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Articles

THE GEOMETRIES OF SITUATION AND EMOTIONS AND THE CALCULUS OF CHANGE IN NEGOTIATION AND MEDIATION

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This article is the fifth in a series authored by Mr. Cooley that describe a new approach to interpersonal problem solving called "Pracademics"—the application of classical methods to achieve practical solutions in negotiation and mediation. Citations to the other four articles appear in *infra* notes 1, 27, 185, and 336.

I. INTRODUCTION

This Article is a sequel to *Descartes' Analytic Method and the Art of Geometric Imagineering in Negotiation and Mediation*, published in Volume 28 of the *Valparaiso University Law Review* in the Fall of 1993.[1] While a review of that article may be helpful in more quickly grasping the ideas presented below, it is not absolutely necessary. As in the Descartes article, definitions of mathematical terms and expressions are liberally provided throughout this article to enlighten the reader as to the simple essences of the pertinent mathematical concepts. Every effort has again been made to minimize the pure mathematics aspects of the discussion, to simplify mathematical concepts actually discussed, and to analogize, in an uncomplicated way, those aspects of the concepts which have direct or indirect application to solving real-life problems presented in transactions or in disputes. Most everyone having a basic understanding of high school level mathematics and of the psychology of visual perception should have no trouble understanding the material presented here.

Every dispute or transaction has two principal elements which must be addressed by a negotiator or mediator: (1) the *substance* of the dispute or transaction and (2) the *relationship* between or among the parties.[2] The Descartes article presented an analytical and geometric imagineering paradigm for use in addressing the *substance* of a dispute or transaction in the context of collaborative negotiation.[3] This Article presents an analytical and geometric imagineering paradigm for use in addressing the *relationship* element of a dispute or transaction.

The primary distinction between the two paradigms exists in the timing and sequence of their use in relation to collaborative negotiation. The Descartes paradigm is normally used *during the course* of collaborative negotiation to

1. The word "imagineering" here means the mental and/or graphical construction and use of visual images in problem solving. The expression "geometric imagineering" connotes visual images that are in the shape, form, or nature of geometric figures, lines, curves, or constructs. This article is a sequel to the Descartes article in the same sense that D. Burger's book, SPHERELAND: A FANTASY ABOUT CURVED SPACES AND AN EXPANDING UNIVERSE (1965) is a sequel to Edwin Abbott's book, published originally in 1880, entitled FLATLAND: A ROMANCE OF MANY DIMENSIONS (5th ed. 1967). *See* John W. Cooley, *Descartes' Analytic Method and the Art of Geometric Imagineering in Negotiation and Mediation*, 28 VAL. U. L. REV. 83, 130 n.122 (1993). The present article and *Sphereland* each venture into a new dimension of imagination.

2. ROGER FISHER ET AL., GETTING TO YES: NEGOTIATING AGREEMENT WITHOUT GIVING IN 19-21 (2d ed. 1991).

3. For purposes of the Descartes article and the present one, "collaborative negotiation" means group decisionmaking in which emotional issues are not present (non-existent or already dissipated) and positional bargaining (right vs. wrong; rights vs. duties) is not present or has been concluded, and the parties are focusing on their interests (or needs) and resources to satisfy those interests with the goal of achieving a mutually acceptable solution.

facilitate group decisionmaking; the paradigm described in this Article, hereinafter referred to as the Leibniz paradigm, is normally used preliminarily by a negotiator or mediator to perceive ways to *prepare* and to *acclimate* the parties to collaborative negotiation. In short, this Article will ultimately explore the geometric and calculus-based visualization techniques to be used *preliminarily* by negotiators and mediators to address issues pertaining to the *relationship* between or among parties in dispute and transaction settings. This Article will make this journey in four exploratory segments, Sections II through V.

In Section II, this Article shall examine the life and general accomplishments of the inventor of the infinitesimal calculus (differential and integral calculus), Gottfried Wilhelm von Leibniz (pronounced Libe - nits), with special emphasis on his abilities as a lawyer and a problem solver. Rising to prominence in seventeenth-century Europe, Leibniz[4] was a highly celebrated product of what Alfred North Whitehead called "the century of genius."[5] He exemplified the Renaissance ideal of the universal man with his multi-faceted interests and accomplishments. He was a lawyer, scientist, inventor, diplomat, poet, philologist, logician, moralist, theologian, historian, and philosopher.[6] But, above all, he was a problem solver, intent on identifying a universal method for interpersonal communication and for solving problems, which he referred to as the "universal characteristic," an essential ingredient for his dream of human progress and social harmony.[7]

In Section III, this Article shall explore Leibniz's theory of relations and his ideas concerning the geometry of situation, which he documented in 1679,

4. Some readers who took college courses in calculus might be more accustomed to the spelling, "Leibnitz." Actually, during his twenties, Leibniz changed the spelling of his name from the version used by his father ("-utz") to "-iz." He never personally used the "itz" form, which became the publicly accepted spelling during his lifetime and which has only recently gone out of fashion. G. MACDONALD ROSS, LEIBNIZ 3 (1984). The first occasion that Leibniz was referred to as "von" Leibniz was in 1700, when he was made Life President of the Brandenburg Society of the Sciences. However, there is apparently no historical information available to document any official elevation of him to the barony. *Id.* at 22.

5. Phillip P. Wiener, *Introduction* to LEIBNIZ: SELECTIONS at xi (P. Wiener ed., 1951). In order to put Leibniz's life in chronological perspective, we should observe that Francis Bacon died twenty years prior to Leibniz's birth, and that Newton was born in 1642, four years prior to Leibniz's birth and in the same year that Galileo died. When Leibniz was born, Descartes was 50, Hobbes was 58, and Locke and Spinoza were each 14. During his lifetime, Leibniz corresponded with experimenters in physics, Huygens, Von Guericke, Boyle, Newton, Papin, Perrault, Mariotte, and with biologist Leeuwenhoek. In mathematics, Leibniz was thoroughly familiar with the works of Pascal, Fermat, and Descartes, and he corresponded with Bernouillis, Sturm, Goldbach, Wallis, Wren, and Newton. *Id.*

6. *Id.* at xi. *See generally* IVOR LECLERC, THE PHILOSOPHY OF LEIBNIZ AND THE MODERN WORLD (1973).

7. Wiener, *supra* note 5, at xi, xxvi-xxxiii.

four years after he conceived the foundations of the infinitesimal calculus and three years after he became acquainted with Benedict Spinoza's geometry of emotions as later published in the *Ethics* in 1677. In Section III, this Article shall also familiarize the reader with the relationship between geometry of situation and modern General System Theory, and their application to the assessment of system function in inter-personal situations.

In Section IV, this Article shall examine, in some detail, another dimension of relationship—Spinoza's geometry of emotions—as a prelude to considering a modern theory describing the cognitive structure of emotions. Finally, in Section V, this Article will explore the basics of differential and integral calculus, relating the use of systems models as instruments of change and extrapolating visualization techniques that can be applied as acclimators for collaborative negotiation.

The primary vehicle that this Article will use to explore these topics is the structure and relations extant in a family system. This particular vehicle has been chosen because most readers will be able to identify with conflict, emotions, and behavior within a family system. Knowledge of the behavior of a family system is also quite helpful in negotiating and mediating resolution of divorce and domestic relations conflicts and family-owned business conflicts. Family system behavior also impacts on the development and resolution of conflicts occurring both in the workplace and with respect to two or more companies where there is a long-standing, ongoing business relationship.

This Article will now begin the voyage into the relationship aspect of problem solving by examining the family roots, the educational preparation, and the professional background of Leibniz—our host for our journey into the geometries of situation and emotions and the calculus of change in negotiation and mediation.

II. LEIBNIZ—THE PERSON; THE LAWYER; THE PROBLEM SOLVER

A. Leibniz—The Person

Gottfried Wilhelm von Leibniz was born on July 1, 1646, in Saxony at Leipzig, which had been a prominent seat of German learning and science since the Renaissance.[8] At the time of Leibniz's birth, his father, Friedrich Leibniz, was Vice Chairman of the faculty of philosophy and Professor of Moral

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8. E.J. AITON, LEIBNIZ: A BIOGRAPHY 9 (1985); E.T. BELL, MEN OF MATHEMATICS 120 (1937); JOHN COTTINGHAM, THE RATIONALISTS, 4 A HISTORY OF WESTERN PHILOSOPHY 23 (1988); ROSS, *supra* note 4, at 3.

Philosophy in the University of Leipzig.[9] Leibniz's mother, Catharina Schmuck, was Friedrich's third wife and the daughter of a celebrated Leipzig lawyer.[10] During his early childhood, Leibniz was not inclined to play, preferring instead to read history, poetry, and literature.

When he was seven years old, Leibniz entered the Nicolai School in Leipzig, where he remained until 1661. Upon entering school, he immediately began to teach himself to read Latin, and, through the intercession of a family friend and nobleman a year later, he was permitted access to the locked library of his father, who had died two years earlier. There, while only eight years old, he read the Latin classics and other materials on a variety of subjects, including metaphysics and theology, as impulse guided him. But it was the traditional syllogistic logic of Aristotle which he encountered in the upper classes of his elementary education that first awakened his inventive genius and later inspired "a certain alphabet of human thought"—the universal characteristic.[11]

In 1661, Leibniz entered the University of Leipzig.[12] There he took a two-year arts course which included philosophy, rhetoric, mathematics, Latin, Greek, and Hebrew.[13] After preparatory courses, he opted to study law, an activity in which he engaged for the next three years.[14] Were it not for some unfortunate university politics, described in more detail *infra*, and a fortunate acquaintanceship with the Baron of Boineburg, the former chief minister of the Elector of Mainz, Leibniz would have probably pursued a career in legal academia. Boineburg, impressed with Leibniz's energy and erudition, offered him employment that involved him in the pursuits of the courtier and diplomat.[15]

Under Boineburg's patronage for the next six years, from 1666 to 1672, Leibniz served the well-connected Boineburg in a variety of capacities and made contact with many political thinkers, philosophers, and men of letters of the

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9. AITON, *supra* note 8, at 9. Friedrich had two children, a boy and a girl, from his first marriage. *Id.* at 9-10. It is said that Friedrich was a competent, though not original, scholar who devoted his time to his academic work and to his family. *Id.* Friedrich died six years after Gottfried's birth. COTTINGHAM, *supra* note 8, at 23.

10. AITON, *supra* note 8, at 10. Leibniz's mother was born in Leipzig and was orphaned at the age of eleven. She was raised in the home of Johann Hopner, Professor of Theology. Before her marriage to Friedrich, she lived in the home of her guardian Quirinus Schacher, a Professor of Law. Leibniz's relatives, on both his father's and mother's sides, enjoyed good social standing and scholarly reputations. Gottfried's mother died when he was 17. *Id.*

11. AITON, *supra* note 8, at 10-13. *See also* BELL, *supra* note 8, at 121.

12. AITON, *supra* note 8, at 13.

13. ROSS, *supra* note 4, at 4.

14. *Id.* *See also* AITON, *supra* note 8, at 16-22; BELL, *supra* note 8, at 121-24.

15. COTTINGHAM, *supra* note 8, at 24.

day.[16] Boineburg and other members of his circle were converts from Lutheranism to Catholicism. Although tempted to convert from Lutheranism to Catholicism on several occasions, Leibniz remained a Lutheran and was able to move easily in Catholic circles. He was ideally situated to further one of his life's ambitions—the reunification of the two Churches.[17] As time passed in Boineburg's friendship, Leibniz began to build a formidable circle of correspondents, which eventually numbered more than one thousand.[18]

One major problem of interest to Boineburg and, therefore, to Leibniz at the time, was the imperialistic designs of King Louis XIV of France. A German diplomatic initiative associated with this problem led to Leibniz being sent to Paris in 1672, where he spent four exciting years which proved to be "a source of immense and fruitful intellectual stimulus" for him.[19] He began a serious study of Descartes' philosophy, studied with the famous physicist and mathematician Christian Huygens, and had discussions with the famous logician and theologian Antoine Arnauld, one of Descartes' most searching critics.[20] He also visited London during this time, met with English scientists Henry Oldenburg and Robert Boyle, and discussed ideas he had for the design of a calculating machine.[21] Although he was on a diplomatic mission, mathematics

16. *Id.*

17. ROSS, *supra* note 4, at 6. Leibniz was offered the prestigious librarianship of the Vatican in 1689 and of Paris in 1698. Leibniz turned down both these positions because a condition of employment was the conversion to Catholicism. *Id.*

18. COTTINGHAM, *supra* note 8, at 24-25. In the seventeenth century, it was commonplace for scholars to correspond with each other on various topics concerning science and mathematics. There were several reasons why this practice emerged. Of the two principal reasons, one was based on fear, the other on pragmatism.

The principal reason was that discussions about natural science were usually at odds with the teachings of the Catholic Church. Therefore, publication of a scholar's scientific ideas could often precipitate castigation of the scholar by the Catholic Church. Galileo encountered such treatment when he disagreed with the Catholic Church's false teaching, at the time, that the sun revolved around the earth. *See* COTTINGHAM, *supra* note 8, at 14; T.Z. LAVINE, FROM SOCRATES TO SARTRE: THE PHILOSOPHIC QUEST 88 (1984). For non-Catholic scientists, mathematicians, and scholars, the reason for correspondence was more pragmatic. Printers were reluctant to accept books on mathematics because of difficulties with typesetting and because of the small number of potential readers for the finished publications. Thus, correspondents shared their ideas in writing but, even so, rather cautiously out of fear of plagiarism. Registering their findings or the results of their experiments with the Royal Society or the Paris Academy provided a means for the correspondents to establish a claim to an invention, pending publication at a later time. AITON, *supra* note 8, at 63.

19. COTTINGHAM, *supra* note 8, at 25.

20. *Id. See also* BELL, *supra* note 8, at 124-25.

21. By 1675, Leibniz had developed his calculating machine to a stage permitting its demonstration in the Paris Academy of Sciences. This demonstration brought orders for three production models. In that same year, Leibniz introduced another of his inventions, a chronometer, at the Paris Academy of Sciences. AITON, *supra* note 8, at 53.

became Leibniz's chief interest in his Paris years.[22] In early 1673, Leibniz was notified of Boineburg's sudden death, which occurred in December of 1672, shortly after Boineburg had sent his son to Paris to be educated in Leibniz's charge.[23] By the time his Paris experience was coming to an end in 1675, Leibniz had arrived at one of his most celebrated discoveries—the theory of the infinitesimal calculus.[24] With this discovery, he "marked the paths on which the thoughts of men should travel for centuries after him"[25] and aptly demonstrated the two unique characteristics of his mind: "the love of practical application, and the desire to go to the root of everything."[26]

Although he had sought a research post at the Paris Academy, he was compelled by his growing financial debt to assume, in December of 1676, the post as Court Councillor in Hanover, Germany.[27] In those years, Hanover was one of a hundred or so independent states under the leadership of the German Emperor in Vienna. The autocratic head of Hanover, Duke Johann Friedrich of the House of Brunswick, acted through a council composed largely of lesser aristocrats and law graduates. Leibniz was able to negotiate some relief from normal council duties because of his responsibilities as librarian, political adviser, international correspondent, and technological adviser. His selection of these special responsibilities was a masterful coup which permitted him to pursue his academic endeavors while occupying a government position for approximately the next forty years.[28]

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22. Actually, Leibniz's official reason for being in Paris came to nothing. He never found an opportunity to present the diversionary Egyptian plan (an enticing scheme for a French conquest of Egypt) to the King. ROSS, *supra* note 4, at 11-12.

23. *Id.*

24. COTTINGHAM, *supra* note 8, at 25.

25. JOHN T. MERZ, LEIBNIZ 128 (1948).

26. *Id.*

27. COTTINGHAM, *supra* note 8, at 14. In October of 1676, while traveling on a yacht back to Germany from England, Leibniz composed a dialogue on the philosophical principles of motion. In this dialogue, Leibniz is represented by the character Pacidius, a conciliator gifted in uniting all scholars in a common task. This is just one of several works that Leibniz wrote in dialogue form. In the same year, he composed a dialogue based on the "recollection" theory of learning demonstrated in Plato's *Meno*. In this dialogue, Leibniz, under a pseudonym, takes the role of Socrates and employs dialectic in a conversation with a young boy to teach him a complete introduction to arithmetic and algebra rather than the single geometric problem demonstrated by Socrates in the *Meno*. *See* AITON, *supra* note 8, at 68. For an example of how the Socratic technique of "recollection" can be used by a mediator in resolving a dispute, see John W. Cooley, *A Classical Approach to Mediation—Part II: The Socratic Method and Conflict Reframing in Mediation*, 19 U. DAYTON L. REV. 589 (1994).

28. ROSS, *supra* note 4, at 14; COTTINGHAM, *supra* note 8, at 26. As another commentator has remarked:

> Following . . . a deep-rooted inclination of his nature, he chose positions in life, in which he was comparatively unmolested, where he could observe and sympathise with the work of those around him, without being forced to take a leading part in it. He held

During the period of 1676 to 1686, Leibniz's efforts were primarily focused on technological innovation. During this time, he became practically obsessed with the problem of draining water from the mines in the Harz mountains. Leibniz persuaded the Duke to let him attempt various experiments, and he designed all sorts of windmills, gearing mechanisms, and pumps which included Archimedean screws, syphons, compressed-air power links and even a forerunner of the modern rotary pump. None of these mechanisms was ever approved for the Harz mines because, according to Leibniz, these ideas were deliberately sabotaged by administrators, technicians, and workers who feared that technological progress would cost them their jobs.[29]

In 1686, Leibniz left the Harz mountains to undertake a serious historical research project commissioned by Duke Johann Friedrich's successor, Ernst August. The new duke contracted with Leibniz to write a history of the whole Guelf family of which the House of Brunswick was a branch. Leibniz was away from Hanover from 1687 to 1690, collecting archival information on the Guelf family in Bavaria, Austria, and Italy.[30] On returning to Hanover, he returned to his librarian and other duties and, over the next few years, negotiated part-time paid appointments at several other courts ruled by branches of the Brunswick family.[31]

During the period of 1698 to 1714, Leibniz's innate diplomatic skills and negotiating ability became widely recognized in Europe. During these years, he managed to persuade Anton Ulrich of Brunswick to drop his claim to the ninth Electorate of the German Empire and then, later in 1702, acted as an intermediary between Anton Ulrich and the Hanover Elector Georg Ludwig when relations became strained over what diplomatic approach should be taken

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himself aloof from the crowd, and cultivated those relations in which there was neither much waste of feelings nor food for the emotions.

MERZ, *supra* note 25, at 129.

29. ROSS, *supra* note 4, at 16-17.

30. *Id.* at 16-19. As an interesting aside, one of the by-products of Leibniz's archival work in Italy was a detailed refutation, published after his death, that there had been a female English Pope (*Flowers Scattered on the Grave of Pope Joan*). Another by-product was an edition, published in 1696, of Johann Burchard's scurrilous diary of life at the court of the Borgia Pope Alexander VI. This latter work was the only example of Leibniz's writings to be listed in the Vatican's *Index of Prohibited Books*. *Id.* at 20.

31. During the time of his historical research, Leibniz demonstrated superior negotiating ability and opportunism. While in Italy, Leibniz took advantage of the opportunity to get acquainted with many scholars and scientists, and he was eventually elected to be a member of the Physico-Mathematical Academy of Rome. In Vienna, he engaged in many discussions on church unity and favorably impressed Emperor Leopold I. During this time, he also completed his first successful diplomatic mission which was to negotiate the marriage of Duke Johann Friedrich's daughter (Charlotte Felicitas) to the Duke of Modena. Ross, *supra* note 4, at 18.

by the German states in relation to France.[32] At the turn of the eighteenth century, he also played an important intermediary role in delicate negotiations between London and Hanover over the question of succession to the British throne. The 1689 Bill of Rights had excluded Catholics from the throne of England and had made it almost inevitable that succession would pass to Georg Ludwig, the eldest son of the wife of Ernst August. This inevitability was eventually embodied in the British Act of Succession of 1701, and Leibniz took pride in his partial responsibility for the outcome.[33]

At about this same time, Leibniz became an enthusiastic promoter of scientific academies.[34] He became involved in tentative plans for such academies in Mainz, Hanover, Hamburg, and Poland. His main efforts in this regard, however, were directed toward Berlin, Dresden, Vienna, and St. Petersburg. In 1700, Leibniz negotiated final approval for a scientific academy in Berlin, and he was made Life President of the "Brandenburg Society of the Sciences." Although the Society met regularly to discuss scientific subjects, only one volume of its proceedings was ever published. The Society did, however, enhance the growing prestige of Prussia and later formed the basis of the German Academy of Sciences in Berlin. After this success in Prussia, Leibniz was equally successful in negotiating proposals for scientific academies with the German Emperor in Vienna and with Peter the Great of Russia, though neither of these academies actually came into existence until after Leibniz's death.[35]

By 1712, Leibniz had spread himself quite thin by traveling all over Europe promoting science. He was, at the time, on the payroll of five different courts, and each court believed that Leibniz was not giving it its money's worth in services. Complaints came strongest from Hanover, where Leibniz had been employed the longest and to whose leaders he still owed production of the Guelf family history, which by then had been in preparation for over thirty years. Leibniz stayed in Vienna for nearly two years, ignoring repeated orders to return to Hanover. He returned to Hanover in September of 1714, about the same time that Georg Ludwig acceded to the English throne.[36]

Leibniz never wrote a *magnum opus* comparable to Descartes' *Principles* or Spinoza's *Ethics*, but in his mature years he did produce a formidable number of shorter essays, articles, and pamphlets. Among the most important of these writings were the *Meditations on Knowledge, Truth and Ideas*, published in

32. *Id.* at 20-21.
33. *Id.* at 21.
34. *Id.* at 22-23. *See also* BELL, *supra* note 8, at 129.
35. ROSS, *supra* note 4, at 22-23.
36. *Id.* at 24.

Latin in 1684; the *Discourse on Metaphysics*, composed in French in 1686; the *Remarks on the General Part of Descartes' Principles*, a critical examination of Descartes' system, written in the early 1690s; the *New System* and *Explanation of the New System*, published in French in 1695-96; the *New Essays on Human Understanding*, an extensive dialogue criticizing John Locke's *Essay Concerning Human Understanding*, written in French in the early 1700s, but published posthumously; and a concise summary of Leibniz's metaphysics, the *Monadology*, written in French in 1714, but published posthumously in 1720.[37] The latter work, the *Monadology*, described the theory of his philosophy of metaphysics developed between age forty and his death at age seventy.[38]

Leibniz's last two years of life, from 1714 to 1716, were ones of miserable neglect in Hanover.[39] He was no longer in the good graces of the Court Council, though Georg Ludwig took him on a holiday on one of his visits to Hanover from London. In his late sixties and too infirm either to travel or to start a new life, he made the extraordinary suggestion that he become the historian of England. That did not come to pass. However, at the same time he did receive an invitation from Louis XV to live in Paris, and he would have gone there, had Louis not died in 1715. Although he worked very hard on the Guelf's family history, he never completed it.[40]

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37. COTTINGHAM, *supra* note 8, at 26-27. It is said that a lifelong characteristic of Leibniz was his ability to work anywhere, at any time, under any conditions. As one commentator has observed:

> He read, wrote, and thought incessantly. Much of his mathematics, to say nothing of his other wonderings on everything this side of eternity and beyond, was written out in the jolting, draughty rattletraps that bumped him over the cow trails of seventeenth century Europe as he sped hither and thither at his employers' erratic bidding. The harvest of all this ceaseless activity was a mass of papers, of all sizes and all qualities, as big as a young haystack, that has never been thoroughly sorted, much less published. . . . [As of 1937, it lay] baled in the royal Hanover library waiting the patient labors of an army of scholars to winnow the wheat from the straw.

BELL, *supra* note 8, at 122.

38. In Leibniz's metaphysics, there exist only what he referred to as "monads." As one commentator explains:

> [M]onads are nothing other than actualized sets of perceptions defined by a particular point of view. Every perception is both spontaneous (arising from the essence of the individual monad) and harmonious (adapted to the rich pattern of the whole universe). Form and matter represent these two complementary aspects. A monad is a form or spirit in so far as it is spontaneous, active, and purposeful; it belongs to the realm of material bodies in so far as it is accommodated to the actions of other substances through the laws of mechanics. For all created beings, the bodily dimension is inescapable. Without it, they would be wholly active and perfect, which is a privilege reserved for God alone.

ROSS, *supra* note 4, at 100.

39. *Id.* at 24.

40. *Id.* at 24-25.

After a week-long battle with colic and gout, Leibniz died peacefully in Hanover at the age of seventy, on November 14, 1716, in the presence of his secretary and coachman. The rump of the Hanover Court Council refused to attend his funeral. He was buried in a simple ceremony without fanfare.[41]

Curious about everything and exploring practically every sphere of human potential, Leibniz was once ascribed the nickname "Lovenix," or "believer in nothing." Upon hearing this, Leibniz responded characteristically, that indeed he was a *believer* in nothing, because he believed only what he *knew*.[42] In a similar vein, while summing up Leibniz's life and character, one commentator observed:

> Leibniz's life was dominated by an unachievable ambition to excel in every sphere of intellectual and political activity. . . . His successes were due to a rare combination of sheer hard work, a receptivity to the ideas of others, and supreme confidence in the fertility of his own mind. Whenever he tackled a new subject, he would read everything he could lay his hands on, but without submitting to orthodox concepts and assumptions. . . . Despite all his notes, letters and articles, he never wrote a systematic treatise on any of his special interests. His assistant Eckhart put it nicely when he said of the Guelf project that, as with numbers, Leibniz knew how to extend his historical journey to infinity.[43]

B. Leibniz—The Lawyer

In 1663, just before beginning his specialist studies in law at the University of Leipzig, Leibniz, age seventeen, spent the summer semester studying at the University of Jena under Professor of Mathematics Erhard Weigel, a celebrated component of Greek mathematical systems.[44] This mathematical experience was later to have a profound influence on the way he approached the study of law, on his legal writings, and on the way he applied inventiveness and imagination in problem solving. Because of his background in history and philosophy, Leibniz found the *theory* of law in his specialist studies easy to understand, and so he devoted his attention to concepts involving law *practice*. He was quite "attracted to the function of a judge but repelled by the intrigues

41. *Id.* at 25-26.

42. *Id.* at 27.

43. *Id.* at 26. The Guelf (Brunswick) family was so tangled in its marital adventures that Leibniz was able to cover only three hundred years of its history (to the year 1005) by the time of his death. A history of the family was eventually published in 1843. BELL, *supra* note 8, at 130.

44. AITON, *supra* note 8, at 15-16; COTTINGHAM, *supra* note 8, at 24.

of lawyers."[45] This was one reason why he never desired to be a trial lawyer. During his law schooling, he eventually befriended an assistant judge at the High Court in Leipzig who often invited the young Leibniz to his home and taught him by examples how to draft decisions.[46] The judicial function better mapped his personal philosophy, refined in his later years, of problem solving, conciliation, and seeking universal harmony.

In February of 1664, Leibniz graduated as Master of Philosophy[47] and proceeded in the next two years toward achieving his bachelor's and master's degrees in law.[48] At about that time, his settled ambition was to acquire a doctor's degree in law and to join the faculty of law at the University of Leipzig.[49] As a prelude to fulfilling this ambition, he began work on his *Dissertation on the Art of Combination*, which was published in 1666. This method of translating logical combinations of ideas into symbolic language was lauded for its originality and foreshadowed some of Leibniz's greatest discoveries and projects.[50] Despite the erudition and scholarship of his writings, the University of Leipzig refused to award Leibniz a doctor's degree

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45. AITON, *supra* note 8, at 16.

46. *Id.*

47. Leibniz's master's dissertation was entitled, *Specimen quaestionum philosophicarum ex jure collectarum*, a study of the relations of philosophy and law. His stated purpose in choosing this subject for his dissertation was to eliminate what he perceived as law students' contempt for philosophy. Leibniz believed that without philosophy, most legal questions would be a labyrinth without exit. *Id.*

48. Leibniz's bachelor of law dissertation was entitled *De conditionibus* and described his theory of hypothetical or conditional judgments applied to law. In that work, he noted that a hypothetical judgment affirms nothing categorically, neither the hypothesis nor the thesis. He believed that every law was subject to a certain condition. In his view, if this condition was impossible, the law would be null; if necessary (and satisfied), the law would be absolute; if contingent or uncertain, the law would be conditional. He set out this theory in a table like the one shown here, using numerical values of 0 (null), 1 (absolute), and ½ (symbolizing a fraction between 0 and 1 and signifying uncertainty):

Conditio:	*impossibilis*	*contingens*	*necessaria*
	0	½	1
Jus:	*nullum*	*conditionale*	*purum*

In this simple yet elegant table, was the novel suggestion of a calculus of probabilities and of conditional judgments, the latter of which was introduced again by George Boole in the nineteenth century under the label "secondary judgments." *Id.* at 17.

49. COTTINGHAM, *supra* note 8, at 24.

50. *Id. See also* AITON, *supra* note 8, at 17. Leibniz's art of combination is also referred to variously as the art of combinations (plural) and the combinatorial art. This art echoed Descartes' notion of a universal method of problem solving (*mathesis universalis*), and was thought by Leibniz to serve as a key to understanding a whole range of subjects. It was applicable not just to arithmetic, but also to metaphysics, physics, and jurisprudence, among others. *See* COTTINGHAM, *supra* note 8, at 65.

in law and to admit him to its law faculty. One version of the cause for the rejection is that the Dean's wife persuaded her husband to refuse Leibniz a doctorate out of malice towards him.[51] Another version is that the "Leipzig faculty, bilious with jealousy, refused Leibniz his degree, officially on account of his youth, [but] actually because he knew more about law than the whole dull lot of them."[52] Whatever the actual facts, in the Fall of 1666, Leibniz quickly matriculated in the faculty of law of the University of Altdorf, located in the republic of Nuremberg. Within a short period of time, he presented his doctoral dissertation, *On difficult cases in law*, and formally received this doctor's degree in law.[53] Listening to the twenty-one-year-old Leibniz defend his dissertation, the "audience admired the clarity and penetration of his exposition and even his opponents declared themselves to be extraordinarily satisfied."[54] Although offered a full professorship appointment by the University of Altdorf, Leibniz declined it in favor of becoming more involved in the outside world under the patronage of his newly found admirer, Boineburg.[55]

As Boineburg's protege, Leibniz persisted in his interest in the law. In 1667, for example, he published a short treatise entitled *New Method of Teaching and Learning Jurisprudence*.[56] Written on a journey from Nuremberg to Frankfurt, this diminutive treatise encompassed topics, advanced for their time, of the psychology of learning, the organization of knowledge, and the logical bases of law.[57] The book also influenced Leibniz's later appointment to assist in revising the legal code of Mainz. In Part I of the book, Leibniz offered some sage advice about teaching law which still has relevance today:

> Teaching is to the soul as medicine is to the body of an animal. Just as the physician aims to heal (1) carefully, (2) swiftly, and (3)

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51. AITON, *supra* note 8, at 21-22.

52. BELL, *supra* note 8, at 121.

53. AITON, *supra* note 8, at 22. Discussing the idea of "case" in his dissertation, Leibniz, quoting his former mathematics teacher Weigel, drew a parallel between geometers (who first used the term) and jurists. As one commentator observed:

> In opposition to those lawyers who believed no solution to be possible in the kind of difficult case . . . [Leibniz] had in mind, or who advocated a decision by drawing lots, or accepting the personal opinion of an arbitrator, Leibniz held that the law always had an answer. For in cases that were uncertain, the natural reason should be brought in to help and the decision based on the principles of natural justice and international law, which limit and determine the civil law. . . . [S]uch difficult cases . . . [were] resolved by Leibniz with much technical skill.

Id.

54. *Id.*

55. ROSS, *supra* note 4, at 5-6.

56. *Id.* at 7.

57. 1 LEROY E. LOEMKER, GOTTFRIED WILHELM LEIBNIZ: PHILOSOPHICAL PAPERS AND LETTERS 134 (1956).

> pleasantly, so the same things are required in the care of the soul; teaching should be (1) sound, (2) swift, and (3) pleasant. . . . Learning is pleasant, . . . not only if the *ends* proposed are pleasant, but also if the *methods* of learning are pleasant.[58]

Though his interests drifted away from law as he matured, Leibniz kept returning to this youthful project during the course of his lifetime.[59]

Soon after meeting Leibniz and becoming aware of Leibniz's treatise, Boineburg managed to get Leibniz appointed as a legal assistant to the Elector of Mainz's legal advisor, Herman Lasser, who was working on a reform and recodification of civil law. Within a year and a half, Leibniz was promoted to the rank of Assessor in the Court of Appeal.[60] As Lasser's legal assistant, a sort of corporation lawyer,[61] Leibniz undertook to draft the first two parts on *Rational Jurisprudence*, which he titled the *Elements of Natural Law* and the *Elements of Contemporary Civil Law*. The *Elements of Natural Law* related the project of legal reform to the cultural state of Europe and explained Leibniz's principles as they applied to law, ethics, and aesthetics. At the time, it was the most complete of the many studies of natural law.[62] Leibniz's "early writings on jurisprudence . . . reveal[ed] clearly the mixture of scholasticism, novelty of insight, and scientific analysis that was to characterize his mature studies."[63] His success in applying his logical genius to human questions of law cleared the way for a career in the diplomatic world.[64]

C. Leibniz—The Problem Solver

Leibniz's reputation for being a superior problem solver arises not only in relation to his intellectual contribution to the invention of the calculus but also in connection with his demonstrated superb abilities as a conciliator and mediator. Both of these aspects of Leibniz—his mathematical and psychological skills—were highly developed manifestations of a mind cognitively rich with abilities in informational analysis and synthesis.

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58. *Id.* at 138-39 (emphasis in original).
59. ROSS, *supra* note 4, at 7.
60. *Id.* at 6.
61. BELL, *supra* note 8, at 124.
62. 1 LOEMKER, *supra* note 57, at 203.
63. H. CAIRNS, LEGAL PHILOSOPHY FROM PLATO TO HILGER 297, 299 (1949), *quoted in* Wiener, *supra* note 5, at xvii.
64. Wiener, *supra* note 5, at xviii.

1. Solver of Mathematical Problems

To say that Leibniz alone invented infinitesimal[65] calculus would be an overstatement. However, his contribution to the invention was monumental and is recognized today as the starting point of modern mathematics, without which the development of physics beyond the seventeenth century would have been virtually impossible.[66] Actually, prior to Leibniz, the calculus had been developing along two historical threads: one, atomistic, from Democritus, Kepler, Fermat, Pascal, Huygens, to Leibniz; the other, kinematic, from Plato, Archimedes, Galileo, Cavalieri, Barrow, to Isaac Newton, a contemporary of Leibniz.[67] Newton eventually accused Leibniz of plagiarism and contended that the development of his own "fluctionary" calculus, conceived in 1665-66 and first published in 1711, was first in priority over that of Leibniz's infinitesimal calculus, conceived in 1675 and published in 1684. Historical events have proved Newton's claim to be unsustainable.[68] As one commentator remarked:

> Even when Newton's "method of fluxions" was eventually unveiled in print, it was by no means obvious that it was essentially the same as Leibniz's infinitesimal calculus. His approach was basically geometrical; his terminology was suspiciously reminiscent of the scholastic jargon of the "flowing" of points and lines; and his notation, which involved the addition and subtraction of dots over letters, was clumsy and difficult to work with. Leibniz's approach, on the other hand, was algebraical; his language fresh and appropriate, incorporating such terms as *differential, integral, coordinate*, and *function*; and his notation, which we still use today, was clear and elegant.[69]

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65. In mathematics, the word "infinitesimal" signifies "approaching zero as a limit" or "being less than any assignable quantity or magnitude except zero." R. BARNHART, DICTIONARY OF SCIENCE 316 (1986).

66. ROSS, *supra* note 4, at 30-31.

67. CARL B. BOYER, THE HISTORY OF THE CALCULUS AND ITS CONCEPTUAL DEVELOPMENT 188 (1959).

68. *Id.* at 187-223.

69. ROSS, *supra* note 4, at 35 (emphasis in original). The irony of this situation was that Newton—not Leibniz—was considered to be the honored inventor of calculus until the second decade of the nineteenth century. Leibniz never enjoyed the rightful honor associated with his magnificent accomplishment. As one author observed:

> [L]ong after both Newton and Leibniz were dead and buried (Newton in Westminster Abbey, a relic to be reverenced by the whole English-speaking race; Leibniz, indifferently cast off by his own people, in an obscure grave where only the men with shovels and his own secretary heard the dirt thudding down on the coffin), Newton carried off all the honors—or dishonors, at least wherever English is spoken.

BELL, *supra* note 8, at 119. For over one hundred years (1711-1816), British mathematicians, paying excessive deference to Newton's reputation, struggled with "the prevailing confusion in the interpretation of the conceptual bases of the [Newtonian] calculus" and ultimately opted for the

Leibniz's intellectual achievement in crystalizing the infinitesimal calculus can best be understood in the context of his designing a new *problem solving process*.[70] Prior to his discovery, constants, straight lines, and certain curves easily constructible from straight lines, such as circles and conic sections, had been subject to mathematical treatment. The infinitesimal calculus extended mathematical treatment to other types of curves and variable quantities. In essence, calculus provided a general problem solving technique for mathematically analyzing physical quantities, which exist in a state of regular variation.[71]

In developing the infinitesimal calculus, Leibniz consumed himself mentally in trying to understand the nature of infinitesimal quantities. At that time, mathematicians believed that objects of mathematics should be real—that is, possess the capacity of being representable geometrically. Imaginary quantities, not constructible with ruler and compass, such as the square root of minus one, were highly suspect. Infinitesimals, such as rates of change at an instant, also fell into this category. For example, speed was known to be the change of distance divided by time, but at an instant, no time elapsed and thus no distance was travelled. Thus, in a strict sense, the notion of distance divided by time was meaningless. Although Leibniz subscribed to the prejudice that infinitesimals needed a geometrical foundation, he eventually arrived at the calculus by an algebraic rather than a geometric route. Through his inventiveness he uncovered the concept of an infinite series converging on a limit; the differential calculus was a technique for determining the *limit* of such a series, and the integral calculus was a technique for finding its *sum*.[72] As one author has described his discovery:

> What he discovered was that the gradient of a curve at a point (corresponding to a rate of change at an instant) could be treated as the limiting value of an infinite series generated by the gradients of shorter and shorter straight lines. It was from this that he developed the process of differentiation as a general technique. Similarly, the area under a curve could be treated as the sum of an infinite series generated by the varying lengths of infinitely many, infinitely thin strips under the curve. It was the generalisation of this process which

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clarity and elegance of the Leibnizian model. BOYER, *supra* note 67, at 223. For additional information on the Newton-Leibniz priority controversy, see AITON, *supra* note 8, at 59-67.

70. Leibniz is quoted as once saying, "Nothing is more important than to see the *sources* . . . of invention which are, in my opinion, more interesting than the inventions themselves." HAROLD P. FAWCETT & KENNETH B. CUMMINS, THE TEACHING OF MATHEMATICS FROM COUNTING TO CALCULUS 385 (1970) (emphasis in original).

71. ROSS, *supra* note 4, at 31.

72. *Id.* at 32.

led to the technique of integration.[73]

Thus, as fully developed and refined, Leibniz's problem solving process ultimately consisted of two distinct problem solving techniques: the differential calculus and the integral calculus, both of which are described in more detail *infra* in Section V.A. The differential calculus provided a general technique for determining the rate of change at any instant of a quantity which was continuously changing in relation to another quantity of which it was a "function." This technique of course had a meaningful and direct application to the solving of certain types of problems, including the calculation of planetary orbits; describing the motion of pendulums, waves, or vibrating strings; finding values of otherwise insoluble equations; establishing the highest and lowest values of functions; calculating the bending of loaded beams, etc.[74] The integral calculus, on the other hand, was the reverse of the differential calculus. Integration—the opposite of differentiation—consisted in reconstructing a whole from a given value at an instant. That is, with the integral calculus, from a rate of change at a point one could reconstruct a whole line, from a line one could reconstruct an area it defined, and from an area one could determine the volume created by rotating it. This technique had direct application in solving problems, such as determining centers of gravity, moments of inertia of rotating bodies such as fly-wheels, and other, more complex problems.[75]

2. Solver of People Problems

Leibniz's unique prowess in solving mathematical problems had a parallel in his widely acclaimed ability to resolve interpersonal problems, both on a local scale and on an international scale. Some examples of this ability have already been presented earlier in Section II.A. According to one expert, Leibniz's mother—who died when he was just fifteen years of age—had a profound influence upon the development of his expert conflict resolution skills. It is said of Gottfried's mother:

> Striving to live with all in peace and harmony, she thought evil of no-one, and lightly forgiving those who had offended her, excelled all in patience. By this example, the young Leibniz had implanted early in his life the seeds of virtue and religion. The conspicuous traits in his own moral being almost agree with the character of his mother . . . .[76]

Leibniz carried his mother's example into the development of his

73. *Id.* at 33-34.
74. *Id.* at 31.
75. *Id.* at 31-32.
76. AITON, *supra* note 8, at 10.

philosophy and into the way he viewed the world in general. He viewed himself in the scheme of nature as a grand synthesizer who, through his universal characteristic, would reconcile the seemingly diverse elements of earthly existence and the metaphysical, and thereby expose what he believed to be the underlying universal harmony in all of nature. In Leibniz's mind, the problems of medicine, morals, law, theology, and metaphysics could all be resolved by his new symbolism. He always seemed to be in the middle—out of synch with the scientific, legal, political, and religious communities of his time—yet constantly probing and challenging them to reconcile and resolve the conflicting ideas that stymied progress in each of them. His whole philosophy overflowed with dualities concerning the reconciliation of the temporal and the spiritual: the kingdoms of nature and of grace, the contingent truths of fact and the necessary truths of reason, the mechanical order of sufficient causes and the teleological order of final causes, and the empirical and the rational elements of knowledge. In his later years, Leibniz studied the Chinese culture and institutions intently in an effort to corroborate with facts his theory of universal culture. He eventually came to believe that Chinese political and social administration was far superior to the rule of favorites and the balance-of-power politics extant in the European monarchies.[77] He had "a genuine love of peace and was imbued with the spirit of reconciliation, hoping to reunite the churches and join the cultures of West and East by the interchange of ideas with distant China."[78]

With the preceding biographical and historical information as a background, this Article will now turn to a more detailed examination of Leibniz's geometry of situation and its usefulness to negotiators and mediators in enhancing the effectiveness of collaborative problem solving.

III. LEIBNIZ'S GEOMETRY OF SITUATION

A. *Problem Solving—General*

Before this Article embarks on a journey into the geometry of situation and beyond, it would be well advised to agree as to the purpose of the journey, to take inventory of the current knowledge, and to consider what the goals of the expedition should be. It easily can be agreed that the purpose of the journey is to learn more about the landscape of problem solving generally—with a special focus on the field of the emotional, behavioral, and relationship aspects of

77. *See generally* Wiener, *supra* note 5, at xxi-li. For a while, he held the belief that the Chinese language might provide the solution to his search for a universal philosophic language. *Id.* at l.

78. *Id.* at xlviii-xlix. He also approved of a project for perpetual European peace consisting of a holy alliance among the European rulers who would ensure continual peace by cooperative organization of the arts and sciences. *Id.* at xlviii.

conflict resolution in dispute and transaction situations. As to our inventory of current knowledge, it is known from prior similar expeditions that problem solving, in general, consists of three principal decisionmaking processes—Problem Design, Process Design, and Solution Design—plus a final review and decisionmaking process termed Reflection.[79] At this point, there is no reason to believe that problem solving with respect to *relationships* in a conflict situation would not also be governed by these same four decisionmaking processes. Thus, the first step in resolving the *relationship* aspect of a particular conflict would be to decide what the problem is—to engage in problem design. This stage of interpersonal problem solving, like its counterpart in mathematical problem solving, is quite challenging and often requires a great deal of intuition and creative thinking.[80] It is safe to predict that in *relationship* analysis, techniques of geometric imagineering will be quite helpful in problem design in at least two ways: in identifying and assessing the relationship situation and in perceiving the *potential* for change.[81] The imagineering techniques may also be of value in the Process Design, Solution Design, and Reflection stages, but their primary benefit will be aiding perception of the geometry of the situation and its potential for change. The goals of this Article will be to proceed on this journey with an open mind, to perceive as many helpful applications of the techniques as possible. With this understanding, this Article will now cross the approaching boundaries of imagination into the geometry of situation.

B. The Origin of Geometry of Situation

It should not come as a surprise that Leibniz's concept of geometry of situation resulted from a *collaboration* among himself, Spinoza (a friend and fellow-philosopher), and Euclid. More specifically, it was one of Leibniz's earliest works (*Dissertation on the Art of Combination* published in 1666), a famous meeting with the philosopher Benedict Spinoza in 1676 shortly after Leibniz's conception of the infinitesimal calculus, and his re-reading of the first book of Euclid's *Elements* in 1679 that inspired his concept of geometry of situation. In generally describing his art of combination, Leibniz wrote:

> The art of combination is . . . the science which treats of the forms of

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79. Cooley, *supra* note 1, at 110 n.99. *See also* Cooley, *supra* note 27; JOHN W. COOLEY, THE APPELLATE ADVOCACY MANUAL 40-49 (Lawyer's ed. 1989).

80. With respect to the concept of problem design in mathematics, see STEPHEN I. BROWN & MARION I. WALTER, THE ART OF PROBLEM POSING 1-8 (2d ed. 1990). *See also* D. SCHÖN, THE REFLECTIVE PRACTITIONER (1983); D. SCHÖN, EDUCATING THE REFLECTIVE PRACTITIONER (1987) (discussing the need to reconceptualize the preparation of professionals in all fields so as to view problem posing—or problem design—as a central phenomenon).

81. This article will return to the concept of perceiving potential for change in the discussion of geometric imagineering of an interpersonal relationship in *infra* section III.D, and in addressing the catastrophe theory of mathematics in *infra* section V.C.2.

> things or of formulae in general. That is, it is the science of quality in general, or of the like and the unlike, according as various formulae arise from the combination of a, b, c, etc., whether they represent quantities or something else. Consequently algebra (which is concerned with formulae applied to quantity) is subordinate to the art of combinations and follows its rules. These rules, however, are much more general, and can be applied not only in algebra but also in cryptography, in various kinds of games, in geometry itself . . . and in all matters where similarity is involved.[82]

In discussing the applicability of his art of combination to the field of jurisprudence, and showing the similarity between jurisprudence and geometry, Leibniz wrote in his dissertation:

> Moreover, the art of forming cases is founded in our doctrine of complexions [combinations]. For as jurisprudence is similar to geometry in other things, it is also similar in that both have elements and both have cases. The elements are simples [sic]; in geometry figures, a triangle, circle, etc.; in jurisprudence an action, a promise, a sale, etc. Cases are complexions [combinations] of these, which are infinitely variable in either field. Euclid composed the *Elements of Geometry*, the elements of law are contained in the *Corpus Juris*, but in both works more complicated cases are added. . . . To us it seems thus: the terms from whose complexions there arises the diversity of cases in the law are persons, things, acts, and rights . . . .[83]

Toward the end of 1676, when Leibniz was thirty, he had several lengthy discussions with Spinoza in The Hague. Leibniz was able to question Spinoza in detail concerning his yet unpublished *Ethics*, which incorporated, in the format of Euclid's *Elements*, his geometry of emotions. A continuing dialogue

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82. GOTTFRIED WILHELM VON LEIBNIZ, ON UNIVERSAL SYNTHESIS AND ANALYSIS, OR THE ART OF DISCOVERY AND JUDGMENT (1679), *quoted in* COTTINGHAM, *supra* note 8, at 65. *See also* 1 LOEMKER, *supra* note 57, at 351-59. In his *Discourse Touching the Method of Certitude, and the Art of Discovery in Order to End Disputes and Make Progress Quickly*, Leibniz noted that "[e]ven in the games of children there are things to interest the greatest Mathematician." LEIBNIZ: SELECTIONS, *supra* note 5, at 47.

83. GOTTFRIED WILHELM VON LEIBNIZ, DISSERTATION ON THE ART OF COMBINATIONS, *quoted in* 1 LOEMKER, *supra* note 57, at 133. In the same passage, Leibniz asserts that judicial decisionmaking is not the best way to resolve disputes concerning the law:

> For one cannot always wait for the lawmaker when a case arises, and it is more prudent to set up the best possible laws without defects, from the first, than to intrust their restriction and correction to fortune; not to mention the fact that, in any state whatsoever, a judicial matter is the better treated, the less is left to the decision of the judge (Plato *Laws*, Book ix; Aristotle *Rhetoric*, Book i . . . .).

Id. at 132-33.

on the subject was rendered impossible, however, as Spinoza, age forty-four, died of consumption within a few weeks after Leibniz's visit.[84] By 1679, Leibniz's ideas concerning the relationship between geometry and situational relationships were beginning to coalesce. In September of that year, apparently having been refreshed by re-reading the first book of Euclid's *Elements*, Leibniz confided in correspondence to Christian Huygens:

> I am still not satisfied with algebra, because it does not give the shortest methods or the most beautiful constructions in geometry. This is why I believe that, so far as geometry is concerned, we need still another analysis which is distinctly geometrical or linear and which will express *situation* [*situs*] directly as algebra expresses *magnitude* directly. And I believe that I have found the way and that we can represent figures . . . and movements by characters, as algebra represents numbers or magnitudes.[85]

The "analysis" Leibniz was referring to in this letter to Huygens was a concept adjunct to his art of combination,[86] but a technique requiring considerably more visualization and imagination. Shortly afterwards, he produced his essays on the geometry of situation.

Leibniz perceived his "geometry of situation" to be a new form of graphic representation "which is entirely different from algebra and which will have great advantages in representing to the mind, exactly and in a way faithful to its nature, even without [numbers], everything which depends on sense perception."[87] This new technique, according to Leibniz, "which follows the visual figures, cannot fail to give the solution, the construction, and the geometric demonstration all at the same time, and in a natural way and in one analysis, that is, through determined procedure."[88]

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84. COTTINGHAM, *supra* note 8, at 22.

85. 1 LOEMKER, *supra* note 57, at 382 (emphasis in original).

86. Leibniz's art of combination was later reflected in the mathematical concept of Group Theory, introduced by the French Mathematician Evariste Galois in about 1832. Group Theory has developed into one of the most imaginative branches of mathematics, and it, along with the Theory of Logical Types, has been proposed as an analogical basis for achieving change in interpersonal problem solving. *See* P. WATZLAWICK ET AL., CHANGE: PRINCIPLES OF PROBLEM FORMATION AND PROBLEM RESOLUTION 2-11 (1974). For an interesting description of the discovery of Group Theory and of Galois—the mathematical genius who suffered an untimely death in a duel at age 20, just after he put his thoughts about Group Theory in a long letter to a friend—see K. DEVLIN, MATHEMATICS: THE NEW GOLDEN AGE 101-04 (1988).

87. 1 LOEMKER, *supra* note 57, at 384.

88. *Id.* Leibniz believed that "by this method one could treat mechanics almost like geometry." *Id.* at 385.

The ten propositions of Leibniz's geometry of situation are listed in the next table.[89]

GEOMETRY OF SITUATION

1. All points in the world are *congruent* to each other, that is, one can always be put in place of another.

2. The situation of one point in relation to another can be thought of without expressing the straight line joining them, provided they are thought of as joined by some line, whatever it may be; if this line is assumed to be rigid, the situation of the two points in relation to each other will be immutable.

3. Two points can be thought of as having the same situation in relation to each other as two other points have, if the one pair can be joined by a line which is congruent with the line joining the other pair.

4. There is a difference between the endpoints of a line and the *situation* of the endpoints in *relation to* each other, and the line itself.

5. Let the character δ signify congruence. In Figure 1, let $AB \; \delta \; BY \; \delta \; CY$. Then the locus of all *Y*s will be a straight line. That is, given three points, *A*, *B*, and *C*, all points which have the *same situation in relation to A* as it has to *B* and to *C* will fall on an infinite straight line *Y(Y)*. If this were all limited to a plane, two given points would suffice to determine the line satisfying the conditions in this way.[90]

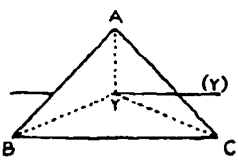

Figure 1

89. *See id.* at 384-90.

90. This proposition constitutes what Leibniz called a relation of *comparison*, as described *infra* in the text. Figures 1 and 2 are reprinted from 1 LOEMKER, *supra* note 57, at 389.

6. In Figure 2, let *AY* δ *BY* δ *CY* δ *DY*. In this three dimensional figure, find the point or points which has the *same situation in relation* to *A* as it has to *B*, and to *C*, and to *D*. Only *one point*, *Y*, satisfies this condition.[91]

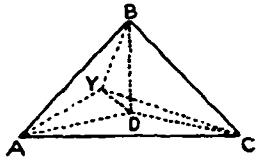

Figure 2

7. The intersection of two spherical surfaces is a circle.

8. The intersection of a plane and a sphere is a circle.

9. The intersection of two planes is a straight line.

10. The intersection of two straight lines is a point.

After documenting his initial concepts of geometry of situation, Leibniz, apparently without appreciating the implications of his discovery, conceived an advanced theory of geometry of situation which posited the basic principles of topology. Today, that is the area of mathematics which explores what remains unchanged of infinitely pliable geometric figures, however much they may have been stretched, compressed, or otherwise deformed (or subjected to continuous transformations). Labeled by him as *analysis situs* (analysis of situation), Leibniz viewed this concept of topology as complementing the analytic geometry of Descartes.[92] Analytic geometry abstracted the quantitative aspect from geometrical figures drawn in space and reduced them to purely algebraic terms. In comparison, Leibniz's *analysis situs* abstracted in turn from all quantitative features of geometrical figures, such as lengths, angles, degrees of curvature, and so on, and dealt only with such relationships as were left. Thus, topology was born. Leibniz's concept of analysis of situation, which lay dormant for two centuries, ultimately became instrumental in the development of non-Euclidean geometries and remains today an important underpinning of modern-day

91. This proposition constitutes what Leibniz describes as relations of *concurrence* or *connection*, as described *infra* in the text.

92. Topology is discussed in more detail in relation to the catastrophe theory of mathematics in *infra* section V.C.2 of the text.

mathematics.[93]

In his advanced thinking on geometry of situation, Leibniz, through his theory of *analysis* of situation, distinguished between mathematical analysis and situation analysis. Mathematical analysis, in Leibniz's thinking, deals with *magnitude*, not situation. Situation, on the other hand, deals with *data* and with the *positions* of the unknown entities, or their *loci*. In Leibniz's mind, analysis must be carried through to first principles and elements of situation—the "perfect analysis."[94]

As Leibniz wrote in his essay entitled *Analysis Situs*:

> Besides quantity, [visual] figure[s] in general include . . . also quality or form. And as those [visual] figures are *equal* whose magnitude is the same, so those are *similar* whose form is the same. The theory of similarities or of forms lies beyond mathematics and must be sought in metaphysics. Yet it has many uses in mathematics also, being of use even in the algebraic calculus itself. But *similarity* is seen best of all in the *situations* or [visual] figures of geometry. Thus a true geometric analysis ought not only consider equalities and proportions

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93. ROSS, *supra* note 4, at 28-29. An aspect of topology, relevant to our discussion of interpersonal problem solving, is the mathematics of knots. The objective of knot theory is to investigate "knottedness." Knots are closed loops and can be distinguished from mere entanglements in that a knot can be undone only by a process which involves pulling a free end through a loop at some stage, whereas a tangle (or a trivial knot analogous to the number zero in arithmetic) can be removed even if the two ends of the string are held fixed. In knot theory, two knots are said to be *equivalent* if it is possible to transform one into the other by a topological process that does not involve cutting. In investigating knots, one attempts to find a collection of knot *invariants* which is adequate for a distinction to be made between any two non-equivalent knots. Knots are classified according to their crossing number, and the crossing number may be used to distinguish between non-equivalent knots. For example, if two knots have different crossing numbers, then they cannot be equivalent. By the end of the nineteenth century, many prime (indivisible into simpler) knots with crossing numbers up to 10 had been distinguished. Eventually, it was realized that every knot could be associated with a group, called a *knot group*, thereby making Group Theory, see *supra* note 86, applicable to knot theory. In 1910, M. Dehn discovered a way to ascribe a simple and concise algebraic description to the *knot group* associated with any knot. By 1928, J. Alexander determined a way to assign a polynomial description to any *knot*. For example, the polynomial for a trefoil knot (an overhand knot with closed ends) is $x^2 - x + 1$; for a four-knot (figure eight knot with closed ends), $x^2 - 3x + 1$; for a granny knot, $(x^2 - x + 1)^2$. The implications of knot theory as applied to interpersonal problem solving—particularly in light of the polynomial-based visualization techniques described in *infra* section V.A—should be apparent. However, this article's time and space limitations regrettably do not permit a detailed discussion of these implications. For a detailed discussion of knot theory, see DEVLIN, *supra* note 86, at 229-61. *See generally* RICHARD H. CROWELL & RALPH H. FOX, INTRODUCTION TO KNOT THEORY (1963); 1 LOUIS H. KAUFFMAN, KNOTS AND PHYSICS (1991).

94. 1 LOEMKER, *supra* note 57, at 390-91.

which are truly reducible to equalities but also similarities and, arising from the combination of equality and similarity, congruences.[95]

Leibniz's five propositions of analysis of situation are:[96]

ANALYSIS OF SITUATION	
1.	Things which cannot be distinguished through their determinants (or through data adequate to define them) cannot be distinguished at all, since all other properties arise from these data. The corresponding Euclidean proposition is that equiangular triangles are similar.
2.	Although subjects can be distinguished by magnitude, magnitude can be known only by observing together either both subjects at the same time or each with some other unit of measure.
3.	In a similar vein, similar triangles are equiangular, triangles are similar if their sides are proportional, and equiangular triangles have proportional sides.
4.	Circles are to each other as the squares of their diameters.
5.	Spheres are to each other as the cubes of their diameters.

At the end of his essay on *analysis situs*, Leibniz wrote: "[T]his calculus of situation which I propose will contain a supplement to sensory imagination and perfect it, as it were. It will have applications hitherto unknown not only in geometry but also . . . in the descriptions of the mechanisms of nature."[97] Although it is not known for certain what exactly Leibniz was referring to in this last quoted passage, we may acquire some clues about where he was going, conceptually, with his geometry, or calculus, of situation by examining his theory of relations.

C. Leibniz's Theory of Relations

Leibniz's theory of relations, of which the geometry of situation is part, is one of the most misunderstood of all of Leibniz's doctrines.[98] Part of the misunderstanding has been caused by some commentators' misinterpretations and

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95. *Id.* at 392 (emphasis in original).

96. *Id.* at 391-96.

97. *Id.* at 396.

98. LEIBNIZ: A COLLECTION OF CRITICAL ESSAYS 191 (H. Frankfurt ed., 1972) [hereinafter LEIBNIZ: CRITICAL ESSAYS].

by some successful efforts by academicians to make a simple theory difficult.[99] As a mathematician, Leibniz recognized that mathematics contains many relational propositions, and "he intended to extend traditional logic so that it would include a calculus of relations—a logical calculus which would account for the special entailments that held between relational propositions."[100] In present-day terminology, Leibniz viewed the relation between two subjects, for example A and B, to consist of two separate and distinct aspects. The first aspect of a relation between two subjects, in Leibniz's view, is abstract—a mental structure—a label representing, in Leibniz's words, "a mere ideal thing." This first aspect describes the relation between two subjects *as viewed from outside* the relation. The first aspect of relation between two subjects occurs as a result of what Leibniz refers to as "accident," that is, chance. The second aspect of a relation between two subjects consists of two parts—one being the *actual* structure of the relation (termed "extrinsic denominations" by Leibniz),[101] and the other being the *actual* substance (termed by Leibniz "intrinsic properties" or "intrinsic denominations" or the "foundation") of relation. This two-pronged second aspect of relation describes the relation between two subjects *as viewed from the perspective of each subject* in the relation.[102] It is quite interesting to note that apart from using mathematical and geometrical examples,[103] Leibniz routinely uses examples of *relations between human beings* to explain this theory of relations.[104] For example, in a letter to one of his correspondents explaining the relation existing in the statement "David was the father of Solomon," Leibniz wrote: "My judgment about relations is that paternity in David is one thing, sonship in Solomon another, but that the relation common to both is a merely mental thing whose

99. Happily, some of these misunderstandings and obfuscations have been cleared up by other academicians. *See generally* NICHOLAS RESCHER, THE PHILOSOPHY OF LEIBNIZ 71-79 (1967); NICHOLAS RESCHER, LEIBNIZ, AN INTRODUCTION TO HIS PHILOSOPHY 54-61 (1979); MASSIMO MUGNAI, LEIBNIZ' THEORY OF RELATIONS 134-37 (1992); LEIBNIZ: CRITICAL ESSAYS, *supra* note 98, at 192-213.

100. LEIBNIZ: CRITICAL ESSAYS, *supra* note 98, at 192.

101. Leibniz recognizes that these extrinsic denominations are, in part, "accidents," in that they occur by chance. MUGNAI, *supra* note 99, at 36-37.

102. *See id.* at 133-37.

103. One noteworthy geometrical example appeared in his fifth letter to British mathematician Clarke. There, Leibniz identified three different ways of considering the relationship that exists between two lines, L and M. Leibniz noted that, depending on one's perspective (extrinsically or intrinsically), the relationship may be conceived of: (1) as a ratio of the greater L to the lesser M; (2) as a ratio of the lesser M to the greater L; and (3) as something abstracted from both, that is to say, without considering which comes first or second—that is, which is the subject and which is the object. The property being compared in (1) and (2) is length; in (3), the property could be angle or orientation with respect to the horizontal. Two important principles can be gleaned from this: (1) to think of a relation implies thinking of it in a given *direction*, and (2) each correlated term involves a *property* which *characterizes* the other correlate. *See* MUGNAI, *supra* note 99, at 45-46.

104. This should not be too surprising, since Leibniz spent a good part of his life researching the relations of the Guelf family.

basis is the modifications of the individuals."[105]

In an essay written around 1690, using a geometric (physical displacement) analogy in describing how change is perceived in the structure of an extrinsic denomination of a relation, Leibniz says: "'Consequently if I say that Peter is 100 paces from me, this is an extrinsic denomination, and if I move and Peter stands still, certainly Peter's distance from me *changes* but *without any change* taking place *in* Peter . . . .'"[106] In another essay, he further advanced his theory as to the *relation* between extrinsic and intrinsic denominations using an example of a human relation combined with a geometric analogy as follows:

> On my view, all extrinsic denominations are grounded in intrinsic denominations, and a thing which is seen really differs from one which is not seen: *for the radii which are reflected by the thing which is seen bring about a change in the thing itself.* What is more, . . . the Emperor of China as known by me differs in intrinsic qualities from himself as not yet known by me. Further, there is no doubt that each thing undergoes a *change* at the very same time, and it is needed time, in order for him, once not known by me to become known by me.[107]

By this passage, Leibniz intends to show that the actual structure (extrinsic denomination) of a relation as viewed from the perspective of each of the two subjects is dependent on the actual intrinsic denominations (properties) or foundation of the relation. Thus, even though I, as one subject of a relation, perceive another subject of the relation in a structural (extrinsic) sense, I will not perceive *change* in the structural (extrinsic) relation until (a) intrinsic change occurs, over time, and (b) I *perceive* the intrinsic change.

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105. MUGNAI, *supra* note 99, at 46. I interpret this passage to mean that the extrinsic denominations of paternity and sonship describe the relation from the perspective of David and Solomon, respectively, and thus would be categorized as the first part of the second aspect of relation. The relation that is common to them both which is a mental thing would be the "family" or "blood" relation which would describe a "mere ideal thing" as viewed from outside the relation. Leibniz further seems to be saying that to make the second aspect of the relation (actual) congruent with the first aspect (ideal) requires modification of the intrinsic properties or foundation of the second aspect of the relation.

106. *Id.* at 52 (quoting Leibniz) (emphasis added). By this Leibniz shows how a change in the actual *structure* of an extrinsic denomination does not necessarily effect a change in the intrinsic denomination or foundation of a subject of a relation.

107. *Id.* at 53 (emphasis added).

What does all this really mean? In a nutshell, with respect to all relations, but particularly human relations, Leibniz is saying:

1. Society ascribes labels to many types of human relations: parent-child, employer-employee, family, husband-wife, partner-to-partner or partnership, master-servant, business-to-business or corporate, government-citizen, etc. These are the paradigm structures—the *ideal* structures of relations. This is the first aspect of relation.

2. The second aspect of relation consists of:

   (a) the *actual* structure of the relation *and*
   (b) the *actual* substance of the relation—the *emotional and communicational* foundation, which includes adaptability to *change.*

3. Change can occur in the *actual* structure of the relation only when, over time, there is change in the substance—or emotional and communicational foundation—of the relation.

Before proceeding, it is important to understand the two broad categories into which, Leibniz says, all actual structural relations fall. Leibniz wrote:

> [A]ll relation involves either comparison or concurrence. Relations of comparison yield identity and diversity, in all respects or in some only, which makes things the same or different, like or unlike. [See Proposition 5 of the geometry of situation, discussed earlier, for a geometric example]. Concurrence includes what you call coexistence, i.e. connectedness of existence. [See Proposition 6 of the geometry of situation, discussed earlier, for a geometric example.] . . . So I believe we can say that there is only comparison and concurrence; but that the comparison which indicates identity or diversity, and the concurrence of the thing with myself, are the relations which deserve to be singled out from all the others.[108]

108. MUGNAI, *supra* note 99, at 73. The diagram in Figure 3 is reprinted from *id.* at 74 with the permission of the publisher, Franz Steiner Verlag Wiesbaden GmbH.

A diagram of this passage would be as shown in Figure 3.

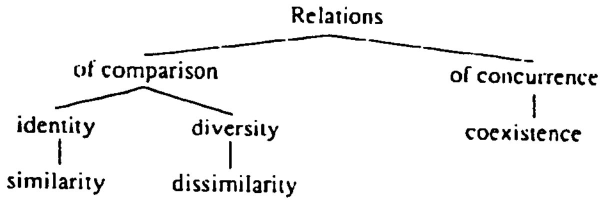

Figure 3

Read in conjunction with Propositions 5 and 6 of Leibniz's geometry of situation, it appears that he believed that relations of comparison could best be analyzed two-dimensionally and that relations of concurrence or connectedness could be best analyzed three-dimensionally.

In the discussions that follow, this Article will refer to the first aspect of relation as the *nominal relation*; the second aspect of relation, collectively, as the *relationship*; the actual *structure* of the relation as the *structure of the relationship*; and the actual *substance* of the relation as the *substance of the relationship*. Implicit in Leibniz's theory of relations is that both the structure and the substance of a relationship can be represented graphically and geometrically. This Article now turns to that topic.[109]

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109. Leibniz believed that the clarity of logic and reason played a very important role in resolving conflict. In an essay published in 1677, *Towards a Universal Characteristic*, Leibniz wrote:

> [Considering parties to a dispute,] there is hardly anyone who is ever able to weigh and figure out the whole table of pros and cons on both sides, i.e., not only to count the advantages and disadvantages, but also to weigh them accurately against one another. Hence, I regard the two disputants as though they were two merchants who owe each other various moneys and have never drawn up a financial statement of their balance, but instead, always cross out the various postings of their outstanding debts and insist on inserting their own claims with respect to the legitimacy and magnitude of their debts. . . . We need not be surprised then that most disputes arise from lack of clarity in things . . . .

LEIBNIZ: SELECTIONS, *supra* note 5, at 24.

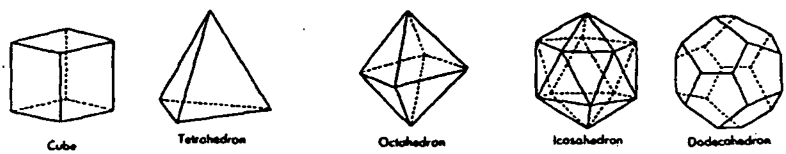
Leibniz also recognized that in problem solving, one "must distrust reason alone" and "have some experience or . . . consult those who have it." Graphical representation (or to use my term, "geometric imagineering") of the structure and substance of a relationship seeks to bring logical clarity to the problem solving process, and its effectiveness is enhanced as the problem solver becomes more experienced.

D. Geometric Imagineering of an Interpersonal Relationship

Leibniz's geometry of situation and theory of relations, together with his palette of metaphysical rules relating to time, space, motion, and relations,[110] provide the tools and materials to geometrically imagineer any particular interpersonal relationship. How relevant or useful the final product will be in a particular negotiation or mediation will depend, of course, on the artisan's skill and craftsmanship in accurately perceiving the aspects of the human relationship and imagining analogies to relevant geometrical relationships. This Article offers a few examples later to stimulate the reader's thinking in this regard.

1. Nominal Relations

In his essay *On Natural Law*, Leibniz described five types of "natural societies" or human relations: the husband-wife relation, the parent-child relation, the employer-employee relation, the family relation, and the government-citizen relation.[111] Each of these basic types of human relations is what Leibniz would categorize as a "nominal relation." Geometrically, these five nominal relations could be analogized to the five Platonic solids: cube, tetrahedron, octahedron, icosahedron, and dodecohedron as shown in Figure 4, together with their assigned nominal relation.[112]



Husband-Wife	Parent-Child	Government-Citizen	Family	Employer-Employee

Figure 4

110. *See infra* Appendix A. RUDY RUCKER, MIND TOOLS: THE FIVE LEVELS OF MATHEMATICAL REALITY (1987), identifies five patterns or archetypes of mathematics, which Rucker calls mind tools. They are: number, space, logic, infinity, and information. He relates these five archetypes, respectively, to the five basic psychological activities: perception, emotion, thought, intuition, and communication. *Id.* at 3, 5. It is interesting—from a geometric imagineering perspective—that he correlates space with emotion. Later in the book, he correlates arithmetic with number, geometry with space, algebra with logic, and calculus with infinity. *Id.* at 15.

111. *See* 1 LOEMKER, *supra* note 57, at 702-06.

112. Named after Plato, the five Platonic solids are three-dimensional figures whose polygonal surfaces are all congruent and whose corners all meet at the same angle. JOHN A. PAULOS, BEYOND NUMERACY 181 (1991). Figure 4 is reprinted from BEYOND NUMERACY by John Allen Paulos at 182. Copyright © 1991 by John Allen Paulos. Reprinted by permission of Alfred A. Knopf, Inc.

Of course, any combination of these nominal relations could be possible in any particular situation, as, for example, when a family operates a business and combines a family relation with an employer-employee relation as shown by the combined icosahedron-dodecahedron in Figure 5 on the right. Another example arises when a husband and wife both assume political office and intertwine the spousal with the government-citizen relation, and all the potential conflicts, as illustrated by the combined cube-octahedron in Figure 5 on the left.[113]

Figure 5

A negotiator's or mediator's imagining these kinds of geometrical figures when analyzing a particular relationship situation will assist that person in appreciating the potential complexity of the situation and raise the negotiator's or mediator's level of awareness of how to deal with the presenting nominal relationship and what approach to take with the individual members of the relationship to achieve one's goal.

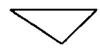
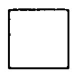
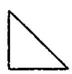
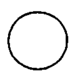
2. Analysis of a Particular Relationship

Having considered the concept of nominal relations generally, this Article will now geometrically "imagineer" a particular interpersonal relationship, in three stages: this Article will first imagineer the *structure* of the relationship, then the *substance* of the relationship, and, finally, the *potential for change*. Recall from the earlier discussion that the first five propositions of Leibniz's geometry of situation and all five propositions of his analysis of situation comprise rules for relational *comparison*, and that comparison (identity vs. diversity) is the process used to analyze the *structure* of a relationship. The *substance* of a relationship can be analyzed by taking into account these same propositions and, additionally, by considering Leibniz's metaphysical rules of mathematics.[114] The potential for change and agreement, or concurrence, in a relationship can be analyzed geometrically through application of Propositions 6 through 10 of Leibniz's geometry of situation and his metaphysical rules of mathematics.

113. These illustrations in Figure 5 appear in H.M. CUNDY, MATHEMATICAL MODELS 130-31 (2d ed. 1961) and are reprinted by permission of Oxford University Press.

114. *See infra* appendix A.

For simplicity, the vehicle we will use for analysis is the typical parent-child relationships existing within the family setting. Assume that the particular family under consideration consists of a Father (*F*), Mother (*M*), and Daughter (*D*). The particular issue causing the present conflict is the Father's requirement that the Daughter, who is a junior in high school, be home by midnight on Friday and Saturday evenings. Assume further that through extensive discussion with all three family members, a neutral third party, hereafter referred to as the mediator, has discovered much about the conflict behavior of each of them and about the family's rule-creating and rule-enforcing processes. From what can be presently deciphered, the family is hierarchical in structure, though the individual perceptions of the family members vary somewhat on this matter. Aware of the five types of conflict behavior (avoiding, accommodating, competing, compromising, and collaborating),[115] the mediator ascribes geometrical symbols to each of them as shown in the next chart.

CONFLICT BEHAVIOR	DESCRIPTIVE WORD	GEOMETRIC SYMBOL
Avoiding	Obtuse	
Accommodating	Form-fitting	
Competing	Squared-off	
Compromising	Half-a-slice	
Collaborating	Roundly satisfying	

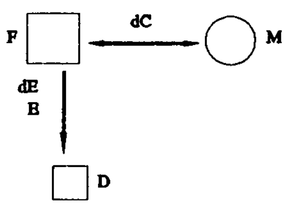
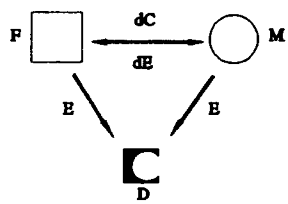
Through discussions, the mediator has determined the Father to be predominantly competitive; the Daughter, compromising; and the Mother, collaborative. But, as shall be seen in the analysis, the individual family members may perceive that the members have conflict behavior styles different from these. Some ideas about the structure of the parent-child relationships existing within

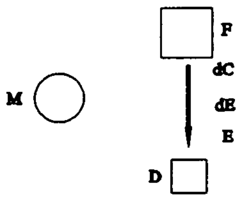
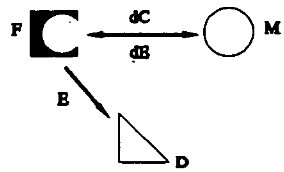
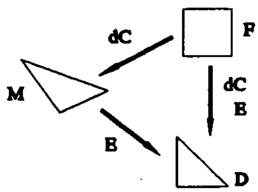
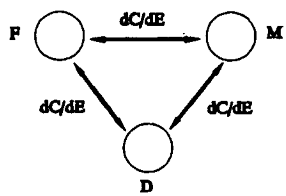
115. *See* Kenneth W. Thomas & Ralph H. Kilmann, *Interpersonal Conflict Handling Behavior as Reflections of Jungian Personality Dimensions*, 37 PSYCHOL. REP. 971, 971-80 (1975).

this family and about the family relationship overall are now ready to be geometrically "imagineered."

a. Structure of Relationship

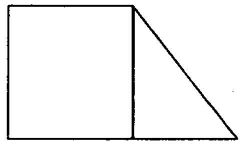
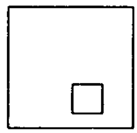
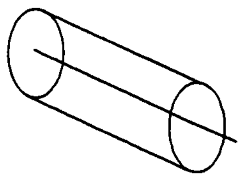
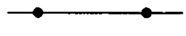
Keeping in mind Propositions 1 through 4 of Leibniz's geometry of situation, the above-described family situation is perceived from the perspective of each endpoint, or family member. However, instead of using points to identify the family members, use the geometrical symbols described in the previous chart. Thus, in graphically illustrating the structure of the relationship, the father could be represented by a square, the daughter by a right triangle, and the mother by a circle. The *size* of these symbols may vary in relation to the perspective of each individual family member, depending on the amount of power or authority each member perceives himself or herself and the other two members to possess. The family authority structure, as perceived by each individual family member, appears in the next chart, along with the authority structure each family member believes would improve the current rule-creating (*C*) and rule-enforcing (*E*) process. The arrows indicate the source of responsibility for rule-creating and rule-enforcing as directed toward the member who is expected to conform. The letters *dC* and *dE* stand for participation in the decisionmaking processes for creating rules and enforcing rules, respectively.

Present Authority Structure	Proposals for Improvement
Father's Perception	*Father's Proposal*
 	
The Father perceives himself in the situation as the authority figure who, with his wife's input, creates rules, which he alone enforces against his Daughter.	The Father would prefer a situation where his wife would not only participate more equally in creating the rules, but also in enforcing them. He also would like the Daughter to be more accommodating to himself and the Mother.

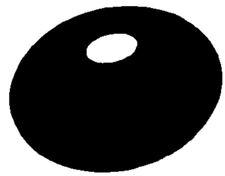
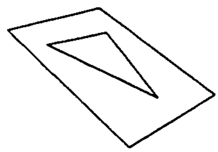
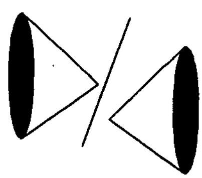
Mother's Perception	*Mother's Proposal*
	
The Mother perceives herself out of the rule-creating and rule-enforcing loop completely. She views the Father and Daughter to be mutually competitive.	The Mother would like to participate equally with the Father in decisions regarding rulemaking and rule-enforcing, but she prefers letting the Father do the enforcing. She would like for the Father to be more accommodating and the Daughter to be more compromising.
Daughter's Perception	*Daughter's Proposal*
	
The Daughter perceives that her Father does all the rule-creating, that he does most of the enforcing, and that the Mother enforces by not intervening when the rules are unfair (that is, by avoiding).	The Daughter proposes that each family member be collaborative and participate in joint rule-creating and joint decisionmaking about rule-enforcement.

b. Substance of Relationship

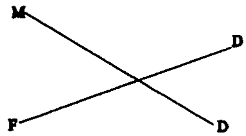
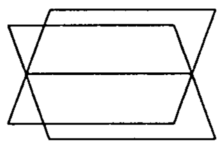
Having completed a "same plane" comparison to determine the authority *structure* of the relationship situation, the mediator can now proceed to conduct a "same plane" comparison analysis of the *substance* of the separate relationships. The next chart illustrates how the substance of the relationship existing between the Daughter and Father could be geometrically imagineered. The left column contains statements made to the mediator by the Daughter and Father in private conversations. The middle column contains the relevant geometrical interpretations of the statements *in words*,[116] and the right column contains geometrical interpretations of the statements *in drawings*. Clues to felt emotions arise from these statements and are indicated in parentheses after the statements in the left column.

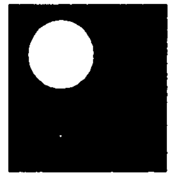
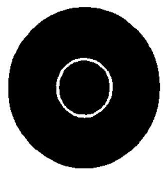
DAUGHTER'S STATEMENTS	GEOMETRICAL WORD DESCRIPTION	GEOMETRICAL DRAWING
"My Dad and I have very little in common." (Disappointment)	B10: Common boundaries of two structures means something contained in both, yet without their having a common part.	
"My Dad never listens to me. He just bosses me around and tells me to conform." (Anger)	B13: Whatever is equal to a part of *A* is less than *A*. If *B* is identical with a part of *A*, *B* is less than *A*.	
"My Mom just avoids the tension between my Dad and me. She won't help me." (Frustration)	D18(k): If a rigidly extended structure is moved in such a way that two points in it remain fixed, all of its stationary points fall collectively on the straight line or axis through the fixed points, but every movable point describes a circle around the axis.	
"My Dad thinks I am irresponsible. I'm a very responsible person. (Pride) I just want some independence."	D18(i): A straight line cannot be moved if two points in it are fixed.	

116. The geometrical word descriptions are taken from Leibniz's metaphysical foundations of mathematics, as presented in *infra* Appendix A. Each code (B10, B13, etc.) represents a specific metaphysical rule from the appendix.

FATHER'S STATEMENTS	GEOMETRICAL WORD DESCRIPTION	GEOMETRICAL DRAWING
"My Daughter is part of *my* family." (Gloating)	B9: A structure which is contained in another which is homogeneous to it is called a part—the other in which it is contained is called the whole. Thus, the part is a homogeneous ingredient of the whole.	
"My wife and I agree that as long as *my* Daughter lives in *my* house, she will do as *I* say." (Anger)	D19(b): The plane within its boundary is uniform because from the nature of its origin (three points) there is no ground for deriving any sort of diversity.	
"I'm tired of my Daughter disobeying the rules." (Anger, Resentment)	D18(g): The straight line is uniform on all sides, i.e., it does not—like a curved line—possess a concave and convex side.	
"Besides, my Daughter may get into trouble if she stays out past midnight. (Fear) She just stays out late and doesn't tell me where she is, or whom she's with. She should listen to me and not be so independent."	C2: An object is self-moved when it alters its position and at the same time contains within itself the ground for this change.	

Finally, examine how the potential for change and concurrence (agreement) could be geometrically imagineered in this situation. Recall that from Proposition 6 of Leibniz's geometry of situation, one could conclude that relations of concurrence or connectedness are best analyzed three-dimensionally. The next chart illustrates how progressive concurrences in the family situation described above could result, over a period of time, in an overall change in the structure of the separate relationships and of the whole family relationship. The column on the left contains types of possible concurrences, the middle column contains in words the proposition of the geometry of situation relating to the particular type of concurrence(s) described, and the right column contains geometrical drawings describing, progressively, the growth of the relationship as the structure of the relationship is changed.

DESCRIPTION OF CONCURRENCES	GEOMETRICAL WORD DESCRIPTION	GEOMETRICAL DRAWING
M and *D* agree that *M* should be equally involved with *F* in rule-creating. *F* and *D* agree to permit *M* to have an equal voice in rule-creating, with *M* and *F* having joint veto power.	Prop. 10: The intersection of two lines is a point.	
As an experiment, *F* and *M* agree that for one month, *D* can stay out past midnight ***on the condition that***: *D* agrees with *F* and *M* to let them know where she is and with whom. The agreement is limited both in duration and on the fulfillment of a specified condition by *D*. Figuratively, *M*, *F*, and *D* have created a line of conditional concurrence.	Prop. 9: The intersection of two planes is a straight line.	

The above experiment is successful. *F*, *M*, and *D* agree to have a weekly family meeting. They further agree that the matters of the creation, relaxation, and enforcement of rules will be discussed more globally with the goal of reaching concurrence.	Prop. 8: The intersection of a plane and a sphere is a circle.	
The above experiment is successful and *F*, *M*, and *D* agree to listen to one another, respect each others viewpoints, and collaborate in decisionmaking on issues that jointly affect all family members.	Prop. 7: The intersection of two spherical surfaces is a circle.	

This Article will, for the time being, move from discussing the application of Leibniz's theories to discussing interpersonal relationships and more modern theories and applications. This Article will return to Leibniz's theories once again when the calculus of change is explored later in Section V.

E. *Geometry of Situation and Modern General System Theory*

Although it will never be certain, it is conceivable that Leibniz was at the threshold of developing what we now refer to as General System Theory (GST) when he was developing his geometry of situation, his theory of relations, and the rudiments of topology. In the modern world, systems theorists seek to explain the *behavior* of complex, organized systems of all sorts—from thermostats to missile guidance computers, from amoebas to families.[117] However,

117. SOURCEBOOK OF FAMILY THEORIES AND METHODS: A CONTEXTUAL APPROACH 325 (Pauline G. Boss et al. eds., 1993) [hereinafter FAMILY THEORIES SOURCEBOOK]. GST is closely related to information theory and cybernetics. Information theory focuses primarily on uncertainty reduction through the gathering of information. Cybernetics, a field of study originally pertaining to the control of antiaircraft guns during World War II, concerns the communication and manipulation of information in controlling the behavior of many kinds of physical, chemical, biological, and social systems. Cybernetics is the basis for such GST concepts as feedback, open and closed loop systems, wholeness, interdependence, self-regulation, and interchange with the environment. *Id.* at 331-32.

GST is probably most accurately defined as a "program of theory construction aimed at building concepts, postulates, principles, and derived theorems that apply universally across all domains of application."[118] Three core assumptions of GST are: (1) systems theories can unify science, (2) a system must be understood as a whole, rather than in component parts, and (3) human systems are unique in their self-reflexivity.[119] The family, as a social system, is particularly suitable for application of GST because a family's transactional patterns (redundant, recurring sequences of behaviors) repeat consistently over time.[120] GST has been used to understand intrafamily processes such as family functioning, communication patterns, family conflict, separateness and connectedness among members, cohesion, integration, and adaptation to change.[121] GST concepts applicable to assessing a particular family system include hierarchy, interdependence and mutual influence, boundaries and open/closed systems, equifinality,[122] and feedback and control.[123] Of course, understanding a particular family system also entails a familiarity with its subsystems (such as the sibling system) and suprasystems (such as the extended family).[124] In the following two subsections, this Article will briefly examine the assessment features of two family system analytical models and their related geometry. Later, Section V.B shall return to these models to view their respective geometric elements in the context of the calculus of behavioral change.

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118. *Id.*

119. *Id.* at 328-30. As to assumption (1), systems theorists have discovered isomorphism among concepts and systems of various disciplines. Isomorphism means equivalence of form. Thus, to say that two systems are isomorphic means that the elements and relationships of one system have a one-to-one correspondence with the elements and relationships of the other. For example, similar patterns of system behavior in response to an overload of information can be observed whether the system is electronic, neurological, or human. *Id.* at 328. As to assumption (2), a corollary assumption of GST is that "the whole is *greater* than the sum of its parts." *Id.* (emphasis added). In a family, for example, the system is something more than parents plus children, as, by analogy, a cake is something more than merely its individual ingredients. *Id.* at 328-29. Finally, as to assumption (3), by self-reflexivity it is meant that human systems have the ability to examine *themselves* and their own *behavior* and to establish goals for themselves. *Id.* at 329.

120. *Id.* at 330.

121. *Id.*

122. Equifinality is defined as "the ability of a system to achieve the same goals through different routes." *Id.* at 334. It is discussed in more detail in *infra* section V.C.1.a, notes 274-86.

123. *See* FAMILY THEORIES SOURCEBOOK, *supra* note 117, at 330-36. For an excellent overview of the family as a system, see KATHLEEN M. GALVIN & BERNARD J. BROMMEL, FAMILY COMMUNICATION: COHESION AND CHANGE 33-48 (3d ed.1991). For a discussion of the family as an interactive system, emphasizing a geometrical approach to its description and an understanding of its dynamics, see WILLIAM C. NICHOLS & CRAIG A. EVERETT, SYSTEMIC FAMILY THERAPY: AN INTEGRATIVE APPROACH 117-41 (1986).

124. FAMILY THEORIES SOURCEBOOK, *supra* note 117, at 332-33.

1. The Satir Model: Assessment of Family System

Over a fifty-year period, Virginia Satir (pronounced Sa-Teer') developed a family therapy model through countless hours of observing clients, testing hypotheses, and creating interventions.[125] The basic premise of her work was that "there is nobody in the world, no matter what the conditions are on the outside, who cannot change."[126] Moving away from the older Aristotelian, linear, singular cause-and-effect approach and toward systems thinking, Satir successfully experimented with systems techniques designed to bring about change in the entire family system by improving the way its members *communicated* with each other.[127] Central to the success of her model was her insistence on viewing a particular family system and issues "from all angles: from underneath, from the top, from all sides, from close by, and from afar," because "[t]hese perspectives help us see the whole." [128] In the Satir model, there are two principal steps: assessment of the *current* system and its behavior, which is analogous to Leibniz's geometry of situation, and implementing new behaviors to achieve overall system and structural change. This subsection will discuss the assessment step; Section V.B.1 will address the implementation step.

Like Leibniz, who believed that, to be fully understood, a relationship had to be viewed from the perspective of each subject,[129] Satir believed that the first important step in assessing a family system is to view that system and its environment from the perspective of each member—to determine how each member perceives the world.[130] In the Satir model, each person in the family system may have one of two possible perceptions: a hierarchical perception or a growth perception.[131] Each of these perceptions has four aspects relating to how a person defines a relationship, defines a person, explains an event, and views attitudes toward change.[132] Hierarchical relationships imply a dominant/submissive arrangement—a threat-and-reward model. These relationships are often described in terms of roles, such as father-child, boss-worker, priest-parishioner, and teacher-student.[133] Emotions that normally arise within hierarchical relationships include emptiness, anger, fear, and helplessness.[134] In the growth perception, the thrust is toward parity, that is, "person equals

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125. JOHN BANMEN ET AL., THE SATIR MODEL: FAMILY THERAPY AND BEYOND 1 (1991).
126. *Id.* at xvi.
127. *Id.* at 2-3.
128. *Id.* at 6.
129. *See supra* section III.C.
130. BANMEN ET AL., *supra* note 125, at 6.
131. *Id.*
132. *Id.*
133. *Id.* at 7.
134. *Id.*

person."[135] Charts depicting persons' hierarchical and growth perceptions (models) in relation to the four aspects of ways to perceive the world appear in Figures 6a and 6b.[136]

According to Satir, most relationships in the Western culture are based on the hierarchical model—dominance and submission.[137] Because of this, many people routinely handle reality, self-worth, and communication by assuming one or more of four "survival stances" or dysfunctional patterns of communicating: placating, blaming, being super-reasonable, or being irrelevant.[138] These four communication patterns are actually manifestations of a person's coping with his or her low self-esteem.[139]

Placating occurs when one disregards one's own feelings of worth, hands power over to someone else, and says "yes" to everything.[140] Other characteristics of placating are a person's being nice when the person does not feel nice, being a rescuer by rushing in to rectify any kind of trouble, and blaming oneself for things that go wrong.[141] In other words, placating involves discounting self.

Blaming is opposite to placating and it normally activates the emotion of fear in others. Blaming involves standing up for oneself and not accepting excuses, inconvenience, or abuse from anyone. Persons who engage in blaming are often described as hostile, tyrannical, nagging, or violent.[142] Blaming involves discounting others.

The super-reasonable pattern of communicating involves being inhumanly objective. This pattern does not allow oneself or others to focus on feelings; rather, it implies functioning with respect to context only, most frequently at the level of data or logic.[143] It involves discounting self and others.[144]

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135. *Id.* at 8.

136. The charts in Figures 6a and 6b are reproduced from BANMEN ET AL., *supra* note 125, at 14, 15 and are reprinted by permission of the authors and publishers. THE SATIR MODEL is available by contacting: Science & Behavior Books, Inc., Palo Alto, CA 94306, 800/547-9982.

137. BANMEN ET AL., *supra* note 125, at 8.

138. *Id.*

139. *Id.* at 65.

140. *Id.* at 36.

141. *Id.* at 36.

142. *Id.* at 41.

143. *Id.* at 45.

144. *Id.* at 45.

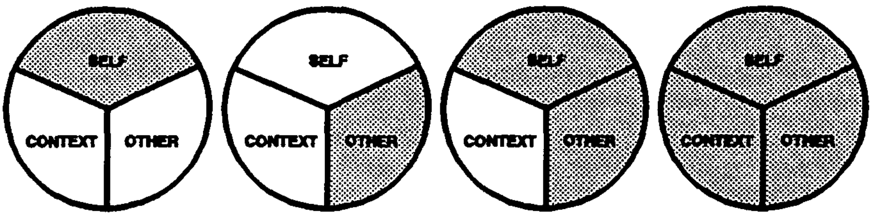
Ways of Perceiving the World	
DEFINITION OF A RELATIONSHIP (How we perceive a pair)	
Hierarchical Model	Growth Model
People are of unequal value.	People are of equal value.
People dominate or submit to each other.	Relationships are between equals in value.
Roles and status are confused and blurred with identity.	Roles and status are distinct from identity.
Roles imply superiority and power, or minority status and powerlessness.	Roles imply a function in a specific relationship at a particular time.
The hierarchical view implies superiority and submissiveness.	Equality is manifested in: equality of persons, connection, interest and acceptance of samenesses and differences.
People have power over each other but feel isolation, fear, anger, resentment, isolation, and distrust.	People feel love, ownership of self, respect of others, freedom of expression, and validation.
DEFINING A PERSON	
Hierarchical Model	Growth Model
People need to conform and obey "shoulds" for physical and emotional survival and acceptance.	Each person is unique and can define him- or herself from an inner source of strength and validation.
People are born with the potential to be evil.	People have an inborn spiritual base and sacredness, and they manifest a universal life force.
People are expected to think, feel, and act like each other, and to live up to external norms by competing, judging, comforting, and imitating.	Combining and respecting samenesses and differences, people delight in discovering themselves and others by cooperating, observing, and sharing.
People devalue or deny their feelings and differences.	People articulate their feelings and accept their differences.

Figure 6a

Ways of Perceiving the World	
DEFINING AN EVENT	
Hierarchical Model	Growth Model
A causes B in a linear, cause-and-effect fashion.	Any event is the outcome of many variables and events. A = B + C + D . . . etc.
Only one right way exists to do something, and the dominant person knows what it is.	Many ways usually exist, and we can use our own criteria to choose an approach.
People deny their own experiences so as to accept the voice of authority.	People look beyond the obvious event to understand its context and its many contributing factors.
Thinking such as "That's the way it is" and "It's black and white" generates manipulation and shuts down originality and discovery.	Circular thinking and a systems approach (action-reaction-interaction) generate relevance, discovery, information, order, and connection.
ATTITUDES TOWARD CHANGE	
Hierarchical Model	Growth Model
Security requires maintaining the status quo.	Security grows out of confidence in the process of change and growth.
People view change as undesirable and abnormal. They therefore reject and resist it.	People view change as ongoing, essential, and inevitable. They therefore welcome and expect it.
The familiar is more valued than the comfortable, even if the price is painful.	People view discomfort or pain as a signal for change.
People fear the unknown.	People take risks and opportunities to move into the unknown.
People judge changes as being right or wrong.	People delight in discovering new choices and resources.
People feel fear and anxiety when they face the prospect of change.	People feel excitement, connectedness, and love when they encounter the prospect of change.

Figure 6b

Finally, the fourth survival stance, being irrelevant, is the opposite of the super-reasonable stance. Being irrelevant is commonly confused with being amusing or clownish. Persons who employ this communication pattern believe they can survive as long as they can direct attention away from topics that carry any degree of stress for them. Normally, they keep changing their ideas and want to engage in many activities simultaneously. When persons are being irrelevant, they discount themselves, other persons, and the context of their interaction.[145] The circles in Figure 7 depict these four patterns of communication used as survival mechanisms in relationships and indicate which aspects of interaction are discounted when they are being used.[146]



Placacting Blaming Super-Reasonable Irrelevant

Figure 7

These two perceptions of the world and the four survival stances of the Satir model are important items of information for negotiators and mediators to have in the repertoire of problem solving resources. I say this from the standpoint not only of one's ability to better understand family systems generally and the behavior of other parties to a negotiation or mediation, but of one's ability to understand the behavior of *oneself* in the role of negotiator or mediator in a particular situation. Later, Section V.B.1.a of this Article will discuss a fifth pattern of communication called *congruence* which, in the Satir model, is the precursor to positive change in relationships. But for now, this Article will briefly examine the assessment aspect of another model for analyzing family systems—the Circumplex Model.

2. The Circumplex Model: Assessment of Family System

The Circumplex Model was originally developed to integrate the fields of

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145. *Id.* at 48-49.

146. The circles in Figure 7 are reproduced from BANMEN ET AL., *supra* note 125, at 36, 41, 45, 49, respectively, and are reprinted by permission of the authors and publishers. THE SATIR MODEL is available by contacting: Science & Behavior Books, Inc., Palo Alto, CA 94306, 800/547-9982.

family system theory, family research, and family therapy.[147] Actually, the Model's three primary dimensions—family cohesion, family adaptability, and family communication—were deduced directly from these three fields. The dimensions of family cohesion and adaptability are built directly into the Model and are related in a curvilinear way to family functioning. That is to say that too much or too little of these two dimensions are seen as problematic if the family system is locked at the dimension extremes.[148] The family communication dimension facilitates movement of the system on the other two dimensions. The communication dimension has a linear relationship to family functioning in that higher levels of good communication facilitate better family functioning.[149]

In the Circumplex Model, family cohesion—what Leibniz might call the extrinsic property or actual structure of the relation—is defined as "the emotional bonding that family members have toward one another," and some of the components of this dimension are emotional bonding, boundaries, coalitions, time, space, friends, decisionmaking, interests, and recreation.[150] The four defined levels of cohesion are: disengaged (very low), separated (low to moderate), connected (moderate to high) and enmeshed (very high). The extreme levels of cohesion—disengaged and enmeshed—are generally viewed as problematic, whereas the central levels—separated and connected—contribute to optimal family functioning.[151] In the central levels, family members are able to experience and balance, being both independent from and connected to the family system.[152]

The second primary dimension of the Circumplex Model, the adaptability to change—what Leibniz might call the intrinsic property or substance of the relation—is defined as "the ability of a family system to change its power structure, role relationships, and relationship rules in response to situational and developmental stress."[153] The four levels of the adaptability dimension are: rigid (very low), structured (low to moderate), flexible (moderate to high), and

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147. Froma Walsh & David H. Olson, *Utility of the Circumplex Model with Severely Dysfunctional Family Systems*, *in* CIRCUMPLEX MODEL: SYSTEMIC ASSESSMENT AND TREATMENT OF FAMILIES 51 (David H. Olson et al. eds., 1989).

The word "Circumplex" is derived from two Latin words, "circum" and "plex." "Circum" means around or round about, and "plex" means to interweave or to form a plexus. "Plexus" means any intertwined or interwoven mass. In mathematics, a plexus is the system of equations required for the complete expression of the relations existing between a set of quantities. 3 & 11 THE OXFORD ENGLISH DICTIONARY 233, 1048 (1989).

148. Walsh & Olson, *supra* note 147, at 54.

149. *Id.*

150. *Id.* at 54.

151. *Id.* at 55.

152. *Id.*

153. *Id.*

chaotic (very high). The extreme levels—rigid and chaotic—are viewed as problematic to the family system; the central levels—structured and flexible—are seen as more conducive to family functioning.[154] It is said that well-functioning family systems maintain a balance between structure and flexibility.[155]

The four levels of cohesion and the four levels of adaptability are depicted in Figure 8.[156] This descriptive model illustrates sixteen types of marital and family systems. These sixteen types include four balanced types, eight mid-range types, and four extreme types. The four balanced types are those in which there are balanced levels on both cohesion and adaptability. Mid-range types are extreme on one dimension and balanced on the other dimension. Extreme types appear as extremes of both dimensions.[157]

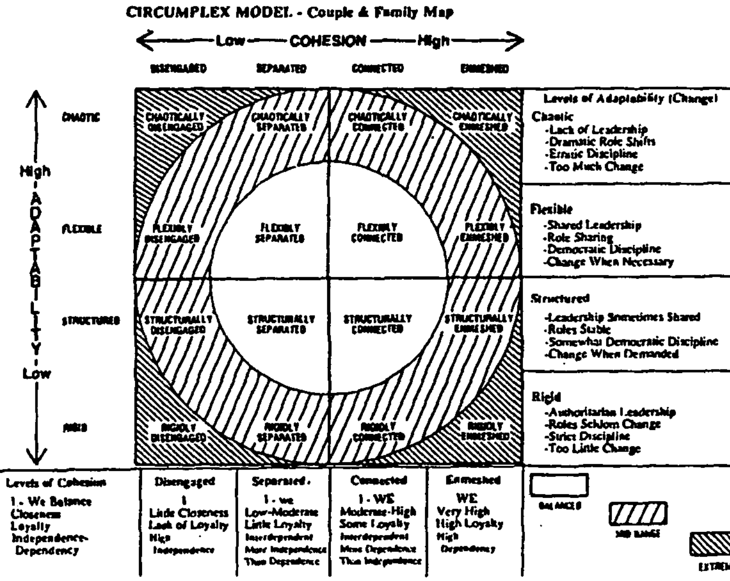

Figure 8

The concepts of balance and curvilinear dynamics are two central and interrelated aspects of the Circumplex Model.[158] On the cohesion dimension,

154. *Id.*
155. *Id.*
156. Figure 8 is reprinted from Walsh & Olson, *supra* note 147, at 56. © By The Haworth Press, Inc. All rights reserved. Reprinted with permission. For copies of the complete work, contact Marianne Arnold at The Haworth Document Delivery Service (Telephone 1-800-3-HAWORTH; 10 Alice Street, Binghamton, N.Y. 13904). For other questions concerning rights and permissions, contact Wanda Latour at the above address.
157. *Id.* at 55.
158. *Id.*

balance involves maintaining an equilibrium between separateness and togetherness, "where too much separateness can lead to a disengaged system and too much togetherness can lead to an enmeshed system."[159] On the adaptability dimension, balance involves maintaining an equilibrium between stability and change. Too much stability can lead to a rigid system, and too much change can often lead to a chaotic system.[160] The curvilinear aspects of cohesion and adaptability are illustrated in Figure 9.[161] The extreme levels of cohesion (disengaged and enmeshed) and adaptability (rigid and chaotic) are at the lowest level of the parabolic arch, indicating that they are the most dysfunctional. Extreme family systems tend to oscillate between these two extremes.

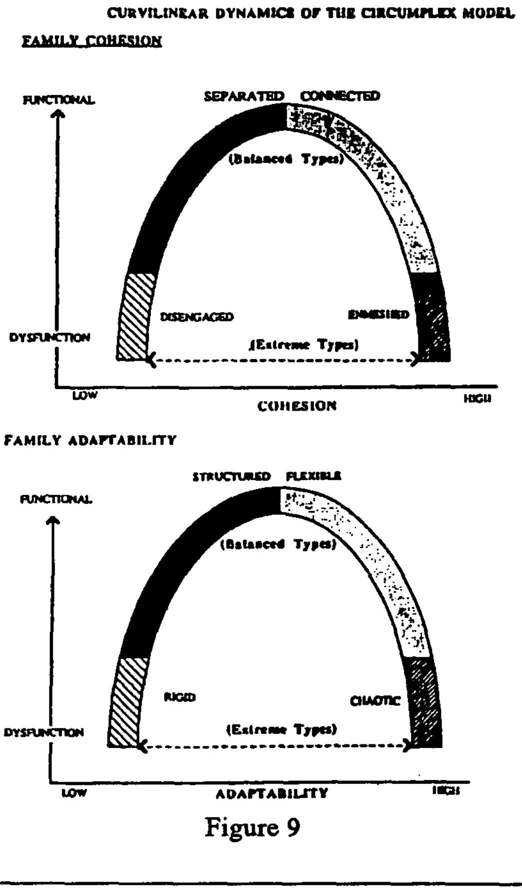

Figure 9

159. *Id.* at 57.

160. *Id.*

161. Figure 9 is reprinted from Walsh & Olson, *supra* note 147, at 58. © By The Haworth Press, Inc. All rights reserved. Reprinted with permission. For copies of the complete work, contact Marianne Arnold at The Haworth Document Delivery Service (Telephone 1-800-3-HAWORTH; 10 Alice Street, Binghamton, N.Y. 13904). For other questions concerning rights and permissions, contact Wanda Latour at the above address.

There is, dynamically, a greater similarity among the four balanced types and among the four extreme types than between any of the balanced or extreme types.[162] Figure 10 illustrates this phenomenon graphically.[163] In the figure, the four extreme types are grouped in the lower box, and the balanced types are grouped together in the upper level.

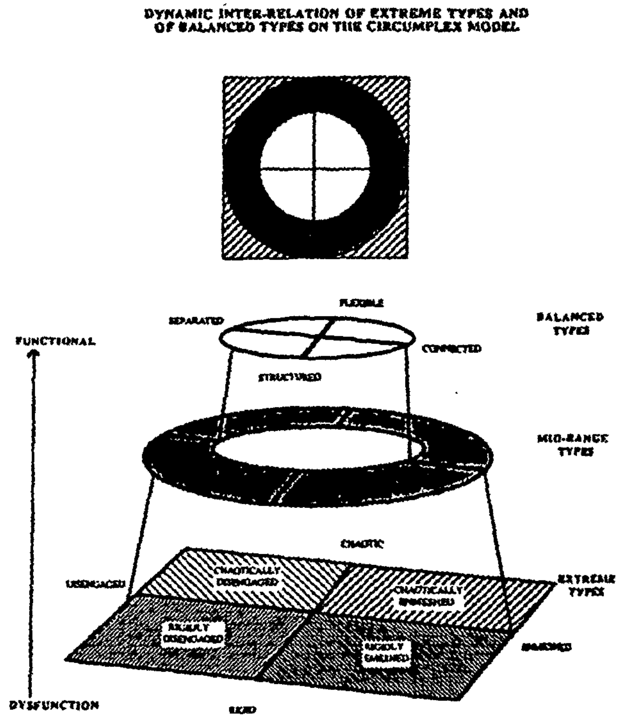

Figure 10

Balanced family systems tend to have a larger behavioral repertoire than extreme types, which enables them to change their system in smaller increments

162. *Id.* at 57.

163. Figure 10 is reprinted from Walsh & Olson, *supra* note 147, at 59. For copies of the complete work, contact Marianne Arnold at The Haworth Document Delivery Service (Telephone 1-800-3-HAWORTH; 10 Alice Street, Binghamton, N.Y. 13904). For other questions concerning rights and permissions, contact Wanda Latour at the above address.

in order to cope with stress and problems.[164] In contrast, extreme family types often function on an "all-or-none" principle. To correct a family system, extreme types react in an extreme fashion—taking a diametrically opposite approach. For example, if a family system of the extreme type encounters an event like an adolescent child being arrested by the police, the family might first react by getting much more rigid and establishing additional rules. If that does not bring the system into balance, the family, exasperated, might then decide to drop all rules, becoming more chaotic.[165]

Having now surveyed Leibniz's geometry of situation in the context of modern General System Theory, this Article now proceeds to explore the geometry of emotions through the philosophy of his contemporary, Spinoza, and through consideration of a modern theory of the structure of emotions.[166] Later, this Article will return to Leibniz's theories when, in Section V, it delves into his basic concepts of differential and integral calculus to derive some analytical and visualization techniques that are helpful in achieving change and resolution in negotiation and mediation.

IV. THE GEOMETRY OF EMOTIONS

A. *Spinoza's Euclidean Structure of Emotions*

As pointed out earlier, it was Spinoza—not Leibniz—who developed the geometry of emotions as described in the *Ethics*, but scholars generally agree that Leibniz was greatly intrigued by this concept.[167] Although Leibniz's writings indicate that he had reservations about the logical soundness of the Euclidean-like demonstrations in Spinoza's *Ethics*,[168] scholars also generally agree that Leibniz's conduct in partially appropriating and incorporating Spinoza's ideas into his own philosophy of metaphysics somewhat belies Leibniz's remarks critical of the *Ethics*.[169] In his lifetime, Leibniz never fully integrated the concepts of the geometry of situation, the geometry of emotions, and the infinitesimal calculus, although an argument could be made that insinuations existing variously in his works indicate that he gave thought to this

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164. *Id.* at 59.

165. *Id.* at 62.

166. The identity, nature, and effect of human emotions are subjects not directly or specifically addressed by the family systems models, and they therefore require separate coverage here in order to round out our examination of the "relationship" aspect of dispute and transactional negotiation and mediation.

167. *See* generally COTTINGHAM, *supra* note 8, at 22.

168. *See* LEIBNIZ: SELECTIONS, *supra* note 5, at 55.

169. BELL, *supra* note 8, at 126-27. It should be noted that Leibniz, in his "Refutation of Spinoza," written in about 1708, said nothing critical of Spinoza's definitions or cataloguing of human emotions contained in the *Ethics*. LEIBNIZ: SELECTIONS, *supra* note 5, at 485-97.

potential, and that he believed it to be possible. Before exploring Spinoza's geometry of emotions, this Article first discusses a few things about Spinoza the person.

1. Spinoza—The Person

In 1632, Spinoza was born into a family which commanded much influence in the Jewish community in Amsterdam. His early education was dominated by the traditional studies of the Torah and the Talmud, influencing his mastery of the Hebrew language. Eventually, however, he achieved the status of polyglot, being fluent additionally in Portuguese, Spanish, Dutch, and Latin. The latter language opened a whole new world to him in the study of the works of the scholastic philosophers under the tutelage of a gentile, Francis van den Enden, a noted intellectual of the time.

By his early twenties, Spinoza became increasingly alienated by the rigid dogmatism of orthodox Jewish learning, and, by age twenty-four, because of his outspokenness, he was formally accused of heresy by his Jewish elders, solemnly cursed, and expelled from the synagogue. In the years that followed his excommunication, his philosophical interests grew, and, by 1660, he had acquired a close circle of friends and correspondents with whom he discussed ethical, metaphysical, and scientific ideas. The early 1660s were intensely intellectually productive for Spinoza. In this decade, he undertook several major writing projects, only one of which, *Descartes' Principles of Philosophy*, was published during his lifetime. By 1665, the bulk of his work on the *Ethics* was completed, and it was revised by him in the early 1670s; but, on advice of Oldenburg, a mutual correspondent of Spinoza and Leibniz, he withheld publication of it so as not to risk the perception that he advocated the overthrow of the practice of religious virtue. The *Ethics* was published shortly after his death in 1677.[170]

2. Overview of the *Ethics*

The *Ethics* reflects Spinoza's fascination with the geometrical method of presentation, and it is reputedly the only major philosophical work to follow such a pattern.[171] This *magnum opus* is divided into five parts titled as listed here.

Part I: Concerning God
Part II: Of the Nature and Origin of the Mind

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170. COTTINGHAM, *supra* note 8, at 20-21. The full title for the *Ethics* is *Ethics Demonstrated in Geometrical Order*. *Id.*
171. *Id.* at 50.

Part III: Of the Origin and Nature of the Emotions
Part IV: Of Human Bondage or the Strength of the Emotions
Part V: Of the Power of the Understanding, or of Human Freedom

The parts most relevant to later discussions in this Article are Parts III and IV.

In justifying his geometrical approach in the introduction to Part III, Spinoza explains as follows:

> For the present I wish to revert to those, who would rather abuse or deride human emotions than understand them. Such persons will, doubtless think it strange that I should attempt to treat of human vice and folly geometrically . . . . However, such is my plan. Nothing comes to pass in nature, which can be set down to a flaw therein; for . . . nature's laws and ordinances . . . are everywhere always the same; so that there should be one and the same method of understanding the nature of all things whatsoever, namely, through nature's universal laws and rules. Thus the passions of hatred, anger, envy, and so on, considered in themselves, follow from this same necessity and efficacy of nature . . . . I shall consider human actions and desires . . . as though I were concerned with lines, planes, and solids.[172]

In keeping with this geometrical approach, each of the five parts opens typically with a numbered list of "definitions," followed by a numbered list of "axioms," followed still by a long sequence of numbered "propositions"—more than thirty for each part. Each proposition is supported by a "proof"—a demonstration showing how the proposition in question is derived either directly or indirectly from the previously demonstrated propositions and from the definitions and axioms. The obvious inspiration for the pattern followed by Spinoza is Euclid's *Elements*, written in Alexandria around 300 B.C., and widely available in Latin in the seventeenth century. Euclid's *Elements* was universally admired as a paradigm of rigorous reasoning. Spinoza unquestionably believed that his axioms were self-evidently and necessarily true. He was not so naive, however, to think that every reader of the *Ethics* would be struck with the blinding conviction immediately that all of the axioms were true. In fact, his expectation was that readers would suspend judgment until they read the axioms in context.[173] Spinoza wrote: "Here no doubt my readers will come to a halt and think many things which will give them pause. For this reason I ask them to continue on with me slowly, step by step, and to make no

172. THE RATIONALISTS: DESCARTES, SPINOZA, LEIBNIZ 262-63 (R.H.M. Elwes trans., 1974) [hereinafter THE RATIONALISTS].

173. COTTINGHAM, *supra* note 8, at 50-55.

judgment on these matters until they have read through everything."[174]

Parts III and IV of the *Ethics* then constitute an orderly deductive proof of emotions, their causes, their interrelationships, their relative strengths, and their effects—taken together, a symphonic syllogism of human nature. It is not necessary that, in later analyses, the reader accept Spinoza's definitions as absolute, his axioms as true, or his propositions as the result of valid reasoning. It is only necessary that, in developing this Article's visual calculus of change, the reader recognize the historical and practical significance of Spinoza's work in laying the foundation for modern logical analysis of emotions in terms of facilitative and inhibitory influences in the resolution of conflict. Knowledge of the modern theories of the cognitive structure or geometry of emotions, discussed later in Section IV.B, is an essential tool for every person who engages the negotiator or mediator craft.

3. Spinoza's Definitions of Emotions

In Part III of the *Ethics*, Spinoza describes the nature and the types of emotions through a series of definitions, postulates, propositions, corollaries, and proofs. Initially, he provides a general definition of the word "emotion," identifies and defines what he calls the three primitive or primary emotions from which all other emotions spring (pleasure, pain, and desire),[175] and, in an appendix to Part III, he sets forth definitions of forty-three specific emotions and of two emotion-related "conceptions" (wonder and contempt). He defines "emotion" generally as "modifications of the body, whereby the active power of the said body is increased or diminished, aided or constrained, and also the ideas of such modifications."[176] "Desire" Spinoza defines as "the actual essence of man, insofar as it is conceived, as determined to a particular activity by some given modification of itself."[177] For Spinoza, the pleasure and pain emotions are grouped together and are separate from the desire emotions.[178] "Pleasure" he defines as "the transition of a man from a less to a greater perfection," and he defines "pain" as "the transition of man from a greater to a less perfection."[179] He also categorizes emotions in two groups: those caused externally and those caused internally.[180] If a human being can be the adequate cause of any of the "modifications of the body" relating to a specific

174. *Id.* at 53 (quoting Spinoza).

175. *Id.* at 309-10. *See generally* JONATHON BENNETT, A STUDY OF SPINOZA'S *ETHICS* 253 (1984).

176. THE RATIONALISTS, *supra* note 172, at 264.

177. *Id.* at 308.

178. BENNETT, *supra* note 175, at 253. *See generally* 2 HARRY A. WOLFSON, THE PHILOSOPHY OF SPINOZA: UNFOLDING THE LATENT PROCESSES OF HIS REASONING 180-220 (1969).

179. THE RATIONALISTS, *supra* note 172, at 308.

180. *Id.* at 263-64.

emotion, then the emotion is an "activity"; otherwise, it is a "passion" or a state in which the mind is passive. In the next chart the forty-five names that Spinoza ascribes to the three categories of primary emotions (including the two conceptions), his definitions of them, and some of his explanatory comments appear.[181]

Spinoza's Definitions of Emotions

EMOTIONS	DEFINITIONS	SPINOZA'S COMMENTS
Ambition	the immoderate desire of power	This is the emotion whereby all the emotions are fostered and strengthened.
Anger	the desire, whereby through hatred we are induced to injure one whom we hate	
Approval	love towards one who has done good to another	
Avarice	the excessive desire and love of riches	
Aversion	pain, accompanied by the idea of something which is accidentally the cause of pain	
Benevolence	the desire of benefiting one whom we pity	
Confidence	pleasure arising from the idea of something past or future, wherefrom all cause of doubt has been removed	
Consternation	attributed to one whose desire of avoiding evil is checked by amazement at the evil that he fears	This is a species of cowardice. It arises from a double fear, which keeps a man so bewildered and wavering that he is not able to remove the evil.

181. This chart has been developed based on information appearing in *id.* at 308-319.

EMOTIONS	DEFINITIONS	SPINOZA'S COMMENTS
Contempt	the conception of anything which touches the mind so little that its presence leads the mind to imagine those qualities which are not in it rather than such as are in it	Spinoza passes over "veneration" and "scorn" here, because he is "not aware that any emotions are named after them."
Courtesy	also called "deference," is the desire of acting in a way that should please men and refraining from that which should displease them	
Cowardice	attributed to one whose desire is checked by the fear of some danger, which his equals dare to encounter	This is not among the emotions springing from desire. It is opposite to the emotion of daring.
Cruelty	also called "savageness," whereby a man is impelled to injure one whom we love or pity	This is opposite to clemency, which is a power whereby man restrains his anger and revenge.
Daring	the desire whereby a man is set on to do something dangerous which his equals fear to attempt	
Derision	pleasure arising from our conceiving the presence of a quality which we despise in an object which we hate	Insofar as we despise a thing which we hate, we deny existence thereof, and to that extent rejoice.
Despair	pain arising from the idea of something past or future, wherefrom all cause of doubt has been removed	Confidence springs from hope and despair from fear, when all cause for doubt as to the issue of an event has been removed.
Devotion	love towards one whom we admire	Devotion readily degenerates into simple love.
Disappointment	pain accompanied by the idea of something past, which has had an issue contrary to our hope	
Disparagement	thinking too meanly of anyone because we hate him	

EMOTIONS	DEFINITIONS	SPINOZA'S COMMENTS
Emulation	the desire of something, engendered in us by our conception that others have the same desire	It is customary to speak of emulation only in him who imitates that which we deem to be honorable, useful, or pleasant.
Envy	hatred, insofar as it induces a man to be pained by another's good fortune, and to rejoice in another's evil fortune	This is generally opposite to sympathy.
Fear	an inconstant pain arising from the idea of something past or future, whereof we to a certain extent doubt the issue	There is no hope unmingled with fear and no fear unmingled with hope.
Hatred	pain accompanied by the idea of an external cause	
Honor	pleasure accompanied by the idea of some action of our own, which we believe to be praised by others	
Hope	an inconstant pleasure arising from the idea of something past or future whereof we to a certain extent doubt the issue	
Humility	pain arising from a man's contemplation of his own weakness of body or mind	This is opposite to self-complacency.
Inclination	pleasure accompanied by the idea of something that is accidentally a cause of pleasure	
Indignation	hatred towards one who has done evil to another	
Intemperance	the excessive desire and love of drinking	
Joy	pleasure accompanied by the idea of something past that has had an issue beyond our hope	

EMOTIONS	DEFINITIONS	SPINOZA'S COMMENTS
Love	pleasure accompanied by the idea of an external cause	
Lust	desire and love in the matter of sexual intercourse	
Luxury	excessive desire or even love of living sumptuously	
Partiality	thinking too highly of anyone because of the love we bear him	
Pity	pain accompanied by the idea of evil that has befallen someone else whom we conceive to be like ourselves	Pity is used in reference to a particular action; sympathy is used in reference to a disposition.
Pride	thinking too highly of one's self from self-love	This should be distinguished from partiality which is used in reference to an external object, rather than the self.
Regret	desire or appetite to possess something, kept alive by the remembrance of the said thing and at the same time constrained by the remembrance of other things which exclude the existence of it	
Repentance	pain accompanied by the idea of some action that we believe we have performed by the free decision of our mind	All of those actions which are commonly called *wrong* are followed by pain; and all those which are called *right* are followed by pleasure.
Revenge	desire whereby we are induced, through mutual hatred, to injure one who, with similar feelings, has injured us	

EMOTIONS	DEFINITIONS	SPINOZA'S COMMENTS
Self-abasement	thinking too meanly of one's self by reason of pain	Those who are believed to be most self-abased and humble are generally the most ambitious and envious.
Self-approval	pleasure arising from a man's contemplation of himself and his own power of action	
Shame	pain accompanied by the idea of some action of our own, which we believe to be blamed by others	In contrast, modesty is the fear or dread of shame, which restrains a man from committing a base action.
Sympathy	love, insofar as it induces a man to feel pleasure at another's good fortune, and pain at another's evil fortune	
Thankfulness	or gratitude, is the desire or zeal springing from love, whereby we endeavor to benefit him who, with similar feelings of love, has conferred a benefit on us	
Timidity	the desire to avoid a greater evil, which we dread, by undergoing a lesser evil	
Wonder	the conception of anything wherein the mind comes to a stand because the particular concept in question has no connection with other concepts	

Spinoza's organization of the forty-three emotions in terms of whether their source is pleasure, pain, or desire, and of whether their cause is external (outside of the experiencer) or internal (from within the experiencer) is shown in the next chart.[182]

182. This chart was developed from information contained in *id.* at 308-19.

Spinoza's Categories of Sources and Causes of Emotions

Pleasure Emotions		Pain Emotions		Desire Emotions
Externally Caused or Induced	Internally Caused or Induced	Externally Caused or Induced	Internally Caused or Induced	Internally Caused or Controlled
Love	Self-Approval	Hatred	Humility	Regret
Inclination	Pride	Aversion	Repentance	Emulation
Devotion	Honor	Fear	Self-abasement	Thankfulness
Derision		Despair	Shame	Benevolence
Hope		Disappointment		Anger
Confidence		Pity		Revenge
Joy		Indignation		Cruelty
Approval		Disparagement		Timidity
Partiality		Envy		Daring
Sympathy				Cowardice
				Consternation
				Courtesy
				Ambition
				Luxury
				Intemperance
				Avarice
				Lust

4. Spinoza's Propositions on Intensity of Emotions

In Part IV of the *Ethics*, Spinoza presents seventy-three propositions regarding the intensity of emotions, some of which appear in the next chart.[183]

Proposition VI: The force of any passion or emotion can overcome the rest of a man's activities or power, so that the emotion becomes obstinately fixed to him.
Proposition VII: An emotion can only be controlled or destroyed by another emotion contrary thereto and with more power for the controlling emotion.
Proposition XI: An emotion towards that which we conceive as necessary is, when other conditions are equal, more intense than an emotion towards that which is possible or contingent or non-necessary.

183. The information contained in this chart was derived from *id.* at 321-71.

Proposition XII: An emotion towards a thing, which we know not to exist at the present time, and which we conceive as possible, is more intense, other conditions being equal, than an emotion towards a thing contingent.
Proposition XIII: Emotion towards a thing contingent, which we know not to exist in the present, is, other conditions being equal, fainter than an emotion towards a thing past.
Proposition XIV: A true knowledge of good (that which is useful to us) or evil (hindrance to that which is useful to us) cannot check any emotion by virtue of being true, but only insofar as it is considered as an emotion.
Proposition XV: Desire arising from the knowledge of good and bad can be quenched or checked by many of the other desires arising from the emotions whereby we are assailed.
Proposition XVI: Desire arising from the knowledge of good and evil, insofar as such knowledge regards what is future, may be more easily controlled or quenched, than the desire for what is agreeable at the present moment.
Proposition XVII: Desire arising from the true knowledge of good and evil, insofar as such knowledge is concerned with what is contingent, can be controlled far more easily still than desire for things that are present.
Proposition XVIII: Desire arising from pleasure is, other conditions being equal, stronger than desire arising from pain.
Proposition XXX: A thing cannot be bad for us through the quality which it has in common with our nature, but it is bad for us insofar as it is contrary to our nature.
Proposition XXXII: Insofar as men are a prey to passion, they cannot, in that respect, be said to be naturally in harmony.
Proposition XXXIII: Men can differ in nature, insofar as they are assailed by those emotions which are passions or passive states; and to this extent one and the same man is variable and inconstant.
Proposition XXXIV: Insofar as men are assailed by emotions which are passions, they can be contrary to one another.
Proposition XXXV: Insofar only as men live in obedience to reason, do they always necessarily agree in nature.

5. Spinoza's Axioms and Propositions on the Control of Emotions

In Part V of the *Ethics*, Spinoza presents two axioms on the control of emotions, shown next, along with forty-two propositions on the same topic, some of which also appear in the next chart.[184]

Axiom I: If two contrary actions are started in the same subject, a change must necessarily take place, either in both or in one of the two, and continue until they cease to be contrary.
Axiom II: The power of an effect is defined by the power of its cause, insofar as its essence is explained or defined by the essence of its cause.
Proposition III: An emotion, which is a passion, ceases to be a passion as soon as we form a clear and distinct idea thereof.
Proposition V: An emotion towards a thing which we conceive simply and not as necessary or as contingent or as possible, is, other conditions being equal, greater than any other emotion.
Proposition VI: The mind has greater power over the emotions and is less subject thereto, insofar as it understands all things as necessary.
Proposition VII: Emotions which are aroused or spring from reason, if we take account of time, are stronger than those which are attributable to particular objects that we regard as absent.
Proposition VIII: An emotion is stronger in proportion to the number of simultaneous concurrent causes whereby it is aroused.
Proposition IX: An emotion which is attributable to many and diverse causes which the mind regards as simultaneous with the emotion itself is less hurtful, and we are less subject thereto and less affected towards each of its causes than if it were a different and equally powerful emotion attributable to fewer causes or to a single cause.
Proposition XII: The mental images of things are more easily associated with the images referred to things which we clearly and distinctly understand than with others.
Proposition XIII: A mental image is more often vivid in proportion as it is associated with a greater number of other images.

B. *A Modern Theory of the Structure of Emotions*

Of all the theories of emotions advanced over the centuries since Aristotle

184. The information contained in this chart was derived from *id.* at 382-90.

unveiled his dyad theory in the *Rhetoric*,[185] one of the most compelling and useful theories to emerge in recent years is that described by Andrew Ortony, Gerald Clore, and Allan Collins in a book entitled *The Cognitive Structure of Emotions*.[186] Considered to be the first systematic, comprehensive, and computationally tractable description of the cognitions that underlie a broad spectrum of human emotions,[187] the theory's primary thrust is directed to the question of emotional *differentiation*—that is, what distinguishes one emotion from another.[188] Ortony and his colleagues have arranged types of emotions into emotion groups along with their *eliciting conditions*—a situational description of the conditions under which the emotions can be triggered.[189] The groups of emotions that they identify have two important characteristics: (1) emotions in the same group have eliciting conditions that are structurally related and, (2) each distinct emotion type represented in these groups is thought of as representing a "family" of closely related emotions.[190] Emotions in each "family" are related in that they share the same basic eliciting conditions but differ in terms of their intensity and sometimes in terms of the weights assigned to their behavioral components.[191] Intensity of emotions depends on local variables, such as desirability, praiseworthiness, and appealability, which affect particular groups of emotions, and on global variables, such as sense of reality, proximity, unexpectedness, and arousal, which affect all emotions.[192]

Underlying the structure of this theory is the assumption that there are three major aspects of the world to which a person may react, namely, *events*, *agents*, and *objects*.[193] Central to this theory is the notion that emotions are positively or negatively valenced reactions,[194] and the notion that any specific valenced

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185. As to Aristotle's dyad theory, see COOLEY, *supra* note 79, at 306-30 (1993 supp.). *See also* John W. Cooley, *A Classical Approach to Mediation—Part I: Classical Rhetoric and the Art of Persuasion in Mediation*, 19 U. DAYTON L. REV. 83 (1993). As to a description of the variety of theories of emotions, see generally KEITH OATLEY, BEST LAID SCHEMES: THE PSYCHOLOGY OF EMOTIONS (1992).

186. ANDREW ORTONY ET AL., THE COGNITIVE STRUCTURE OF EMOTIONS (1988).

187. *Id.* at ix, 14.

188. *Id.* at 1.

189. *Id.* at 15.

190. *Id.* at 15-16.

191. *Id.* at 16.

192. *Id.* at 83-84.

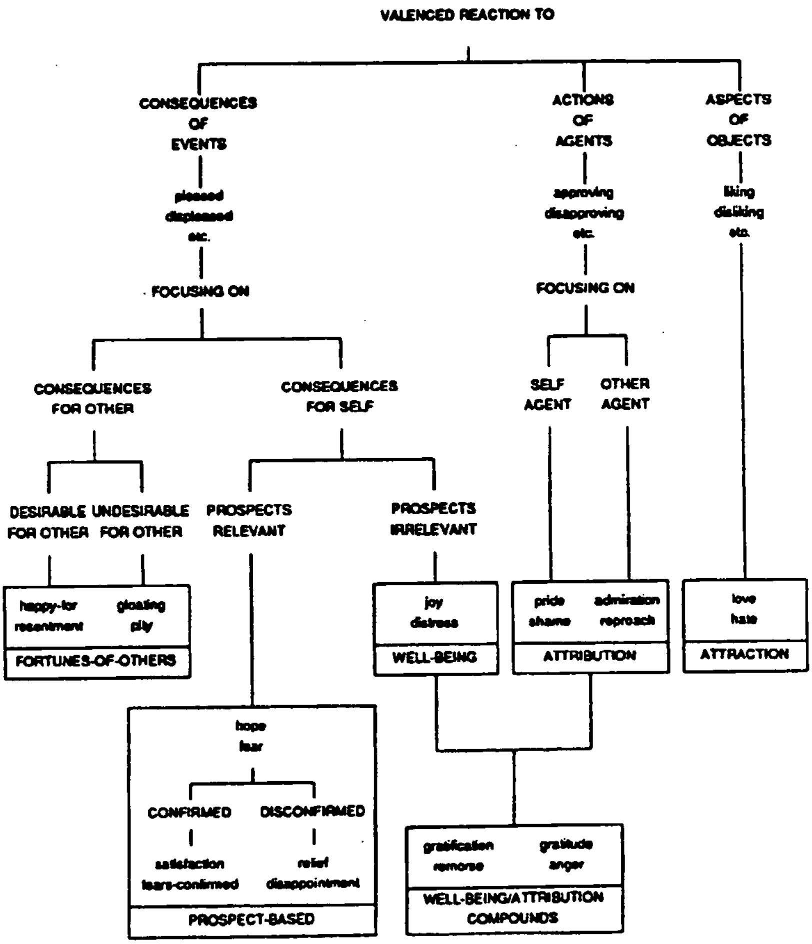
193. *Id.* at 18.

194. In defining valence, one commentator has observed:

> Events, objects, and situations may possess positive or negative valence; that is, they may possess intrinsic attractiveness or aversiveness. The adjective *intrinsic* serves to distinguish these features from derived attractiveness or aversiveness: Loss derives its aversiveness from the positive valence of the object lost.

NICO FRIJDA, THE EMOTIONS 207 (1986) (emphasis in original).

reaction is always a reaction to either an event, an agent, or an object.[195] The chart in Figure 11 depicts the overall structure of emotion types, organized as valenced reactions to events, agents, and objects.[196]



Global structure of emotion types.

Figure 11

Note in Figure 11 that hope and fear—emotions routinely operating in

195. ORTONY ET AL., *supra* note 186, at 18.

196. This chart appears in ANDREW ORTONY ET AL., THE COGNITIVE STRUCTURE OF EMOTIONS 19 (1988) and is reprinted with the permission of Cambridge University Press.

negotiation and mediation situations—are valenced reactions to (that is, being pleased or displeased about) the prospect of an event.[197] Related emotions are prospect-confirmed emotions (satisfaction and fears-confirmed) and prospect-disconfirmed emotions (relief and disappointment).[198] Another emotion often operating in a negotiation or mediation situation is anger, depicted as a compound well-being/attribution emotion arising as a valenced reaction, which combines disapproval of an agent's action with displeasure at the undesirable outcome.[199]

In this theory of the structure of emotions, each emotion specification has five components: type identification, type specification, tokens (a list of words or phrases that constitute the family of emotions of that type), variables affecting intensity, and a prototypical example. The next chart depicts the emotion specification for the following emotions: hope, fear, satisfaction, fears-confirmed, relief, disappointment, anger, and appreciation.[200]

Type Identification	Type Specification	Tokens	Variables Affecting Intensity	Example
Hope Emotions	pleased about the prospect of a desirable event	anticipation, anticipatory excitement, excitement, expectancy, hope, hopeful, looking forward to, etc.	(1) degree to which the event is desirable, (2) likelihood of the event	As she thought about the last settlement offer, the plaintiff was filled with hope that the case would soon settle.

197. *Id.* at 109.
198. *Id.* at 110-11.
199. *Id.* at 154-55.
200. The information contained in the first four columns of the chart appears in *id.* at 112, 118-19, 121-22, 145, 148. The examples provided in column 5 are the creation of this author.

Type Identification	Type Specification	Tokens	Variables Affecting Intensity	Example
Fear Emotions	displeased about the prospect of an undesirable event	apprehensive, anxious, cowering, dread, fear, fright, nervous, petrified, scared, terrified, timid, worried, etc.	(1) degree to which the event is undesirable, (2) likelihood of the event	The defendant, knowing that he ran the red light, feared that a jury would surely find him guilty of negligence.
Satisfaction Emotions	pleased about the confirmation of the prospect of an event	gratification, hopes-realized, satisfaction, etc.	(1) intensity of the attendant hope emotion, (2) effort expended in trying to attain the event, (3) degree to which the event is realized	When she realized that she was indeed being offered a reasonable settlement amount, she was gratified.
Fears-confirmed Emotions	displeased about the confirmation of the prospect of an undesirable event	fears-confirmed, worst fears realized	(1) intensity of the attendant fear emotion, (2) effort expended in trying to prevent the event, (3) degree to which the event is realized	The defendant's fears were confirmed when the rabbi who witnessed the accident was located and agreed to testify against the defendant.

Type Identification	Type Specification	Tokens	Variables Affecting Intensity	Example
Relief Emotions	pleased about the disconfirmation of the prospect of an undesirable event	relief	(1) the intensity of the attendant fear emotion, (2) effort expended in trying to prevent the event, (3) degree to which the event is realized	The defendant was relieved to learn that the principal witness against him could not be located.
Disappointment Emotions	displeased about the disconfirmation of the prospect of a desirable event	dashed-hopes, despair, disappointment, frustration, heartbroken, etc.	(1) intensity of the attendant hope emotion, (2) effort expended in trying to attain the event, (3) degree to which the event is realized	The lawyer was disappointed when she was told that the defense counsel would not budge from his last settlement offer.
Appreciation Emotions	approving of someone else's praiseworthy action	admiration, appreciation, awe, esteem, respect, etc.	(1) degree of judged praiseworthiness (2) deviations of the agent's action from person/role-based expectations (i.e., unexpectedness)	The plaintiff appreciated the insurance adjuster's offer to pay the settlement amount within five days.

Type Identification	Type Specification	Tokens	Variables Affecting Intensity	Example
Anger Emotions	disapproving of someone else's blameworthy action and being displeased about the related undesirable event	anger, annoyance, exasperation, fury, incensed, indignation, irritation, livid, offended, outrage, rage, etc.	(1) degree of judged blame-worthiness, (2) deviations of the agent's action from person/role-based expectations, (3) degree to which the event is undesirable	The plaintiff was angry with her lawyer when she learned that he failed to advise her about her right to have the dispute mediated prior to trial.

In all, the Ortony-Clore-Collins model of the structure of emotions distributes 130 tokens across twenty-two emotion types.[201] This model also contemplates the opportunity for the eventual computer-analysis of emotions. The model's architects believe that the emotion characterizations that they have proposed provide a first step in developing a system of rules and representations about the elicitation of emotions—a computer program to analyze emotions.[202] They propose that rules be stated in terms as functions, as shown below in relation to the prospect-based emotion of fear:[203]

(1) IF PROSPECT (p,e,t) AND DESIRE $(p,e,t) < 0$
THEN *set* FEAR-POTENTIAL $(p,e,t) =$
f_f [| DESIRE (p,e,t) | , LIKELIHOOD (p,e,t), I_g (p,e,t)]

(2) IF FEAR-POTENTIAL (p,e,t) > FEAR-THRESHOLD (p,t)
THEN *set* FEAR INTENSITY $(p,e,t) =$
FEAR-POTENTIAL (p,e,t) - FEAR-THRESHOLD (p,t)
ELSE *set* FEAR-INTENSITY $(p,e,t) = 0$

These rules may be stated in ordinary language as follows. First, let us consider Rule (1). Rule (1) basically provides the statement (equation) for describing when the potential for fear emotions will be triggered. It indicates that when it is true that p (a person) entertains the PROSPECT of e (an event) at time t, and if p considers e to be undesirable (< 0), then the potential for fear will be

201. *Id.* at 173.
202. *Id.* at 182.
203. These rules appear in *id.* at 185.

triggered. The magnitude of the emotion potential (described by the symbols to the right of the equal sign (=)) is a function of the *absolute value* of the *desirability* of the event, the subjective *likelihood* that it will be realized, and the contribution of the *global factors*.[204] Rule (2) then provides a basis for determining whether the magnitude of FEAR-POTENTIAL exceeds the current threshold for fear emotions, and, if so, it sets the intensity of fear equal to the difference between the current value of FEAR-THRESHOLD and the just set value of FEAR-POTENTIAL.[205] If the magnitude of FEAR-POTENTIAL emotions does not exceed the current threshold for fear emotions, it resets the value of FEAR-INTENSITY to zero to indicate that *p* did not experience fear in response to event *e* at time *t*.[206] It is not difficult to understand how similar routines could be developed for the other twenty-one emotion types, and how computer analysis of emotions could be helpful in conflict resolution and in many other aspects of the behavioral sciences and sociology.[207] As the model's architects observe, computerizing this emotions model raises the prospect of being able to experiment with some of the parameters of the model in ways that are impossible in the real world or in the psychological laboratory.[208]

V. The Calculus of Change in Negotiation and Mediation

As noted earlier in Section II, the genius of Leibniz as a problem solver is best demonstrated by his crystalization of the mathematical problem solving process called the infinitesimal calculus. The purpose of this Section is *not* to make calculus experts out of readers who have no such interest, nor to test the mathematical prowess of those who are mathematically inclined. Rather, its purpose is to examine a few of the general principles of calculus as a background for further inquiry into the utility of a visual calculus for enhancing problem solving in negotiation and mediation contexts.

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204. *Id.* at 185.

205. *Id.* at 185-86

206. *Id.* at 186.

207. As Ortony, Clore, and Collins have observed, "[t]here are many AI [artificial intelligence] endeavors in which the ability to understand and reason about emotions or aspects of emotions could be important. Obvious examples include natural language understanding, cooperative problem solving, and planning." *Id.* at 182.

208. *Id.* at 190. Looking to the future, some experts theorize that artificial intelligence will someday be viewed as having an emotional component. *See* Nico Frijda & J. Swagerman, *Can Computers Feel? Theory and Design of an Emotional System*, 1 COGNITION AND EMOTION 235-37 (1987); Aaron Sloman & Monica Croucher, *Why Robots Will Have Emotions*, PROCEEDINGS OF THE SEVENTH INTERNATIONAL JOINT CONFERENCE ON ARTIFICIAL INTELLIGENCE 197-202 (1981).

A. Calculus—An Instrument for Visualizing and Measuring Change

In the English language, the only expression that arouses more fear than the three-word phrase "Internal Revenue Service" is the three-syllable word "calculus." It should not, however. Calculus is actually a very friendly Latin word, whose literal English translation is "a small stone" or "pebble." In science, it has been, ironically, a large building block. Before proceeding to discuss the basics of differential and integral calculus, this Article will review a few simple definitions.

First of all, a line can be considered either straight or curved, and it can be viewed either as a locus of points[209] or the path of a single point.[210] When considering "change" as represented by a straight line or a curve, it is usually easier to consider a *line* as consisting of the *path of a point*. Furthermore, every straight line or curve has a unique *equation* (or *function*), which defines it in terms of *independent* and *dependent variables* and which equation (or function) can be represented graphically.[211] Thus, the path of "change" has both a quantitative mathematical description in terms of symbols (an equation) and a qualitative visual description in terms of a line, area, volume, etc. (a graph). Geometric imagineering of the *relationship* in conflict situations is more concerned with the qualitative rather than the quantitative aspects of change.[212] Some of the many different types of change, together with graphical representations of their typical time series, are depicted in Appendix B of this Article.

To gain a clearer understanding of the meaning of the terms "function," "independent variable," and "dependent variable," consider Figure 12.[213] Depicted in that figure is a steel rod, clamped at the left with unit weights being added progressively to the right end. In the top position (rod horizontal), the unweighted system is at rest, and the elements composing it are in equilibrium.

209. Descartes defined a locus as a collection of an infinite number of points, all of which satisfied a condition, that is, the equation of a curve. *See* Cooley, *supra* note 1, at 95 n.51.

210. Leibniz defined a line as the path of a point. *See infra* appendix A, para. C8.

211. EDWARD KASNER & JAMES R. NEWMAN, MATHEMATICS AND THE IMAGINATION 323 (1989).

212. However, it must be emphasized that calculus can be very useful to the negotiator or mediator in addressing the *substance* of a dispute or transaction in the collaborative negotiation phase of problem solving. For example, calculus can be used to solve demand and supply problems, to determine an equilibrium price, to calculate rates of change of quantities in business transactions, to conduct a marginal analysis, to solve optimization problems, to solve problems requiring the minimizing of average costs, etc. *See generally* BARBARA L. BLEAU, FORGOTTEN CALCULUS: A REFRESHER COURSE FOR BUSINESS APPLICATIONS (1988).

213. Figure 12 is reprinted from *Mathematics and the Imagination* by Edward Kasner and James Newman, at 303. Copyright © 1989 by Ruth G. Newman. Reprinted by permission of Microsoft Press. All rights reserved.

As unit weights are added, the system moves and arrives at three successive equilibrium points. The bending in the rod is a *function* of the unit weights being added to the right side. The degree of bending depends on the number of unit weights added and is, therefore, said to be the *dependent variable*; the weight may be added arbitrarily—by independent selection—and is, therefore, the *independent variable*. From position one to position four, the rod underwent a change continuously which could be depicted visually by a graph on x and y axes. That is, at the top position, where the independent variable (x)—the unit weight—is 0, the degree of bend or the dependent variable (y) is 0; where the independent variable is one unit weight, the dependent variable is about 1.75; where the independent variable is two unit weights, the dependent variable is 2.25; when the independent variable is three unit weights, the dependent variable is 4.0. Of course, in this example, if too many unit weights were added, the steel rod might reach its elastic limit and either snap or remain in a bent position, creating a discontinuity in the behavior of the system.[214]

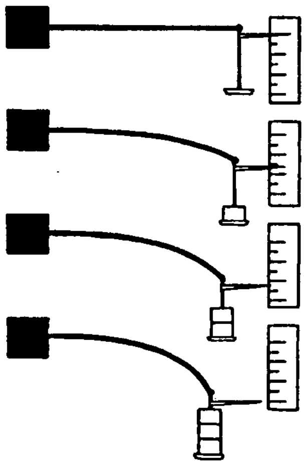


Figure 12

Now move from the physical world of steel rods and weights to the world of interpersonal relations and consider these three terms in the context of this example.[215] First, assume that the force of attraction between two people, John and Mary, in a friendship behaves inversely to the force of attraction

214. *See infra* section V.C.2 concerning the catastrophe theory of behavioral systems.

215. This example is adapted from KASNER & NEWMAN, *supra* note 211, at 313-14.

between two magnets. That is, whereas the force of attraction between two magnets increases when the distance between them decreases, let us assume that the force of attraction between human beings increases when the distance between them increases. Absence makes the heart grow fonder. Another way to say this is that the force of attraction between John and Mary is a *function* of the distance between them according to a defined rule (equation).[216] If the distance between John and Mary is permitted to assume arbitrary values, distance is considered to be an *independent* variable. The force of attraction then becomes the *dependent* variable, dependent upon distance and is uniquely determined, both mathematically and graphically, by assigning values to the independent variable. In describing functional relations, the letter x usually denotes the independent variable (as measured along the horizontal axis), and the letter y denotes the dependent variable (as measured along the vertical axis). The dependency relationship "y is a function of x" is written symbolically by the equation: $y = f(x)$. That equation determines a value of y for every value of x. Each pair of values which satisfies this equation defines a point on a unique straight line or curve depicting the function. Let us assume that through observation over time, we determine that the force of attraction in the friendship between John and Mary varies in relation to the distance between them plus the constant 3—that is, $y = f(x) = x + 3$. In another friendship that John has with Cynthia, we note that the attractive force varies in relation to the square of the distance between them—that is, $y = f(x) = x^2$. The graphs of the functions showing the respective attraction/distance relationships of the two friendships appear in Figures 14 and 15.[217]

---

216. A function may be defined, simply, as "a rule that associates with each object in a set A, one and only one object in a set B." LAURENCE D. HOFFMANN, APPLIED CALCULUS 1 (McGraw-Hill 1983). Figure 13 is a visuai representation of a function. This figure is reprinted from HOFFMAN, *supra*, and is reproduced with the permission of McGraw-Hill, Inc. © 1983 by McGraw-Hill, Inc.

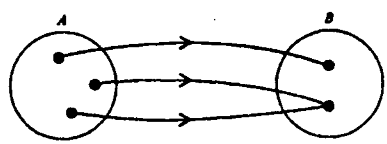


Figure 13

217. Figures 14 and 15 are reprinted from *Mathematics and the Imagination* by Edward Kasner and James Newman, at 313. Copyright © 1989 by Ruth G. Newman. Reprinted by permission of Microsoft Press. All rights reserved.

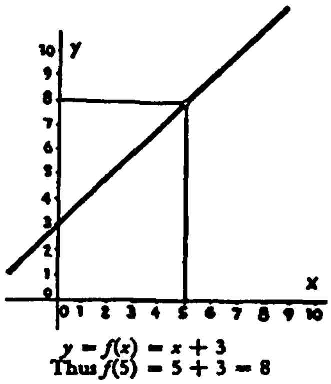


Figure 14

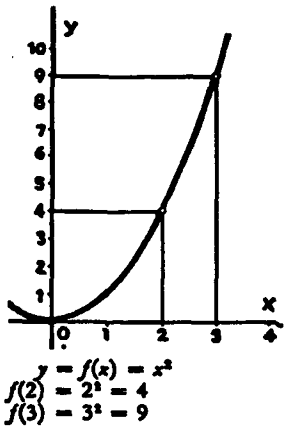


Figure 15

What the graph in Figure 14 reveals is that the force of attraction between John and Mary is at a "3" level when they are together and that it steadily increases in a predictable, straight-line manner as the distance between them increases. John and Cynthia's friendship, depicted by the graph in Figure 15, seems parabolically dysfunctional in that when they are together, the force of attraction between them is zero, but it increases to exaggerated levels when they are only short distances apart.

The chart below presents a brief review of the definitional relationships existing among *function*, *independent variables*, and *dependent variables* in mathematical, physical, and interpersonal contexts:[218]

Mathematics	*Physics*	*Interpersonal*
Independent variable, x	Amount of weight	Amount of distance
Dependent variable, y	Amount of bend of steel rod	Amount of attractive force
Function is the relation between x and y	Function is the relation between the weight and the degree of bend	Function is the relation between amount of distance and the attractive force

218. This chart was adapted from KASNER & NEWMAN, *supra* note 211, at 316.

Mathematics	*Physics*	*Interpersonal*
Increase or decrease of x (i.e., change)	Addition or dimunition of weight (i.e., change)	Increase or decrease of distance (i.e., change)
Increase or decrease of y (i.e., change)	Increase or decrease in the degree of bend of the steel rod (i.e., change)	Increase or decrease in the amount of attractive force (i.e., change)
Limiting value of y (the function of x) equals a number	Limiting value of degree of bend (function of weight) equals a position	Limiting value of amount of attractive force (function of distance) equals a position (feeling)

With the aid of these basic definitions of terms from analytical geometry, this Article now reviews, briefly, the rudiments of differential and integral calculus.

1. Differential Calculus—Visualization Techniques

It should not be a surprise to the readers that learning something about *differentiating* could enhance their prowess in negotiating and mediating. Differentiating may be the negotiator-mediator's most valuable skill in that almost every aspect of each process requires some type of differentiation of information—discerning the meaning and accuracy of verbal and nonverbal language; distinguishing reality from unreality, fact from fiction; and, as we learned from Proposition 5 of Leibniz's Geometry of Situation, and Proposition 2 of his Analysis of Situation,[219] *comparing* information with other information or with some known standard information and making or attempting to make appropriate adjustments (changes) toward a specific goal. This skill is necessary when a negotiator or mediator is dealing with both the substance and the relationship of a particular human interaction. Another of the negotiator-mediator's valuable skills is *integrating*, which is discussed in the following subsection.

While functions describe change over an interval and can be represented graphically, as shown earlier, differential calculus is concerned with change occurring at a particular point on a curve. Specifically, it is concerned with the instantaneous rate of change. Mathematically, an "instantaneous rate of change" is defined as the slope of the tangent line to the particular point in question on a curve.[220] "Slope of a line" is defined as the change of vertical distance (on

219. *See supra* section III.B.
220. HOFFMANN, *supra* note 216, at 57-58.

the y axis) divided by the change of horizontal distance (on the x axis).[221] In Figure 16, the short line segments represent tangent lines at various points along the curve, and the plus and minus signs indicate whether the slope is positive (increasing) or negative (decreasing).[222]

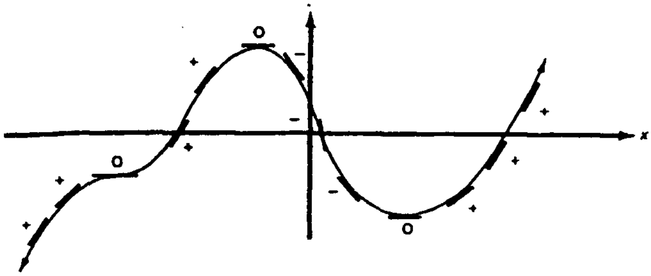


Figure 16

It is apparent that unlike a straight line with a constant slope, the slope of a curve is not constant. Also, there are three points on the above curve where the slope is zero (that is, the tangent is parallel to the x axis). The first zero, from left to right on the figure, is at a point of inflection, the second zero is at the point called a local maximum, and the third zero is at the point called the local minimum.[223] By differentiating the original function (that is, determining the first derivative function or equation), the original function can be quickly sketched because the *behavior* of the original function can be discerned by the increasing and decreasing slopes of tangents at its critical (maximum, minimum, or inflection) points. But before using an example of differentiation in geometric imagineering to aid in visualizing change, one must first know: (1) the rules for determining derivatives and (2) the rules for curve sketching.

Symbolically, given the function $y = f(x)$, the instantaneous rate of change equals $dy/dx = f'(x)$, which is also called the first derivative (function) of f.[224] The symbol for the second derivative (function) is $f''(x)$.[225]

221. *Id.* at 58. *See also* DANIEL KLEPPNER & NORMAN RAMSEY, QUICK CALCULUS 17 (1965).

222. This figure is reprinted from BARBARA L. BLEAU, FORGOTTEN CALCULUS: A REFRESHER COURSE FOR BUSINESS APPLICATIONS 121 (1988) with the permission of the publisher, Barron's Educational Series, Inc.

223. EDWIN J. PURCELL, CALCULUS WITH ANALYTIC GEOMETRY 194, 212 (3d ed. 1978).

224. HOFFMANN, *supra* note 216, at 78.

225. *Id.* at 137.

RULES FOR DETERMINING DERIVATIVES[226]

Power Rule

If $f(x) = x^n$, then $f'(x) = nx^{n-1}$ for all n, n being a real number.

***Example*:** **If $f(x) = x^4$, then $f'(x) = 4x^3$; $f''(x) = 12x^2$**

Derivative of a Constant

If $f(x) = c$, then $f'(x) = 0$, where c is a constant.

***Example*:** **If $f(x) = 5$, then $f'(x) = 0$**

Derivative of a Constant Times a Function

If $f(x) = (c)(u(x))$, then $f'(x) = (c)(u'(x))$

Example*:** ***If **$f(x) = 5x^2$, then $f'(x) = 5(2x) = 10x$**

Derivative of Sums or Differences

If $f(x) = u(x) + v(x)$, then $f'(x) = u'(x) + v'(x)$

***Example*:** **If $f(x) = x^3 + 7x^2 - 3x + 10$, then**

$f'(x) = 3x^2 + 14x - 3$ and $f''(x) = 6x + 14$

---

226. This chart has been developed from information appearing in BLEAU, *supra* note 212, at 121-25. To understand how these rules can be developed and proven mathematically, see SILVANUS P. THOMPSON, CALCULUS MADE EASY 15-44 (3d ed. 1987).

RULES FOR CURVE SKETCHING[227]

Step 1: Find the derivative $f'(x)$.

Step 2: Find the x coordinates of the critical points by setting $f'(x)$ equal to zero and solving for x. Also include any values of x for which the derivative is undefined. Substitute these values of x into the function $f(x)$ to obtain the y coordinates of the critical points.

Step 3: Plot the critical points on the graph. These are the only points at which relative minima and maxima points (extrema) can possibly occur.

Step 4: Determine where the function is increasing or decreasing by checking the sign of the derivative on the intervals whose endpoints are the x coordinates of the critical points.

Step 5: Draw the graph so that, with respect to the x axis, it increases on the intervals on which the derivative is negative, and levels off where $f'(x) = 0$.

Example: Determine where the function $f(x) = 2x^3 + 3x^2 - 12x - 7$ is increasing and where it is decreasing, then find its relative extrema, and draw the graph.

Step 1—Differentiate (find the derivative):

$$f'(x) = 6x^2 + 6x - 12 = 6(x + 2)(x - 1)$$

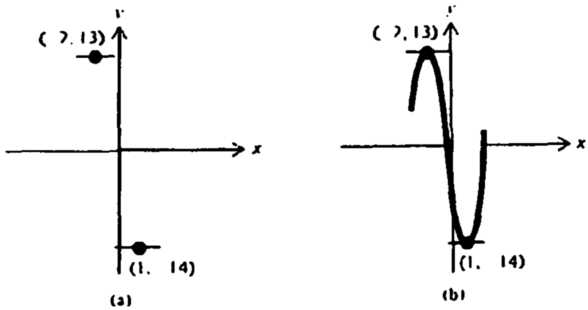
Step 2—Find the critical points:

From the factored form of the derivative, it is obvious that $f'(x) = 0$ when $x = -2$ and when $x = 1$. Since $f(-2) = 13$ and $f(1) = -14$, it follows that the critical points are (-2, 13) and (1, -14).

Steps 3 and 4: See Figure 17 in Step 5.

227. The information contained in this chart was adapted from HOFFMANN, *supra* note 216, at 97.

Step 5—Draw the Graph:[228]



Steps leading to the graph of $y = 2x^3 + 3x^2 - 12x - 7$.

Figure 17

Let us now consider the question-and-answer pattern of a mediator's caucus[229] and imagineer a cubic function[230]—similar to that plotted in the

228. The graphs in Figure 17 appear in LAURENCE HOFFMANN, APPLIED CALCULUS 99 (McGraw-Hill 1983) and are reproduced with the permission of McGraw-Hill, Inc. © 1983 by McGraw-Hill, Inc.

229. Normally, there are five principal levels of ordinary conversations and most, if not all of them, occur during a typical caucus. The five levels are: formal ("testing the waters"), contact maintenance (induction, strong resistance), standard (information collecting, moderate resistance), critical occasions (ideal working), and intimacy (peak times). JAMES F.T. BUGENTAL, THE ART OF THE PSYCHOTHERAPIST 28-29 (Norton ed. 1992). In the example caucus appearing in the text, *infra*, the conversation begins at the standard level and progresses into the critical occasions level.

230. First, second, and third-degree polynomial functions are called linear, quadratic, and cubic functions, respectively. Their graphs are predictable in that they have particular characteristics depending on the terms of the function. Predictability ends with the cubic function. Starting with the fourth-degree polynomial functions, graphs of polynomial functions result in a variety of shapes with only very general characteristics. *See* BLEAU, *supra* note 212, at 117-19. Examples of the linear, quadratic, and cubic functions, together with their particular characteristics, appear in the following chart.

LINEAR	QUADRATIC	CUBIC
$y = f(x) = mx + b$	$y=f(x)=ax^2 + bx + c$	$y=f(x)=ax^3+bx^2+cx+d$
straight line	parabola	S-shaped, on side
slope is m	If a is positive, curve opens up (like a cup)	If a is positive, the curve opens up at the right end

preceding chart—and perceive how the function $y = f(x) = x^3 + 3x^2 - 9x + 3$ and its first (y') and second (y'') derivatives can be used to visualize "frame paralleling" during the conversation.[231] Assume that the mediator is caucusing with a plaintiff-employee who has alleged racial discrimination in firing. The plaintiff-employee is quite angry with the company. The mediator has just previously caucused with the president of the defendant-company—Perfectseal—a small business with fifty employees that manufactures seals for piston engines. The company president was adamant that she fired the plaintiff for his general incompetence and his repeated absences from work—*not* for racial reasons. In this example, the mediator (*M*) asks the plaintiff (*P*) initial broadening questions, achieves paralleling, begins narrowing and gains control of the situation, and finally loses control by reverting to broad questions again. The dialogue appears in the left column below and the graphical representation of the curve (continuous function) of the conversation appears on the right (Figure 18), with an explanation (on the left) of the first and second derivatives of the conversation (on the right).[232] The unconnected points shown in the first graph represent the nine questions being asked by the mediator during this phase of the caucus. The symbols [B], [N], and [P] indicate a broadening question, a narrowing question, and a parallel question, respectively.

y-intercept at *b*	If *a* is negative, curve opens down (like a dome)	If *a* is negative, the curve opens down at the right end
	y-intercept is at *c*	*y*-intercept is at *d*
	There are, at most, two *x*-intercepts	There are, at most, three *x*-intercepts
	x-intercepts may be found by setting quadratic function equal to zero and solving for *x*	*x*-intercepts may be found by setting the cubic function equal to zero and solving for *x*

231. Frame paralleling is concerned with how abstractly or concretely the speaker handles the subject matter. BUGENTAL, *supra* note 229, at 13. The questioner is able to control the degree of concreteness of responses by selectively asking broad or narrow questions. *Id.* at 121-31.

232. This dialogue was adapted from *id.* at 129. The graphical representations in Figure 18 are reprinted from DOUGLAS DOWNING, CALCULUS THE EASY WAY 29, 33 (1982) with the permission of the publisher, Barron's Educational Series, Inc.

M: Tell me a little about your educational background. [B]
P: Oh, well, I went through Catholic grade school and three years of public high school and then went into the Army. I got my GED while I was in the service.

M: How did you like the military service? [B]
P: It was okay. I didn't like the discipline much. I was glad to get out and try civilian employment.

M: According to the work history you provided, you had quite a number of different jobs after you left the military service. [P]
P: Yeah, had a bunch of bad breaks. Couldn't find something that suited me.

M: Could you give me an example? [N]
P: Oh sure, like the job I had at the fairgrounds. I really thought that would be a good job, but the boss was a wino who didn't know what he was doing. He always cussed me out for not doing the right things or enough work or something.

M: How did you leave there? [N]
P: What do you mean? I just had enough and got out of the situation. Believe me it was bad news.

M: Were you fired? [N]
P: I've never had to leave a job I wanted to keep.

M: I'm not sure what you mean by "wanted to keep." [P]
P: If I wanted to keep a job, I would conform and stay.

M: How did you come to be employed at Perfectseal? [B]
P: I ran across their ad in the newspaper for a stamping machine operator.

M: Describe your prior experience doing that kind of work. [B]
P: Well, I had none. But the ad said there would be on-the-job training.

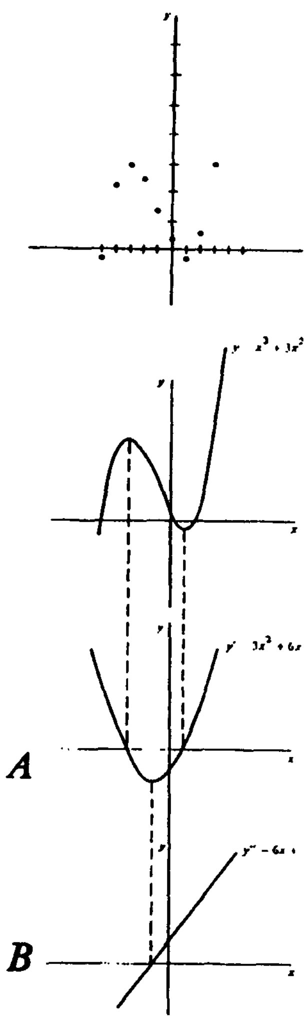


Figure 18

Graph *A* represents the first derivative of the conversation appearing above. As shown, the first derivative is a parabola which helps us to visualize the mediator's critical decision points in the main conversation (function). In fact, it can be imagined as the graph of the mediator's mental conversation with *himself* or *herself* during which critical *decisions* were made to change the relative degree of abstractness/concreteness of the conversation. Graph *A* further illustrates that the mediator gradually decided to decrease the abstractness of the conversation, reaching zero abstractness at the left x-intercept, then continuing concreteness through the vertex of the parabola until a decision was made at the right x-intercept to revert to a more abstract conversation with the plaintiff. Graph *B* represents the second derivative of the conversation and can be imagined to illustrate the critical point (at the x-intercept) on the continuum (line) of the entire conversation between mediator and plaintiff for controlling the conversation's direction and result.

In the example caucus above, continued narrowing of the questioning could have produced the following answers—as it in fact did in the real-life situation on which the dialogue was based.[233]

> M: [By saying you never had to leave a job you wanted to keep], [y]ou still aren't saying you've never been fired.
>
> P: I'm saying that a lot of different things go into whether or not a particular job is satisfying to me and whether I'm the man the boss wants. I have my own ideas of what kind of place I want to work in.
>
> M: I can understand that, but it doesn't answer the question I asked. It must be hard for you to give me a straight answer about whether you've been fired from any job.
>
> P: No, it's not hard. I just don't see the point of it. (Pause). So okay, yeah, I've been fired twice from lousy jobs that I didn't really want anyway.

Thus, a decision to continue with narrowing questioning at the critical opportunity point of conversation-control would have increased the mediator's probability of influencing a more successful direction and result in the caucus.

Having considered some uses of differential calculus in geometric imagineering, this Article is now ready to explore some visualization techniques associated with integral calculus.

2. Integral Calculus—Visualization Techniques

As noted earlier, another important skill of the effective negotiator-mediator is integrating, and, as with the skill of differentiating, it must be used in dealing with both the substance and the relationship of a particular human interaction. The negotiator-mediator must be able to synthesize discrete bits of information to form one whole accurate message, to combine differentiated parts into a new or unperceived form, or, as learned from Proposition 6 of Leibniz's Geometry of Situation, to see the points, lines, or areas where various parts or elements (information) *concur*, *coexist*, or *intersect*.

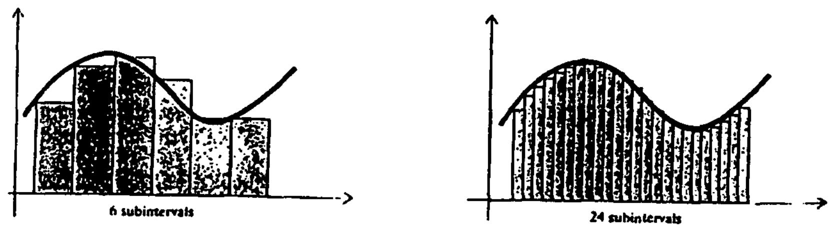
Having reviewed the principles of differential calculus, this Article has simultaneously reviewed much of what must be known about integral calculus for purposes of geometric imagineering. This is so because integrating, or

233. BUGENTAL, *supra* note 229, at 129.

finding the *indefinite* integral, is merely the reverse of differentiating. Finding areas and volumes can be accomplished by determining the *definite* integral.[234] The rules set out earlier, for example, taught that if $y = x^4$, then y' (the first derivative) $= 4x^3$. Thus, if asked to integrate the function $4x^3$, one could easily reverse the procedure, or determine the antiderivative, and determine the indefinite integral to be x^4. But because there is always a possibility of an added constant, the proper answer would be $x^4 + C$. Thus, the general rule for finding the indefinite integral of x^n is:

$$\frac{x^{n+1}}{n+1} + C$$

As the figure below suggests, the sum of the areas of the rectangles approaches the actual area under the curve as the number of rectangles increases without limit.[235]



The approximation improves as the number of subintervals increases.

Figure 19

234. For a simplified description of integration, and definite and indefinite integrals, see THOMPSON, *supra* note 226, at 159-87.

235. The illustrations in Figure 19 appear in LAURENCE HOFFMANN, APPLIED CALCULUS 278 (McGraw-Hill 1983) and are reproduced with the permission of McGraw-Hill, Inc. © 1983 by McGraw-Hill, Inc.

From this is derived the fundamental theorem of calculus:

$$f(x)dx = F(b) - F(a)$$

where F is an antiderivative of f, and a and b are the endpoints of the interval over which the area is to be determined.[236] The procedure for determining a definite integral has four straightforward steps:[237]

1. Integrate the function.
2. Evaluate the antiderivative at the upper limit.
3. Evaluate the antiderivative at the lower limit.
4. Subtract the value found in Step 3 from the value found in Step 2.

To understand how integral calculus can be used as a visualization tool to perceive the emotional content of transactional situations, consider the following example, which closely parallels an analysis appearing in *The Cognitive Structure of Emotions*.[238] In the left column of the next figure, a situation is described in which a young man is simultaneously experiencing emotions of hope and fear and their various tokens (including anticipatory excitement and apprehensiveness, respectively). In the right column appears the statement and solution of a problem requiring the determination of the area of a region bounded by two functions—a parabola and a straight line—over a specified interval. Imagining that the graphs of the functions represent hope and fear, one is able to visualize the intensity of their simultaneous elicitation.

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236. *Id.* at 278. This theorem is valid only for nonnegative functions, and it holds for any function that is continuous on the interval a, and is less than or equal to x, which is less than or equal to b. *Id.*

237. *See* BLEAU, *supra* note 212, at 308-09.

238. ORTONY ET AL., *supra* note 186, at 113-14. Figure 20, *infra* in text, appears in LAURENCE HOFFMANN, APPLIED CALCULUS 272 (McGraw-Hill 1983) and is reproduced with the permission of McGraw-Hill, Inc. © 1983 by McGraw-Hill, Inc.

Consider a young man, Tom, who, having finally plucked up the courage to ask Sally for a date, is waiting for her arrival at an agreed upon place. As the time of her expected arrival approaches, he is filled with hope, or perhaps more accurately, anticipatory excitement. [Refer to left, upper part of parabola.] As time passes, any doubts he may have had about whether or not she would indeed show up begin to loom larger. Slowly, his anticipatory excitement starts to turn to concern or worry [refer to vertex of parabola], and eventually to resignation, hopelessness, and disappointment. [Refer to right branch of parabola]. Initially, Tom reacted to the prospect of Sally arriving; later he reacted to the prospect of her *not* arriving. Tom's experiencing of hope-disappointment emotions is quite symmetrical.

Simultaneously, Tom is experiencing fear that his "steady" girlfriend Kathy will appear and he will have to explain what he is doing there—or, worse yet, that Sally and Kathy will appear at the same time. This fear continuously increases in a straight-line manner during the period Tom is waiting. Thus, insofar as hope and fear have complementary objects (an event occurring and not occurring), the contribution of desirability to the intensity of the two emotions will also be the same, so that one might expect the intensity of fear and of decreasing hope to sum to a constant (visualized as the region between the parabola and the straight line) for the occurrence or non-occurrence of any prospective event.

Statement of Problem

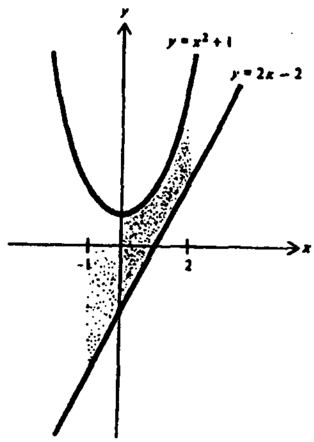
Given: Area of Region Between Two Graphs =

$$\int [f(x) - g(x)]dx$$

Find: The area of the region bounded by the curves:

$y = x^2 + 1$ and $y = 2x-2$ over the interval between $x = -1$ and $x = 2$

Solution



So that you can visualize the situation, begin by sketching the region as shown. Then apply the integral formula with $f(x) = x^2 + 1$, $g(x) = 2x - 2$, $a = -1$, and $b = 2$ to get

$$Area = \int_{-1}^{2} [(x^2 + 1) - (2x - 2)]dx = \int_{-1}^{2} (x^3 - 2x + 2)$$
$$= (\frac{1}{3}x^3 - x^2 + 3x) \Big|_{-1}^{2} = \frac{14}{3} - (-\frac{13}{3}) = 9$$

Figure 20

This is only one example of how integral calculus can be used to visualize interpersonal situations. Those who are more adept at calculus and who have an interest in physics and mechanics may want to experiment with other types of visual analogies and constructs.[239]

B. Psycho-Geometric Models for Effecting Behavioral Change

In Section III.E, this Article examined the assessment feature of two models of family systems—the Satir Model and the Circumplex Model. We now return to those psycho-geometric models briefly to explore their implementation features—their calculus for effecting change. It should be noted preliminarily that the process of structural change in family systems is called morphogenesis.[240] Similar to Leibniz's description of change in his theory of relations, there are two basic types of change possible in family systems: first order change and second order change. First order change occurs when there is comparatively minor structural change among a system's components, such as when family members modify their individual behaviors without completion of positive feedback loops associated with morphogenesis. A system that has undergone first order change still has its original structure, with cosmetic or superficial modifications, and has the potential for relapse.[241] Second order change is a more significant, higher level change in which the system itself is altered. This type of change is dramatic and enduring, as when the entire system is reorganized into new transactional patterns.[242]

It should be emphasized again that although the two models discussed below concern methods and techniques for achieving change therapeutically in a family system over a period of time, some of these same methods and techniques can be of assistance to the negotiator or mediator in planning for a negotiation or mediation or when participating in pre-collaborative negotiation.

239. Other opportunities for applying integral calculus in geometric imagineering may be found in MARY L. BOAS, MATHEMATICAL MODELS IN THE PHYSICAL SCIENCES (1966): 167-68 (volume of a solid), 169-70 (moment of inertia), 241 (electric fields), 242-43 (water flow), 637-40 (heat flow), 640-41 (vibrating string); PURCELL, *supra* note 223, at 296-97 (work done by a variable force), 299-301 (liquid pressure), 309-19 (center of gravity), 323 (center of mass of solid).

240. FAMILY THEORIES SOURCEBOOK, *supra* note 117, at 331.

241. *Id.*

242. *Id.*

1. The Satir Model: Calculus for Change

a. Congruence

The concept of congruence is one of the main constructs of the Satir model, and it manifests itself both as a state of being and as a way people communicate with themselves and others.[243] It is also a precondition to change in a family system.[244] People choose congruence as a state of being when they choose to be themselves, to relate to and contact others, and to connect with people directly.[245] "State of being" congruence has three levels as depicted in Figure 21.[246]

CONGRUENCE

Level 1: Feelings	Awareness Acknowledgment Ownership Management Enjoyment	High self-esteem
Level 2: The Self ("I am")	Centeredness Wholeness Harmony	High self-esteem
Level 3: Life-Force	Universality Spirituality	High self-esteem

Figure 21

Apart from a state of being, congruence in the Satir Model also refers to a way of conveying information.[247] When people communicate, they have at

243. BANMEN ET AL., *supra* note 125, at 65.

244. *Id.* at 85.

245. *Id.* at 66. Compare the discussion of congruence in ROGER FISHER & STEVEN BROWN, GETTING TOGETHER: BUILDING RELATIONSHIPS AS WE NEGOTIATE 173-93 (1988).

246. The chart in Figure 21 is reproduced from BANMEN ET AL., *supra* note 125, at 68 and is reprinted by permission of the authors and publishers. THE SATIR MODEL is available by contacting: Science & Behavior Books, Inc., Palo Alto, CA 94306, 800/547-9982.

247. The concept of "paralleling" is related to congruence in communication and is one with which negotiators and mediators should become familiar. Paralleling is a way of thinking about the content of a conversation. It provides a vehicle for a participant's (a mediator's) understanding conversational content more completely and for using its development through the ongoing conversation to form predictions about the other participants' intentions, the relationship between the participants, and the likely course of the conversation. Various types of paralleling include:

least three choices: (1) using incongruent words and congruent affect, (2) using congruent words and incongruent affect, and (3) using congruent words and congruent affect.[248] Where there are discrepancies between words and affect, as in choices (1) and (2), the message people intend to communicate is not always the message received.[249] Incongruent communication can send a double message, be confusing, and evoke distrust in the recipient.[250] It is important that intervenors, such as mediators, be aware of their own communicational incongruencies, make efforts to become congruent, and model congruent communication when conducting sessions.[251] It is, of course, equally important for a mediator to note congruent and incongruent communication of participants in a mediation as an aid in assessing the participants' true feelings about a topic under discussion, either in a joint session or in caucus.

b. Process of Change

In the Satir Model, change is defined as an internal shift that in turn brings about external modification.[252] In that model, regardless of motivation, people who desire change, such as those who want to resolve conflict, go through the following six steps:[253]

1. *Status quo*: Within the person's or system's existing state, the need for change emerges.

2. *Introduction of a foreign element*: The system or individual articulates the need for change to another person—someone outside the system, such as a mediator.

3. *Chaos*: The system or individual begins moving from a status quo into a state of disequilibrium.[254] An example would be a mediator

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topical, feeling, frame, and locus. *See* BUGENTAL, *supra* note 229, at 95-146.

248. BANMEN ET AL., *supra* note 125, at 69.

249. *Id.* at 73.

250. *Id.* at 70-71.

251. *Id.* at 69, 82.

252. *Id.* at 85.

253. *Id.* at 98-99.

254. In mathematics, chaos theory addresses the behavior of arbitrary, nonlinear systems, and is related to the catastrophe theory discussed in *infra* section V.C.2. As one author has noted: "[S]ystems whose development is governed by nonlinear rules and equations can be extremely sensitive to . . . minuscule changes, often manifesting unforeseeable and 'chaotic' behavior as a result. Linear systems, by contrast, are much more robust, small differences in initial conditions leading only to small differences in final outcomes." PAULOS, *supra* note 112, at 33. *See also* RUCKER, *supra* note 110, at 130. Another concept in mathematics related to chaos theory is fractals. A fractal is "a curve or surface (or a solid or higher-dimensional object) that contains more but

asking questions whose answers alter a participant's *feelings* about the participant's attitudes regarding other person(s) or past events.

4. *Integration*: New learnings are integrated and a new state of being evolves. An example would be when a participant in a mediation, exposed to new information, achieves new insights into the participant's own feelings and behavior, the feelings and behavior of others, and possible behavioral solutions.

5. *Practice*: The new state is strengthened by practicing new learnings. An example would be when parties to a mediated settlement actually implement an agreement.

6. *New Status Quo*: The new status quo represents a more functional state of being. An example would be a successful implementation of a settlement agreement which achieves, for the time being at least, a new stable equilibrium between or among the parties.

The Satir Model proposes that an intervenor acclimate the participants to change by: (1) *accepting and connecting* with each of them, by direct eye contact, shaking hands, being at the same physical level, asking each person's name, and asking how he or she would like to be called, thereby modeling equality of value by sharing; (2) *conveying hope* by empowering their participation in a process toward a positive, achievable goal; (3) *establishing one's own credibility*, by displaying confidence, competence, and high self-esteem, by creating a comfortable atmosphere, free of interruptions and distractions, and by refraining from being judgmental or diagnostic; (4) *instilling awareness* that change is always occurring by noting when participants agree to minor, even inconsequential matters as they proceed through the process; and (5) *following the process* by allowing the process to control the outcomes and not allowing *expectations* about outcomes to control the process.[255]

2. The Circumplex Model: Calculus for Change

In the Circumplex Model for systems, assessed at either extreme on the cohesion and adaptability dimensions, intervention strategies can be tailored to fit the particular pattern of organization and to guide change progressively

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similar complexity the closer one looks." PAULOS, *supra* note 112, at 84. Fractals are playing an increasingly important role in the chaos theory where they can be used to describe a nonlinear system's collection of possible graphic trajectories. *Id.* at 86.

255. BANMEN ET AL., *supra* note 125, at 94-98.

toward a more balanced system.[256] In systems experiencing severe and chronic dysfunction, an attainable goal would be arriving at higher functioning at the next, adjacent pattern.[257] An example would be a system shift from disengaged to separated or from enmeshed to connected. Attempts to shift system patterns into a quite different type of organization, such as assisting a disengaged family to be strongly connected or an enmeshed family to become separated, are normally unrealistic and unsuccessful.[258] A common error when dealing with severely dysfunctional systems is to assume either that patterns are unchangeable or that change toward the opposite pattern is necessary and desirable.[259] The two charts in Figures 22 and 23 illustrate Circumplex intervention techniques that may be used with respect to two types of systems frequently encountered by intervenors, particularly mediators.[260] These techniques have been successfully employed by intervenors to move a rigid system to a structured system (Figure 22) and a chaotic system to a flexible system (Figure 23).

C. Geometric Imagineering and Behavioral Change

1. A Two-Dimensional Method for Visualizing Behavioral Change

In the late 1950s, Fritz Heider, Professor of Psychology at the University of Kansas, published a book entitled *The Psychology of Interpersonal Relations*, which described a common sense, philosophical, and research-based approach to the analysis of interpersonal relations.[261] His approach, still vibrant after four decades of advances in the field of psychology,[262] was predicated on selected elements of everyday language. With a goal reminiscent of Leibniz's "universal characteristic," Heider proposed a universal, short-hand notational method of situational analysis for interpersonal relations, having some features of symbolic logic.[263] He also created a two-dimensional, geometric method of analyzing balance in relations and the stresses inducing change.[264]

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256. Walsh & Olson, *supra* note 147, at 60. For a discussion of the Circumplex Model, see *supra* section III.E.2.

257. *Id.* at 61-62.

258. *Id.*

259. *Id.*

260. Figures 22 and 23 are reproduced from Walsh & Olson, *supra* note 147, at 63, 66, respectively. © By The Haworth Press, Inc. All rights reserved. Reprinted with permission. For copies of the complete work, contact Marianne Arnold at The Haworth Document Delivery Service (Telephone 1-800-3-HAWORTH; 10 Alice Street, Binghamton, N.Y. 13904). For other questions concerning rights and permissions, contact Wanda Latour at the above address.

261. FRITZ HEIDER, THE PSYCHOLOGY OF INTERPERSONAL RELATIONS 10 (1958).

262. *See* ORTONY ET AL., *supra* note 186, at 97-99; *see also* Cooley, *supra* note 1, at 130 n.123.

263. HEIDER, *supra* note 261, at 10-15.

264. *Id.* at 182-212.

FAMILY CHANGE: GOALS AND TECHNIQUES - RIGID TO STRUCTURED

COMPONENTS	CURRENT SYSTEM: RIGID →	GOAL: STRUCTURED	INTERVENTION TECHNIQUES
LEADERSHIP (Control)	Authoritarian leadership. Parent(s) highly controlling.	Primarily authoritarian but some egalitarian leadership.	Model & promote more flexible, equalitarian leadership without (feared) loss of control.
DISCIPLINE (For Families Only)	Autocratic, "law & order". Strict, rigid consequences. Not lenient.	Somewhat democratic. Predictable consequences. Seldom lenient.	Increase democratic process. Negotiate more flexible, yet predictable consequences.
NEGOTIATION	Limited negotiations, solutions. Decisions imposed by parents.	Structured negotiations. Decisions mainly made by parents.	Increase flexibility, experimentation. Expand repertoire of solutions.
ROLES	Limited repertoire; strictly defined rules.	Roles stable, but may be shared.	Increase role flexibility.
RULES	Unchanging rules. Rules strictly enforced.	Few rule changes. Rules firmly enforced.	Increase flexibility of rules and enforcement.

Figure 22

FAMILY CHANGE: GOALS AND TECHNIQUES - CHAOTIC TO FLEXIBLE

COMPONENTS	CURRENT SYSTEM: CHAOTIC →	GOAL: FLEXIBLE	INTERVENTION TECHNIQUES
LEADERSHIP (Control)	Limited and/or erratic leadership. Parental control unsuccessful, rebuffed.	Egalitarian leadership with fluid changes.	Model & promote strong leadership and control.
DISCIPLINE (For Families Only)	Laissez-faire and ineffective. Inconsistent consequences. Very lenient.	Usually democratic. Negotiated consequences. Somewhat lenient.	Set clear limits, appropriate consequences. Follow through consistently.
NEGOTIATION	Endless negotiations with closure; impulsive decisions.	Flexible negotiations. Agreed upon decisions.	Increase ability to reach compromise, closure.
ROLES	Lack of role clarity, role shifts and role reversals.	Role sharing and making. Fluid changes of roles.	Increase role clarity, stability, and continuity.
RULES	Frequent rule changes. Rules inconsistently enforced.	Some rule changes. Rules flexibly enforced.	Increase predictability of rules & enforcement.

Figure 23

a. Notational Method of Situational Analysis

The everyday language aspect of Heider's notational method for analyzing interpersonal relations had these elements: subjective environment (or life space); perceiving, suffering (or experiencing or being affected by); causing, can, trying, wanting, sentiments, belonging, ought, and may.[265] His notations for representing interpersonal relations were relatively simple. Some of them are shown in the next chart:[266]

p	the person whose life space is being considered
o	other person
q	other person
r	an undetermined person
x,y,z	impersonal entities, things, situations, changes, etc.
a,b,c	actions, attitudes, etc.
R	a relation exists
L	likes
DL	dislikes
U	belongs to
B	benefits
H	harms
C	causes
Cd	caused
Can C	can cause
Tr C	tries to cause
ought C	ought to cause
Pres	presents or informs
W	wants
S	suffers or experiences
not	negative or opposite

In the following chart, examples of notations for simple interpersonal relations are first shown, then more complex ones are demonstrated:[267]

265. HEIDER, *supra* note 261, at 18-19
266. This information was extrapolated from an explanation appearing in *id.* at 299-301.
267. *Id.*

Simple Representational Examples :

p C *x*	*p* causes *x*
a R *b*	a relation holds between actions *a* and *b*
p Can C *x*	*p* can cause *x*
p L *x*	*p* likes *x*
p U *x*	*x* belongs to *p* in some way
p B *o*	*p* benefits *o*
p H *o*	*p* harms *o*

More Complex Representational Examples:

(*NOTE*: The first term of a relation is often a person; the second term can be a personal or impersonal entity, or another relation. If the second term is itself a relational proposition, a colon (:) is inserted between the symbol for the first relation and the expression for the second relation.)

p C : *o* C *x*	*p* causes *o* to cause (or do) *x*
p C : *o* Can C *x*	*p* makes it possible for *o* to do *x*
p C : *o* S *x*	*p* causes something that affects *o*
p C : *o* L *x*	*p* causes *o* to like *x*
p W : *o* B *q*	*p* wants *o* to benefit *q*
p Pres *o* : *q* Cd *x*	*p* tells *o* that *q* caused *x*
p Pres *o* : *p* not L *x* (*o* Cd *x*)	*p* tells *o* that he does not like the *x* that *o* caused

Heider believed that his notation system would not only aid in the analysis of another person's present action or conduct (that is, that the other person is trying to do something, intends to do something, or has the ability to do something, etc.),[268] but also in the analysis of how one person can induce

268. HEIDER, *supra* note 261, at 79.

another to do something by producing conditions of action in the other person.[269] These kinds of analyses, of course, are of particular interest to negotiators and mediators. Heider posited, similar to Spinoza, that the result of any human action depends on two sets of conditions: factors within the person and factors within the environment. To explain this phenomenon and to provide a sample of the expressions used to refer to factors that are significant to the outcome of an action, Heider provided this example:

> Consider . . . [that] a person [is] rowing a boat across a lake. . . . We say, "He is *trying* to row the boat across the lake," "He has the *ability* to row the boat across the lake," "He *can* row the boat across the lake," "He *wants* to row the boat across the lake," "It is *difficult* to row the boat across the lake," "Today there is a good *opportunity* for him to row the boat across the lake," "It is sheer *luck* that he succeeded in rowing the boat across the lake."[270]

These descriptive statements in everyday language refer to personal factors on the one hand and to environmental factors on the other. Heider noted that we could speak of the effective force (*ff*) of the person or of the environment to describe the totality of the forces originating from one or the other source and that the occurrence of the desired action outcome, x, was, therefore, dependent upon, or a function of, a combination of the effective personal force and the effective environmental force. According to Heider, this situation could be expressed by the following formula:

$$x = f\,(ff \text{ person}, ff \text{ environment})$$

Heider was also quick to point out that there is a natural temptation to predict that the occurrence of the desired action outcome or goal is merely the *sum* of the effective personal force and the effective environmental force. This is true in some cases, but other cases require a more in-depth analysis. For example, in situations where the effective environmental force is zero—where the combination of environmental factors neither hinders nor furthers the result x—then the relation between the two forces could be said to be additive and the occurrence of x would depend only on whether the effective personal force was favorable to achieving the goal—directed toward the goal. If so, there would exist a virtual certainty that the boat would eventually arrive on the other side of the lake. Similarly, if the effective personal force were zero (such as if the rower was asleep), and the effective environmental forces (such as wind and current) were greater than zero and favorable in relation to achieving the goal,

269. *Id.* at 244.
270. *Id.* at 82.

then x would, to a virtual certainty, occur. However, in situations where the effective environmental force is unfavorable or adverse to achieving the goal, its *degree* of adversity must be considered in conjunction with two contributing factors of the effective personal force: a power factor and a motivational factor. The power factor primarily includes *ability*. The motivational factor relates to what a person is *trying* to do—intention and exertion. Thus, the contribution of the rower to the outcome x depends on the rower's *ability* to maneuver the boat and on how hard the rower *tries* to accomplish the goal. This is graphically illustrated in Figure 24.[271]

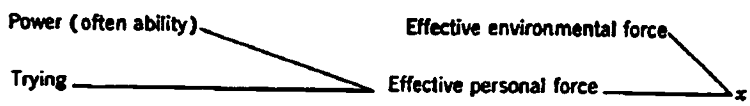


Figure 24

Thus, the original equation expressing the relationship between a goal and the effective personal and effective environmental forces can be simplified as follows:[272]

$$x = f(\textit{trying, power, environment})$$

It should be noted in this restated equation that *power* and *trying* are related as a multiplicative combination since the effective personal force is zero if either *power* or *trying* is zero. For example, as Heider pointed out, it is common knowledge that if a person has the ability but does not try at all, he or she will make no progress toward the goal.[273]

Heider also distinguished between situations of personal causality and those of impersonal causality. He defined personal causality as "instances in which p causes x intentionally"[274] and explained that it is characterized by equifinality which defines what a person "can" do if he or she tries.[275] In situations of personal causality, a person is the "local cause."[276] Impersonal causality is

271. This figure appears in HEIDER, *supra* note 261, at 83 and is reprinted with permission.
272. *Id.*
273. *Id.* at 83.
274. *Id.* at 100.
275. *Id.* at 102. Equifinality also is defined as "the invariance of the end and variability of the means." *Id.* at 101.
276. *Id.* at 107.

comprised of instances where environmental conditions cause certain effects and a person can avoid or change these effects by changing certain conditions.[277] It is characterized by multifinality, meaning the existence of a wide range of outcomes or effects.[278] The principal differences between personal and impersonal causality are illustrated in Figures 25 and 26. Figure 25 illustrates multifinality in the case of impersonal causality.[279]

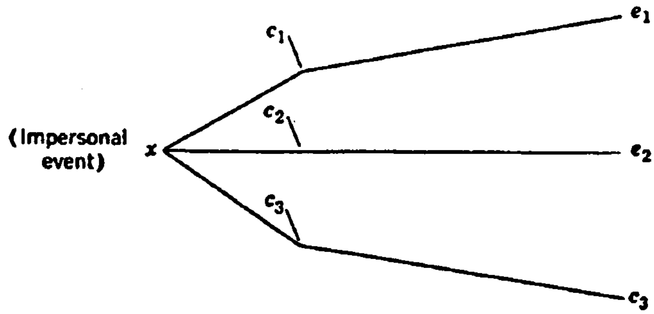


Figure 25

In Figure 25, x represents an impersonal event which, with the circumstance $c1$ results in effect e_1, with circumstance c_2 leads to effect e_2, etc. In a case of impersonal causality, the effects are usually all different—that is, multifinal. For example, a falling stone will hit a man (e_1), fall on the ground (e_2), or start an avalanche (e_3).[280]

In contrast, Figure 26 illustrates equifinality in the case of personal causality.[281]

277. *Id.* at 102.
278. *Id.* at 101-02.
279. Figure 25 appears in HEIDER, *supra* note 261, at 107 and is reprinted with permission.
280. *Id.*
281. Figure 26 appears in HEIDER, *supra* note 261, at 108 and is reprinted with permission.

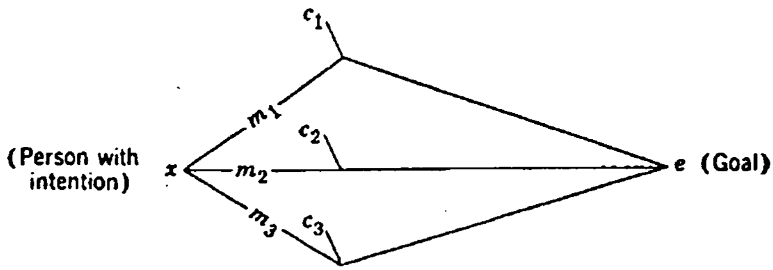


Figure 26

In Figure 26, x depicts a source of personal causality—a person who intends to produce effect or goal e. Confronted with circumstance c_1, x will choose means m_1 to reach goal e. Similarly, if circumstance c_2 is prevalent, x will choose m_2, etc. Equifinality exists here because the *means* are variable but the *end* is the same. Local causality also exists. The causal lines emanate from x and are controlled by x to their final outcome e.[282]

In further explaining the distinctions between Figures 25 and 26, Heider wrote:

> The consequences of the represented differences are significant. For example, in the case of impersonal causality [Figure 25], a source outside the given situation can influence the outcome by altering any one of the circumstances c_1 to c_3. Thus, if a person exposed to the effect of x [an impersonal event] does not like e_1, he can change c_1 to c_2, as when he steps aside in order to avoid the falling stone. On the other hand, where personal causality operates [as in Figure 26], a source outside the situation cannot as simply change the outcome. The outcome will not be altered merely by changing c_1 to c_2 or to c_3. Another person will succeed in influencing e only by altering x, that is the intention of the agent, or by creating circumstance c_4 that makes it impossible for the agent to produce e.[283]

282. HEIDER, *supra* note 261, at 108.
283. *Id.*

Figure 27 depicts multifinality following goal achievement.[284]

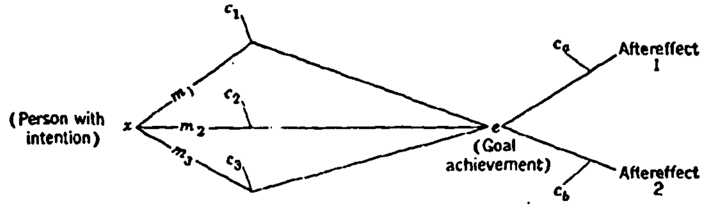


Figure 27

This figure illustrates that any resulting aftereffect of goal achievement, even if it is an inevitable consequence under a variety of intervening circumstances, cannot strictly be considered a component of personal causality. With the passage of time, goal achievement can be construed as an impersonal event yielding multifinality of aftereffects.[285] The causal lines will not converge to equifinality without the reintroduction of *x*, a person with intention.[286]

This Article will now consider how Heider's notational and analytical method could be used in a negotiation situation.

b. Geometrical Analysis of Balance in Relations

In developing his method of geometrical analysis of relations, Heider dealt with the concept of "emotions" or "feelings" under the label "sentiment."[287] He recognized that many of the actions that occur between people (that is, human behavior) can be understood only if one has an appreciation of the *feelings* that guide them.[288]

Heider defined sentiment as "the way a person *p* feels about or evaluates something."[289] The "something" can be another person, *o*, or an impersonal entity, *x*, and sentiments can be classified roughly as positive or negative. In Heider's model of relations, positive sentiments described a relation of *liking* between *p* and another entity; negative relations described a relation of *disliking*.[290] According to Heider, relations between *p* (a person), *o* (another person), and *x* (an impersonal entity) consist of two types, a unit relation and a

284. Figure 27 appears in HEIDER, *supra* note 261, at 109 and is reprinted with permission.
285. *Id.* at 109-10.
286. *Id.* at 109.
287. *Id.* at 174-75.
288. *Id.* at 174.
289. *Id.*
290. *Id.*

sentiment relation, and they bear a striking resemblance to Leibniz's two-pronged second aspect of relations—the intrinsic denominations, described earlier in Section III.C. A unit relation, in the Heider model, refers to the instance of persons and objects *belonging together* (U) in a specially close way.[291]

In essence, a unit relation is a combination or structure. Unit-forming factors include: similarity (of beliefs, values, goals), familiarity, proximity, interaction, common fate, and ownership.[292] On the other hand, a sentiment relation refers to a person's *evaluation* of something, as when *p* likes (L) *o*, or when *p* dislikes (DL) *x*.[293] In a typical triad *p-o-x*, three specific dyads are possible: *p* and *o*, *o* and *x*, and *p* and *x*. Each of these dyads has four possibilities of unit and sentiment relations: U, notU, L, DL. Thus, in any triadic situation, there exist sixty-four combinations (4x4x4) of possible triadic relations.[294] Some of these triadic relations represent balanced states, while others represent imbalanced states.[295]

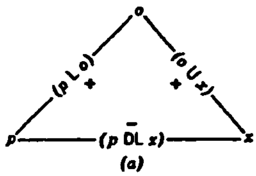
By balanced state, Heider meant a "situation in which the relations among the entities fit together harmoniously; there is no stress towards change."[296] A basic assumption, based on behavioral research, is that unit and sentiment relations tend toward a balanced state.[297] Other assumptions include: (1) unit and sentiment relations are mutually interdependent, and (2) if a balanced state does not exist, then forces will arise tending to change the situation in the direction of balance, and if a change in an imbalanced state is not possible, tension will result.[298] In Heider's model, a dyad is balanced if the relations between two entities are all positive (L and U), designated by a plus sign (+); or all negative (DL and notU), designated by a negative sign (-).[299] Disharmony results in a dyad when unit and sentiment relations have different signs (that is, + and -).[300] Similarly, a triad is balanced when all three of the dyads have positive signs or when two of the relations have negative signs and one has a positive sign.[301] Imbalance occurs when two of the dyads have positive signs and one has a negative sign.[302] The most important aspect of

291. HEIDER, *supra* note 261, at 200-01.
292. *Id.* at 177-201.
293. *Id.* at 200.
294. *Id.* at 201.
295. *Id.*
296. *Id.*
297. *Id.* at 201-05.
298. *Id.* at 201.
299. *Id.* at 202.
300. *Id.*
301. *Id.*
302. HEIDER, *supra* note 261, at 202.

Heider's theory of interpersonal relations, however, is that in situations of imbalanced relations, "*the situation can be made harmonious either by a change in the sentiment relations or in the unit relations.*"[303] Consider the following example.[304]

Basic Interpersonal Situation

Let us suppose that *p* likes *o*, and *p* perceives that *o* has done something, *x*, which is disliked by *p*. Triadic imbalance will result. The triad contains two positive relations (*o* U *x*), (*p* L *o*) and one negative relation (*p* DL *x*), as shown in Figure 28a. This is an unpleasant situation for *p*. Tension will arise and forces will arise in a direction of annulling the tension and toward achieving balance.



The given situation is unbalanced: two positive relations and one negative relation.

Figure 28a

This situation can be made harmonious either by a change in the sentiment relations or in the unit relations.

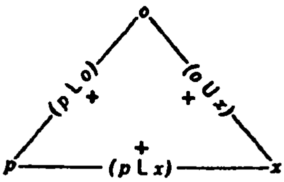
Change in Sentiment Relations

p can begin to feel that *x* is really not so bad, thereby producing a triad of three positive relations (Figure 28b).

p can admit that *o* is not quite as good as *p* thought he was, thereby producing a balanced triad of two negative relations and one positive relation (Figure 28c).

303. *Id.* at 207 (emphasis added).

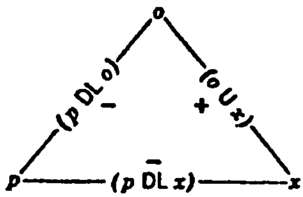
304. Figures 28(a)-(e) and explanatory material contained in this chart appear in *id.* at 207-09 and are reprinted with permission.



(b)

Change in sentiment relation resulting in a balance of three positive relations.

Figure 28b



(c)

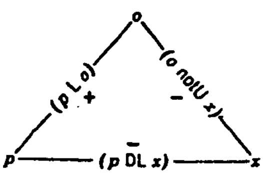
Change in sentiment relation resulting in a balance of two negative relations and one positive relation.

Figure 28c

Change in Unit Relations

p can begin to feel that *o* is not really responsible for *x*. In this way, *x* cannot be attributed to *o* and the unit connection between *o* and *x* is destroyed, producing two negative relations and one positive relation (Figure 28d).

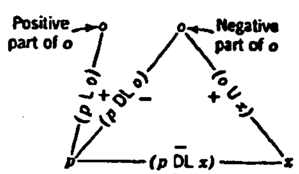
p can resolve the situation by gaining a more *differentiated* picture of *o*. *p* concludes in effect that *o*, like everyone, has good points and bad points; *p* still likes *o* because of his good points, though *p* dislikes part of *o*'s personality. *o* has been differentiated in such a way that the unit with the negative *x* now consists of just the negative part of *o*. A unit of two negative entities is thereby formed and the triad now revolves around two negative relations ((*p* DL *o*) (*p* DL *x*)) and one positive relation (*o* U *x*). Of course, the *o* in this triad refers to that part of *o* which is negative. Since the total *o* must then consist of a positive part and a negative part, to this extent, imbalance still exists (Figure 28e).



(d)

Change in unit relation resulting in a balance of two negative relations and one positive relation.

Figure 28d



(e)

Change in unit relation through differentiation resulting in a balance of two negative relations and one positive relation.

Figure 28e

Now that the reader has become familiar with a two-dimensional model for analyzing interpersonal relations and behavior, the reader is prepared to consider a three- (and higher) dimensional model for visualizing and understanding balance in interpersonal situations and the potential for and the dynamics of change in conflict situations.

2. A Multi-Dimensional Method for Visualizing Behavioral Change

In 1972, René Thom, a highly respected expert in the field of mathematics,[305] made a significant contribution, perhaps unwittingly, toward advanced human understanding of the potential for and the dynamics of change in conflict situations by publishing a book entitled *Structural Stability and Morphogenesis: An Essay on the General Theory of Models*.[306] A classic example of the expression of creativity in pure mathematics,[307] Thom's ideas came to be known collectively as the "catastrophe theory"—an unfortunate label that invited much initial controversy[308] and that obscured the beauty of its underlying mathematics and its broad utility as a visual tool in understanding behavioral phenomena in many diverse aspects of the natural and social sciences.[309] At the root of the theory was Thom's ingenious melding of Leibniz's concepts of calculus and topology to reveal a seemingly universal truth—that there are only seven fundamentally different ways for change or discontinuity to occur in a "something's got to give" situation.[310] But it was

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305. Thom had been a professor at the University of Grenoble until 1957, when he moved to the University of Strasbourg. ALEXANDER WOODCOCK & MONTE DAVIS, CATASTROPHE THEORY 16 (1978). At the time he published his book in 1972, Thom was on the faculty of the Institut des haut Études Scientifique at Bures-sur-Yvette in France. E.C. Zeeman, *Catastrophe Theory*, SCI. AM., Apr. 1976, at 65. A leading mathematician, Thom was well known in mathematical circles, and the ideas expressed in his 1972 book spread very quickly to scientists in general, and the public at large. IVAR EKELAND, MATHEMATICS AND THE UNEXPECTED 76 (1988).

306. *See* EKELAND, *supra* note 305, at 76.

307. WOODCOCK & DAVIS, *supra* note 305, at 14.

308. *Id.* at 15-16.

309. *Id.* For an interesting article describing graphic visualization methods for explaining social stability and change in sociology and population courses, see Donald R. Ploch & Donald W. Hastings, *Cohort Surfaces: A Graphic Approach to Teaching Social Stability and Change*, TEACHING SOC., July 1992, at 192-200.

310. WOODCOCK & DAVIS, *supra* note 305, at 19, 42. Thom was not the first mathematician to see a connection between calculus and topology. In the 1880s and 1890s, the french mathematician, Henri Poincaré, had linked these two aspects of mathematics to create *qualitative* dynamics and apply it to unsolved problems of planetary motion. *Id.* at 17-18. It was Thom, however, who discovered that, in nature where there are sudden changes in form, a limited number of archetypal structures could occur; that is, for a very wide range of processes, only seven stable unfoldings—seven "elementary catastrophes"—are possible. *Id.* at 24-25. The existence of these seven stable unfoldings has been compared to the mathematical restrictions on the combination of certain regular polygons. For example, the Greeks found that of all the regular polygons, only three of them (the triangle, square, and hexagon) can be packed edge to edge to fill a plane. This fact is

the mathematician E. Christopher Zeeman, a contemporary of Thom and a lecturer at Cambridge University, who eventually theorized the application of the catastrophe theory to conflict situations.[311] Before this Article can consider the application of catastrophe theory to conflict situations, the reader must first gain a better understanding of the theory's basic tenets.

a. Basic Tenets of the Catastrophe Theory

Catastrophe theory is applicable to dissipative systems.[312] The dynamics of dissipative systems are very simple: (1) every motion tapers off toward some final rest position, (2) the possible rest positions are called equilibria, and (3) a particular dissipative system may have one or several equilibria.[313] As one commentator explains: "A catastrophe . . . is any discontinuous transition that occurs when a system can have more than one stable state, or can follow more than one stable pathway of change. The catastrophe is the "jump" from one state or pathway to another."[314]

A simple, perceptual example of such a "jump" from one state to another is provided by Figure 29, called a "Necker cube."[315] If one stares at the corner of the Necker cube marked "A" for some time, it will alternate between appearing to face forward and appearing to face back. One's perception can oscillate—jump suddenly back and forth—between these two states of equilibria, literally at will.[316]

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known by anyone who has ever tiled a wall or floor. The Greeks also found that if the regular polygons are assembled as the faces of three-dimensional solids, only five such solids—known as the Platonic solids (see *supra* section III.D in the text)—can be constructed. These polygons and solids appear throughout nature in snowflakes, crystals, and honeycombs, not because geometry dictates to nature, but because there is no other way for certain natural processes to turn out or unfold. *Id.* at 25.

311. WOODCOCK & DAVIS, *supra* note 305, at 19-21.

312. EKELAND, *supra* note 305, at 79.

313. *Id.*

314. WOODCOCK & DAVIS, *supra* note 305, at 32.

315. The "Necker cube" receives its name from geologist Louis Necker, who, in 1832, reported that line drawings of crystals appeared to reverse spontaneously in depth. P.T. SAUNDERS, AN INTRODUCTION TO CATASTROPHE THEORY 93 (1980). Figure 29 appears in ANDREW ORTONY ET AL., THE COGNITIVE STRUCTURE OF EMOTIONS 93 (1988) and is reprinted with the permission of Cambridge University Press.

316. *Id.* at 93-96.

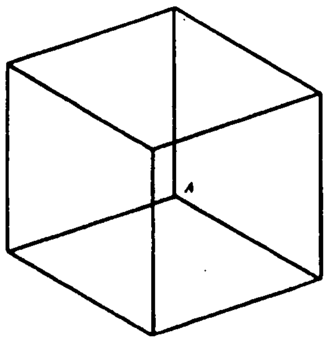

Figure 29

This "jump" between states can also be exemplified, physically, by considering the behavior of a pendulum.[317] Visualize a rod hanging vertically from a nail with a small metal sphere attached to its lower end. Two equilibria positions are possible. The most obvious one is the rod positioned vertically, with the sphere down, and at zero speed. This is called the system's stable equilibrium. Another less obvious equilibrium position is the pendulum positioned upside down, with the metal sphere on top. This is the system's unstable equilibrium. The pendulum will stay in this unstable position indefinitely if the pendulum's speed remains zero. However, the slightest impulse on the metal sphere will result in the pendulum falling away from this position of equilibrium, swinging back and forth in ever decreasing oscillations, finally coming to rest at the stable equilibrium. Note that the dissipative system's *environment* has a great impact on the speed with which the pendulum reaches its stable equilibrium. In ordinary atmosphere, the pendulum will make numerous oscillations back and forth before coming to a rest at its stable equilibrium. If the air environment is replaced by a water environment (that is, the resistance to movement is increased), the oscillations will not occur, and the pendulum will fall directly into its stable equilibrium position.

To fully understand the concept of catastrophe, one must be familiar with the concept of *potential* and its relationship to the idea of equilibria.[318]

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317. The pendulum example that follows in the text is adapted from EKELAND, *supra* note 305, at 80-81.

318. Examples of potential in a physical system are: the tendency of a spring to contract (mechanical potential), the tendency of two chemicals in a battery to react (chemical potential), and the tendency of a ball to roll downhill (gravitational potential). WOODCOCK & DAVIS, *supra* note 305, at 33-34.

Consider the graph of the roller-coaster type of curve in Figure 30.[319]

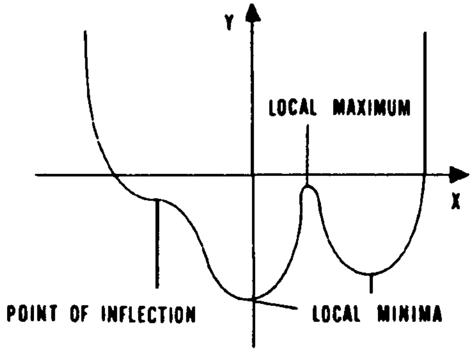


Figure 30

The y (vertical) axis represents levels of potential (height) and, for our purposes, is equivalent to gravitational potential. The x (horizontal) axis represents straight line distance potential. Now visualize placing a ball at any point on the curve in Figure 30. At all except four of the points, the ball will immediately begin to roll. Those four points—the four points at which the curve has neither upward nor downward slope—are called equilibrium points. As learned earlier in Section V.A, in Figure 30, the "ledge" is called a point of inflection and is a semi-stable equilibrium, the "hilltop" is called a local maximum and is an unstable equilibrium, and the "valleys" are called local minima or points of minimum *potential* and are stable equilibria.[320] Extending this situation into three dimensions, we would have a landscape similar to the one appearing in Figure 31.[321]

319. Figure 30 is reprinted from CATASTROPHE THEORY by Alexander Woodcock and Monte Davis, at 17. Copyright © 1978 by Alexander Woodcock and Monte Davis. Used with the permission of Dutton Signet, a division of Penguin Books USA Inc. A similar type of curve was examined in *supra* section V.A.

320. *Id.* at 34.

321. Figure 31 is reprinted from CATASTROPHE THEORY by Alexander Woodcock and Monte Davis, at 36. Copyright © 1978 by Alexander Woodcock and Monte Davis. Used with the permission of Dutton Signet, a division of Penguin Books USA Inc.

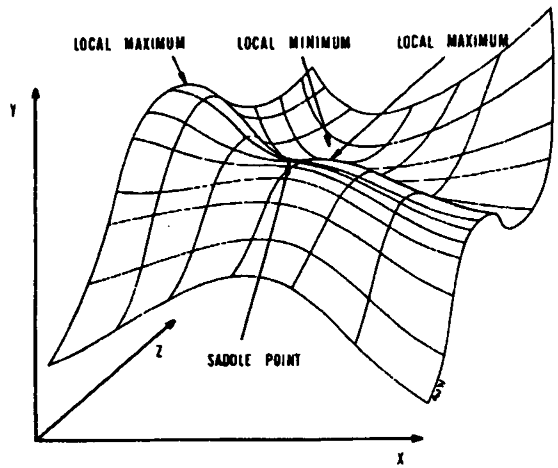


Figure 31

Figure 31 allows us to visualize a dissipative system, for example, a ball rolling around and around a basin (a friction surface) surrounding a local minimum. Because of the friction, the system will dissipate, spiral down towards the local minimum, and come to rest at its stable equilibrium—its point of minimum *potential*. The third dimension allows the introduction of a new feature—the saddle point—which replaces the "point of inflection" feature on the two-dimensional graph. The saddle point is a point of semi-stable equilibrium in a three-dimensional construct.[322]

In terms of *potential*, the basic theorem of the catastrophe theory can be stated as follows: In any system governed by a potential and in which the system's behavior is determined by no more than four different control factors, only seven *qualitatively* different types of discontinuity are possible.[323] The word "qualitatively" has been emphasized in the theorem statement for a reason. The catastrophe theory can only be applied *qualitatively*, not quantitatively; the best it can do is reveal patterns of behavior for which to look.[324] To represent

322. *Id.* at 35-36.
323. *Id.* at 42.
324. EKELAND, *supra* note 305, at 79. Mathematical models can be classified as quantitative vs. qualitative, probabilistic vs. determinate, ready-made vs. custom-built, descriptive vs. optimizing, and analytic mode vs. numeric mode. *See* CLIFFORD H. SPRINGER ET AL., ADVANCED METHODS AND MODELS, 2 MATHEMATICS FOR MANAGEMENT SERIES 7-14 (1965).

these patterns of behavior, a special kind of graph is needed. The graph must have one dimension, or axis, for each control factor that determines the system's behavior and an additional axis or two to represent the behavior itself.[325] In the space defined by these dimensions, every possible equilibrium state is represented by a single point, and the points together form a smooth line or surface.[326] In this behavior model, a continuous change in behavior appears as a movement *within* the line or surface; a discontinuous change—a catastrophe—appears as a movement that *leaves* the line or surface.[327] The simplest elementary catastrophe, called the "fold," is two-dimensional and has only one control axis and one behavior axis. The most complex of the seven, called the "parabolic umbilic," has four control axes and two behavior axes and is therefore six-dimensional.[328] The chart in Figure 32 identifies the seven elementary catastrophes by name, the number of their respective control dimensions (control factors) and behavior dimensions (behavior axes), the potential functions of each, and the first derivatives of the potential functions which define the behavior surfaces.[329]

CATASTROPHE		CONTROL DIMENSIONS	BEHAVIOR DIMENSIONS	FUNCTION	FIRST DERIVATIVE
CUSPOIDS	FOLD	1	1	$\frac{1}{3}x^3 - ax$	$x^2 - a$
	CUSP	2	1	$\frac{1}{4}x^4 - ax - \frac{1}{2}bx^2$	$x^3 - a - bx$
	SWALLOWTAIL	3	1	$\frac{1}{5}x^5 - ax - \frac{1}{2}bx^2 - \frac{1}{3}cx^3$	$x^4 - a - bx - cx^2$
	BUTTERFLY	4	1	$\frac{1}{6}x^6 - ax - \frac{1}{2}bx^2 - \frac{1}{3}cx^3 - \frac{1}{4}dx^4$	$x^5 - a - bx - cx^2 - dx^3$
UMBILICS	HYPERBOLIC	3	2	$x^3 + y^3 + ax + by + cxy$	$3x^2 + a + cy$ $3y^2 + b + cx$
	ELLIPTIC	3	2	$x^3 - xy^2 + ax + by + cx^2 + cy^2$	$3x^2 - y^2 + a + 2cx$ $-2xy + b + 2cy$
	PARABOLIC	4	2	$x^2y + y^4 + ax + by + cx^2 + dy^2$	$2xy + a + 2cx$ $x^2 + 4y^3 + b + 2dy$

SEVEN ELEMENTARY CATASTROPHES describe all possible discontinuities in phenomena controlled by no more than four factors. Each of the catastrophes is associated with a potential function in which the control parameters are represented as coefficients (*a, b, c,d)* and the behavior of the system is determined by the variables (*x, y*). The behavior surface in each catastrophe model is the graph of all the points where the first derivative of this function is equal to zero or, when there are two first derivatives, where both are equal to zero.

Figure 32

325. WOODCOCK & DAVIS, *supra* note 305, at 43.

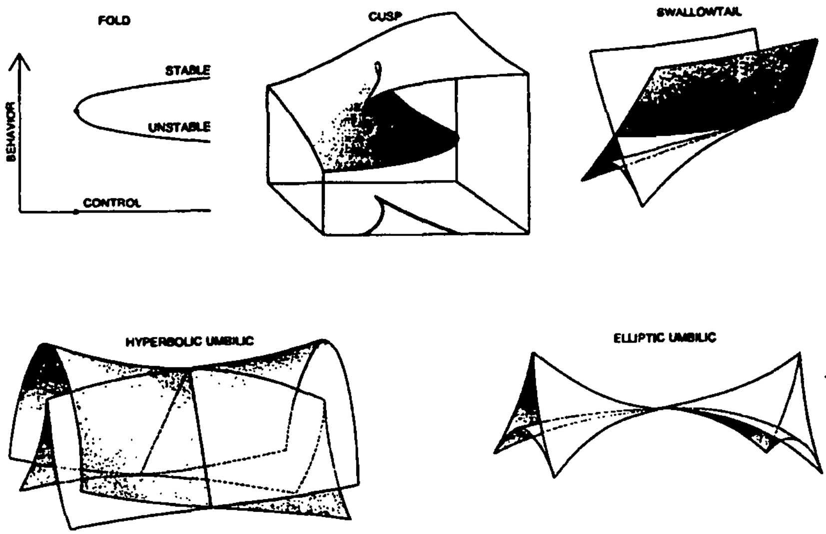
326. *Id.*

327. *Id.*

328. *Id.*

329. The chart in Figure 32 is reprinted from E.C. Zeeman, *Catastrophe Theory*, SCI. AM., Apr. 1976, at 78.

The graphs of the five elementary catastrophes having three control factors or less are pictured in Figure 33.[330]

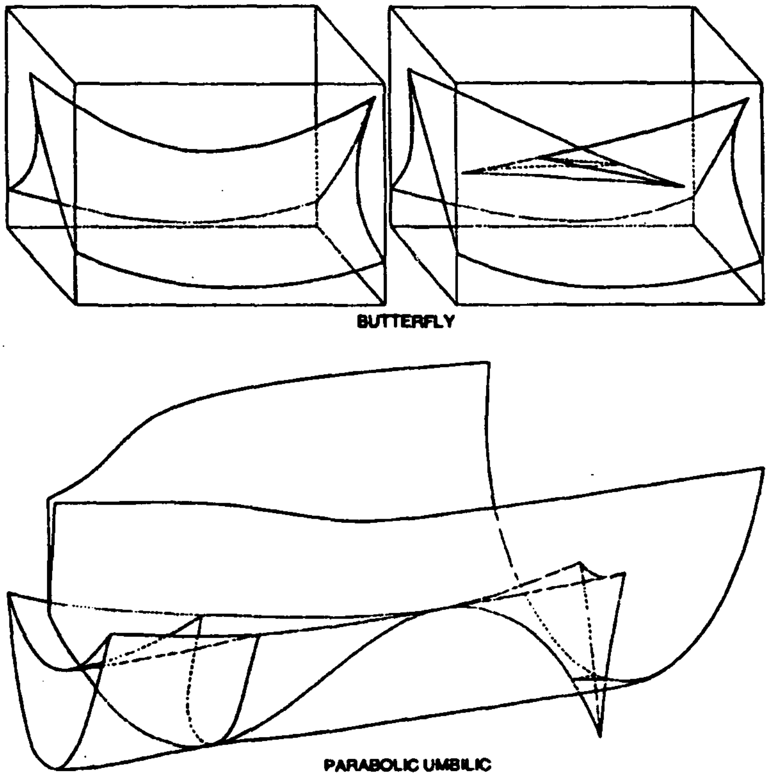


GRAPHS of five of the elementary catastrophes suggest the nature of their geometry. The fold catastrophe is a transverse section of a fold curve of the cusp catastrophe, and its bifurcation set consists of a single point. The cusp is the highest-dimensional catastrophe that can be drawn in its entirety. The swallowtail is a four-dimensional catastrophe and the hyperbolic umbilic and the elliptic umbilic catastrophes are five-dimensional. For these graphs only the three-dimensional bifurcation sets can be drawn; the behavior surfaces are not shown.

Figure 33

330. The five graphs in Figure 33 are reprinted from E.C. Zeeman, *Catastrophe Theory*, SCI. AM., Apr. 1976, at 78.

The graphs of the two elementary catastrophes having four control factors are pictured in Figure 34.[331]



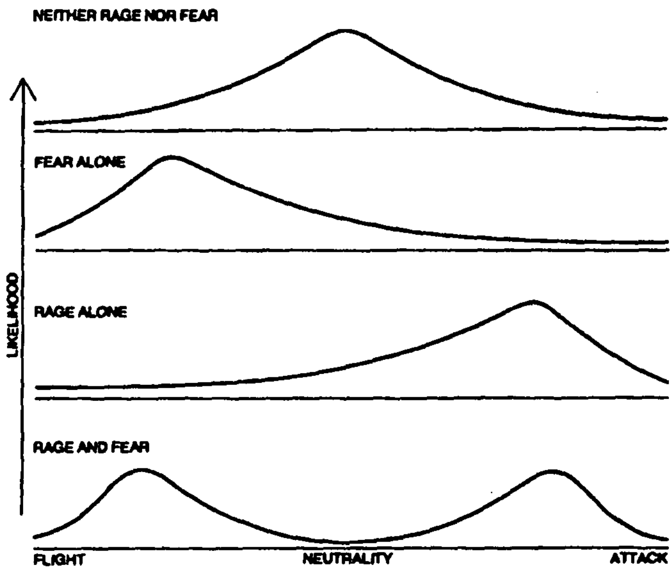
SECTIONS are the only recourse for illustrating the remaining two catastrophes, since even their bifurcation sets have more than three dimensions. The four-dimensional bifurcation set of the butterfly catastrophe is shown in three-dimensional sections; the fourth dimension is the butterfly factor, and if it happens to be time, then one configuration evolves into the other. Moving from left to right in each drawing is equivalent to changing the bias factor; two-dimensional "slices" reveal the effect of this factor more clearly. The four-dimensional bifurcation set of the parabolic umbilic catastrophe is also shown in a three-dimensional section. It is based on a drawing prepared with a computer by A. N. Godwin of Lanchester Polytechnic in England.

Figure 34

331. Figure 34 is reprinted from E.C. Zeeman, *Catastrophe Theory*, SCI. AM., Apr. 1976, at 79. Copyright © 1976 by Scientific American, Inc. All rights reserved.

b. Catastrophe Theory and Conflict Behavior

One of the earliest and best known of Zeeman's applications of the catastrophe theory to conflict behavior is that dealing with the behavior of a dog under stress.[332] The graphs below and the accompanying text in Figures 35 and 36 explain the behavior of a dog under conflicting influences of rage and fear.[333]



LIKELIHOOD FUNCTION determines the behavior of the dog under the conflicting influences of rage and fear. When neither stimulus is present, the most likely behavior is neutrality; rage alone elicits aggression, fear alone submission. When the dog is made both angry and fearful, the likelihood graph becomes bimodal: attack and flight are both favored, and neutrality is the least likely response. The bimodality is reflected in the behavior surface of the cusp catastrophe, which has two sheets representing most likely behavior where both stimuli are present.

Figure 35

332. SAUNDERS, *supra* note 315, at 86.

333. Figure 35 is reprinted from E.C. Zeeman, *Catastrophe Theory*, SCI. AM., Apr. 1976, at 67.

Zeeman's fear-rage dog behavior model based on the cusp catastrophe is pictured in Figure 36.[334]

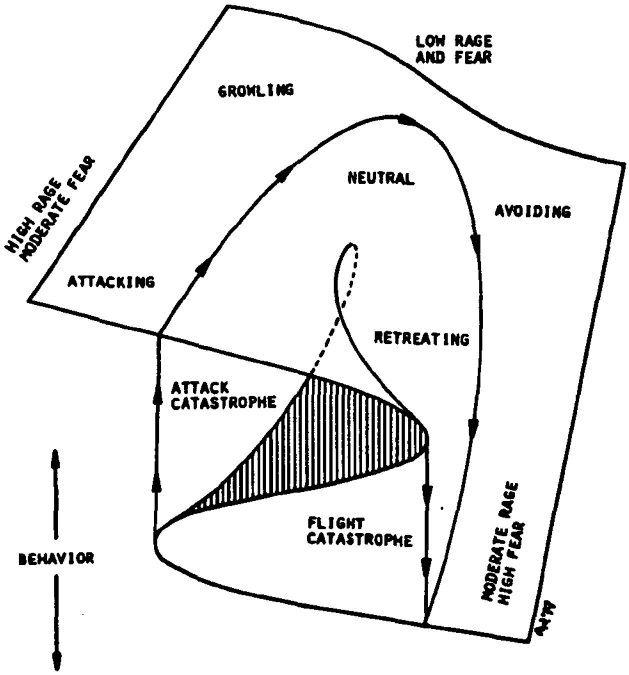


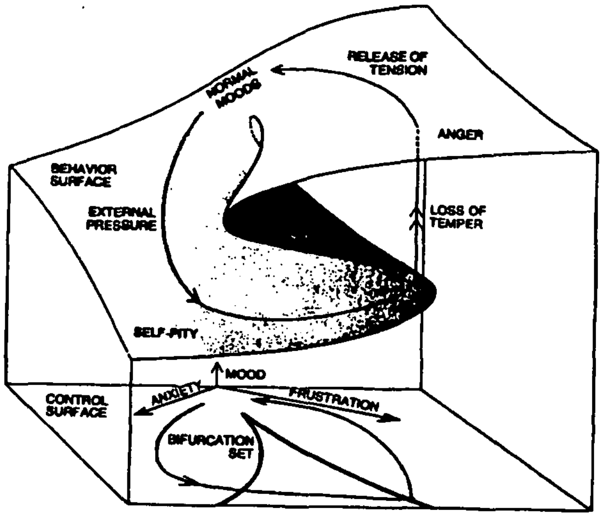
Figure 36

In Figure 36, rage and fear are shown as the control factors and the most likely level of aggression as the resultant behavior. Rage alone causes the dog to attack, and fear alone causes it to submit or flee. Neutral behavior is most likely at low levels of the control factors but is very unlikely when both are strong. The attack catastrophe path and the flight catastrophe path indicate sudden changes (discontinuities) in behavior.[335]

334. Figure 36 is reprinted from CATASTROPHE THEORY by Alexander Woodcock and Monte Davis, at 101. Copyright © 1978 by Alexander Woodcock and Monte Davis. Used by permission of Dutton Signet, a division of Penguin Books USA Inc.

335. *Id.*

Figures 37 and 38, and the accompanying text that follows, explain: (1) the control (conflicting) factors of human anxiety and frustration influencing mood as described by a cusp catastrophe, and (2) the emergence of compromise opinion in a model of the development of war policy as described by a butterfly catastrophe, which could easily be analogized to a litigation situation.[336]

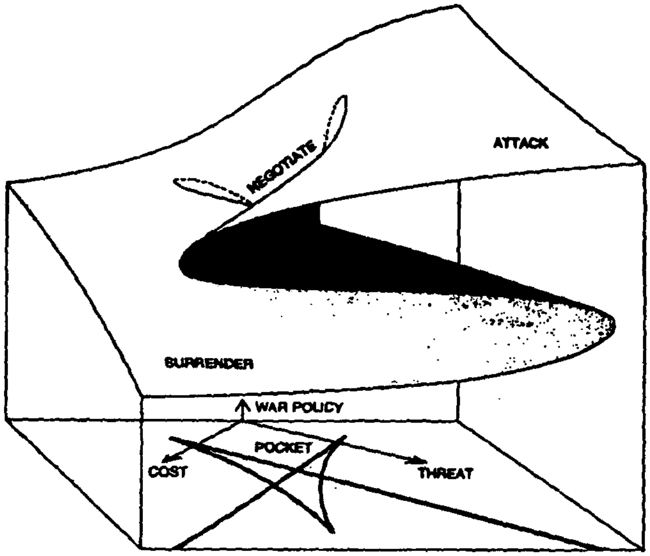


CATHARTIC RELEASE FROM SELF-PITY is described by a cusp catastrophe in which anxiety and frustration are conflicting factors influencing mood. Self-pity is induced by an increase in anxiety; it can be relieved by some event, such as a sarcastic remark, that causes an increase in frustration. As the control point crosses the cusp the mood changes catastrophically from self-pity to anger; the resulting release of tension gives access to calmer emotional states.

Figure 37

336. Figures 37 and 38 are reprinted from E.C. Zeeman, *Catastrophe Theory*, SCI. AM., Apr. 1976, at 75, 80, respectively. Copyright © 1976 by Scientific American, Inc. All rights reserved.

Compromise behavior is further described in terms of the elementary butterfly catastrophe in SAUNDERS, *supra* note 315, at 89-93. Humor has also been described in terms of the catastrophe theory. *See* JOHN A. PAULOS, MATHEMATICS AND HUMOR (1980). *See generally* John W. Cooley, *Mediation and Joke Design: Resolving the Incongruities*, 1992 J. DISP. RESOL. 249.



BUTTERFLY CATASTROPHE provides for the emergence of compromise opinion in a model of the development of war policy. In the butterfly four controlling parameters are required, but here only two are shown (threat and cost) and the other two are assumed to remain constant. The bifurcation set is one of the sections on the preceding page; it is a complex curve with three cusps and a "pocket" in the middle. On the behavior surface above the pocket is a new sheet that provides for a new intermediate mode of behavior. If both the threat and the cost of war are high, the cusp model would allow for only the extreme positions advocating attack or surrender. The new sheet in the butterfly model represents a compromise opinion, advocating negotiation.

Figure 38

The present utility of the catastrophe theory in negotiation and mediation is not so much an asset in the prediction of possible outcomes as it is an aid in *visualizing* the inventory of them.[337] With the advent of computer-drawn projections of the elementary catastrophes,[338] use of this visual behavior model as an aid in conflict resolution may be greatly expanded.[339] If nothing else, the catastrophe theory, which emphasizes the qualitative over the quantitative, heralds the comeback of geometry—the new preeminence of pictures over

337. EKELAND, *supra* note 305, at 107.

338. *Id.*

339. *See generally* CLIFFORD A. PICKOVER, COMPUTERS AND THE IMAGINATION: VISUAL ADVENTURES BEYOND THE EDGE (1991).

computations.[340] And its related graphics are nothing less than a visual Strauss waltz.[341] As one author put it: "The main contribution of catastrophe theory lies in the . . . seven beautiful figures in geometry, to be carried around in one's mind and to be matched against what we observe around us."[342]

VI. CONCLUSION

The contributions of Gottfried Wilhelm von Leibniz to human problem solving are only beginning to be recognized today and may, through applications perceived by social imagineers of the future, be, in fact, as limitless as the infinity which occupied his life's studies. The year 1996 will mark the 350th anniversary of Leibniz's birth,[343] and the year 2016 will mark the 300th anniversary of his death. It is only fitting that during one or both of these years we should celebrate Leibniz as a true genius of the ages. Unfortunately, his universal legacies advocating dispute settlement and social harmony, historians, up to now, have habitually ignored, suppressed, or relegated to footnote status.[344] One of Leibniz's more supportive commentators has said:

> Leibniz would have made an ideal Director for an international organization like Unesco whose program in many ways embodies his cosmopolitan ideas, far-sighted spirit of collaboration and co-operation for a more enlightened, peaceful, and morally progressive world civilization. Whether Leibniz was too optimistic about world harmony is no longer a question for academic dispute or literary wit, but one on which the very survival of civilization depends.[345]

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340. EKELAND, *supra* note 305, at 107. *See generally* ARTHUR I. MILLER, IMAGERY IN SCIENTIFIC THOUGHT CREATING 20TH-CENTURY PHYSICS (1984).

341. The physicist and engineer Lord Kelvin (1824-1907), who was the scientific consultant for the first transatlantic telegraph cable, once remarked that "a single curve . . . can depict all that the ear can possibly hear in the most complicated musical performance." E.T. BELL, MATHEMATICS: QUEEN AND SERVANT OF SCIENCE 121 (1987).

342. EKELAND, *supra* note 305, at 101.

343. The 300th anniversary of Leibniz's birth was celebrated anomalously on July 1, 1946, by the exploding of the fourth atomic bomb at Bikini, an event which most would view as a tragic irony in light of the man himself, whose life was dedicated to creating world harmony. 1 LOEMKER, *supra* note 57, at 1. It is also noteworthy that the year 1996 will mark the 400th year of Descartes' birth.

344. It is inexplicable that *The Great Books of the Western World*, published by the Encyclopaedia Britannica with editorial advice from the prestigious University of Chicago, does not contain a segment describing the life and collected writings of Leibniz. It is hoped, by this author at least, that this embarrassing omission will be corrected in the next few years.

345. Wiener, *supra* note 5, at li. For examples of Leibniz's writings on dispute resolution and universal peace, see *Discourse Touching the Method of Certitude, and the Art of Discovery in Order to End Disputes and to Make Progress Quickly*, *in* LEIBNIZ: SELECTIONS, *supra* note 5, at 46-50; *Precepts for Advancing the Sciences and Arts*, *in id.* at 29-46. In the latter essay, Leibniz draws some unusual analogies between problem solving and musical composition on the one hand and the

In an uncharacteristicly pessimistic mood, Leibniz once lamented that "his age . . . [was] not ripe for his ideas."[346] Our best hope is that our age now is.

APPENDIX A. LEIBNIZ'S METAPHYSICAL FOUNDATIONS OF MATHEMATICS[347]

A. *Time*

1. *Time* is the order of non-contemporaneous things.

2. *Duration* is the quantity of time.

3. A multiplicity of concrete circumstances which are not mutually exclusive are called *contemporaneous* or co-existing.

4. If one of two non-contemporaneous elements contains the ground for the other, the former is regarded as the *antecedent* and the latter as the *consequent*.

5. Time is an infinite process. Every temporal whole is similar to its parts.

B. *Space*

1. *Space* is the order of co-existing things.

2. *Extension* is the quantity of space.

3. *Position* is a determination of togetherness and includes both quantity and quality.

4. *Quantity* or magnitude is that determination of things which can be known in things only through their immediate contemporaneous togetherness or through their simultaneous observation.

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science of mechanics on the other. He also makes critical distinctions between theory and practice in problem solving. *See also* LEROY E. LOEMKER, STRUGGLE FOR SYNTHESIS: THE SEVENTEENTH CENTURY BACKGROUND OF LEIBNIZ'S SYNTHESIS OF ORDER AND FREEDOM 177-202 (1972) (ch. 8, "On Universal Harmony"). *See generally* DAVID E. MUNGELLO, LEIBNIZ AND CONFUCIANISM: THE SEARCH FOR ACCORD (1977).

346. Wiener, *supra* note 5, at xlv.

347. This appendix is adapted from LEIBNIZ: SELECTIONS, *supra* note 5, at 201-17.

5. *Quality* is that determination of things which may be known in them when we consider them individually and for themselves, without any necessity for their being given together. To quality belongs all attributes which can be explained through a definition or through a group of characters which they entail.

6. *Equal* is whatever has the same quantity.

7. *Similar* is whatever has the same quality.

8. Two elements are *homogeneous* when we can produce two others of the kind which are equal to the first pair and similar to each other. Given A and B, if we can find an element $L = A$ and another $M = B$ such that L and M are similar to each other, then A and B are homogeneous.

9. A structure which is contained in another which is homogeneous to it is called a *part*—the other in which it is contained is called the *whole*. Thus, the part is a homogeneous ingredient of the whole.

10. *Common boundaries* of two structures means something contained in both, yet without their having a common part.

11. Time and an instant, space and a point, boundary and the bounded, are not homogeneous, but nevertheless they are cognate or *homogonous*, insofar as through a continuous change one can be transformed into the other.

12. A spatial structure said to be contained *within* another is thought of as *homogonous* with it, but, if a part of it is formed or is equal to one of its parts, then it is not only homogonous but also homogeneous. The angle, though situated at a point, is nonetheless not contained in that point but might describe a magnitude at the point.

13. Whatever is equal to a part of A is less than A. If B is identical with a part of A, B is less than A.

14. All proofs may be reduced to two foundations: (a) to the definitions of the ideas and (b) to the original identical propositions. For example, B is the same as B.

C. Motion

1. *Motion* is change of position.

2. An object is *self-moved* when it alters its position and at the same time contains within itself the ground for this change.

3. The *movable* is homogonous with extension; for the point is also viewed as movable.

4. A *path* is the locus of the continuous, successive positions of a movable object.

5. *Place* is the position which the movable object occupies at a fixed instant.

6. The *boundary of a movable object* is given by the cross-section of the path which the boundary prescribes, assuming that the object has a path and does not move in one and the same place.

7. An *object moves in one and the same place* when every one of its points, except its boundary, occupies continually one or another position of the points belonging to the object itself.

8. A *line* is the path of a point.

9. A *surface* is the path of a line.

10. The *space of three-dimensional bodies* is the path of a surface.

11. The magnitudes of the paths in which a point describes a line, the line a surface, and the surface a volume, are called *dimensions*, of which there are only three: length, breadth, and depth.

12. A structure has *breadth* when its boundary has extension.

13. The boundary on all sides of a given structure, extended with breadth, is called its *periphery*. The periphery of a circle is its circumference. The periphery of a sphere is the surface of the sphere.

14. A structure has *depth* when it cannot be viewed as the boundary of another structure.

15. The line is the *ultimate boundary of extension*.

16. Three dimensional structures are the *ultimate extended and bounded structures*.

D. Relations

1. *Absolute space* is the locus of all loci.

2. A *spatial point* is the simplest locus—the locus of no other locus.

3. From a *single point* nothing results.

4. From *two points* a new structure results.

5. From *three points* a plane results.

6. From *four points not lying in the same plane*, absolute space results.

7. "Results" means that a new structure is determined through the fixing of defined original elements and the structure stands to these elements in a *unique relationship*.

8. Space, time, straight lines, and *every continuous structure* can be subdivided infinitely.

9. For *every given motion*, there can be found another which is definitely related to it as faster or slower.

10. There are *two kinds of measurement*: imperfect and perfect.

11. *Imperfect measurement* occurs when we set up a relation of greater and less between two elements, even though they do not stand in a numerical relation to one another, such as "a line is greater than a point" or "a surface is greater than a line."

12. *Perfect measurement* of homogeneous content is covering in a continuous transition from one end-point to the other all intermediate links.

13. Among *quantities*, there are many different sorts of *relations*. Two examples are: (a) the relation between two straight lines which makes their sum exactly equal to a constant length (there are infinitely many pairs of straight lines, x and y, which satisfy the condition $x + y = a$), and (b) the relation between two straight lines such that the square

root of the sum of their squares is equal to a constant line ($x^2 + y^2 = a^2$).

14. *Continuity* is present in time as well as in extension; in qualities as well as in motions. It lurks in every process in Nature, and it is a process that never takes place by sudden jumps.

15. *Place* is a *relation* of coexistence among a plurality of elements.

16. The *order of transition* from one element to another is the shortest path from one element to another. This shortest path is called the distance of the elements.

17. The *path of a point* is a line, which has no breadth, because its cross-section, the point, has no length.

18. By means of one given point, no further structure can be determined. Through *two given points*, the *simplest path* from one to the other determines a straight line.
    (a) A straight line is the shortest line from one point to the other; it is the magnitude or *distance between the points*.
    (b) The straight line between the endpoints is *uniform*.
    (c) The *different successive positions of a point* moving in a straight line can be distinguished only through their *different relations to the endpoints*.
    (d) *Every part of a straight line* is itself a straight line and *internally similar to itself* throughout, and two of its parts cannot be distinguished except through their endpoints.
    (e) If the endpoints are assumed as similar, congruent, or coincident, then the straight lines themselves are respectively *similar*, *congruent*, and *coincident*.
    (f) A straight line goes through those points which stand in a *unique relation* to the position of the *end-points*.
    (g) The straight line is uniform on all sides, i.e., it does not, like a *curved line*, possess a *concave and convex side*.
    (h) If two arbitrary points, *L* and *M*, are situated outside a straight line and stand in the same relation to a pair of points on the straight line, *A* and *B*, such that *L* is related to *A* and *B* as *M* is to *A* and *B*, then *their relation to the whole straight line will also be identical*. That is, *L* will have exactly the same relation as *M* to the straight line through *A* and *B*.
    (i) A straight line cannot be moved if two points on the line are *fixed*.

(j) *All other points not lying on the straight line* between *A* and *B*, or in the direction *AB*, are *movable* without changing their situation in relation to the fixed points *A* and *B*.

(k) If a rigidly extended structure is moved in such a way that two points in it remain fixed, all its stationary points fall collectively on the straight line or axis through the fixed points, but every movable point describes a *circle* around the axis.

19. If *three points not lying in the same straight line are given*, then the structure that is determined by them is a *plane*.

(a) The plane is the *smallest surface* of all those which are possible within a given boundary.

(b) The plane within its boundary is *uniform* because from the nature of its origin (three points) there is no ground for deriving any sort of diversity.

(c) The plane is *everywhere similar* to itself.

(d) Planes whose *perimeters* are similar or congruent or coincide are themselves similar or congruent or coincide.

(e) By definition of a plane, the locus of all points whose *positions in relation to the three given points* are determined.

(f) The plane is uniform in its relations on both sides that it has *neither a concave nor convex side.*

(g) If there exists *a point outside the plane* in an *arbitrary relation* to A, B, and C, and consequently to the plane determined by them, then *there is always another arbitrary point which shows the same relation* to these three points, since no reason exists for any diverse form of relationship.

(h) The plane possesses *breadth* because it can be cut by a straight line which passes through any two given points.

20. If four points not lying in the same plane are given, then we obtain *depth* or a structure in which something appears which cannot be treated as a boundary and which, therefore, cannot itself have a boundary in common with another structure unless the latter is included in it as a part.

21. *Similarity or dissimilarity* of two spatial wholes results from their boundaries.

22. *Volumes* whose ends or surfaces touch, cover, or resemble one another, belong themselves to a *class of congruent, similar things*. The same is true for a *plane*, which is a uniform surface, similar in all its parts. It is true also for the *straight line*, which is essentially a uniform line.

APPENDIX B. GRAPHICAL EXAMPLES OF TYPES OF CHANGE[348]

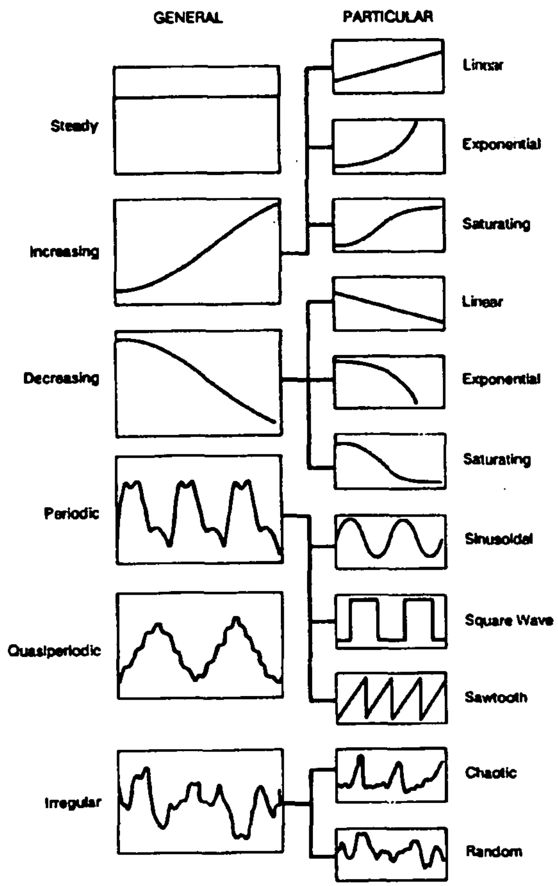

Figure 39

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