Distributed Adaptive Fault-Tolerant Control of Uncertain Multi-Agent Systems ⋆

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Abstract

This brief paper presents a distributed adaptive fault-tolerant leader-following consensus control scheme for a class of *nonlinear uncertain* multi-agent systems under a bidirectional communication topology with possibly asymmetric weights and subject to process and actuator faults. A local fault-tolerant control (FTC) component is designed for each agent using local measurements and suitable information exchanged between neighboring agents. Each local FTC component consists of a fault diagnosis module and a reconfigurable controller module comprised of a baseline controller and two adaptive fault-tolerant controllers activated after fault detection and after fault isolation, respectively. By using an appropriately chosen Lyapunov function, the closed-loop stability and asymptotic convergence property of leader-follower consensus are rigorously established under different operating modes of the FTC system.

Key words: Fault-Tolerant Control; Adaptive Control; Multi-Agent Systems; Nonlinear Uncertain Systems.

1 Introduction

The study of distributed multi-agent systems (MAS) focuses on the development of control algorithms that enable a team of interconnected agents to accomplish desired team missions (see, for instance, Ren and Beard (2008), and the references cited therein). Adaptive control methods for achieving consensus in uncertain MAS have also been proposed by assuming the absence of faults. For instance, interesting adaptive algorithms have been presented recently to handle unstructured uncertainty for undirected graphs by Wang et al. (2017) and parametric uncertainty for directed graphs by Wang et al. (2014) and Ding and Li (2016), respectively.

In order to ensure reliable and safe operations of MAS, there have been significant research interest in the development of distributed fault diagnosis and accommodation schemes. A distributed fault detection and isolation (FDI) strategy is proposed by Arrichiello et al. (2015) for a team of first-order networked robots, and Shames et al. (2011) developed a distributed fault detection method for interconnected second-order linear time-invariant systems. Distributed fault diagnosis and estimation schemes for systems with more general structures have also been proposed (see, for instance, Reppa et al. (2015); Davoodi et al. (2014); Zhang et al. (2015, 2016)). Additionally, several researchers have also investigated the problem of distributed fault-tolerant control (FTC) of MAS. Semsar-Kazerooni and Khorasani (2010) and Li (2013) focus on fault-tolerant consensus control of MAS with linear dynamics. A fault-tolerant tracking control method for accommodating actuator faults in linear and Lipschitz nonlinear MAS was developed by Zuo et al. (2015). The aforementioned distributed FTC results are derived based on a critical assumption regarding the interconnection topology, i.e., the corresponding Laplacian matrix is symmetric. Moreover, detailed fault information acquired by the fault diagnosis procedure is

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very valuable to FTC, since the objective of FTC is to compensate for the effect of such faults. There exist limited results on the systematic design of integrated fault diagnosis and FTC schemes for MAS with *both actuator and process faults*, especially for *nonlinear uncertain* agents under an interconnection topology whose Laplacian matrix is *asymmetric* .

In this paper, we investigate the problem of integrated design of fault diagnosis and fault-tolerant leader-follower consensus control for a class of nonlinear uncertain MAS, which are interconnected via a bidirectional communication topology with possibly asymmetric weights and are subject to process faults, actuator faults, and unstructured modeling uncertainties. In a previous paper (Zhang et al., 2004), a centralized adaptive FTC method for a class of nonlinear systems was presented, where the centralized fault-tolerant controller has access to all the measurements in the overall system. In contrast, the distributed FTC problem for leader-follower multi-agent systems considered in this paper is much more challenging. First, the interconnection topology between follower agents are considered to be bidirectional but possibly with asymmetric weights. The resulting asymmetric Laplacian matrix significantly increases the complexity of the stability analysis. For instance, the methods for stability analysis presented in (Khalili et al., 2015; Wang et al., 2012; Cao and Ren, 2012; Wang et al., 2017), which utilizes the symmetric property of the Laplacian matrix to solve the leader-follower consensus problem for undirected symmetric graphs, are no longer applicable. It is also worth noting that the asymmetric weights of the graph under consideration don't assume the critical detail-balanced condition considered in the literature (Chen et al., 2011; Zhang et al., 2013b), which makes the stability analysis more challenging. Second, in the leader-following topology considered in this paper, the time-varying leader only communicates to a small subset of followers, and each follower exchanges measurement information only with its neighbors through an unbalanced interconnection topology. This makes it more difficult to accomplish the asymptotic convergence property of leader-following consensus error in the presence of faults and modeling uncertainty. For instance, the well-known Lyapunov function given in Zhang et al. (2012) (Lemma 12) would only guarantee uniformly ultimately bounded (UUB) results, where the consensus errors will be dependent on bounds on the fault functions and modeling uncertainties.

In the presented fault diagnosis and accommodation architecture, a local FTC component is designed for each agent by utilizing local measurements and state information exchanged between neighboring agents. Each local FTC component consists of a baseline controller and two adaptive fault-tolerant controllers. The baseline controller guarantees robust leader-following performance with respect to modeling uncertainty. A decentralized fault diagnosis component is used for detecting and isolating faults in each local agent. Based on local fault diagnostic information, two adaptive fault-tolerant controllers are utilized after fault detection and after fault isolation, respectively. An appropriately chosen Lyapunov function is presented to circumvent the technical difficulty in the design and analysis of the fault-tolerant leader-following controllers. Based on adaptive approximation and adaptive bounding control techniques, the closed-loop asymptotic stability property of leader-following consensus is rigorously established under different operating modes of the FTC system, including the time-period before fault occurrence, between fault detection and possible isolation, and after fault isolation.

The rest of this brief paper is organized as follows. The problem formulation is given in Section 2. The design and analysis of the fault-tolerant control algorithms between fault detection and isolation, and after fault isolation are rigorously investigated in Sections 3 and 4, respectively. In Section 5, a simulation example is used to illustrate the effectiveness of the FTC method. Finally, Section 6 provides some concluding remarks.

2 Problem Formulation

2.1 Graph Theory Notations

A directed graph $\mathcal{G}$ is a pair $(\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{v_1, \cdots, v_P\}$ is a set of nodes, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of edges, and P is the number of nodes. An edge is an ordered pair of distinct nodes (v_j, v_i) meaning that the ith node can receive information from the jth node, and v_j is a neighbor of v_i. An undirected graph is a special case of a directed graph where $(v_i, v_j) \in \mathcal{E}$ implies $(v_j, v_i) \in \mathcal{E}$ for any $v_i, v_j \in \mathcal{V}$. A graph contains a directed spanning tree if there exists a node called the root such that the node has directed paths to all other nodes in the graph.

The set of neighbors of node v_i is denoted by $N_i = \{j : (v_j, v_i) \in \mathcal{E}\}$. The weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in \Re^{P\times P}$ associated with the directed graph $\mathcal{G}$ is defined such that $a_{ii} = 0$, $a_{ij} > 0$ if $(v_j, v_i) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. The topology of an intercommunication graph $\mathcal{G}$ is said to be fixed if each node has a fixed neighbor set and a_{ij} is fixed. For undirected graphs $a_{ij} = a_{ji}$ and for balanced graphs $\sum_{j=1}^{P} a_{ij} = \sum_{j=1}^{P} a_{ji}$. The Laplacian matrix $L = [l_{ij}] \in \Re^{P\times P}$ is defined as $l_{ii} = \sum_{j\in N_i} a_{ij}$ and $l_{ij} = -a_{ij}$, $i \neq j$. Both $\mathcal{A}$ and L are symmetric only for balanced undirected graphs. The sum of the elements on each row of the Laplacian matrix is zero, therefore 0 is an eigenvalue of the Laplacian matrix. The directed graph $\mathcal{G}$ has a spanning tree if and only if the Laplacian matrix of the graph $\mathcal{G}$ has a simple zero eigenvalue. More detailed description of graph theory can be found in Ren and Beard (2008).

2.2 Distributed Multi-Agent System Model

Consider a set of M follower agents with the dynamics of the ith agent, $i = 1, \cdots, M$, being described by

$$\dot{x}_i = \phi_i(x_i) + u_i + \eta_i(x_i, t) + \beta_i(t - T_{iu})\theta_i u_i + \beta_i(t - T_{if}) f_i(x_i) \quad (1)$$

where $x_i \in \Re^n$ and $u_i \in \Re^n$ are the state vector and input vector of the ith agent, respectively, $\phi_i : \Re^n \mapsto \Re^n$, $\eta_i : \Re^n \times \Re^+ \mapsto \Re^n$ and $f_i : \Re^n \mapsto \Re^n$ are smooth vector fields. Specifically, η_i and ϕ_i represent the modeling uncertainty and known nonlinearity, respectively.

The term $\beta_i(t - T_{if})f_i(x_i)$ in (1) denotes the change in the dynamics of ith agent due to the occurrence of a process fault. Specifically, $\beta_i(t - T_{if})$ represents the time profile of the process fault which occurs at some unknown time T_{if}. In this paper, the time profile function $\beta_i(\cdot)$ is assumed to be a step function, which represents an abrupt fault. Additionally, for isolation purposes, we assume that there are $r_i - 1$ types of possible nonlinear process fault functions associated with the ith agent; Specifically, each process fault function f_i^w, $w = 1, \cdots, r_i - 1$, is described by

$$f_i^w(x_i) \triangleq [(\theta_{i1}^w)^T g_{i1}^w(x_i), \cdots, (\theta_{in}^w)^T g_{in}^w(x_i)]^T, \qquad (2)$$

where θ_{ip}^w, for $i = 1, \cdots, M$, and $p = 1, \cdots, n$, is an unknown parameter vector assumed to belong to a known compact set Θ_{ip}^w (i.e., $\theta_{ip}^w \in \Theta_{ip}^w \subseteq \Re^{z_{ip}^w}$), and $g_{ip}^w : \Re^n \mapsto \Re^{z_{ip}^w}$ is a known smooth vector field. As described in Zhang et al. (2004), the process fault model described by (2) characterizes a general class of nonlinear process faults where the vector field g_{ip}^w represents the functional structure of the wth process fault, and the unknown parameter vector θ_{ip}^w characterizes the fault magnitude.

Furthermore, the term $\beta_i(t - T_{iu})\theta_i u_i$ in (1) represents the changes in the dynamics of ith agent due to the occurrence of an actuator fault. Specifically, $\theta_i = \text{diag}\{\theta_{i1}, \cdots, \theta_{in}\}$ represents the actuator fault magnitude, where $\theta_{ip} \in [\bar{\theta}_{ip}, 0]$ is an unknown parameter characterizing the occurrence of a partial loss of effectiveness fault in actuator u_{ip}, for $i = 1, \cdots, M$, $p = 1, \cdots, n$. Additionally, $\beta_i(t - T_{iu})$ is the time profile of the actuator fault which occurs at some unknown time T_{iu}. The case of $\theta_{ip} = 0$ corresponds to a healthy actuator, whereas the case of $\bar{\theta}_{ip} \leq \theta_{ip} < 0$ implies that the actuator is partially faulty. Note that the constant $\bar{\theta}_{ip} \in (-1, 0)$ is a known lower bound needed to maintain the controllability of the distributed agents.

Based on the process faults described by (2) and actuator fault model described above, the fault class associated with agent i under consideration is given by

$$\mathcal{F}_i \triangleq \{f_i^1(x_i), \cdots, f_i^{(r_i-1)}(x_i), \theta_i u_i\}. \qquad (3)$$

The system model (1) allows the occurrence of faults in multiple agents, but it is assumed that there is only a single fault in each agent at any time. In this paper, we denote $\mathcal{G}$ as the fixed graph of the overall MAS and k_{ij} as weights of the adjacency matrix. The leader only communicates to a small subset of followers, and each follower only communicates to its neighboring agents. The following assumptions are made throughout the paper:

Assumption 1 The unstructured modeling uncertainty of each agent, represented by $\eta_i(x_i, t)$ in (1), has a known upper bound, i.e., $\forall p = 1, \cdots, n$, $\forall x_i \in \Re^n$,

$$|\eta_{ip}(x_i, t)| \leq \bar{\eta}_{ip}(x_i, t),$$

where $\bar{\eta}_{ip}$ is a known bounding function.

Assumption 2 Consider $\mathcal{G}$ to be the fixed communication topology of the overall system including the leader, where the intercommunication among followers is bidirectional but with possibly asymmetric weights. It is assumed that the leader has directed paths to all followers.

Assumption 3 The derivative of the leader's time-varying state is bounded, i.e., $\forall p = 1, \cdots, n$, $|\dot{x}_p^r(t)| \leq \kappa_p$, where κ_p is an unknown constant.

Assumption 1 characterizes the class of modeling uncertainty under consideration. The bound on the modeling uncertainty is needed in order to distinguish between the effects of faults and modeling uncertainty during the fault diagnosis process (Zhang et al., 2004). Note that the bounding function $\bar{\eta}_{ip}$ can possibly be obtained by making use of certain limited knowledge on the modeling uncertainty under the worst-case scenario (see, for instance, an aircraft engine fault diagnosis application considered in Zhang et al. (2013a)). Assumption 2 is needed to ensure that the information exchange among agents is sufficient for the team to achieve the desired team goal. The leader is only required to be a neighbor of a subset of followers but has paths to all followers through the intercommunication topology. Additionally, note that the intercommunication graph under consideration is more general than the detail-balanced graph considered in the literature (Chen et al., 2011; Zhang et al., 2013b), because the critical detail-balanced condition on graph weights is not required. Assumption 3 requires an *unknown* constant bound on the derivative of the leader's time-varying state, which is needed to achieve consensus (Li et al., 2013). An adaptive algorithm will be designed to estimate the unknown bound.

The research objective is to design a distributed fault-tolerant leader-following control algorithm that guarantees each agent's state converges to the time-varying bounded reference state of the leader. More specifically, the distributed FTC architecture is designed to achieve the following objectives:

(1) In the absence of faults, a distributed local baseline controller designed for each agent guarantees the state of the ith agent $x_i(t)$ should track the leader's time-varying state $x^r(t)$, for all $i \in \{1, \cdots, M\}$, even in the presence of modeling uncertainties.
(2) If a fault is detected in an agent, the local baseline controller is reconfigured, and the first fault-tolerant controller is utilized to recover some control performance by exploiting the information that a fault has been detected, even though the fault type is still unknown before fault isolation.
(3) If the fault type in the local agent is isolated, then the local controller is reconfigured again. The second local fault-tolerant controller is designed by exploiting the information on the functional structure

of fault (see (2) and (3)) that has been isolated so as to further improve the control performance.

Due to space limitation, this paper focuses on the design and analysis of the distributed adaptive FTC algorithms using FDI information.

3 Accommodation between Fault Detection and Fault Isolation

3.1 Decentralized Fault Detection

Under normal operating conditions, each local fault detection estimator (FDE) monitors the corresponding agent to detect the occurrence of any faults. The decentralized fault detection method can be easily designed using the results in Zhang et al. (2004). Based on the agent model described by (1), the FDE for each agent is chosen as:

$$\dot{\hat{x}}_i = -H_i(\hat{x}_i - x_i) + \phi_i(x_i) + u_i \,, \tag{4}$$

where $\hat{x}_i \in \Re^n$ denotes the estimated local state, $H_i = \text{diag}\{h_{i1}, \cdots, h_{in}\}$ is a positive definite matrix, and $-h_{ip} < 0$ is the estimator pole, for $p = 1, \cdots, n$, $i = 1, \cdots, M$. Without loss of generality, let the observer gain be $H_i = h_i I_n$, and I_n is a $n \times n$ identity matrix. For each local FDE, we define $\epsilon_i^0 \triangleq x_i - \hat{x}_i$ as the state estimation error of the ith agent. Then, before fault occurrence (i.e., for $0 \le t < T_i$), a bounding function on each component of the state estimation error ϵ_{ip}^0 can be derived. Specifically, it can be shown that $|\epsilon_{ip}^0| \le \nu_{ip}(t)$, where $\nu_{ip}(t) \triangleq \int_0^t e^{-h_i(t-\tau)}\, \bar{\eta}_{ip}(x_i, \tau)\, d\tau + \bar{\epsilon}_{ip}^0\, e^{-h_i t}$, and $\bar{\epsilon}_{ip}^0$ is a possibly conservative bound on the initial state estimation error (i.e., $|\epsilon_{ip}^0(0)| \le \bar{\epsilon}_{ip}^0$). Thus, the decision on the occurrence of a fault in the ith agent is made when the absolute value of at least one component of the state estimation error (i.e., $\epsilon_{ip}^0(t)$) generated by the local FDE exceeds its threshold $\nu_{ip}(t)$.

3.2 Distributed Fault-Tolerant Controller

Now, assume that a fault is detected in the ith agent at some time T_d; accordingly, the nominal controller is reconfigured to ensure the system stability after fault detection. However, before the fault is isolated, no information about the fault type is available. Adaptive approximators such as neural-network models can be used to estimate the unknown process fault function $\beta_i f_i$ (Farrell and Polycarpou, 2006). Specifically, we consider linearly parametrized network (e.g., radial-basis-function networks with fixed centers and variances) described as follows: for $p = 1, \cdots, n$,

$$\hat{f}_{ip}(x_i, \hat{\vartheta}_{ip}) = \hat{\vartheta}_{ip}^T \varphi_{ip}(x_i) \,, \tag{5}$$

where $\varphi_{ip}(\cdot)$ represents the fixed basis functions, and $\hat{\vartheta}_{ip}$ is the adjustable weights of the nonlinear approximator. Therefore, the system dynamics (1) can be rewritten as

$$\dot{x}_{ip} = \phi_{ip} + (1 + \theta_{ip})u_{ip} + \eta_{ip} + \hat{f}_{ip}(x_i, \vartheta_{ip}) + \delta_{ip}, \tag{6}$$

where $\delta_{ip} \triangleq f_{ip}(x_i) - \hat{f}_{ip}(x_i, \vartheta_{ip})$ is the network approximation error, and ϑ_{ip} is the optimal weight vector defined as $\vartheta_{ip} \triangleq \arg \inf_{\hat{\vartheta}_{ip} \in \Theta_{ip}} \left\{ \sup_{x_i \in \mathcal{X}_i} |f_{ip}(x_i) - \hat{f}_{ip}(x_i, \hat{\vartheta}_{ip})| \right\}$, where $\mathcal{X}_i \subseteq \Re^n$ denotes the set to which the variable x_i belongs for all possible modes of behavior of the controlled system. For each network, we make the following assumption:

Assumption 4 *The network approximation error*

$$|\delta_{ip}(x_i)| \le \alpha_{ip}\, \bar{\delta}_{ip}(x_i) \,, \ \ \forall x_i \in \Re^n \tag{7}$$

where $\bar{\delta}_{ip}$ is a known positive bounding function, and α_{ip} is an unknown constant.

Let the leader be the $(M + 1)$th agent. The following lemma is needed for the design and analysis of the adaptive fault-tolerant controllers:

Lemma 1 *Consider the graph $\mathcal{G}$ satisfying Assumption 2. Suppose $\Psi \in \Re^{(M+1)\times(M+1)}$ is the Laplacian matrix of the graph as if the communication between the leader and followers is bidirectional. Then, the matrices*

$$\bar{\Psi} \triangleq Q\Psi + \Psi^T Q \,, \tag{8}$$

$$\bar{\mathcal{L}} \triangleq \bar{\Psi}\hat{\mathcal{L}} + \hat{\mathcal{L}}^T \bar{\Psi} \,, \tag{9}$$

and $\hat{\mathcal{L}}$ are positive semidefinite and have a simple zero eigenvalue with $\mathbf{1}_{M+1}$ as its right eigenvector, where $Q = \text{diag}\{q_1, q_2, \cdots, q_{(M+1)}\}$ is a diagonal matrix consisting of the elements of the left eigenvector of Ψ associated with the eigenvalue zero (i.e., $\Psi^T \bar{q} = 0$, $\bar{q} = [q_1, q_2, \cdots, q_{(M+1)}]^T$), the matrix $\hat{\mathcal{L}} \in \Re^{(M+1)\times(M+1)}$ has the same rows as $\bar{\Psi}$ for the ith row, $i = 1, \cdots, M$, while the last row is a $1 \times (M + 1)$ row vector of zeros, and $\mathbf{1}_{M+1}$ is a $(M + 1) \times 1$ column vector of ones.

Proof: Based on Assumption 2 and considering the communication between the leader and followers to be bidirectional, i.e., by adding edges with gains $\bar{k}_i$ connecting agent i to the leader if agent i receives information from the leader, the augmented graph topology of all the agents including the bidirectional leader becomes strongly connected. Therefore, based on Lemma 6 and Lemma 9 in Zhang et al. (2012), the symmetric matrix $\bar{\Psi}$ defined in (8) is positive semidefinite and has a simple zero eigenvalue with $\mathbf{1}_{M+1}$ as its right eigenvector. Moreover, based on Lemma 9 in Zhang et al. (2012), $\bar{\Psi}$ can be considered as the Laplacian matrix of an augmented undirected graph $\bar{\mathcal{G}}$, which has the same node set as the graph corresponding to Ψ, but with weights $q_i k_{ij} + q_j k_{ji}$ connecting agent i and agent j, for $j \ne M + 1$, as well as weights $q_i k_{i(M+1)} + q_{(M+1)}\bar{k}_i$ connecting agent i and the leader. Based on the definition of $\hat{\mathcal{L}}$ given in Lemma 1, we can decompose the matrices $\bar{\Psi}$ and $\hat{\mathcal{L}}$ as follows:

$$\hat{\mathcal{L}} = \begin{bmatrix} \hat{\mathcal{L}}_{11} & \hat{\mathcal{L}}_{12} \\ 0_{1\times M} & 0_{1\times 1} \end{bmatrix}, \quad \bar{\Psi} = \begin{bmatrix} \hat{\mathcal{L}}_{11} & \hat{\mathcal{L}}_{12} \\ \hat{\mathcal{L}}_{12}^T & \hat{\mathcal{L}}_{22} \end{bmatrix}, \tag{10}$$

where $\hat{\mathcal{L}}_{11} \in \Re^{M\times M}$, $\hat{\mathcal{L}}_{12} \in \Re^{M\times 1}$, and $\hat{\mathcal{L}}_{22} \in \Re$. From the specific structures of $\hat{\mathcal{L}}$ and $\bar{\Psi}$ given in (10), we can see that the topology graph corresponding to $\hat{\mathcal{L}}$ for the $M+1$ agents (with the leader being agent $M+1$) has the same nodes as $\bar{\mathcal{G}}$ and a spanning tree with the leader as its root. Thus, the matrix $\hat{\mathcal{L}}$ has M positive eigenvalues and has a simple zero eigenvalue with $\mathbf{1}_{M+1}$ as its right eigenvector (Ren and Beard, 2008). Now, by using the specific structure of $\hat{\mathcal{L}}$ given in (10), we can conclude $\hat{\mathcal{L}}_{11}$ is a positive definite matrix.

Let χ be an eigenvalue of $\bar{\mathcal{L}}$. From (9) and (10), we have $|\chi I_{M+1} - \bar{\mathcal{L}}| = \begin{vmatrix} \chi I_M - 2\hat{\mathcal{L}}_{11}^2 & -2\hat{\mathcal{L}}_{11}\hat{\mathcal{L}}_{12} \\ -2\hat{\mathcal{L}}_{12}^T\hat{\mathcal{L}}_{11} & \chi - 2\hat{\mathcal{L}}_{12}^T\hat{\mathcal{L}}_{12} \end{vmatrix}$, where I_{M+1} represents a $(M+1)\times(M+1)$ identity matrix. Using $\begin{vmatrix} A & B \\ C & D \end{vmatrix} = |A|\cdot|D - C\,A^{-1}B|$, we obtain

$$\begin{aligned} |\chi I_{M+1} - \bar{\mathcal{L}}| = {} & |\chi I_M - 2\hat{\mathcal{L}}_{11}^2| \cdot |\chi - 2\hat{\mathcal{L}}_{12}^T\hat{\mathcal{L}}_{12} \\ & -4\hat{\mathcal{L}}_{12}^T\hat{\mathcal{L}}_{11}(\chi I_M - 2\hat{\mathcal{L}}_{11}^2)^{-1}\hat{\mathcal{L}}_{11}\hat{\mathcal{L}}_{12}|\,. \end{aligned}$$

Note that the eigenvalues of $\bar{\mathcal{L}}$ satisfying $|\chi I_M - 2\hat{\mathcal{L}}_{11}^2| = 0$ are all positive, because $\hat{\mathcal{L}}_{11}$ and $2\hat{\mathcal{L}}_{11}^2$ are positive definite as proved above. Furthermore, $\chi = 0$ satisfies $|\chi - 2\hat{\mathcal{L}}_{12}^T\hat{\mathcal{L}}_{12} - 4\hat{\mathcal{L}}_{12}^T\hat{\mathcal{L}}_{11}(\chi I_M - 2\hat{\mathcal{L}}_{11}^2)^{-1}\hat{\mathcal{L}}_{11}\hat{\mathcal{L}}_{12}| = 0$. Additionally, it can be shown $\bar{\mathcal{L}}\mathbf{1}_{M+1} = \bar{\Psi}\hat{\mathcal{L}}\mathbf{1}_{M+1} + \hat{\mathcal{L}}^T\bar{\Psi}\mathbf{1}_{M+1} = 0$, since it has been proved that $\hat{\mathcal{L}}$ and $\bar{\Psi}$ both have a simple zero eigenvalue with $\mathbf{1}_{M+1}$ as its right eigenvector. Thus, the proof can be concluded. □

Remark 1. This important lemma provides a skillfully chosen symmetric positive definite matrix $\bar{\Psi}$, which is crucial for designing an appropriate Lyapunov function for deriving the asymptotic fault-tolerant leader-following consensus property of the MAS under a bidirectional communication topology with general asymmetric weights. The appropriately constructed Laplacian matrix Ψ for the augmented graph with a bidirectional leader makes the graph topology of all the agents including the leader strongly connected and therefore allows us to compute the positive gains q_l, $l = 1, \cdots, M+1$, which can be used to derive appropriate controller gains for ensuring the convergence of leader-following consensus error. It is worth noting that the Laplacian matrix Ψ for a bidirectional leader is only considered for the purpose of controller performance analysis, while the underlying communication topology has a directed leader since the leader is only sending the data and does not receive any data from other agents.

Now, based on (5), (6), and Assumption 4, the following distributed control algorithm can be chosen:

$$u_{ip} = \frac{1}{1+\hat{\theta}_{ip}}\bar{u}_{ip}\,, \tag{11}$$

$$\begin{aligned} \bar{u}_{ip} = {} & -\phi_{ip} - \sum_{j\in N_i}\left(\rho_{ij}\tilde{x}_{ij}\right) - \hat{f}_{ip}(x_i, \hat{\vartheta}_{ip}(t)) - \psi_{ip} \\ & -(\bar{\eta}_{ip} + \hat{\kappa}_{ip})sgn\Big(\sum_{j\in N_i}\rho_{ij}\,\tilde{x}_{ij}\Big), \end{aligned} \tag{12}$$

$$\dot{\hat{\vartheta}}_{ip} = \Gamma_{ip}\sum_{j\in N_i}\rho_{ij}\,\tilde{x}_{ij}\varphi_{ip}(x_i)\,, \tag{13}$$

$$\psi_{ip} = \hat{\alpha}_{ip}\bar{\delta}_{ip}(x_i)sgn\Big(\sum_{j\in N_i}\rho_{ij}\,\tilde{x}_{ij}\Big), \tag{14}$$

$$\dot{\hat{\alpha}}_{ip} = \Upsilon_{ip}\,\Big|\sum_{j\in N_i}\rho_{ij}\,\tilde{x}_{ij}\Big|\,\bar{\delta}_{ip}(x_i)\,, \tag{15}$$

$$\dot{\hat{\kappa}}_{ip} = \bar{\Upsilon}_{ip}\,\Big|\sum_{j\in N_i}\rho_{ij}\,\tilde{x}_{ij}\Big|\,, \tag{16}$$

$$\dot{\hat{\theta}}_{ip} = \mathcal{P}_{\bar{\theta}_{ip}}\Big\{\bar{\Gamma}_{ip}\sum_{j\in N_i}\rho_{ij}\,\tilde{x}_{ij}u_{ip}\Big\}, \tag{17}$$

where $\tilde{x}_{ij} \triangleq x_{ip} - x_{jp}$, $p = 1, \cdots, n$, $i = 1, \cdots, M$, u_{ip} and x_{ip} are the pth component of the input and state vectors of the ith agent, respectively, ρ_{ij} are constant design gains to be defined later in (20), $\hat{\theta}_{ip}$ is an estimation of the actuator fault magnitude parameter θ_{ip} with the projection operator $\mathcal{P}$ restricting $\hat{\theta}_{ip}$ to the corresponding set $[\bar{\theta}_{ip}, 0]$, $\hat{\vartheta}_{ip}$ is an estimation of the neural network parameter vector ϑ_{ip}, $\varphi_{ip} \triangleq col(\varphi_j : j = 1, \cdots, \varrho)$ is the collective vector of fixed basis functions, $\hat{\alpha}_{ip}$ is an estimation of the unknown constant α_{ip} (see Assumption 4), $\hat{\kappa}_{ip}$ is an estimation of the unknown positive constant bound κ_p on $|\dot{x}_p^r|$ (see Assumption 3), Γ_{ip} is a symmetric positive definite matrix, $\bar{\Gamma}_{ip}$, Υ_{ip} and $\bar{\Upsilon}_{ip}$ are positive learning rate constants, and $sgn(\cdot)$ is the sign function defined to take zero value at zero. Note that we can rewrite (11) as $u_{ip} = \bar{u}_{ip} - \hat{\theta}_{ip}u_{ip}$. Therefore, Based on (6), the closed-loop system dynamics are given by

$$\begin{aligned} \dot{x}_{ip} = {} & \phi_{ip} + \bar{u}_{ip} - \hat{\theta}_{ip}u_{ip} + \theta_{ip}u_{ip} + \eta_{ip} + \delta_{ip}(x_i) \\ & + \hat{f}_{ip}(x_i, \vartheta_{ip})\,. \end{aligned} \tag{18}$$

By using (12) and (18), we have

$$\begin{aligned} \dot{x}_{ip} = {} & -\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij} - \hat{f}_{ip}(x_i, \hat{\vartheta}_{ip}) + \hat{f}_{ip}(x_i, \vartheta_{ip}) + \delta_{ip} - \psi_{ip} \\ & + \eta_{ip} - (\bar{\eta}_{ip} + \hat{\kappa}_{ip})sgn\Big(\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}\Big) + \tilde{\theta}_{ip}u_{ip}\,, \end{aligned} \tag{19}$$

where $\tilde{\theta}_{ip} = \theta_{ip} - \hat{\theta}_{ip}$ is the actuator fault magnitude estimation error.

We choose the following positive distributed controller gains: for $i=1,\cdots,M$, and $j \in N_i$,

$$\rho_{ij} = \begin{cases} q_i k_{ij} + q_j k_{ji}\,, & \text{for} \quad j \neq M+1 \\ q_i k_{i(M+1)} + q_{(M+1)}\bar{k}_i\,, & \text{for} \quad j = M+1 \end{cases} \tag{20}$$

where q_i is defined in Lemma 1, $\bar{k}_i$ is a constant designed to make the graph corresponding to Ψ strongly connected (i.e., $\bar{k}_i > 0$ if the leader directly communicates to follower agent i in the graph $\mathcal{G}$, and $\bar{k}_i = 0$ otherwise), k_{ij} and k_{ji} are positive constants denoting the weights on the intercommunication graph $\mathcal{G}$. Using (19) and (20) and the definition of $\hat{\mathcal{L}}$ in Lemma 1, we can represent the collective closed-loop state dynamics as

$$\dot{x}^p = -\hat{\mathcal{L}}x^p + \zeta^p - \bar{\zeta}^p + \tilde{f}^p + \delta^p - \psi^p + \varpi^p\,, \tag{21}$$

where $x^p \in \Re^{M+1}$, $p=1,\cdots,n$, is comprised of the pth component of the $M+1$ agents, including the leader as the $(M+1)$th agent, i.e., $x^p = [\,x_{1p}, x_{2p}, \cdots, x_{Mp}, x_p^r\,]^T$, $\hat{\mathcal{L}}$ is defined in Lemma 1, the terms $\zeta^p \in \Re^{M+1}$, $\bar{\zeta}^p \in \Re^{M+1}$, $\tilde{f}^p \in \Re^{M+1}$, $\delta^p \in \Re^{M+1}$, $\psi^p \in \Re^{M+1}$ and $\varpi^p \in \Re^{M+1}$ are defined as

$$\zeta^p \triangleq [\,\eta_{1p}, \cdots, \eta_{Mp}, \dot{x}_p^r\,]^T\,, \tag{22}$$

$$\bar{\zeta}^p \triangleq [\,\bar{\zeta}_{1p}, \cdots, \bar{\zeta}_{Mp}, 0\,]^T\,, \tag{23}$$

$$\tilde{f}^p \triangleq [\,(\tilde{\vartheta}_{1p})^T\varphi_{1p}, \cdots, (\tilde{\vartheta}_{Mp})^T\varphi_{Mp}, 0\,]^T\,, \tag{24}$$

$$\delta^p \triangleq [\,\delta_{1p}, \cdots, \delta_{Mp}, 0\,]^T\,, \tag{25}$$

$$\psi^p \triangleq [\,\psi_{1p}, \cdots, \psi_{Mp}, 0\,]^T\,, \tag{26}$$

$$\varpi^p \triangleq [\,\tilde{\theta}_{1p}u_{1p}, \cdots, \tilde{\theta}_{Mp}u_{Mp}, 0\,]^T\,, \tag{27}$$

and $\bar{\zeta}_{ip} \triangleq (\bar{\eta}_{ip} + \hat{\kappa}_{ip})sgn\big(\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}\big)$, $i=1,\cdots,M$, $\tilde{\vartheta}_{ip} \triangleq \vartheta_{ip} - \hat{\vartheta}_{ip}$ and φ_{ip} are the neural network parameter estimation error and basis functions, respectively.

To derive the adaptive FTC algorithm, we consider the following Lyapunov function candidate:

$$V_p = x^{pT}\bar{\Psi}\,x^p + (\tilde{\vartheta}^p)^T(\Gamma^p)^{-1}\tilde{\vartheta}^p + (\tilde{\alpha}^p)^T(\Upsilon^p)^{-1}\tilde{\alpha}^p + (\tilde{\theta}^p)^T(\bar{\Gamma}^p)^{-1}\tilde{\theta}^p + (\tilde{\kappa}^p)^T(\bar{\Upsilon}^p)^{-1}\tilde{\kappa}^p\,, \tag{28}$$

where $\bar{\Psi}$ is defined in (8), $\tilde{\vartheta}^p = [\,\tilde{\vartheta}_{1p}^T, \cdots, \tilde{\vartheta}_{Mp}^T\,]^T$ is the collective parameter estimation errors, $\tilde{\alpha}^p = [\,\tilde{\alpha}_{1p}, \cdots, \tilde{\alpha}_{Mp}\,]^T$ is the collective bounding parameter estimation errors defined as $\tilde{\alpha}_{ip} = \alpha_{ip} - \hat{\alpha}_{ip}$, $\tilde{\theta}^p = [\,\tilde{\theta}_{1p}, \cdots, \tilde{\theta}_{Mp}\,]^T$ is the collective actuator fault magnitude parameter estimation errors, $\tilde{\kappa}^p = [\,\tilde{\kappa}_{1p}, \cdots, \tilde{\kappa}_{Mp}\,]^T$ is the collective bounding parameter estimation errors defined as $\tilde{\kappa}_{ip} = \kappa_p - \hat{\kappa}_{ip}$, and $\Gamma^p = \text{diag}\{\Gamma_{1p}, \cdots, \Gamma_{Mp}\}$, $\Upsilon^p = \text{diag}\{\Upsilon_{1p}, \cdots, \Upsilon_{Mp}\}$, $\bar{\Gamma}^p = \text{diag}\{\bar{\Gamma}_{1p}, \cdots, \bar{\Gamma}_{Mp}\}$, and $\bar{\Upsilon}^p = \text{diag}\{\bar{\Upsilon}_{1p}, \cdots, \bar{\Upsilon}_{Mp}\}$ are constant matrices.

Remark 2. Based on the definition of $\bar{\Psi}$ in (8), it can be shown that $x^{pT}\bar{\Psi}\,x^p = \sum_{i=1}^{M}\sum_{j\in N_i}\rho_{ij}(\tilde{x}_{ij})^2$, which is positive definite with respect to the relative tracking error $\tilde{x}_{ij}$ by following a similar reasoning logic given in (Ren, 2009). Additionally, based on Lemma 1, $\bar{\Psi}$ is positive semidefinite and have a simple zero eigenvalue with $\mathbf{1}_{M+1}$ as its right eigenvector, which implies the only x^p that makes $x^{pT}\bar{\Psi}\,x^p$ in (28) zero satisfies $x_{ip} = x_{jp} = x_r^p$, for $i=1,\cdots,M$, $j\in N_i$.

Then, the time derivative of the Lyapunov function (28) along the solution of (21) is

$$\dot{V}_p = -x^{pT}\bar{\mathcal{L}}\,x^p + 2x^{pT}\bar{\Psi}(\zeta^p - \bar{\zeta}^p + \tilde{f}^p + \delta^p - \psi^p + \varpi^p) + (\tilde{\vartheta}^p)^T(\Gamma^p)^{-1}\dot{\tilde{\vartheta}}^p + (\tilde{\alpha}^p)^T(\Upsilon^p)^{-1}\dot{\tilde{\alpha}}^p + (\tilde{\theta}^p)^T(\bar{\Gamma}^p)^{-1}\dot{\tilde{\theta}}^p + (\tilde{\kappa}^p)^T(\bar{\Upsilon}^p)^{-1}\dot{\tilde{\kappa}}^p\,, \tag{29}$$

where $\bar{\mathcal{L}}$ is defined in (9). Based on (22) - (27), and by using the left eigenvector property, i.e., $\sum_{j\in N_i} q_i k_{ij} = \sum_{j\in N_i} q_j k_{ji}$, we have

$$x^{pT}\bar{\Psi}\zeta^p = \sum_{i=1}^{M}\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}\eta_{ip} + \dot{x}_p^r\sum_{i=1}^{M}\rho_{i(M+1)}(x_p^r - x_{ip}),$$

$$x^{pT}\Psi\bar{\zeta}^p = \sum_{i=1}^{M}\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}(\bar{\eta}_{ip} + \hat{\kappa}_{ip})sgn\Big(\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}\Big)\,,$$

$$x^{pT}\Psi\tilde{f}^p = \sum_{i=1}^{M}\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}(\tilde{\vartheta}_{ip})^T\varphi_{ip}\,,$$

$$x^{pT}\Psi\delta^p = \sum_{i=1}^{M}\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}\delta_{ip}\,,$$

$$x^{pT}\Psi\psi^p = \sum_{i=1}^{M}\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}\psi_{ip}\,,$$

$$x^{pT}\Psi\varpi^p = \sum_{i=1}^{M}\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}\tilde{\theta}_{ip}u_{ip}\,.$$

Note that we have $\sum_{i=1}^{M}\sum_{d}\rho_{id}\tilde{x}_{id} = 0$, for $d\in N_i, d\neq M+1$. Therefore, we have

$$\dot{x}_p^r\sum_{i=1}^{M}\rho_{i(M+1)}(x_p^r - x_{ip}) = -\dot{x}_p^r\sum_{i=1}^{M}\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}\,.$$

By using the above seven equations and (29), we obtain

$$\dot{V}_p = -x^{pT}\bar{\mathcal{L}}\,x^p + 2\sum_{i=1}^{M}\Bigg[\tilde{\vartheta}_{ip}^T\Big(\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}\varphi_{ip} - (\Gamma_{ip})^{-1}\dot{\hat{\vartheta}}_{ip}\Big) + \sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}\Big(\eta_{ip} - \bar{\eta}_{ip}sgn\big(\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}\big)\Big) + \sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}\Big(-\dot{x}_p^r - \hat{\kappa}_{ip}sgn\big(\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}\big)\Big)$$

$$-\tilde{\kappa}_{ip}(\bar{\Upsilon}_{ip})^{-1}\dot{\hat{\kappa}}_{ip} + \tilde{\theta}_{ip}\Bigg(\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}u_{ip} - (\bar{\Gamma}_{ip})^{-1}\dot{\hat{\theta}}_{ip}\Bigg)$$
$$+\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}(\delta_{ip}-\psi_{ip}) - \tilde{\alpha}_{ip}(\Upsilon_{ip})^{-1}\dot{\hat{\alpha}}_{ip}\Bigg]. \tag{30}$$

Based on Assumption 1, we have

$$\eta_{ip}\Big(\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}\Big) - \bar{\eta}_{ip}\Big(\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}\Big)sgn\Big(\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}\Big) \le 0\,. \tag{31}$$

Therefore, by applying the above inequality to (30) and selecting the adaptive algorithm for $\hat{\vartheta}_{ip}$ and $\hat{\theta}_{ip}$ as (13) and (17), respectively, we have

$$\dot{V}_p \le -x^{pT}\bar{\mathcal{L}}\,x^p + 2\sum_{i=1}^{M}\Bigg[-\tilde{\kappa}_{ip}(\bar{\Upsilon}_{ip})^{-1}\dot{\hat{\kappa}}_{ip}$$
$$+\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}\Bigg(-\dot{x}_p^r - \hat{\kappa}_{ip}sgn\Big(\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}\Big)\Bigg)\Bigg]$$
$$+2\sum_{i=1}^{M}\Bigg(\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}(\delta_{ip}-\psi_{ip}) - \tilde{\alpha}_{ip}(\Upsilon_{ip})^{-1}\dot{\hat{\alpha}}_{ip}\Bigg).$$

Since the parameter projection modification can only make the Lyapunov function derivative more negative, the stability properties derived for the standard algorithm still hold (Farrell and Polycarpou, 2006). Note that by using (14) and Assumption 4, we have

$$\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}(\delta_{ip}-\psi_{ip}) \;\le\; \Big|\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}\Big|\hat{\alpha}_{ip}\bar{\delta}_{ip}\,. \tag{32}$$

By using (32) and Assumption 3, we obtain

$$\dot{V}_p \le 2\sum_{i=1}^{M}\Bigg[\Bigg(\Big|\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}\Big|\tilde{\alpha}_{ip}\bar{\delta}_{ip} - \tilde{\alpha}_{ip}(\Upsilon_{ip})^{-1}\dot{\hat{\alpha}}_{ip}\Bigg)$$
$$+\Bigg(\Big|\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}\Big|\tilde{\kappa}_{ip} - \tilde{\kappa}_{ip}(\bar{\Upsilon}_{ip})^{-1}\dot{\hat{\kappa}}_{ip}\Bigg)\Bigg] - x^{pT}\bar{\mathcal{L}}\,x^p.$$

Therefore, by using (15), (16) and after some algebraic manipulations, we have

$$\dot{V}_p \le -x^{pT}\bar{\mathcal{L}}\,x^p = -2\sum_{i=1}^{M}\Bigg(\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}\Bigg)^2. \tag{33}$$

Using Lemma 1, we know $\dot{V}_p$ is negative semidefinite. Based on the definition of V_p, we conclude that $\tilde{x}_{ij}$, $\hat{\vartheta}_{ip}$, $\hat{\theta}_{ip}$, $\hat{\kappa}_{ip}$ and $\hat{\alpha}_{ip}$ are uniformly bounded. Integrating both sides of (33), we know that $\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij} \in L_2$. Additionally, x_{ip} is bounded because $\tilde{x}_{ij}$ and the leader's state x_p^r are bounded. Therefore, based on (12), (18), and the smoothness of the function ϕ_{ip}, we have $\bar{u}_{ip} \in L_\infty$, $\dot{x}_{ip} \in L_\infty$, and $\sum_{j\in N_i}\rho_{ij}\dot{\tilde{x}}_{ij} \in L_\infty$. Now, based on Barbalat's Lemma, we can conclude that $\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij} \to 0$ as $t \to \infty$, which implies $\hat{\mathcal{L}}x^p \to 0$. Using Lemma 1, we know that $\hat{\mathcal{L}}$ has a single zero eigenvalue with $\mathbf{1}_{M+1}$ as its right eigenvector. Thus, $\hat{\mathcal{L}}x^p \to 0$ if and only if $x_{ip}(t) \to x_p^r(t)$, which implies the leader-follower consensus is reached asymptotically. The aforementioned design and analysis procedure is summarized as follows:

Theorem 1 *Suppose that Assumptions 1–4 hold. Then, if a fault is detected, by using the distributed controller gains given by (20), the adaptive fault-tolerant law (11), the weight parameter adaptive law (13), the bounding parameter adaptive laws (14)-(16), and the actuator fault parameter adaptive law (17) guarantee*

(1) all the signals and parameter estimates are uniformly bounded, i.e., $\tilde{x}_{ij}$, $\hat{\vartheta}_{ip}$, $\hat{\theta}_{ip}$, $\hat{\alpha}_{ip}$ and $\hat{\kappa}_{ip}$ are bounded;
(2) the leader-follower consensus is achieved asymptotically with a time-varying reference state, i.e., $x_i(t) - x^r(t) \to 0$ as $t \to \infty$, for all $i = 1, \cdots, M$.

Remark 3. Interesting neural-network-based adaptive consensus algorithms for MAS with first-order agent dynamics have been presented in Cheng et al. (2010) and Das and Lewis (2010). The Laplacian matrix of the topology is assumed to be symmetric in Cheng et al. (2010), while bidirectional topology with possibly asymmetric weights is considered in this paper, which makes the design of adaptive consensus algorithms more challenging. Additionally, UUB results are derived in Das and Lewis (2010), where the consensus error bound is shown to be a function of several constant bounds on modeling uncertainty, neural network approximation error and optimal weights, neural network activation functions, and the derivative of the leader's state, respectively. In contrast, adaptive algorithms guaranteeing asymptotic convergence of the consensus tracking error is derived in this paper. Moreover, it is also worth noting that this paper considers both process and actuator faults, while the latter is not considered in Cheng et al. (2010) and Das and Lewis (2010).

4 Accommodation after Fault Isolation

4.1 Decentralized Fault Isolation

Once the fault is detected at time $t = T_d$, the fault isolation estimators (FIEs) in the local FDI component designed for the ith agent are activated for determining the fault type that has occurred. Each local FIE is designed based on the functional structure of one potential fault type associated with the agent (see (2) and (3)). Specifically, based on the fault class described by (3), the following r_i nonlinear adaptive estimators are designed as FIEs: for $s = 1, \cdots, r_i$, and $p = 1, \cdots, n$,

$$\dot{\hat{x}}_{ip}^s = -\lambda_{ip}^s(\hat{x}_{ip}^s - x_{ip}) + \phi_{ip} + u_{ip} + \hat{\theta}_{ip}^{sT}\bar{g}_{ip}^s(x_i, u_i) \tag{34}$$
$$\dot{\hat{\theta}}_{ip}^s = \mathcal{P}_{\Theta_{ip}^s}\{\gamma_{ip}^s\,\bar{g}_{ip}^s(x_i, u_i)\,\epsilon_{ip}^s\}\,, \tag{35}$$

where $\bar{g}_{ip}^s$ for $s = 1, \cdots, r_i$ represents the functional structure of the sth fault (see (3), i.e., $\bar{g}_{ip}^s \stackrel{\triangle}{=} g_{ip}^s$ for process faults and $\bar{g}_{ip}^s \triangleq u_{ip}$ for the actuator fault), $\hat{\theta}_{ip}^s$ is the estimate of the fault parameter vector for the pth state component in the ith agent, $\epsilon_{ip}^s(t) \triangleq x_{ip} - \hat{x}_{ip}^s$ is the pth component of the state estimation error generated by the sth FIE, and γ_{ip}^s and λ_{ip}^s are positive constants. For notational simplicity, we assume that $\lambda_{ip}^s = \lambda_i$. The adaptive law (35) for updating each $\hat{\theta}_{ip}^s$ is derived by using the Lyapunov synthesis approach (Farrell and Polycarpou (2006)), with the projection operator $\mathcal{P}_{\Theta_{ip}^s}$ restricting $\hat{\theta}_{ip}^s$ to the corresponding known set Θ_{ip}^s.

By following the reasoning logic given in (Zhang et al., 2004), a bound on each component of the state estimation error can be obtained as $|\epsilon_{ip}^s| \le \mu_{ip}^s(t)$, where $\mu_{ip}^s(t) \triangleq \int_{T_d}^t e^{-\lambda_i(t-\tau)}\left[\bar{\eta}_{ip}(x_i,\tau) + \xi_{ip}^s \left|\bar{g}_i^s\right|\right] d\tau + \bar{\epsilon}_{ip}^s\, e^{-\lambda_i(t-T_d)}$, and $\bar{\epsilon}_{ip}^s$ is a possibly conservative bound on the initial state estimation error (i.e., $|\epsilon_{ip}^s(T_d)| \le \bar{\epsilon}_{ip}^s$), and ξ_{ip}^s represents the maximum fault parameter vector estimation error, i.e., $|\theta_{ip}^s - \hat{\theta}_{ip}^s(t)| \le \xi_{ip}^s$. The form of ξ_{ip}^s depends on the geometric properties of the compact set Θ_{ip}^s (Zhang et al., 2004). Thus, based on the generalized observer scheme, the following fault isolation decision scheme is devised: If for each $b \in \{1, \cdots, r_i\}\backslash\{s\}$, there exist some finite time $t^b > T_d$ and some $p \in \{1, \cdots, n\}$, such that $|\epsilon_{ip}^b(t^b)| > \mu_{ip}^b(t^b)$, then the occurrence of fault s in agent i is concluded.

4.2 Adaptive FTCs after Fault Isolation

Now, assume that the aforementioned isolation procedure provides the information that fault s has been isolated at time T_{isol}. The controller is reconfigured again to further improve the control performance based on the information of isolated fault type. Below, we investigate the cases of process fault and actuator fault, respectively.

4.2.1 Adaptive FTC of Process Faults

After the isolation of process fault type s , i.e., $t \ge T_{isol}$, the dynamics of the system for $p = 1, \cdots, n$, can be represented as

$$\dot{x}_{ip} = \phi_{ip} + u_{ip} + \eta_{ip}(x_i, t) + \theta_{ip}^{sT} g_{ip}^s(x_i)\,. \qquad (36)$$

The following adaptive FTC scheme is chosen:

$$u_{ip} = -\phi_{ip} - \sum_{j\in N_i} \left(\rho_{ij}\tilde{x}_{ij}\right) - \hat{\theta}_{ip}^T g_{ip}^s(x_i)$$
$$-(\bar{\eta}_{ip} + \hat{\kappa}_{ip}) sgn\bigg(\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}\bigg), \qquad (37)$$

$$\dot{\hat{\theta}}_{ip} = \Gamma_{ip} \sum_{j\in N_i} \rho_{ij}\tilde{x}_{ij} g_{ip}^s(x_i)\,, \qquad (38)$$

$$\dot{\hat{\kappa}}_{ip} = \bar{\Upsilon}_{ip} \,\Big|\sum_{j\in N_i} \rho_{ij}\tilde{x}_{ij}\Big|\,, \qquad (39)$$

where the controller gains ρ_{ij} are defined in (20), $\hat{\theta}_{ip}$ is the estimated unknown fault parameter vector, $\hat{\kappa}_{ip}$ is an estimation of the unknown constant bound κ_p on $|\dot{x}_p^r|$ (see Assumption 3), Γ_{ip} is a symmetric positive definite learning rate matrix, and $\bar{\Upsilon}_{ip}$ is a learning rate constant. Then, we have the following:

Theorem 2 *Assume that process fault s occurs at time T_{if} and that it is isolated at time T_{isol}. Then, the fault-tolerant controller (37), the fault parameter adaptive law (38), and the bounding parameter adaptive law (39) using the distributed gains (20) guarantee that the leader-follower consensus is achieved asymptotically with a time-varying leader, i.e., $x_i(t) - x^r(t) \to 0$ as $t \to \infty$, for all $i = 1, \cdots, M$.*

Proof: Based on (36) and (37), the closed-loop system dynamics are given by

$$\dot{x}_{ip} = -\sum_{j\in N_i} (\rho_{ij}\tilde{x}_{ij}) + \eta_{ip}(x_i,t) + \tilde{\theta}_{ip}^T\, g_{ip}^s(x_i)$$
$$-(\bar{\eta}_{ip} + \hat{\kappa}_{ip}) sgn\bigg(\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}\bigg)\,.$$

We can represent the collective output dynamics as

$$\dot{x}^p = -\hat{\mathcal{L}}x^p + \zeta^p - \bar{\zeta}^p + \tilde{f}^{sp}\,, \qquad (40)$$

where $x^p \in \Re^{M+1}$, $p = 1, \cdots, n$, is comprised of the pth component of the M agents and the leader as the $(M+1)$th agent, i.e., $x^p = [\,x_{1p}, x_{2p}, \cdots, x_{Mp}, x_p^r\,]^T$, the terms $\zeta^p \in \Re^{M+1}$ and $\bar{\zeta}^p \in \Re^{M+1}$ are defined in (22) and (23), and $\tilde{f}^{sp} \triangleq [\,\tilde{f}_{1p}^s, \cdots, \tilde{f}_{Mp}^s, 0\,]^T$. Note that $\tilde{f}_{ip}^s \triangleq (\tilde{\theta}_{ip})^T g_{ip}^s$, where $\tilde{\theta}_{ip} \triangleq \theta_{ip} - \hat{\theta}_{ip}$ and g_{ip}^s are the parameter estimation error and process fault functions corresponding to the pth component of the ith agent, respectively.

We consider the following Lyapunov function candidate:

$$V_p = x^{pT}\bar{\Psi}x^p + \tilde{\theta}^{pT}(\Gamma^p)^{-1}\tilde{\theta}^p + \tilde{\kappa}^{pT}(\bar{\Upsilon}^p)^{-1}\tilde{\kappa}^p\,, \qquad (41)$$

where $\bar{\Psi}$ is defined in (8), $\tilde{\theta}^p = [\,\tilde{\theta}_{1p}^T, \cdots, \tilde{\theta}_{Mp}^T\,]^T$ is the collective parameter estimation errors, $\tilde{\kappa}^p = [\,\tilde{\kappa}_{1p}^T, \cdots, \tilde{\kappa}_{Mp}^T\,]^T$ is the collective bounding parameter estimation errors, and $\Gamma^p = \mathrm{diag}\{\Gamma_{1p}, \cdots, \Gamma_{Mp}\}$ and $\bar{\Upsilon}^p = \mathrm{diag}\{\bar{\Upsilon}_{1p}, \cdots, \bar{\Upsilon}_{Mp}\}$ are positive definite adaptive learning rate matrices. Then, the time derivative of the Lyapunov function (41) along the solution of (40) is given by

$$\dot{V}_p = -x^{pT}\bar{\mathcal{L}}\,x^p + 2\sum_{i=1}^{M}\bigg[-\tilde{\kappa}_{ip}(\bar{\Upsilon}_{ip})^{-1}\dot{\hat{\kappa}}_{ip}$$
$$+\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}\bigg(\eta_{ip} - \dot{x}_p^r - (\bar{\eta}_{ip} + \hat{\kappa}_p) sgn\big(\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}\big)\bigg)$$
$$+\tilde{\theta}_{ip}^T\bigg(\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}g_{ip}^s - (\Gamma_{ip})^{-1}\dot{\hat{\theta}}_{ip}\bigg)\bigg]\,,$$

where $\bar{\mathcal{L}}$ is defined in (9). By choosing the adaptive laws as (38) and (39), we have $\dot{V}_p \le -x^{pT}\bar{\mathcal{L}}\,x^p$. Therefore, the proof can be concluded based on the similar reasoning logic as reported in the proof of Theorem 1. □

4.2.2 Adaptive FTC of Actuator Faults

In the case of an actuator fault, i.e., $t \ge T_{isol}$, the dynamics of the system takes on the following form: for $p = 1, \cdots, n$,

$$\dot{x}_{ip} = \phi_{ip} + (1+\theta_{ip})u_{ip} + \eta_{ip}(x_i, t)\,. \tag{42}$$

The following adaptive FTC scheme is adopted:

$$u_{ip} = \frac{1}{1+\hat{\theta}_{ip}}\bar{u}_{ip}\,, \tag{43}$$

$$\dot{\hat{\theta}}_{ip} = \mathcal{P}_{\bar{\theta}_{ip}}\Big\{\bar{\Gamma}_{ip}\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}u_{ip}\Big\}\,, \tag{44}$$

$$\dot{\hat{\kappa}}_{ip} = \bar{\Upsilon}_{ip}\,\Big|\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}\Big|\,, \tag{45}$$

where $\bar{u}_{ip} \triangleq -\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij} - (\bar{\eta}_{ip}+\hat{\kappa}_{ip})sgn\big(\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}\big) - \phi_{ip}$, the controller gains ρ_{ij} are defined in (20), $\hat{\theta}_{ip}$ is an estimation of the unknown actuator fault magnitude parameter θ_{ip} with the projection operator $\mathcal{P}$ restricting $\hat{\theta}_{ip}$ to the corresponding set $[\bar{\theta}_{ip}, 0]$, $\hat{\kappa}_{ip}$ is an estimation of the unknown positive constant bound κ_p on $|\dot{x}_p^r|$ (see Assumption 3), and $\bar{\Gamma}_{ip}$ and $\bar{\Upsilon}_{ip}$ are positive learning rate constants. Then, we have the following:

Theorem 3 *Assume that an actuator fault occurs at time T_{iu} and that it is isolated at time T_{isol}. Then, the fault-tolerant controller (43), fault parameter adaptive law (44), and the bounding parameter adaptive law (45) using the distributed gains (20) guarantee that the leader-follower consensus is achieved asymptotically with a time-varying leader, i.e., $x_i(t) - x^r(t) \to 0$ as $t \to \infty$, for all $i = 1, \cdots, M$.*

Proof: Using some algebraic manipulations, we can rewrite (43) as $u_{ip} = \bar{u}_{ip} - \hat{\theta}_{ip}u_{ip}$. By substituting u_{ip} into (42), the closed-loop system dynamics are given by

$$\dot{x}_{ip} = -\sum_{j\in N_i}(\rho_{ij}\tilde{x}_{ij}) + \eta_{ip}(x_i,t) + \tilde{\theta}_{ip}u_{ip} - (\bar{\eta}_{ip}+\hat{\kappa}_{ip})sgn\bigg(\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}\bigg)\,.$$

We can represent the collective output dynamics as

$$\dot{x}^p = -\hat{\mathcal{L}}x^p + \zeta^p - \bar{\zeta}^p + \varpi^p\,, \tag{46}$$

where $x^p \in \Re^{M+1}$, $p = 1, \cdots, n$, is comprised of the pth component of the M agents and the leader as the $(M+1)$th agent, i.e., $x^p = [\,x_{1p}, x_{2p}, \cdots, x_{Mp}, x_p^r\,]^T$, and the terms $\zeta^p \in \Re^{M+1}$, $\bar{\zeta}^p \in \Re^{M+1}$ and $\varpi^p \in \Re^{M+1}$ are defined in (22), (23) and (27), respectively.

Consider the following Lyapunov function candidate:

$$V_p = x^{pT}\bar{\Psi}x^p + \tilde{\theta}^{pT}(\bar{\Gamma}^p)^{-1}\tilde{\theta}^p + \tilde{\kappa}^{pT}(\bar{\Upsilon}^p)^{-1}\tilde{\kappa}^p\,, \tag{47}$$

where $\bar{\Psi}$ is defined in (8), $\tilde{\theta}^p = [\,\tilde{\theta}_{1p}, \cdots, \tilde{\theta}_{Mp}]^T$ is the collective actuator fault magnitude parameter estimation errors, $\tilde{\kappa}^p = [\,\tilde{\kappa}_{1p}, \cdots, \tilde{\kappa}_{Mp}]^T$ is the collective bounding parameter estimation errors, and $\bar{\Gamma}^p = \mathrm{diag}\{\bar{\Gamma}_{1p}, \cdots, \bar{\Gamma}_{Mp}\}$ and $\bar{\Upsilon}^p = \mathrm{diag}\{\bar{\Upsilon}_{1p}, \cdots, \bar{\Upsilon}_{Mp}\}$ are positive definite adaptive learning rate matrices. Then, using the same reasoning logic reported in the proof of Theorem 1, the time derivative of the Lyapunov function (47) along the solution of (46) is given by

$$\begin{aligned}\dot{V}_p = &-x^{pT}\bar{\mathcal{L}}\,x^p + 2\sum_{i=1}^{M}\bigg[-\tilde{\kappa}_{ip}(\bar{\Upsilon}_{ip})^{-1}\dot{\hat{\kappa}}_{ip}\\ &+\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}\bigg(\eta_{ip} - \dot{x}_p^r - (\bar{\eta}_{ip}+\hat{\kappa}_p)sgn\big(\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}\big)\bigg)\\ &+\tilde{\theta}_{ip}\bigg(\sum_{j\in N_i}\rho_{ij}\tilde{x}_{ij}u_{ip} - (\bar{\Gamma}_{ip})^{-1}\dot{\hat{\theta}}_{ip}\bigg)\bigg]\,,\end{aligned}$$

where $\bar{\mathcal{L}}$ is defined in (9). By choosing the adaptive laws as (44) and (45), we have $\dot{V}_p \le -x^{pT}\bar{\mathcal{L}}\,x^p$. Then, the proof can be concluded by using the same reasoning logic as reported in the analysis of Theorem 1. □

5 Simulation Results

In this section, a simulation example of a team of 5 agents is considered to illustrate the effectiveness of the distributed fault-tolerant control method. The dynamics of each agent, $i = 1, \cdots, 5$, is given by

$$\dot{x}_i = u_i + \eta_i + \beta_i(t - T_{if})f_i(x_i) + \beta_i(t - T_{iu})\theta_i u_i\,, \tag{48}$$

where $x_i = [x_{i1}, x_{i2}]^T$ and $u_i = [\bar{\nu}_i cos(\bar{\psi}_i), \bar{\nu}_i sin(\bar{\psi}_i)]^T$ are the state and input vector of ith agent, respectively, $\bar{\psi}_i$ and $\bar{\nu}_i$ in the input vector u_i are the orientation and the linear velocity of each agent representing a ground vehicle. The ground vehicle model given in (48) is a standard unicycle-like model that can be controlled with the orientation $\bar{\psi}_i$ and vehicle linear velocity $\bar{\nu}_i$. Using the developed algorithms, the desired orientation and linear velocity of the ground vehicle can be obtained uniquely. Then, a low-level controller can be designed to track the desired orientation and linear velocity for driving the ground vehicles to desired positions. The unknown modeling uncertainty in the local dynamics of the agents are assumed to be sinusoidal signals $\eta_i = [0.5sin(t), 0.5sin(t)]^T$ bounded by $\bar{\eta}_i = [0.6, 0.6]^T$.

The intercommunication graph of agents plus leader is shown in Figure 1. As can be seen, the leader only communicates with agent 2. It can be shown that the detail-balanced condition described in (Chen et al., 2011; Zhang et al., 2013b) is not satisfied. We choose $\bar{k}_2 = 0.5$. Then, the left eigenvector of $\bar{\Psi}$ associated with the zero eigenvalue is $\bar{q} = [0.142, 0.212, 0.402, 0.521, 0.566, 0.425]^T$. The objective is for each agent to follow the leader's position described by $x^r = [x_1^r, x_2^r]^T = [5 + sin(t), 5 + cos(t)]^T$.

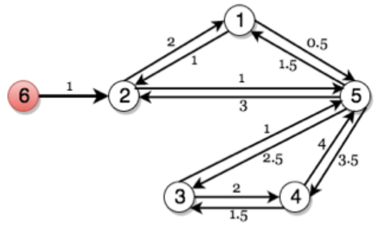

Fig. 1. Communication graph

Before the fault is detected in the ith agent, the following nominal controller is employed which guarantees the leader-follower consensus in the absence of faults.

$$u_{ip} = -\sum_{j\in N_i} \rho_{ij}\tilde{x}_{ij} - \phi_{ip} - (\bar{\eta}_{ip} + \hat{\kappa}_{ip})\, sgn\Big(\sum_{j\in N_i} \rho_{ij}\tilde{x}_{ij}\Big)$$

$$\dot{\hat{\kappa}}_{ip} = \bar{\Upsilon}_{ip} \,\Big|\sum_{j\in N_i} \rho_{ij}\tilde{x}_{ij}\Big|\,.$$

Note that the baseline controller is a special case of the adaptive FTC described in Section 3. The fault class under consideration is defined as follows

(1) A process fault described by $f_i^1 = \theta_i^1 g_i^1$, where $g_i^1 = x_i^2 cos(x_i)$, and the fault magnitude $\theta_i^1 \in [0, 1]$.
(2) An actuator fault described by $f_i^2 = \theta_i^2 g_i^2$, where $g_i^2 = u_i$, and the fault magnitude $\theta_i^2 \in [-0.8, 0]$.

A radial basis function (RBF) neural network (see (5)) used for approximation of the process fault function consists of 21 neurons. The center of RBFs are equally distributed on the interval $[-10, 10]$ with a variance of 0.5. We set the learning rate as $\Gamma_i = 10$ and consider a constant bound on the network approximation error, i.e., $\bar{\delta}_i = 1$. The adaptive gains in (15), (16) and (17) are chosen as $\Upsilon_i = 3$, $\bar{\Upsilon} = 1$ and $\bar{\Gamma}_i = 1$, respectively. After fault isolation, the controller is reconfigured to accommodate the specific fault that has been isolated. We set the adaptive gain $\Gamma_i = 10$ with a zero initial condition (see (38)).

The first fault type (i.e., $f_1^1 = \theta_1^1 g_1^1$) with a magnitude of 0.3 occurs to agent 1 at $T_i = 5$ second and agent 3 at $T_i = 10$ second, respectively. The fault detection and isolation results are omitted here due to space limitation. Regarding the performance of the adaptive fault-tolerant controllers, as can be seen from Figure 2, the leader-following consensus is achieved using the proposed adaptive FTC scheme, while the agents cannot follow the leader without the FTC controllers after fault occurrence (see Figure 3). Thus, the benefits of the FTC method can be clearly seen. The tracking error of the agents is shown in Figure 4, when the second adaptive fault tolerant controller is not exploited. Compared with Figure 2, it can be seen that the second FTC designed with simpler structure and less assumptions provides better tracking performance.

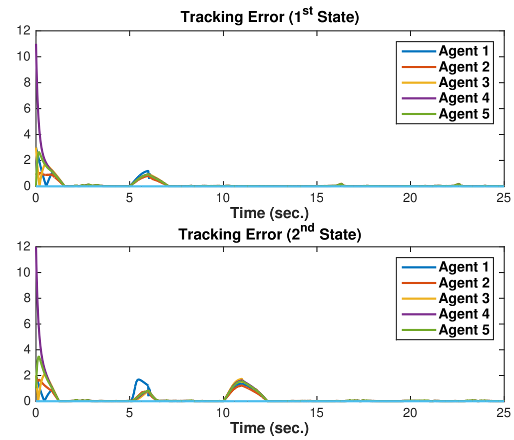

Fig. 2. Tracking errors with both FTCs

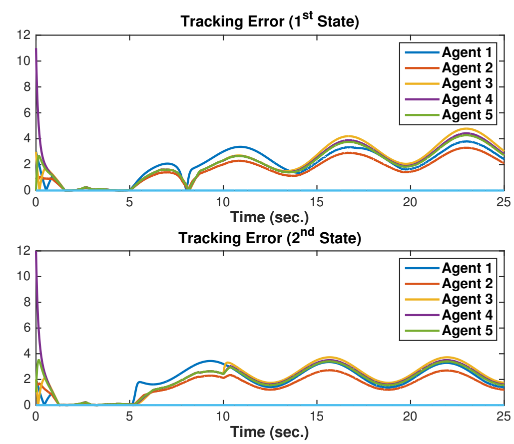

Fig. 3. Tracking errors without the adaptive FTC algorithm

6 Conclusion

Distributed integrated FTC and FDI design for a class of nonlinear uncertain multi-agent systems was investigated in this paper. Under certain assumptions, asymptotic leader-follower consensus properties were rigorously established in the presence of faults and modeling uncertainty. The extension to high-order systems (e.g., the class of systems in output feedback form considered by Wang et al. (2017)) and directed graphs (see, for instance, Wang et al. (2014); Ding and Li (2016)) are interesting topics for future research.

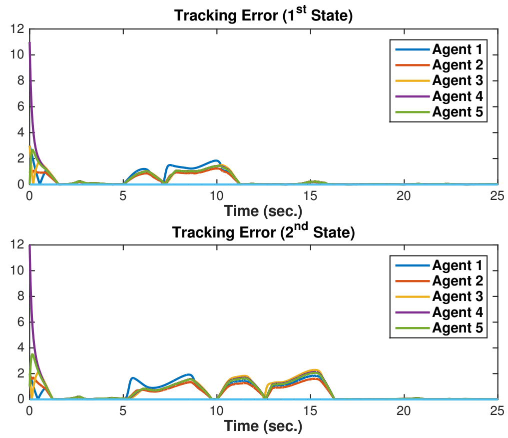

Fig. 4. Tracking errors without the second adaptive FTC

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