

# FRP STRUCTURE DESIGN METHOD BASED ON THE STIFFNESS EQUIVALENCE: CASE STUDY AND PRACTICE

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## Abstract:

Nowadays, fiber reinforced polymer materials are getting widely used in civil engineering. They have some advanced engineering characteristics like high stiffness-to-weight ratio, high strength-to-weight ratio, anti-electromagnetic interference, electrical isolation and their long term durability. Metal and concrete, two main kinds of construction materials, are prone to failure during their structure service lives because of environmental adverse effects and atmospheric weathering. Those harsh attacks/impacts are oxidization, acidification, high temperature and alkali corrosion. After various maintenance methods like isolating the attacks or protecting plastic skins, an interface crack between protections can easily propagate and cause some durability concern. Fiber reinforced polymer emerges in a new era of civil engineering and can surely be added to traditional materials and/or traditional materials can surely be substituted by this material. This paper sets out to unfold a case study on how fiber reinforced polymer works in civil structures.

## 1 Introduction

Long span bridges were widely built in China nearly two decades ago. With the increased bridge span length, heavy self-weight can be controlled in accordance with the load. Advanced composite materials (ACM) used for airplanes and satellites like CFRP (Carbon Fiber Reinforced Polymer) seems to be too expensive for civil engineering applications [1-3]. However, there exists another option, GFRP (Glass Fiber Reinforced Polymer), which is much cheaper and has been very extensively used in engineering constructions in China since 1980s. This paper shows how to design

FRP structures and presents a brief case of a detailed design work.

## 2 Design process

### 2.1 Basic theory

The stiffness parameter is a critical factor that matters more than strength in FRP structures [4]. ACI committee in the U. S. and AASHTO in Canada set 1/800 span length as the control thread, and after considering many other impact factors, GFRP was designed without exceeding the strength of 221MPa.

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The focus of this case study is on designing a bridge model on the basis of a real cable - stayed bridge in China. The total span length of this bridge is 345 meters, the main span is 180 meters long, and the assistant spans are 90 and 75 meters long. Apart from steel decks and steel cables, it has also a pre-

stressed steel reinforced concrete tower to sustain the two-way sprayed cable force. By exchanging steel and concrete with FRP, advantages can clearly be seen even in physical behavior (Table 1).

Table 1. General physical target of FRP and steel

Types	Density	Thermal Coefficient	Tensile strength	Young's modulus	Fracture strain
	g/cm <sup>3</sup>	10 <sup>-6</sup> /K	MPa	GPa	%
Steel	7.9	11.7	483 ~ 690	200	6 ~ 12
GFRP	1.3 ~ 2.1	6.0 ~ 10.0	483 ~ 1600	35 ~ 51	1.2 ~ 3.1
CFRP	1.5 ~ 1.6	-9.0 ~ 0.0	600 ~ 3690	120 ~ 580	0.5 ~ 1.7
AFRP	1.3 ~ 1.4	-6.0 ~ -2.0	1720 ~ 2540	41 ~ 125	1.9 ~ 4.4



Figure 1. A real bridge in China

According to the basic similarity theory of physical parameters provided by E. Buckingham in 1914, the physical meaning of all the parameters for one complete multi-parameter function (1) could be illustrated by non-dimensional equations (2):

$$F(X_1, X_2, X_3, \dots, X_n) = 0 \tag{1}$$

$$G(\pi_1, \pi_2, \pi_3, \dots, \pi_m) = 0 \tag{2}$$

where  $\pi_1, \pi_2, \pi_3, \dots, \pi_m$  are the non-dimension parameter products ( $X_1, X_2, X_3, \dots, X_n$ ).

A transformation or solution could be made by applying the equation (3):

$$\pi_1 = \phi(\pi_2, \pi_3, \dots, \pi_m) \tag{3}$$

The equivalence of the similarity calculation is based on this famous function, so that  $m$  stands for a design model, and  $p$  stands for an original model:

$$\frac{\pi_{1m}}{\pi_{1p}} = \frac{\phi(\pi_{2m}, \pi_{3m}, \dots, \pi_{nm})}{\phi(\pi_{2p}, \pi_{3p}, \dots, \pi_{np})} = 1 \tag{4}$$

Equation (4) is set to indicate equivalence of single/particular physical parameters, since structure stiffness is a complex physical parameter. Therefore, research into and an improvement of a current concept of similarity calculation should be made.

Equation (5) is a transformation of equation (4):

$$\begin{aligned} \pi_{1m} &= \pi_{1p} = \pi_1 \\ \pi_{2m} &= \pi_{2p} = \pi_2 \\ &\dots \\ \pi_{nm} &= \pi_{np} = \pi_n \end{aligned} \tag{5}$$

A complex physical parameter should be transformed under power closure / when power is closed using a multiscale product method Put another way, it has a mathematical form as follows:

$$\Omega_k = \prod_{i=1}^n (r_i \pi_i)^{L_i} \tag{6}$$

Where  $i$  stands for the parameter number,  $r_i$  is the coefficient of the product, and  $L_i$  is the power coefficient.  $K$  is a sequence number of the complex physical parameter. Subsequently, the criterion of similarity for complex physical parameter is formed in equation (7):

$$\Omega_{mK} = \Omega_{pK} = \Omega_K \tag{7}$$

## 2.2 Profile and cross section design

Table 2. General information on material

Components	Original model material	Young's modulus	Scaled model material	Young's modulus	$\lambda_E$
	N/A	MPa	N/A	MPa	1
Foundation	C30	$0.3 \times 10^5$	C30	$0.3 \times 10^5$	1/1
Tower	C50	$0.345 \times 10^5$	C50	$0.345 \times 10^5$	1/1
Beam	Q345	$2.06 \times 10^5$	GFRP	$0.3 \times 10^5$	7/1
Cable	High strength wire	$2.05 \times 10^5$	GFRP	$0.5 \times 10^5$	4/1

The design process is conducted according to the similarity factor of material  $\lambda_E$ , and calculated by Yong's modulus's division.

Mechanical behavior of the bridge is different for different components so that different main physical parameters are chosen according to their application. Details of design criteria are given:

- Strict similarity of the length scale concept for bridge span/ tower height/ cable length/;
- Stiffness equality section based on mechanical behavior of the structure components. For instance, bending structure using bending stiffness and tension structures using tensile stiffness etc.
- Since the gravitational field cannot be scaled down, and material density cannot be designed for different components, it follows that gravity compensation should be considered in the design.

With reference to structure model design, a scale factor can be chosen between 1 and 1/50. Having considered economic conditions and construction advantages, 1/45 was chosen for this purpose. Afterwards, the total length of the bridge was changed into 7.67 meters in span and 2.96 meters in height.

Main span length:  $0.4+11 \times 0.27+0.67=4.0$  m;

Assistant span length:  $1.63+11 \times 0.07+1.3=3.67$ m.

While the cable interval at the tower side is 0.088m, the angle of the tower is  $80^\circ$ , and the inner reinforcement of the tower is GFRP rebar.

It is worth noting that during building the tower and foundation, it was not necessary to change the original material:

$$\lambda_E^{(1)} = \lambda_E^{(2)} = 1 \tag{8}$$

whereas, for the beam and cables, materials were changed:

$$\lambda_E^{(3)} = \frac{E_{GF}}{E_{Q345}} = \frac{1}{7} \tag{9}$$

$$\lambda_E^{(4)} = \frac{E_{GF}}{E_{WIRE}} = \frac{1}{4} \tag{10}$$

The following terms mean :

- $\lambda_E^{(1)}$  : Young's modulus similarity constant in the tower;
- $\lambda_E^{(2)}$  : Young's modulus similarity constant in the foundation;
- $\lambda_E^{(3)}$  : Young's modulus similarity constant in the beam;
- $\lambda_E^{(4)}$  : Young's modulus similarity constant in the cables.

Given the fact that there are three control sections for the tower, there is a difference between two of them. The first cross section is NO.1 in Fig. 2 and it stands for m-m, NO.2 stands for j-j and NO.3 stands for o-o. As these three sections have critically changed the section, they are chosen in this analysis. In contrast to the m-m section, a theoretically scaled section is different from the real designed section (Fig. 3, Fig. 4). Another point is that the errors for different design results are minimally controlled less than 5% (Table 3).

The other two control sections are designed in the same sequence, and the errors are represented in Table 4 and Table 5.

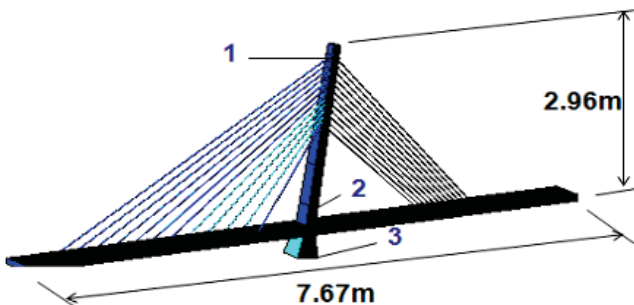


Figure 2. A theoretically scaled model

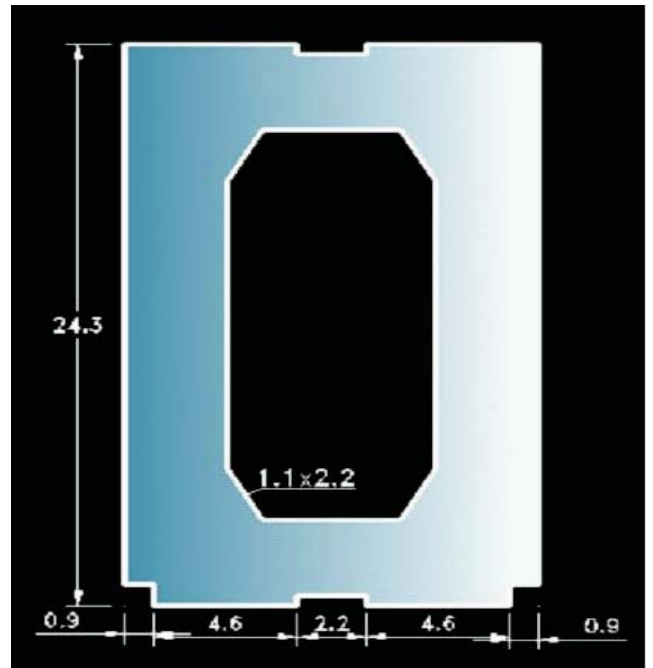


Figure 3. A theoretically scaled section m-m

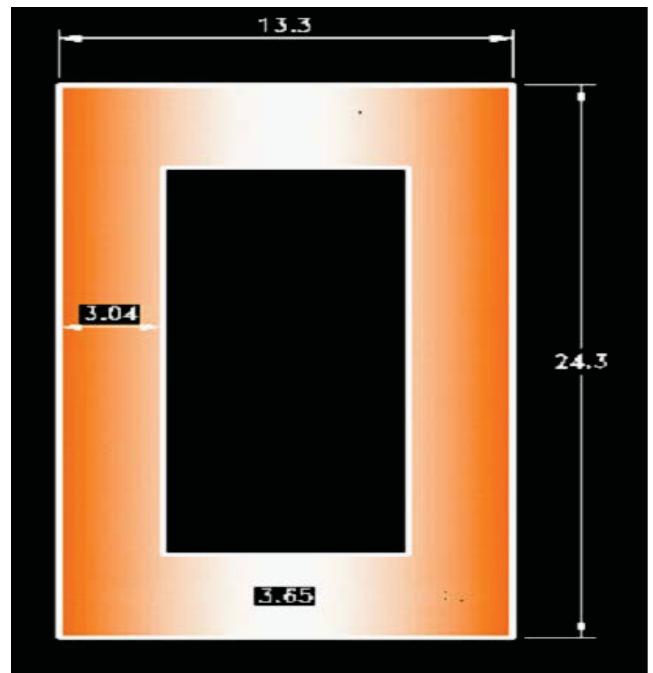


Figure 4. A real scaled section m-m

The equivalence/comparison of bending stiffness should be made provided that FRP is substituted with steel. The cross section of the original bridge deck was very complicated; the reinforced strips in the shape “I” or “U” cannot be ignored but should be considered in compensation (Fig. 7, Fig. 8, and Table 6).

Table 3. Errors classified for cross section m-m

m-m	A	$I_x$	$I_y$
	$m^2$	$m^4$	$m^4$
Theory value	2.1E-02	1.2E-04	4.2E-05
Real value	2.0E-02	1.2E-04	4.2E-05
Theory Scale	1/2025	1/4100625	1/4100625
Real Scale	1/2128	1/4099437	1/4102099
Error	-4.86%	0.03%	-0.04%

Table 4. Errors classified for the cross section j-j

j-j	A	$I_x$	$I_y$
	$m^2$	$m^4$	$m^4$
Theory value	8.64E-03	1.67E-05	7.77E-06
Real value	8.39E-03	1.70E-05	7.76E-06
Theory Scale	1/2025	1/4100625	1/4100625
Real Scale	1/2086	1/4021490	1/4103272
Error	-2.93%	1.97%	-0.06%

Table 5. Errors classified for the cross section o-o

o-o	A	$I_x$	$I_y$
	$m^2$	$m^4$	$m^4$
Theory value	2.1E-02	1.2E-04	4.2E-05
Real value	2.0E-02	1.2E-04	4.2E-05
Theory Scale	1/2025	1/4100625	1/4100625
Real Scale	1/2128	1/4099437	1/4102099
Error	-4.86%	0.03%	-0.04%

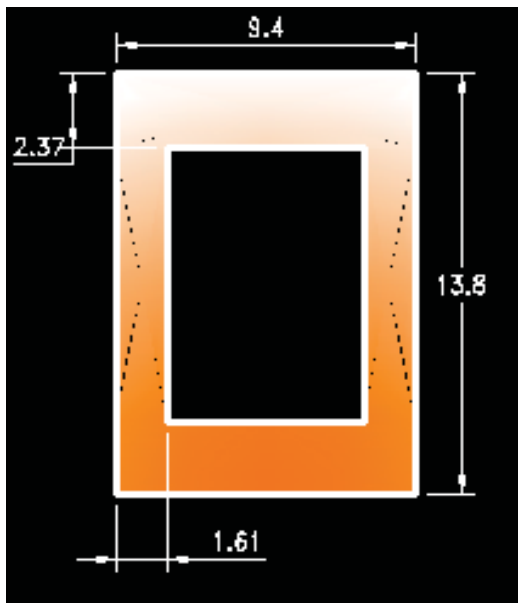


Figure 5. The real scaled section j-j

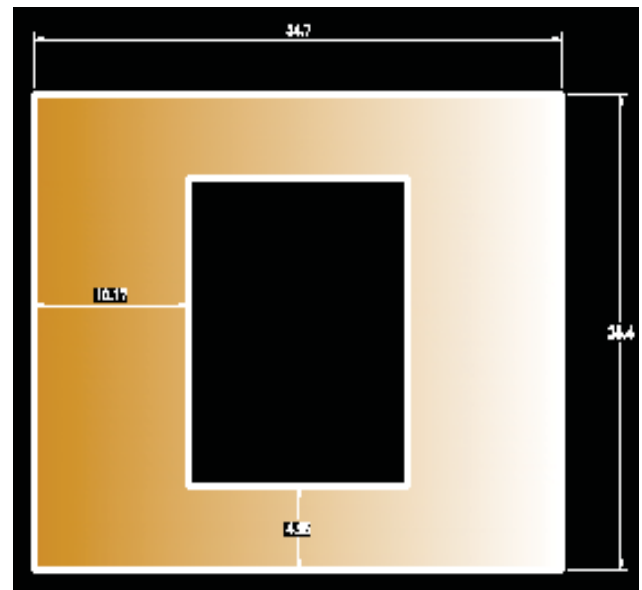


Figure 6. The real scaled section o-o

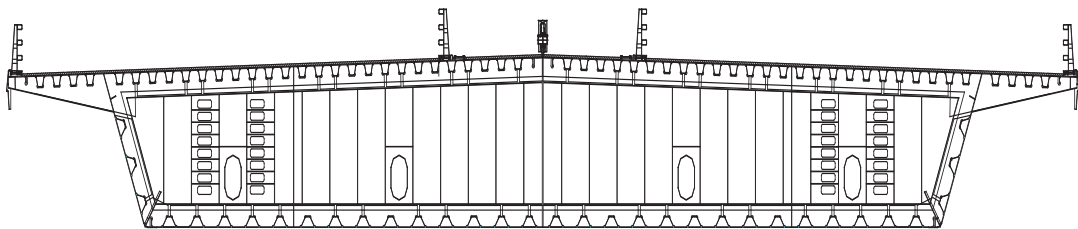


Figure 7. An original steel bridge deck

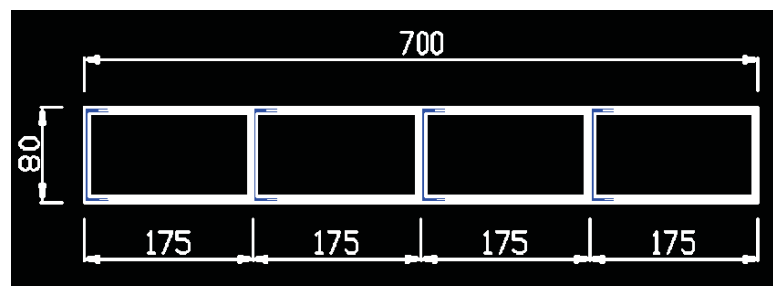


Figure 8. A scaled FRP bridge deck

### 3 Construction of FRP Bridge in Site

Construction stages are divided into five parts: first, the fundamental location survey and foundation handling; second, concrete pre-casting for bases,

piers and towers; next, Pultrusion of FRP decks and cables in the factory (Fig. 9, Fig. 10 and Fig. 11); finally, assembling all the components and shaping a full view of the bridge (Fig. 11).

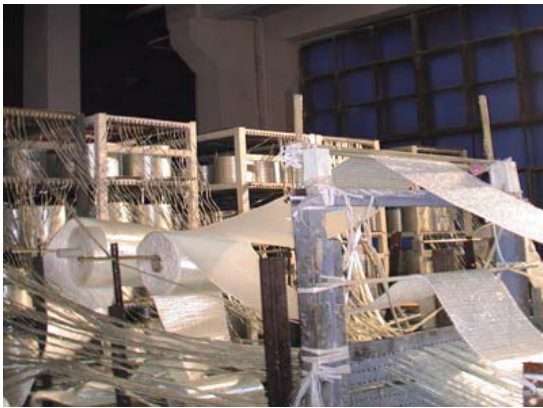


Figure 9. FRP deck pultrusion-fiber converged



Figure 10. FRP deck pultrusion-Pull out



Figure 11. FRP cable anchorage



Figure 12. A constructed FRP bridge

#### 4 Conclusion

FRP structure design is still a novel subject for researchers in the world. The equivalence of stiffness design methods has some advantages as follows:

- 1) Abundant residual strength will maintain the structure at high safety stage;
- 2) Light weight components of the structure can make the construction easier and the construction period shorter;
- 3) Maintenance fees will be further reduced due to the FRP's potential durability performances;
- 4) As there is no carbon circling during the entire design-construction-service life, it could be considered as an efficient way of energy conservation building. .

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