

SPECIAL FINITE ELEMENTS: THEORETICAL BACKGROUND AND APPLICATION

Dušan Kovačević, Igor Budak, Aco Antić, Borut Kosec

Subject review

Special finite elements (FEs) are used for modeling of certain phenomena in structural behavior. The term "special" denotes FEs with specific characteristics (stiffness parameters, behavior under load, etc.) and a specific role in modeling, which is their main difference from "common" FEs. There are many types of special FEs, but the following are often used: so-called "nonlinear spring" FE, "gap" FE and "link" FE. Various examples will show many advances in the application of these FEs, especially in modeling of boundary and interface conditions. Theoretical background is given only in the amount necessary for proper understanding of numerical aspects of the application of special FEs.

Keywords: boundary and Interface conditions, FE modeling, special FE

Specijalni konačni elementi: Teorijska podloga i primjena

Pregledni članak

Specijalni konačni elementi (KE) koriste se za modeliranje posebnih fenomena u ponašanju konstrukcija. Izraz "specijalni" označava KE sa specifičnim svojstvima (parametri krutosti, ponašanje pod opterećenjem, itd.) i posebnom ulogom u modeliranju, po čemu se i bitno razlikuju od "standardnih" KE. Postoji više tipova specijalnih KE, ali se najčešće koriste: "nelinearna opruga" KE, "zazor" KE i "veza" KE. Na različitim će primjerima biti pokazane prednosti primjene specijalnih KE, posebno u modeliranju konturnih i prijelaznih uvjeta. Teorijska podloga data je samo u neophodnom obimu, zbog dobrog razumijevanja numeričkih aspekata primjene specijalnih KE.

Ključne riječi: konturni i prijelazni uslovi, modeliranje KE, specijalni KE

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Introduction

Finite Element Method (FEM) evolves a particular "technology" for structural analysis, mostly because of intensive development of the FEM based CASA (Computer Aided Structural Analysis) software. The use of CASA software for modeling of structural behavior strongly depends on possible selection of analysis and available type of FEs. One possible classification of FEs [1], based on FE type and role in modeling is shown in Fig. 1.

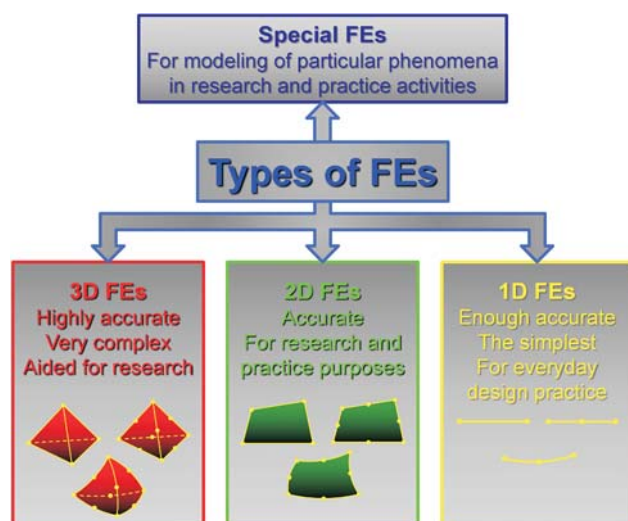


Figure 1 FEs categorization with regard to the type and role in modeling

So-called "special" FEs have an important place in this categorization. The term "special" denotes FEs with specific performances (immateriality, defining parameters, behavior, etc.) and a specific role in modeling, which is their main difference from "common" FEs, [2-4]. Special FEs are used for modeling of wide range of compatibility conditions

of all degrees of freedom between FE system joints or between FE system joints and some external joints. Possibilities are several:

- various types of indirect connections between FEs or between FEs and structural supports,
- various types of shear, sliding or frictional conditions between FEs and
- changing of behavior of connection between FEs because of change of structural conditions, etc.

2

Theoretical background

The mentioned connection models can be categorized as "boundary conditions" (related to supports) and "interface conditions" models (for relations between FE joints). In numerical treatment of these conditions so-called "removal" and "restraint" methods are used, [5].

In that sense, the term "single-joint restraint" is used for boundary condition while the term "multi-joint restraint" is used for definition of interface conditions. Usual example for a "multi-joint restraint" is a model of connection between 1D beam FE and 2D plate FE, Fig. 2.

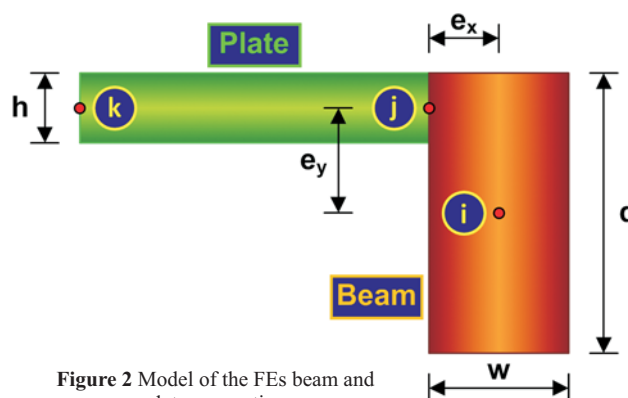


Figure 2 Model of the FEs beam and plate connection

Beam and plate FEs have no common nodes and their axes/edges do not coincide in this model. Offsets in local FE axes direction are e_x and e_y . Mathematically, it is necessary to equalize degrees of freedom (DOF) i.e. displacements of the beam and plate FEs. Matrix relation between corresponding DOF in "i" (so-called "master" joint) and "j" (so-called "slave" joint) is:

$$\begin{Bmatrix} u_i \\ v_i \\ \theta_i \end{Bmatrix} = \begin{bmatrix} 1 & 0 & e_y \\ 0 & 1 & e_x \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{Bmatrix} u_j \\ v_j \\ \theta_j \end{Bmatrix} \quad (1)$$

Possibilities of numerical treatment of this "multi joint restraint" generic equation are, [2]:

- "master-slave elimination",
- "penalty function augmentation" and
- "Lagrange multiplier adjunction".

Regardless of the method for achievement of proper numerical realization of "multi-joint restraint" i.e. "interfaces conditions" modeling, special FEs could be categorized in the following generic groups:

- "link" special FEs,
- "nonlinear spring" special FE and
- "gap" special FE.

The mentioned special FEs are usually two nodes 1D FEs with all six DOF in each node. The field of use of special FEs with necessary explanations will be given in the following sections.

There is one alternative concept for the modeling of boundary and, especially, interface conditions in the FEM technology. It is based on standard FEs adapted for the special conditions. Such example is standard beam FE adapted with the possibility for modeling of contact.

Common opinion is that the approach based on special FEs is more universal and gives wider opportunities for modeling of various phenomena.

Probably the best representative of this approach is ANSYS® CASA software with a rich library of FEs for modeling of connections: 2D/3D interface FEs, node/node, node/surface and surface/surface, edge/surface contact FEs. SAP® and AxisVM® contain 1D link, gap and spring FEs as well as Scia Engineer® (only gap and nonlinear spring FEs). CASA software for everyday structural design office practice (Tower®, for example) uses adapted standard FEs with "offset" and "eccentricity" modeling capability, which is described in Fig. 2.

3 Indirect connections in FE models

It is well known that FEs can be connected directly (in common nodes) and indirectly. Indirect connections can be a consequence of "offset" i.e. distance between axes, edges or nodes of FEs in model, Fig. 3.

The examples are: longitudinal/lateral [6] connections between beam FEs, with/without eccentricity, between elements with various functions in structural system (wall-plate, column-beam, beam-plate...), etc.

The link FEs is ultimate for modeling in these cases. Definition of the link FE needs a setup of the following:

- six parameters (three for translational and three for

rotational DOFs) which define stiffness of the modeled indirect connection and

- position of "interface" i.e. common point in the modeled indirect connection.

It is possible to define ideal rigid connection by the choice of proper stiffness values of all six stiffness parameters. Setup of the link FE parameters in AxisVM® CASA software is presented in Fig. 4.

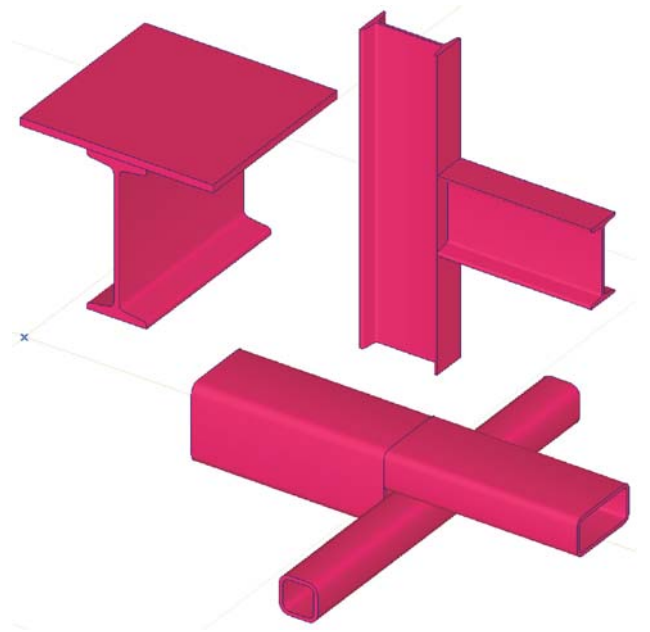


Figure 3 Examples of indirect connection in models

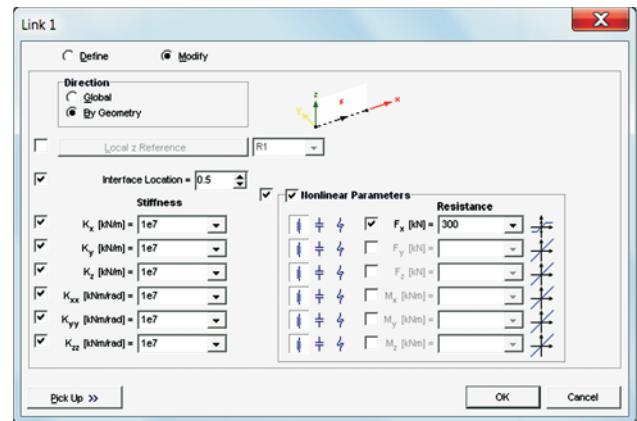


Figure 4 Definition of the link FE parameters in AxisVM®

This dialog box offers a very useful possibility of the setup of various levels of the link FEs functionality (regardless of stress state, only for tension state, only for compression state) and limit (failure) force. Of course, it needs the application of nonlinear analysis which will be widely discussed in Section 5.

Similar case is with the other mentioned CASA software. Definition of the FEs parameters is very simple and given in intuitive way. Of course it needed proper knowledge in the FEM technology field, what is standard in the engineering education, as well as some changes in technical regulations (EC standards) for a full success of the FEs practice.

4 Shear and hinged connections in FE models

Many connections in civil and mechanical engineering are based on shear/frictional stiffness and sliding/rolling capability between two or more parts made by one or more materials [7, 8]. Fig. 5 shows some of these cases. Typical examples of these connections are coupling of structural parts made by different or same materials (reinforced concrete, steel-concrete composite, glue-laminated wood) and rolling/hinged connection (axle/bearing joint) [9], etc.

Typical use of special FEs (link FEs especially) is characteristic for this area of modeling. It is possible, by appropriate choice of shear/friction stiffness parameters of the FE link, to model various types of connections, which will be illustrated by a few examples.

It is clear that the use of such types of models needs accurate information about friction/adhesion properties of the connection which can be achieved only by experiment.

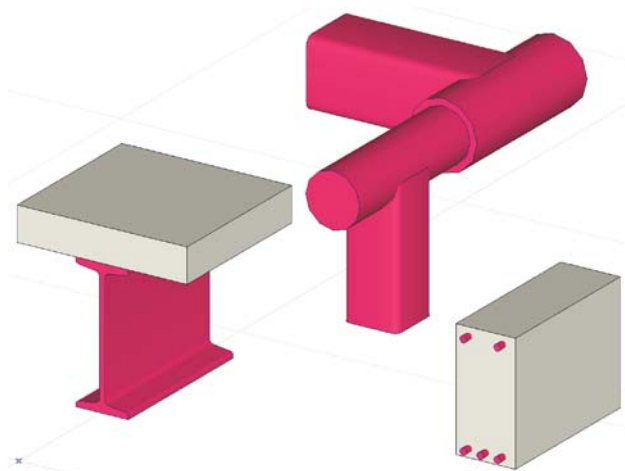


Figure 5 Examples of shear and hinged connections

5 Behavior changing of connection in FE models

This group of models has connections with variable behavior under certain circumstances. The most frequent situations of such kind are the cases of breakup or merge of model connections in some load or displacement conditions. Fig. 6 shows an example of the model's nodes which will be merged when the distance between them almost completely disappears (e.g. steel tube below the concrete beam). Similarly, when the force between the merged joints reaches some limit state, the connection state will be changed. (e.g. wood insert between steel tube and concrete beam). The "gap" and "nonlinear spring" special FEs are good solution for modeling in mentioned cases.

Modeling of these phenomena needs some considerations in nonlinear theory domain. For that reason it is necessary to explain some details about nonlinear structural analysis of boundary/interface conditions.

There are many various classifications of nonlinear phenomena/effects. According to the influence of boundary/interface conditions to nonlinear behavior there are: phenomena of so-called "smooth" (continual) nonlinearity and phenomena of so-called "rough" (non-continual) nonlinearity.

In general, the behavior of boundary/interface conditions can be described by the application of some "smooth" nonlinear model if these conditions are constant

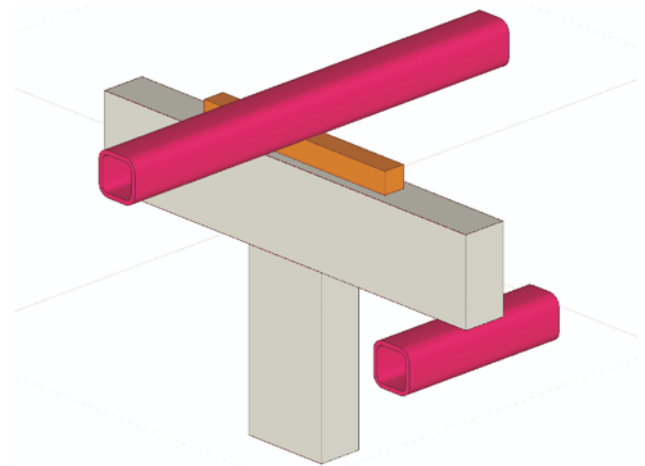


Figure 6 Connections with changeable behavior in some conditions

or continually variable. If the change of the boundary/interface conditions is non-continual, it is necessary to apply some "rough" nonlinear model.

One of the causes of nonlinear behavior in structural systems is so-called "contact nonlinearity" [9]. This nonlinear phenomenon appears due to the changes in structural systems according to the "activation" or "deactivation" of boundary/interface conditions. Usual appearance of contact problem is due to the loss of support because of the lifting of structural joint upwards or due to the realization of connection between previously non-connected structural joints. The first reason for contact problem is in the domain of the boundary conditions change, while the second one is in the interface conditions domain. The FE gap is used only in nonlinear analysis for modeling of changes in compatibility conditions of displacements between structural elements.

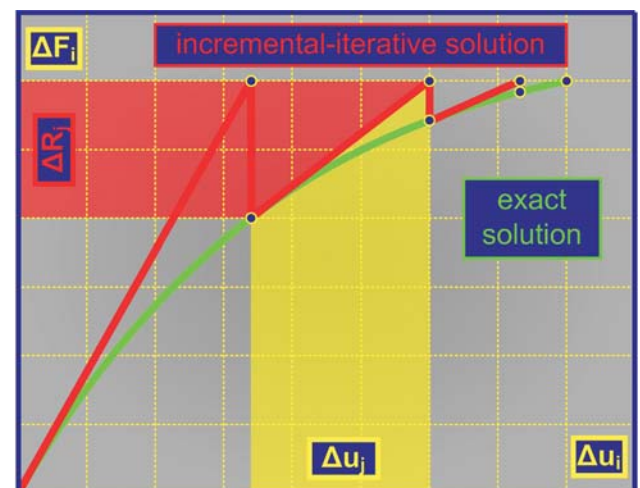


Figure 7 Incremental solution with an iterative improvement

The formulation of adequate FE models for discontinued nonlinearity is not only a formal (mathematical) problem, but it requires understanding of the causes of discontinuity of nonlinear phenomenon. For non-continual nonlinear problems, there are no common solutions, and it is necessary to formulate special techniques for every specific case of such nonlinear behavior. The only unquestionable fact is that one incremental-iterative solution technique should be used for practical solving of nonlinear boundary/interface condition problems. Fig. 7 shows a diagram of incremental-iterative nonlinear solution.

6 Numerical examples

Examples in this section will show, both in numerical and qualitative sense, several advantages of use of special FEs in modeling of structural behavior.

Fig. 8 shows stress distribution for cantilever girder which is made from the beam and plate structural elements. The model includes beam FEs, shell FEs and link FEs in between.

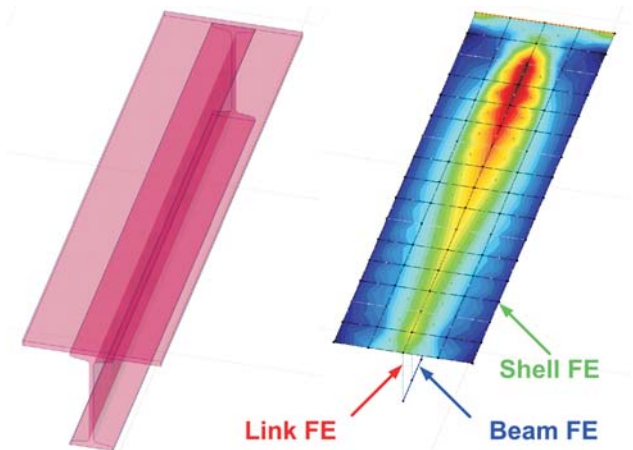


Figure 8 Link FEs for coupling of beam and plate FEs

This model and the corresponding results of analysis much better simulate the behavior of real system in comparison with the widely used model based on the 1D beam FEs of "T" shape of cross-section. Additional improvement of modeling could be achieved by the application of shell FEs for all structural parts and link FEs for connections.

Reinforced concrete floor surface structural element, Fig. 9, can be modeled by the use of shell FEs for plate, beam FEs for reinforcement and link FEs for connection between them. Bearing capacity of this plate is based on adhesive connection (so-called "bond") between steel and reinforcement and shear connection between slab and plate parts in this structural system.

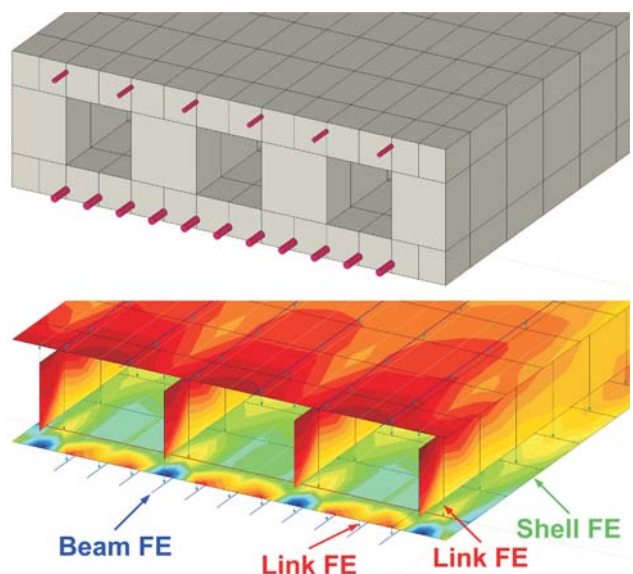


Figure 9 Link FEs in RC plate model

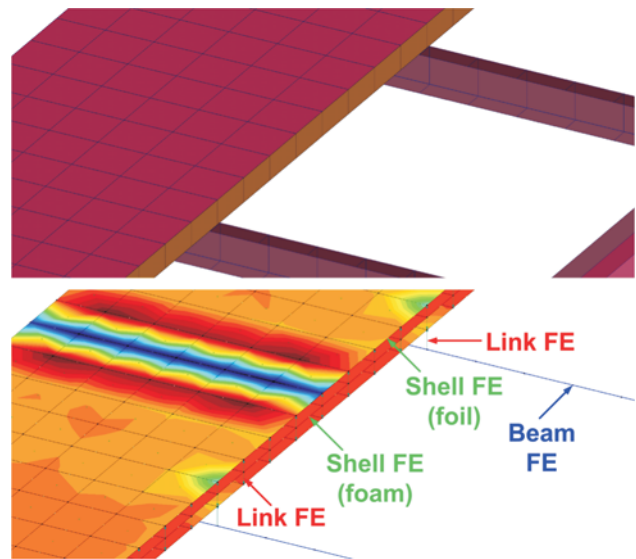


Figure 10 Link FEs in "sandwich" plate model

Behavior of so-called "sandwich" plate can be modeled in the same way, Fig. 10. This plate consists of three layers: steel foils (top and bottom face) and polystyrene foam core (between steel foils), [9].

Stiffness and bearing capacity of this plate is based on shear bond connection between foils and core. The model contains top and bottom steel foil shell FE, polystyrene foam core shell FE and link FE in between. The modeling of bond is performed by increasing or decreasing stiffness parameters of link FEs. The figure shows the stress peaks in zones with bond deterioration. This bond behavior is modeled by sufficiently small value of the link FEs shear component stiffness parameter.

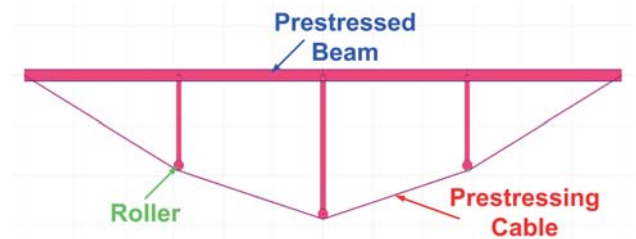


Figure 11 Externally pre-stressed beam with friction decreasing rollers

The next two examples are used to explain the possibilities of hinged/rolling connections modeling. The first one is very illustrative and presents one externally prestressed beam, Fig. 11.

Prestressing force is applied by externally located cable. High friction forces appear in the cable/column joint area, as unwanted consequence. In order to decrease friction, rollers are usually installed in cable/column joint.

In this example the connection between rollers and cables is modeled with and without link FE (for comparison reasons) and results of computation are given in Fig. 12.

The model with link FEs gives a more realistic moment and especially axial forces distribution. It is well known that the cable has a constant axial force along its length. Only with the link FEs is it possible to model the phenomenon of friction between roller and cable.

Next example shows the possibility of modeling of hinged connection. The "bearing-axle" connection of a jib structure [10] is actually a cylindrical hinge which allows only the rotation around the own axis, Fig. 13. Model of

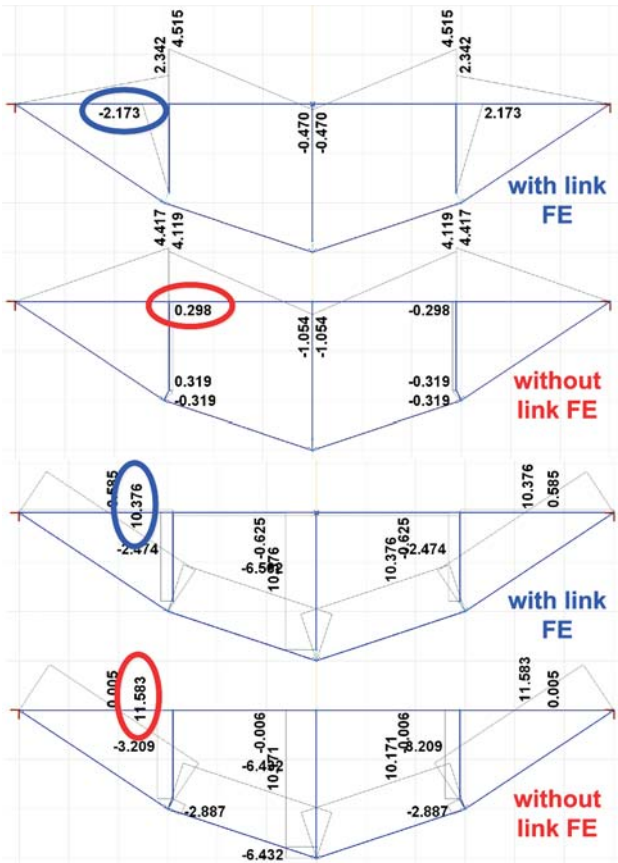


Figure 12 Moments and axial forces - model with and without link FEs

hinge with the distribution of link FEs is given in Fig. 14.

Of the six stiffness parameters, only the one for the rotation around the own axle axis has zero value (the perfect rotation without friction). All other parameters have the value corresponding to frictional stiffness. Parasite forces/moments would appear in every other case of modeling, which is not the correct presentation of the real hinge behavior.

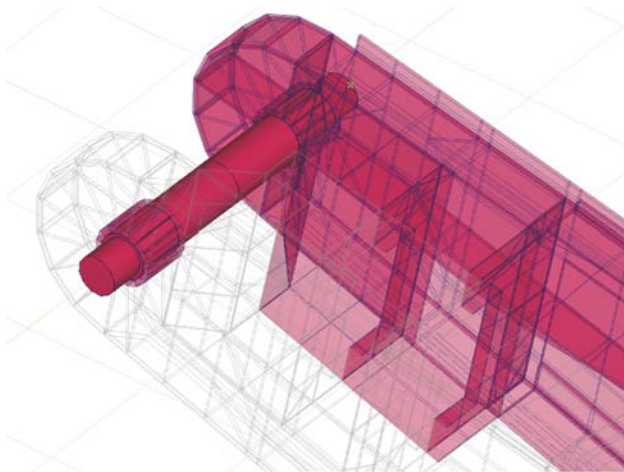


Figure 13 Hinge support of dredger's jib structure

As it can be observed, link FEs radially join a node of the FE axle and adjoining nodes of the shell FEs of the structure hinge parts.

Following examples show the possibilities of gap FE and nonlinear spring FE in modeling of the change of connection in certain circumstances.

Two beams in space with a gap located in the half of

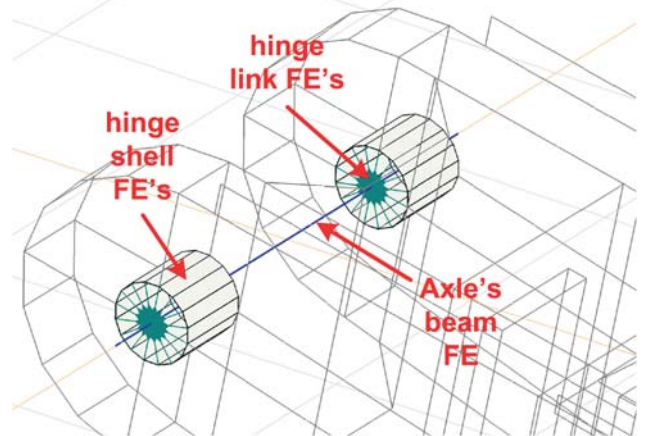
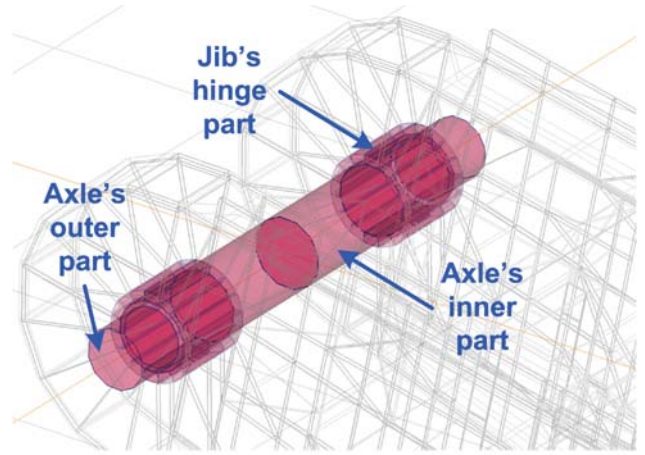


Figure 14 Modeling the hinge support by link FEs



Figure 15 Beams in space with a gap in between

their span, Fig. 15, is a numerical example which illustrates the application of gap FE in modeling of the connection activation.

Structural behavior is modeled by the gap FE with stiffness of $s_{BC}=10^{-1}$ kN/m and $s_{AC}=10^7$ kN/m before and after the realization of contact and with the length according to geometry, Fig. 16.

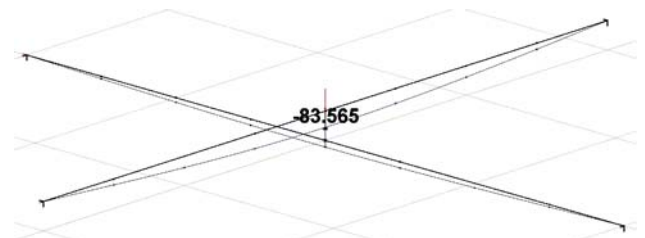


Figure 16 Gap FE for modeling of contact establishing

For load of $P = 50$ kN the system has displacement of $\delta_{NL} = 83,565$ mm. Fig. 17 shows the system's nonlinear response curve with one break point ($P = 21,4$ kN, $\delta = 50$

mm) which represents the contact realization.

The slope of the second curve segment represents the phase after the contact realization when the stiffness increases.

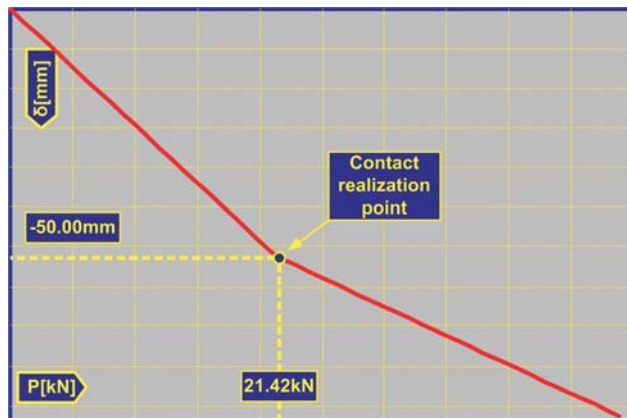


Figure 17 Nonlinear response of model with gap FE

The following numerical example is an illustration of the application of nonlinear spring FE in modeling of the connection-failure phenomenon. Two beams in space are joined only at cross-section located in the half of their span, Fig. 18.

A piece of wood ($d = 10$ cm) is located in between these beams. The connection is modeled by the nonlinear spring FE with stiffness of $k = 10^5$ kN/m and with failure force of $N_f = 15$ kN, Fig. 19.

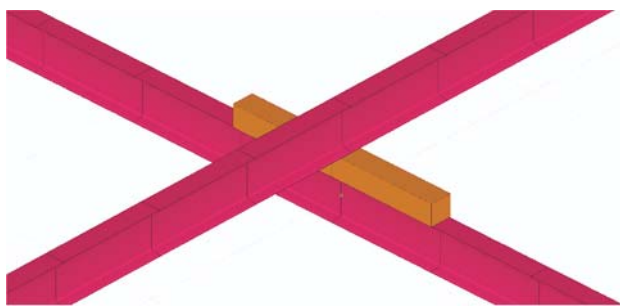


Figure 18 Beams with wood insert in between

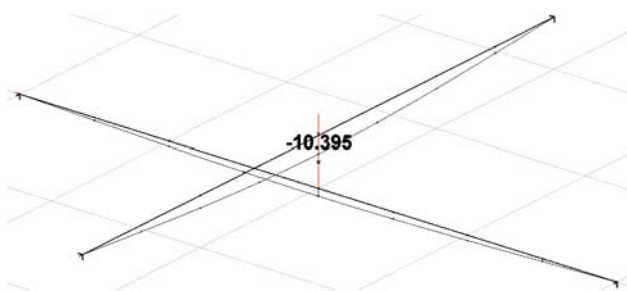


Figure 19 Nonlinear spring FE for modeling of connection failure

Under the load of $P = 50$ kN the system has maximum displacement of $\delta_{NL} = 10,395$ mm. Nonlinear response of the model is given in Fig. 20.

Nonlinear response curve has one break point ($P = 30,5$ kN, $\delta_{BP} = -4,634$ mm) which represents failure limit, because of the loss of wood bearing capacity. The second segment represents the stiffness decreases after the failure of wooden piece.

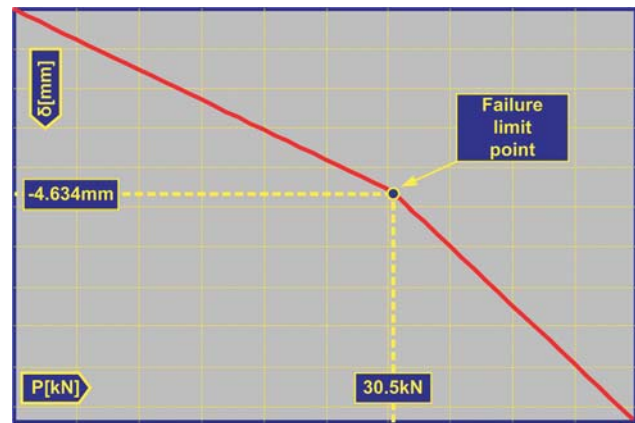


Figure 20 Nonlinear response of model with nonlinear spring FE

7 Conclusions

The goal of this paper is to emphasize the possibilities of creative use of special FEs. Convenient choice of their parameters makes the modeling of most different structural behavior possible.

In contrast with the so-called "brutal force" style, oriented to the sometimes worthless application of very complex models, the use of non-conventional models with special FEs could be very effective.

Engineer-like approach by the use of special FEs enables relatively simple models, which corresponds to the requests of everyday design and research practice.

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Authors' addresses:

Prof. Dr Dušan Kovačević, Civ. Eng.

Faculty of Technical Sciences
University of Novi Sad
Trg Dositeja Obradovića 6
21000 Novi Sad, Serbia
e-mail: dusan@uns.ac.rs

Ass. Prof. Dr Igor Budak, Mech. Eng.

Faculty of Technical Sciences
University of Novi Sad
Trg Dositeja Obradovića 6
21000 Novi Sad, Serbia
e-mail: budaki@uns.ac.rs

Ass. Prof. Dr Aco Antić, Mech. Eng.

Faculty of Technical Sciences
University of Novi Sad
Trg Dositeja Obradovića 6
21000 Novi Sad, Serbia
e-mail: antica@uns.ac.rs

Assoc. Prof. Dr Borut Kosec, Mech. Eng.

Faculty of Natural Sciences and Engineering
University of Ljubljana
Aškerčeva cesta 12
1000 Ljubljana, Slovenia
e-mail: borut.kosec@ntf.uni-lj.si