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A Simple FSPN Model of P2P Live Video Streaming System

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Peer to Peer (P2P) live streaming is relatively new paradigm that aims at streaming live video to large number of clients at low cost. Many such applications already exist in the market, but, prior to creating such system it is necessary to analyze its performance via representative model that can provide good insight in the system's behavior. Modeling and performance analysis of P2P live video streaming systems is challenging task which requires addressing many properties and issues of P2P systems that create complex combinatorial problem. Inspired by several related articles, in this paper we present our Fluid Stochastic Petri Net (FSPN) model for performance analysis of a mesh based P2P live video streaming system.

Keywords: P2P live streaming, modeling, performance analysis, fluid stochastic Petri nets

1. Introduction

Nowadays, more and more visitors are attracted to web locations for live video content which leads to sustainability issues when clients rise above the upload capabilities of the streaming server. Since IP Multicast failed to satisfy these requirements, in the last decade the science community intensively works in the field of P2P networking technologies for live video broadcast. In this paradigm every user (peer) maintains connections with other peers and forms an application level logical network on top of the physical network. Video stream originates at a source and every peer acts as a client as well as a server forwarding the received video packets to the next peer. Two types of data circulate in these networks. *Control data* is used for network formation and maintenance (*Control Scheme*), and video data is the disseminated data (*Data Scheme*). *Control scheme* forms several types of networks: single or multiple multicast trees where the root of the tree is the video source [1], [3], [5], [6], [8], [9], mesh (unstructured P2P network) where peers are organized in swarming like environments [2], [4], [7], [10], [11], [14], or hybrid systems that combine tree and mesh network constructions [12], [13].

Combinations of *Control Scheme* and *Data Scheme* form several approaches for data dissemination. *Source driven* (Push Based) – data scheme is built as a tree on top of the control scheme, where data is pushed down the tree from the source to the peers [9], [2]. *Data driven* (Pull Based) approach is data oriented, doesn't form data distribution trees, and every piece of data is explicitly requested [10], [13], [14]. *Receiver driven* approach means that the data scheme is a tree rooted at the receiver [1], [7], [11]. In our previous work [15] we have introduced all the technical characteristics of these systems. We also reviewed numerous P2P live streaming applications, but found only several research papers that propose mathematical models for performance analyses of such systems. Therefore, this particular research is concerned with modeling and performance analysis of P2P live streaming systems. We base our idea on [18] and propose our modeling approach with Fluid Stochastic Petri Nets [28], [29]. We also introduce several extensions to [18].

This paper is organized as follows: Section 2 gives a survey of related work. In Section 3 we present our analytical model for a mesh P2P live video streaming system and in Section 4 we give our summary of contribution and future work.

2. Related Work

D. Qiu, et al. [16] modeled mesh based, file sharing P2P system, developing simple deterministic fluid model that provides insights in the system's performance. This deterministic fluid model is described by a set of differential equations which are solved in steady state. Then, simple stochastic fluid model is developed which characterizes the variability of the number of peers around the equilibrium values predicted by the deterministic model. All in all, this model provides expressions for the average number of downloaders, average number of seeds, average download time as a function of the peer arrival rate, dewnloaders leaving rate, seeds leaving rate, uploading bandwidth etc., which gives good insight on how the performance of the network is affected by various parameters.

S. Tewari et al. [17] proposed analytical model for BitTorrent (mesh) based live video streaming. In this paper, importance of well designed P2P system is pointed out, with a conclusion that peer group size has no influence on system efficiency when group size exceeds 7-8 peers. This paper provides analytical expressions that concentrate on: the fraction of the total peer upload capacity that can be utilized, the number of fragments available for sharing, fragment size and video playback latency. Inspired by [16], they use the equation describing the fragment exchange efficiency, inferring that the efficiency of fragment exchange heavily depends on the number of available fragments (except for large size fragments). In this paper, fragment size influence on the playback delay is emphasized. It also shows that peer group size of 15 to 20 peers is sufficient to achieve any benefits that a large peer group size can offer.

R. Kumar et al. [18] developed stochastic fluid theory for mesh P2P streaming systems. Their simple model exposes the fundamental characteristics of such systems as well as its limitations. It represents general base for modeling such systems while interpreting peer churn as multiple levels Poisson processes and adopting fluid flow model for video streaming. The analysis includes modeling a system with streaming without buffering and churn, bufferless system with churn and P2P model with peer churn and buffering, while its analytical expressions bring insights in service degradation in all these cases. The performance of the P2P system, as the joining rate of new peers becomes very large, is studied and explicit expressions for the probability of degraded service for large systems are obtained.

Y. Zhou et al. [19] developed a simple stochastic model for data driven system. The model can be used for comparison of different data-driven, chunk selection, downloading strategies based on two performance metrics: continuity and startup delay. It formulates tractable analytical model that helps in understanding the design issues importance in P2P streaming systems. Assuming independent and homogeneous peers, meaning that they have same size playback buffer and chunk selection strategy in a symmetric network setting, simple analytical model that allows computation of the buffer content distribution is constructed. This simple stochastic model is given by differential equations in discrete and continuous case that are solved numerically. Both, discrete model and continuous model are then compared with simulation.

J. Wu et al. [20] presented an extension of [18] focusing on the problem of maximizing universal streaming rate in P2P streaming networks. The difference is in taking into consideration neighborhood constraints. So, the fluid model is extended and new P2P streaming network model is presented with neighbor constraints taken into account. In addition, as a solution, after formulating the problem of maximizing the universal streaming rate in the new model and proving its NP hardness, a heuristic optimum algorithm, using the method of multiple multicast trees packing, is proposed. Similarly, as in one part of [18] this model, is based on the assumption that all peers are bufferless. Situations with peer churn, video packets buffering and heterogeneous peers aren't taken into consideration.

Y. Yue et al. [21] developed a general fluid model to study the performance and fairness of BitTorrent-like networks. In its basics, this model is an extension of [16], taking into consideration the diversity of peers' bandwidth capacities instead of identical ones. This fluid model is defined by a set of differential equations which are then solved in steady state. It is a model of a BitTorrent-like file sharing P2P system where the emphasis is laid on the performance and fairness in steady state.

F. C. Perronnin et al. [22] proposed a stochastic fluid model for analysis of scalable file sharing systems. In more precision, this model is developed for performance evaluation of Squirrel, a P2P cooperative Web Cache. The basic idea in this modeling is presenting HTTP requests as fluid flow modulated by random arrivals and departures of Squirrel nodes. More specifically, cached objects are replaced with fluid. Under the assumption that all objects are equally popular, the model is presented with first order, linear differential equations which are solved in steady state. The model is extended to include unequal popularity by grouping the documents into classes and implementing the same steady state solution. The accuracy of the model is validated by a comparison with discrete event simulations.

Y. Lu et al. [23] presented analytical fluid model for mesh based P2PVoD system. Similarly like previously described fluid models, this model is described with ordinary differential equations, solved in steady state. The model doesn't consider peer selection strategies, incentives management, failure management, chunk scheduling etc. The fluid model is used for computation of the time-dependent average number of downloaders and seeds in the system. Any fluctuations around the average are not computed.

A. Yazici et al. [24] introduces Markov chain based model to study mesh based, multi-stream, P2P Video on Demand systems. The model accounts for peer churn, and query and setup times for a new connection with exponential probability distribution.

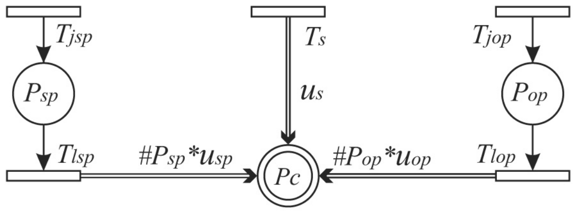
Y. C. Tu et al. [25] presents a study of the capacity growth of P2P media streaming systems using a discrete time analytical model. The capacity is defined as the total streaming bandwidth available from both the servers and the peers. It is concluded that bandwidth capacity increases exponentially in time. These analyses include single file and multi file systems, as well as failure free and system with peer failures. The explicit expressions for the load transition time from servers to peers are derived using exponential and Pareto distributions. Finally, this analysis is validated through simulations and effects of changing some parameters, as requests rate, average peer bandwidth contribution, average peer lifespan on system performance, are studied.

F. Liu et al. [26] studies the inherent relationship between time and scale in P2P streaming systems during a flash crowd. For this study a mathematical framework is developed. This analysis also brings in depth understanding of peer's competition for limited bandwidth resources and important insight in other critical factors. It also shows that system scaling behavior and peer startup delay are strongly affected by different peer arrival patterns and peer churn.

D. Wu et al. [27] developed a tractable analytic theory for multi-channel P2P live streaming systems. The developed models are: mesh based isolated channel system where the peer uploads the same channel it views (ISO) and a model where viewing and uploading channels are separate ones (VUD). The systems performance is studied using closed and open queuing network models. These analytic models capture the essentials of multi-channel video streaming systems including channel switching, peer churn, upload bandwidth heterogeneity and channel popularity.

3. FSPN Model for P2P Live Video Streaming System

Our model is mostly inspired by [18] and introduces few extensions to it. Similar to [18], we assume two types of peers – *Super peers* with upload bitrate higher than the video rate, and *Ordinary peers* with upload bitrate lower than the video rate. The basic FSPN representation of our model with peer churn is given in Figure 1. Single line circles are discrete places that can contain discrete tokens which represent peers. The tokens move via single line arcs into and out of the discrete places (representing peers joining and leaving the system). Double line arcs represent pipes through which fluid (bits) is transferred. The double line circle represents fluid place that in this case doesn't play any significant role. The important thing is the rate at which the fluid is pushed through fluid arcs to the place P_c. All transitions are timed transitions with exponentially distributed firing times.



T_{jsp} – Joining of super peers with rate λ_s
T_{lsp} – Leaving of super peers with rate μ_s
T_{jop} – Joining of ordinary peers with rate λ_o
T_{lop} – Leaving of ordinary peers with rate μ_o
T_s– Represents the server streaming with rate u_s
P_{sp} – Discrete place for super peers
$\#P_{sp}$ – The number of super peers in P_{sp} (further denoted by X)
P_{op} – Discrete place for ordinary peers
$\#P_{op}$ – The number of ordinary peers in P_{op} (further denoted by Y)
P_c – The fluid place
u_s – Upload bitrate of the server
u_{sp} – Upload bitrate of super peers
u_{op} – Upload bitrate of ordinary peers
r_v – Bitrate of the video stream

Figure 1. Basic FSPN model of P2P live video streaming system

T_{jsp} , T_{jop} and T_s are always enabled. T_{lsp} and T_{lop} are enabled only when there are tokens in P_{sp} and P_{op} respectively. When enabled, these transitions may consume tokens from P_{sp} and P_{op} and constantly push fluid to the continuous place P_c with the rate linearly dependant to the number of tokens in discrete places ($\#P_{sp}$ and $\#P_{op}$).

We define a fluid function (Φ) as the maximum bitrate that can be streamed to each individual peer as (1):

$$\Phi = \min\left\{u_s, \frac{u_s + \#P_{sp} * u_{sp} + \#P_{op} * u_{op}}{\#P_{sp} + \#P_{op}}\right\} \tag{1}$$

$$\text{clearly, } \Phi = \phi\left(u_s, \#P_{sp}, \#P_{op}\right) \tag{2}$$

Universal streaming is possible if and only if $\phi(u_s,\#P_{sp},\#P_{op}) \geq r_v$, so, to calculate the performance of the system we need to answer the question: what is the probability for universal streaming for a given scenario?

Since, $u_{sp} > r_v > u_{op}$, the $\#P_{op}$ that one super peer can support is $(u_{sp} - r_v)/(r_v - u_{op}) = k$, and the $\#P_{op}$ that the server can support is $u_s/(r_v - u_{op}) = c$. $\#P_{op}$ that one ordinary peer can support is

$$u_{op}/(r_v - u_{op}) = l \text{ (for buffered system).}$$

Universal streaming is achievable only if $\#P_{sp}$ is sufficient to support $\#P_{op} - c$ peers i.e. the probability for universal streaming ($P_{UniStream}$) is:

$$P_{UniStream} = P\left(X \geq \frac{Y - c}{k}\right) \tag{3}$$

In our model we have two independent $M/M/\infty$ processes where $X(t)$ and $Y(t)$ are independent random variables with Poisson probability distribution. The total number of peers is a two dimensional random variable with parameters X, and Y. The joint cumulative distribution function for the two independent random variables is given by (4):

$$F(x, y) = F(x) * F(y) \tag{4}$$

Then the joint probability density function of this bivariate is given by (5):

$$f(x, y) = \frac{\partial^2 F(x, y)}{\partial x \partial y} \tag{5}$$

If we express the Y_{max} for which universal streaming is achievable for any value of X, then Y would be limited by (6):

$$Y \leq kX + c \tag{6}$$

Represented in a Cartesian coordinate system, $P_{UniStream}$ would be the area presented in Figure 2.

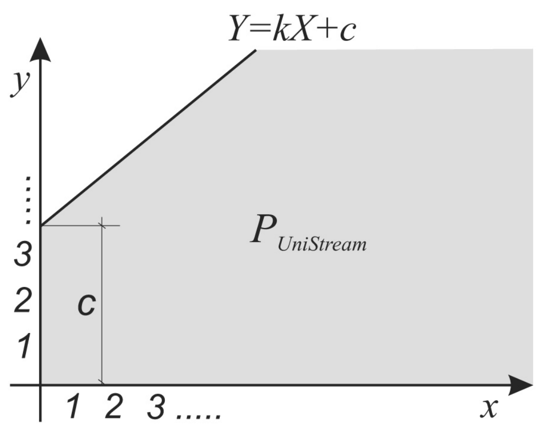
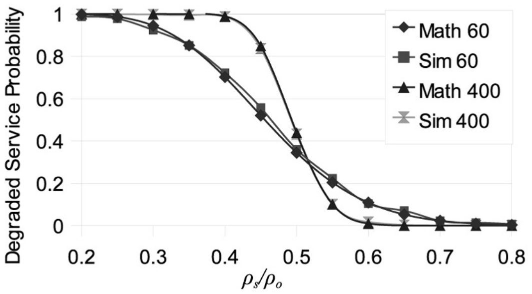


Figure 2. Probability for universal streaming

Next, we can calculate the probability for universal streaming by (7):

$$P(Y \leq kX + c) = \int_0^x \int_0^{kx+c} f(x, y)\, dy dx \tag{7}$$

Varying the rates at which peers arrive and depart, we can calculate $P_{UniStream}$ for a variety of scenarios. In Figure 3. mathematical results verified through simulation are presented. These results comply with the results given in [18].



ρ_s – Mean number of Super peers in the system (λ_s/μ_s)
ρ_o – Mean number of Ordinary peers in the system (λ_o/μ_o)
Math (#) – Mathematical calculation for $\rho_o = (\#)$
Sim(#) – Simulation for $\rho_o = (\#)$

Figure 3. Probability for degraded service

Next, we extend our model to include buffering. This part differs significantly from [18] because we propose analytical expression opposed to their simulation technique.

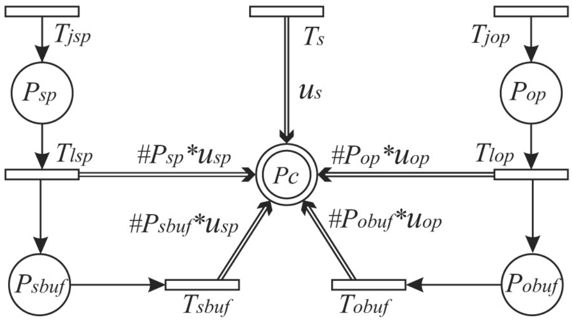
To model this P2P system with buffering, P_{sbuf} and P_{obuf} are needed on both sides of the system. We assume that when a peer leaves a buffered system, it virtually stays in the system and leaves its resources for a certain amount of time. Assuming that the average amount of bits in every peer's buffer is approximately half of its capacity, we can also model the buffer size as twice the amount of time of the peer's virtual stay. In time units, the buffer size is given by (8):

$$Buffer_{size} = \frac{2}{\mu_{sbuf}} = \frac{2}{\mu_{obuf}} \text{ [sec]} \quad (8)$$

Our FSPN model with peer buffering is presented in Figure 4.

Now, the average number of peers in any buffer place can be calculated from the tandem $M/M/\infty$ queue. We denote the average number of peers in buffer places as Z_s and Z_o for P_{sbuf} and P_{obuf} respectively. Extending equation (3), we get:

$$P_{UniStream} = P\left(X \geq \frac{Y - c - l \cdot Z_o}{k} - Z_s\right) \quad (9)$$



P_{sbuf} – place for virtual stay of super peers
P_{obuf} – place for virtual stay of ordinary peers
T_{sbuf} – super peers leave buffer with rate μ_{sbuf}
T_{obuf} – ordinary peers leave buffer with rate μ_{obuf}

Figure 4. FSPN model with buffering

$$P_{UniStream} = P\left(Y \leq k\left(X + Z_s\right) + c + l \cdot Z_o\right) \quad (10)$$

Then, probability for universal streaming for a buffered system is given by (11):

$$P_{UniStream} = \int_0^x \int_0^{k(x+z_s)+c+l\cdot z_o} f\,(x,y)\,dydx \quad (11)$$

The second extension of [18] is application of some admission control for peers with low contribution of resources (in this case ordinary peers). To solve this problem, we need to set Guard (Boolean) function to transition T_{jop} such as:

$$G = \left\{\phi\left(u_s, \#P_{sp}, \#P_{op}, \#P_{buf}\right) \geq r_v\right\} \quad (12)$$

If G evaluates to true, T_{jop} is enabled. Otherwise T_{jop} is disabled. This remains as a challenge for our future work.

4. Summary of Contribution

In this paper we presented several basic characteristics of P2P live video streaming systems and made a survey of existing mathematical models for performance analysis of such systems. We also proposed new FSPN model for performance analysis of P2P live streaming systems with peer bandwidth heterogeneity, peer churn, peer buffering, and admission control. In our future work we intend to mathematically sustain our model and use it in performance analysis of new P2P live video streaming concept that we will propose in another paper. For further acknowledgment of our model, all our analytical results will be evaluated via discrete event simulations.

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