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MODEL UPDATE WITH THE OBSERVER/KALMAN FILTER AND GENETIC ALGORITHM APPROACH

UDC 519.6

Summary

The discrete time Observer/Kalman model identification technique is implemented in order to identify the structure pulse response. The model updating procedure based on the finite element model pulse response of the test structure and the genetic algorithm is developed. The objective function evaluates the difference between the system and the model pulse responses. The modal assurance criteria implementation is considered. The model reduction in order to match the model degrees of freedom (dofs) and the test structure dofs involved in the experiment is discussed. A case study on the frame test structure is provided.

Key words: *system identification, model update, observer, Kalman filter, genetic algorithm*

1. Introduction: theoretical background

In practice, the Experimental Modal Analysis (EMA) comprises three phases/elements: the experiment/process supervision, the data measurement and processing, and the model validation. However, the experimental data are not final goal for engineers. Modal parameters, i.e. vibration frequencies, vibration modes, and modal damping are often of major interest. Finally, the reliable simulation of the structure behaviour has typically high priority and a model update is required. The model update is often inevitable in the damage detection or the Structural Health Monitoring (SHM) where sensitivity to the structural change plays an important role. Also, the model update is an essential part of the inverse analysis when structural parameters are unknown or uncertain, [1-4].

Typical structural systems, providing that the boundaries, the geometry, and the material do not involve nonlinearities, have the linear governing equation:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{Q} \qquad (1)$$

where **M**, **D** and **K** are the nxn system mass, damping and stiffness matrices, respectively, and **q** and **Q** are generalized coordinates and generalize forces. The generalized displacements are defined as the minimal set of parameters-coordinates that describes the position of the structure in a unique way. To solve the direct problem one needs to find motion (**q**) for the known forces (**Q**) that satisfies the governing equation (1) and the initial conditions: $\mathbf{q}(0)=\mathbf{q}_0$, $\dot{\mathbf{q}}(0)=\dot{\mathbf{q}}_0$. The indirect problem, i.e. the finding of forces **Q** for the given motion **q,** is straightforward.

The measurement, control and model development processes often require inverse tasks: to define the mass, damping and stiffness matrices for the given motion and the given forces. Typically, a new terminology and new aspects are involved: inputs, outputs and the governing equation are said to be in a state space:

$$\dot{\mathbf{x}}=\mathbf{A}_c\mathbf{x}+\mathbf{B}_c\mathbf{u}(t) \quad (2)$$

where $\mathbf{A}_c$ and $\mathbf{B}_c$ are the continuous-time state and the input system matrices, $\mathbf{u}(t)$ is the input and **x** is the vector of state variables (also: state vector). The state vector typically has the following form: $\mathbf{x}^T=\left[\mathbf{q}^T \quad \dot{\mathbf{q}}^T\right]$, [5]. For the linear mechanical system (1) the corresponding system in the state space is the linear time invariant i.e. the system matrices $\mathbf{A}_c$ and $\mathbf{B}_c$ are constant.

The system output **y** is detected by means of the output matrix **C** and the direct transfer matrix **D**:

$$\mathbf{y}=\mathbf{C}\mathbf{x}+\mathbf{D}\mathbf{u}(t) \quad (3)$$

where the matrix **C** basically depends on sensor properties.

1.1 Discrete model

Model discretization has two independent motivation sources. In the *numerical analysis,* we discretize the continuous model to solve the boundary problem when a closed analytical solution is not available. In the experimental procedures, we perform data acquisition with the digital equipment that involves discretization. Matching the two aspects is necessary in the model validation, the modal update and other similar experiment-analytic (numeric) model correlation dependent techniques. Without going into more detail, we assume that the values of variables are constant during discrete time increments (zero order hold assumption).

Consider a discrete-time linear state space dynamic system:

$$\begin{aligned}\mathbf{x}(k+1)&=\mathbf{A}\mathbf{x}(k)+\mathbf{B}\mathbf{u}(k)\\ \mathbf{y}(k)&=\mathbf{C}\mathbf{x}(k)+\mathbf{D}\mathbf{u}(k)\end{aligned} \quad (4)$$

where **A** and **B** are the state and input system matrices, respectively. It should be noted that the state at the k+1 step depends on the state at step k only.

The observer state space presentation has the form [6]:

$$\begin{aligned}\mathbf{x}(k+1)&=\overline{\mathbf{A}}\mathbf{x}(k)+\overline{\mathbf{B}}\mathbf{v}(k)\\ \mathbf{y}(k)&=\mathbf{C}\mathbf{x}(k)+\mathbf{D}\mathbf{u}(k)\end{aligned} \quad (5)$$

where the observer input reads:

$$\mathbf{v}(k)=\begin{bmatrix}\mathbf{y}(k)\\ \mathbf{u}(k)\end{bmatrix} \quad (6)$$

The correlation of the observer presentation to the system state space is given by:

$$\begin{aligned}\overline{\mathbf{A}}&=\mathbf{A}+\mathbf{G}\mathbf{C}\\ \overline{\mathbf{B}}&=\left[\mathbf{B}+\mathbf{G}\mathbf{D} \quad -\mathbf{G}\right]\end{aligned}$$

The *Kalman filter* is an observer system model with the optimized gain matrix **G** so that it whitens the response.

The impulse response as the inverse Fourier transform of the Frequency Response Function (FRF) is very important in the continuous-time system characterization. However, in the discrete-time procedure we rely on the pulse response for the characterization of the linear system.

Given the discrete-time linear system (4), the pulse response reads:

$$\mathbf{Y}_0 = \mathbf{D} \quad \mathbf{Y}_1 = \mathbf{CB} \quad \mathbf{Y}_2 = \mathbf{CAB} \quad \dots \quad \mathbf{Y}_k = \mathbf{CA}^{k-1}\mathbf{B} \tag{7}$$

where $\mathbf{Y}_k$, k = 0,1,2,… are the m x r matrices for the systems with r inputs and m outputs and are called the system Markov parameters.

The observer Markov parameters read:

$$\bar{\mathbf{Y}}_0 = \mathbf{D} \quad \bar{\mathbf{Y}}_1 = \mathbf{C}\bar{\mathbf{B}} \quad \bar{\mathbf{Y}}_2 = \mathbf{C}\bar{\mathbf{A}}\bar{\mathbf{B}} \quad \dots \quad \bar{\mathbf{Y}}_k = \mathbf{C}\bar{\mathbf{A}}^{k-1}\bar{\mathbf{B}} \tag{8}$$

where the correlation to the system Markov parameters is given by:

$$\begin{aligned}
&\bar{\mathbf{Y}}_0 = \mathbf{D}\\
&\bar{\mathbf{Y}}_1 = \mathbf{C}\bar{\mathbf{B}} = \mathbf{C}\left[\mathbf{B}+\mathbf{GD} \quad -\mathbf{G}\right]\\
&\bar{\mathbf{Y}}_2 = \mathbf{C}\bar{\mathbf{A}}\bar{\mathbf{B}} = \mathbf{C}\left[\mathbf{A}+\mathbf{GC}\right]\left[\mathbf{B}+\mathbf{GD} \quad -\mathbf{G}\right]\\
&\dots\\
&\bar{\mathbf{Y}}_k = \mathbf{C}\bar{\mathbf{A}}^{k-1}\bar{\mathbf{B}} = \mathbf{C}\left[\mathbf{A}+\mathbf{GC}\right]^{k-1}\left[\mathbf{B}+\mathbf{GD} \quad -\mathbf{G}\right]
\end{aligned} \tag{9}$$

From the last expressions, it is easy to note that the observer Markov parameters can be calculated from the system Markov parameters and vice versa by means of the recursive expression (9).

2. The computation of system Markov parameters and observer Markov parameters from experimental data

The finite difference equation, also known as single-step autoregressive model with the exogenous input (ARX) input-output relation, reads:

$$\mathbf{y}(k) + \boldsymbol{\alpha}_1\mathbf{y}(k-1) + \dots + \boldsymbol{\alpha}_p\mathbf{y}(k-p) = \boldsymbol{\beta}_0\mathbf{u}(k) + \boldsymbol{\beta}_1\mathbf{u}(k-1) + \dots + \boldsymbol{\beta}_p\mathbf{u}(k-p) \tag{10}$$

where the output at step k is given by:

$$\begin{aligned}
\mathbf{y}(k) = &\\
&-\boldsymbol{\alpha}_1\mathbf{y}(k-1) - \dots - \boldsymbol{\alpha}_p\mathbf{y}(k-p)\\
&+\boldsymbol{\beta}_0\mathbf{u}(k) + \boldsymbol{\beta}_1\mathbf{u}(k-1) + \dots + \boldsymbol{\beta}_p\mathbf{u}(k-p)
\end{aligned} \tag{11}$$

where the index range is: $k = k–p, k–p+1,…, k$. The same input-output relation can be rewritten with the Markov parameters:

$$\begin{aligned}
\mathbf{y}(l-1) &= \sum_{i=1}^{l-1}\mathbf{CA}^{i-1}\mathbf{Bu}(l-1-i) + \mathbf{Du}(l-1)\\
\mathbf{y}(l-1) &= \sum_{i=1}^{l-1}\mathbf{Y}_i\mathbf{u}(l-1-i) + \mathbf{Y}_0\mathbf{u}(l-1)
\end{aligned} \tag{12}$$

with the index range: i = 0,1,2,…l – 1. For the same expression in the matrix form we have:

$$\mathbf{y}(l-1)=\begin{bmatrix}\mathbf{Y}_0 & \mathbf{Y}_1 & \mathbf{Y}_2 & \ldots & \mathbf{Y}_p & \ldots & \mathbf{Y}_{l-1}\end{bmatrix}\begin{Bmatrix}\mathbf{u}(l-1)\\ \mathbf{u}(l-2)\\ \mathbf{u}(l-3)\\ \ldots\\ \mathbf{u}(l-p-1)\\ \ldots\\ \mathbf{u}(0)\end{Bmatrix} \tag{12a}$$

So, using the system Markov parameters, we can collect the output sequence in:

$$\begin{bmatrix}\mathbf{y}(0) & \mathbf{y}(1) & \mathbf{y}(2) & \ldots & \mathbf{y}(p) & \ldots & \mathbf{y}(l-1)\end{bmatrix}=$$
$$=\begin{bmatrix}\mathbf{Y}_0 & \mathbf{Y}_1 & \mathbf{Y}_2 & \ldots & \mathbf{Y}_p & \ldots & \mathbf{Y}_{l-1}\end{bmatrix}\begin{bmatrix}\mathbf{u}(0) & \mathbf{u}(1) & \mathbf{u}(2) & \ldots & \mathbf{u}(p) & \ldots & \mathbf{u}(l-1)\\ & \mathbf{u}(0) & \mathbf{u}(1) & \ldots & \mathbf{u}(p-1) & \ldots & \mathbf{u}(l-2)\\ & & \mathbf{u}(0) & \ldots & \mathbf{u}(p-2) & \ldots & \mathbf{u}(l-3)\\ & & & \ldots & \ldots & \ldots & \ldots\\ & 0 & & & \mathbf{u}(0) & \ldots & \mathbf{u}(l-p-1)\\ & & & & & \ldots & \mathbf{u}(1)\\ & & & & & \ldots & \mathbf{u}(0)\end{bmatrix} \tag{13}$$

where $\mathbf{Y}_i$ are the system Markov parameters (7).

If we assume that the system was at rest for the first p steps due to the zero initial conditions and the zero excitation inputs, the system truncates to:

$$\begin{bmatrix}\mathbf{y}(p) & \mathbf{y}(p+1) & \mathbf{y}(p+1) & \ldots & \mathbf{y}(l-1)\end{bmatrix}$$
$$=\begin{bmatrix}\mathbf{Y}_0 & \mathbf{Y}_1 & \mathbf{Y}_2 & \ldots & \mathbf{Y}_p\end{bmatrix}\begin{bmatrix}\mathbf{u}(p) & \mathbf{u}(p+1) & \mathbf{u}(p+2) & \ldots & \mathbf{u}(l-1)\\ \mathbf{u}(p-1) & \mathbf{u}(p) & \mathbf{u}(p+1) & \ldots & \mathbf{u}(l-2)\\ \mathbf{u}(p-2) & \mathbf{u}(p-1) & \mathbf{u}(p) & \ldots & \mathbf{u}(l-3)\\ \ldots & \ldots & \ldots & \ldots & \ldots\\ \mathbf{u}(0) & \mathbf{u}(1) & \mathbf{u}(2) & \ldots & \mathbf{u}(l-p-1)\end{bmatrix} \tag{13a}$$

$$\mathbf{y}=\mathbf{YU} \tag{13b}$$

where we assume additionally that any (excitation input) response dies out due to the system damping after p steps.

As far as the low damped system requires a relatively large number of steps (which leads to a low conditioned system) we can consider an artificially damped system. So we use the observer Markov parameters that describe the related system (14).

$$
\begin{aligned}
&[\mathbf{y}(0) \quad \mathbf{y}(1) \quad \mathbf{y}(2) \quad \ldots \quad \mathbf{y}(p) \quad \ldots \quad \mathbf{y}(l-1)] = \\
&= \left[\bar{\mathbf{Y}}_0 \quad \bar{\mathbf{Y}}_1 \quad \bar{\mathbf{Y}}_2 \quad \ldots \quad \bar{\mathbf{Y}}_p \quad \ldots \quad \bar{\mathbf{Y}}_{l-1}\right]
\begin{bmatrix}
\mathbf{u}(0) & \mathbf{u}(1) & \mathbf{u}(2) & \ldots & \mathbf{u}(p) & \ldots & \mathbf{u}(l-1) \\
 & \mathbf{v}(0) & \mathbf{v}(1) & \ldots & \mathbf{v}(p-1) & \ldots & \mathbf{v}(l-2) \\
 & & \mathbf{v}(0) & \ldots & \mathbf{v}(p-2) & \ldots & \mathbf{v}(l-3) \\
 & & & \ldots & \ldots & \ldots & \ldots \\
 & 0 & & & \mathbf{v}(0) & \ldots & \mathbf{v}(l-p-1) \\
 & & & & & \ldots & \mathbf{v}(1) \\
 & & & & & \ldots & \mathbf{v}(0)
\end{bmatrix}
\end{aligned}
\tag{14}
$$

where $\bar{\mathbf{Y}}_i$ are the observer Markov parameters (8). After truncation in the same manner as above we have:

$$
\begin{aligned}
&[\mathbf{y}(p) \quad \mathbf{y}(p+1) \quad \mathbf{y}(p+1) \quad \ldots \quad \mathbf{y}(l-1)] \\
&= \left[\mathbf{Y}_0 \quad \mathbf{Y}_1 \quad \mathbf{Y}_2 \quad \ldots \quad \mathbf{Y}_p\right]
\begin{bmatrix}
\mathbf{u}(p) & \mathbf{u}(p+1) & \mathbf{u}(p+2) & \ldots & \mathbf{u}(l-1) \\
\mathbf{v}(p-1) & \mathbf{v}(p) & \mathbf{v}(p+1) & \ldots & \mathbf{v}(l-2) \\
\mathbf{v}(p-2) & \mathbf{v}(p-1) & \mathbf{v}(p) & \ldots & \mathbf{v}(l-3) \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
\mathbf{v}(0) & \mathbf{v}(1) & \mathbf{v}(2) & \ldots & \mathbf{v}(l-p-1)
\end{bmatrix}
\end{aligned}
\tag{14a}
$$

$$\mathbf{y} = \bar{\mathbf{Y}}\mathbf{V} \tag{14b}$$

The details of the solution techniques for (14a) and (14b) can be found in [6]. Once the observer Markov parameters are found, the system Markov parameters can be computed by (9). The system Markov parameters have imbedded the system state space matrices that we extract with the procedure based on the SVD decomposition of the Henkel matrix. At this point, the system pulse response is available in a few simple steps.

3. Model update

The model update is the optimization procedure that minimizes discrepancy between the analytical/numerical model (typically a FEM model) and the model identified experimentally from the measured input-output time histories of a real system, [7-11]. The model update optimization procedure is based on the objective function and constraints that can be implemented in the frequency domain or/and in the time domain. For the model characterizations in the frequency domain we typically use modal parameters: natural frequencies, vibration modes, and modal damping. In this paper, for the model characterization in time we consider pulse response.

The typical objective function of a structure in the frequency domain reads:

$$f_1 = \sum_{i=1}^{k} \frac{\left|\omega_{\mathrm{A}i} - \omega_{\mathrm{X}i}\right|}{\omega_{\mathrm{X}i} \cdot MAC_i} \tag{15}$$

with the Modal Assurance Criterion (MAC):

$$MAC_i = \frac{\left|\mathbf{\Phi}_{\mathrm{X}i}^T \mathbf{\Phi}_{\mathrm{A}i}\right|^2}{\left(\mathbf{\Phi}_{\mathrm{X}i}^T \mathbf{\Phi}_{\mathrm{X}i}\right)\left(\mathbf{\Phi}_{\mathrm{A}i}^T \mathbf{\Phi}_{\mathrm{A}i}\right)} \tag{16}$$

where ω_{Ai} and ω_{Xi} are the i-th natural frequencies, $\mathbf{\Phi}_{Ai}$ and $\mathbf{\Phi}_{Xi}$ are the corresponding natural modes, and indexes A and X mark analytical and experimental models. Alternatively, instead of MAC_i, SCO_i can be used:

$$SCO_i = \frac{\left|\mathbf{\Psi}_{Xi}^T \mathbf{M}_R \mathbf{\Psi}_{Ai}\right|^2}{\left(\mathbf{\Psi}_{Xi}^T \mathbf{M}_R \mathbf{\Psi}_{Xi}\right)\left(\mathbf{\Psi}_{Ai}^T \mathbf{M}_R \mathbf{\Psi}_{Ai}\right)} \tag{17}$$

where $\mathbf{\Psi}_{Ai}$ and $\mathbf{\Psi}_{Xi}$ are the i-th mass normalized vibration modes and $\mathbf{M}_R$ is the model mass matrix.

The objective function in the time domain, based on the system pulse response, reads:

$$f_2 = \frac{1}{Nm} \sum_{i=1}^{N} \sum_{j=1}^{m} \frac{\left|y_{Aij} - y_{Xij}\right|}{\left|y_{Xij}\right|} \tag{18}$$

where y_{Ai} and y_{Xi} are the analytical and experimental pulse response of the j-th output at the i-th time step, m is the number of system outputs and N is the number of the considered time steps. The alternative pulse error norms can be considered.

These objective functions can be used independently or can be combined in one unique objective function. Also, the objective function in the frequency domain can be used with constraint in the time domain and vice versa.

In this paper, the Genetic Algorithm (GA) optimization technique is used. The method is known to be very robust: the knowledge is kept in population and the optimization is a specific random search strategy based on crossover and mutation. The details of the GA optimization procedure can be found in [12, 13].

4. Illustrative example

A simple illustrative example with two dofs is considered first. As long as we consider all system outputs, the system reduction is not required.

Let us consider the given system:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{Q}$$

where:

$$\mathbf{K} = \begin{bmatrix} 35000 & -14000 \\ -14000 & 20000 \end{bmatrix},\ \mathbf{M} = \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix},\ \mathbf{D} = \begin{bmatrix} 8.0515 & -1.7617 \\ -1.7617 & 4.7051 \end{bmatrix}.$$

The damping is an orthogonal, damping matrix calculated as: $\mathbf{D} = \alpha\mathbf{M} + \beta\mathbf{K}$, α=0.7294, β=1.2584*10^{-4}. The α and β are so selected that the modal damping coefficients are: ξ_1=0.01 and ξ_2=0.01.

The eigenvalues and eigenvectors of a corresponding undamped system are:

$$\mathbf{\Lambda} = \begin{bmatrix} 3215 & 0 \\ 0 & 10452 \end{bmatrix},\ \mathbf{\Phi} = \begin{bmatrix} -0.3089 & -0.3234 \\ -0.4175 & 0.3987 \end{bmatrix}$$

The natural frequencies are: ω_1 = 56.6984, ω_2 = 102.2348.

The input-output histories will be reproduced by means of a discrete-time state space model:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k)$$
$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{u}(k)$$

where:

$$\mathbf{A} = \begin{bmatrix} 0.8719 & 0.0504 & 0.0059 & 0.0001 \\ 0.0840 & 0.8779 & 0.0002 & 0.0059 \\ -40.4352 & 15.6394 & 0.8626 & 0.0523 \\ 26.0657 & -38.5733 & 0.0871 & 0.8688 \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} 0.0000 & 0.0000 \\ 0.0000 & 0.0000 \\ 0.0012 & 0.0000 \\ 0.0000 & 0.0020 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \ \mathbf{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Relying on the state space model, for the given input **u** we generate the output **y**, as presented in Fig. 1. The input and output are additionally blurred with a controlled level of white noise that simulates measurement error. In that way, we approach real conditions and have a more realistic experiment: we measure input and output time histories. The detection procedure identifies the system properties from measured time histories.

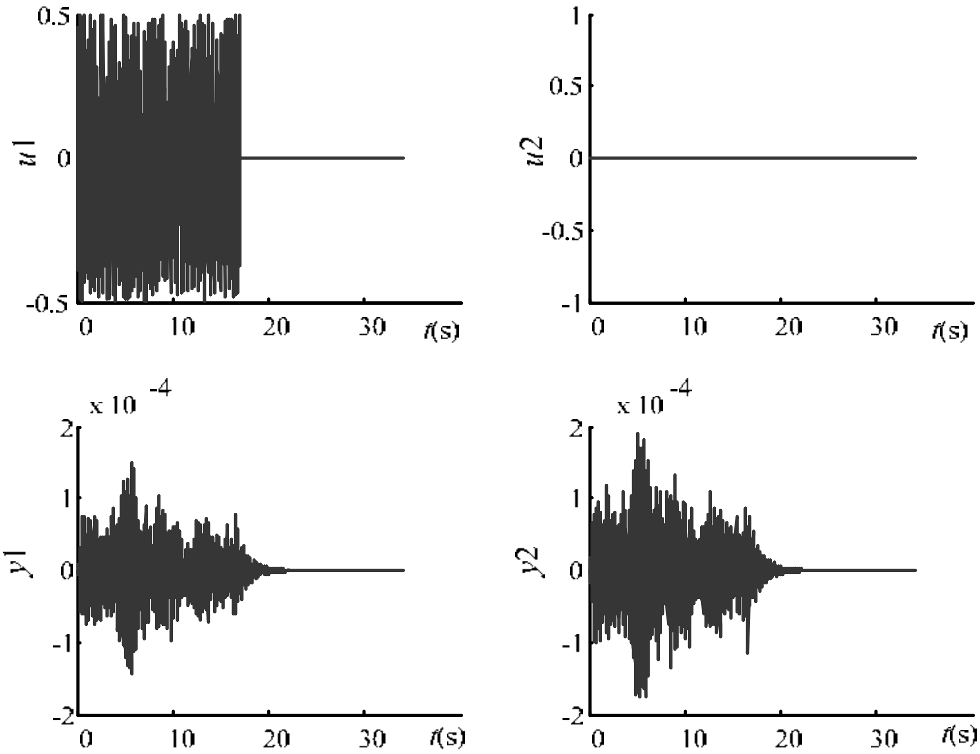


Fig. 1 Input-output time histories for random input at dof 1 and zero input at dof 2, SIMO (Single Input Multi Output)

In practice, we perform a number of experiments in order to minimize the influence of the measurement error by means of averaging.

4.1 Analysis in the frequency domain

Once measuring data are available, we proceed with data processing. The analysis in the time and frequency domain is possible. First, we present the results of the Fourier analysis.

Each experiment consists of three successive excitations in order to perform averaging. Applying the window functions and the Fourier transform we get frequency response functions (FRFs). Then, the least square approach is typically applied in order to detect modal parameters of the system.

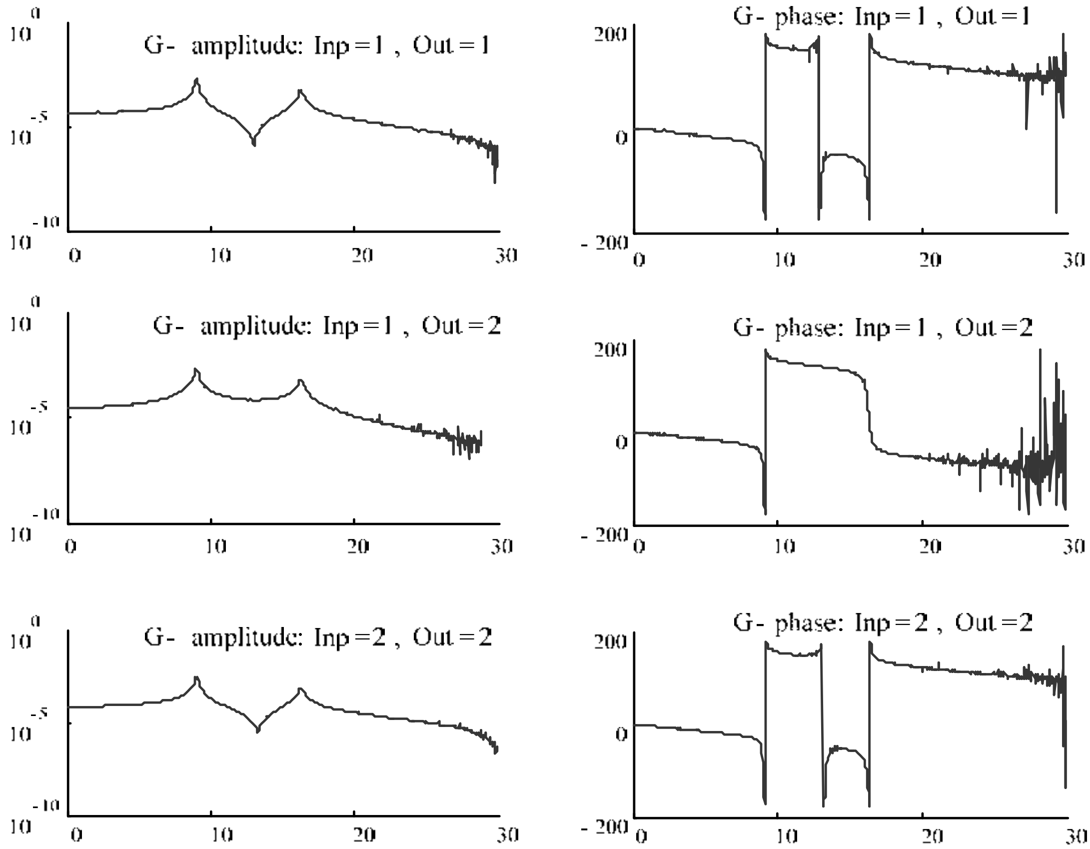


Fig. 2 Frequency response function for the given 2 dof system (horizontal axes frequency ω (s^{-1}))

A reliable detection of the system modal parameters is crucial for the modal update based on equations (15)-(17). However, these modal parameters are not required in the model update based on (15).

4.2 Analysis in the time domain

In the time domain we use the very same experiment data as in the frequency domain. The analysis is based on the observer/Kalman approach to calculate observer Markov parameters and then on the recursion to calculate system Markov parameters. The system Markov parameters present the system pulse response, Figure 3.

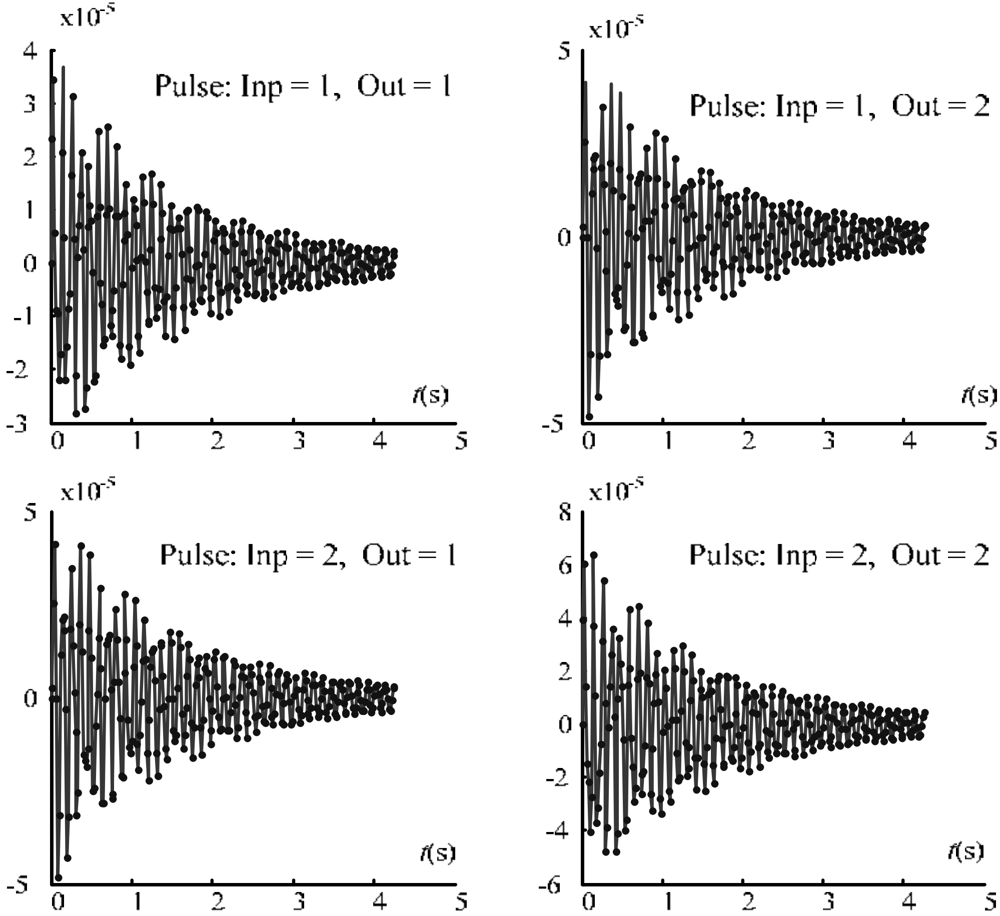


Fig. 3 Pulse response based on the system identified with the observer approach, p=4.

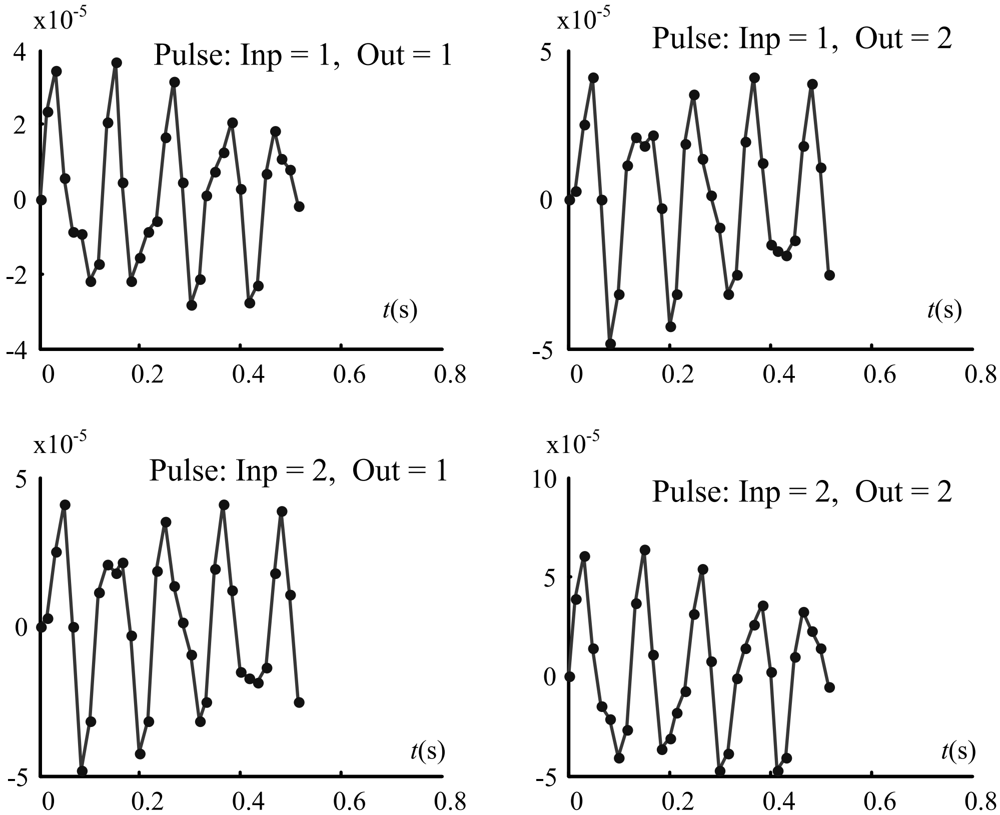


Fig. 4 First 32 steps of pulse response

The identified pulse response of the system is presented in Figures 3 and 4 with a line. The dots in these figures are analytical pulse response given for reference. A very good agreement of the identified and analytical pulse can be observed.

4.3 Model update

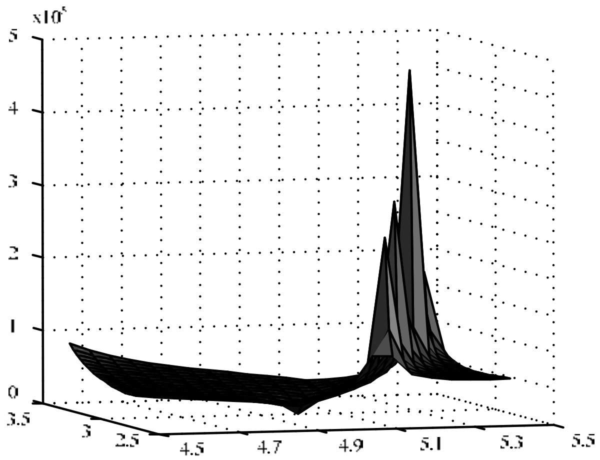
The unconstrained Genetic Algorithm (GA) optimization procedure is used in the model update.

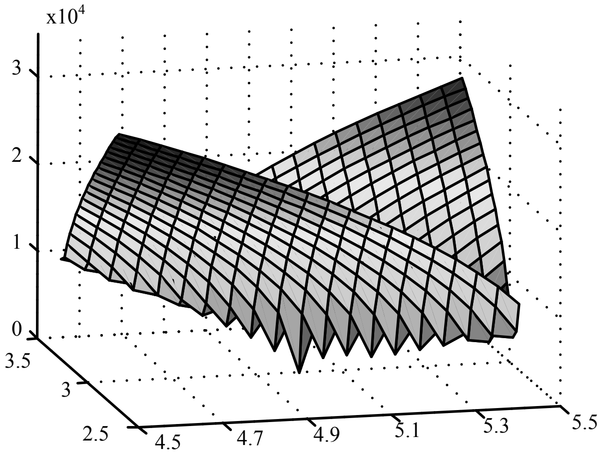
Structural model parameters: m1, m2, k1, k2,… can be used as optimization variables in the model update. The circumstances of the equivalence of a mass and a stiffness structural change under criterion (15) are discussed in [14].

The objective functions discussed in paragraph 3 are implemented as standalone and in combination with weighting parameters to prove the algorithm and implementation.

In order to efficiently visualize the objective function only 2 optimization variables are used in this illustrative example.

The graphical presentation of the objective functions based on (15) and (18) criteria are presented in Figures 5 and 6.





The identified mass of the 2 dof system

Fig. 5 Based on the criteria (15) **Fig. 6** Based on the criteria (18)

Optimization algorithms typically require a starting point/variant (or a set of starting points, population). In our case we started the procedure with a number of randomly selected starting variants and the genetic algorithm detected the optimum repeatedly.

5. Case study: frame structure model update

The frame structure according to Figure 7 is modelled with beam elements. The concentrated masses are added at the end nodes (25, 26, 27, 28) of the structure. The amount of the masses added at the end nodes is taken as model update variables. The degrees of freedom that are considered in the analysis are at nodes 15, 27 and 28 in the directions of the global coordinate axes.

The natural frequencies and modes are available from the FEM model of the structure.

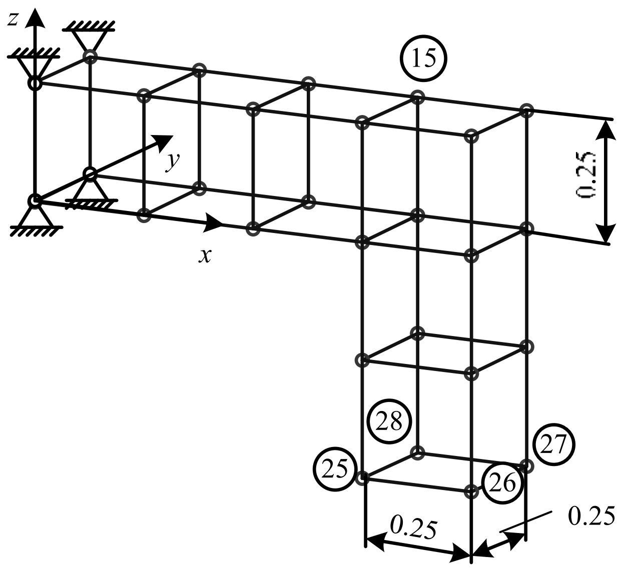


Fig. 7 Model of the test structure, dimensions given in m

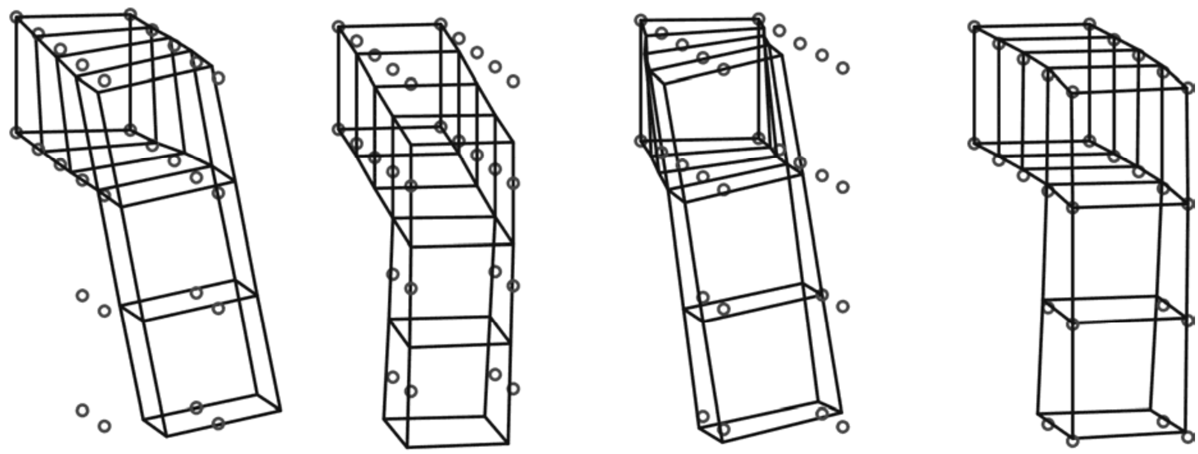
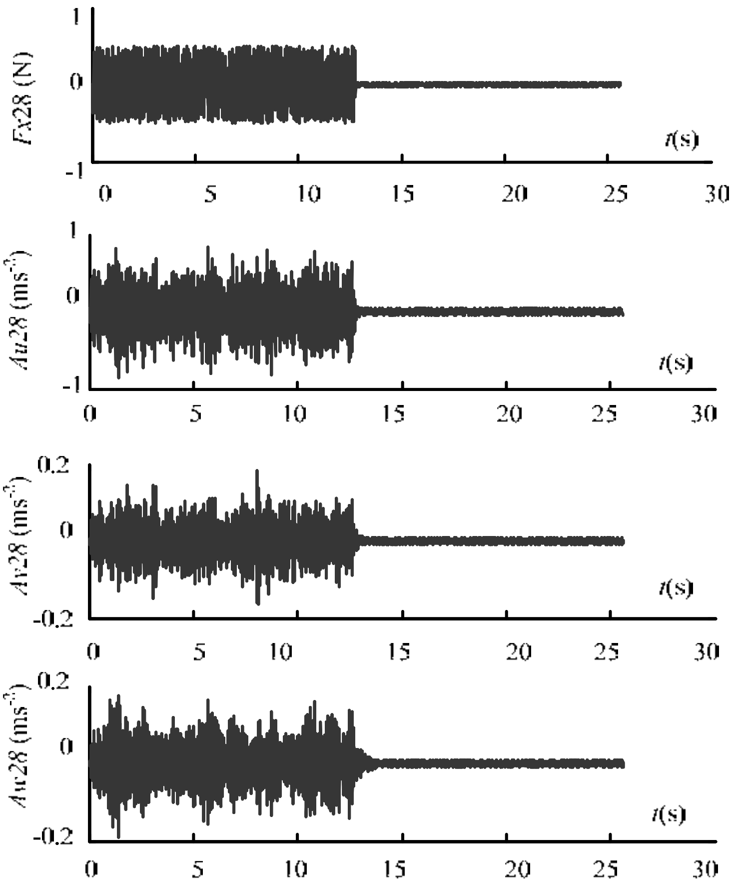


Fig. 8 Natural modes of the test structure FEM model

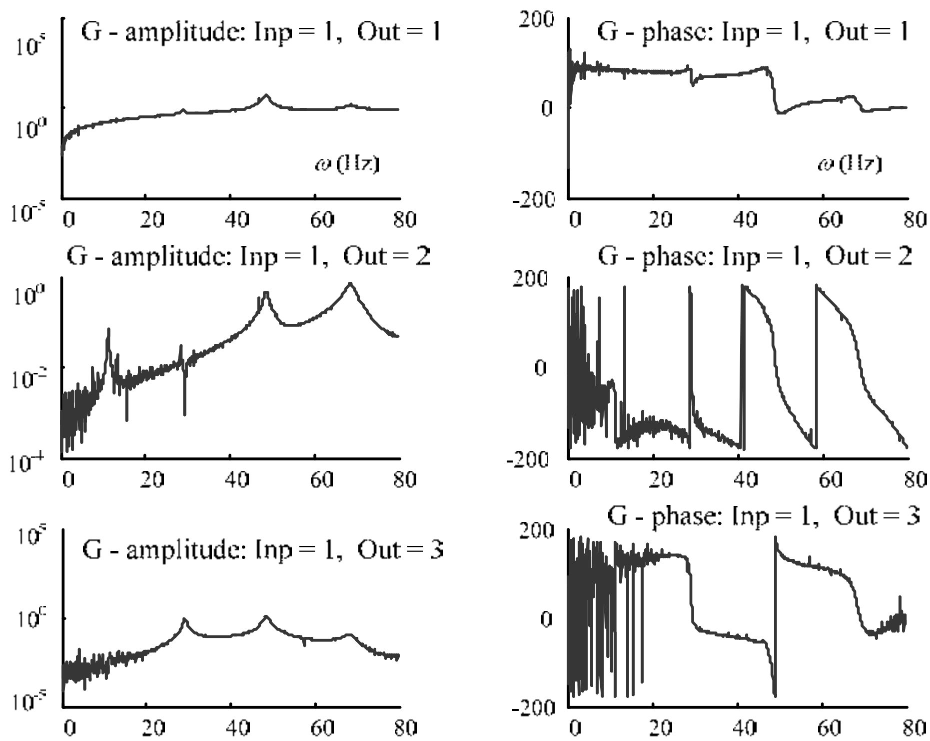
The test structure FEM model frequencies (Hz) are as follows:

1. 11.4958
2. 29.7222
3. 44.6608
4. 50.1884
5. 69.3742

The Single Input Multi Output (SIMO) virtual experiment has been performed for the considered dofs with random excitation. The force has been applied along one dof at a time and response (acceleration) has been recorded for all considered dofs. The procedure has been repeated three times in order to enable the averaging procedure take place. The excitation force at node 28 in the *x* direction and responses at node 28 (*x*, *y* and *z* directions) and the corresponding FRFs are presented in Figure 9.



a) Force F_x at node 28 and acceleration time history of dofs at node 28



b) Frequency response functions for excitation in x direction, dofs at node 28

Fig. 9 Virtual experiment performed on the laboratory test structure: Input force at node 28 in x direction and accelerations of node 28 in x, y, and z directions: a) input-output time histories and b) corresponding transfer functions (FRFs).

The model update according (15) is efficient only when measurements at significant dofs are available. We first examine effects of the model update in the case when measurement is available only for one dof, i.e. displacement in the direction of the *x* axis at node 28. In that case, the model update can be performed for criteria (18) only.

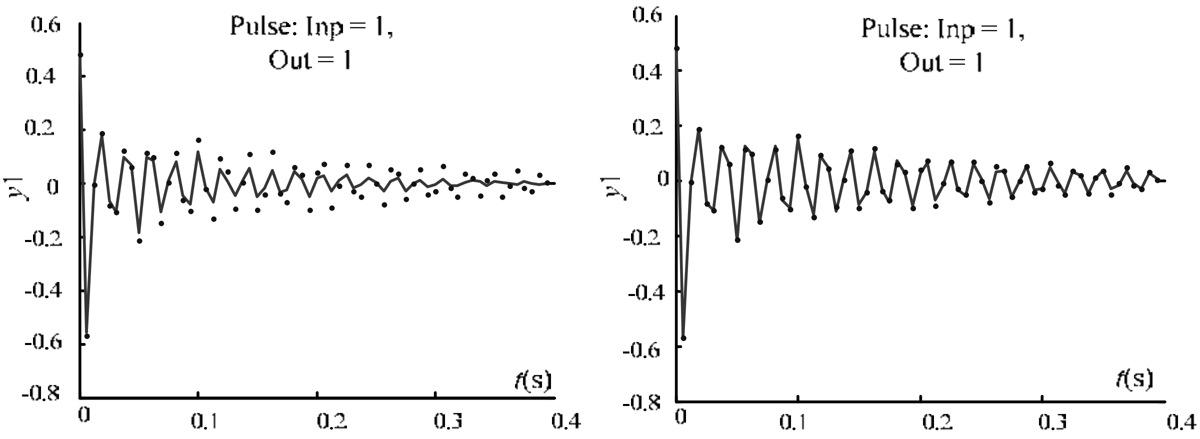


Fig. 10 Comparison in time domain: pulse response of node 28, *x* direction: virtual experiment (dots) and FEM model before update (line, left) and FEM model after model update (line, right)

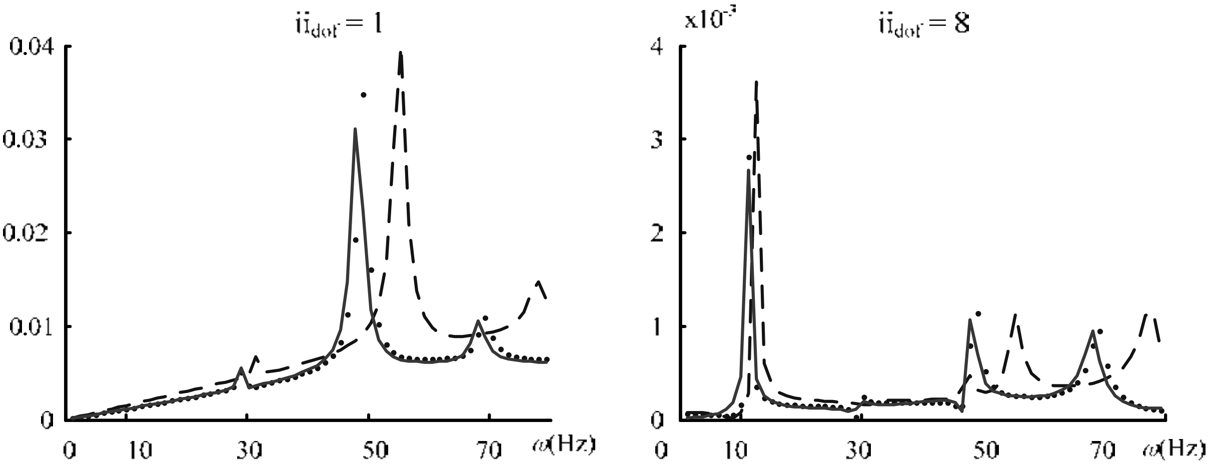


Fig. 11 Comparison in frequency domain of virtual experiment (dots), FEM model before update (dash line), and updated FEM model (solid line)

We observe a substantial improvement of the time response of the updated structure at the considered dof, Fig 10. In the frequency domain, the FRF is improved too, Fig 11, however, the solution is far to be unique. For a unique solution, additional requirements-constraints should be included. Including additional constraints (or optimization criteria) progressively decreases the efficiency of the approach. The efficiency depends on the number of steps of the pulse response involved in the optimization. A smart selection of the parameters (that is the trade-off between efficiency and accuracy) such as considered dofs, the number of included time steps etc., can make the model update acceptably efficient.

The modal update in the time domain only, without a MAC constraint, certainly does not guarantee a global optimum/solution, however, a substantial improvement can be expected at optimized dofs. For a more reliable global solution, a MAC constraint should be included. However, the implementation of MAC criteria requires a model reduction or experiment expansion to the dofs of the mathematical model which requires an additional numerical effort. The procedure that includes a model reduction is typically more efficient when the reduced model is used in the state space time domain for the pulse response calculation.

6. Conclusion

The discrete time model identification technique is presented. The observer/Kalman procedure is implemented in order to detect system pulse response. The model update based on the genetic algorithm and objective function that weights the discrepancy between the pulse response of the test structure and the model is developed. A model reduction is not necessarily required in this approach. Advantages and disadvantages of the proposed approach are discussed. The proposed approach is especially useful when the time excitation and response are available for a limited number of dofs that are of our major interest. However, this approach can miss the response of unsupervised parts of the structure and additional constraints to the mode shapes may be necessary. The provided case study has shown that the finite element model of the structure can be efficiently updated by using the genetic algorithm and the pulse response.

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