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Fault Detection and Diagnosis for Non-Gaussian Singular Stochastic Distribution Systems via Output PDFs

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Original scientific paper

This paper investigates the problem of fault detection and diagnosis (FDD) problem for non-Gaussian singular stochastic distribution control (SDC) systems via the output probability density functions(PDFs). The PDFs can be approximated by using square-root B-spline expansion, via this expansions to represent the dynamics weighting systems between the system input and the weights related to the output PDFs. In this work, an optimal fault detection and diagnosis algorithm is presented by introducing the parameter-updating. When the fault occurs, an adaptive network parameter-updating law is designed to approximated the fault. Finally, the simulation result are given to show that the approach can detect fault and estimate the size of fault.

Key words: Probability density fuctions, Non-gausian singular stochastic distribution control, Fault detection and diagnosis, Adaptive network parameter-updating

Detekcija pogreške i dijagnostika za singularne stohastičke sustave s ne-Gaussovom razdiobom preko izlazne funkcije gustoće razdiobe. U ovome radu prikazan je problem detekcije pogreške i dijagnostike (FDD), za singularne stohastičke sustave upravljanja koji nemaju normalnu razdiobu, preko izlazne gustoće vjerojatnosti, koja se može aproksimirati koristeći *square-root B-spline* ekspanziju. Prikazan je optimalni algoritam za detekciju i dijagnostiku grešaka uvodeći postupak izmjene parametara. U trenutku pojave greške, adaptivna mreža za izmjenu parametara je dizajnirana u svrhu aproksimacije nastale pogreške. U radu su dani simulacijski rezultati koji prikazuju ispravnost ovakvog načina detekcije pogreške i procjene njezine veličine.

Ključne riječi: funkcija gustoće vjerojatnosti, upravljanje sustavima sa stohastičkom razdiobom koja nije normalna, detekcija pogreške i dijagnostika, adaptivna mreža podešavanja parametara

1 INTRODUCTION

Fault detection and diagnosis (FDD) are important area of research in recent years from the viewpoint of improving control systems reliability. In the past two decades many effective methods have been presented for various types of system faults([1]-[5]). For stochastic system,the two kinds of approaches include the system identification techniques[6] and the statistic approaches based on the Likelihood methods, Bayesian theorem,and Hypothesis test techniques ([1]) can be used to deal with the related FDD problems. Besides, we have known that observers or filters have been extensively applied to generate the residual signal for fault detection and diagnosis([7],[8]), and many significant approaches of them have been applied to practical processes successfully([9],[10]).

Up to now, most of the existing observer-based or filter-based FDD algorithms have only been concerned with those stochastic system subjected to Gaussian distribution. However, nonlinearity may lead to non-Gaussian output,where (especially for asymmetric distributions with multiple peaks) mean and variance of the system output are insufficient to characterize their statistical behavior precisely ([11],[12],[13]). As such, there is need to further develop fault detection and diagnosis algorithms that can be applied to the stochastic system subjected to random parameter.On the other hand,along with the development of advance instruments and data processing technique, the measurements for feedback are the stochastic information which can be described by the probability density functions(PDFs) of the stochastic distribution system output rather than the actual output values.For such non-Gaussian stochastic system, we call stochastic distribution control(SDC) systems ([3],[7],[10]-[16]).Different from any other previous stochastic control approaches,the stochastic variables are not confined to be Gaussian and the output PDFs of the stochastic system is concerned([13]-[18]).while the control objective of SDC systems is to control the shape of the output PDFs, the aim of FDD in SDC system is to use the shape of the measured input and output PDFs to obtain information of the fault.

It is noted that if only output PDFs can be measured rather than the output vector itself,most existing observer-based FDD approaches are invalid. As a result, an observer-based fault detection and diagnosis approaches have been developed in reference([3],[7],[11],[14],[15],[16],and reference therein) to detect and diagnose the fault in the non-Gaussian SDC system. However, concerning the FDD problem, few literatures have been reported to the singular non-gaussian stochastic distribution control systems. This forms the main purpose of the current work in this paper.

In this paper, this study extends the results established in reference [17-18]. The problem of FDD for non-Gaussian singular stochastic systems is studied. Sufficient conditions for the stability of the error system and the FDD are presented by using LMI. At last, paper-making process example is given to demonstrate efficiency of proposed approach.

Notations: * denotes the elements below the main diagonal of a symmetric block. I is the identity matrix with appropriate dimensions. ||·|| refers to the induced matrix 2-norm of a given vector . diag{. . . } stands for the block diagonal matrix. A^{-1} represents the inverse of matrix A. tr (A) denotes the trace of matrix A. A^T refers to the transpose of vector x or matrix A.

2 PROBLEM FORMULATION AND PRELIMINARIES

Consider a dynamic stochastic system where $u(t) \in R^m$ is the control input, $z \in [a,b]$ represents the system output,and F(t) stands for the fault to be detected and diagnosed. The output PDFs is denoted by γ(z,u(t),F(t)), which is conditional under the occurrence and influence of the known control input and the unknown fault.The objective in the FDD context is to use the informance of γ(z,u(t),F(t)) to design an observer such that the fault F(t) can be detected and diagnosed.

As shown in reference [14] and [15], the output PDFs γ(z,u(t),F(t)) can be approximated by using square root B-spline expansions as the following form:

$$\sqrt{\gamma(z,u(t),F)} = \sum_{i=1}^{n} v_i(u(t),F)b_i(z) + \omega_0(z,u(t),F) \tag{1}$$

where b_i(z)(i=1,2,. . . ,n) are a pre-specified basis function defined on [a,b], v_i(u(t),F)(i=1,2,. . . ,n) are the corresponding weights of such an expansion, and ω_0(z,u(t),F) stands for the model uncertainty or the error on the approximation of PDFs, which is supposed to satisfy $|\omega_0$(z,u(t),F) $| \leq \delta_0$, δ_0 is assumed to be known positive constant. Denote

$$\begin{aligned} B_0(z) =&[b_1(z) \quad b_2(z)\cdots b_{n-1}(z)] \\ V(t) =&V(u(t),F) \\ =&[v_1(u(t),F) \quad v_2(u(t),F)\cdots v_{n-1}(u(t),F)]^T \end{aligned}$$

and

$$\Lambda_1 = \int_a^b B^T(z)B(z)dz, \Lambda_2 = \int_a^b B^T(z)b_n(z)dz, \Lambda_3 = \int_a^b b_n^2(z)dz \neq 0, \Lambda_0 = \Lambda_1\Lambda_3 - \Lambda_2^T\Lambda_2$$

In this paper, similar to references [14]and [15], we adopt the following model:

$$\begin{aligned} \sqrt{\gamma(z,u(t),F)} =&B(z)V(t) + h(V(t))bn(z) \\ &+ \omega(z,u(t),F) \end{aligned} \tag{2}$$

where

$$\begin{aligned} B(z) &= B_0(z) - \frac{\Lambda_2}{\Lambda_3}b_n(z) \\ h(V(t)) &= \frac{1}{\Lambda_3}[-\Lambda_2V(t) + \sqrt{\Lambda_2(t) - V^T(t)\Lambda_0V(t)}] \end{aligned} \tag{3}$$

From the boundedness of ω_0(z,u(t),F) and reference [15], it can be assumed that $|\omega$ (z,u(t),F) $| \leq \delta$ holds for all {z,u(t),F},where δ is also a known positive constant.

Once square root B-spline expanxions have been made for the PDFs, the next step is to find the dynamic relationship between the control input and weights related to the PDFs corresponds to a further modeling. As such,in this paper the following nonlinear dynamic model will be considered between V(t) and u(t) :

$$\left\{ \begin{array}{ll} E\dot{x}(t) &= Ax(t) + Gg(x(t)) + Du(t) + F \\ V(t) &= Cx(t) \end{array} \right. \tag{4}$$

where x(t)$\in R^m$ is the unmeasured state, and A,G,D,C represent the known parametric matrices of the dynamic part of the weight system, In fact, these matrices can be obtained either by physical modeling or the scaling estimation technique described in [1] and [6]; E$\in R^{m\times m}$ is a known singular matrix, i.e., rank(E)= r < m; g(x(t))$\in$Rm is a nonlinear vector function that represents the nonlinear dynamics of the weight model and is supposed to satisfy g(0)=0.

The following network can be used to approximate the continuous unknown function F(t):=F(x,u)

$$F(x,u) = TWS(x,u) + \theta(x,u) \tag{5}$$

where T is given matrix,W is the ideal weight matrix, θ(x,u) is a neural network approximation error, S(x,u) is the basis function. Since the state x is immeasurable, then the output of neural network can be expressed as

$$\hat{F}(\hat{x}, u) = T\hat{W}S(\hat{x}, u) \tag{6}$$

where $\hat{x}(t)$ is the estimated state, $\hat{W}_1$ is an estimated matrix

In addition, similar to [7] and [16],In the rest of this paper, the following assumptions are needed.

Assumption 1. For any x_1(t) and x_2(t),g(x(t)) satisfies the following condition:

$$\begin{array}{ll} & \|g(x_1(t)) - g(x_2(t))\| \le \|U_1(x_1(t) - x_2(t))\| \\ or & \|g(x_1(t)) - g(x_2(t))\| \le \delta_1 \|(x_1(t) - x_2(t))\| \end{array}$$

where U_1 is a known matrix, δ_1 is a known positive constant, $||\cdot||$is denoted as the Euclidean norm.

Assumption 2. For any V_1(t) and V_2(t), h(V(t)) denoted by (3) satisfies the following condition:

$$\begin{array}{ll} & \|h(V_1(t)) - h(V_2(t))\| \le \|U_2(V_1(t) - V_2(t))\| \\ or & \|h(V_1(t)) - h(V_2(t))\| \le \delta_2 \|C(x_1(t) - x_2(t))\| \end{array}$$

where U_2 is a known matrix, δ_2 is a known positive constant.

3 FAULT DETECTION AND DIAGNOSIS VIA OUTPUT PDFS

3.1 Observer-based fault detection

Since the measured informance is the output probability distribution, in order to detect the fault based on the changes of output PDFs,the following full-order observer is applied to detect the fault.

$$\left\{ \begin{array}{l} E\dot{\hat{x}}(t) = A\hat{x}(t) + Gg(\hat{x}(t)) + Du(t) + L\varepsilon(t) \\ \varepsilon(t) = \int_a^b \sigma(z)[\sqrt{\gamma(z, u(t), F)} - \sqrt{\hat{\gamma}(z, u(t), F)}]dz \\ \sqrt{\hat{\gamma}(z, u(t), F)} = B(z)C\hat{x}(t) + h(C\hat{x}(t))b_n(z) \end{array} \right. \tag{7}$$

where $\hat{x}(t)$ is the estimated state, L$\in R^{m\times p}$ is the gain to be determined and the rsidual ε(t) is formulated as an integral of the difference between the measured PDFs and the estimated ones, $\sigma\ (z) \in R^{m\times m}$ can be regarded as a pre-specified weight vector lying [a,b] and makes the integration simple or adjust the scale of ε(t).

By defining $e(t) = x(t) - \hat{x}(t)$,$\tilde{g}(t) = g(x(t)) - g(\hat{x}(t))$,$\tilde{h}(t) = h(Cx(t)) - h(C\hat{x}(t))$, the estimation error system can be described

$$E\dot{e}(t) = (A - L\Gamma_1)e(t) + G\tilde{g}(t) - L\Gamma_2\tilde{h}(t) - L\Delta(t) + F \tag{8}$$

where

$$\Gamma_1 = \int_a^b \sigma(z)B^T(z)Edz, \Gamma_2 = \int_a^b \sigma(z)b_n(z)Edz, \Delta(t) = \int_a^b \sigma(z)\omega(z, u(t), F)dz$$

It can be seen that

$$\varepsilon(t) = \Gamma_1 e(t) + \Gamma_2\tilde{h}(t) + \Delta(t) \tag{9}$$

From $|\omega(z, u(t), F| \le \delta$,it can be verified that

$$\|\Delta(t)\| = \left\| \int_a^b \sigma(z)\omega(z, u, F)dz \right\| \le \alpha$$

$$\forall\alpha = \delta \int_a^b \sigma(z)dz$$

E is a singular matrix,hence exist two orthogonal matrices U and V such that

$$UEV = \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} \tag{10}$$

where $\Sigma = diag(\lambda_i)$and λi > 0 are the singular values of singular matrix E.

As shown in reference [17] and [18],Denote

$$U(A - L\Gamma_1)V = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \begin{bmatrix} [e_1(t)] \\ e_2(t) \end{bmatrix} = V^{-1}e(t)$$
$$UG = [G_1 G_2]^T,\ UL\Gamma_2 = [H_1 H_2],\ UL = [L_1 L_2]^T \tag{11}$$

Assumption 3[17][18]**.** If A_{22} is invertible,then the following inequality

$$\frac{\|A_{22}\|}{K(A_{22})} > \delta_1 \|G_2\| + \delta_2 \|C\| \|H_2\|$$

holds, where K(A_{22}) is the condition number of A_{22}.

In the absence of F, Eq. (4) is transformed into

$$\left\{ \begin{array}{rl} \Sigma\dot{e}_1(t) = & A_{11}e_1(t) + A_{12}e_2(t) + G_1\tilde{g}(t) \\ & -H_1\tilde{h}(t) - L_1\Delta(t) \\ 0 = & A_{21}e_1(t) + A_{22}e_2(t) + G_2\tilde{g}(t) \\ & -H_2\tilde{h}(t) - L_2\Delta(t) \end{array} \right. \tag{12}$$

First, when there is no fault, our objective is to find L such that the system (12) is stable, which can be formulated in the following theorem.

Theorem 1. For the parameters $\gamma > 0$ and $\kappa_i > 0,\ (i = 1, 2)$, if there exist matrices R and P with P being nonsingular,satisfying

$$E^T P = P^T E \ge 0 \tag{13}$$

$$\begin{bmatrix} \Pi & P^TG & R\Gamma_2 & C^TU_1^T & U_2 \\ * & -\frac{1}{\kappa_1} & 0 & 0 & 0 \\ * & * & -\frac{1}{\kappa_2} & 0 & 0 \\ * & * & * & -\kappa_1 & 0 \\ * & * & * & * & -\kappa_2 \end{bmatrix} < 0 \quad (14)$$

where $\Pi = A^TP + P^TA - R\Gamma_1 - \Gamma_1^TR^T + \gamma I$,then in the absence of F(x,u),the error system (11) with gain $L = P^{-T}R$ is asymptotically stable.

Proof. Substituting Eq. (10) and (11) into (13), it can be seen that $\Sigma P_{11} = P_{11}^T\Sigma^T > 0$ and $P_{12} = 0$. A_{22} and P_{22} are inveritible.

Define the Lyapunov candidate function as follows

$$\begin{aligned} V(t) &= e^T(t)E^TPe(t) + \kappa_1\int_0^t [\|U_1Ce(s)\|^2 \\ &- \left\|\tilde{h}(s)\right\|^2]ds + \kappa_2\int_0^t [\|U_2e(s)\|^2 - \|\tilde{g}(s)\|^2]ds \\ &= \dot{e}^T(t)E^TPe(t) + e^T(t)E^TP\dot{e}(t) \\ &+ \kappa_1\int_0^t [\|U_1Ce(s)\|^2 - \left\|\tilde{h}(s)\right\|^2]ds \\ &+ \kappa_2\int_0^t [\|U_2Ee(s)\|^2 - \|\tilde{g}(s)\|^2]ds \end{aligned}.$$

Along the trajectories of (12) in the absence of F, it can be shown that

$$\begin{aligned} \dot{V}(t) &= e^T(t)[(A - L\Gamma_1)^TP + P^T((A - L\Gamma_1)]e(t) \\ &+ 2e^T(t)P^TG\tilde{g}(t) - 2e^T(t)R\Gamma_2\tilde{h}(t) \\ &- 2e^T(t)P^TL_2\Delta(t) \\ &+ \kappa_1[\|U_1Ce(s)\|^2 - \left\|\tilde{h}(s)\right\|^2] \\ &+ \kappa_2[\|U_2e(s)\|^2 - \|\tilde{g}(s)\|^2] \end{aligned}$$

Since

$$\begin{aligned} 2e^T(t)P^TG\tilde{g}(t) &\le \frac{1}{\kappa_1}e^T(t)P^TGG^TPe(t) \\ &+ \kappa_1\tilde{g}^T(t)\tilde{g}(t) \\ -2e^T(t)P^TL\Gamma_2\tilde{h}(t) &\le \frac{1}{\kappa_2}e^T(t)P^TL\Gamma_2\Gamma_2^TL^TP \\ &+ \kappa_2\tilde{h}^T(t)h(t) \end{aligned}$$

Using Assumptions 1 and 2, then we can get that

$$\begin{aligned} \dot{V}(t) \le & e^T(t)[(A - L\Gamma_1)^TP + P^T(A - L\Gamma_1) \\ &+ \frac{1}{\kappa_1}P^TGG^TP + \frac{1}{\kappa_2}P^TL\Gamma_2\Gamma_2^TL^TP \\ &+ \kappa_1C^TU_1^TU_1C \\ &+ \kappa_2U_2^TU]e(t) - 2e^T(t)P^TL_2\Delta(t) \end{aligned}$$

Thus, under (14),it can be seen that

$$\begin{aligned} \dot{V}(t) &< -\gamma\|e(t)\|^2 - -2e^T(t)P^TL_2\Delta(t) \\ &\le -\gamma\|e(t)\|^2 + 2\alpha\|e(t)\|\|R\| \end{aligned}$$

Therefore, it can be claimed that

$$\|e_1(t)\| \le \eta_1 = \max\{\|e_1(0)\|, 2\gamma^{-1}\alpha\|R\|\} \quad (15)$$

As before, A_{22} is inveritible, by Eq. (12) and assumption 1,2 and 3, we can calculated that

$$\begin{aligned} &\|e_2(t)\| \le \left\|A_{22}^{-1}\right\| \\ &\cdot \left\|A_{21}e_1(t) + G_2\tilde{g}(t) - H_2\tilde{h}(t) - L_2\Delta(t)\right\| \\ &= \frac{K(A_{22})\left\|A_{21}e_1(t) + G_2\tilde{g}(t) - H_2\tilde{h}(t) - L_2\Delta(t)\right\|}{\|A_{22}\|} \\ &\le \frac{(\|A_{21}\| + \delta_1G_2 + \delta_2\|H_2\|\|C\|)\eta_1 + \|L_2\|\alpha}{\delta_1\|G_2\| + \delta_2\|C\|\|H_2\|} \end{aligned}$$

It can be seen that

$$\begin{aligned} &e_2(t) \le \eta_2 = \max \\ &\left\{\|e_2(0)\|, \frac{(\|A_{21}\| + \delta_1G_2 + \delta_2\|H_2\|\|C\|)\eta_1 + \|L_2\|\alpha}{\delta_1\|G_2\| + \delta_2\|C\|\|H_2\|}\right\} \end{aligned} \quad (16)$$

From Eq. (15) and (16), we can seen that the error system is stable.

Theorem 1 presents a necessary condition for fault detection. In order to detect F, we select ε(t) as residual signal and propose the following result to determine the threshold.

Corollary 1: If for the parameters $\kappa_i > 0$ (i=1,2), there exists matrices R and P with being non-singular and constant $\gamma > 0$ satisfying (14), then fault F can be detected by the following criterion:

$$\varepsilon(t) > \eta = (\|\Gamma_1\| + \delta_2\|C\|\|\Gamma_2\|)(\eta_1 + \eta_2) + \alpha \quad (17)$$

Which means that there exists fault in the system.

Proof: From Eq. (9) and based on theorem 1, when F=0, we can seen that

$$\begin{aligned} \varepsilon(t) &= \Gamma_1e(t) + \Gamma_2\tilde{h}(t) + \Delta(t) \\ &\le \|\Gamma_1\|\|e(t)\| + \delta_2\|\Gamma_2\|\|C\|\|e(t)\| + \|\Delta(t)\| \\ &\le (\|\Gamma_1\| + \delta_2\|\Gamma_2\|\|C\|)(\eta_1 + \eta_2) + \alpha \end{aligned}$$

This means that the residual vector ε(t) should satisfying Eq. (17) in the presence of F.

After square root B-spline expansion are used to approximated the measured output PDFs, and the nonlinear weight dynamics model is established, the fault detection procedures for general singular stochastic system can be summarized as follows:

1) Constructer the observer with nonlinear system (7).

2) Compute the observer gain L using theorem 1.

3) Use ε(t) as the residual signal and calculate the corresponding threshold to detect the fault.

3.2 Observer-based Fault Diagnosis

After the fault is detected based upon the results in section 3.1,the fault diagnosis need to be carried out in order to estimate the size of fault F.when a fault occurs,we construct the following adaptive observer.

$$\begin{cases} E\dot{\hat{x}}(t) = & A\hat{x}(t) + Gg(\hat{x}(t)) + Du(t) + L\varepsilon(t) \\ & +T\hat{W}S(\hat{x},u) \\ \varepsilon(t) = & \int_a^b \sigma(z)[\sqrt{\gamma(z,u(t),F)} \\ & -\sqrt{\hat{\gamma}(z,u(t),F)}]dz \\ \sqrt{\hat{\gamma}(z,u(t),F)} & = B(z)C\hat{x}(t) + h(C\hat{x}(t))b_n(z) \end{cases} \quad (18)$$

The system (4) can be rewritten as

$$\begin{cases} E\dot{x}(t) = & Ax(t) + Gg(x(t)) + Du(t) + TWS(x,u) \\ & +\theta(x,u) \\ V(t) = & Cx(t) \end{cases} \quad (19)$$

By defining $\tilde{W} = W - \hat{W}, \hat{S} = S(x,u) - S(\hat{x},u), \theta_1 = TW\hat{S} + \theta$, and $\|\theta_1\| \le \vartheta$, the estimation system can be described as

$$E\dot{e}(t) =(A - L\Gamma_1)e(t) + G\tilde{g}(t) - L\Gamma_2\tilde{h}(t) - L\Delta(t) + T\tilde{W}S(\hat{x},u) + \theta_1 \quad (20)$$

Then, an adaptive fault estimation algorithm is presented by the following theorem.

Theorem 2 For the parameters $\kappa_i > 0$ (i=1,2), if there exist matrices R, P with P being non-singular,and $\beta > 0$,such that the following LMI holds:

$$E^T P = P^T E \ge 0 \quad (21)$$

$$M = \begin{bmatrix} \Pi & \frac{1}{\kappa_1}P^T G & R\Gamma_2 & C^T U_1^T & U_2 \\ * & -\frac{1}{\kappa_1} & 0 & 0 & 0 \\ * & * & -\frac{1}{\kappa_2} & 0 & 0 \\ * & * & * & -\kappa_1 & 0 \\ * & * & * & * & -\kappa_2 < 0 \end{bmatrix} \quad (22)$$

Where $\Pi = A^T P + P^T A - R\Gamma_1 - \Gamma_1^T R^T + \gamma I$,Then the error system (20) with gain L=P-TR is stable and the fault estimation algorithm is as

$$\dot{\hat{W}} = -\pi T^T Pe(t)S^T(\hat{x},u)$$

Proof The Lyapunov candidate function can be choose as

$$V(t) =e^T(t)E^T Pe(t) + tr\{\tilde{W}^T\pi^{-1}\tilde{W}\} + \kappa_1\int_0^t [\|U_1Ce(s)\|^2 - \left\|\tilde{h}(s)\right\|^2]ds + \kappa_2\int_0^t [\|U_2e(s)\|^2 - \|\tilde{g}(s)\|^2]ds$$

By Eq. (20) and using $\dot{\tilde{W}} = \dot{\hat{W}} = -\pi T^T Pe(t)S^T(\hat{x},u)$, we can obtain

$$\begin{aligned} \dot{V}(t) =&\dot{e}^T(t)E^T Pe(t) + e^T(t)E^T P\dot{e}(t) + 2tr\{\dot{\tilde{W}}^T\pi^{-1}\tilde{W}\} \\ &+ \kappa_1(e^T(t)C^T U_1^T U_1Ce(t) - \tilde{h}^T(t)h(t)) \\ &+ \kappa_2(e^T(t)U_2^T U_2e(t) - \tilde{g}^T(t)g(t)) \\ =&e^T(t)[P^T A + A^T P - R\Gamma_1 - \Gamma_1^T R^T]e(t) \\ &+ 2e^T(t)P^T G\tilde{g}(t) - 2e^T(t)R\Gamma_2\tilde{h}(t) \\ &- 2e^T(t)R\Delta(t) + 2e^T(t)PT\tilde{W}S(\hat{x},u) \\ &+ 2e^T(t)P\theta_1 - 2tr\{S(\hat{x},u)e^T(t)P^T T\tilde{W}\} \\ &+ \kappa_1(e^T(t)C^T U_1^T U_1Ce(t) - \tilde{h}^T(t)h(t)) \\ &+ \kappa_2(e^T(t)U_2^T U_2e(t) - \tilde{g}^T(t)g(t)) \end{aligned}$$

It is noted that

$$\begin{aligned} e^T(t)PT\tilde{W}S(\hat{x},u) &= tr\{e^T(t)PT\tilde{W}S(\hat{x},u)\} \\ &= 2tr\{S(\hat{x},u)e^T(t)P^T T\tilde{W}\} \end{aligned}$$

Then ,we can obtained that

$$\begin{aligned} \dot{V}(t) \le& e^T(t)[P^T A + A^T P - R\Gamma_1 - \Gamma_1^T R^T \\ &+ \frac{1}{\kappa_1}P^T GG^T P + \frac{1}{\kappa_2}R\Gamma_2\Gamma_2^T R^T \\ &+ \kappa_1C^T U_1^T U_1C + \kappa_2U_2^T U]e(t) \\ &- 2e^T(t)R\Delta(t) + 2e^T(t)P\theta_1 \\ \le& e^T(t)\mathrm{M}e(t) - 2e^T(t)R\Delta(t) + 2e^T(t)P\theta_1 \\ \le& -\beta e^T(t)e(t) - 2e^T(t)R\Delta(t) + 2e^T(t)P\theta_1 \\ =& -\frac{\beta}{2}e^T(t)e(t) - \frac{\beta}{4}[e^T(t)e(t) + \frac{8}{\beta}e^T(t)R\Delta(t)] \\ &- \frac{\beta}{4}[e^T(t)e(t) - \frac{8}{\beta}e^T(t)P\theta_1] \end{aligned}$$

Since

$$\begin{aligned} &\tfrac{8}{\beta}e^T(t)P\theta \le e^T(t)e(t) + \tfrac{16}{\beta^2}\theta^T P^T P\theta_1 \\ &-\tfrac{8}{\beta}e^T(t)R\Delta(t) \le e^T(t)e(t) + \tfrac{16}{\beta^2}\Delta^T R^T R\Delta \end{aligned}$$

It is noted that

$$\dot{V}(t) \le -\frac{\beta}{2}\|e(t)\|^2 + \frac{4\varsigma_1}{\beta}\vartheta^2 + \frac{4\varsigma_2}{\beta}\alpha^2 \quad (23)$$

Where $\varsigma_1 = \lambda_{\max}(P^T P), \varsigma_2 = \lambda_{\max}(R^T R)$are the maximum eigenvalue of PTP and RTR.

In the presence of F,the following inequality can be obtained that

$$\begin{aligned} &\|e_1(t)\|^2 \le \tau_1 \\ &= \min\{\|e_1(0)\|^2, \frac{8}{\beta^2}(\lambda_{\max}(P^T P)\vartheta^2 + \lambda_{\max}(R^T R)\alpha^2)\} \end{aligned} \quad (24)$$

From Eq. (12), we have

$$\|e_2(t)\| \leq \tau_2 = \max\{\ \|e_2(0)\|, \frac{(\|A_{21}\| + \delta_1 G_2 + \delta_2 \|H_2\| \|C\|)\sqrt{\tau_1} + \|L_2\| \alpha}{\delta_1 \|G_2\| + \delta_2 \|C\| \|H_2\|}\} \quad (25)$$

Eq. (20) with diagnosis observer Eq.(18) based on gain $L = P^{-T}R$ is stable and the estimation error satisfies

$$\|e(t)\|^2 \leq \tau_1 + \tau_2^2 \quad (26)$$

If $\|e(t)\|^2 > \tau_1 + \tau_2^2$is satisfied,which indicates that $\|e_1(t)\|$or $\|e_2(t)\|$ is larger than $\tau_1$or τ_2. it can be seen that that fault occurs in this case. Eq. (26) means that the estimated error be guaranteed to be small if we selects suitable design parameters.This implies that the estimated error can be made arbitrarily small by choosing the suitable design parameters $\beta, \varsigma_1, \varsigma_2$ and L. In terms of the optimal fault detection and diagnosis algorithm, the key issue is to construct the above algorithm that is maximum sensitive to the faults while minimum sensitive to the model uncertainty or the error on the approximation of PDFs.

4 SIMULATION

An application of paper-making process is given to demonstrate the applicability of the proposed method. Suppose the output PDFs for singular stochastic system can be described by using a square root B-spline model:

$$\sqrt{\gamma(z, u(t)} = \sum_{i=1}^{3} v_i(t) b_i(z)$$

$$b_1(y) = (y+1)^2 I_1 + (-y^2 - 3y - 2) I_2 + (y-2)^2 I_3$$
$$b_2(y) = (y+2)^2 I_1 + (-y^2 - 2y - 3) I_2 + y^2 I_3$$

Where

$$I_i = \begin{cases} 1, & \forall y \in [i-4, i-3] \quad (i = 1, 2, 3) \\ 0 & other \end{cases}$$

It can be verified that

$$\sqrt{\gamma(z, u(t)} = B(z)V(t) + b_3 h(V(t))$$

Furthermore,it can be seen that

$$\Lambda_1 = diag\{0.40, 0.40\}, \Lambda_2 = [0 \quad 0], \Lambda_3 = 0.40$$

It is assumed the identified weighting system is formulated by (2) with the following coefficient matrices

$$g(x(t)) = \sin x(t), E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$G = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

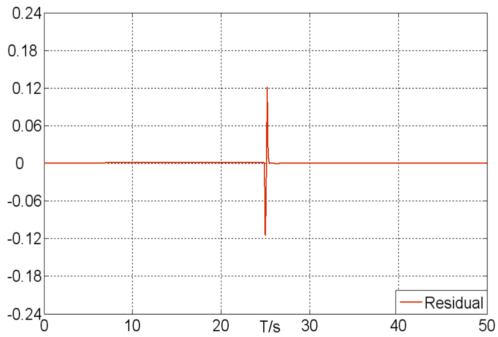

Fig. 1. Response of the residual signal

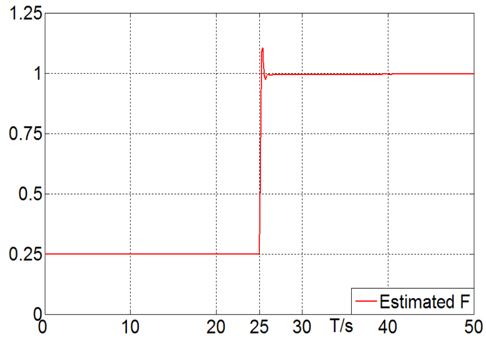

Fig. 2. Response of $\hat{F}$

It is assumed that the upper bounds of nonlinearity exists and satisfies $\delta_1 = 0.50, \delta_2 = 0.50$.

Let $\sigma(z) = 1, then, \Gamma_1 = [1/\pi \quad 1/\pi], \Gamma_2 = 1/\pi$.

We suppose that fault occurs at 25s,matrix L is set to be L=[-0.1 0.5] .the initial condition of fault diagnosis observer and fault detection observer are selected as $x(0) = [0.25 \quad 0]^T, \hat{x}(0) = [0 \quad 0]^T$

The residual signal response is shown in Fig.1. In Fig.2 the fault diagnosis result are given. It is shown in Fig.3 that the fault can be well estimated throngh the fault diagnostic observer after its occurrence .From these figure it is shown that the desire fault diagnosis results have been obtained.

5 CONCLUSION

In this paper, a new fault detection and diagnosis method is investigated for a class of non-gaussian singular stochastic system,where only the output PDFs can be measured rather than an output value. It is developed from the technology of PDFs,which is modeled by a square root B-spline expansion. Based on LMI techniques, the complexity FDD problem of non-gaussian singular stochastic system is transformed into the classical nonlinear FDD problem. A new adaptive network parameter-updating law is

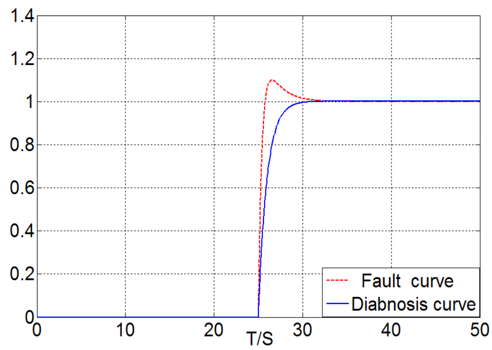

Fig. 3. Response of diagnosis observer

presented in this work,by introducing the tuning parameters,the corresponding estimation error system is guaranteed to be stable.Simulation is given to demonstrate the efficiency of the proposed approach.

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