

PROBABILISTIC ASSESSMENT OF CALCULATION RESISTANCE MODELS OF COMPOSITE SECTION WITH PARTIAL SHEAR INTERACTION

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Original scientific paper

Composite beams in buildings with partial shear connection have been researched. The procedure of determining the exact resistance of such composite beams in bending with partial shear interaction is iterative and tedious. Four different calculation resistance models of various authors for non-iterative determination of such cross-section resistance have been analysed. Bending resistance of cross-section obtained from laboratory tests is compared with the resistance calculated from different models after which the calculation models are probabilistically evaluated. The result is the most certain non-iterative calculation resistance model of cross-section in bending for a beam with partial shear interaction. Furthermore, a new resistance model is proposed. The proposed model is based on a linear model which is then modified according to the most favourable non-iterative calculation resistance model. Correction coefficient is obtained on a deterministic level, after which probabilistic evaluation of the new model is conducted. The result is a very simple non-iterative model with the reliability levels close to the target value of the reliability index.

Keywords: composite beam, model uncertainty, partial shear interaction, reliability index

Probabilističko vrednovanje proračunskih modela otpornosti spregnutog nosača s djelomičnom posmičnom vezom

Izvorni znanstveni članak

Istraživani su spregnuti nosači u zgradarstvu s djelomičnom posmičnom vezom. Postupak određivanja točne otpornosti presjeka na savijanje takvih spregnutih nosača iterativan je i dugotrajan. Analizirana su četiri različita proračunska modela otpornosti, različitih autora, koji omogućuju neiterativno određivanje otpornosti ovakvih nosača. Laboratorijski utvrđena otpornost poprečnog presjeka na savijanje uspoređena je s otpornostima dobivenim korištenjem različitih modela, nakon čega su računski modeli probabilistički evaluirani. Rezultat ovakve analize je najpouzdaniji neiterativni proračunski model za određivanje otpornosti na savijanje poprečnih presjeka s djelomičnom posmičnom vezom. Nadalje, predlaže se novi proračunski model otpornosti. Predloženi model se temelji na linearnom modelu koji se zatim prilagođava najpouzdanijem neiterativnom proračunskom modelu otpornosti. Koeficijent korekcije određuje se na determinističkoj razini, nakon koje se provodi probabilistička analiza novog modela. Rezultat istraživanja je vrlo jednostavan neiterativni model koji daje razinu pouzdanosti u skladu s ciljanom vrijednosti indeksa pouzdanosti.

Ključne riječi: spregnuti nosač, nepouzdanost modela, djelomična posmična veza, indeks pouzdanosti

1

Introduction

In simply-supported composite beams built-up from concrete slab and usually hot-rolled I steel section and subjected to sagging bending longitudinal shear force is transmitted through shear stud connectors. The design bending resistance with full shear connection may be determined by rigid-plastic theory where local slenderness of cross section steel components is sufficiently small or neutral axis lies within concrete slab, which is often case in buildings in regions of sagging bending (Point B in Fig. 1). It is quite often, in practice, that design bending moment M_{Ed} is lower than design value of the plastic resistance moment with full shear connection $M_{pl,Rd}$ of selected cross-section. It would mean that in such cases the number of connectors n_f for full shear connection can be reduced to a smaller number n in such manner that:

$$M_{Ed} \cong M_{Rd}, \quad (1)$$

where M_{Rd} is design value of the resistance moment of a critical composite section with reduced number of shear connectors n i.e. with partial shear connection.

Degree of shear connection η is defined with the following equation:

$$\eta = \frac{n}{n_f} = \frac{N_c}{N_{c,f}}, \quad (2)$$

where:

$N_{c,f}$ – design value of the compressive normal force in the concrete flange with full shear connection;

N_c – design value of the compressive normal force in the concrete flange with partial shear connection.

This is usually allowed in buildings and in regions of sagging bending. Relation between design value of the resistance moment M_{Rd} and degree of shear connection η is, for ductile shear connectors, presented with ACB curve in Fig. 1. Minimum value (point A) corresponds to design value of the plastic resistance moment of the structural steel section alone $M_{pl,a,Rd}$ and point B corresponds to design value of the plastic resistance moment of the composite section with full shear interaction $M_{pl,Rd}$.

Precise determination of bending resistance with partial shear connection is an iterative and long lasting procedure. Hence, it is common, in everyday practice, to use simplified procedure assuming linear relationship between cross-section bending resistance and number n of shear connectors as shown in Fig. 1 with straight line AB.

Several authors proposed their simplified calculation models for determining bending resistance of composite section with partial shear connection. These models will be probabilistically evaluated in order to find the most favourable non-iterative calculation resistance model.

Beside these models this paper will propose a new very simple calculation model which will be based on the linear model with a correction to the model which in the previous analyses proves to have the most favourable reliability level. In such a way, the proposed model will retain the application simplicity known for simplified linear model and at the same time provide resistances that

are closer to the exact value of cross-section bending resistance. Naturally, for proper assessment of the new model it is necessary to conduct probabilistic evaluation

which would determine whether acceptable reliability level is achieved.

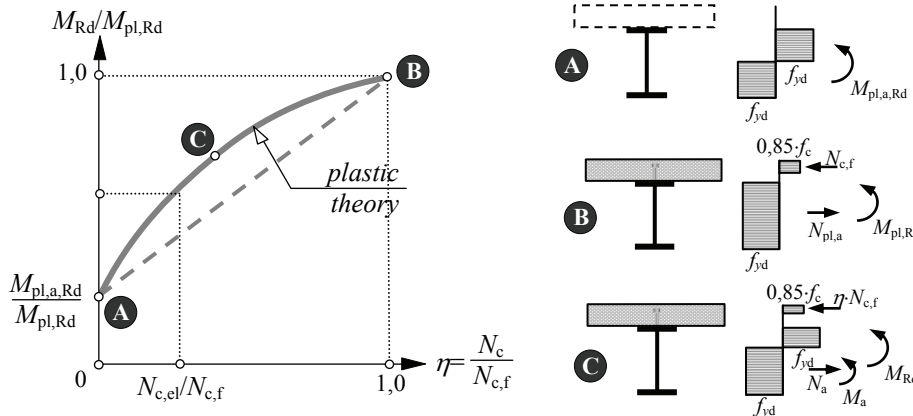


Figure 1 Relation between M_{Rd} and η for ductile shear connectors

2 Calculation resistance models

2.1 Johnson-Anderson model (Model 1)

Johnson-Anderson model is only applicable in cases where neutral axis falls within concrete part of the cross-section when full shear connection is present. It is based on the following premises (Fig. 2.a):

- flange area of hot-rolled I steel section $A_{a,f}$ is at least 30 % of the entire hot-rolled I steel section area A_a
- degree of shear connection η is at least 40 %
- second neutral axis is always within top flange of steel I section.

- difference between concrete slab height h_c and distance between bottom edge of steel top flange to upper edge of concrete slab (h_c+t_f) is negligible compared to the total height of composite section.

Calculation method of resistance moment of composite section M_{Rd} with partial shear connection is shown in detail in [1] and is defined with the following equation:

$$M_{Rd} = N_a \cdot \frac{h_a}{2} + 0,85 \cdot b_{eff} \cdot x_{pl,1} \cdot f_{cd} \cdot \left(h_p + h_c - \frac{x_{pl,1}}{2} \right) \quad (3)$$

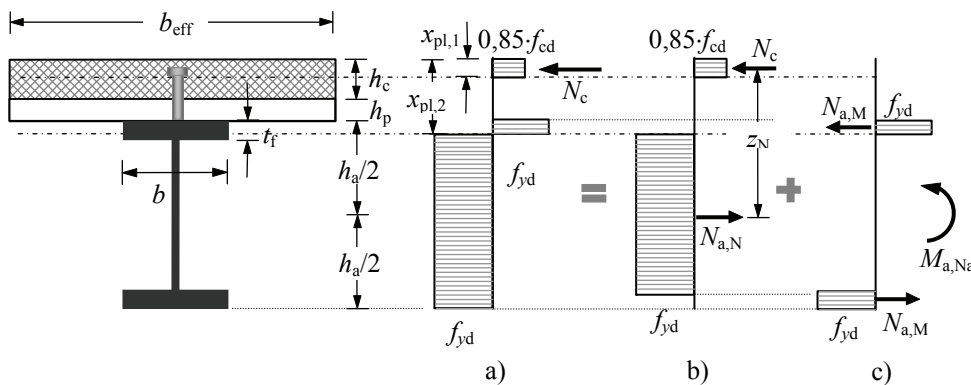


Figure 2 Johnson-Anderson and Bode calculation model

2.2 Bode model (Model 2)

This calculation resistance model is shown in detail in [2]. Following assumptions are made:

- compressive normal force in the concrete flange N_c is balanced with tensile normal force in central part of steel I section $N_{a,N}$ (Fig. 2.b)
- additional part of bending moment is carried only by steel part of cross-section (Fig. 2.c).

Reduced resistance moment of cross-section is then calculated as:

2.3 Vayas model (Model 3)

This model is in detail shown in [3]. It is based on the assumption that degree of shear connection is already known and that compressive normal force in the concrete flange N_c is balanced with tensile normal force in steel part of composite section $N_{a,red}$ such that only the most distant part of steel section is activated, as shown in Fig. 3.

$$M_{Rd} = N_c \cdot \left(\frac{h_a}{2} + h_p + \frac{h_c}{2} \right) + 1,11 \cdot M_{pl,a,Rd} \cdot \left(1 - \frac{N_c}{N_{pl,a,Rd}} \right) \quad (4)$$

cross-section in bending obtained from experiments M_{exp} were compared to resistance moments calculated from

four calculation models mentioned in section 2. Results are given in Tab. 1.

Table 1 Resistances of composite beam cross-section in bending obtained from experiments M_{exp} and resistance moments calculated from four calculation models

Beam	N1	N2	N3	N4	N5	N6	N7	N8	N9	N10	N11
Source	[6]	[6]	[6]	[6]	[7]	[7]	[7]	[7]	[7]	[7]	[7]
% shear connection	0,38	0,65	0,45	0,58	0,65	0,50	0,57	0,47	0,52	0,65	0,61
M_{exp} / kN·m	461	433	402	407	827	705	1027	946	683	814	699
$M_{R,Model 1}$ / kN·m	443	433	409	439	865	716	874	941	697	863	756
$M_{R,Model 2}$ / kN·m	409	406	378	408	793	651	804	860	642	793	700
$M_{R,Model 3}$ / kN·m	306	289	325	396	708	602	579	770	652	708	738
$M_{R,Model 4}$ / kN·m	392	406	365	401	795	633	801	832	624	793	682

3.3 Limit state equations and statistical parameters of basic variables

Limit state equations for reliability analysis are formed separately for each calculation model in the following manner:

$$k_R \cdot f_R(x_i) - k_E \cdot f_E(y_i) = 0, \quad (10)$$

where:
 k_R – basic variable which takes into account uncertainty of calculation resistance model
 $f_R(x_i)$ – resistance function of independent basic variables, separately for every calculation model
 k_E – basic variable which takes into account uncertainty of calculation model of action effects
 $f_E(y_i)$ – function of action effects with appropriate basic variables.

Table 2 Statistical parameters of basic variables

BASIC VARIABLE	MEAN VALUE	C.O.V.	DISTRIBUTION
Beam span	nominal	0,01	normal
Concrete slab height	nominal	0,1	normal
Bottom flange width of the I section	nominal	0,014	normal
Bottom flange thickness of the I section	nominal	0,065	normal
Top flange thickness of the I section	nominal	0,065	normal
Web height of the I section	nominal	0,009	normal
Web thickness of the I section	nominal	0,068	normal
Yield strength of structural steel	1,19 · nom. (S235)	0,068	lognormal
	1,18 · nom. (S355)	0,054	
Concrete cube strength	35,67 (C 25/30)	0,119	lognormal
	45,1 (C 30/37)	0,087	
Reduction factor for concrete strength	0,68	0,125	lognormal
Diameter of the shank of the stud	nominal	0,03	normal
Ultimate strength of the material of the stud connector	1,15 · nominal	0,07	lognormal
Permanent load	nominal	0,15	normal
Imposed load / kN/m ²	0,80	0,56	Gumbel
	1,00	0,56	
Model-factor of Model 1 (k_{R1})	0,992	0,070	normal
Model-factor of Model 2 (k_{R2})	1,077	0,072	normal
Model-factor of Model 3 (k_{R3})	1,251	0,197	normal
Model-factor of Model 4 (k_{R4})	1,098	0,073	normal
Model-factor for action effects (k_E)	1	0,05	normal

Tab. 2 shows statistical parameters of basic variables. Design degree of shear connection in composite beam is determined using design values of basic variables. In such manner the obtained degree of shear connection differs from the degree of shear connection calculated from the measured values of basic variables. Therefore, for determination of degree of shear connection, needed for reliability analyses, measured values were taken as mean values and design values for steel yield strength and concrete cylinder strength were taken as 2,3 % and 5 % fractile respectively.

3.4 Reliability indices

Probabilistic analysis is carried out using FORM and SORM methods included in VaP software. Reliability index target value for 50 year period and second reliability class is 3,8. Minimum, average and maximum values of reliability index β for single calculation models are shown in Fig. 4.

Fig. 5 shows average values of reliability indices β according to construction purpose for different calculation resistance models.

Reliability indices obtained through probabilistic evaluation show that the highest reliability levels result from simplified linear model (Model 4) whilst the lowest

reliability levels, which are also insufficient, are the result of Vayas model (Model 3). The Johnson-Anderson model, compared to the reliability index target value of 3,8, has the most favourable reliability level, therefore it can be used as a replacement for iterative procedure to quickly determine bending resistance of cross-section with partial shear interaction.

The major sensitivity factor α is related to basic variable k_R which takes into account uncertainty of the calculation resistance model. It ranges from 0,325 (Model 1) to 0,944 (Model 4). That refers to database extension of conducted laboratory experiments for composite beams with partial shear connection.

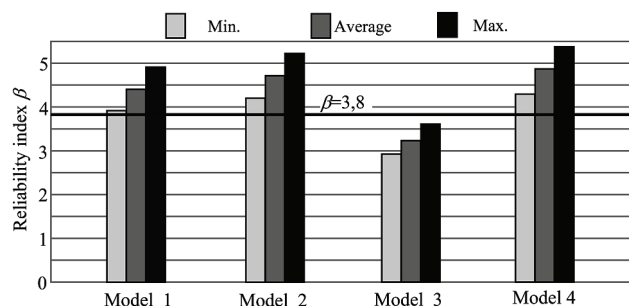


Figure 4 Reliability indices for different calculation resistance models

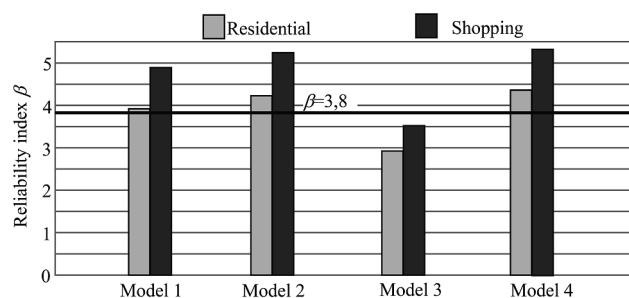


Figure 5 Reliability indices for different construction purposes

4

New non-iterative calculation resistance model proposition

4.1

General

The basic idea is to propose a new very simple calculation model for bending resistance determination of cross-section with partial shear interaction. This new model would be formed by adjustment of the simplified linear model as in [8]. As reference model Johnson-Anderson model was chosen since, in earlier probabilistic evaluation of different resistance models, it proved to have the most favourable reliability level with regard to the target value of the reliability index β . It is important that this new model retains application simplicity and promptness of computing known for simplified linear model, and provides bending resistances that are close to the exact values.

4.2

Corrected linear model (Model 5)

The newly proposed calculation model is obtained by the application of the appropriate correction coefficient Δ to the existing simplified linear model. At first, it is necessary to determine design bending resistances of

cross-section with partial shear interaction according to Johnson-Anderson and simplified linear model. From Fig. 1 and from probabilistic evaluation of all calculation resistance models the following relationship between the two models can be written as:

$$M_{Rd}^{J-A} = (\Delta + 1) \cdot M_{Rd}^{LIN} \quad (11)$$

Substituting corresponding expression (8) in equation (11) a new resistance model equation is derived and can be given as:

$$M_{Rd}^* = (\Delta + 1) \cdot \left[M_{pl,a,Rd} + (M_{pl,Rd} - M_{pl,a,Rd}) \cdot \frac{N_c}{N_{c,f}} \right] \quad (12)$$

where:

M_{Rd}^* – design value of the resistance moment of the composite section with partial shear connection calculated with corrected linear model

$\Delta = \Delta(\eta)$ – correction coefficient as a function of degree of shear connection η .

4.3

Correction coefficient determination

As is explained earlier the value of the correction coefficient is calculated on deterministic level. For this purpose most commonly steel I sections used in buildings were chosen – five different cross-section profiles from each group of IPE, HE A and HE B. Reinforced concrete slab was taken as solid with thickness of 12 cm, and degree of shear connection ranged from $0,4 < \eta < 1,0$ with an increment of 0,1. Steel grade used is S 235, and concrete grade C 30/37.

After selection of all necessary parameters correction coefficient can be calculated according to (12), wherefrom the following expression can be written:

$$\Delta = \frac{M_{Rd}^{J-A}}{M_{Rd}^{LIN}} - 1 \quad (13)$$

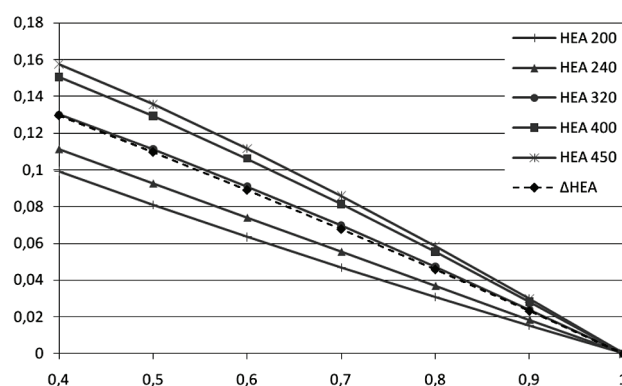


Figure 6 Correction coefficient values for IPE profiles with their approximation function

Given (13), for each steel profile and appropriate degree of shear connection η corresponding correction coefficient value is obtained. As was expected, largest differences in cross-section bending resistances between

the two models occur for low degrees of shear connection, which in practice rarely occur. The calculated values of correction coefficient are given in Fig. 6, Fig. 7 and Fig. 8.

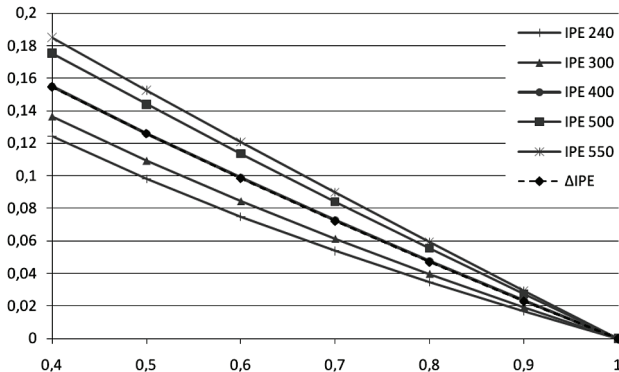


Figure 7 Correction coefficient values for HE A profiles with their approximation function

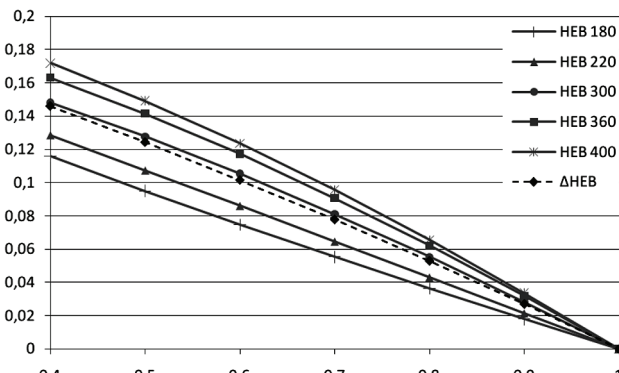


Figure 8 Correction coefficient values for HE B profiles with their approximation function

For each steel profile group functional relationship between the correction coefficient Δ and degree of shear connection η , which varies from 0,4 to 1,0, can well be described with quadratic equation. These are, for various steel profile groups, shown in Fig. 6, Fig. 7 and Fig. 8 with dashed line, and are given with their exact values as follows:

$$\Delta_{IPE} = 0,058 \cdot \eta^2 - 0,339 \cdot \eta + 0,281 \quad \text{for IPE} \quad (14.a)$$

$$\Delta_{HEA} = -0,033 \cdot \eta^2 - 0,170 \cdot \eta + 0,203 \quad \text{for HEA} \quad (14.b)$$

$$\Delta_{HEB} = -0,053 \cdot \eta^2 - 0,169 \cdot \eta + 0,222 \quad \text{for HEB.} \quad (14.c)$$

Substituting corresponding expression (14) in the previously proposed resistance model equation (12) provides bending resistances of cross-sections with partial shear connection that quite well approximate resistances calculated according to Johnson-Anderson model.

5
Corrected linear model reliability analysis
5.1
General

It is necessary to conduct probabilistic analysis of the newly proposed model for its proper evaluation. As before, evaluation is conducted comparing reliability indices β obtained for each of the three calculation models. For this purpose fifteen different cross-sections were analysed with two different degrees of shear connection -40 % and 70 %. Analysed beams were simply-supported and the imposed loads in construction were taken as for office areas. For proper comparison of reliability indices β in all analysed beams cross-sections must be completely utilised meaning that in all of the analysed beams bending moment as a result of external action was taken as equal to the value of the resistance moment of composite section. Design of composite beams was carried out according to European standards EN.

5.2
Limit state equations and statistical parameters of basic variables

Limit state equations were formed for corresponding calculation resistance models that were used. The most common form is already given in Section 3.3. Statistical parameters used for probabilistic analysis are shown in Tab. 3.

Table 3 Statistical parameters of basic variables

BASIC VARIABLE	MEAN VALUE	C. O. V.	DISTRIBUTION
Beam span	nominal	0,01	normal
Beam offset	nominal	0,01	normal
Concrete slab height	nominal	0,10	normal
Steel section profile area	nominal	0,04	normal
Plastic section modulus of steel profile	nominal	0,04	normal
Steel section profile height	nominal	0,009	normal
Composite slab effective width	nominal	0,01	normal
Diameter of the shank of the stud	nominal	0,03	normal
Yield strength of structural steel	1,19 · nom. (S235)	0,068	lognormal
Ultimate strength of the material of the stud connector	1,15 · nominal	0,07	lognormal
Concrete cube strength / MPa	45,1 (C 30/37)	0,087	lognormal
Conversion coefficient of the concrete cube strength to the concrete cylinder strength	0,83	0,10	normal
The coefficient of reduction of the concrete cylinder strength	0,68	0,125	lognormal
Permanent load	nominal	0,15	normal
Imposed load / kN/m ²	1,7535	0,3426	Gumbel
Model-factor of model 1 (k_{R1})	0,992	0,070	normal
Model-factor of model 4 (k_{R4})	1,098	0,073	normal
Model-factor of model 5 (k_{R5})	0,992	0,070	normal
Model-factor for action effects (k_E)	1	0,05	normal

5.3 Reliability indices

Reliability indices of composite cross-sections were obtained through FORM and SORM included in VaP software. Fig. 9 shows their mean values with respect to particular steel profile group and corresponding degree of shear connection for three different resistance models.

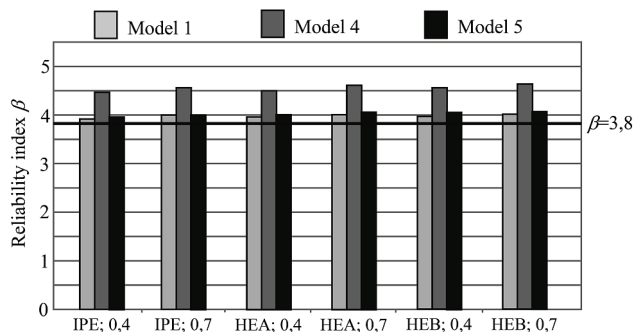


Figure 9 Reliability indices of three calculation resistance models for different steel profile groups and degrees of shear connection η

It was expected that all of resistance models would result with reliability indices whose values are higher than target value which is for a 50 year time period and second structure reliability class given as 3,8. Based on the results it can be seen that the corrected linear model provides reliability levels which are consistent with the original Johnson-Anderson model, and which compared to the ones obtained from the simplified linear model is significantly improved.

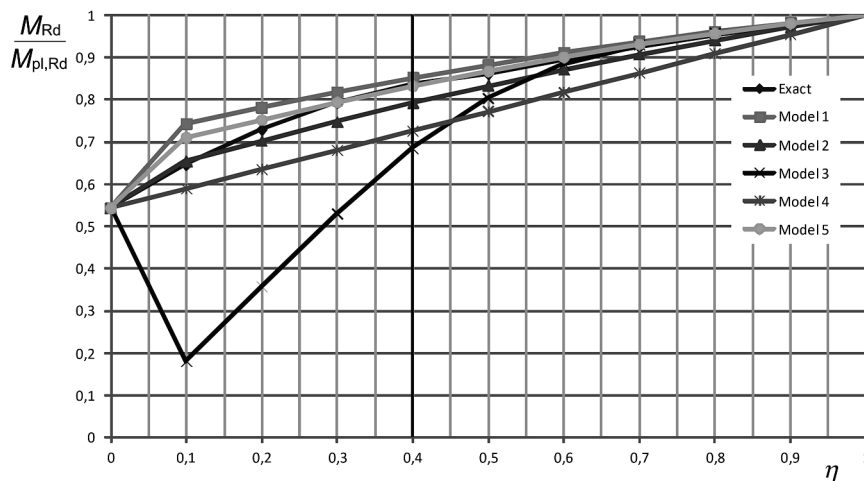


Figure 10 Comparison of calculation resistance models for various degree of shear connection η

Finally, for a comparison on deterministic level the resistance of a sample, made out of a hot rolled steel I section IPE 400 and solid concrete slab with thickness of 12 cm, is calculated for each resistance model mentioned as is shown in Fig. 10. As it can be seen from comparison of Fig. 4 and Fig 5 with Fig. 10 probabilistic evaluation, with regard to deterministic evaluation, can be taken as the only valid assessment of different calculation resistance models.

6 Conclusion

First part of the research included the research involved in probabilistic evaluation of different calculation models. It was found that among all simplified linear model gives the highest and the Vayas model gives the lowest reliability levels. The Johnson-Anderson model proved to have the most favourable reliability level compared to the reliability index target value of 3,8, hence it can be used as a replacement for the iteration procedure to quickly determine bending resistance of composite section with partial shear interaction.

Second part of the paper proposes a new calculation resistance model which aims to provide a procedure with the reliability level close to the reliability index target value of 3,8. The new model is based on a simplified linear model but has additional correction coefficient whose value is calculated in reference to the Johnson-Anderson model which previously proved to have the most favourable reliability level. Probabilistic analysis of the proposed model gave back lower reliability levels compared to the simplified linear model and slightly and negligibly higher compared to the most reliable non-iterative resistance model – Johnson-Anderson. Therefore implementation of this new model is justified for it retains application simplicity and promptness of computing known for the simplified linear model, but still provides very favourable reliability levels that are consistent with the Johnson-Anderson model.

7 References

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