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CONSIDERATION OF A MOVING MASS EFFECT ON DYNAMIC BEHAVIOUR OF A JIB CRANE STRUCTURE

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Original scientific paper

This work examines the dynamics of a two-dimensional jib crane structure subjected to a moving trolley with hoist and payload. Dynamic responses of the structure, both in the vertical (Y) and horizontal direction (X), are calculated using the finite element method and the direct integration method. Instead of the conventional moving force problem, this paper deals with the two-dimensional inertial effects due to the masses of trolley, hoist and payload. For this purpose, the *moving mass matrix* has been used to give contribution to the overall mass matrix of the entire system. The title problem was solved by calculating the forced vibration responses of the jib crane structure with time-dependent overall mass while subjected to an equivalent moving force. Factors as magnitude, speed and acceleration of the moving trolley were studied as well. Numerical results reveal that the approach used herein is usefull and can be used to draw conclusions for the structural design purposes of jib cranes.

Keywords: *dynamics of structures, finite element model, jib crane, moving mass*

Analiza utjecaja pokretne mase na dinamičko ponašanje konstrukcije stupne konzolne dizalice

Izvorni znanstveni članak

U radu se razmatra dinamičko ponašanje ravninske konstrukcije stupne konzolne dizalice izložene djelovanju gibanja kolica elektromotornog vitla koje nosi teret. Dinamički odzivi konstrukcije, u vertikalnom i u horizontalnom pravcu, dobiveni su uporabom metode konačnih elemenata i metode direktne integracije. Umjesto konvencionalnog pristupa uporabe modela pokretne sile, u ovom su radu obuhvaćeni utjecaji inercije masa kolica, vitla i tereta. Radi toga, koncept matrice pokretne mase je implementiran u matrici masa cijelog sustava. Određene su prinudne oscilacije konstrukcije stupne konzolne dizalice zbog djelovanja ekvivalentnog pokretnog opterećenja pri čemu je matrica masa sustava promjenjiva u vremenu. Razmatran je utjecaj intenziteta, brzine i ubrzanja pokretnog opterećenja. Rezultati daju korisne zaključke za konstruiranje konzolnih dizalica.

Ključne reči: *dinamika skonstrukcija, metoda konačnih elemenata, pokretna masa, stupna konzolna dizalica*

1 Introduction

The moving load problem is considered as a special topic in structural dynamics. Structures subjected to moving bodies have been analysed ever since the first railway bridges were built in the early 19th century. Since then, the moving load problem has become more dynamic in character mainly due to increased vehicle speeds and structural flexibility. A large number of studies dealing with the moving load problem refer to the excellent monograph by Fryba [1].

Irrespective of many viewpoints and analytical methods proposing to solve the dynamic problems, the majority of research can be grouped into two categories: the moving force problem and the moving mass problem. The additional approach is moving oscillator model which is only reasonable to be used in some special structures because of its complexity. The basic understanding of the moving force phenomena is given in [2]. In most moving force models the magnitude of the contact force is constant in time which implies that the inertia forces of a moving body are neglected. Even so, the moving force models are simple to use and yield reasonable structural results in some cases [3].

The moving mass problem implies the existence of an interaction force between the moving mass and the structure during the time the mass travels along the structure, to which the following factors contribute: the inertia of the mass, the centrifugal force, the Coriolis force and the time-varying speed-dependent forces. Hence, the speed of the moving mass, structural flexibility and the ratio of the moving mass and structure mass are important factors that contribute to the creation of the interaction force. Michaltsos et al. [4] have studied the effect of a moving mass and other parameters, such as magnitude and speed of the moving mass, on the dynamic response of a simply supported beam. Akin and Mofid [5] presented an analytical-numerical method that can be used to determine the dynamic behaviour of a fixed-free beam carrying a moving mass and showed the importance of the moving mass problem in the responses of structures.

Typical structures under a moving load in mechanical engineering are bridge cranes, jib cranes, gantry cranes, elevators, cableways, guideways, shipunloaders and container cranes. The modelling and simulation of these machines dynamic behaviour has an important role in the early phases of design [6, 7]. Also, dynamic interaction between the structure and the moving object should be properly considered in order to obtain the dynamic amplification factors for deflection as well as the bending moment values [8, 9].

The research [10] is according to the author's best knowledge the first attempt to increase the understanding of the dynamics of bridge cranes due to the moving load. Oguamanam et al. [11] studied the vibrations of a two-member open frame structure with tip mass (payload) on the second beam which is related to the dynamics of the L-shaped structure. Jerman [12, 13] studied the dynamics of the slewing cranes using the non-linear mathematical model along with the verification derived from measurements of the physical model of the crane. The vibrations of frame structures in comparison to standard codes are investigated in [14]. Paper [15] presents a technique developed for using standard finite element packages for analysing the dynamic response of gantry crane structures to time-variant moving loads. Paper [16] analyzes three-dimensional responses of a gantry crane structure due to moving loads with introduction of the moving mass matrix concept. As it is conclusive from the last two references considerations of moving load problem by using and developing FEM is actual because modern FEM packages are not suited for the moving load problem, especially when the structure-vehicle interaction is to be considered.

This paper studies the dynamic responses of a jib crane structure subjected to the moving mass. It includes the following contributions: (i) Both the horizontal and the vertical response of a jib crane structure under a moving load are investigated; (ii) The moving mass problem and moving mass matrix are applied for the first time at the vibrations of the jib crane structure; (iii) Results can be used for verification of dynamic responses at L-shaped structures.

2 Model description

Jib cranes are widely used in industrial facilities all over the world. A typical jib crane consists of a top beam which is rotating around a fixed column. This configuration may be reffered to as an L-shaped structure. The top beam is attached to the column at two points, directly on top and with down support. The trolley, with the hoist and payload, is moving along the top beam, Fig. 1(a). Because the rotating speed of the top beam is usually very low and constant, the vibration components due to this motion are assumed negligible in this paper. In such a case, the dynamic behaviour of the crane structure may be predicted with two-dimensional model. In normal usage, the separation between the moving load and the loaded structure is prevented and it is reasonable to assume here that the moving trolley is always in contact with the crane structure.

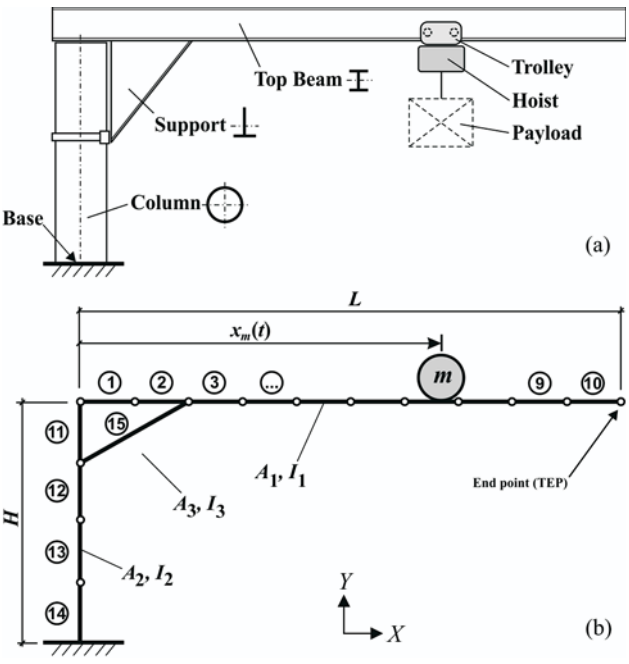

Figure 1 (a) Sketch for a column jib crane, (b) finite element model of a crane structure with moving mass

The moving trolley, hoist and payload are modelled as a moving lumped mass (m). It is assumed that the moving mass is travelling from the left end of the top beam with position defined by coordinate $x_m(t)$. The finite element model of the jib crane structure i.e. framework, is shown in Fig. 1(b). The top beam (with length L) is composed of 10 identical frame elements with cross section A_1, and sectional moment of inertia I_1. The column (with height H) is composed of 4 identical elements with properties A_2, I_2 and the support is presented with one element with properties A_3, I_3. All the frame elements are made of steel with mass density ρ and Young's modulus E. The whole system has (14×3=) 42 DOF's.

In the following, the formulation for the moving mass matrix associated with 2-D inertial effects of the moving trolley with hoist and payload is presented. Then, the overall property matrices and the equivalent force vector for the framework of the crane are determined, and the title problem is solved by means of the finite element model and direct integration method.

3 Overall property matrices and formulation of the problem

The equation of motion for the multiple degree of freedom undamped structural system is given by

$$[M]\{\ddot{q}\}+[K]\{q\}=\{F(t)\} \tag{1}$$

where $[M]$ is the overall mass matrix, $[K]$ is the overall stiffness matrix; $\{\ddot{q}\},\{q\}$ are acceleration and displacement vectors for the whole system respectively, while $\{F(t)\}$ is the external force vector. Matrix $[M]$ is called the instantaneous overall mass matrix because it is a time-dependent matrix composed of the constant mass matrix due to the crane structure itself and the time-dependent moving mass matrix.

3.1 Moving mass matrix

The element mass matrix associated with the moving mass m is called the moving mass matrix because the location of the mass is time-dependent. Its contribution to the overall mass matrix of the crane structure is therefore also time-dependent. When the moving mass m is located at the (s)th beam element of the structure (Fig. 2), then the moving mass matrix takes the form

$$[m]=[m_{ij}(t)]_{6\times 6}. \tag{2}$$

Since the title problem is two-dimensional, the beam element is a planar frame element and the order of the mass matrix for the (s)th beam element is 6×6. Moreover, the moving mass is a concentrated mass (without mass moment of inertia) and the associated moving mass matrix is diagonal with all the non-diagonal components being equal to zero. Among the diagonal components, those associated to the mass moment of inertia are equal to zero

$$m_{ii}(t)=0 \qquad \text{(for } i=3 \text{ and } 6) \tag{3}$$

while the non-zero ones (associated with the lumped mass m) are determined by

$$m_{ii}(t)=mN_5^s(t) \quad \text{(for } i=1 \text{ and } 2) \tag{4}$$

$$m_{ii}(t)=mN_6^s(t) \quad \text{(for } i=4 \text{ and } 5) \tag{5}$$

where superscript "s" denotes the numbering of the beam element at which the lumped mass m is located at time t (Fig. 2) and linear interpolating functions are defined by [17]

$$N_5^s(t)=1-x(t)/l \tag{6}$$

$$N_6^s(t) = x(t)/l \tag{7}$$

noting that l is the length of the (s)th beam element of the top beam and $x(t)$ is distance between the location of the lumped mass, at time t, and the left end of the element.

Thus, this concept takes into account the two-dimensional inertial effects on the dynamic responses of the crane structure due to the moving mass.

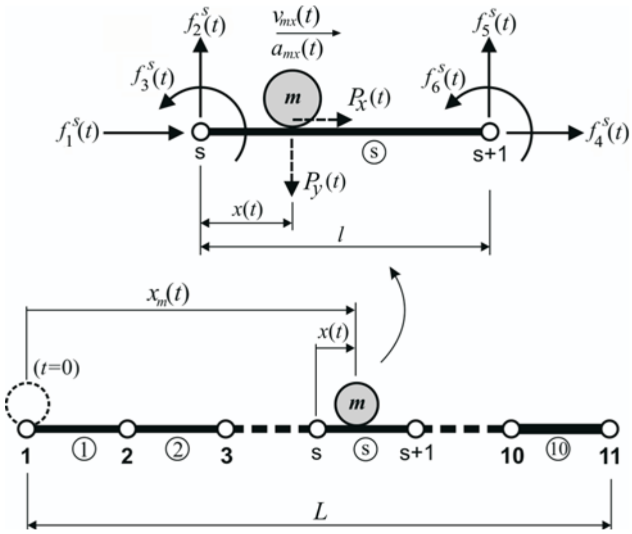

Figure 2 Equivalent nodal forces of the (s)th beam element subjected to the moving mass

3.2 Overall stiffness and mass matrices

The overall stiffness matrix for the postulated system is given by

$$[K] = [Kst]_{42\times 42} \tag{8}$$

where $[Kst]_{42\times42}$ represents the stiffness matrix of the framework itself and can be obtained by assembling all the element stiffness matrices [18].

In order to take inertial effects of the moving load into consideration, one must add the contribution of the moving mass matrix due to moving mass m, to the overall mass matrix of the framework itself [Mst] so as to construct the instantaneous overall mass matrix [M] i.e.

$$[M] = [Mst] + [m]. \tag{9}$$

The overall mass matrix of the framework itself $[Mst]_{42x42}$ is obtained by assembling all the element mass matrices of the framework [18]. In order to execute the Equation (9) one must augment the moving mass matrix $[m]_{6\times6}$ by adding zero rows and columns to the order of 42×42 except for the 6 degrees of freedom of the 2 nodes of the (s)th beam element at which the equivalent mass m is located at time t. Hence,

$$M_{ij} = Mst_{ij}\text{, for } i,j = 1\div42 \tag{10}$$

except for the following diagonal components

$$M_{s_i s_i} = Mst_{s_i s_i} + m_{ii}(t)\text{, } i=1\div6 \tag{11}$$

where subscript s_i, i=1÷6, respectively represents the numbering for the 6 DOF's of the two nodes of the (s)th beam element at which the moving mass is positioned.

3.3 Equivalent nodal forces and external force vector

Force vector $P(t)$ induced by the moving lumped mass at any time t is given by

$$\vec{P}(t) = \vec{i}\,P_x(t) + \vec{j}P_y(t) \tag{12}$$

where $\vec{i}$, $\vec{j}$ are, respectively, unit vectors in the X, Y directions, while the corresponding force components are given by

$$P_x(t) = ma_{mx}(t) \tag{13}$$

$$P_y(t) = mg\,. \tag{14}$$

In Equation (13), $a_{mx}(t)$ represents the acceleration of the lumped mass at any instant of time and g is gravitational acceleration. The force vector, because of its components, changes location on the framework from time to time. For the convenience of the finite element analysis, it is replaced by an equivalent nodal force vector

$$\{f^s(t)\} = \{f_1^s(t)\ f_2^s(t)\ f_3^s(t)\ f_4^s(t)\ f_5^s(t)\ f_6^s(t)\} \tag{15}$$

where superscript s refers to the numbering of the beam element at which the moving lumped mass m is located. If the longitudinal compression or extension of the beam due to the axial force component is neglected, then the two axial (X) nodal forces can be evaluated by

$$f_1^s(t) = P_x(t)/2 \tag{16a}$$

$$f_4^s(t) = P_x(t)/2. \tag{16b}$$

Nodal forces related to the vertical (transverse, Y) force component are given by [11]

$$[\,f_2^s\ f_3^s\ f_5^s\ f_6^s\,] = P_y[N_1(t)\,N_2(t)\,N_3(t)\,N_4(t)] \tag{17}$$

with $N_i(t)$ representing the shape functions

$$N_1 = 1 - 3\xi^2 + 2\xi^3 \tag{18a}$$

$$N_2 = l(\xi - 2\xi^2 + \xi^3) \tag{18b}$$

$$N_3 = 3\xi^2 - 2\xi^3 \tag{18c}$$

$$N_4 = l(-\xi^2 + \xi^3), \tag{18d}$$

$$\text{where } \xi = x(t)/l. \tag{19}$$

In (19), the definitions for $x(t)$ and l are the same as those given by (6) and (7).

One can find the element number s, which the moving mass is applied to at any time t ($t \neq 0$), as

$$s = \text{Integer part}[\frac{x_m(t)}{l}] + 1. \quad (20)$$

Equation (19) can be rewritten in terms of the global $x_m(t)$ instead of the local $x(t)$:

$$\xi = \frac{x_m(t) - (s-1)l}{l} . \quad (21)$$

Instantaneous forces (and moments) due to the moving force $P(t)$ can now be obtained at any time by using (15)-(21).

Since all nodal forces of the entire crane structure are equal to zero except those at the two nodes of the (s)th beam element at which the lumped mass m is located (according to Clough and Penzien [17]), the external force vector in Equation (1) takes the following form:

$$\{F(t)\} = \{0\ 0 .. f_1^s\ f_2^s\ f_3^s\ f_4^s\ f_5^s\ f_6^s .. 0\ 0\}^{\mathrm{T}} \quad (22)$$

where f_i^s (i=1÷6) represent the nodal forces equivalent to $P(t)$ and are determined by (16) and (17).

They are the s_i (i=1÷6), component of the $\{F(t)\}$, where s_i represents the numbering for the 6 degrees of freedom of the (s)th beam element at which the mass is located at time t.

3.4 Solution of the problem

The procedure for calculating the dynamic responses of the stationary jib crane structure undergoing a moving trolley with hoist and payload are as follows:

- Consider the total time of load travelling over the top beam, τ, divided in p time steps and choosing a time interval Δt ($\tau = p\ \Delta t$), where time t is represented with r time step such that $t = r\,\Delta t$ ($r = 1 \div p$).
- Calculate the position of the moving load, relative to the left end of the top beam, $x_m(t)$ at time step r.
- Calculate the moving mass matrix [m] associated with the moving lumped mass m with Equation (2).
- Calculate the overall stiffness matrix [K] and the instantaneous overall mass matrix [M] of the entire crane structure by using Equations (8) and (9).
- Calculate the overall external force vector {$F(t)$} at time t using expressions of Section 4.3.
- Solve the equation of motion at time t, Equation (1), with software for numerical solution of linear differential equations of second order.
- Repeat the previous points at any time t, i.e. for every time step r (r = 1 ÷ p), to obtain the dynamic responses of the structure.

4 Numerical results and discussion

Equation (1) is used for studying the dynamic response of a jib crane structure subjected to a moving load and is solved by means of direct step-by-step-integration method. An original in-house software is created, based on the Newmark integration method [19], to solve the title problem. The time interval is Δt = 0,01 s, unless stated otherwise.

4.1 Implementation and validation

In order to confirm the reliability of the presented technique, the authors used the research from Akin [5] who studied the dynamics of simple structures-beams subjected to the moving mass. In particular it is given responses of fixed-free (cantilever) beam due to moving load which is travelling from the left end to the free end.

The algorithm and more complex model presented in this paper, for evaluation purposes, is adjusted to comply with fixed-free (cantilever) beam (H<<L, I_2>>I_1, support element, No. 15, is neglected). The properties of the verification model of fixed-free beam with moving mass are given in original units from Reference [5] (for the reason of easier comparison of the results): v = 2000 in/s, L = 300 in, m =3 lb·s²/in, $E\ I = E\ I_1$ = 3,3 e+09 lb·in². Fig. 3 shows the deflections of the cantilever free end under the moving force and moving mass.

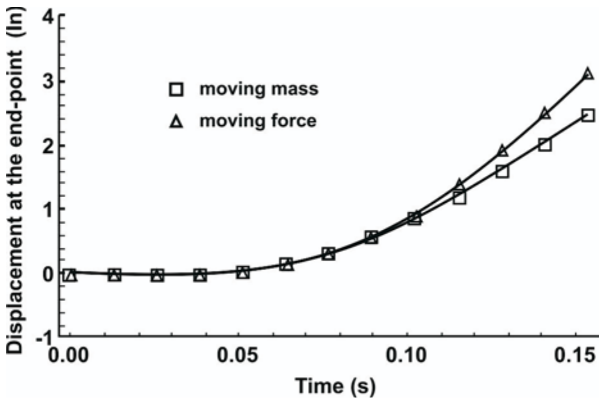

Figure 3 Fixed-free beam under moving load

Comparasion is done with the results from Reference [5 (Fig. 3)]. The solid line with square (-□-) represents deflection of the end point for fixed-free beam under moving mass while solid line with triangle (-△-) considers the same deflection when inertia effects are ignored (moving force). The same markers are used as in original research and one can see that lines are very close to their corresponding curves. It has thus been accepted that the presented technique is viable and it will be used in the further study of the title problem.

4.2 Influence of the moving load magnitude on the dynamic response of the structure

The example model of jib crane structure studied in the following subsections is shown in Figure 1(a). Characteristics of the steel-made structure of the jib crane are: Young's modulus E = 2,1×10¹¹ N/m² and value for mass density is ρ = 7850 kg/m³. All results presented here are based on the acceleration of gravity g = 9,81 m/s².

Overall (catalogue) dimensions of the jib crane structure are: top beam length L = 10 m and column height H = 6 m. Within these dimensions rated loads of the jib cranes are 4000 kg and 8000 kg. The section properties of elements are: A_n = 0,0191 m², I_n = 0,00084 m⁴ (n = 1÷10); A_n = 0,0248 m², I_n = 0,001936 m⁴ (n = 11÷14); and A_n = 0,019 m², I_n = 0,00064 m⁴ (n = 15).

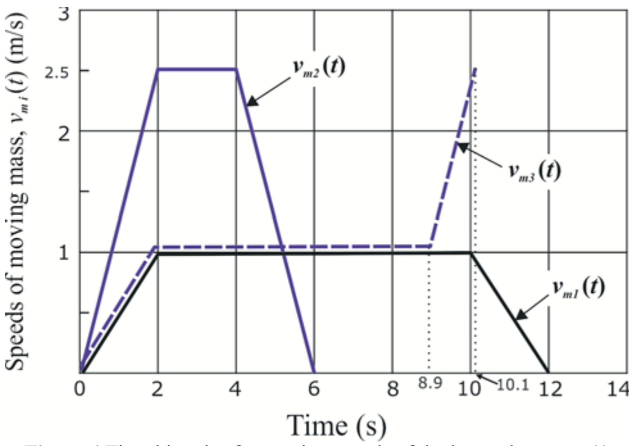

Figure 4 Time histories for moving speeds of the lumped mass $v_{mi}(t)$

The structure is subjected to a load moving with speed history $v_{m1}(t)$, defined in Fig. 4. The TEP (Top beam End Point) is considered relevant for presenting dynamic responses. The influence of moving load magnitude on the TEP vertical displacement is shown in Fig. 5(a), where solid thick lines (—) denote the dynamic responses of the TEP due to the moving load with magnitudes m = 4000 and 8000 kg with inertial effects included, while the solid thin lines (—) denote corresponding ones by neglecting inertial effects. It can be noticed that the difference between the curve considering the inertial effects of the moving load and corresponding one neglecting the inertial effects increases with the increase of the moving load magnitude. The vibration frequency decreases with the increase of the moving load magnitude because the mass of the moving load is included. Also, vertical displacements are higher with the increase of the moving load magnitude, which is known from physical intuition.

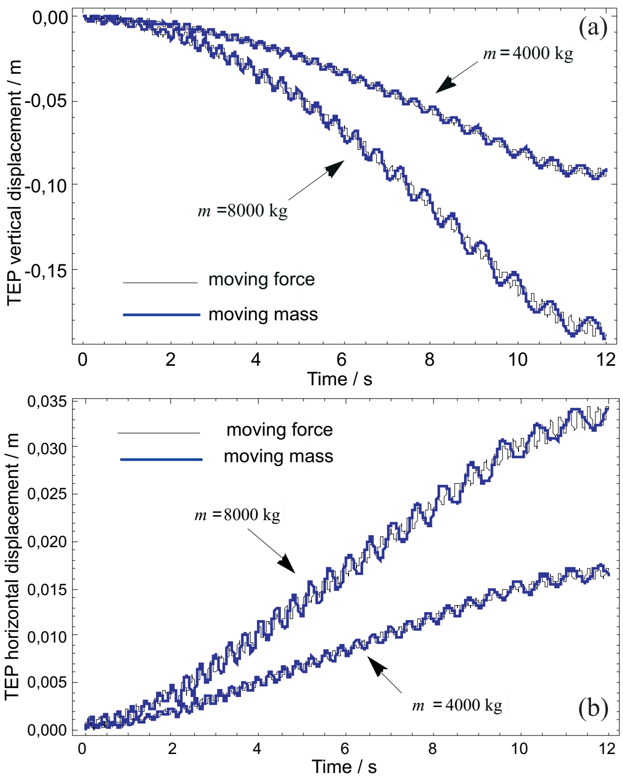

Figure 5 Time histories for: (a) vertical and (b) horizontal displacement of the top beam end point subjected to a moving load with magnitudes m = 4000 kg and 8000 kg; moving speed is $v_{m1}(t)$

Horizontal TEP displacements are shown in Fig. 5 (b), where solid thick lines (—) denote the dynamic responses of the TEP due to the moving load with magnitudes m = 4000 and m = 8000 kg by considering the inertial effects, while solid thin lines (—) denote the corresponding ones by neglecting inertial effects. Vibration amplitudes are higher for the curves that include inertial effects. In periods of acceleration (0÷2 s) or deceleration (10÷12 s), one can notice the influence of the horizontal force component $P_x(t)$ on the vibration amplitude, especially for the curves that include inertial effects of the moving mass.

4.3 Influence of the moving load speed

The structure is hereby subjected to the moving load with a magnitude of m = 4000 kg, moving with two speeds, $v_{m1}(t)$ and $v_{m2}(t)$. Fig. 6 shows TEP responses by taking into consideration inertial effects of the moving load, where the dashed line (---) and solid line (—) respectively, represent the displacements due to speeds $v_{m1}(t)$ and $v_{m2}(t)$. Fig. 6 reveals that the vibration amplitude increases with the increase of the moving speed and the vibration frequency decreases with the increase of the moving speed (acceleration).

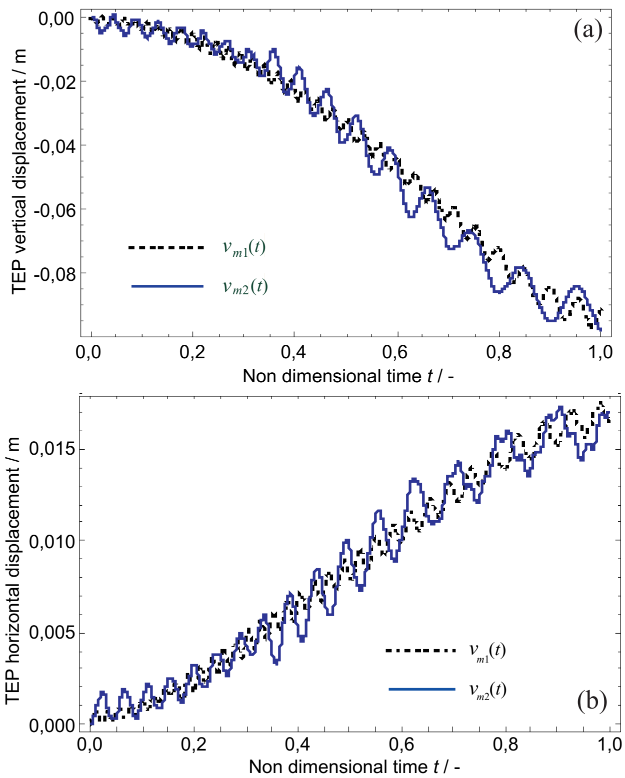

Figure 6 Time histories for: (a) vertical and (b) horizontal displacement of the top beam end point subjected to a moving load with magnitude m = 4000 kg and moving speeds $v_{m1}(t)$ and $v_{m2}(t)$

Fig. 6(b) shows that vibrations are having additional influence in acceleration (or deceleration) period, that being the result of horizontal force component $P_x(t)$.

4.4 Influence of the moving load character on the base of the structure

This subsection studies the effect of the moving load

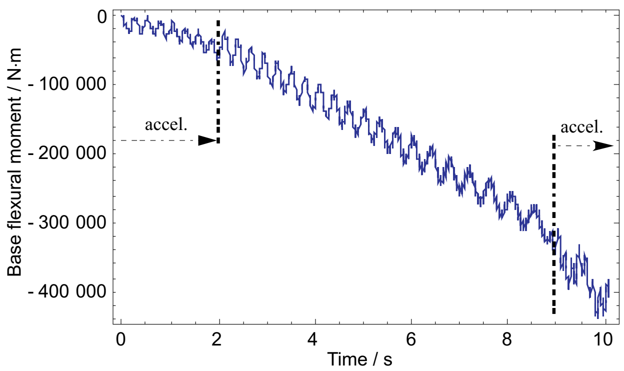

Figure 7 Time histories for the base flexural moment when structure is subjected to a moving load with magnitude m = 4000 kg and moving speed $v_{m3}(t)$

character of the trolley on the flexural moment at the base of the jib crane's structure and bending deflection of the column top point. The flexural moment at the base of the structure is one of the basic parameters in the structural desing process of jib cranes and is used for the design of bolted connection. The example of the jib crane structure is the same as in the previous section. The moving load is of magnitude m = 4000 kg and is moving with speed history $v_{m3}(t)$ representing the situation when the trolley accelerates from fine to main speed near the free end of the jib. Fig. 7 shows the flexural moment in the base of the jib crane structure.

One can see that oscillations are free in the constant speed period (2÷8,9 s), but in the acceleration period the values are higher because of the horizontal force. For this case the maximum absolute value of the flexural moment is 438 096,00 N·m which gives dynamic amplification factor of DAF_1=1,11 compared with the static value when acceleration of the trolley is not included in the structural calculation and DAF_2=1,04 when the acceleration of the trolley is included.

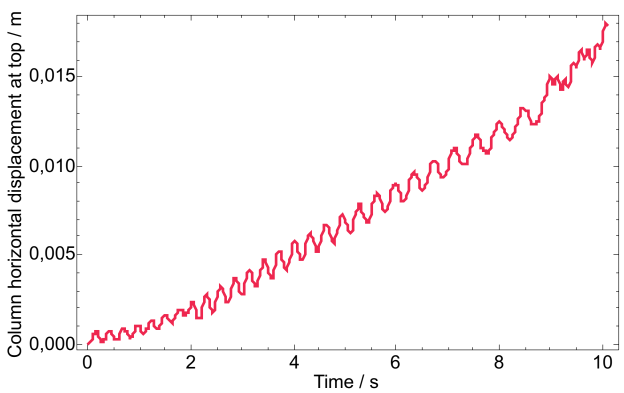

Figure 8 Time histories for horizontal deflection of the column top subjected to a moving load with magnitude m = 4000 kg and moving speed $v_{m3}(t)$

Fig. 8 shows the horizontal displacement of the column top which is related to the bending of the column and its cantilever character. It is obvious that the behavioural trend is similar as in Fig. 7. The maximal deflection for this case is 0,0182 m when the mass is located at the free end of the top beam, which stands for dynamic amplification factor DAF_3=1,08. This result is due to the fact that column elements have higher stiffness than elements of the top beam.

5 Conclusion

By means of the moving mass matrix, two-dimensional inertial effects due to the mass of the moving load are taken into consideration for obtaining the dynamical responses of the jib crane structure in both the vertical and horizontal direction. It is shown that magnitude, speed and acceleration (or deceleration) have effect on the dynamic responses of the structure. The vibration amplitude increases with the increase of magnitude of the moving mass. The same applies for the moving speed and the acceleration. The responses of the structure in horizontal direction have significant influence due to the increase of the moving speed and acceleration of the moving load. This needs to be studied because most of the researchers use conventional methods for showing vertical responses of a simple structure, e.g. beam, induced by a moving mass. The influence of the moving load character on the flexural moment on the base anchor plate of the jib crane structure is presented with dynamic amplification factors which, along with responses of the top beam end point, can be used in the structural design process of column jib cranes.

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Nomenclature

$[M]$	structural mass matrix
$[K]$	structural stiffness matrix
$\{F(t)\}$	overall external force vector
$\{q\}$	displacement vector
$\{\ddot{q}\}$	acceleration vector
$[m]$	moving mass matrix
m	magnitude of moving mass
N_i	shape functions (i = 1-6)
x	distance between the contact position of moving load and left end of top beam element
l	top beam element length
$[Kst]$	framework stiffness matrix
$[Mst]$	framework mass matrix
P_x, P_y	components of dynamic force vector $P(t)$
v_{mx}	velocity of moving mass
a_{mx}	acceleration of moving mass
x_m	distance of the moving system from the left end of a top beam
τ	time required for moving system to travel from left end to right end of top beam
p	total number of time steps
Δt	time interval
L	length of a top beam structure
H	column height
g	gravitational constant
A	cross-sectional area of a beam
I	sectional moment of inertia
E	Young's modulus
ρ	mass density of beam