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[Note]

Comment on "Bulletin of Nippon College of Physical Education, 2, 19 (1972)"

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Using the numerical data given in the paper due to Enda, Y. entitled "On the relation of muscular activity and brain clearness (1)—Change of flicker fusion frequencies during exercise—" (Bulletin of Nippon College of Physical Education, 2, 19 (1972)), we obtain that numerically well defined experimental function $x(t)$ describing the variations with time of the critical flicker fusion frequencies during the exercises on the treadmill which is a solution of an initial value problem in the time interval $[0, \infty)$ for a linear ordinary differential equation.

Introduction

Through the observations of the variations with time of the critical flicker fusion frequencies (in the followings, we abbreviate to c.f.f.f.) during the exercises on the treadmill with various speeds below 150 m/min, Enda, Y.[1] has found the facts that the more the exercise intensity increases, the higher the c.f.f.f. remains. Table 1, which is a part of the data due to Enda, Y. describing the variations with time $f(t)$ of the c.f.f.f. during the exercises for a 60 min period, supports the above statements in a qualitative sense.

In the present paper, we propose such a theoretical model described in terms of an initial value problem in $[0, \infty)$ for a first order linear ordinary differential equation as generates the function $f(t)$ given in Table 1 as a solution of the initial value problem.

Linear model

The numerical data listed in Table 1 implies that the function $f(t)$ which describes the variations with time of the c.f.f.f. can be adequately approximated by the following experimental function (1)

Table 1. Variations with time $f(t)$ of the critical flicker fusion frequencies during the exercises on the treadmill with speeds 50 m/min, 100 m/min and 150 m/min
(Reproduction of the data due to Enda, Y. given in "Bulletin of Nippon College of Physical Education, 2, 19 (1972))

No. j	Time t_j (min)	$f(t)$ (arbitrary unit) 50 m/min	100 m/min	150 m/min
0	0	0	0	0
1	4	1.1	1.1	2.3
2	8	1.3	1.7	5.3
3	12	1.1	1.5	6.2
4	16	2.3	2.3	5.4
5	20	2.7	3.3	6.2
6	24	2.1	3.8	6.9
7	28	2.4	3.8	5.6
8	32	3.0	4.0	6.7

$$x(t)=A(1-\exp -at)\,,\quad A>0\,,\quad a>0\,,\quad t\in[0,\infty) \qquad (1)\,.$$

$x(t)$ represents the variations of the c.f.f.f. and the positive constant A corresponds to the value of $x(t)$ in saturation, that is, the supremum*) (least upper bound) of $x(t)$ in the

*) The monotone increasing function $x(t)$ has on maximum in $0\leqq t$.

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interval $[0, \infty)$. Differentiating the function (1) with respect to t, we obtain the following linear ordinary differential equation (2).

$$\frac{dx}{dt}=a(A-x)\,, \qquad t\in[0, \infty) \qquad (2)$$

The pair composed of the differential equation (2) and the initial condition $x(0)=0$ is the linear model which will be emploied in the present work to explain the manner of the variations with time of the c.f.f.f.. The equation (2) indicates that the gradient of the function $x(t)$ at any fixed time t_0 is proportional to the difference between $x(t_0)$ and the supremum A with the proportional constant a. We may approximately take as the supremum A the maximum value which is attained during the exercises. The maximum value during the exercises for a 60 min period is, according to Enda, Y.[1], 4.5 in the same unit as emploied in Table 1 for the exercises on the treadmill with the speed 50 m/min, 5.0 for the speed 100 m/min and 9.3 for the speed 150 m/min. Using these values, we can calculate the proportional constant a as the arithmetic mean of a_j $(j=1, 2, \ldots, 8)$ each of which is obtained from the function (1′), a logarithmic form of the function (1) with the actual data $f(t)$,

$$a_j=\frac{-1}{t_j}\log\left(1-\frac{f(t_j)}{A}\right), \qquad j=1, 2, \ldots, 8$$

where $f(t_j)$ is the observed value listed in Table 1. Substituting these calculated values into (1), we have the experimental function (3) equipped with the numerical value

$$\begin{aligned} x(t)&=4.5(1-\exp-0.039t) && \text{for } 50\text{ m/min} \\ x(t)&=5.0(1-\exp-0.049t) && \text{for } 100\text{ m/min} \\ x(t)&=9.3(1-\exp-0.066t) && \text{for } 150\text{ m/min}\,. \end{aligned} \qquad (3)$$

The levels of approximation, or the degree of approximation, of the experimental function (3) is estimated by the root mean square error (Appendix). The calculated root mean square error for the exercises on the treadmill with the speed 150 m/min is 1.13. For the exercises on the treadmill with the speed 100 m/min or 50 m/min, the root mean square error in a lower level is obtained; that is, 0.32 for 100 m/min and 0.41 for 50 m/min. In these levels of approximation, the theoretical model described in terms of the initial value problem for a linear ordinary differential equation—the equation (2) and the initial condition $x(0)=0$—gives the conceptional basis to the manner of variations with time of the c.f.f.f. given in Table 1. Furthermore, the successive increase of the numerical value of the proportional constant a shown in the function (3) quantitatively supports the qualitative statements due to Enda, Y. concerning the relationships between the brain clearness described in terms of the c.f.f.f. and the intensities of exercises.

Appendix

The root mean square error of the experimental function $x(t)$ is defined by

$$\left(\sum_{j=0}^{m}\{f(t_j)-x(t_j)\}^2/m+1\right)^{1/2}$$

where $f(t_j)$ is the actual data obtained at the time t_j. The total number $m+1$ of the data effectively utilized in the present work is nine.

Reference

1) Enda, Y.: *Bulletin of Nippon College of physical Education*, 2, 19 (1972).