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Long memory in the Croatian and Hungarian stock market returns*

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Abstract

The objective of this paper is to analyze and compare the fractal structure of the Croatian and Hungarian stock market returns. The presence of long memory components in asset returns provides evidence against the weak-form of stock market efficiency. The starting working hypothesis that there is no long memory in the Croatian and Hungarian stock market returns is tested by applying the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) (1992) test, Lo's (1991) modified rescaled range (R/S) test, and the wavelet ordinary least squares (WOLS) estimator of Jensen (1999). The research showed that the WOLS estimator may lead to different conclusions regarding long memory presence in the stock returns from the KPSS and unit root tests or Lo's R/S test. Furthermore, it proved that the fractal structure of individual stock returns may be masked in aggregated stock market returns (i.e. in returns of stock index). The main finding of the paper is that both the Croatian stock index Crobex and individual stocks in this index exhibit long memory. Long memory is identified for some stocks in the Hungarian stock market as well, but not for the stock market index BUX. Based on the results of the long memory tests, it can be concluded that while the Hungarian stock market is weak-form efficient, the Croatian stock market is not.

Key words: *stock market, long memory, efficient-market hypothesis*

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1. Introduction

Long memory (or long-range dependence) describes the correlation structure of a series at long lags. Sibbertsen (2004) noted that the autocorrelation of a process with a long memory decays slowly, by hyperbolic rate. If a series exhibits long memory (or the "biased random walk"), there is persistent temporal dependence even between distant observations (Barkoulas et al., 2000). This has important implications for numerous paradigms in modern financial economics (Maheswaran and Sims, 1992). Optimal consumption/savings and portfolio decisions would become extremely sensitive to the investment horizon if stock returns were long-range dependent (Lo and MacKinlay, 2001). Problems arise in the pricing of derivative securities with martingale methods, since the continuous time stochastic processes most commonly employed are inconsistent with long memory (Sims, 1984; Maheswaran and Sims, 1992). According to LeRoy (1989), the CAPM and the APT are also not valid, because the usual forms of statistical inference do not apply to time series that exhibit such persistence.

The presence of long memory in stock return series would also provide evidence against the weak-form of financial market efficiency since it would imply nonlinear dependence in the first moment of the distribution and hence a potentially predictable component in the series dynamics (Barkoulas and Baum, 1996). According to Fama (1970), financial market can be called efficient only if the security prices always fully reflect the available information. The weak-form of financial market efficiency, which in empirical studies is the usually tested financial market efficiency hypothesis, asserts that the only relevant information set to the determination of current security prices is the historical prices of that particular security. In this regard, investors cannot expect to find any patterns in the historical sequence of stock prices or returns that will provide insight into future price movements and allow them to earn abnormal rates of returns. In empirical literature, the random walk behavior of security prices is used as the basis to test for the weak-form of stock market efficiency (Dima and Miloş, 2009). The random walk behavior of stock (market) prices (i.e. white noise process of stock market returns) posits that today´s returns time series is un-forecastable transformation of its preceding return. An empirical finding that a returns time series is fractionally integrated is thus inconsistent with a white noise hypothesis of return series and efficient market hypothesis (see Eitelman and Vitanza, 2008).

The basic hypothesis of this paper is that there is no long memory in the Croatian and Hungarian stock market returns. We apply the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) (1992) test, the modified rescaled range (R/S) test of Lo (1991), and the wavelet ordinary least squares (WOLS) estimator of Jensen (1999) to answer whether Croatian and Hungarian stock market returns exhibit fractal structure. As the fractal structure of individual stock returns may be masked in the fractal structure of a stock index that incorporates the stock, the fractal structure of individual stock's returns included in the calculation of the stock index will also be calculated. The results will be informative of the weak-form efficiency hypothesis of the stock markets and individual shares in the Croatian and Hungarian stock markets.

The paper consists of six related sections. After the introduction, the second section of the research provides the literature review of long memory studies in stock markets. The third sections provides the definition of long memory and the fourth section outlines the methodology applied in this paper to test long memory of Croatian and Hungarian stock markets. The fifth section describes the data and analyzes the empirical results of long memory tests. The paper ends by outlining proposals and recommendations, as well as final reflections.

2. Literature review

The subject of long memory (or long-range dependence) in time series was brought to prominence by the seminal work of hydrologist Hurst (1951). The works of Mandelbrot (1965, 1971) extended the study of long memory to economic and financial time series. Two measures are commonly used in estimating the strength of the long memory (long-range dependence). The parameter H, known as the Hurst or self-similarity parameter, was introduced to applied statistics by Mandelbrot and van Ness (1968). They focused on self-similar processes such as fractional Brownian motion and fractional Gaussian noise in their studies of long memory. The other measure, the fractional integration parameter, d, arises from the generalization of the Box-Jenkins ARIMA (p, d, q) models from integer to non-integer values of the integration parameter d (Autoregressive Fractionally Integrated Moving Average – ARFIMA models) and were introduced by Granger and Joyeux (1980) and Hosking (1981)[5].

Due to their flexibility, ARFIMA (Autoregressive Fractionally Integrated Moving Average) models have been often used to model financial time series (Baillie, 1996; Dark, 2007). They have been applied to interest rates (Harmantzis and Nakahara, 2006; McCarthy et al., 2004; Tkacz, 2001), exchange rates (Jin et al., 2006; Karuppiah and Los, 2005), prices of financial derivatives (Fang et al. 1994), and returns and volatilities of stocks and stock indices. For stock returns and their volatilities the results are mixed. The studies that found supportive evidence of long memory in stock market returns or their volatilities include: Ding et al. (1993) for the S&P500 index in the period 1928-1992, Lobato and Savin (1998) for the S&P500 in the period 1962-1994 and Ray and Tsay (2000) for the companies listed on the S&P500 index for the period 1962-1995. Barkoulas et al. (2000) found significant and robust

[5] The fractional integration parameter d is also the discrete time counterpart to the self-similarity parameter H and the two are related by the simple formula $d = H – 0.5$ (Rea et al., 2007).

evidence of positive long-range persistence for the stocks traded in Athens stock exchange for the period 1981-1990. Assaf and Cavalcante (2005) provide empirical evidence of the long-range dependence in the returns and volatility of the Brazilian stock market in the period from the start of 1994 until May 2002. Supportive evidence of long memory is provided by Ozdemir (2007) and Chan and Feng (2008) for the DJI, the S&P500, the FTSE, DAX and NIKKEI (for different time periods), Bilel and Nadhem (2009) for G7 countries stock indices for high frequency data between 2003-2004 and Mariani et al. (2010) for international stock indices.

Mixed results were obtained by Henry (2002), who investigated nine developed stock market indices for the period from 1982-1998. He only found strong evidence of long memory in the South Korean returns and some evidence of long memory in the German, Japanese and Taiwanese returns. Tolvi (2003), investigating stock market indices of 16 OECD countries for the period from 1960-1999, only found evidence of long memory in stock indices returns for three smaller stock markets: in Finland, Denmark and Ireland. Jagric et al. (2005) found mixed evidence of long memory presence in the stock indices of six Central and Eastern European (CEE) countries for the period between 1991-2004: strong long-range dependence was identified in the returns of the Czech, Hungarian, Russian and Slovenian stock markets, whereas weak or no long-range dependence in the returns of the Slovakian and Polish stock markets. Jagric et al (2006) investigated long memory in stock returns of ten Central and Eastern European economies. Their results suggested strong long memory presence in eight stock markets, among them in the Croatian and Hungarian stock markets. Another study investigating fractal structure of the CEE stock markets is Kasman et al. (2009a). Their results indicate existence of long memory in five of eight studied markets. Furthermore, Kasman et al. (2009b), investigating four CEE stock markets, found significant long memory in the return series of the Slovak Republic, weak evidence of long memory for Hungary and the Czech Republic, and no evidence for Poland.

Studies that found no evidence of long memory presence in stock market returns and return volatilities are Barkoulas and Baum (1996) for the Dow Jones index returns, sectoral stock returns, and stock returns included in the Dow Jones Industrials index; Chow et al. (1996), examing 22 international stock indices; Lux (1996) for the DAX and some individual shares in the DAX; Grau-Carles (2005) for S&P500 and Dow Jones Industrial; and Oh et al. (2006) for stock indices in seven developed countries.

The majority of the empirical studies on long memory have used returns series on stock indices, whose construction entails a great deal of aggregation. As argued by Barkoulas and Baum (1996), if fractal structure does exist in individual stock returns series, its presence may be masked in aggregate returns series. It is therefore important to test for long memory presence in individual stock returns as well as in the returns of stock indices.

3. The long memory property of time series

The long memory property of a time series can be defined in the time domain by a hyperbolically decaying autocovariance function. Alternatively, it can be defined in the frequency domain by a spectral density function that approaches infinity at near zero frequencies.

Let X_t be a stationary process with an autocovariance function γ_τ (τ is time lag). The long memory is present in the process, if its autocovariance function decays hyperbolically (DiSario et al. 2008):

$$\gamma_\tau \approx |\tau|^{2d-1} \text{ as } |\tau| \to \infty, \tag{1}$$

where $d \in (0, 0.5)$ is a long memory parameter. The spectral density function $\omega(\lambda_s) \approx c|\lambda_s|^{-2d}$ of such a process, where $\lambda_s \in (-\pi, \pi]$, has the following property:

$$\omega(\lambda_s) \approx c|\lambda_s|^{-2d} \text{ as } \lambda_s \to 0, \tag{2}$$

where $c > 0$ and $d \in (0, 0.5)$.

A general class of fractional processes ARFIMA (*p, d, q*) is described as (Sadique and Silvapulle, 2001):

$$\Phi(B)\,(1 - B)^d X_t = \Theta(B)\varepsilon_t, \tag{3}$$

where X_t is a time series, $\Phi(B) = 1 - \varphi_1 B - \cdots - \varphi_p B^p$ and $\Theta(B) = 1 + \theta_1 B + ... + \O_q B^q$ are autoregressive and moving average polynomials in the lag operator B with all roots of $\Phi(B)$ and $\Theta(B)$ being stable, and ε_t is a Gaussian white noise $\varepsilon_t \sim D\,(0,\sigma^2)$. For $d = 0$ the process is stationary and the effect of a shock to ε_t on X_t decays geometrically with time. For $d = 1$, the process is said to have a unit root, and the effect of a shock to ε_t on X_t persists into the infinite future.

When $p = q = 0$, X_t becomes a simple fractional differenced process, proposed by Joyeux (1980) and Hosking (1981). The function $(1 - B)^d$ can be defined for non-integer values of d:

$$(1 - B)^d X_t = \varepsilon_t, \tag{4}$$

where B is a lag operator, innovation is a Gaussian white noise $\varepsilon_t \sim D\,(0,\sigma^2)$, d a fractional integration parameter varying in the interval (–0.5, 0.5) and $(1 - B)^d$ is the fractional differencing operator.

Following Hosking (1981) and Granger and Joyeaux (1980), a more general definition of $(1 - B)^d$ can be derived from a power series expansion as follows:

$$(1-B)^d = \sum_{k=0}^{\infty} \binom{d}{k} (-B)^k = 1 - dB - \frac{1}{2} d(1-d)B^2 - \frac{1}{6} d(1-d)(2-d)B^3 - \cdots . \quad (5)$$

This power expansion can be re-expressed in terms of the gamma function as (Jin et al. 2006):

$$(1-B)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)B^k}{\Gamma(-d)\Gamma(k+1)} . \quad (6)$$

When $d = 1$, we have $(1 - B)X_t = X_t - X_{t-1}$, meaning X_t follows a unit root process with a zero mean and infinite variance. When $d = 0$, the process X_t equals ε_t so we have $\varepsilon_t \sim D(0,\sigma^2)$, or $X_t \sim I(0)$. The process is stationary and the effect of a shock in ε_t geometrically decays with time.

It may be shown that for $-0.5 < d < 0.5$ the process X_t is stationary and invertible (Hoskings 1981). An ARFIMA process with the parameter $0 < d < 0.5$ is stationary, but the effects of a shock in ε_t on X_t decay at a much slower rate than for a process integrated of order zero – the process exhibits long memory. A process with the parameter $-0.5 < d < 0$ also exhibits long memory (Lo and MacKinlay, 2001)[6]. The autocovariance function for zero integrated processes decays geometrically, while the autocovariance function for a fractionally integrated process decays hyperbolically with the sign of the autocovariances being the same as the sign of d (Pons Fanals and Suriñach Caralt, 2002). When d is positive the sum of autocorrelations diverges to infinity, and collapses to zero when d is negative (Lo and MacKinlay, 2001). A process with the parameter $d \geq 0.5$ is not stationary and a shock in ε_t on X_t decays even more slowly.

4. Methodology

More methods of the fractional integration parameter estimation have been applied in empirical studies. Ding et al. (1993) used autocorrelation function, GARCH and APGARCH model. Granger and Ding (1995) applied the method of Geweke and Porter-Hudak (1983) (the GPH method). Lobato and Savin (1998) checked the time series for long memory by Robinson's (1995) method of local Whittle approximation. Lobato and Velasco (2000) developed a semi-parametric method,

[6] In the literature the processes with the parameter $-0.5 < d < 0$ are mostly defined as a long memory processes (e.g. Lo and MacKinlay, 2001; Lo, 1991). However, as the autocorrelation function is summable, some call these processes also intermediate processes (e.g. Baillie, 1996). Processes with parameter $-0.5 < d < 0$ are also called antipersistent; this terminology is applied by Mandelbrot in numerous works, Rea et al. (2007), and Künze and Strohe (2010)). The term short-term memory processes is reserved for the processes with $d = 0$.

based on Whittle approximation to the Gaussian log-likelihood. Ray and Tsay (2000) used three methods: the GPH method, Robinson's (1995) method of local Whittle approximation and method of Breidt et al. (1998). Assaf and Cavalcante (2005) used the modified rescaled range statistics of Lo (1991), rescaled variance method of Giraitis et al. (2000) and Robinson's (1995) method of local Whittle approximation. Atilla and Ozun (2008) applied GPH method and wavelet ordinary least squares method of Jensen (1999), while Bilel and Nadhem (2009) the GPH and Robinson's (1995) method of local Whittle approximation. Jagric et al. (2005) applied a wavelet method, Chan and Feng (2008) the GPH and Robinson's (1995) method of local Whittle approximation, Oh et al. (2006) detrended fluctuation analysis, while Grau-Carles (2005) used four different methods: rescaled range, modified rescaled range, the GPH and detrended fluctuation analysis method. Kasman et al. (2009b) applied GPH and parametric method of parametric method of Sowell (1992), while Kasman et al. (2009a) the GPH and ARFIMA (ARFIMA-FIGARCH, ARFIMA-HYGARCH) methods.

The long memory presence in stock and stock index returns in our study will be calculated by the test of Kwiatkowski-Phillips-Schmidt-Shin (KPSS) (1992), modified R/S of Lo (1991), and the wavelet ordinary least squares estimator of Jensen (1999).

Despite the widespread use of the ARFIMA model, estimation is difficult because the ARMA structure is unknown (Dark, 2007). The advantage of semiparametric estimates of *d*, based on the spectral density (for example Geweke and Porter-Hudak, 1983; Robinson, 1995) and wavelet multiresolution analysis (the WOLS of Jensen, 1999), is that they remain agnostic about the ARMA structure of the underlying model.

4.1. KPSS test in combination with unit root tests

Lee and Schmidt (1996) proposed the test of the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) (1992) to test the null hypothesis of *I*(0) against the fractional alternative. According to Kwiatkowski et al (1992), the test assumes that a time series (X_t) can be decomposed into three components, a deterministic time trend (*ct*), a random walk (r_t) and a stationary error (ε_t):

$$X_t = r_t + tc + \varepsilon_t, \tag{7}$$

where r_t is a random walk $r_t = r_{t-1} + v_t$, v_t is $IID(0,\sigma_v^2)$. The initial value r_0 can be regarded as an intercept for the equation above. The null hypothesis of stationarity implies H : $\sigma_v^2 = 0$. Under the alternative hypothesis $\sigma_v^2 > 0$ the time series X_t is a fractionally integrated process. This test may be conducted under the null hypothesis of either trend stationarity or level stationarity. Using the residuals from the regression of X_t on intercept and time (or on intercept only in the case of level stationarity), the test statistic is computed as:

$$KPSS = \sum_{t=1}^{T} (s(t)^{\;2} / (s_{nv}^{2} T^{2}) \,, \tag{8}$$

where $s(t) = e_1 + ... + e_t$, e is a vector of residuals, s_{nw}^2 is the Newey-West estimator of the long-run variance σ^2 of the errors ε_t, and T the sample size.

According to Lee and Schmidt (1996), the two KPSS tests (trend stationarity or level stationarity) are consistent against an $I(d)$ alternative, and can be used in conjunction with the usual stationarity tests, like the Augmented Dickey-Fuller (ADF) test and the Phillips-Perron (PP) test, to investigate the possibility that a series is fractionally integrated (i.e. neither $I(1)$ nor $I(0)$).

- Following Lee and Schmidt (1996) and Barkoulas et al. (1998), combining the results of the KPSS, ADF and PP tests suggests these deductions:
- The rejection of the null hypothesis for the ADF and PP test and the non-rejection of the null hypothesis of the KPSS test lead to the conclusion of no unit-root in a time series.
- The alternative hypothesis, of stationary time series, $I(0)$, can be accepted.
- Non-rejection of the null hypothesis of the ADF and PP test and the rejection of KPSS test leads to the conclusion of a unit root in time. The time series is integrated to the order 1 ($I(1)$).

When the null hypothesis of the ADF, PP, and KPSS test are not rejected we cannot draw a conclusion about the (non-) stationarity of a time series.

The rejection of the null hypothesis of the ADF, PP, and KPSS test leads to the conclusion that the time series is neither $I(1)$ nor $I(0)$, wherefrom it follows that parameter d takes some non-integer value (i.e. the time series is fractionally integrated).

4.2. Lo's rescaled/range statistics

To detect long memory in economic or financial time series Mandelbrot (1972) has suggested using the range over standard deviation or R/S statistic, also called the "rescaled range" (the test was developed by Hurst (1951)). The R/S statistic is the range of partial sums of deviations of a time series from its mean, rescaled by its standard deviation (Lo and MacKinlay, 2001). For the time series $X_1, X_2, ..., X_n$ the R/S statistics is defined the following way (Lo and MacKinlay, 2001):

$$Q_n \equiv \frac{1}{s_n}\left[\max_{1\le k\le n}\sum_{j=1}^{k}(X_j-\overline{X}_n)-\min_{1\le k\le n}\sum_{j=1}^{k}(X_j-\overline{X}_n)\right], \tag{9}$$

where s_n is the maximum likelihood estimator of the standard deviation of $\{X\}$ ($s_n \equiv \left[\frac{1}{n}\sum_j (X_j-\overline{X}_n)^2\right]^{-\frac{1}{2}}$) and $\overline{X}_n$ is the sample mean. The first term in the brackets of equation (9) is the maximum of the partial sums of the first k deviations of X_j from the full sample mean (which is nonnegative). The second term in the equation (9) is the corresponding minimum, which is non-positive. The difference of these two terms is thus non-negative ($Q_n \geq 0$). The R/S statistics has been shown to be sensitive to short memory features of the data. Lo (1991) shows that a sizeable AR(1) component in the data generating process will seriously bias the R/S statistics. He modifies the R/S statistics to account for the effect of short memory by applying a Newey-West (using Bartlett kernel window) correction to derive a consistent estimate of the long memory variance of the time series:

$$Q_n \equiv \frac{1}{\hat{\sigma}_n(q)}\left[\max_{1\le k\le n}\sum_{j=1}^{k}(X_j-\overline{X}_n)-\min_{1\le k\le n}\sum_{j=1}^{k}(X_j-\overline{X}_n)\right], \tag{10}$$

where

$$\hat{\sigma}_n(q) \equiv \frac{1}{n}\sum(X_j-\overline{X}_n)^2+\frac{2}{n}\sum_{j=1}^{q}\omega_j(q)\left\{X_i-\overline{X}_n)(X_{i-j}-\overline{X}_n\right\}=\hat{\sigma}_x^2+2\sum_{j=1}^{q}\omega_j(q)\hat{\gamma}_j\,,$$

$\omega_j(q) \equiv 1-\frac{j}{q+1}$, $q < n$. $\hat{\sigma}_x^2$ and γ_j are variance and autocovariance estimators of $\{X\}$.

By allowing q to increase with (but at a slower rate than) the number of observations n, the denominator of Q_n adjusts appropriately for general forms of short-range dependence. Andrews (1991) provided a data-dependent rule for choosing q.

Although Lo's test is a significant improvement over the original R/S test, the test has a strong preference for accepting the null hypothesis of no long-range dependence, irrespective of whether long-range dependence is present in the data or not (Teverovsky et al., 1999). It is therefore advised to provide robustness check by other tests of long memory (Teverovsky et al., 1999).

4.3. Wavelet ordinary least squares method (WOLS method)

Jensen (1999) developed a wavelet estimator of parameter d assuming the zero mean ARFIMA(0,d,0) process $(1-B)^d X_t = \varepsilon_t$, with $|d|$ …< 0.5.

A wavelet is defined as any function ψ, whose collection of dilations, for $j \in Z$ (which compress the function $\psi_{j,k}$), and translations, for $k \in Z$ (which shift the function $\psi_{j,k}$) given by

$$\psi_{j,k}(t) = 2^{\frac{j}{2}} \psi(2^j t - k), \tag{11}$$

form a basis for the Hilbert space $L^2(R)$ of square integrable functions.

The wavelet coefficient $w_{j,k}$ of a process X_t is a function of the scale parameter, j, and translation parameter, k, equals to

$$w_{j,k} = 2^{\frac{j}{2}} \int X(t)\psi(2^j t - k)dt \tag{12}$$

and is the representation of X_t at different levels of resolution (scales).

Jensen (1999) proved that the wavelet coefficients, $w_{j,k}$, associated with a zero mean $I(d)$ process with $|d| < 0.5$, are distributed $N(0, \sigma^2\ 2^{-2jd})$, where σ^2 is a finite constant.

If $R(j)$ is defined as a variance of the wavelet coefficients on the scale j, $R(j) = \sigma^2\ 2^{-2jd}$, one can take advantage of the fact that this variance is independent of the translations, k. Taking the logarithmic transformation of $R(j)$ relationship, we obtain (Tkacz, 2000):

$$\ln R(j) = \ln \sigma^2 - d \ln 2^{2j}, \tag{13}$$

from which the parameter d of a fractionally integrated series can be estimated by the ordinary least squares estimator (OLS). It was proved by Jensen (1999) that the OLS estimate $\hat{d}$ provides a consistent estimate of d.

Percival and Walden (2000), Tkacz (2001), Gencay et al. (2002) and Vuorenmaa (2004) show that WOLS performs better than the GPH and alternative estimators of the fractional integration parameter d. Jensen (1999), using Monte Carlo analysis, shows that MSE (mean squared error) of the WOLS estimator is 4-6 times smaller than MSE of the GPH estimator.

5. Empirical results

5.1. Data

The returns of stocks and stock indices are calculated as differences of logarithmic daily closing prices: $\ln(P_t) - \ln(P_{t-1})$, where P is a closing price of the stock or stock index. The stocks considered are those that are included in the main national stock market index, have a great share in the stock market capitalization and are regularly traded in the stock market. The longest possible time period is taken, therefore the observation period differs among the stocks (the observation periods are denoted in Tables 1 and 2). The data of the stock and stock index prices were obtained from web pages of Zagreb and Budapest stock exchanges[7].

Table 1: Descriptive statistics for returns of the Croatian stocks and the stock index Crobex

Name of a stock/ stock index	Observation period	Min	Max	Mean	Std. deviation	Skewness	Kurtosis	Jarque-Bera statistics
Adris Grupa	31.7.2003-21.4.2011	-0.1226	0.2346	0.0002941	0.02081	1.4342	20.0509	24,103.62***
Dalekovod	5.3.2001-21.4.2011	-0.5015	0.635	0.0007458	0.03795	1.5000	64.7576	296,123,41***
Ericsson Nikola Tesla	16.6.1997-21.4.2011	-0.3262	0.4745	0.0007124	0.03927	1.7961	32.6214	110,475,01***
Hrvatski Telekom	5.10.2007-21.4.2011	-0.111	0.1039	-0.0003249	0.01535	-0.5146	13.3542	3,969.82***
Ina	1.12.2006-21.4.2011	-0.1185	0.5032	0.0005357	0.02678	6.0608	119.8994	614,651.60***
Končar	12.11.1996-21.4.2011	-0.9167	1.373	0.0006583	0.0554	5.7409	207.1289	4.655,525.38***
Kraš	19.3.1996-21.4.2011	-0.3102	0.5104	0.0004185	0.03644	1.0913	30.4157	103,246.39***
Podravka	30.6.1994-21.4.2011	-0.4627	0.4354	0.0003053	0.03059	1.0955	57.3244	448,564.69***
Zagrebačka Banka	25.6.1996-21.4.2011	-0.356	0.469	0.0007505	0.03803	-0.0016	24.9465	584,40.03***
Crobex (index)	2.9.1997-21.4.2011	-0.1942	0.1757	0.0001971	0.0178	-0.0484	19.9528	40,835.80***

Note: The skewness of the normal distribution (or any perfectly symmetric distribution) is zero. If the statistic is negative, then the data are spread out more to the left of the mean than to the right. If skewness is positive, the data are spread out more to the right. The kurtosis of the normal distribution is 3. Fat-tailed distributions have kurtosis greater than 3; distributions that are less outlier-prone than normal distribution have kurtosis less than 3. Jarque-Bera test: the null hypothesis is that the sample data come from a normal distribution with unknown mean and variance, against the alternative that it does not come from a normal distribution. Jarque-Bera statistics: *** indicate that the null hypothesis (of normal distribution) is rejected at a 1% significance, ** that the null hypothesis is rejected at a 5% significance, and * that the null hypothesis is rejected at a 10% significance level

Source: Own calculations

[7] All calculations, but Lo's R/S statistics, were performed in Matlab.

As table 1 and 2 suggests, the return series appear extremely non-normal and Jarque-Bera test rejects the hypothesis of normal distribution for all stocks and for the stock indices[8].

Table 2: Descriptive statistics for the returns of the Hungarian stocks and the stock index BUX

Name of a stock/ stock index	Observation period	Min	Max	Mean	Std. deviation	Skewness	Kurtosis	Jarque-Bera statistics
ANY	8.12.2005-21.4.2011	-0.1316	0.1214	0.000176	0.01949	-0.4428	10.4804	3,177.44***
Egis	1.4.1997-21.4.2011	-0.3567	0.1944	0.0001761	0.02625	-0.9651	20.6111	45,969.75***
Fotex	1.4.1997-21.4.2011	-0.3365	0.2346	0.0002633	0.03197	0.4055	14.0875	18,100.81***
MOL	1.4.1997-21.4.2011	-0.2231	0.1403	0.0005951	0.02418	-0.1880	9.6598	6,516.58***
MTelekom	14.11.1997-21.4.2011	-0.1257	0.1199	-0.00005357	0.0209	-0.1929	6.8011	2,042.44***
OTP	1.4.1997-21.4.2011	-0.2513	0.2092	0.0007981	0.02759	-0.2478	10.6610	8,631.63***
Pannergy	1.4.1997-21.4.2011	-0.2076	0.2343	-0.0001497	0.02613	0.1969	11.9828	11,840.55***
Raba	17.12.1997-21.4.2011	-0.2501	0.1999	-0.0003963	0.02531	-0.1423	13.0103	13,935.68***
Richte	1.4.1997-21.4.2011	-0.231	0.2178	0.0003489	0.02559	-0.6279	16.7910	28,086.05***
Synergon	5.5.1999-21.4.2011	-0.1625	0.1526	-0.0006251	0.0291	0.4147	8.8941	4,425.65***
TVK	1.4.1997-21.4.2011	-0.2231	0.2068	0.0001003	0.027	-0.1433	12.0129	11,909.24***
BUX (index)	1.4.1997-21.4.2011	-0.1803	0.1362	0.0004285	0.01894	-0.6278	13.1805	15,410,10***

Notes: See notes for Table 1

Source: Own calculations

5.2. The results of long memory tests

For the Croatian stock market, the unit root tests (ADF and PP tests) clearly reject the null hypothesis of unit root in the time series; results are robust to model

[8] Žiković (2008) found that developing stock markets indices from Croatia and other ex-Yugoslavia stock markets significantly differ in statistical characteristics from the developed markets. Distribution of returns on these indexes is not symmetrical and shows significant positive asymmetry (except BIRS index). Žiković and Atkan (2009) note that this is due to low liquidity, frequent internal and external shocks (inflation, depreciation of local currency, credit rating changes, etc.) as well as high degree of insider trading that causes these markets to be more volatile and deviate more from the normal distribution.

specifications (Table 3). The null hypothesis of the KPSS test (i.e. the time series is stationary) is rejected for the majority of investigated stocks. There is some evidence of fractal structure in returns of Hrvatski Telekom and Dalekovod shares. However the KPSS model for the later share had a significant trend (at the 1% significance level), therefore the null hypothesis of stationarity could not be rejected. Based on the KPSS and unit root tests there is no significant evidence of long memory in the Croatian stock market returns.

Table 3: Results of the KPSS and other tests of stationarity for the Croatian stock market return series

Name of a stock/ stock index	KPSS test	PP test	ADF test
Adris Grupa	0.445* (0)	-39.266*** (7)	- 39.344*** (L=0)
Dalekovod	0.894*** (12)	-51.864*** (15)	- 20.627*** (L=3)
Ericsson Nikola Tesla	0.162 (14)	-61.377*** (16)	- 43.865*** (L=1)
Hrvatski Telekom	0.359* (8)	-28.778*** (8)	- 22.587*** (L=1)
Ina	0.325 (6)	-30.532*** (4)	- 30.511*** (L=0)
Končar	0.086 (18)	-64.628*** (19)	- 44.138*** (L=1)
Kraš	0.098 (11)	-67.822*** (7)	-67.974*** (L=0)
Podravka	0.064 (8)	-68.395*** (4)	-45.771*** (L=1)
Zagrebačka Banka	0.154 (11)	-62.261*** (14)	-62.719*** (L=0)
Crobex (index)	0.163 (9)	-59.049*** (9)	-30.762*** (L=2)

Notes: The presented unit root and KPSS tests include only a constant. For the KPSS and PP tests the Bartlett Kernel estimation method is used with Newey-West automatic bandwidth selection. Optimal bandwidth is indicated in parenthesis under statistics. For ADF test, the number of lags to be included (L) in the ADF test were selected by SIC criteria (30 was a maximum lag). Exceeded critical values for rejection of null hypothesis are marked by *** (1% significance level), ** (5% significance level) and * (10% significance level)

Source: Own calculations

For the Hungarian stock market the null hypotheses of ADF and PP tests can also be rejected (Table 4), proving that the returns of the BUX and shares listed in the BUX are not unit root. KPSS model with a constant is given advantage over KPSS model with a constant plus trend since trend is not significant for any of the time series.

Combining unit root and KPSS test results, we conclude that returns of stock and stock index in the Hungarian stock market are stationary (d = 0).

Table 4: Results of KPSS and other tests of stationarity for Hungarian stock market return series

Name of a stock/ stock index	KPSS test (a constant)	PP test (a constant)	ADF test (a constant)
ANY	0.290 (14)	-41.032*** (9)	-30.516*** (L=1)
Egis	0.063 (10)	-58.682*** (10)	- 58.677*** (L=0)
Fotex	0.068 (16)	-59.345*** (16)	- 59.112*** (L=0)
MOL	0.075 (28)	-57.222*** (29)	- 57.189*** (L=0)
MTelekom	0.121 (15)	-57.344*** (16)	- 57.318*** (L=0)
OTP	0.043 (6)	-55.864*** (8	- 36.396*** (L=2)
Pannergy	0.262 (5)	-59.381*** (5)	- 59.380*** (L=0)
Raba	0.099 (13)	-56.801*** (13)	-56.783*** (L=0)
Richte	0.037 (27)	-55.668*** (28)	-35.837*** (L=2)
Synergon	0.190 (11)	-50.456*** (9)	-50.287*** (L=0)
TVK	0.104 (14)	-61.303*** (13)	-61.289*** (L=0)
BUX (index)	0.063 (12)	-56.585*** (13)	-56.642*** (L=0)

Notes: See notes for Table 3

Source: Own calculations

The Lo's (1991) modified R/S statistics for the Croatian stock (market) return time series indicates a possible long memory in the Crobex returns and the returns of Adris Grupa, whereas there is no evidence against the null hypothesis of short-term memory in the Hungarian stock (market) returns (Tables 5 and 6).

Table 5: Results of Lo's R/S analysis for Croatian stock market

Name of a stock/ stock index	Lo's R/S statistics
Adris Grupa	1.83* (lag=5)
Dalekovod	1.66 (lag=0)
Ericsson Nikola Tesla	1.48 (lag=0)
Hrvatski Telekom	1.47 (lag=1)
Ina	1.64 (lag=3)
Končar	1.05 (lag=0)
Kraš	1.43 (lag=0)
Podravka	1.1 (lag=0)
Zagrebačka Banka	1.16 (lag =0)
Crobex (index)	2.00** (lag=0)

Notes: The lag parameter used for the modified R/S test is determined by Andrew's (1991) data-dependent rule and is indicated in parentheses. According to Lo (1991), the critical values for rejection of the null hypothesis (that a return time series is not a long memory process) at 5 % significance level are [0.809, 1.862]. The return time series for which the null of no long memory can be rejected at the 5% significance level are indicated by **, and for the 10% level by *. The Lo's R/S statistics were calculated with Stata

Source: Own calculations

Table 6: Results of Lo's R/S analysis for Hungarian stock market

Name of a stock/ stock index	Lo's R/S statistics
ANY	1.41 (lag=0)
Egis	1.36 (lag=1)
Fotex	1.09 (lag=0)
MOL	1.12 (lag=2)
MTelekom	1.28 (lag=1)
OTP	1.54 (lag=4)
Pannergy	1.56 (lag=0)
Raba	1.33 (lag=1)
Richte	1.23 (lag=4)
Synergon	1.71 (lag=5)
TVK	1.01 (lag=0)
BUX (index)	1.18 (lag=3)

Notes: See notes for Table 5

Source: Own calculations

Next, we apply the WOLS estimator of Jensen (1999) to estimate the fractional integration parameter *d*. As in studies of Jensen (1999) and Tkacz (2001), we choose to use three representative Daubechies and the Haar wavelet functions to verify the robustness of our results to different degrees of smoothing. The Daubechies-20 wavelet is the smoothest wavelet used, followed by the Daubechies-12 and the Daubechies-4, with the Haar wavelet being the least smooth of all.

The WOLS estimates provide evidence of long memory presence in return series of Adris Grupa, Atlantska plovidba, Hrvatski Telekom, Končar, Podravka and for the stock index Crobex (Table 7).

Table 7: The WOLS estimates of the parameter *d* for the Croatian stock market return series

Name of a stock/ stock index	Haar	Daubechies-4	Daubechies-12	Daubechies-20
Adris Grupa	0.1089*** (0.0274)	0.0478 (0.0454)	0.0574 (0.0423)	0.0696* (0.0374)
Atlantska plovidba	0.0198 (0.0429)	0.0088 (0.0409)	0.0468* (0.0246)	0.0573*** (0.0202)
Dalekovod	-0.0344 (0.0382)	-0.0677 (0.0850)	0.0006 (0.0450)	0.0252 (0.0251)
Ericsson Nikola Tesla	-0.0495 (0.0739)	0.1024* (0.0563)	0.0612 (0.0578)	0.0301 (0.0562)
Hrvatski Telekom	0.1027 ** (0.0430)	-0.0205 (0.0653)	-0.0138 (0.0411)	-0.0019 (0.0328)
Ina	0.0954* (0.0569)	-0.0041 (0.1212)	0.0144 (0.0945)	0.0622 (0.0617)
Končar	0.1209*** (0.0321)	0.1047*** (0.0370)	0.1029*** (0.0325)	0.1057*** (0.0336)
Kraš	0.0190 (0.0474)	-0.0186 (0.0485)	-0.0057 (0.0497)	0.0107 (0.0529)
Podravka	0.0706* (0.0414)	0.0541 (0.0381)	0.0458* (0.0269)	0.0542** (0.0270)
Zagrebačka Banka	0.0620 (0.0448)	0.0304 (0.0581)	0.0307 (0.0480)	0.0390 (0.0447)
Crobex (index)	0.1408*** (0.0217)	0.1260*** (0.0364)	0.1346*** (0.0265)	0.1389*** (0.0244)

Notes: To obtain the WOLS estimates of parameter *d*, time series of indices returns have to be of dyadic length, as the WOLS estimator is based on the discrete wavelet transform. The last 2048 (or shorter time period of 1024 or 612 if the time series of stock returns is shorter) days of the return time series are taken in the estimation of the parameter *d*. In parentheses standard errors of parameter *d* are presented. Significance levels for rejection of the null hypothesis that the time series is stationary are denoted by: *** for a 1% significance level ** for a 5% significance level and * for a 10% significance level

Source: Own calculations

The strongest evidence of long memory, insensitive to wavelet function, is provided for the returns of Končar and stock index Crobex. An important finding is that the test of long memory based on the WOLS estimator may lead to different conclusions, regarding long memory presence in stock returns, than KPSS and unit root tests or Lo's modified R/S test. Given the strong evidence of efficiency of WOLS test (see Walden, 2000;, Tkacz, 2001; Gencay et al., 2002; Vuorenmaa, 2004), we give more weight to the results of WOLS test.

For the Hungarian stock market, the long memory presence is found in the return series of the following shares: Egis, MTelekom, OTP, Pannergy, Raba, Richte and Synergon (Table 8). The strongest evidence in support of long memory was found for the shares of MTelekom, Synergon and Richte shares. No fractal structure is identified for the returns of stock market index BUX.

Table 8: The WOLS estimates of the parameter *d* for the Hungarian stock market return series

Name of a stock/ stock index	Haar	Daubechies-4	Daubechies-12	Daubechies-20
ANY	-0.0458 (0.0512)	-0.0486 (0.0780)	-0.0484 (0.0893)	0.0104 (0.0439)
Egis	0.0342** (0.0154)	-0.0463 (0.0549)	-0.0227 (0.0350)	0.0091 (0.0146)
Fotex	0.0478 (0.0302)	0.0304 (0.0352)	0.0264 (0.0386)	0.0325 (0.0428)
MOL	0.0137 (0.0324)	-0.0852 (0.0648)	-0.0334 (0.0423)	0.0087 (0.0273)
MTelekom	-0.0483** (0.0200)	-0.1373*** (0.0355)	-0.1363*** (0.0267)	-0.1096*** (0.0193)
OTP	0.0550** (0.0271)	-0.0426 (0.0492)	-0.0393 (0.0508)	0.0104 (0.0269)
Pannergy	0.0397*** (0.0136)	0.0398 (0.0491)	0.0544 (0.0500)	0.0643* (0.0362)
Raba	0.0367 (0.0217)	0.0740** (0.0325)	0.0699** (0.0331)	0.0597 (0.0419)
Richte	-0.0313* (0.0189)	-0.2627** (0.1246)	-0.1871** (0.0867)	-0.0843** (0.0353)
Synergon	0.0521** (0.0213)	0.0454 (0.0341)	0.0633** (0.0246)	0.0683*** (0.0257)
TVK	0.0417 (0.0274)	0.0326 (0.0369)	0.0264 (0.0364)	0.0451 (0.0322)
BUX (index)	0.0521 (0.0282)	-0.0361 (0.0472)	-0.0204 (0.0407)	0.0232 (0.0216)

Notes: See notes for Table 7

Source: Own calculations

We can confirm the findings of Jagric et al. (2006), who found long memory in returns of the Croatian stock index returns. However, regarding Hungarian stock market, our results are different as we find strong evidence against long memory in the BUX returns. As already noted, the differences in findings may be due to different methods used and the different time period of observation.

Our study complements the existent empirical literature on long memory in the Hungarian and Croatian stock market by analyzing individual stocks listed in national stock markets. Evidence is provided that, as firstly argued by Barkoulas and Baum (1996), the long memory of individual stocks may be masked in the aggregated stock market returns (i.e. returns of the stock index). For investors that are considering only individual stocks in the investigated stock markets, it is therefore reasonable to investigate for a possible long memory in returns of the individual stocks.

Long memory, found in the returns of Croatian stock market, implies that the stock returns follow a predictable behavior, which is inconsistent with the weak-from market efficiency hypothesis. The results of long memory tests do not reject the weak-form efficiency hypothesis for the Hungarian stock market.

6. Conclusion

Based on the presented results, the hypothesis of no long memory in Croatian stock market index (Crobex) returns and returns of individual stocks in the index could be rejected. The hypothesis of no long memory could also be rejected for returns of some individual stocks in the Hungarian stock market, but not for the BUX returns. The results implicate that the stock returns in Croatian stock market follow a predictable behavior, which is inconsistent with the weak-from market efficiency hypothesis, while no such conclusion could be derived for the Hungarian stock market. The existent empirical literature on long memory in stock market returns in Hungarian and especially Croatian stock market is scarce. Our results confirm the findings of the existent literature that the returns of Croatian stock market exhibit long memory. However, our results contrast regarding Hungarian stock market as we find strong evidence against long memory in the BUX returns. The differences in findings may be due to different methods used and the different time period of observation. The present paper is the first in the empirical literature that has applied the most efficient method (i.e. the WOLS method) of estimating the fractal structure of stock returns in Central and Eastern Europe.

Our study complements the existent empirical literature on long memory in the Hungarian and Croatian stock market by analyzing individual stocks listed in national stock markets. The research proved that the fractal structure of individual stock returns may be masked in aggregated stock market returns (i.e. in returns of stock index). Therefore, it is advisable for individual investors in the investigated stock markets to examine fractal structure not only for the stock market index but also for the individual stocks they invest in. One of the findings of the study is that different long memory tests may lead to different results. Furthermore, fractal structure of stock (market) returns may be time-varying, i.e. changing through time. This might be another reason why the results of the different studies lead to different conclusion and leaves open space for future research of long memory in Croatian and Hungarian stock market.

The study has some limitations. Firstly, only two methods of testing the long memory are applied. According to existent literature the results may be dependent on the method chosen. The future research that would provide evidence on the fractal structure of the Croatian and the Hungarian stock markets returns should additionally apply other long memory tests. Secondly, the results are static in sense that they only test long memory in stock (indices) returns for the whole observed period. The fractal structure of the time series may, however, change through time. The future research should therefore develop and apply the dynamic versions of the long memory tests as well.

The main implication of the research findings are for the financial investors and risk managers operating with investments in the two investigated stock markets. According to the literature, the finding of long memory presence in Croatian stock market returns renders the application of CAPM or APT in Croatian stock invalid. Problems additionally arise in the pricing of derivative securities with martingale methods, since the continuous time stochastic processes most commonly employed are inconsistent with long memory.

References

Andrews, T. (1991) "Heteroskedasticity and autocorrelation consistent covariance matrix estimation. *Econometric*a", Vol. 59, No. 3, pp. 817-858.

Assaf, A., Cavalcante, J. (2002) "*Long-range Dependence in the Returns and Volatility of the Brazilian Stock Market*". [Internet]. Available from: <http://www.long-memory.com/volatility/CavalcanteAssaf2002.pdf> [Accessed March 20, 2011].

Baillie, T.R. (1996) "Long memory processes and fractional integration in economics", *Journal of Econometrics*, Vol.73, No.1, pp. 5-59.

Barkoulas, T.J., Baum, C.F. (1996) "Long Term Dependence in Stock Returns", *Economics Letters*, Vol. 53, No. 3, pp. 253–259.

Barkoulas, T.J., Baum, C.F., Travlos, N. (2000a) "Long memory in the Greek stock market", *Applied Financial Economics*, Vol. 10, No. 2, pp. 177-84.

Barkoulas, T.J., Baum, C.F., Oguz, G. S. (1998) “Stochastic Long Memory in traded goods prices”, *Applied Economics Letters*, Vol. 5, No. 3, pp. 135-138.

Bilel, T.M., Nadhem, S. (2009) “Long Memory in Stock Returns: Evidence of G7 Stocks Markets”, *Research Journal of International Studies*, January, Issue No. 9, pp. 36-46.

Breidt, F.J., Crato, N., de Lima, P. (1998) “The Detection and Estimation of Long Memory in Stochastic Volatility”, *Journal of Econometrics*, Vol. 83, No. 1-2, pp. 325-348.

Cajueiro, O.D., Tabak, B.M. (2004) “Evidence of long range dependence in Asian equity markets: the role of liquidity and market restrictions”, *Physica A,*Vol. 342, No. 3-4, pp. 656 – 664.

Chan, H.W., Feng, L. (2008), “*Extreme News Events, Long-Memory Volatility, and Time Varying Risk Premia in Stock Market Returns*”. [Internet]. Available from: <http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1183173> [Accessed April 2, 2011].

Chow, K.V., Ming-Shium, P., Sakano, R. (1996) “On the long-term or short-term dependence in stock prices: Evidence from international stock markets”, *Review of Quantitative Finance and Accounting*, Vol. 6, No. 2, pp. 181-194.

Dark, J. (2007) “*Estimation of the fractional differencing parameter using wavelets and temporal aggregation*”. [Internet]. Available from: <http://69.175.2.130/~finman/Barcelona/Papers/wavelets.pdf> [Accessed March 28, 2011].

Dima, B., Miloş, L.R. (2009) “Testing the efficiency market hypothesis for the Romanian stock market”, *Annales Universitatis Apulensis Series Oeconomica*, Vol. 11, No. 1, pp. 402-415.

Ding, Z., *Granger, C.W.J.*, *Engle, R.F. (*1993) “A long memory property of stock returns and a new model”, *Journal of Empirical Finance*,Vol. 1, No. 1, pp. 83–106.

DiSario, R., Saraoglu, H., McCarthy, J., Li, H.C. (2008) “An investigation of long memory in various measures of stock market volatility, using wavelets and aggregate series”, *Journal of Economics and* Finance, Vol. 32, No. 2, pp. 136-147.

Eitelman, P. S., Vitanza, J. T. (2008) “*A Non-Random Walk Revisited: Short- and Long memory in Asset Prices*”, Board of Governors of the Federal Reserve System International Finance Discussion Papers No. 956. Internet]. Available from: < http://www.federalreserve.gov/pubs/ifdp/2008/956/ifdp956.pdf> [Accessed May 28, 2011].

Fama, E. (1970) “Efficient Capital Markets: A Review of Theory and Empirical Work”, *Journal of Finance*, Vol. 25, No. 2, pp. 383-417.

Fang, H., Lai, K.S., Lai, M. (1994) “Fractal Structure in Currency Futures Price Dynamics”, *The Journal of Futures Markets*, Vol. 14, No. 2, pp. 169-181.

Geweke, J., Porter-Hudak, S. (1983) "The Estimation and Application of Long Memory Time Series Models", *Journal of Time Series Analysis,* Vol. 4, No. 4, pp. 221-238.

Granger, C.W.J., Joyeux, R. (1980) "An introduction to long-memory time series models and fractional differencing", *Journal of Time Series Analysis* No. 1, pp. 15-39.

Granger, C.W.J., Ding, Z. (1995) "Some properties of absolute value return an alternative measure of risk", *Annales d'Economie et de Statistique*, No. 40, pp. 67-91.

Grau-Carles, P. (2005) "Tests of Long Memory: A Bootstrap Approach", Computational conomics, 25(1-2): 103-113

Harmantzis, C.F., Nakahara, J.S. (2006) "*Evidence of Persistence in the London Interbank Offer Rate*", The Financial Management Association Working Paper. [Internet]. Available from: <Available: http://www.fma.org/Barcelona/Papers/Libor.pdf> [Accessed March 28, 2011].

Henry, Ó.T. (2002) "Long memory in stock returns: some international evidence", *Applied Financial Economics*, Vol. 12, No. 10, pp. 725-729

Higuchi, T. (1988) "Approach to an irregular time series on the basis of fractal theory", *Physica D*, Vol. 31, pp. 277-283.

Hosking, J.R.M. (1981) "Fractional differencing", *Biometrika,* Vol 68, No. 1, pp. 165-176.

Hurst, H.E. (1951) "Long-term storage capacity of reservoirs", *Transactions of the American Society of Civil Engineers*, Vol. 116, pp. 770-799.

Limam, I. (2003) "Is Long Memory a Property of Thin Stock Markets? International Evidence Using Arab Countries", *Review of Middle East Economics and Finance*, Vol. 1, No. 3, pp. 251-266.

Jagric, T., Podobnik, B., Kolanovic, M. (2005) "Does the efiicient market hypothesis hold? Evidence from six transition economies", *Eastern European Economics*, Vol. 43, No. 4, pp. 79-103.

Jagric T, Rodobnik B, Kolanovic M, Jagric V (2006) "Modelling Some Properties of Stock Markets in Transition Economics", *Journal of Economics*, Vol. 54, No. 8, pp. 816-829.

Jensen, J.M. (1999) "Using Wavelets to Obtain a Consistent Ordinary Least Squares Estimator of the Long-memory Parameter", *Journal of Forecasting*, Vol. 18, pp. 17-32.

Jin, J.H., Elder, J., Koo, W.W. (2006) "A reexamination of fractional integrating dynamics in foreign currency markets", *International Review of Economics & Finance*, Vol. 15, No. 1, pp. 120-135.

LeRoy, S. (1989) "Efficient capital markets and martingales", *Journal of Economic Literature*, Vol. 27, No. 4, pp. 1583-1621.

Lo, W. A. (1991) "Long term memory in stock market prices", *Econometrica*, Vol. 59, No. 5, pp. 1279-1313.

Lo, W.A., MacKinlay, C.A. (2001) "*A Non-Random Walk Down Wall Street*". New Jersey: Princeton University Press.

Lux, T. (1996) "Long-term stochastic dependence in financial prices: evidence from the German stock market", *Applied Economics Letters*, Vol. 3, No. 11, pp. 701-706.

Karuppiah, J., Los, A.C. (2005) "Wavelet multiresolution analysis of high-frequency Asian FX rates", *International Review of Financial Analysis*, Vol. 14, No. 2, pp. 211–246.

Kasman, A., Kasman, S., Torun, E. (2009a) "Dual long memory property in returns and volatility: Evidence from the CEE countries' stock markets", *Emerging Markets Review*, Vol. 10, No. 2, pp. 122-139.

Kasman, S., Turgutlu, E., Ayhan, A.D. (2009b) "Long memory in stock returns: evidence from the major emerging Central European stock markets", *Applied Economics Letters*, Vol. 16, No. 17, pp. 1763-1768.

Künsch, H.R. (1987) "Statistical aspects of self-similar processes". In: Prohorov, Y., Sazonov, V.V. (ed), *Proc. 1st World Congress Bernoulli Soc.* 1, Utrecht: VNU Science Press, pp. 67—74

Kwiatkowski, D., Phillips, P.C.B., Schmidt, P., Shin, Y. (1992) "Testing the Null Hypothesis of Stationarity against the Alternative of a Unit Root", *Journal of Econometrics*, Vol. 54, No. 1-3, pp. 159-178.

Lee, D., Schmidt, P. (1996) "On the Power of the KPSS Test of Stationarity against Fractionally Integrated Alternatives", *Journal of Econometrics*, Vol. 72, No. 1, pp. 285-302.

Lobato, N.I., Savin, N.E. (1998) "Real and spurious long memory in stock market data", *Journal of Business & Economic Statistics*, Vol. 16, No. 3, pp. 261–268.

Lobato, N.I., Velasco, C. (2000) "Long memory in stock market trading volume", *Journal of Business & Economic Statistics*, Vol. 18, No. 4, pp. 410–426.

Maheswaran, S., Sims, C.A. (1992), "*Empirical Implications of Arbitrage Free Asset Markets*", Cowles Foundation Discussion paper No. 1008.

Mandelbrot, B.B. (1965) "*Une classe de processus stochastiques homothetiques a soi; application a loi climatologique de H.E. Hurst* ", C R Acad Sci, Paris 240, pp. 3274–3277.

Mandelbrot, B.B. (1971) "When can price be arbitraged efficiently? A limit to the validity of the random walk and martingale models", *Review of Economics and Statatistics*, Vol. 53, No. 3, pp. 225–236.

Mandelbrot, B.B., van Ness, J.W. (1968) "Fractional brownian motions, fractional noises and applications, Parts 1, 2, 3". *SIAM Review,* Vol. 10, No. 4, pp. 422–437.

Mariani, M.C., Florescub, I., Beccar Varelaa, M., Ncheuguim, E. (2010), "Study of memory effects in international market indices", *Physica A: Statistical Mechanics and its Applications*, Volume 389, No. 8, pp. 1653-1664.

McCarthy, J., DiSario, R., Saraoglu, H., Li, H. (2004) "Tests of long-range dependence in interest rates using wavelets", *The Quarterly Review of Economics and Finance*, Vol. 44, No. 1, pp. 180–189.

Oh, G., Cheol-Jun, U., Kim, S. (2006) "*Long memory and Volatility Clustering in Daily and High-frequency Price Changes*" Quantitative Finance Papers, Physics No. 0601174. [Internet]. Available from: <http://arxiv.org/PS_cache/physics/pdf/0601/0601174v2.pdf> [Accessed April 10, 2011].

Ozdemir, Z.A. (2009) "Linkages between international stock markets: A multivariate long memory Approach", *Physica A: Statistical Mechanics and its Applications*, Vol. 388, No. 12, pp.: 2461-2468

Palma, W.(2007) "*Long-Memory Time Series. Theory and Methods*", Hoboken, New Jersey: John Wiley & Sons.

Pons Fanals, E., Suriñach Caralt, J. (2002) "*An analysis of inflation rates in the european union using wavelets: strong evidence against unit roots*", Documents de Treball de la Divisió de Ciències Jurídiques Econòmiques i Socials. [Internet]. Available from: <http://www.ere.ub.es/dtreball/E0281.rdf/at_download/file > [Accessed March 18, 2011].

Ray, B., Tsay, R. (2000) "Long-range dependence in daily stock volatilities", *Journal of Business & Economic Statistics*, Vol. 18, No. 2, pp. 254–262.

Rea, W., Reale, M.., Brown, J. (2007), "Estimators for Long Range Dependence: An Empirical Study", New Zealand Econometric Study Group Paper. [Internet]. Available from: < http://yoda.eco.auckland.ac.nz/nzesg/PDFs/paper/Rea.pdf> [Accessed March 19) 2011].

Robinson, P.M. (1995) "Gaussian semiparametric estimation of long range dependence", *Annals of Statistics,* Vol. 23, No. 5, pp. 1630-1661.

Sadique, S., Silvapulle, P. (2001) "Long memory in stock market returns: International evidence. *International Journal of Finance and Economics*, Vol. 6, No. 1, pp. 59-67.

Sibbertsen, P. (2004), "Long memory in volatilities of German stock returns" *Empirical Economics,* Vol. 29, No. 3, pp. 477-488.

Sims, A.C. (1984) "*Martingale-like behavior of asset prices and interest rates*" Department of Economics, University of Minnesota Discussion paper 205.

Sowell, F. (1992) "Maximum Likelihood Estimation of Stationary Univariate Fractionally Integrated Time Series Models", *Journal of Econometrics*, Vol. 53, No. 1-3, pp. 165-188.

Tkacz, G. (2001) "Estimating the Fractional Order of Integration of Interest rates Using a Wavelet OLS Estimator" Bank of Canada Working paper No. 2000-5. [Internet]. Available from: <http://www.bankofcanada.ca/en/res/wp/2000/wp00-5.pdf>[Accessed March 29) 2011].

Tolvi, J. (2003) "Long memory and outliers in stock market returns" *Applied Financial Economics*, Vol. 13, No. 7, pp. 495-502.

Teverovsky, V, Taqqu, M. S., Willinger, W. (1999) "A critical look at Lo´smodified R/S statistics", *Journal of Statistical Planning and Inference*, Vol. 80, No. 1, pp. 211-227.

Žiković, S. (2008) "Quantifying extreme risks in stock markets: A case of former Yugoslavian states", *Zbornik radova Ekonomskog fakulteta u Rijeci*, Vol. 26, No., pp. 41-68.

Žiković, S., Atkan B. (2009) "Global financial crisis and VaR performance in emerging markets: A case of EU candidate states - Turkey and Croatia", *Zbornik radova Ekonomskog fakulteta u Rijeci*, Vol. 27, No. 1, pp. 149-170.

Dugoročna memorija u prinosima hrvatskog i madžarskog dioničkog tržišta

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Sažetak

U ovom radu analizira se dugoročna memorija prinosa hrvatskog i madžarskog dioničkog tržišta. Prisutnost dugoročne memorije u prinosima dokaz je neučinkovitosti dioničkog tržišta. Pod pretpostavkom da je moguće prinose modelirati kao ARFIMA (engl. Autoregressive Fractionally Integrated Moving Average) procese, aplicirani su Kwiatkowski-Phillips-Schmidt-Shin (KPSS) (1992) test i Jensenova (1999) valna metoda klasičnih najmanjih kvadrata (engl. wavelet ordinary least squares – WOLS) kako bi se dobila ocjena parametra integriranosti prinosa dioničkih tržišta. Rezultati ove studije ukazuju na to da WOLS, KPSS i R/S metoda vode do različitih konstatacija o dugoročnoj memoriji u prinosima dioničkog tržišta. Nadalje, utvrđeno je da dugoročna memorija u prinosima pojedinačnih dionica može biti ″zamaskirana″ u agregatnim prinosima dioničkog indeksa, koji uključuje ove dionice. Stoga je za investitore bitno da istovremeno testiraju i potencijalnu prisutnost dugoročne memorije u prinosima indeksa dioničkog tržišta i pojedinačne dionice u koje investiranju. Ključni rezultat studije je dokaz o dugoročnoj memoriji u prinosima hrvatskog dioničkog indeksa Crobex i pojedinačnih dionica u indeksu. Dugoročna memorija identificirana je i za pojedinačne dionice na madžarskom dioničkom tržištu, ali ne i za sam indeks BUX. Na temelju rezultata testova dugoročne memorije, odbačena je hipoteza slabe tržišne učinkovitosti za hrvatsko, ali ne i za madžarsko dioničko tržište.

Ključne riječi: *dioničko tržište, dugoročna memorija, hipoteza učinkovitog tržišta*

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