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## MODELING OF LUBRICATING LAYER OF STRIP DRESSING WITH THE INFLUENCE OF SURFACE ROUGHNESS

### Abstract

*In the paper the influence of transversal roughness of the strip on dressing processes with lubricants is analyzed. The analysis begins with Reynolds differential equation for lubrication, in which transversal roughness of the strip is incorporated. In the estimation, the height of lubricant on the strip is taken into account, as well as its influence on the height of lubricant at the inlet section of the deformation zone. The research has shown that transversal roughness has a twofold influence on the height of lubricant at the inlet section of the deformation zone. If roughness is small of the strip the height of lubricating layer has a tendency of slight decreasing related to the nominal height (when the process is described by smoothness of surfaces) but with an increase of roughness, the thickness of lubricating layer has a tendency to increase. The nominal height of lubricant is considered to be the case of changing the concave surface into the convex one, which seems to look straight as if the process is described by smoothness of surfaces. Lubricating layer modeling in the friction area on insufficiently lubricated surfaces was also performed. The basis of the analysis was the Monte-Carlo numerical method, and an approximate analytical solution, that gave good match in comparison with the numerical method, was established. The results of this theoretical research can clarify some phenomena of lubrication in plastic deformation of metal and the fact that the shape of strip roughness determines the form of lubricating layer.*

### 1. Introduction

Nowdays, the mathematical modeling [1-3] has been greatly developing in metallurgy. It is very specific in the area of plastic deformation of metals. It starts with Reynolds differential equation for lubrication of smooth surfaces [4-5], with pressure gradient along x axis being:

$$\frac{dp}{dx} = \frac{6\mu(v_0 + v_R)}{\varepsilon^2(x)} - \frac{12\mu Q}{\varepsilon^3(x)} \quad (1)$$

Symbols:  $\mu$  – dynamic viscosity of lubricating layer under the pressure  $p$ ,  $v_0$  – strip velocity,  $v_R$  – rolling velocity,  $Q$  – volume lubricant consumption per perimeter.

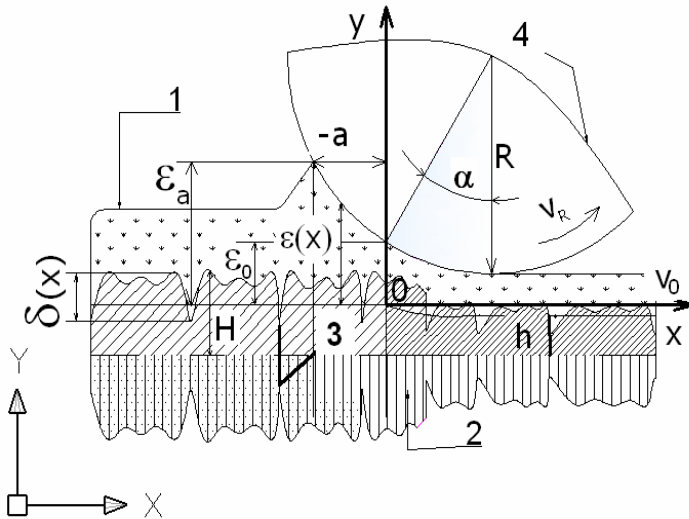


Figure 1: Description of tribomechanical system model:

1 – lubricating layer;  $\varepsilon(x)$  - nominal height when the process is defined by surface smoothness, 2 – strip – in processes of dressing it has a small grasping angle  $\alpha$ ; 3 – transversal roughness of a strip;  $\delta(x)$  – random height due to the roughness of a strip;  $dp/dx$  – pressure gradient,  $x, y$  – Descartes coordinate system;  $H$  and  $h$  – strip height before and after the deformation, 4 – roll – defined by the surface smoothness, added some roughness in Figures 8, 9 and 10, height of lubricating layer on the strip is  $\varepsilon_a$ , and  $R$  is radius of a roll.

Contact geometry of lubricant [8,9,10] is expressed by:

$$\varepsilon(x) = \varepsilon_0 + R \left[ \cos \alpha - \sqrt{1 - \left( \sin \alpha - \frac{x}{R} \right)^2} \right] \quad (2)$$

Which can be transformed into MacLaurin series:

$$\varepsilon(x) = \varepsilon_0 - \alpha x + \frac{x^2}{2R} - \frac{\alpha x^3}{2R^2} + \frac{x^4}{8R^3} \quad (3)$$

The length of lubricating wedge is given by the expression:

$$a = R \cdot \left[ \sqrt{1 - \left( \cos \alpha - \frac{\varepsilon_a}{R} + \frac{\varepsilon_0}{R} \right)^2} - \sin \alpha \right] \tag{4}$$

The height of lubricating layer along with the roughness of the surface is given in:

$$\langle \varepsilon(x_0) \rangle = \varepsilon(x) + \delta(x) \tag{5}$$

Here the twofold roughness is included:  $\delta(x)$  – random height determined by the roughness of the strip and the roll. The transversal roughness of the strip according to the Gauss distribution law transforms the differential equation (1) into a new form:

$$\left\langle \frac{dp}{dx_0} \right\rangle = 6\mu(v_0 + v_R) \left[ \left\langle \frac{1}{\varepsilon^2(x_0)} \right\rangle - \frac{\left\langle \frac{1}{\varepsilon_0^2} \right\rangle}{\left\langle \frac{1}{\varepsilon_0^3} \right\rangle} \left\langle \frac{1}{\varepsilon^3(x_0)} \right\rangle \right] \tag{6}$$

Where  $\langle \rangle$  is a designation for the operator of mathematic hope. Theoretically, the following approximation for the random roughness is taken:

$$\delta(x) = \frac{4}{\pi} \left( \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x \right) R_z \tag{7}$$

where  $R_z$  stands for roughness of surfaces,  $R_z \sim 6 \times \delta$  according to GOST 2789-73. The graphical representation is shown in the Figure 2. This is actually the development of quadratic profile to the third member in the Fourier series. Symbols  $N$  and  $P$  represent the positive and negative range of the roughness of the strip when related to the zero line, when the process is described by the smoothness of surfaces.

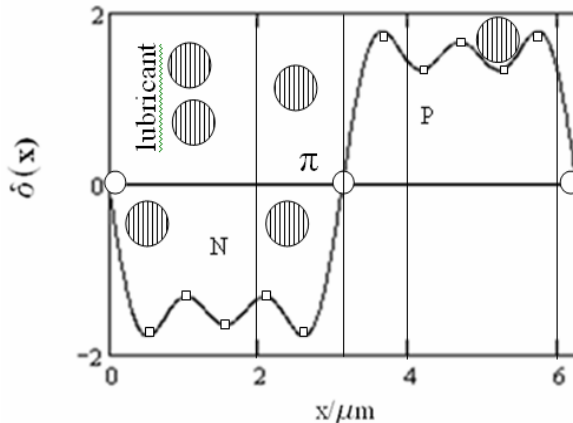


Figure 2: Profile of strip roughness (0-2π)

## 2. Analysis of the positive range $[\pi-2\pi]$ related to smooth rolls

In the analysis of a differential equation (6) the following moderator is included:

$$\frac{\varepsilon_0 [\varepsilon_0^2 + 3(\delta(x))]^2 (\pm p)}{\varepsilon_0^2 + 6[\delta(x)p]^2} \quad (8)$$

Here  $p$  stands for strip roughness class (0-10) along the profile of roughness shown in Figure 2  $[0-2\pi]$ , while  $\varepsilon_0$  is the height of lubricant at the inlet section of the deformation zone. The analysis is shown in two approaches. In the first approach the positive range of roughness is analyzed in the Figure 2 from  $[\pi-2\pi]$ , while in the second approach the supporting profile of roughness is analyzed  $[0-2\pi]$ . Such approaches are not common in the reference literature since the analyses are done mostly through isotropic [11-15] homogenous roughness of the strip shown in the Figure 3 or through smooth surfaces. The solutions of differential equations are directed towards the calculation of the height of lubricating layer at the inlet section of the deformation zone  $\varepsilon_0$ , in the Figure 1 for smooth surfaces and in the Figure 8 for rough surfaces.

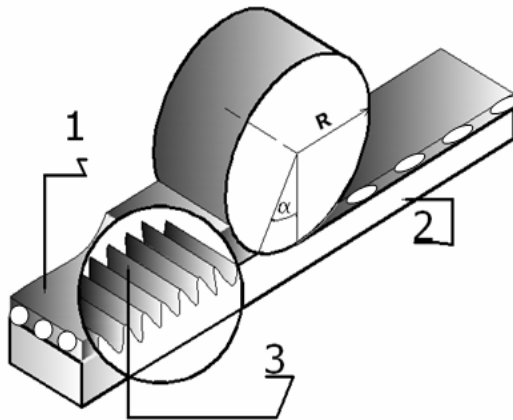


Figure 3: Homogenous isotropic strip roughness (3); a strip being dressed (2) with a grasping angle  $\alpha$ ; lubricating layer (1) and radius of the roll  $R$

The differential equation has been solved by the Monte-Carlo method (6) in Mathematica and MAT-LAB programs. Parameters are shown in Table 1. The calculation results according to the Table 1 are to be presented in the form of pictures. The analyses have shown that the positive range of strip roughness usually takes less lubricant at the inlet section of the deformation zone as oppose to the negative range of roughness. In other words, the calculated lubricating layer in the positive range of roughness is always smaller than the calculated layer at the profile  $[0-\pi]$  in the Figure 2.

Table 1: Common characteristics of lubricant for theoretical calculations

Parameter	Value	Unit
$\gamma$ - piezocoefficient of lubricant viscosity	$2.18 \times 10^{-7}$	$\text{Pa}^{-1}$
$p_0$ - rolling pressure	$20 \times 10^6$	Pa
$v_R$ - orbital velocity of a roll	10	$\text{m s}^{-1}$
$v_0$ - strip velocity	6	$\text{m s}^{-1}$
$R$ - radius of a roll	0.35 - 0.25	m
$\mu_0$ - dynamic viscosity of lubricant $\mu = \mu_0 \exp(\gamma \times p_0)$	0.024 - 0.048	Pa s
$\alpha$ - pressure angle	0-0.02	rad
$\epsilon_A, \epsilon_a$ - lubricant height on a strip	0.001 - 0.0001	m
$A$ - technological parameter	1965512 - 3934525	$\text{m}^{-1}$
$R_z \approx 6 \times \delta$	1 - 10	$\mu\text{m}$
$S$ - roughness classes	0 - 10	$\mu\text{m}$
$L$ - supporting profile of roughness	0 - $2\pi$	$\mu\text{m}$

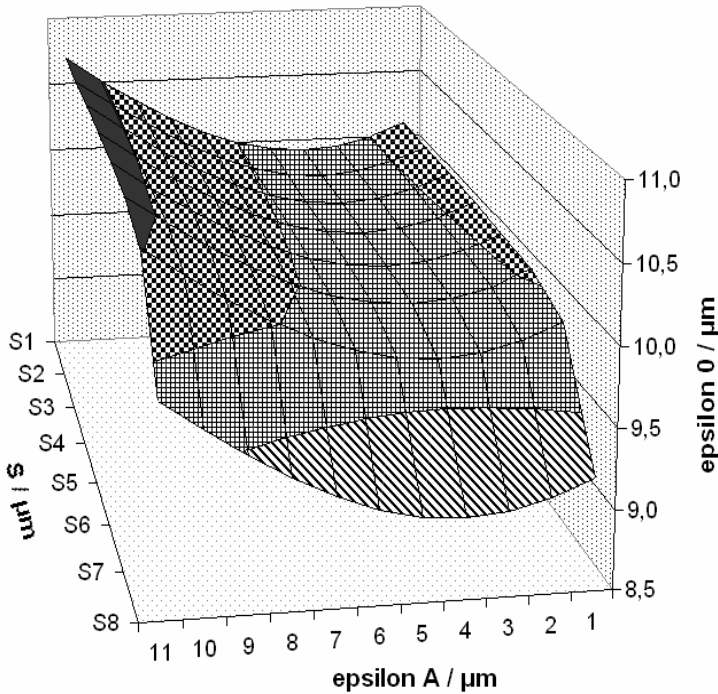


Figure 4: Influence of  $\epsilon_A$  on  $\epsilon_0$  in the roughness function in  $\mu\text{m}$ , with classes from (0-10)  $\mu\text{m}$

The Figure 4 shows the influence of  $\varepsilon_A$  on  $\varepsilon_0$  in the roughness function. Here the  $\varepsilon_A$  is within range 0,001 - 0,0001 m;  $p$  within (0,0 - 10)  $\mu\text{m}$  increment 2;  $R_z = 10 \mu\text{m}$ ,  $\varepsilon_0$  within (9 - 10,8)  $\mu\text{m}$ , angle  $\alpha = 0,02 \text{ rad}$ . The main conclusion is that by reducing the height of lubricant on the strip also the height of lubricant at the inlet section of the deformation zone is reduced, but non-uniformly regarding the roughness classes. The Figure 5 shows the 3D display of lubricating layer which was modeled through the differential equation (6) at the applicate; x-axis contains 10 roughness classes (0-10)  $\mu\text{m}$  not taking roughness  $R_z \rightarrow 0$  into consideration. The ordinate contains 32 classes per formula (7). Marker P indicates an apparent line of the 6<sup>th</sup> order of roughness class through which the concavity of the lubricating layer mirrors into convexity.

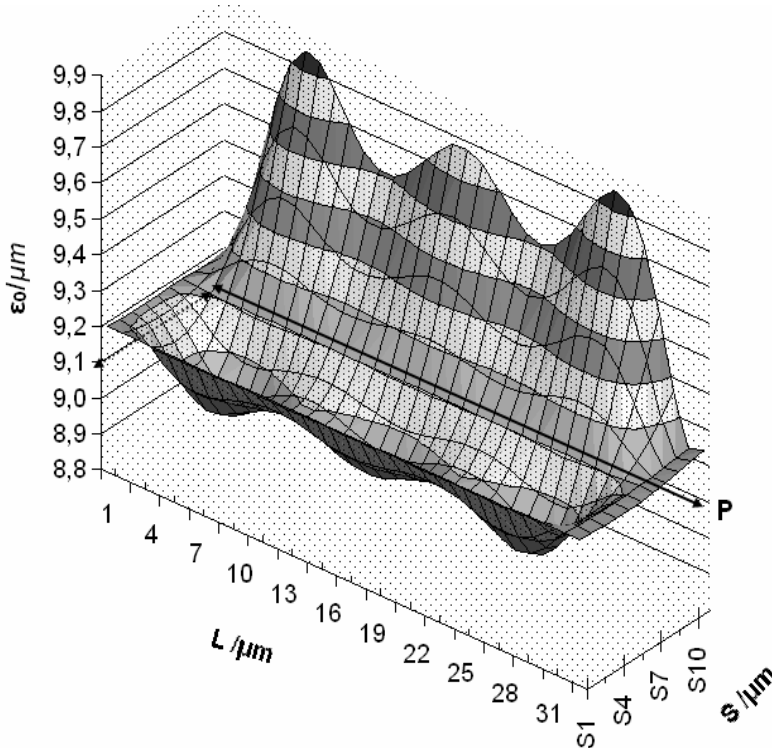


Figure 5: 3D display of lubricating layer

The Figure 5 also shows that the lubricating layer has a tendency of slight decrease with the increase of transversal roughness of the strip (drop depth). After the third class of roughness the lubricating layer starts to increase coming close to the nominal layer P.

In fact, this phenomenon is synergetically connected to the second range of roughness of the strip, which is the negative range of roughness according to the Figure 2 at the x axis of [0-π], with the tendency to supply and lubricate the positive range of roughness. Taken that L is supporting roughness profile in the area [0-a] of the Figure 1, while S is roughness class of [0-10] μm. The differential equation (6) has approximate analytical solution in the first approximation:

$$315 AR^3\alpha^7 - 168 R^2\alpha^4 - 1824 \delta^2 = 0 ; \varepsilon_0 = 0,5 R \alpha^2 \dots \tag{9}$$

$$A = \frac{1 - \exp(-\gamma \times p_0)}{6 \mu \gamma (v_0 + v_R)} \tag{10}$$

Explanation of symbols:  $\gamma$  -piezo-coefficient of viscosity of a lubricant,  $\exp$  – basis of natural logarithm,  $p_0$  –pressure of rolls, A – technological parameter. An approximate solution was checked at the marker P of the Figure 5. The Table 2 gives the calculated values. A good match of numerical method and approximate analytical solution can be seen.

Table 2: The comparison of numerical Monte-Carlo method and approximate analytical solution of equations (9) and (10) in starting profile (x = 0)

Example parameters	Approximate analytical solution (9)	Monte-Carlo method, diff. equ. (6)
x = 0 $R_z = 1 \mu\text{m}$ $R_z \approx 6 \delta$ $A = 1965512 \text{ m}^{-1}$ $R = 0.35 \text{ m}$	$\varepsilon_a = 0.001 \text{ m}$ $\varepsilon_0 = 14.771 \mu\text{m}$	$\varepsilon_a = 0.001 \text{ m}$ $\varepsilon_0 = 14.772 \mu\text{m}$
	$\alpha = 0.00918759 \text{ rad}$	
x = 0 $R_z = 10 \mu\text{m}$ $R_z \approx 6 \delta$ $A = 1965512 \text{ m}^{-1}$ $R = 0.35 \text{ m}$	$\varepsilon_a = 0.001 \text{ m}$ $\varepsilon_0 = 15.092 \mu\text{m}$	$\varepsilon_a = 0.001 \text{ m}$ $\varepsilon_0 = 15.077 \mu\text{m}$
	$\alpha = 0.0092867 \text{ rad}$	
x = 0 $R_z = 10 \mu\text{m}$ $R_z \approx 6 \delta$ $A = 3934525 \text{ m}^{-1}$ $R = 0.25 \text{ m}$	$\varepsilon_a = 0.001 \text{ m}$ $\varepsilon_0 = 8.838 \mu\text{m}$	$\varepsilon_a = 0.001 \text{ m}$ $\varepsilon_0 = 8.755 \mu\text{m}$
	$\alpha = 0.00840867 \text{ rad}$	

In the Figure 6 the histogram 3D display of results of modeling lubrication layer is shown for the parameters given in the Table 1. The marker W represents 8<sup>th</sup> roughness class, which is additionally marked with balls. According to that specific roughness class modeling of lubricating layer is given in the Figure 7. L stands for supporting roughness profile in the area [0-a] of the Figure 1.

Modeling was conducted in AutoCAD program according to solutions of differential equation (6) and parameters given in the Table 1. Volume consumption of lubricant in the differential equation is (1):

$$Q = \frac{1}{2}(v_0 + v_R) \quad (11)$$

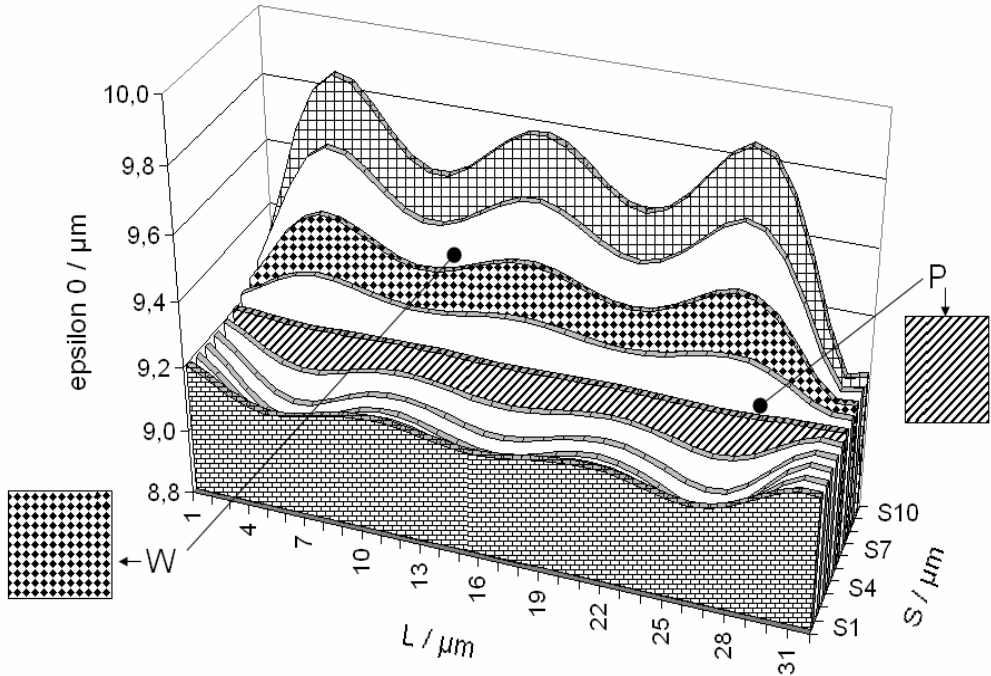


Figure 6: Histogram 3D-display of results of lubrication layer modeling

Symbols in the Figure 7 are as follow: 1 – solid state of roughness of the strip (A, B, C), not lubricated or partly lubricated; 2 – inert lubricating layer (K, L), lubricating layer without synergetic effect and influence of moderator according to the equation (8); 3 – impossibility of physical access of the lubricant (D, F, H); 4 – effect of transversal roughness of the strip, actually the increase of the lubrication layer (I-J, J-I) influenced by a moderator; 5 - nominal height of lubricating layer along  $P$  in the Figures 5 and 6 (E, G). As modeling was conducted in the area of friction due to insufficiently lubricated surfaces, the effect of transversal roughness on the lubricating layer is smaller (I, J). It needs to be emphasized that the effect of roughness in the Figure 7 would be bigger if longitudinal roughness of rolls was added.



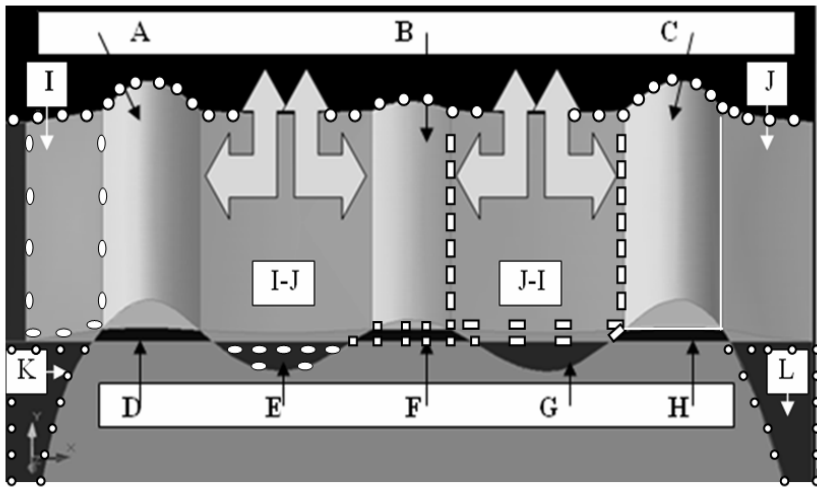


Figure 7: Modeled lubricating layer according to marker *W* from the Figure 6

### 3. Analysis of the whole range $[0-2\pi]$ in relation to rough rolls

In the further analysis the longitudinal roughness is added and the lubricating layer is calculated at the whole theoretical profile according to the Figure 2. It was assumed that there is not any phase shift between roughness of the strip and rolls, and since the grasping angles tend to the zero, it is not necessary to take Gauss-Krüger corrections into the consideration. The expression (5) is transformed into:

$$\langle \varepsilon(x_0) \rangle = \varepsilon(x) + \delta(x_1) + \delta(x_2) \quad (12)$$

which means that in the expression (3) random roughness of rolls  $\delta(x_1)$  and the strip  $\delta(x_2)$  are added. Their mutual position is illustrated in the Figure 8.

The results of differential equation (6) in the case of the Figure 8 are shown in the Figure 9, where: *A* - positive ranges of roughness (in the Figure 8 markers 4), *B* - negative ranges of roughness (in the Figure 8 markers 1 and 5), *C* – area of amorphous lubricating layer, *L* – length of supporting profile of a strip, *S* – classes of roughness of a strip (0 - 10)  $\mu\text{m}$ . In the *A* area the increase of surface roughness makes the lubricant flow at the inlet section of the deformation zone difficult, while in the *B* area negative range is able to accumulate lubricant which increases the height of lubricating layer ( $\varepsilon_0$ ) at the inlet section of the deformation zone. Amorphous lubricating layer in the *C* area shows up at the conditions of high roughness of a strip and a roll where the laws of boundary lubrication are actually broken, which indicates the plastic condition of surface roughness of a strip. The explanation is shown in the Figure 10.

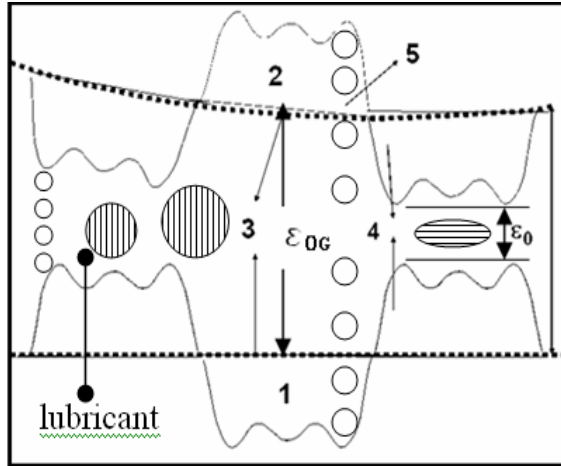


Figure 8: Dressing from the roughness of a roll (longitudinal) and a strip (transversal): 1 - strip, 2 - roll, 3 - referent lines ( $R_z \rightarrow 0$ ),  $\varepsilon(x)$  is expressed in the formula (3), 4 - positive range of roughness, 5 - negative range of roughness, analogue strips,  $\varepsilon_{0G}$  – calculated height of lubricant (nominal),  $\varepsilon_0$  – calculated height of lubricant with twofold roughness

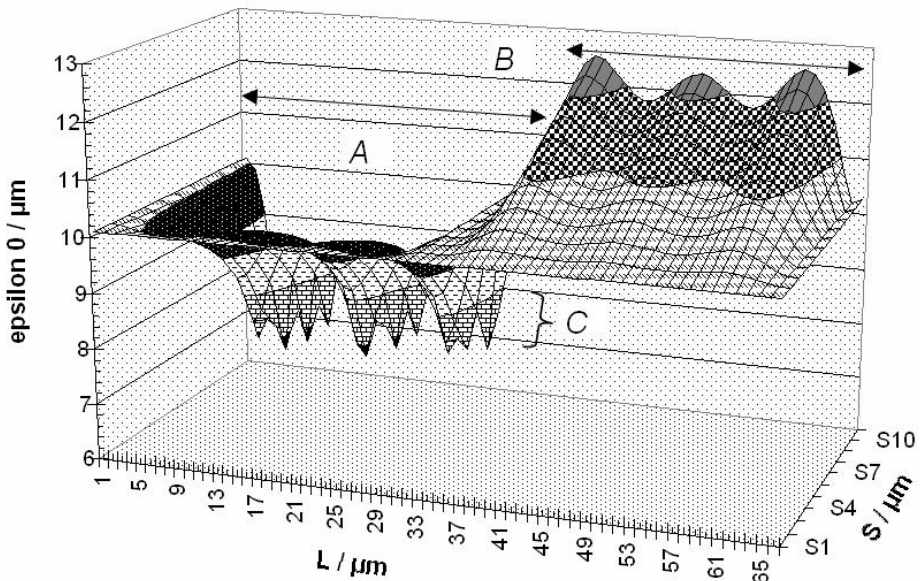


Figure 9: Calculation of lubrication layer with transversal roughness of the strip and longitudinal roughness of the roll

Damaged homogenous lubricating layer is clearly seen in the domain B of the ribbon of B (markers D), where the continuous lubricating layer is becoming dotted in the "island" display (marker C in the Figure 9). The surface condition of the strip at the inlet section of the deformation zone in the plastic condition caused the amorphous lubricating layer inside of ribbon B in the Figure 10. Marker A represents the stable lubrication. The classes of roughness  $S$  start here from  $[0-20] \mu\text{m}$  in order to prove the presence of amorphous lubricating layer, by theoretical approach and announce the possibility of forming the island lubricating layers (above marker B), for  $L = 9-14 \mu\text{m}$ , which appear in classes of roughness  $S = 19 - 21$ , as mini domains. For the theoretical explanation of this effect we could look in the practical technological process of rolling where we could compare it with the effect of compressed burntout scale in the surface layer of the rolling metal. The form of amorphous lubricating layer could offer information on the process of lubrication, when the lubricated surfaces in the plastic condition were treated with conventional industrial lubricants and emulsions.

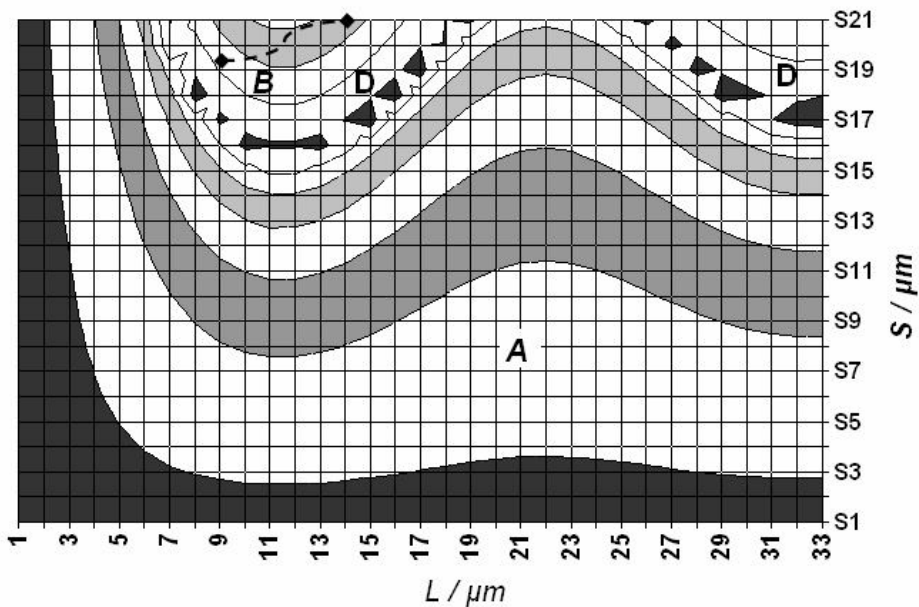


Figure 10: Contour display of positive ranges of roughness in the abscissa domain,  $L = [0-\pi]$  from the Figure 9 along the A area

The numerical methods of solving the Reynolds differential equations related to this topic are used in the papers [16-19] as well as the methods of computer processing of surface roughness [20] by introducing moderators similar to a moderator given in the equation (8), with less clear theoretical basis. Nevertheless, the moderator given in the equation (8) has the core in the Gauss distribution.

## 4. Conclusion

The moderator (equation (8)) introduced in the differential equation (6) started the analysis of the nominal lubricating layer which is marked as  $P$  in the Figures 5 and 6. Mirroring of the concave surface into the convex one is conducted through the nominal layer which shows that the lubricating layer with transversal roughness of a strip can increase and decrease. This especially relates to the positive range of roughness of the strip. Amorphous lubricating layer was detected in case of transversal roughness of the strip and longitudinal roughness of the roll in the friction area on insufficiently lubricated surfaces and in the presence of plastic deformation of lubricated metal surfaces. The analysis of this layer could offer some information on the mechanism of plastic deformation in the surface metal layer for extremely small grasping angles. Approximate analytical solutions, when compared to numerical method, give a good match on a neutral line of roughness (marker 3 in the Figure 8) and represent the control of the Monte-Carlo method.

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.001.575	ledgište ispitivanja na materijalnom modelu	real model investigation viewpoint
621.771.8	dresiranje traka	strip dressing
621.771.23	valjanje limova	plate rolling
62-408.66	hrapavost površine	roughness of surface
621.891.275	Reynoldsova diferencijalna jednačnja	Reynolds' differential equation
519.21	Monte-Carlo metoda	Monte-Carlo method
517.52	Fourierovi redovi	Fourier series

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