



# VISUALIZATION OF COMPLEX NETWORKS BASED ON DYADIC CURVELET TRANSFORM

Marjan Sedighi Anaraki<sup>1,\*</sup>, Fangyan Dong<sup>1</sup>, Hajime Nobuhara<sup>2</sup> and Kaoru Hirota<sup>1</sup>

<sup>1</sup>Department of Computational Intelligence & Systems Science, Tokyo Institute of Technology  
Yokohama – Tokyo, Japan

<sup>2</sup>Department of Intelligent Interaction Technologies,  
Graduate School of Systems and Information Engineering, University of Tsukuba  
Ibaraki, Japan

*Regular paper*

*Received: 28 April, 2006. Accepted: 5 July, 2006.*

## SUMMARY

A visualization method is proposed for understanding the structure of complex networks based on an extended Curvelet transform named Dyadic Curvelet Transform (DClet). The proposed visualization method comes to answer specific questions about structures of complex networks by mapping data into orthogonal localized events with a directional component via the Cartesian sampling sets of detail coefficients. It behaves in the same matter as human visual system, seeing in terms of segments and distinguishing them by scale and orientation. Compressing the network is another fact. The performance of the proposed method is evaluated by two different networks with structural properties of small world networks with  $N = 16$  vertices, and a globally coupled network with size  $N = 1024$  and 523 776 edges. As the most large scale real networks are not fully connected, it is tested on the telecommunication network of Iran as a real extremely complex network with 92 intercity switching vertices, 706 350 E1 traffic channels and 315 525 transmission channels. It is shown that the proposed method performs as a simulation tool for successfully design of network and establishing the necessary group sizes. It can clue the network designer in on all structural properties that network has.

## KEY WORDS

visualization, complex network, human visual system

## CLASSIFICATION

PACS: 89.75.-k

\*Corresponding author, *η*: [msedighi@hrt.dis.titech.ac.jp](mailto:msedighi@hrt.dis.titech.ac.jp); +81 45 924 5686, +81 45 924 5682;  
G3-49, 4259 Nagatsuta, Midori-ku, Yokohama-city 226-8502, Japan

## INTRODUCTION

Complex networks are being studied across many fields of science [1 – 4] inspired by empirical studies of networked systems such as the internet, biological networks like brain neural networks and so on. Scientists think they know most of the elements and forces of networks [5, 6]. But networks are inherently difficult to understand because of structural complexity. For a network with millions of vertices direct analyzing by sketching the structure of network and looking at them by eye is hopeless. Having some static displays for instance vertex map or matrix, data filtering as an interactive control, or smooth zooming can be useful.

The recent development of statistical methods for quantifying huge networks is to a large extent an attempt to find something to play the part played by eye for complex networks. Furthermore it usually is assumed that the network architecture is static. These simplifications allow us to sidestep any issues of structural complexity and propose a visualization method based on an extended Curvelet transform named Dyadic Curvelet Transform (DClet) for understanding the topology of complex networks. The proposed method gives us the matrix of network in dyadic scales, filters data by the coarse to fine strategy and brings us smooth zoom without losing the structural properties of a network. This modification is done to solve the Curvelet transform [7, 8] inconvenience of decomposition into components at different scales.

One potential of this proposal might be the optimization of a network by removing redundant edges in the network without changing the desired properties of it.

The aim of this research is showing that the proposed visualization method with directional sensitivity can be considered as an efficient tool for investigating the structural properties of complex networks. It can be considered as a simulation tool, or a compression method. It does not have the geographical displays of a network. As the topology of network is robust in any scales of the DClet, it can be worked on a smaller matrix and modeled the real network with the DClet reconstruction.

The performance of the proposed method is evaluated in two different networks with structural properties of small world networks with  $N = 16$  vertices, and a globally coupled network with size  $N = 1024$  and 523 776 edges. As the most large scale real networks are not fully connected, it is tested on the telecommunication network of Iran as a real extremely complex network with 92 intercity switching vertices, 706 350 E1 traffic channels and 315 525 transmission channels. It is presented that the proposed method provides a natural way of analyzing a complex network due to analyzing the network at a coarse level and then increasing the resolution in dyadic scale, if necessary. The experiments have been done in Matlab on a personal computer with processor Pentium IV 2.4 MHz, RAM 256 M, and HDD 60GB. The details of evaluation of the proposed method on the telecommunication network of Iran is not shown here just the final outputs are considered as an Annex.

The proposed method based on the DClet is presented in second section. In third section, experimental results are shown. The article is concluded with some discussion and an outlook.

## VISUALIZATION METHOD BASED ON THE DCLET

The idea of the DClet is first to decompose the input data into a set of wavelets bands and analyze each band by a non-redundancy Finite Ridgelet Transform (FRIT) [9]. This idea makes a directional wavelet that outperforms wavelet for orientation. It is scale sample of wavelet transform following a geometric sequence of ratio 2. The input is decomposed into smooth blocks of side length  $l$  units in such a way that adjacent blocks are square array of

size  $l \times l$  although it is possible to be rectangular array of size  $l \times l/2$  without overlapping. Fig. 1 shows the general concept of the DClet. It achieves non-redundancy transformation with invertibility via the FRIT. The FRIT is ridgelet transform based on the Finite RADon Transform (FRAT) that uses the orthogonal symmlet wavelet with four vanishing moments. For the FRAT, first the set of normal vectors is obtained which indicate the represented directions.

The decomposition transform for an input matrix with size  $M \times M$  is done according to the following procedure:

First, the 2D wavelet transforms  $(\bullet)W_A^r(b)$  of a matrix  $A$  with size  $M \times M$ ,  $A \in Z^{M \times M}$  at scale  $r$  and translation  $b$  with the mother wavelet  $\psi$  is given by:

$$(\bullet)W_A^r(b) = A(p_1, p_2) * (\bullet)\psi_b^r(p_1, p_2), \quad (1)$$

where  $b = \{1, 2, \dots, M/2^r\}$  and  $(\bullet)$  shows the approximation or details and  $*$  denotes convolution operation:

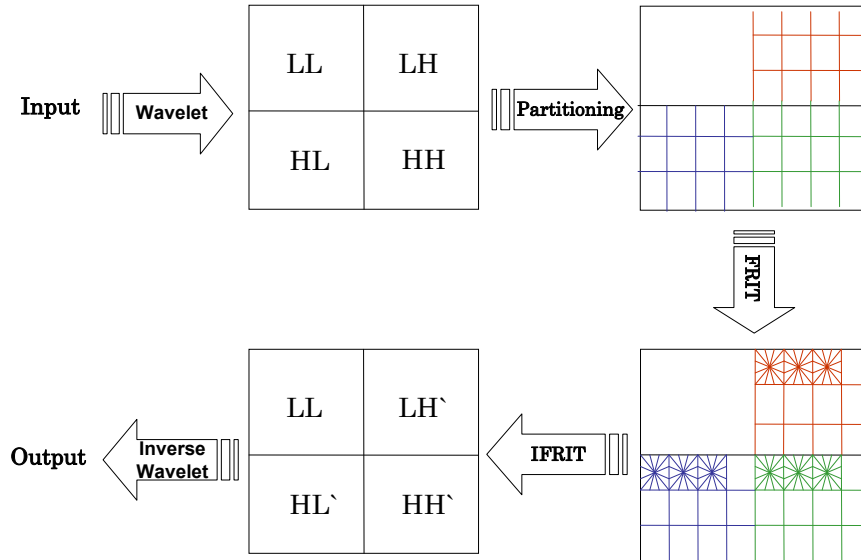
$$(\bullet)W_A^r(b) = \sum_{p_1=1}^M \sum_{p_2=1}^M A(p_1, p_2) (\bullet)\psi_b^r(p_1, p_2). \quad (2)$$

Then, subband decomposition:

$$A(p_1, p_2) \Rightarrow (LL)W_A^r(b) + (LH)W_A^r(b) + (HL)W_A^r(b) + (HH)W_A^r(b), \quad (3)$$

where a high pass filter (H) and a low pass filter (L) are applied to the input in both horizontal and vertical directions, and three orientation selective high pass subbands, HH, HL, LH, and a low pass subband LL are generated. Afterwards, smooth partitioning is done via windowing each subband into  $\Delta$  windows of size  $l \times l$  as well as  $l \times l/2$ . The coarse description of the input data is not processed that it makes an anisotropic (bi-)orthogonal transformation without redundancy:

$$(\bullet)\Delta(p_1, p_2) = \Delta[(\bullet)W_A^r(b)], \quad (4)$$



**Fig. 1.** The conceptual model of the extended Curvelet named the DClet.

Finally, the function is oriented at an angle  $\theta$  and the FRIT is applied to each window:

$${}^{(\bullet)}R_{\Delta}^r(b, \theta) = \sum_{p_1=1}^{M/2^r} \sum_{p_2=1}^{M/2^r} {}^{(\bullet)}\Delta(p_1, p_2) * {}^{(\bullet)}\psi_{b, \theta}^r(p_1, p_2), \quad (5)$$

$${}^{(\bullet\Delta)}R_{\Delta}^r(b, \theta) = {}^{\bullet}\Delta(p_1, p_2) * \psi_{b, \theta}^r(p_1, p_2), \quad (6)$$

therefore, for  $M = 2^r$ ,  $r - 1$  level of the DClet will be held.

There is an analogy scaling with wavelet algorithm. Using (bi-)orthogonal wavelet causes to have dyadic scaling layers instead of fixed scaling numbers for analyzing and decomposition. The construction of (bi-)orthogonal wavelets is equivalent to the synthesis of perfect reconstruction filters having a stability property. This class of wavelets is characterized by a maximal number of vanishing moments for some given support. With each wavelet of this class, there is a scaling function called father wavelet which generates an orthogonal multiresolution analysis.

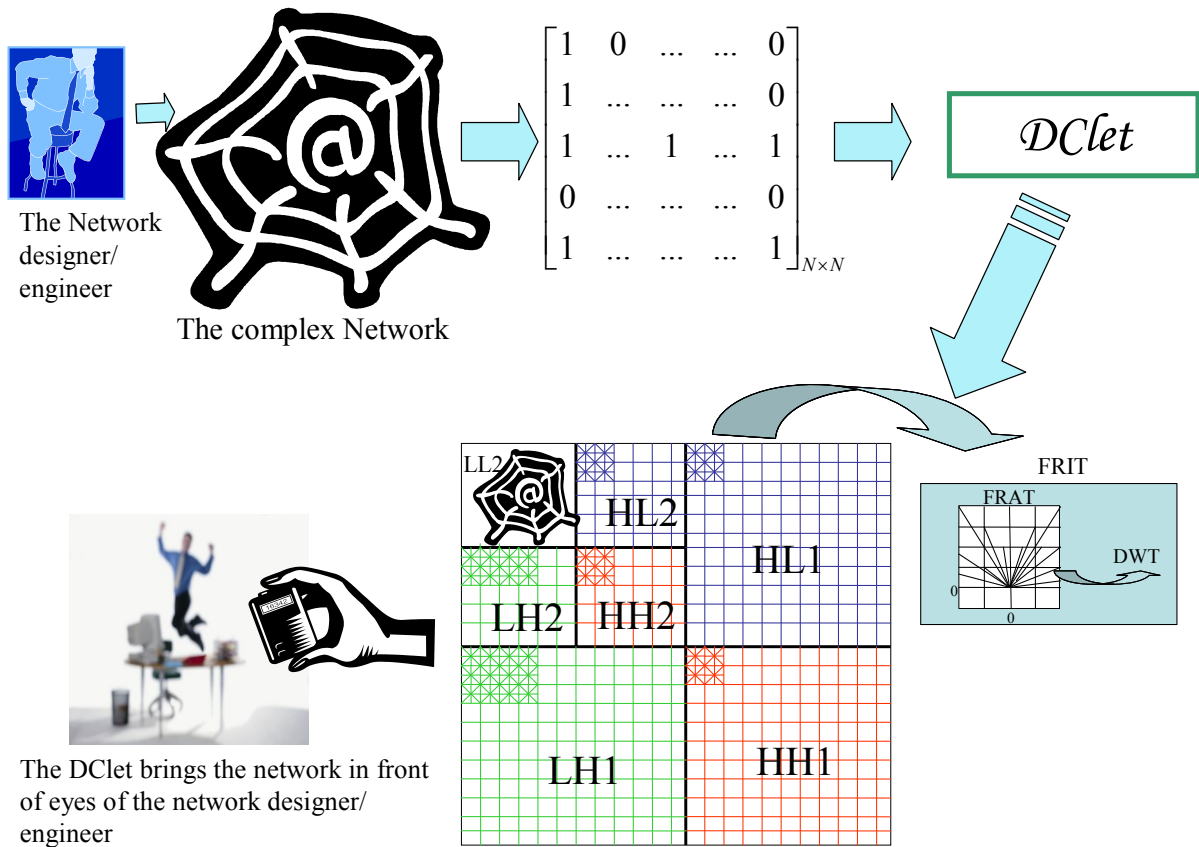
This extension helps the Curvelet transform to make smoother strategy, also may solve a significant redundancy problem of Curvelet:

- Fixed subband definitions is changed to muliresolution transformation using the DClet. Via Curvelet transform an input matrix with size  $256 \times 256$  partitioned the frequency domain into only 3 subbands indexed by  $r = 1, 2, 3$  [7].
- The need of subband filtering is slightly removed via the DClet, while to actually implement Curvelet decomposition into subbands the wavelet transform should be used. First the object must be decomposed into its 8 wavelet subbands, then to form Curvelet subband  $r$ , partial reconstruction should be performed [7].
- The Curvelet transform is a redundancy transformation. The spatial partitioning introduces a redundancy factor equal to four for it. Using non-redundant wavelet and applying non-redundant FRIT makes the DClet applicable for different applications such as compressing a complex network, representing a huge network with less data as possible.
- The theory imposes a down sampling on the set of orientations by a factor two as one proceeds to the coarser scale via the FRIT. In some sense, the digital implementation increasingly over samples the angular variable at coarser scales.

Fig. 2 shows how the proposed method filters the input data in dyadic scales and optimizes a network by removing redundant edges. Using the DClet a high pass filter (H) and a low pass filter (L) are applied to a the network in both horizontal and vertical directions, and the filter outputs sub-sampled by a factor of two, generating three orientation selective high pass subbands, HH, HL, LH, and a low pass subband LL. Afterwards smooth partitioning is done via windowing each subbband, and the FRIT is applied to each window. As the FRIT uses the FRAT, first we obtain the set of normal vectors which indicate the represented directions. The information contained in subband  $LL_1$  can be viewed as a coarse description of the network; the others correspond to higher resolution and involve the finer details of the network at different smaller scales.

The method enjoys exact reconstruction and stability, because each step of the transform is both invertible and stable. For reconstruction, first subband recomposing is done. Then we apply the Inverse FRIT (IFRIT) to each block, finally via subband synthesis the input is reconstructed.

This method gives the possibility to work on a small among of data or information in dyadic scales and get the real modified data that shows the exact points of changes on the network.



**Fig. 2.** Visualization of complex networks via the DClet filtering. The upper left square labeled *LL* corresponds to the lowest resolution subspace. The other regions involve higher resolution subspace.

To analyze complex networks via the DClet, it is assumed a network with vertices makes the input matrix with size  $N \times N$ . In this approach, each vertex is represented as a pixel and the edge weight is assumed as the distance between the visual properties of each pair of pixels in the matrix for instance gray level in an image.

In the construction of the FRIT, we need wavelet bases for prime length signals. Let  $p$  be the nearest dyadic number to prime length that is smaller than or equal to the prime length window used as the input. This is because of definition of the FRAT as summations of pixels of input matrix over a certain set of lines. Those lines are defined in a finite geometry in a similar way as the lines for the continuous Radon transform in the Euclidean geometry.

Compared to wavelet [10, 11], the DClet adds orientation and operates by measuring information about an object at specified orientation. This provides more details information about the network and outperforms wavelet.

## EXPERIMENTAL RESULTS FOR VISUALIZATION METHOD

A set of vertices joined by edges is only simple type of network, there are many ways in which networks may be more complex. To evaluate performance of the proposed method, it is applied to two networks with different structural properties, first a network with structural properties of small world with size  $N = 16$ , and afterwards a globally coupled network with  $N = 1024$  vertices. The globally coupled network has the smallest average path length and the largest clustering coefficient. As most real networks are neither fully connected nor entirely

random a small world network is chosen as a network that describes a transition from a regular fully connected network to a random graph.

### A SMALL WORLD NETWORK

A network with structural properties of small world network is represented with rewiring probability  $p$  by its adjacency matrix  $\mathbf{A} \in \mathbb{Z}^{N \times N}$  and represents the coupling configuration of the network, if there is a connection between vertices  $i$  and  $j$  ( $i \neq j$ ) then  $A_{ij} = A_{ji} = 1$ , otherwise  $A_{ij} = A_{ji} = 0$ . The DClet is applied to this network with frame elements indexed by scale and location parameters. As it is a multiresolution dyadic transform, it is repeated on the LL subband, generating the next level of the decomposition.

Suppose that a vertex  $i$  in the network has  $k_i$  edges. Clearly the most  $k_i(k_i - 1)/2$  edges can exist among them and this occurs when every neighbor of vertex  $i$  connected to every other neighbor of vertex  $i$ . The average path length  $L$  of the network is defined as the mean distance between two vertices, averaged over all pairs of vertices:

$$L = \frac{2}{N(N+1)} \sum d_{ij}, \quad (7)$$

where  $d_{ij}$  is distance between two vertices. More precisely, one can define a clustering coefficient  $C$  as the average fraction of pairs of neighbors of a vertex that is also neighbors of each others. The clustering coefficient  $C_i$  of vertex  $i$  is defined:

$$C_i = \frac{2E_i}{k_i(k_i - 1)}, \quad (8)$$

where  $E_i$  is the number of edges that actually exist among these  $k_i$  vertices. They found that for small  $p$ ,  $L$  drops rapidly while  $C$  remains almost unchanged.

To show that the properties of the network can be derived from coarse part of the subband,  $N$  vertices of the complex network is divided to  $N/2^r$  groups while the detail parts are divided to  $N/(l \cdot 2^r)$  where  $l$  is number of windows  $\Delta$ , therefore:

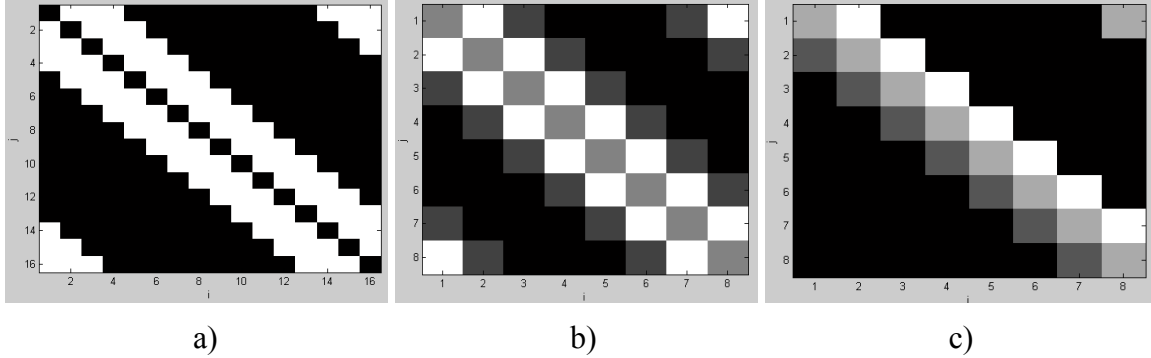
$${}^{(LL)}W_A^r(b) = \sum_{i=1}^{N/2^r} \sum_{j=1}^{N/2^r} A(i, j) {}^{(LL)}\psi_b^r(i, j) = \sum_{i=1}^{N/2^r} \sum_{j=1}^{N/2^r} E(i, j) {}^{(LL)}\psi_b^r(i, j), \quad (9)$$

$i, j = 1, 2, \dots, 2^r$  where  $E(i, j)$  is the number of edges between group  $i$  and  $j$  ( $i \neq j$ ) or is the number of edges within group  $i$ . It is noted that  $E(i, j)$  is considered as one pixel of  ${}^{(LL)}W_A^r(b)$ . The detail parts are calculated based on (6).  $\mathbf{A}$  in  $r$  scale corresponds to a division of the lattice into  $N/2^r$  groups and the diagonal elements in the lowest frequency subband LL of the DClet transformation represents the normalized number of edges within each group, while the non-diagonal elements in LL represent the normalized number of edges between different groups. Here the normalized constant is  $2^r$ .

The non-diagonal element  $W_a^r(b)$  characterizes the connection property between two different groups  $i$  and  $j$  ( $i \neq j$ ).  ${}^{(LL)}W_a^r(b) \neq 0$  if and only if there is at least one edge between the two groups. Higher value of  ${}^{(LL)}W_a^r(b)$  implies more edges between the two groups and among vertices within the group illustrated in Fig. 3. A vertex in the new network corresponds to a group in the original network, and there is an edge between two different vertices  $i$  and  $j$  in the new network if and only if there is at least one edge between groups  $i$  and  $j$  in the original network for example  ${}^{(LL)}W_a^r(b) \neq 0$ . The figure represents details of original network and helps the reader observes the qualitative similarities between the proposed method with different probabilities and original data. The white blocks stand for edges and the black parts no edges. In the DClet view the gray parts represent the negative number that in the network

analyzing can be considered as no connection or on the other word no edge. Light gray, with a high approximation, can consider as an edge; by this order the network designer can extract exact meaning of different levels of grayness. Also the size of output is considerable. The DClet decomposition gives us a smaller matrix of the input data with the same properties of the real network. It is a solution for visualizing complex networks in different smooth zooms.

The clustering property of the network can be derived from LL. If value of element  $^{(LL)}W_a^r(b)$  of LL is much bigger than the value of  $^{(LL)}W_a^r(b)$  for all  $i$  and  $j$  ( $i \neq j$ ), then the edges within each group would be much more than the edges between different groups, which implies that the network is highly clustered. In simulation, the DClet decomposition of a network with  $N = 16$  an average degree of  $k = 6$  edges per vertex is represented.



**Fig. 3.** The DClet representation of the lattice as small world network. The DClet view of the network gives similar structural properties of the network. a) Original matrix ( $p = 0,8$ ), b) LL subband of the DClet ( $p = 0,8, r = 1$ ), c) LL subband of the DClet ( $p = 0,38, r = 1$ ).

Choosing the number of zero non-diagonal elements in  $LL$  instead of  $L$ , and error between the minimum diagonal elements and the maximum non-diagonal elements in  $LL$  instead of  $C$  give us similar properties. Small value of number of zero non-diagonal elements and large value of error imply small characteristic path length and large clustering coefficient of the original network.

### GLOBALLY COUPLED NETWORK

A globally coupled network with  $N = 1024$  vertices is chosen for evaluating the performance of the proposed visualization method. This network with the smallest average path length and the largest clustering coefficient is a dynamical network which is suitable for testing the chaotic state [12]. That is a simple network, which has  $N(N - 1)/2$  edges. It is outlined mainly for sake of clarity of performance of proposed method. The proposed method is applied to the coupling matrix  $A$ :

$$A = \begin{bmatrix} -N+1 & 1 & \dots & \dots & 1 \\ 1 & -N+1 & 1 & \dots & 1 \\ \dots & \dots & -N+1 & \dots & \dots \\ 1 & \dots & \dots & -N+1 & 1 \\ 1 & 1 & \dots & 1 & -N+1 \end{bmatrix}_{N \times N}.$$

Fig. 4 illustrates the results in the different scales. These observations display that the DClet gives an inclusive topology of a complex network via a coarse to fine strategy for characterizing and classifying networks by processing minimum amount of information. In this figure white background is equal to 1 and the diagonal line stands for no edge. The DClet decomposition of the coupled network gives the similar properties of the network to a network designer. It brings a small understandable dimension view of the network to his/ her

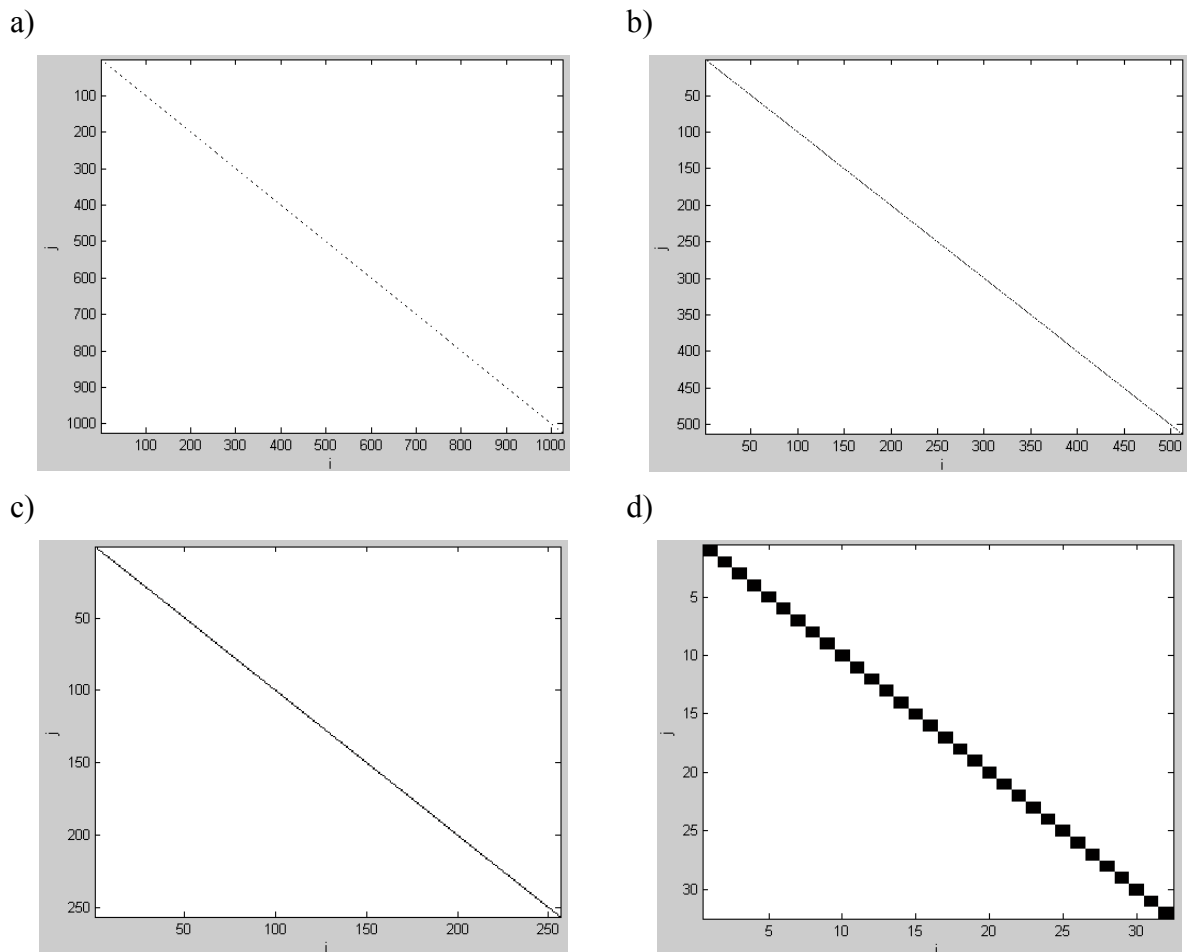
eye. Having a perspective of network is better than designing based on guess. As the structural properties of the network are robust in different scales, it can be a trustable platform for tracking the behavior of the network through the smaller matrix and find the exact points of changes by the DClet reconstruction.

The evaluated coupling network is a fully connected network with 1024 vertices and 523 776 edges, while the most large scale real networks appear to sparse, that is most real networks are not fully connected and their number of edges is generally of order  $N$  rather than  $N^2$ .

As a side comment, it is noted that for the above reason, it is tested on the intercity network of telecommunication network of Iran as a real complex network too. This network has been distributed in a country having the area of 1 648 000 km<sup>2</sup>, more than 18 million fixed telephone lines, 92 primary and secondary switching vertices and 7171 homing. The total 636 900 input and output traffics as totals 706 350 E1 traffic channels are distributed through this network. The result exhibits that the telecommunication network designer can get similar information through the DClet decomposition, make different traffic behaviors, compare the results and make final decision only via a smaller matrix. It yields a general view of network to designer that usually is enough to see the traffic behavior of the network. See Annex A.

It is observed:

- the DClet provides a coarse to fine strategy for characterizing and classifying a complex network by processing minimum amount of information,



**Fig. 4.** Simulation results of visualization of the coupling network via the DClet. a) Original input with size  $1024 \times 1024$ , b) LL subband of the DClet ( $r = 1$ ), c) LL subband of the DClet ( $r = 2$ ) and d) LL subband of the DClet ( $r = 5$ ).



- it behaves like the brain in terms of segments and distinguishes them by scale and orientation,
- the DClet enjoys superior performance over the FRIT, regardless of block size,
- properties of a complex network will be derived using the DClet regardless of the structure of the network,
- information in approximation parts can be viewed as a coarse (large scale) description of the network and the other regions correspond to higher resolution and involve the finer details of the networks in vertical and horizontal directions with different orientations,
- as it is zoomed in smaller scales, details can be seen that before could not. The DClet outperforms wavelet for orientation, deriving more information from the detail parts. Accordingly, highly localized features can be detected without exhaustive searching of the entire high resolution input,
- in comparison, every structure in the input which is visually detectable is clearly displayed in the DClet reconstruction by adding orientation. Indeed, it has been suggested that the analogy between the DClet and human vision is no accident, and that our neurons filter visual signals in a similar way of the DClet.

The results prove that the DClet can be considered as a simulation tool for successfully designed network topology and established necessary group sizes.

As a side comment, it is noted that output size is unexpected sometimes. This mismatching comes via the FRIT which accepts only inputs with the size of prime numbers. Fixing this mismatching issue is a job in the future.

The DClet gives the network designer a general view of network. It helps the designer to configure the network. It as a simulation tool provides the opportunity to see different view of network. It gives the possibility of working with less data to trace the properties and behaviors of network. For example as the topology of the network is robust in any scales of the DClet, we can work on a smaller matrix and sketch the topology of network. It is a trustable efficient tool for surveying. We can model the real network with the DClet reconstruction. Doing some changes in the small dimension of the network, and extract a real dimension of the network with reconstruction and find the exact points of changes for further designs is another fact.

## **CONCLUSION**

Networks are inherently difficult to understand because of structural complexity. Making reasonable the knowledge of scientists about the elements and forces of networks without having a view of network is difficult. Understanding complex networks can be possible by sketching their structures with actual points and links and then looking at them by eyes. However for a network with millions of vertices such a direct analyzing by human eyes is hopeless. It is impossible drawing a meaningful picture of the network. Assuming that the network architecture is static allows us to sidestep any issues of structural complexity and to propose a visualization method for understanding the topology of complex networks. The proposed method is based on an extended Curvelet transform named Dyadic Curvelet Transform (DClet). This extension makes it possible to generate the multiscale non-redundant Curvelet transform that provides a coarse to fine strategy for characterizing and classifying networks by processing the minimum amount of information. This statistical method quantifies huge networks and plays the part played by eyes for small networks.

The performance of the proposed method is evaluated in a network with structural properties of small world network with size  $N = 16$  vertices and a coupled network with  $N = 1024$ . It is observed the connectivity of complex networks is robust in sense that the networks are still connected even when a high percent of randomly selected vertices and or edges are removed so that some desired properties of the network remain unchanged. This strategy reduces the computational complexity by starting at low resolution and increasing the resolution when it is necessary; also provides the minimum among of data which are necessary to perform a qualitative task. Since the coarse coefficient processing can be performed quickly, it gets higher computational efficiency.

It is expected the DClet to be widely applicable, especially in fields of image processing and computer vision. In image analysis for example, the DClet may be used for compression, enhancement and restoration of images, and for post processing applications such as extracting patterns from large digital images and detecting features embedded in noisy image. The experimental results show that the proposed filtering method can behave in the same matter as human eyes, processing an object by filtering the input data into a number of bands and levels. It can be a new way for controlling dynamical characteristics such as synchronization of complex networks. It can suggest a way of compressing complex networks, representing a huge network with as less data as possible.

## REFERENCES

- [1] Wang, X.F. and Chen, G.: *Complex networks small-world, scale-free and beyond*. IEEE Circuits and Systems Magazine, 6-20, 2003,
- [2] Newman, M.E.J.: *Models of the small worlds, a review*. <http://arxiv.org/abs/cond-mat/0001118>,
- [3] Brooks, D.: *Complex systems*. Carleton College, 2004 – 2005, <http://www.cdsp.neu.edu/info/faculty/brooks/stuff/complexnetworks.pdf>,
- [4] Barthelemy, M. and Amaral, L.A.N.: *Small-world networks: evidence for a crossover picture*. Physical Review Letters **82**(15), 3180-3183, 1999, <http://polymer.bu.edu/~amaral/Papers/prl99z.pdf>,
- [5] Strongatz, S.H.: *Exploring complex networks*. Nature **410**, 268-276, 2001,
- [6] Newman, M.E.J.: *The structure and function of complex networks*. SIAM Review 45, 167-256, 2003,
- [7] Donoho, D.L. and Duncan, M.R.: *Digital curvelet transform: strategy, implementation and experiments*. <http://www-stat.stanford.edu/~donoho/Reports/1999/DCvT.pdf>,
- [8] Candes, E.; Demanet, L.; Donoho, D. and Ying, L.: *Fast discrete curvelet transforms*. 2005,
- [9] Do Minh N. and Vetterli, M.: *The finite ridgelet transform for image representation*. IEEE Transactions on Image Processing **12**(1), 16-28, 2003, [http://lcavwww.epfl.ch/%7Eminhdo/publications/FRIT\\_trans.pdf](http://lcavwww.epfl.ch/%7Eminhdo/publications/FRIT_trans.pdf),
- [10] Fan, J. and Wang, X.F.: *A wavelet view of small-world networks*. IEEE Transactions on Circuits and Systems-II **52**(5), 238-241, 2005,
- [11] Ma, S. and Ji, C.: *Modeling heterogeneous network traffic in wavelet domain*. IEEE/ACM Transaction on Networking **9**(5), 634-649, 2001,
- [12] Zhang, H.; Wu, R. and Fu, X.: *The emergence of chaos in complex dynamical networks*. Elsevier, Chaos Solutions and Fractals, pp. 472-479, 2006.

ANNEX A

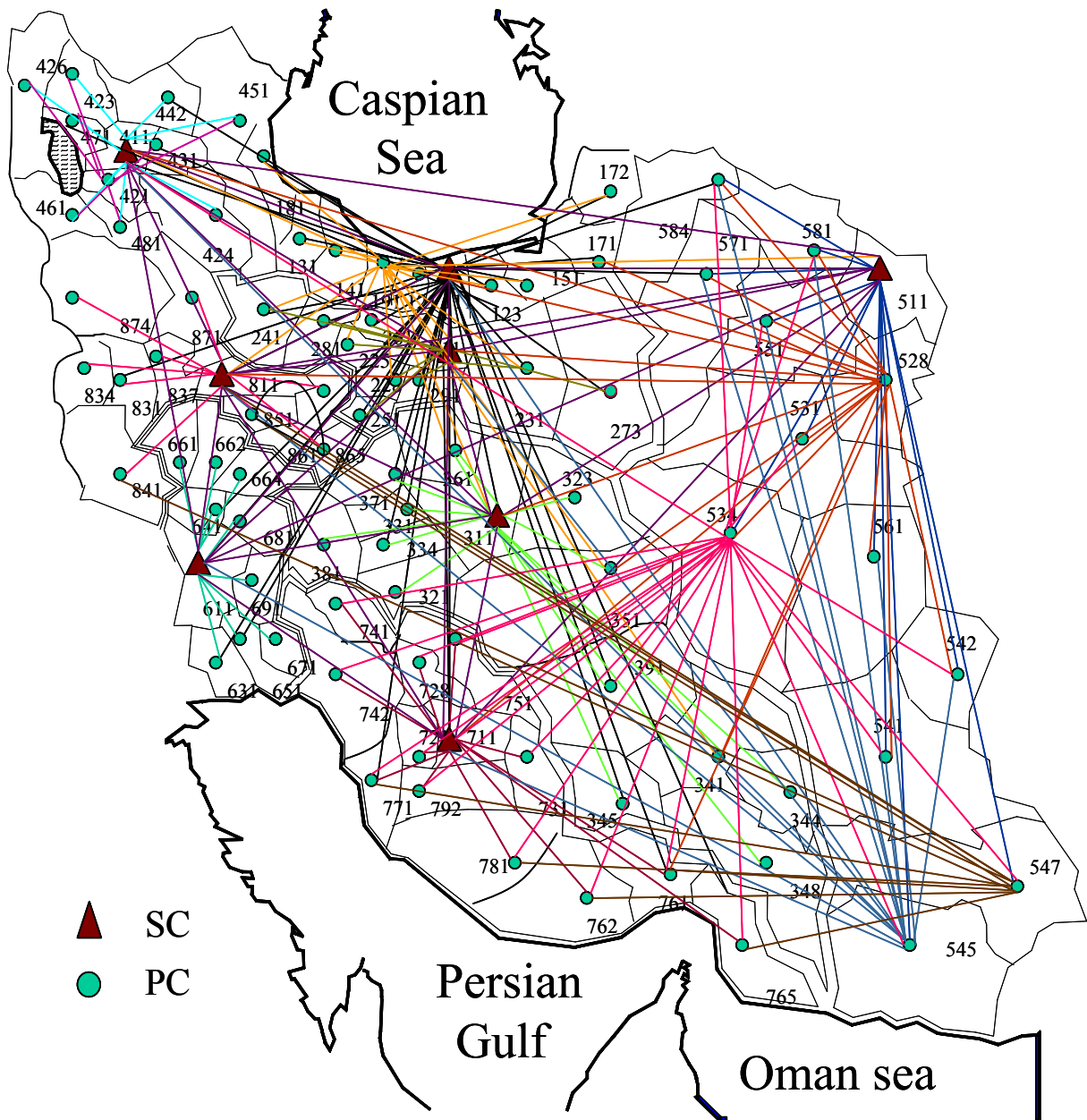


Fig. 5. Configuration of the trunk network of telecommunication network of Iran. The trunk network is extremely complex that understanding the structure of this network by eyes is impossible.

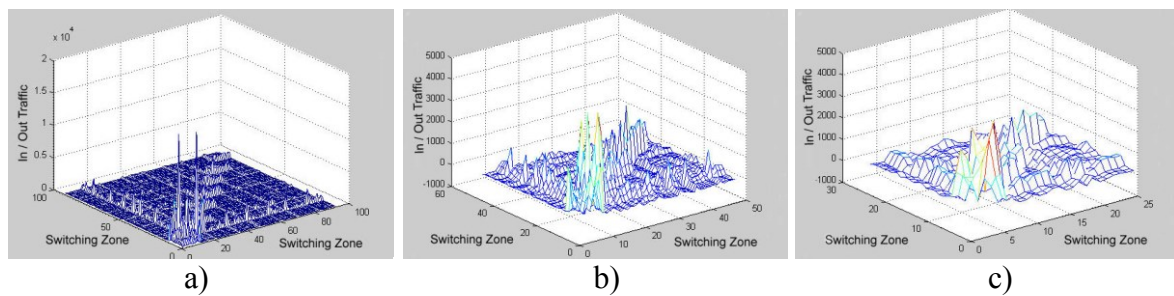


Fig. 6. Simulation results of the trunk network in Iran using the DClet. a) Trunk network, b) Level 1 of the DClet (LL), c) Level 2 of the DClet (LL).

## VISUALIZATION OF COMPLEX NETWORKS BASED ON DYADIC CURVELET TRANSFORM

M. Sedighi Anaraki<sup>1</sup>, F. Dong<sup>1</sup>, H. Nobuhara<sup>2</sup> and K. Hirota<sup>1</sup>

<sup>1</sup>Odjel računalne inteligencije i znanosti o sustavima, Tokijski institut za tehnologiju

Tokio - Jokohama, Japan

<sup>2</sup>Odjel za inteligentne tehnologije međudjelovanja,

Viša škola za sustave i inženjerstvo informacija, Sveučilište u Cukubi,

Ibaraki, Japan

### SAŽETAK

Predložena je metoda vizualizacije za razumijevanje strukture kompleksnih mreža, koja se temelji na proširenoj transformaciji *Curvelet* pod nazivom diadska Curvelet-transformacija. Predložena metoda doprinosi odgovaranju na pitanja o strukturi kompleksnih mreža preslikavanjem podataka u ortogonalne lokalizirane događaje s usmjerenom komponentom određenom putem uzorkovanja skupa koeficijenata. Time je postignuta sličnost sa sustavom za viziju čovjeka, jer vidi u segmentima koje razlikuje po dimenziji i orijentaciji. Posebno je pitanje komprimiranje mreže. Djelovanje predložene metode ispitano je pomoću dvije mreže strukture mreža malog svijeta s  $N = 16$  čvorova i globalno vezane mreže s  $N = 1024$  čvorova i 523 776 bridova. Budući da većina stvarnih mreža velikih skala nije potpuno povezana, metoda je provjerena na telekomunikacijskoj mreži Irana. To je ekstremno kompleksna mreža s 92 čvora, međugradska preklopnika, 706 350 kanala E1 i 315 525 prijenosnih kanala. Pokazano je da predložena metoda djeluje kao simulacijskih alat za učinkoviti dizajn mreže i postavljanje potrebne veličine grupe. Metoda upućuje dizajnera mreže na sva strukturalna svojstva mreže.

### KLJUČNE RIJEČI

vizualizacija, kompleksne mreže, sustav za viziju čovjeka