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On repeated low-density burst error detecting linear codes

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Abstract. The paper presents lower and upper bounds on the number of parity-check digits required for a linear code that is capable of detecting repeated low-density burst errors of length b (fixed) with weight w or less ($w \leq b$). A bound for codes which can correct and simultaneously detect such burst errors has also been derived. An illustration has been provided for the code detecting 2-repeated burst errors of length 3 (fixed) with weight 2 or less over GF(2).

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1. Introduction

Various kinds of errors have been dealt with in coding theory for which codes have been developed to detect and/or correct such errors. Amongst these, burst errors have played a dominant role because of their usefulness. Abramson (1959) developed codes which dealt with the correction of single and double adjacent errors, which was extended by Fire (1959) as a more general concept called 'burst errors'. A burst of length b is defined as follows:

Definition 1. *A burst of length b is a vector whose only non-zero components are among some b consecutive components, the first and the last component of which are non-zero.*

The nature of burst errors differ from channel to channel depending upon the type of channels used during the process of transmission.

Chien and Tang (1965) observed that in many channels errors occur in the form of a burst but errors do not occur towards the end digits of the burst. Channels due to Alexander, Gryb and Nast (1960) fall in this category. A burst of length b defined by Chien and Tang (1965) is as follows:

Definition 2. *A burst of length b is a vector whose only non-zero components are confined to some b consecutive positions, the first of which is non-zero.*

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Such bursts have been termed as CT bursts. Dass (1980) further modified this definition as follows:

Definition 3. *A burst of length b (fixed) is an n-tuple whose only non-zero components are confined to b consecutive positions, the first of which is non-zero and the number of its starting positions in an n-tuple is the first $n - b + 1$ components.*

This definition is useful for channels not producing errors near the end of a code word. In very busy communication channels, errors repeat themselves. Dass, Garg and Zannetti (2008) studied repeated burst errors. They termed such a burst error as an 'm-repeated burst of length b (fixed)' which has been defined as follows:

Definition 4. *An m-repeated burst of length b (fixed) is an n-tuple whose only non-zero components are confined to m distinct sets of b consecutive digits, the first component of each set is non-zero and the number of its starting positions is among the first $n - mb + 1$ components.*

Linear codes capable of detecting '2-repeated burst errors of length b (fixed)' were studied by Dass and Garg (2009). During the process of transmission some disturbances cause occurrence of burst errors in such a way that over a given length, some digits are received correctly while others are corrupted, i.e. not all digits inside a burst are in error. Such bursts have been termed as low-density bursts [A.D. Wyner (1963)]. A study of burst-error correcting linear codes that correct bursts of a specified length, say b, with weight w or less ($w \leq b$) was made by Dass (1983).

A low-density burst of length b (fixed) with weight w or less has been defined as follows:

Definition 5. *A low-density burst of length b (fixed) with weight w or less is an n-tuple whose only non-zero components are confined to b consecutive positions, the first of which is non-zero with at most w ($w \leq b$) non-zero components within such b consecutive digits and the number of its starting positions in an n-tuple is among the first $n - b + 1$ components.*

In this paper, we consider codes which are capable to detect/correct m-repeated low density bursts of length b (fixed) with weight w or less ($w \leq b$). To be specific, a 2-repeated low-density burst of length b (fixed) with weight w or less ($w \leq b$) is defined as follows:

Definition 6. *A 2-repeated low-density burst of length b (fixed) with weight w or less is an n-tuple whose only non-zero components are confined to two distinct sets of b consecutive positions, the first component of each set is non-zero where each set can have at most w non-zero components ($w \leq b$), and the number of its starting positions is among the first $n - 2b + 1$ components.*

For example,

(i) (001111000000100000) is a 2-repeated low-density burst of length up to 6 (fixed) with weight 4 or less.

(ii) (000001000011010) is a 2-repeated low-density burst of length at most 5 (fixed) with weight 3 or less.

It may be noted that according to Definition 6, when the first low-density burst of length b (fixed) with weight w or less starts from the first position of the vector, then the second low-density burst of length b (fixed) with weight w or less is in the last $n-b$ components. When the first low-density burst of length b (fixed) with weight w or less starts from the second position of the vector, then the second low-density burst of length b (fixed) with weight w or less will be in the last $n-b-1$ components. In general, when the first low-density burst of length b (fixed) with weight w or less starts from the i-th position, then the second low-density burst of length b (fixed) with weight w or less will be in the last $(n-b-i+1)$ components where i can take the values from 1 to $n-2b+1$ since the starting positions are among the first $n-2b+1$ components. It may be noted that, in the last $2b-1$ components only a single low-density burst of length b (fixed) with weight w or less can exist. Such a burst can at most start from the $(n-b+1)$-th component.

Development of codes detecting and correcting repeated low-density burst errors will economize the number of parity-check digits in comparison to the usual repeated burst error detecting and correcting codes.

This paper has been organized as follows:

In Section 2, we derive a lower and an upper bound for codes detecting 2-repeated low-density burst errors of length b (fixed) with weight w or less. Section 3 presents a bound for codes which can correct and simultaneously detect 2-repeated low-density bursts of length b (fixed) with weight w or less. In Section 4, we obtain a lower bound for codes detecting an m-repeated low-density burst of length b (fixed) with weight w or less followed by another bound for codes which can correct and simultaneously detect such repeated low-density bursts. Finally, an illustration has been provided for the code detecting 2-repeated burst errors of length 3 (fixed) with weight 2 or less over GF(2).

In what follows, a linear code will be considered as a subspace of the space of all n-tuples over GF(q). The distance between two vectors shall be considered in the Hamming sense.

2. Detection of 2-repeated low-density burst error

In this section, we consider linear codes that are capable of detecting any 2-repeated low-density burst of length b (fixed) with weight w or less. Clearly, the patterns to be detected should not be code words. In other words, we consider codes that have no 2-repeated low-density burst of length b (fixed) with weight w or less a code word. Firstly, we obtain a lower bound on the number of parity-check digits required for such a code.

Theorem 1. *Any (n,k) linear code over* GF(q) *that detects any 2-repeated low-density burst of length b (fixed) with weight w or less must have at least $2w$ parity-check digits $(w \le b)$.*

Proof. The result will be proved on the basis that no detectable error vector can be a code word.

Let V be an (n,k) linear code over GF(q). Consider a set X that has all those vectors which have their non-zero components confined to the first $2b$ components among the first $n-b+1$ components. (It is clear that there should be at least $b-1$ left-out components beyond the first $2b$ components in the n-tuple in view of the definition of burst under consideration). From each set of b consecutive components, i.e., 1-st to b-th and $(b+1)$-th to $2b$-th components, the non-zero components are confined to some fixed $w (w \leq b)$ components. We claim that no two vectors of the set X can belong to the same coset of the standard array, else a code word shall be expressible as a sum or difference of two error vectors.

Assume on the contrary that there is a pair, say x_1, x_2 in X belonging to the same coset of the standard array. Their difference viz. $x_1 - x_2$ must be a code vector. But $x_1 - x_2$ is a vector all of whose non-zero components are confined to the first $2b$ components with non-zero components confining to the same fixed w or less $(w \leq b)$ components each in the 1-st to b-th and $(b+1)$-th to $2b$-th components and so is a member of X, i.e., $x_1 - x_2$ is a 2-repeated low-density burst of length b (fixed) with weight w or less, which is a contradiction. Thus all the vectors in X must belong to distinct cosets of the standard array. The number of such vectors over GF(q) is clearly q^{2w}. The theorem follows since there must be at least this number of cosets. $\square$

Remark 1. *For $w = b$, the lower bound on the number of parity-check digits required for such codes coincides with a result due to Dass and Garg (2009) when bursts considered are 2-repeated bursts of length b (fixed).*

Next, an upper bound on the number of parity-check digits required for the construction of a linear code has been provided. This bound ensures the existence of a linear code that can detect all 2-repeated low-density bursts of length b (fixed) with weight w or less. The bound has been obtained by first constructing a matrix and then by reversing the order of its columns altogether giving rise to a parity-check matrix for the requisite code, a technique given by Dass (1980) which is a suitable modification of the technique used by Sacks (1958) in establishing the well-known Varshamov-Gilbert bound.

Theorem 2. *There exists an (n, k) linear code that has no 2-repeated low-density burst of length b (fixed) with weight w or less $(w \leq b)$ as a code word provided that*

$$q^{n-k} > [1+(q-1)]^{(b-1,w-1)}\{1+(n-2b+1)(q-1)[1+(q-1)]^{(b-1,w-1)}\},$$

where $[1+x]^{(m,r)}$ denotes the incomplete binomial expansion of $(1+x)^m$ up to the term x^r in ascending power of x, viz.

$$[1+x]^{(m,r)} = 1 + \binom{m}{1}x + \binom{m}{2}x^2 + \ldots + \binom{m}{r}x^r .$$

Proof. The existence of such a code will be shown by constructing an appropriate $(n-k) \times n$ parity-check matrix H. Firstly, we construct a matrix H' from which the requisite parity-check matrix H shall be obtained by reversing the order of its columns altogether.

Any non-zero $(n-k)$-tuple is chosen as the first column h_1 of H'. Subsequent columns are added to H' such that after having selected the first $j-1$ columns $h_1, h_2, \ldots, h_{j-1}$, j-th column h_j is added provided that it is not a linear combination of any $w-1$ or fewer columns among the immediately preceding $b-1$ columns together with w or fewer columns among the b consecutive columns from the first $j-b$ columns, i.e.

$$h_j \neq (\alpha_{j_1}h_{j_1} + \alpha_{j_2}h_{j_2} + \cdots + \alpha_{j_{w-1}}h_{j_{w-1}}) + (\beta_{i_1}h_{i_1} + \beta_{i_2}h_{i_2} + \cdots + \beta_{i_w}h_{i_w}),$$

where $h_{j_1}, h_{j_2}, \ldots, h_{j_{w-1}}$ are any $w-1$ columns among $h_{j-b+1}, h_{j-b+2}, \ldots, h_{j-1}$ and h_i are any w columns from a set of b consecutive columns among the first $j-b$ columns such that either all β_i are zero, or if β_t is the last nonzero coefficient among the w or fewer non-zero coefficients, then $b \leq t \leq j-b, \alpha_j$'s and β_i's in GF(q). This condition ensures that no 2-repeated low-density burst of length b (fixed) with weight w or less will be a code word.

The number of ways in which the coefficients α_j can be selected is clearly $[1 + (q-1)]^{(b-1,w-1)}$. To enumerate the coefficients β_i is equivalent to enumerate the number of bursts of length b (fixed) with weight w or less amongst the first $j-b$ components. This number, including the vector of all zeros, is [as to Theorem 1, Dass (1983)]

$$1 + (j-2b+1)(q-1)[1+(q-1)]^{(b-1,w-1)}.$$

Thus, the total number of possible combinations that h_j cannot be equal to, is

$$[1+(q-1)]^{(b-1,w-1)}\{1 + (j-2b+1)(q-1)[1+(q-1)]^{(b-1,w-1)}\}.$$

At worst, all these linear combinations might yield a distinct sum. Therefore a column h_j can be added to H' provided that

$$q^{n-k} > [1+(q-1)]^{(b-1,w-1)}\{1 + (j-2b+1)(q-1)[1+(q-1)]^{(b-1,w-1)}\}.$$

The required parity-check matrix $H = [H_1 H_2 \ldots H_n]$ can be obtained from H' by reversing the order of its columns altogether ($h_i \to H_{n-i+1}$). For a code of length n, replacing j by n gives the result.

Remark 2. *The result obtained holds for $w = b$. If we take $w = b$, the weight consideration over the 2-repeated burst becomes redundant. The bound then reduces to*

$$q^{n-k} > q^{b-1}\{1 + (n-2b+1)(q-1)q^{b-1}\},$$

which coincides with a result due to Dass (1980) and also with a result due to Dass and Garg (2009). The result obtained for $w = b$ indicates that such a code can serve a dual purpose viz. it can either be used to correct bursts of length b (fixed) or it can be used to detect 2-repeated bursts of length b (fixed).

3. Detection/correction of 2-repeated low-density burst errors

In this section we determine extended Reiger's bound [Reiger (1960); also refer to Theorem 4.15, Peterson and Weldon (1972)] for correction and simultaneous detection of 2-repeated low-density bursts of length b (fixed) with weight w or less. The following theorem gives a bound on the number of parity-check digits for a linear code that corrects and simultaneously detects 2-repeated low-density bursts of length b (fixed) with weight w or less.

Theorem 3. *An (n,k) linear code over $\mathrm{GF}(q)$ that corrects all 2-repeated low-density bursts of length b (fixed) with weight w or less $(w \le b)$ must have at least $4w$ parity-check digits. Further, if the code corrects all 2-repeated low-density bursts of length b (fixed) with weight w_1 or less $(w_1 \le b)$ and simultaneously detects 2-repeated low-density bursts of length d (fixed) $(d \ge b)$ with weight w_2 or less $(w_2 \le d)$, then the code must have at least $2(w_1 + w_2)$ parity-check digits.*

Proof. Consider a vector all of whose non-zero components are confined to the first $4b$ components among the first $n-b+1$ components. (As pointed out in Theorem 1, it is clear that there should be at least $b-1$ left-out components beyond the first $4b$ components in the n-tuple in view of the definition of burst under consideration). From each set of b consecutive components, i.e., 1-st to b-th, $(b+1)$-th to $2b$-th, $(2b+1)$-th to $3b$-th and $(3b+1)$-th to $4b$-th components, the non-zero components are confined to some fixed $w(w \le b)$ components. Such a vector is expressible as a sum or difference of two vectors, each of which is a 2-repeated low-density burst of length b (fixed) with weight w or less. These component vectors must belong to different cosets of the standard array because both such errors are correctable errors. Accordingly, such a vector viz. low-density burst of length $4b$ (fixed) with weight $4w$ or less cannot be a code vector. In view of Theorem 1, such a code must have at least $4w$ parity-check digits.

Further, consider a vector all of whose non-zero components are confined to the first $2(b+d)$ components among the first $n-d+1$ components. (It is clear that there should be at least $d-1$ left-out components beyond the first $2(b+d)$ components in the n-tuple in view of the definition of burst under consideration). It may be noted that such a vector is expressible as a sum of four vectors where two of these are bursts of length b each and the other two are bursts of length d each. From each set of b consecutive components, the non-zero components are confined to some fixed $w_1(w_1 \le b)$ components wherein each set of d consecutive components, the non-zero components are confined to some fixed $w_2(w_2 \le d)$ components. Obviously, such a vector is expressible as a sum or difference of two vectors, one of which is a 2-repeated low-density burst of length b (fixed) with weight w_1 or less and the other is a 2-repeated low-density burst of length d (fixed) with weight w_2 or less. Both such component vectors, one being a detectable error and the other being a correctable error, cannot belong to the same coset of the standard array. Therefore such a vector cannot be a code vector, i.e., a low-density burst of length $2(b+d)$ (fixed) with weight $2(w_1+w_2)$ or less cannot be a code vector. Hence the code must have at least $2(w_1 + w_2)$ parity-check digits. □

Remark 3. *When the weight consideration over the bursts becomes redundant, this result coincides with a result due to Dass and Garg (2009) when bursts considered are 2-repeated bursts of length b (fixed).*

4. Detection and correction of an m-repeated low-density burst error

In this section, we consider linear codes that are capable of detecting any m-repeated low-density burst error of length b (fixed) with weight w or less, further we derive extended Reiger's bound [Reiger (1960); also refer to Theorem 4.15, Peterson and Weldon (1972)] for correction and simultaneous detection of m-repeated low-density bursts of length b (fixed) with weight w or less. Clearly, the patterns to be detected should not be code words.

Firstly, we obtain a lower bound over the number of parity-check digits required for such a code.

Theorem 4. *Any (n, k) linear code over* GF(q) *that detects any m-repeated low-density burst of length b (fixed) with weight w or less must have at least mw parity-check digits $(w \leq b)$.*

Proof. The result will be proved on the basis that no detectable error vector can be a code word.

Let V be an (n, k) linear code over GF(q). Consider a set X that has all those vectors which have their non-zero components confined to first mb components among the first $n - b + 1$ components. (As pointed out in Theorem 1, it is clear that there should be at least $b - 1$ left-out components beyond the first mb components in the n-tuple in view of the definition of burst under consideration). From each set of b consecutive components, i.e., $(ib + 1)$-th to $(i + 1)b$-th, where $i = 0, \ldots, m - 1$, the non-zero components are confined to some fixed $w(w \leq b)$ components. We claim that no two vectors of the set X can belong to the same coset of the standard array, else a code word shall be expressible as a sum or difference of two error vectors.

Assume on the contrary that there is a pair, say x_1, x_2 in X belonging to the same coset of the standard array. Their difference viz. $x_1 - x_2$ must be a code vector. But $x_1 - x_2$ is a vector all of whose non-zero components are confined to the first mb consecutive components with non-zero components confining to the same fixed w or less $(w \leq b)$ components each in the $(ib+1)$-th to $(i+1)b$-th components, $0 \leq i \leq m - 1$ and so is a member of X, i.e., $x_1 - x_2$ is a m-repeated low-density burst of length b (fixed) with weight w or less, which is a contradiction. Thus all the vectors in X must belong to distinct cosets of the standard array. The number of such vectors over GF(q) is clearly q^{mw}. The theorem follows since there must be at least this number of cosets. □

Remark 4. *For $m = 2$, the result obtained in Theorem 4 coincides with Theorem 1 of this paper when bursts considered are 2-repeated low-density bursts of length b (fixed) with weight w or less.*

Remark 5. *For $w = b$ the result coincides with Theorem 1 of [Dass, Garg and Zannetti (2008)] when bursts considered are m-repeated bursts of length b (fixed).*

Remark 6. *For $m = 2$ and $w = b$, the result of Theorem 4 reduces to that of the case of 2-repeated bursts of length b (fixed) due to Dass and Garg (2009).*

Remark 7. *For $m = 1$, the result gives a bound in the case of a non-repeating low-density burst of length b (fixed) with weight w or less.*

Remark 8. *For $m = 1$ and $w = b$, the result reduces to the case of non-repeating burst of length b (fixed) which is also the result when bursts are the usual open-loop bursts [refer to Theorem 4.13, Peterson and Weldon(1972)].*

The following theorem gives a bound on the number of parity-check digits for a linear code that corrects and simultaneously detects m-repeated low-density bursts of length b (fixed) with weight w or less.

Theorem 5. *An (n, k) linear code over* GF(q) *that corrects all m-repeated low-density bursts of length b (fixed) with weight w or less ($w \leq b$) must have at least $2mw$ parity-check digits. Further, if the code corrects all m-repeated low-density bursts of length b (fixed) with weight w_1 or less ($w_1 \leq b$) and simultaneously detects m-repeated low-density bursts of length d (fixed) with weight w_2 or less ($w_2 \leq d$), then the code must have at least $m(w_1 + w_2)$ parity-check digits.*

Proof. Consider a vector all of whose non-zero components are confined to the first $2mb$ components among the first $n-b+1$ components. (As pointed out in Theorem 1, it is clear that there should be at least $b-1$ left-out components beyond the first $2mb$ components in the n-tuple in view of the definition of burst under consideration). From each set of b consecutive components, i.e., $(ib+1)$-th to $(i+1)b$-th components, $0 \leq i \leq m-1$, the non-zero components are confined to some fixed $w(w \leq b)$ components. Such a vector is expressible as a sum or difference of two vectors, each of which is an m-repeated low-density burst of length b (fixed) with weight w or less. These component vectors must belong to different cosets of the standard array because both such errors are correctable errors. Accordingly, such a vector viz. low-density burst of length $2mb$ (fixed) with weight $2mw$ or less cannot be a code vector. In view of Theorem 1, such a code must have at least $2mw$ parity-check digits.

Further, consider a vector all of whose non-zero components are confined to the first $m(b+d)$ components among the first $n-d+1$ components. (It is clear that there should be at least $d-1$ left-out components beyond the first $m(b+d)$ components in the n-tuple in view of the definition of burst under consideration). It may be noted that such a vector is expressible as a sum of $2m$ vectors where m of these are bursts of length b each and the other m are bursts of length d each. From each set of b consecutive components, the non-zero components are confined to some fixed $w_1(w_1 \leq b)$ components wherein each set of d consecutive components, the non-zero components are confined to some fixed $w_2(w_2 \leq d)$ components. Obviously, such a vector is expressible as a sum or difference of two vectors, one of which is an m-repeated low-density burst of length b (fixed) with weight w_1 or less and the other is an m-repeated low-density burst of length d (fixed) with weight w_2 or less. Both such component vectors, one being a detectable error and the other being a correctable error, cannot belong to the same coset of the standard array. Therefore such a vector cannot be a code vector, i.e., a low-density burst of length $m(b+d)$ (fixed) with weight $m(w_1 + w_2)$ or less cannot be a code vector. Hence the code must have at least $m(w_1 + w_2)$ parity-check digits. □

Remark 9. *For $m = 2$, the result obtained in Theorem 5 coincides with Theorem 3 of this paper when bursts considered are 2-repeated low-density bursts of length b (fixed) with weight w or less.*

Error vectors	Syndromes	Error vectors	Syndromes
10000000	00010	01000101	11111
10010000	00110	01100000	00010
10011000	01110	01101000	01010
10010100	10110	01101100	11010
10001000	01010	01101010	01101
10001100	11010	01100100	10010
10001010	01101	01100110	10101
10000100	10010	01100101	11100
10000110	10101	01010000	00101
10000101	11100	01011000	01101
11000000	00011	01011100	11101
11010000	00111	01011010	01010
11011000	01111	01010100	10101
11010100	10111	01010110	10010
11001000	01011	01010101	11011
11001100	11011	00100000	00011
11001010	01100	00100100	10011
11000100	10011	00100110	10100
11000110	10100	00100101	11101
11000101	11101	00110000	00111
10100000	00001	00110100	10111
10110000	00101	00110110	10000
10111000	01101	00110101	11001
10110100	10101	00101000	01011
10101000	01001	00101100	11011
10101100	11001	00101110	11100
10101010	01110	00101101	10101
10100100	10001	00010000	00100
10100110	10110	00011000	01100
10100101	11111	00010100	10100
01000000	00001	00001000	01000
01001000	01001	00001100	11000
01001100	11001	00001010	01111
01001010	01110	00000100	10000
01000100	10001	00000110	10111
01000110	10110	00000101	11110

Table 1. *Error vector-syndrome*

Remark 10. *For $w = b$, $w_1 = b$ and also $w_2 = d$, the result coincides with Theorem 2 of [Dass, Garg and Zannetti (2008)] when bursts considered are m-repeated bursts of length b (fixed).*

Remark 11. *For $m = 2$, $w = b$, $w_1 = b$ and also $w_2 = d$, the result of Theorem 5 reduces to that of the case of 2-repeated bursts of length b (fixed)due to Dass and Garg (2009).*

Remark 12. *For $m = 1$, and $w_1 = w_2 = w$, the result gives a bound in the case of a non-repeating low-density burst of length b (fixed) with weight w or less.*

Remark 13. *For $m = 1$, $w = b$, $w_1 = b$ and also $w_2 = d$, the result reduces to the case of non-repeating bursts of length b (fixed), which is also the result when bursts are the usual open-loop bursts [refer to Theorem 4.15, Peterson and Weldon (1972)].*

We conclude the paper with an example.

Example 1. *Consider an $(8, 3)$ binary code with a parity-check matrix*

$$H = \begin{bmatrix} 0&0&0&0&0&1&0&0 \\ 0&0&0&0&1&0&0&1 \\ 0&0&0&1&0&0&1&1 \\ 1&0&1&0&0&0&1&1 \\ 0&1&1&0&0&0&1&0 \end{bmatrix}.$$

This matrix has been constructed by the synthesis procedure, outlined in the proof of Theorem 2, by taking $b = 3, w = 2$. It can be seen from Table 1 that the syndromes of the different 2-repeated low-density bursts of length 3 (fixed) with weight 2 or less are non-zero, showing thereby that the code that is the null space of this matrix can detect all 2-repeated low-density bursts of length 3 (fixed) with weight 2 or less.

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