American Journal of Advanced Research, 2017, 1–2
December, 2017, pages 01-07
doi: 10.5281/zenodo.1012589
http://www.ibii-us.org/Journals/AJAR/
ISBN 2572-8849 (Online), 2572-8830 (Print)

Hierarchical Kernel and Sub-kernels

Francisco Casanova-del-Angel[1, *]

[1]National Polytechnic Institute, Address Miguel Othón de Mendizábal S/N. Col. La Escalera. 07320 Ciudad de México.

Tel. (5255) 57296000. Ext: 53153

* Email: fcasanova49@prodigy.net.mx

Received on August 7, 2017; revised on October 6, 2017; published on October 15, 2017

Abstract

This paper shows the theoretical development of hierarchy by kernels and an algorithm used to obtain an interesting class or partition from a hierarchy. Also shown is the theorem about the *Kernels Optimal Criterion* and how it is expressed as a function of the masses of the points of the vector space and product scale points, the inertia of the cloud formed by those two points or hierarchical nodes, which are called subcores or sub-kernels. The application is made on the terminal efficiency of postgraduate degrees at ESIA, IPN Mexico, along its first 48 years of academic and scientific life and the development of students´ graduation.

Keywords: Hierarchical kernels; sub-kernels; cores; inertia; classes

1 Introduction

From the theoretical standpoint, the purpose of this paper is to analyze, under the precepts of data analysis, the relationship which exists or lies between a hierarchical kernel and the sub-kernels. I also considered, for the construction of the hierarchical classification, the influence that have kernel and sub-kernel concepts at the time of the interpretation of the hierarchical tree.

But why the existence of the purpose? Well, the answer is simple, since one of the major problems that arise at the time of interpretation and validation of results in hierarchical classification is the confidence given to the separation of classes of values in the hierarchical tree, and above the meaning of the height to which are those same classes that build the hierarchy. The height is called level index.

In relation to the application of the theory and interpretation of results on real data, let me explain then what seems to be the classic tedious and gruesome development of education in underdeveloped societies academically and where the accountability is very little practiced.

On December 14, 1961, the General Director of the National Polytechnic Institute Mexico, IPN, presented before to the Technical Advisory Board of the Institute the advantages and rationale to establish the Graduate Section of the Higher School of Engineering and Architecture, ESIA, IPN. On July 16, 1962, registration into postgraduate degrees at ESIA IPN, were formally started, with two master degrees in sciences, with specialty in structures and hydraulics [1]. Courses formally started on Wednesday, August 1, 1962. The syllabi of both master degrees were organized in half-year periods. On December 14, 1966, the School Consulter Technical Committee of the IPN discussed, in the fourth point of its agenda, the creation of six master degrees and a doctorate degree in sciences, among which there was a new master degree in sciences in planning. In 1978, an unprecedented expansion was intended, based on a proposal to create a master degree in sciences for every civil engineering specialty. Despite the boldness of such proposal, four new master degrees in sciences were established: architecture, environmental engineering, geology and soil mechanics. In 1981, mining and oil specialties for the master degree in geology, and architecture and architecture specialty with options for architectonic design and works construction and control were created. In 1983, the structural analysis, steel structures, cement structures, architecture and ports development specialties, for the master degree in hydraulics [1] were created. On June, 1998, through the execution of a General Academic Collaboration Agreement with the Polytechnic University of Madrid, UPM, a joint civil engineering doctorate degree with specialty in environmental hydraulics was established. Such doctorate degree was taught at SEPI ESIA, ALM Unit, IPN, with the support of the Civil Engineering -hydraulics and energy-Department of the UPM. Years later, educational and research institutions must report to the society on the resources provided for the creation and support of these graduate programs. Its usefulness and provided benefits who gave such economic support, since it is never done.

2 Properties of the kernel

A vector space is a set V provided with two operations: the addition of elements of V and the multiplication of elements of V with a scalar.

A mapping T of a vector space V into a vector space W is called a linear transformation of V into W if, for any vectors α, $\beta \in V$ and an arbitrary real number r, the following hold: i) $T(\alpha+\beta) = T\alpha + T\beta$ and ii) $T(r\alpha) = rT(\alpha)$. Si $W = V$, the transformation T is often called an operator on V. Associated with any linear transformation T: $V \to W$, are two very important space in data analysis: the rank space or range denoted by R_T; and the kernel or the null space of T, denoted by N_T, defined in (1) as:

$$N_T = \{T\alpha = 0 \mid \alpha \in V\} \quad (1)$$

We also know that if V is the space on which T operated, we define K_i to be the kernel of the operator T - λ_i; that is, K_i is the subspace of vectors $\alpha \in V$, such that $(T + \lambda_i)\alpha = 0$, and so its nonzero members are the Eigenvalues of T that belong to λ_i. [2] pp. 94-121.

3 Theoretical development of hierarchy by kernel

Based on the fact that factorial correspondence analysis represents, on the same graphic, both sets comprising a tabular correspondence arrangement; sets I of individuals and Q of classes defined for each variable J, and that when such must be taxonomies, a rigid class system must be fixed, then the global and spatial vision provided by factorial analysis allows us to establish through some kind of aggregation method, a type of hierarchy of the data under analysis.

The method herein shown is tributary to three options: i) calculation of the distance between elements where factorial coordinates are known; ii) juxtaposition of mass or weight to each element; and iii) calculation of a distance between element classes, depending on an aggregation criterion based on cores. Since our data include factorial values related to Q classes, we shall retain a small number of A cardinality factors, not higher than 75% of factorial data.

Let us define factorial set of values through set: $\{F_\alpha(q) \mid q \in Q \text{ and } \alpha \in A\}$, with which it is possible to calculate many tabular arrangements for distances between elements. In our case, we shall introduce the following distance. Let q and q' be two classes of a variable $j \in J$ such that q and $q' \in Q$. Classes q and q' belong to a normed factorial space with a fixed set of coordinates. If $d: F \longrightarrow \mathbb{R}$ then (F, d) is a metric space. Factorial distance between $F(q)$ and $F(q')$ is the addition of lengths of projections of line segment between factorial values on the axes system. This is mathematically expressed as follows [3] and [4]:

$$d^2(q,q') = \| q,q' \|^2 = \sum_{\alpha \in A}(F_\alpha(q) - F_\alpha(q'))^2 \quad (2)$$

Where q and q' are classes of variable $j \in J$, d is the distance between classes, α is the axis, A is the set of axes and $F_\alpha(q)$ and $F_\alpha(q')$ are factorial values of classes. In accordance with the second option of the aggregation method defined, the distance between classes is juxtaposed by inertia λ of the set of dots along axis α, which is represented by the own value related to the corresponding axis, because of this equation (2) may be re-expressed as follows:

$$d^2(q,q') = \| q,q' \|^2 = \sum_{\alpha \in A} \lambda_\alpha^{-1}(F_\alpha(q) - F_\alpha(q'))^2 \quad (3)$$

Where q and q' are the classes of variable $j \in J$, d is the distance between classes, α is the axis, λ_α^{-1} is the inverse of distance between classes on axis α and $F_\alpha(q)$ represents factorial value of class q on axis α [4] and [5]. Once the distance between values has been defined, the diameter index of nodes of classification ν of such hierarchy must be calculated, through:

$$\nu(n) = \frac{f_a * f_b}{f_a + f_b} \| F_\alpha(a) - F_\alpha(b) \|^2 \quad \forall\ n \in Nodo \quad (4)$$

Where a and b are barycenter's of elements of the index, f_a and f_b are the mass in a and b barycenter's, and $F_\alpha(a)$ and $F_\alpha(b)$ are factorial values of a and b barycenter's. In addition, $a \cup b = n$ and $a \cap b = \Phi$.

Every time, the distance between elements that are hierarchized must be recalculated with those to be hierarchized, because of this the following diameter index $\nu(n)$ is:

$$\nu(n) = \frac{f_a * f_b}{f_a + f_b} \| \lambda_\alpha^{-1} F_\alpha(a) - \lambda_\alpha^{-1} F_\alpha(b) \|^2 \quad \forall\ n \in Nodo \quad (5)$$

Where $\nu(n)$ is diameter index, f_a and f_b are masses of a and b barycenter's, $F_\alpha(a)$ and $F_\alpha(b)$ are factorial values of a and b barycenter's, and λ_α^{-1} is the square root of total distance of the A set of dots, along axis α.

Now, from equation (4) it may be seen that the addition of values of diameter indexes is equal to the addition of total distance λ of the set of dots along α axis, that is:

$$\sum_{n \in Nodo} \nu(n) = \sum_{\alpha \in A} \lambda_\alpha \quad (6)$$

Where $\nu(n)$ diameter is indexed and λ_α is the total distance of the set of axes. From equation (5) it may be seen that the addition of the values of diameter indexes is equal to A's cardinality.

$$\sum_{n \in Nodo} \nu(n) = Card\ (A) \quad (7)$$

3.1 The algorithm

Classification algorithm looks for two minimum values of the table of factors of classes of the sub-kernels to be hierarchies.

$$\delta(q,q') = \frac{f_q * f_{q'}}{f_q + f_{q'}} \| F_\alpha(q) - F_\alpha(q') \|^2 \quad \forall\ q, q' \in Q \quad (8)$$

From this aggregation, defined as $k = q \cup q'$, a new partition or kernel of the set of Q classes must be updated making: $\mathcal{P} = Q \cup \{k\} - \{q, q'\}$. Distances between this new element k and q'' are recalculated, showing the following minimum value of the factors table, through formula (4), thus making $\nu(n) = \delta(a,b)$.The minimum of the new table is investigated, aggregated and a new partition is updated below. The above is carried out until there are no more than the two last cores to be added, taking into account that the link is the base set [5] and [6].

Theorem Kernels Optimal Criterion. If aggregation kernels are groups of factors with same cardinality and Ω the space of kernels or cores, the optimal election criterion is:

$$d(L,P) = \sum_{i=1}^{k} d(A_i - P_i)$$

Where L is the total set of kernels or cores, A_i is the ith core containing a certain number of objects of P population.

Demonstration. Let $L = \{A_1, \ldots, A_h\}$, $A_i \subset \mathcal{L}$ be the ith kernels or core containing q elements of population. $P = \{P_1, \ldots, P_h\}$ is partition of space Ω into k-classes. Let $\mathcal{L}_k$ be the set of kth cores and $\mathcal{P}_k$ the set of partitions of Ω kernels space into classes. $d(A_i, \mathcal{P}_i)$ measures dissimilarities between kernel or core A_i and class $\mathcal{P}_i$. Based on the above, the principal problem is to look for a $L^* \subset \mathcal{L}_k$ and a population $P \subset \mathcal{P}_k$ that minimize d dissimilarity.

Let $d(q_1, q_2)$ be a measure for dissimilarities between couples of individuals or classes. Let us suppose that:

$$d(q_1, q_2) = \sum_{q_1 \in X} \sum_{q_2 \in Y} d(q_1 - q_2)$$

Where X and Y are parts of the set of Ω individuals, then:

$d(q_2, \{q_1\}) = d(Y, q_1)$ and $d(\{q_1\}, Y) = d(q_1, Y)$

In case that kernels or cores are groups of individuals, the algorithm shall be specified, since such is based on choosing two functions: assignation function and representation function. For the assignation function, given the kernels or cores $\{A_1, \ldots, A_h\}$, partition $P = \{P_1, \ldots, P_h\}$ deducted is defined by:

$$P_i = \{q_1 \in \Omega \mid d(A_i, q_1) \leq d(A_j, q_1) \,\forall\, i, j\}$$

In case of equality, q_1 shall be assigned to the lowest index class. Partitions P thus deducted from L are shown by $P = f(L)$, where f is an application of $\mathcal{L}_k$ in $\mathcal{P}_k$; that is: $f: \mathcal{L}_k \longrightarrow \mathcal{P}_k$, and it is called assignation function.

For the representation function, given partition P, $L = \{A_1, \ldots, A_h\}$ kernels or cores are deducted as:

$$A_i = \{q_1 \in \mathcal{L} \mid q_1 \in \{q\}\, wich\ produce\ lowest\ possible\ dissimilarity\ \ d(q_1, \mathcal{P}_i)\} \quad (9)$$

In order to ensure the unit of A_i, the set of q elements of Ω space minimizing $\sum_{q_1 \in A_i} d(q_1, \mathcal{P}_i) \,\forall\, \mathcal{P}_i \subset \Omega$, exists and is unique. Therefore, the representation function exists. ***QED***

3.2 Sub-kernels

Let a vector space V of W, if it exists $U \subset V$ not empty then U is a vector subspace of V if it complies with the properties given in § 2. Therefore, sub-kernel means a subset $N \subset N_T$. Now that we have seen the principal theorem of hierarchical cores and the implementation of his algorithm, let's see how it is expressed, depending on the masses of the points of the vector space and the scalar product of these points, the inertia of the cloud formed by those two points or hierarchical nodes, which in our case are called sub-cores or sub-kernels forming the principal node of the hierarchy.

Usually the inertia In(g) (or In(h)) part of the cloud N(I) is given by:

$$In(g) = \sum\left\{r_{ii'} \left\|i_V - i_V'\right\|^2 / \, i, i' \in V\right\} = \sum\left\{\frac{m_i m_{i'}}{2m_g} \left\|i_V - i_V'\right\|^2 / \, i, i' \in g\right\} \quad (10)$$

In the first part of (10), the double sum includes (Card g)^2 terms (or (Card h)^2 terms). For the proof of (10), it is enough with to replace $i_V - i_V'$ by $(i_V - g_V) - (i_V' - g_V)$ and to develop the square with what you get, when the sums:

½ In(g) + ½ In(g) + 0 = In(g)

For the people not familiar with the data analysis, it is understood by class or tax on to the taxonomic division of finite size [7] pp. 94.

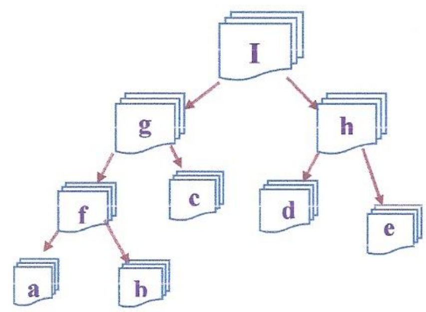

Figure 1. Taxonomic system of sub-kernels.

Finally, to express the inertia of the subspace I fitted with a system of classes or sub-kernels; as shown in Figure 1. The set (a, b, c, d, e, f, g, h) are parts of I that have properties such as a and b, as well as d and e or g and h; among many others, are two to two empty intersections and their union is I, i. e, the set $\{a, b, f, c, d, e\}$ is a partition of I in a number of sub-kernels; which, in the case of Figure 1, are five sub-kernels or classes. In addition is that, for example: $f = a \cup b$, $g = f \cup c$ y $I = g \cup h$.

Given any two parts of I, denoted by $d(a, b)$, the inertia of the point cloud consisting of points a_υ and b_υ with their respective masses m_a and m_b, is possible to write that the indices of diameter are: $\upsilon(I) = d(g, h)$, ….., $\upsilon(f) = d(a, b)$.

In addition, $\upsilon(a) < \upsilon(f) < \upsilon(g) < \upsilon(I)$. With the above, it is possible to express the inertia In(I) depending on the index diameter of the kernel I index; $\upsilon(I)$ and their sub-kernels in the following manner:

In(I) = $d(g, h)$ + In(g) +In(h) = N(I) + In(g) +In(h)

That mind a classic decomposition of total inertia In(I) in inertias inter kernels of the cloud N(I) and the inertia produced by the addition of the inter kernels of g and h; In(g) + In(h). This last also can be expressed as the inertia associated with centers of gravity of the kernels.

4 Hierarchical Kernel in pseudocode

This algorithm in pseudocode (Figure 4) synoptically describes the operating principle for the production of kernels and hierarchical sub-kernels. The one which, based on the theory developed here, can be implemented in any programming language, or you can make use of commercial software of mathematical statistics.

Table 1. Historical terminal efficiency of master's degrees in sciences.

Master Degree in Science	Number of graduated students	Period	Year of defense of the first specialty thesis	Terminal efficiency annual index
Structures	58	1962-2010	1970	1.20
Hydraulics	51	1962-2010	1975	1.06
Planning	66	1966-2010	1977	1.37
Soil Mechanics	32	1981-2010	1987	0.66
Environmental Engineering	114	1977-2010	1979	2.37
Doctorate degree				
Environmental Hydraulics	5	1998-2010	2000	0.50
Master degree in Engineering				
Structures	3	2009-2010	2010	1.5
Hydraulics	5	2009-2010	2010	2.5
Planning	5	2009-2010	2010	2.5
Geotechnics	0	2009-2010	-	0
Environmental Engineering	10	2009-2010	2010	5

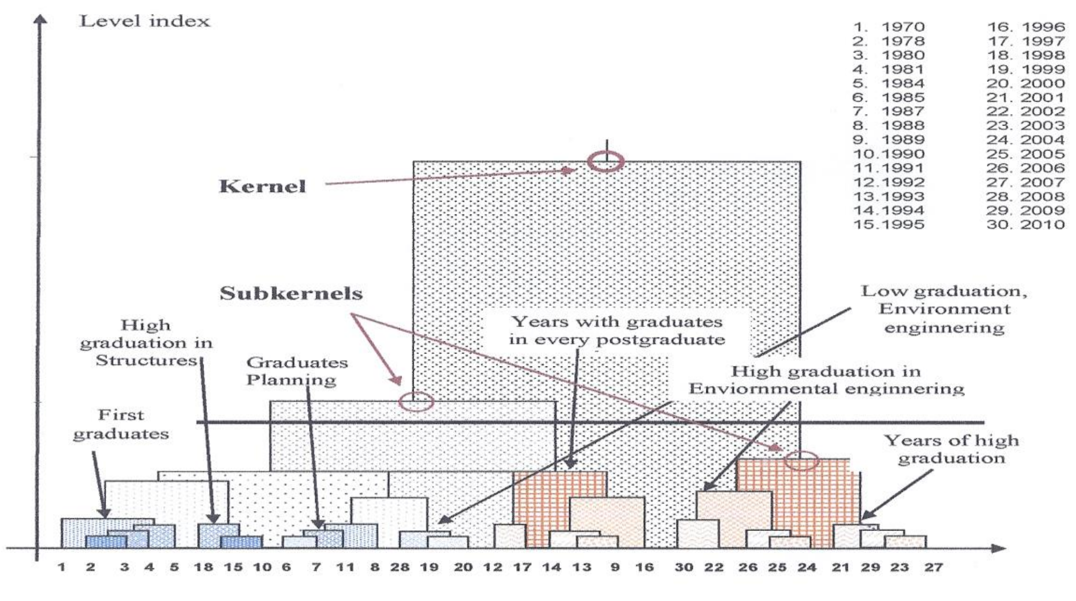

Figure 3. Hierarchical classification of terminal efficiency of postgraduate degree ESIA IPN, Mexico with hierarchical kernels theory.

Table 2. Correlations matrix

	1	2	3	4	5	6	7
1	1.000						
2	0.652	1.000					
3	0.377	0.554	1.000				
4	0.564	0.656	0.695	1.000			
5	0.630	0.502	0.431	0.535	1.000		
6	-0.229	-0.018	-0.044	-0.035	-0.60	1.000	
7	-0.159	-0.098	-0.202	-0.178	-0.086	0.526	1.000

Where 1: Structures, 2: Hydraulics, 3: Planning, 4: Environmental engineering, 5: Soil mechanics, 6: Hydrocarbons administration and 7: Geotechnics.

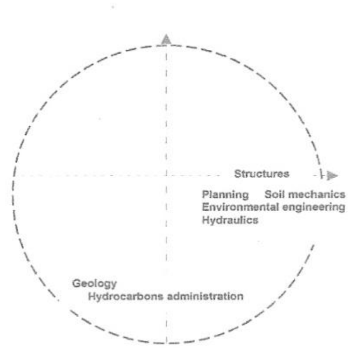

Figure 2. Correlation Circle of first principal component *vs* second principal component.

//For the preparation of the table classes//
For $i \leftarrow 0$ **up to** n **Make**
Input $(r_1 \le j_1 \le r_2)$ ***up to*** $(r_{m-1} \le j_m \le r_m)$ **Make**
For $j_q \leftarrow (r_1 \le j_1 \le r_2)$ ***up to*** $(r_{m-1} \le j_m \le r_m)$
Instruction $k_{IQ} = k(i, j_q) \quad \forall\ i \in I, j \in J, q \in Q - class$
Write $k(i, j_q)$
End for r_m
End for i
//The calculation of correlations or *degree of association* among variables in class, was carried out from usual Euclidian distance//
For $i \leftarrow 1$ **up to** n **Make**
Input j, j' **and define distance**
Instruction $d^2(j, j') = \sum_{i=1}^{n} x_{ij}^2 + \sum_{i=1}^{n} x_{ij'}^2 - 2\sum_{i=1}^{n} x_{ij}\, x_{ij'}$ **Make**
Write and Draw $d^2(j, j')$
End for i
// Calculation of the distance factor data in class//
For $\alpha \leftarrow 1$ **up to 7 Make**
Instruction $d^2(q, q') = \| q, q' \|^2 = \sum_{\alpha \in A} (F_\alpha(q) - F_\alpha(q'))^2$ **Make**
Instruction λ_α^{-1} **Make**
Instruction $d^2(q, q') = \| q, q' \|^2 = \sum_{\alpha \in A} \lambda_\alpha^{-1} (F_\alpha(q) - F_\alpha(q'))^2$ **Make**
Write $d^2(q, q')$
End for α
//Built of the class hierarchy or aggregation criteria and draw level index//
For $\alpha \leftarrow 1$ **up to 7 Make**
Input q **and** q′
Instruction f_q **and** $f_{q'}$ **Make**
Instruction $\delta(q, q') = \frac{f_q * f_{q'}}{f_q + f_{q'}} \| F_\alpha(q) - F_\alpha(q') \|^2 \quad \forall\ q, q' \in Q$ **Make**
Write and Draw $\delta(q, q')$
End for α
//Application of theorem of kernels and sub-kernels//
For $i \leftarrow 1$ **up to** k **Make**
Instruction $d(L, P) = \sum_{i=1}^{k} d(A_i - P_i)$ **Make**
Write and Draw $d(L, P)$
End for i
// Reading and interpretation of the hierarchical kernels//
End of Hierarchical Kernels and Sub-kernels
End of Procedure

Figure 4. Pseudocode

5 Application

Currently, one of the criteria used to assess the functioning of academic and research activities is terminal efficiency, as one of the principal indicators showing the achievements of the corresponding education institution. Since the School of engineering and architecture. Unit Adolfo Lopez Mateos of the Polytechnic Institute National. Mexico, is has been one of the schools of civil engineering with more students in Mexico, it is very important to know its terminal efficiency, both for licentiate and postgraduate degrees, see [8] and [9]. On the top of the table, the number of graduates for each master's degree in sciences that is officially known up to 2007, throughout 48 years, is shown. On the bottom of table 1, the terminal efficiency of the master's degree in civil engineering up to date, which substituted the five previous ones in 2007 is shown. The institution does not update your data automatically, due complicate administrative processes.

5.1 Correlation of terminal efficiency

The calculation of correlations or *degree of association* among variables was carried out from usual Euclidian distance $d(j, j')$ among variables j and j'; that is: $d^2(j,j') = \sum_{i=1}^{n} x_{ij}^2 + \sum_{i=1}^{n} x_{ij'}^2 - 2\sum_{i=1}^{n} x_{ij}\, x_{ij'}$. Since general terms of normed analysis in general terms in real space of dimension p, $\Re^p$, are points x_{ij} we have that: $\sum_{i=1}^{n} v_{ij}^2 = \sum_{i=1}^{n} x_{ij'}^2 = 1$. Every point-variable is on a sphere with radius 1 and center on the origin of principal axes, which the correlation coefficient $c_{ij'}$ among variables j and j' is: $\sum_{i=1}^{n} x_{ij}\ x_{ij'} = c_{ij''}$. The correlation matrix is shown as Table 2.

Best correlated master degrees in sciences are: environmental engineering planning/hydraulics, and structures/ hydraulics, Figure. 2. It must be remembered that, if two meteorological variables are *strongly correlated,* they are near from each other ($c_{ij'} = 1$) or, on the contrary, as far from each other as possible ($c_{ij'} = -1$), in accordance with linear relationship linking them is direct or inverse, and that when $c_{ij'} = 0$ they are considered at an average distance or that variables j and j' are orthogonal.

5.2 Factorial Correspondence Analysis of Gross Data

The factorial method chosen to describe data under study is the Factorial Correspondence Analysis, FCA, since it allows the direct search of simultaneous representation of sets under study I years of graduation and J master degrees in the sciences [3]. The FCA applied on gross data K_{IJ} has the following factorial characteristics: variances on the first five principal axes or own values are: $X_1 = 3.3186$, $X_2 = 1.4787$, $X_3 = 0.7817$, $X_4 = 0.4838$ y $X_5 = 0.4223$, while the inertia percentages explained by such axes are, respectively: 47.4%, 21.1%, 11.2%, 6.9%, and 6.0%. Principal axes are well defined. The first includes master's degrees in sciences in environmental engineering, hydraulics, and structures. The second principal axe includes the master's degrees in science that did not belong to this school of engineering for a long time, hydrocarbons administration, economy and geology while the third axe is planning.

Figure 3, shows the hierarchy of relationship between the years of graduation of master degrees in sciences. In the upper-right corner of Figure 3 are the years or periods of analysis of available information. They are the years that contain record of students graduating in these graduate programs, which at the same time are the classes that define the hierarchy. Reading and interpretation is based on the value of hierarchical level index, shown on the left of the dendrogram, such being understood as the consecutive order of values from the product of the weight of the class under analysis and its diameter (distance $d(i,\ i')$ is the diameter of the smallest part of a hierarchy containing both i and a i') [1].

6 Conclusions

This work is presented in accordance with its development. The theory developed on hierarchical cores is shown, where the method shown is tributary to three options: i) calculation of distance between elements where factorial coordinates are known; ii) juxtaposition of mass or weight to each element; and iii) calculation of a distance between element classes, depending on an aggregation criterion based on hierarchical cores. From the point of view of the theory developed, it may be seen that from various starting points, the problem of looking for stable classes may be resolved. Starting points may be chosen by the user, with the help of a hierarchical classification. The theorem demonstrated and called *Cores Optimal Criterion Theorem* allows to implement f *and* f^{-1} functions from a kth core randomly estimated with the algorithm. In relation to the application of the theory, it is possible to say that the hierarchical dendrogram built is formed by three branches, whose interpretation is absolutely congruent with knowledge on the topic.

To achieve the optimal terminal efficiency of the Section of Postgraduate Degrees, a real connection between professor and student must be fostered, in order that information moves in both ways, since a lot of students, along their lives, carry out professional practice highly contributing to the technological and scientific progress, which, together with professors as knowledge guides, may yield significant progress. As a result of the analysis carried out in this work, it must be noticed that one of the areas of knowledge of the postgraduate degree students are more interested in are environmental engineering and planning, offering the highest number of graduates in such specialties.

Acknowledgements

This document was developed with part of the time devoted to the IPN-SIP 20171058 research project.

Conflict of Interest: none declared.

References

[1] Catálogo de los estudios de posgrado ofrecidos en el IPN. (1984). Dirección de Graduados e Investigación. Instituto Politécnico Nacional. México.

[2] Benzécri, J P and Benzécri F. (1980). *Pratique de L´Analyse des Données. 1. Analyse des Correspondances. Exposé Élémentaire*. Dunod Bordas, Paris. ISBN: 2-04-011227-8

[3] Casanova del Angel, F. (2001). *Análisis multidimensional de datos*. Editorial Logiciels, ISBN: 970-92662-1-7. Mexico.

[4] Marion M and Signonello S. (2011). From Histogram Data to Model Data Analysis. B. Fichet *et al* (eds.). Classification and Multivariate Analysis for Complex Data Structures,

Studies in Classification Data Analysis, and Knowledge Organization. Doi: 10.1007/978-3-642-13312-1_38. Springer-Verlag Berlin Heidelberg.

[5] Diday E and Noirhomme M. (2008). *Symbolic Data Analysis and the SODAS software*. Wiley. ISBN 978-0-470-01883-5, 457 pages.

[6] Diday E. (2008). *Spatial classification*. DAM (Discrete Applied Mathematics) Volume 156.

[7] Benzécri J. P, Benzécri F, Bellier L, Bénier B, Blaise S, Bourgarit Ch, Brian, J. P, Cazes P, Dreux Ph, Escofier B, Fénelon J. P, Forcade J, Giudicelli X, Grosmangin A, Guibert B, Hassan A R, Hathout A, Jambu M, Kamal I. H, Kerbaol M, Lacoste A, Lacourt P, Laganier J, Lebesux M O, Lechat J, Le Chappelier M, Lenoir P, Mahé J, Mann C, Marano Ph, Masson M, Moitry J, Müller J, Nakhlé F, Piétri M, M, Richard J F, Rousseau R, Roux G & M, Salem A, Sandor G, Stérpan S, Tabet N, Thauront G, Volle M, Yagolnitzer E y Zloptowicz M. 1976. *L´Analyse des Données 1. La Taxinomie*. Ed. Dunod Paris. ISBN: 2-04-003316-5.

[8] Casanova del Angel, F. 2010. Structure géométrique des distances hiérarchique No. 41, pp. 27-47. Revue MODULAD. ISSN: 1769-7387.

[9] Casanova del Angel, F. 2012. Terminal Efficiency Analysis of a Postgraduate Degree, Under the Theory of Hierarchical Distances Geometric Structure. The 2012 International Conference on Frontiers in Education: Computer Science & Computer Engineering. Las Vegas, Nevada USA. July 16-19, 2012. ISBN: 1-60132-212-7, 1-60132-213-5 (1-60132-214-3), pp. 143-147.