the investor with a higher loss carryforward in case of negative capital gains. Since the opportunity to earn extra money by locating stocks in the tax-deferred account is more attractive than the opportunity to increase the loss carryforward by holding stocks in the taxable account, the investor prefers stocks in the tax-deferred account when her capital gains are realized short-term.

When the investor attains retirement age and is not endowed with significant tax-deferred wealth, she increases her equity exposure in the taxable account significantly. Having passed that significant increase in equity, however, her equity exposure decreases with age. The strong increase in her equity exposure is due to the fact that having passed retirement age; she no longer has the opportunity to contribute to a tax-deferred account. On the one hand, this causes her to increase her consumption (not shown here), as the premium for consumption deferral is decreasing. On the other hand, she can no longer increase her return by shifting funds from the taxable to the tax-deferred account. To avoid decreasing her return too sharply, she increases her equity exposure. Having increased her fraction of stocks in the taxable account at retirement age, she then decreases this fraction again in time. This decrease is due to the fact that her mortality increases, which in turn increases her probability of potentially generating a loss carryforward that she cannot use anymore.

## 3.4 Conclusion

This paper has analyzed optimal asset location and asset allocation decisions in the presence of tax-deferred investment opportunities and limits on tax rebates for capital losses. It has shown that in contrast to the findings in recent economic literature, it is no longer always optimal to follow a "pecking-order" rule in such tax-systems. Even though utility gains from following an optimal asset location strategy are quite small, it is usually not optimal to hold as many bonds as possible in the tax-deferred account. There are two important reasons for this result.

First, limitations on tax rebates for capital losses worsen stocks' risk-return profile in the taxable account by compensating the investor with a loss carryforward instead of a tax rebate. Especially for aged investors facing high mortality rates, a loss carryforward is potentially worthless if the investor does not survive long enough to offset it with positive capital gains or receive tax rebates on it. Furthermore, in contrast to compensation by tax refunds, a loss carryforward does not pay any interest.

Second, while for positive capital gains holding stocks in the tax-deferred account can result in a higher increase in total wealth, for negative capital gains holding stocks in the taxable account results in the same return on the stock, but leaves the investor with the tax-free return of bonds and a loss carryforward that provides her with a tax-advantage in forthcoming periods. Thus, investors might want to hold stocks in both taxable and tax-deferred accounts to diversify this risk. Hence, limits on tax rebates for capital losses can explain the so-called asset location puzzle, i.e. why private investors hold substantial amounts of their tax-deferred wealth in stocks.

The equivalent increase in both taxable and tax-deferred wealth needed to bring the expected lifetime utility of an investor who is not endowed with an initial loss carryforward to the same level of an investor who is endowed with an initial loss carryforward is convex in the level of the investor's utility. The higher the level of the initial loss carryforward, the lower the increase in both taxable and tax-deferred wealth needed to bring the investor without an initial loss carryforward up to the same level as the investor who is endowed with an initial loss carryforward. This is due to the fact that a loss carryforward does not pay any interest and faces the risk of never being used at all. The higher the level of the initial loss carryforward is, the more important these two types of disadvantages are.

Inevitably this paper has ignored several important factors. In particular, it made a simplifying assumption on the taxation of capital gains, namely that capital gains are taxable when they occur. Many tax-systems around the world do not tax capital gains until they are realized which provides the investor with a tax-timing opportunity. Computations with effective capital gains tax-rates according to Chay et al. (2006) suggest that the optimal asset location decision might change when the investor has a tax-timing opportunity. However, their effective tax-rates rely on some simplifying assumptions that are not met in this paper.

An interesting avenue for further research is to extend our model with an endogenous tax-timing opportunity. Furthermore, due to the forgiveness of capital gains when being bequeathed, aged investors facing higher mortality rates might consider pre-tax capital gains more important than after-tax capital gains and might thus prefer stocks to bonds in their taxable accounts. It would be interesting to know how our results on optimal asset location change in a tax-system that allows for tax-timing. However, this problem is quite challenging from a numerical perspective. We leave this problem for further research.

# Appendix:
# Anmerkungen für Deutschland

Während im vorangehenden Aufsatz eine Erklärung für das Asset Location Puzzle im Rahmen eines Steuersystems gegeben wurde, welches dem US-amerikanischen nachempfunden wurde, wird im Rahmen dieses Appendix auf die Situation einer Anlegerin eingegangen, die sich mit dem deutschen Steuersystem konfrontiert sieht. Neben Änderungen in der Höhe der Steuersätze auf Dividenden und Kursgewinne sieht das deutsche Steuerrecht vor, dass Entnahmen aus Altersvorsorgekonten in der Regel in Form von Leibrenten erfolgen müssen. Im Rahmen so genannter Rürup-Verträge hat die Entnahme in Form einer lebenslangen Leibrente zu erfolgen, im Rahmen so genannter Riester-Verträge muss die Entnahme spätestens ab einem Alter von 85 in Form einer lebenslangen Leibrente erfolgen.

Dem Vorteil einer Leibrente, das Langlebigkeitsrisiko auszuschalten, stehen die Nachteile der mangelnden Vererbbarkeit und der Inflexibilität gegenüber. So kann die Anlegerin etwa bei einem kurzfristig hohen finanziellen Bedarf (z.B. in Folge einer Krankheit, deren Kosten nicht oder nur teilweise von ihrer Krankenkasse getragen werden) nicht auf das in die Leibrente investierte Vermögen zurück greifen. Dies liegt daran, dass Leibrenten praktisch nicht wieder veräußerbar sind, was im Wesentlichen an der asymmetrischen Informationsstruktur liegt, der sich ein möglicher Verkäufer und ein möglicher Käufer gegenüber sehen.

Bedingt auf das eigene Überleben darf eine Anlegerin, die eine Leibrente kauft, eine höhere Rendite erwarten als im Rahmen eines Auszahlplans, welche aus dem Ableben anderer Inhaberinnen desselben Typs von Leibrente resultiert. Diesen Effekt bezeichnet man auch als mortality credit.

Ein wesentlicher Unterschied zwischen Deutschland und den USA besteht darin, dass die Mittel in Altersvorsorgekonten restriktiveren Entnahmeregelungen unterliegen und insbesondere keine Vererbung mehr möglich ist.

Der faire Preis $P_x$ einer variablen Leibrente, die im Alter von $x$ Jahren von einer Anlegerin gekauft wird, die höhstens bis zum Alter 100 lebt und ab dem Alter $x+1$ jährlich eine gleichbleibende Stückzahl $n$ des Fonds, in den investiert wird, auszahlt, ist gegeben durch

$$P_x = \sum_{t=1}^{100-x} {}_{x+t}p_x n, \tag{3.33}$$

worin ${}_{x+t}p_x$ die Wahrscheinlichkeit bezeichnet, dass die Anlegerin vom Alter $x$ bis zum Alter $x+t$ überlebt. Legt man für diese Wahrscheinlichkeiten die Daten der Sterbetafel des Statistischen Bundesamts 2002/2004 für deutsche Frauen im Alter von 66 Jahren zu Grunde, so ergibt sich

$$P_{66} = 18,4\,n. \tag{3.34}$$

Für eine jährliche Zahlung von einer Einheit des Werts eines Fonds bis zu ihrem Lebensende muss die Anlegerin also im Alter von 66 Jahren den Preis von 18,4 Einheiten dieses Fonds bezahlen.

Nachfolgend wird in Anlehnung an das vorstehende Kapitel und die deutschen Gesetzesregelungen untersucht, welchen Einfluss eine erzwungene Umwandlung des gesamten Altersvorsorgevermögens bei Erreichen des 66sten Lebensjahres in eine variable Leibrente hat, die ausschließlich in das risikolose Wertpapier investiert.[13] Auf Grund der erzwungenen Umwandlung des Altersvorsorgevermögens in eine Leibrente stehen dem Vorteil der Steuerfreiheit der Erträge in diesem Konto sowie dem mit der Leibrente verbundenen mortality credit die Nachteile der unflexiblen Entnahmemöglichkeiten sowie der mangelnden Vererbbarkeit gegenüber.

Um die Bedeutung der Zwangsumwandlung in eine Leibrente für die optimale Anlagestrategie einer deutschen Anlegerin abzuschätzen, wurde eine dem Vorgehen des voranstehenden Artikels entsprechende Berechnung mit deutschen Parametern für Steuersätze, Sterblichkeiten, etc. mit Zwangsumwandlung des angesparten Altersvorsorgevermögens in eine Leibrente durchgeführt. Die Ergebnisse der Berechnung lassen sich kompakt wie folgt zusammenfassen:

[13] Die hier vorgenommene Modellierung unterscheidet sich von der steuerlichen Behandlung deutscher Anlegerinnen dadurch, dass statt einer konstanten oder steigenden Leibrente eine variable Leibrente zu Grunde gelegt wird. Eine solche Leibrente wäre nicht Riester-kompatibel, wenn sie risikobehaftete Anlageformen enthält, da sie dann auf Grund von möglichen Wertschwankungen des Fonds keine gleichbleibenden oder steigenden Auszahlungen garantiert. Es wird deshalb davon ausgegangen, dass der Leibrentenanbieter ausschließlich in das risikolose Wertpapier investiert. Ferner erlaubt eine Riester-Rente eine Implementierung eines Entnahmeplans bis 85 mit anschließender Leibrente. Hier wird vereinfachend angenommen, dass die Anlegerin von dieser Möglichkeit keinen Gebrauch macht und stattdessen eine Leibrente bereits mit Ablauf des 65sten Lebensjahres erwirbt. Die beiden wesentlichen Unterschiede zwischen einer Leibrente und einem Entnahmeplan – der mortality credit und die Unvererbbarkeit – lassen sich auch im Rahmen einer variablen Leibrente untersuchen.

- Wie für eine US-Amerikanische Anlegerin ist es auch für eine deutsche Anlegerin ab einem Anteil des Altersvorsorgevermögens von etwa 30% am Gesamtvermögen optimal, in jungen Jahren gemischte Portfolios in beiden Kontentypen zu halten.

- Im Alter nimmt der optimale Anteil von Aktien im Altersvorsorgekonto stetig mit dem Anteil des Gesamtvermögens im Altersvorsorgekonto zu. Dies liegt daran, dass im Altersvorsorgekonto auf Grund der Zwangsumwandlung in eine Leibrente nach Konstruktion der Leibrente keine Aktien gehalten werden können. Um die Aktienquote bezogen auf das Gesamtvermögen nicht zu stark zu senken, erhöht die Anlegerin deshalb mit steigendem Altersvorsorgevermögen ihren Aktienanteil im konventionellen Konto.

- Die Möglichkeit in ein Altersvorsorgekonto zu investieren, bleibt trotz Zwangsumwandlung in eine Leibrente ohne Vererbungsmöglichkeit sehr attraktiv, was man daran erkennt, dass Anlegerinnen mit nicht mehr als 50% ihres Vermögens im Altersvorsorgekontos den maximal zulässigen Beitrag in ihr Altersvorsorgekonto leisten.

Zusammenfassend kann festgehalten werden, dass die Kernergebnisse des Aufsatzes "Are Bonds Desirable in Tax-Deferred Accounts?" auf eine Anlegerin, die sich gesetzlichen Rahmenbedingungen gegenüber sieht, die in groben Zügen den deutschen Regelungen nachempfunden wurden, übertragbar sind.

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# Lebenslauf

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# Ehrenwörtliche Erklärung

Ich habe die vorgelegte Dissertation selbst verfasst und dabei nur die von mir angegebenen Quellen und Hilfsmittel benutzt. Alle Textstellen, die wörtlich oder sinngemäß aus veröffentlichten oder nicht veröffentlichten Schriften entnommen sind, sowie alle Angaben, die auf mündlichen Auskünften beruhen, sind als solche kenntlich gemacht.

Frankfurt am Main, den 02.07.2007

(Marcel Marekwica)