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Wind-induced currents directed to the left of the wind in the northern hemisphere: An elementary explanation and its historical background

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Every beginning student of geophysical fluid dynamics knows that steady-state wind-driven currents are directed to the right of the wind in the northern hemisphere. In contrast, the fact that wind-induced currents may under some specific conditions be of the opposite direction is mentioned in only a few papers. Here, a succession of simple explicit solutions is used to illustrate that the wind varying at subinertial frequencies always generates currents directed to its right, whereas at superinertial frequencies the wind rotating clockwise (counterclockwise) coincides with currents directed to its left (right). The difference is related to a dynamics differing in the two frequency bands. At subinertial frequencies, friction is primarily balanced by the Coriolis acceleration, and therefore the currents vary in phase with the wind while being directed to its right. At superinertial frequencies, the primary balance is between friction and local acceleration, implying that the currents lag behind the wind that causes them while pointing down the forcing. The former dynamic regime was originally considered by Vagn Walfrid Ekman (in 1905), the latter by Karl Jakob Zöppritz (in 1878). The destiny of the two contributions, however, was completely different: Ekman's is widely appreciated today, whereas Zöppritz's is rarely mentioned. Although Zöppritz's findings were adversely influenced by his consideration of the laminar rather than turbulent viscosity, his neglect of the Coriolis acceleration was acceptable and his solution therefore formally correct at superinertial frequencies.

Keywords: wind-driven currents, subinertial frequencies, Ekman, superinertial frequencies, Zöppritz

1. Introduction

That steady-state wind-driven currents are directed to the right of the wind in the northern hemisphere is well known to every beginning student of geophysical fluid dynamics. In contrast, the fact that wind-induced currents may under some specific conditions be of the opposite direction is mentioned in only a few papers (Gonella, 1972; Weller, 1981; Craig, 1989; Elipot and

Gille, 2009). In this note a succession of simple explicit solutions will be used to illustrate the conditions under which this anomalous behavior occurs and to interpret the mechanism behind it. Moreover, the interpretation will be related to theoretical studies published by Karl Jakob Zöppritz, a 19th century geophysicist (Günther, 1900), and by Vagn Walfrid Ekman, the famous Swedish oceanographer (e.g., Jenkins and Bye, 2006; Svansson, 2010).

2. Governing equations

Let us assume that the wind stress acts on a homogeneous sea. If the wind is uniform in space and the sea is unbounded, it may be expected that the horizontal pressure gradient vanishes and that the motion is uniform inside the horizontal slabs of sea. Linearized momentum equations are (e.g., Gill, 1982; Cushman-Roisin, 1994):

$$\frac{\partial u}{\partial t} - fv = K\frac{\partial^2 u}{\partial z^2}$$
$$\frac{\partial v}{\partial t} + fu = K\frac{\partial^2 v}{\partial z^2} \qquad (1)$$

where u and v are x and y current components, respectively, K is the coefficient of turbulent viscosity, and f is the Coriolis parameter. The two parameters are assumed constant. The origin of the coordinate system is positioned at sea level with the z axis pointing vertically upwards. Equations (1) are to be solved subject to the wind stress imposed at the sea surface:

$$\left(K\frac{\partial u}{\partial z}\right)_{z=0} = \frac{\tau_x}{\rho}, \quad \left(K\frac{\partial v}{\partial z}\right)_{z=0} = \frac{\tau_y}{\rho}$$

where τ_x and τ_y are x and y components of the wind stress, respectively, and ρ is the seawater density. Moreover, it is assumed that the currents vanish at large depths:

$$(u)_{z\to-\infty} = (v)_{z\to-\infty} = 0.$$

By defining the complex currents as $w = u + iv$, equations (1) may be compressed into:

$$\frac{\partial w}{\partial t} + ifw = K\frac{\partial^2 w}{\partial z^2} \qquad (2)$$

with the corresponding boundary conditions:

$$\left(K\frac{\partial w}{\partial z}\right)_{z=0} = \frac{\tau_x + i\tau_y}{\rho},$$
$$(w)_{z\to-\infty} = 0.$$

The use of complex quantities considerably simplifies the mathematics and – at the same time – allows the solutions obtained to be easily visualized: one simply has to think of the complex plane as of the horizontal plane. If equations (1) and (2) are now vertically integrated and boundary conditions are taken into account, it follows:

$$\frac{dU}{dt} - fV = \frac{\tau_x}{\rho}$$
$$\frac{dV}{dt} + fU = \frac{\tau_y}{\rho} \tag{3}$$

and:

$$\frac{dW}{dt} + ifW = \frac{\tau_x + i\tau_y}{\rho} \tag{4}$$

where the volume transports are:

$$U = \int_{-\infty}^{0} u dz, \quad V = \int_{-\infty}^{0} v dz, \quad W = \int_{-\infty}^{0} w dz.$$

The above simple equations document a surprising variety of physical processes. If there is no wind acting on the sea, complex currents and transports are obviously proportional to exp(–*ift*) and, therefore, represent inertial oscillations manifested in the clockwise rotation with an angular frequency equal to *f*. The existence of inertial oscillations had been for the first time postulated by Ferrel (1858), and they were subsequently found in the seas worldwide. When the wind drives the sea, its response strongly depends on frequency and this will be illustrated through a progression of elementary solutions.

3. Simple solutions

3.1. Transports due to the wind stress periodically varying along a line

Let us assume that the wind stress is defined by $\tau_x = \tau \cos(\omega t)$ and $\tau_y = 0$, where ω is the angular frequency whereas τ is an arbitrary amplitude, and that the inertial oscillations are not triggered. In the case of very low, sub-inertial frequencies, at which the Coriolis acceleration surpasses local acceleration unless *U* is much larger than *V*, equations (3) imply:

$$-fV = \frac{\tau_x}{\rho}$$

and therefore:

$$V = -\frac{\tau}{\rho f}\cos(\omega t). \tag{5}$$

This transport does not depend on the way the friction is parameterized. It varies in phase with the wind stress and is directed to the right of the wind (Fig. 1a). The balance between friction and the Coriolis acceleration was first explored by Ekman (1905), who – among other things – derived expression for transport given by solution (5) under the limit $\omega \to 0$, the transport that today bears his name. The results presented by V. W. Ekman (Fig. 2) in his seminal paper, concerning not only transports but also vertically variable currents, represent one of the cornerstones of geophysical fluid dynamics.

With the same wind forcing and general conditions as above but for very high, superinertial frequencies, at which local acceleration surpasses the Coriolis acceleration unless U is much smaller than V, equations (3) give:

$$\frac{dU}{dt} = \frac{\tau_x}{\rho}$$

and the solution is now:

$$U = \frac{\tau}{\rho\omega}\sin(\omega t). \tag{6}$$

In this case the transport is directed down the wind stress that causes it, but it lags behind the wind by a quarter of a period (Fig. 1b). Study of the underlying balance between friction and local acceleration was pioneered by Zöppritz (1878a, 1878b), who made much use of the analogy with the heat diffusion prob-

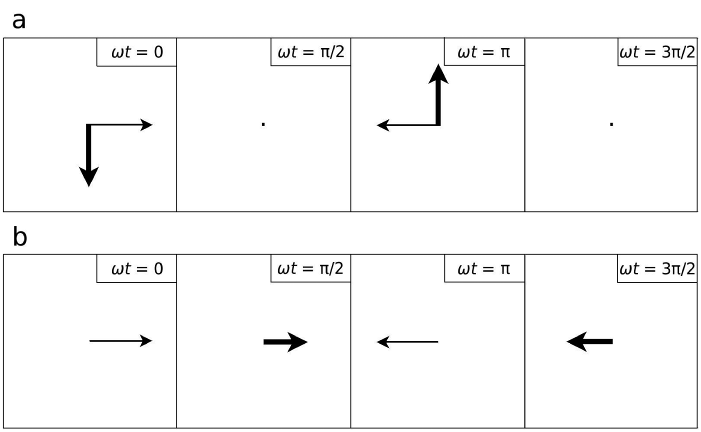

Figure 1. A cycle of the wind (thin arrow) oscillating along an axis and of the transport (thick arrow) induced by it in the northern hemisphere at subinertial frequencies (a) and superinertial frequencies (b).

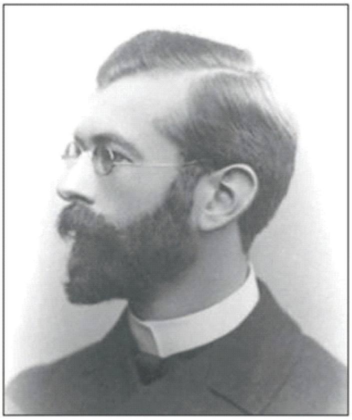

Figure 2. Karl Jakob Zöppritz (1838–1885, left) and Vagn Walfrid Ekman (1874–1954, right). Karl Jakob Zöppritz should not be confused with his nephew Karl Bernhard Zöppritz (1881–1908), who made important contributions to seismology in the beginning of the 20th century (Ritter et al., 2009).

lem. K. J. Zöppritz (Fig. 2), however, considered vertically variable currents rather than transports, and he obtained very slow downward propagation of momentum because he allowed for laminar rather than turbulent viscosity. It may have been this basic deficiency of the theory that discouraged his successors from paying more attention to the local acceleration as possibly equal in magnitude to the friction and from analyzing the resulting transports that, ironically, would not be affected by an inadequate representation of friction.

3.2. Transports due to the rotating wind stress

The wind stress is now assumed to be of a more general form: $\tau_x + i\tau_y = A\exp(i\omega t) + B\exp(-i\omega t)$, with A and B being arbitrary moduli. The form allows the wind stress to rotate clockwise ($A < B$), to be linearly polarized ($A = B$) or to rotate counterclockwise ($A > B$). Equation (4) then results in:

$$W = \frac{A}{i\rho(f+\omega)}\exp(i\omega t) + \frac{B}{i\rho(f-\omega)}\exp(-i\omega t). \tag{7}$$

Obviously, inertial oscillations may be added to this solution if initial conditions require so. It is straightforward to show that in the case of the wind stress periodically varying along the x axis ($A = B$), solution (7) reduces to solution (5) for low subinertial frequencies and to solution (6) for high superinertial frequencies, thus proving that the constraints imposed on the volume transports U and V while deriving the previous solutions are acceptable. Also instructive are the special cases in which the wind stress follows a circular path while rotating clockwise ($A = 0$) or counterclockwise ($B = 0$), originally considered by Gonella (1972). When the wind stress is rotating purely clock-

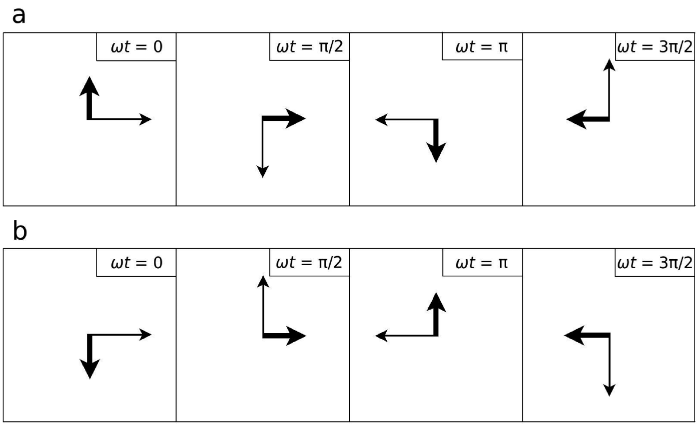

Figure 3. A cycle of the wind (thin arrow) rotating clockwise (a) and counterclockwise (b) and of the corresponding transport (thick arrow) in the northern hemisphere at superinertial frequencies.

wise, the transport is of the Ekman type at subinertial frequencies ($\omega < f$), there is resonance at the inertial frequency ($\omega = f$), and the transport is opposed to the Ekman transport at superinertial frequencies ($\omega > f$, Fig. 3a). For the wind stress rotating purely counterclockwise, the transport is reduced smoothly from the Ekman value at subinertial frequencies to the smaller value at superinertial frequencies, without resonant amplification at the inertial frequency and without changing direction (Fig. 3b). Solution (7) also allows for the wind stress following elliptical paths, either in a clockwise or in a counterclockwise sense, with the resonance occurring at the inertial frequency not only under the former but also under the latter forcing.

3.3. Currents due to the rotating wind stress

Although rich in content, solution (7) does not offer any information on the vertical variability of currents or the depth to which wind-driven currents extend. For this one has to return to equation (2) and corresponding boundary conditions, which – with the wind forcing given above – leads to:

$$w = \frac{A}{\rho\sqrt{K(f+\omega)}} \exp\left(\frac{\pi}{D} z\right) \exp\left[i\left(\omega t + \frac{\pi}{D} z - \frac{\pi}{4}\right)\right] + \frac{B}{\rho\sqrt{K(f-\omega)}} \exp\left(\frac{\pi}{D_1} z\right) \exp\left[-i\left(\omega t - \frac{\pi}{D_1} z + \frac{\pi}{4}\right)\right] \quad \text{(8a)}$$

at subinertial frequencies, and:

$$w = \frac{A}{\rho\sqrt{K(f+\omega)}} \exp\left(\frac{\pi}{D} z\right) \exp\left[i\left(\omega t + \frac{\pi}{D} z - \frac{\pi}{4}\right)\right] + \frac{B}{\rho\sqrt{K(\omega - f)}} \exp\left(\frac{\pi}{D_2} z\right) \exp\left[-i\left(\omega t + \frac{\pi}{D_2} z - \frac{\pi}{4}\right)\right] \quad \text{(8b)}$$

at superinertial frequencies. The three depth scales, related to counterclockwise rotation at all frequencies, clockwise rotation at subinertial frequencies, and clockwise rotation at superinertial frequencies, are respectively:

$$D = \pi\sqrt{\frac{2K}{f+\omega}}, \quad D_1 = \pi\sqrt{\frac{2K}{f-\omega}}, \quad D_2 = \pi\sqrt{\frac{2K}{\omega-f}}. \tag{8c}$$

It is easy to verify that vertical integration of both (8a) and (8b) results in (7). When the wind stress is assumed to periodically vary along the x axis ($A = B$), equation (8a) reduces at vanishing frequencies to the solution obtained by Ekman (1905), whereas equation (8b) reduces at very high frequencies to the solution similar to the one published by Zöppritz (1878a, 1878b). The depth scale is $\pi(2K/f)^{1/2}$ in the first case (which, of course, is the Ekman depth), and $\pi(2K/\omega)^{1/2}$ in the second case (which resembles the depth scale obtained by Zöppritz except that K in his solution was the coefficient of laminar viscosity). Also interesting are the special cases with the wind stress following a circular path while rotating clockwise ($A = 0$) or counterclockwise ($B = 0$), previously studied by Weller (1981) and Craig (1989) in the time domain and by Elipot and Gille (2009) in the frequency domain. They show that when the transport is directed to the right (left) of the wind, the currents are spiraling to the right (left) with the increasing depth. As implied by (8c), the vertical extent of wind-driven currents is limited by the Coriolis effect at low frequencies and by wind variability at high frequencies, except when clockwise rotating wind is resonantly coupled to inertial oscillations. Solution (8) shows that resonance is possible at the inertial frequency not only when the wind stress is rotating purely clockwise but also when it is following elliptical paths (i.e., when $B \neq 0$).

4. Discussion

The above simple solutions show that when a homogeneous, unbounded sea located in the northern hemisphere is subjected to wind forcing that is uniform in space but variable in time, the response strongly depends on frequency of the forcing. At subinertial frequencies, the Ekman-type currents develop because at these frequencies friction is primarily balanced by the Coriolis acceleration. These currents vary in phase with the wind and are directed to the right of the wind. At the inertial frequency, resonance occurs, except when the wind rotates purely counterclockwise. At superinertial frequencies, the wind-induced currents are directed to the left of the wind when it is rotating clockwise and to the right of the wind when it is rotating counterclockwise. In this case, the primary balance is between friction and local acceleration. This balance implies that the currents lag behind the wind that causes them. Moreover, the currents point down the forcing. Consequently, the currents are directed to the left of the simultaneous wind in the case of clockwise rotation simply because the wind was previously so directed, and they are directed to the right of the simultaneous wind in the case of counterclockwise rotation because that was the previous wind direction. Interestingly, the counterclockwise wind rotation supports qualitatively similar response at all frequencies, but for dynamical reasons that differ considerably between subinertial and superinertial frequencies. Of course, all conclusions also pertain to the southern hemisphere, with a simple conversion of “left” and “clockwise” to “right” and “counterclockwise”, respectively, and *vice versa*.

The difference between the wind-driven currents at subinertial and superinertial frequencies has important consequences. For example, the wind rotating clockwise in the northern hemisphere supports currents directed to its right at subinertial frequencies and to its left at superinertial frequencies. The different angles have been observed while performing cross-spectral analysis of wind and current time series collected in the open sea (Gonella, 1972; Weller, 1981). The left-directed currents should also be observable in the time domain, probably in the data sets collected close to the coasts. As is well known, the northern hemisphere sea- and land-breezes are characterized by clockwise rotation unless they are modified by pronounced orographic effects. Their spectra are dominated by diurnal peaks that are subinertial at the latitudes larger than 30° N, superinertial at the smaller latitudes. It may thus be expected that sea- and land-breezes would support currents directed to their left at low latitudes. Apparently, such a cause-and-effect relationship did not receive much attention in the past, but may prove to be of interest to future researchers.

Another important consequence of the different dynamic regimes is that they strongly influence the response of the stratified sea in the vicinity of a straight coast to the periodic wind forcing, with the resulting time-variable upwelling being influenced more by the alongshore wind at subinertial frequencies and by the cross-shore wind at superinertial frequencies (Orlić and Pasarić, 2011). Because the upwelling is supported by an offshore transport in the surface layer, the difference between the two dynamic regimes implies that the alongshore wind efficiently pumps the pycnocline at low frequencies whereas the cross-shore wind is more efficient at high frequencies.

5. Historical context

As already mentioned, study of the dynamics dominating the subinertial variability was pioneered by Ekman (1905), whereas the dynamics controlling the superinertial variability have previously been considered by Zöppritz (1878a, 1878b). The destiny of the two contributions, however, was completely different: Ekman’s is remembered today as one of the milestones in the development of geophysical fluid dynamics, whereas Zöppritz’s is rarely mentioned and then merely as an unsuccessful precursor to Ekman’s work.

It appears, however, that Zöppritz’s contribution requires a more nuanced evaluation. His paper was originally published in German (Zöppritz, 1878a) and was in the same year translated into English (Zöppritz, 1878b) because it could be used in a debate on the origin of ocean circulation carried out at the time by William B. Carpenter and James Croll. In the paper Zöppritz considered three different wind forcings: (1) steady state, (2) transient, i.e., imposed at an initial time and kept constant thereafter, and (3) periodic. On the basis of the first two solutions he concluded that the steady-state wind-driven currents extend to the ocean bottom but also that it takes hundreds of thousands of years for the currents to fully develop from an initial state of rest. The conclusions were invoked in the Carpenter-Croll controversy but were later disproved by Ekman (1905) who made two important improvements – namely, he took into account the Coriolis acceleration and allowed for the turbulent rather than laminar viscosity. In this sense it is true that Zöppritz's work was superseded by Ekman's, as sometimes mentioned in the oceanographic textbooks (Krauss, 1973) and in the histories of oceanography (e.g., Mills, 2009).

On the other hand, it appears that Zöppritz's third solution, allowing for the periodic wind that acts on the sea in which friction is balanced by local acceleration, has a more lasting value. The solution is formally correct at high frequencies at which the Coriolis acceleration is of secondary importance, and requires only that the coefficient of laminar viscosity be substituted by the coefficient of turbulent viscosity in order to be completely right. Zöppritz could hardly be blamed for not allowing for turbulent viscosity since he worked at a time when it was essentially unknown, and the substitution of coefficients could easily have been made by his successors as, for example, it was done by Ekman while considering steady-state and transient problems. It therefore seems that Zöppritz deserves more credit for his work than he has previously received and that his study of wind-driven currents at high frequencies should be considered almost on a par with Ekman's investigation of wind-driven currents at low frequencies.

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SAŽETAK

Vjetrovne struje usmjerene ulijevo od vjetra na sjevernoj hemisferi: jednostavno objašnjenje i njegova povijesna pozadina

Mirko Orlić

Da su na sjevernoj hemisferi stacionarne vjetrovne struje usmjerene udesno od vjetra, to znade svaki početnik u geofizičkoj dinamici fluida. Nasuprot tome, činjenica da vjetrovne struje mogu u nekim posebnim uvjetima biti suprotnog smjera spominje se u tek nekoliko članaka. U ovom radu niz jednostavnih eksplicitnih rješenja pokazuje da vjetar koji se mijenja na frekvencijama manjima od inercijalnih uvijek podržava struje usmjerene na svoju desnu stranu, dok se vjetar koji rotira na frekvencijama većima od inercijalnih podudara sa strujama usmjerenima bilo na svoju lijevu bilo na svoju desnu stranu, ovisno o tome odvija li se njegova rotacija u smjeru kazaljke na satu ili u suprotnom smjeru. Ta je razlika povezana s različitom dinamikom u dva spomenuta frekvencijska područja. Na frekvencijama manjima od inercijalnih trenje je prvenstveno uravnoteženo s Coriolisovim ubrzanjem, tako da se struje mijenjaju u fazi s vjetrom pri čemu su usmjerene na njegovu desnu stranu. Na frekvencijama većima od inercijalnih ravnoteža se prvenstveno ostvaruje između trenja i lokalnog ubrzanja, što znači da struje kasne za vjetrom koji ih uzrokuje te da su usmjerene niz njega. Prvi dinamički režim izvorno je istražio Vagn Walfrid Ekman (1905. godine), a drugi Karl Jakob Zöppritz (1878. godine). Međutim, ta su dva istraživanja imala sasvim različite sudbine: Ekmanov je rezultat danas opće poznat dok se Zöppritzov rijetko spominje. Na Zöppritzove je rezultate nepovoljno utjecalo uvažavanje laminarne umjesto turbulentne viskoznosti – ipak, za frekvencije veće od inercijalnih njegovo je zanemarenje Coriolisova ubrzanja prihvatljivo i stoga je njegovo rješenje za taj slučaj formalno ispravno.

Ključne riječi: vjetrovne struje, frekvencije manje od inercijalnih, Ekman, frekvencije veće od inercijalnih, Zöppritz

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