QUASI-GEOSTROPHIC FLOW PAST MOUNTAINS

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Abstract: Blocked flow at moderate Rossby numbers is studied and an analytic expression is derived for the vertical velocities on each side of the wake:

$$W = (U_o h) / (f\ L_x L_y)$$

Where Uo is the incoming wind speed, h is the mountain height, f is Coriolis parameter, Ly is mountain half-width and Ly is a length scale of the vertical mortion downstream of the edges of the mountains. To illustrate the use of the above expression, a collection of flows over Iceland is presented and the calculated vertical velocities are compared to the number of days with precipitation in two different regions.

Keywords *– quasi-geostrophic flow, Rossby number, Froude number, Nh/U, non-dimensional mountain height, asymetric ascending motion, Iceland, Coriolis force, blocking, conceptual model, wake, corner winds.*

1. INTRODUCTION

In this paper, the vertical motion downstream of the edges of a mountain is explored. The flow is considered to be blocked (high Nh/U where N is the Brunt-Väisälä frequency, h is mountain height and U is the speed of the impinging flow) and the Rossby number is sufficiently large for some geostrophic adjustment to take place. Ascending and descending motion on each side of the wake of a mountain has been described in connection with calculation of mountain pressure drag in Ólafsson (2000) and an analytic expression of the vertical velocities is presented in Hunt et al., (2001). Here, a new and simple approach to the problem is presented. On the basis of a linearized set of equations of blocked flow, a simple equation for the vertical velocity is derived. To illustrate the theory, the equation is applied on southeasterly flow in Iceland.

2. DERIVATION OF AN EXPRESSION FOR THE VERTICAL VELOCITY

Figure 1 shows the pressure field in orographically blocked flow. At moderate values of the Rossby number (O(1)), a greater part of the upstream flow is diverted to the northern edge of the mountain. In the lee, the lateral pressure gradient is not strong enough to sustain such a strong outflow from the mountain edge. The westerly component of the flow in the box shown in Fig. 1 and Fig. 2. is decelerated. To fulfill the requirement of mass conservation, there must either be ascending motion through the top of the box or lateral flow, either into the wake (southwards) or towards the north (or both). Here, we only consider flow up through the top of the box, i.e. V=0. Solving the set of equations in Fig. 4 for the box shown in Figures 1-3 yealds a simple expression for the vertical velocity. The expression is given at the bottom of Fig. 4. The vertical velocity is proportional to the speed of the incoming flow, the height of the box (approximated to be the same as the mountain height) and inversely proportional to the Coriolis parameter, the half-width of the mountain and the length of the box. In a similar box on the left (south) side of the wake, there is descending motion. The qualitative result is summarized in Figure 5.

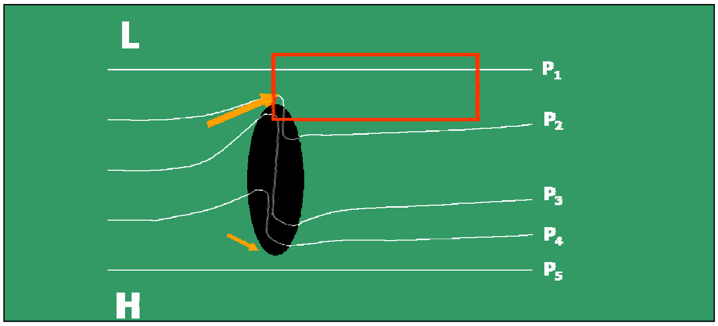

Figure 1. A schematic figure of the surface pressure field in blocked flow at moderate Rossby number

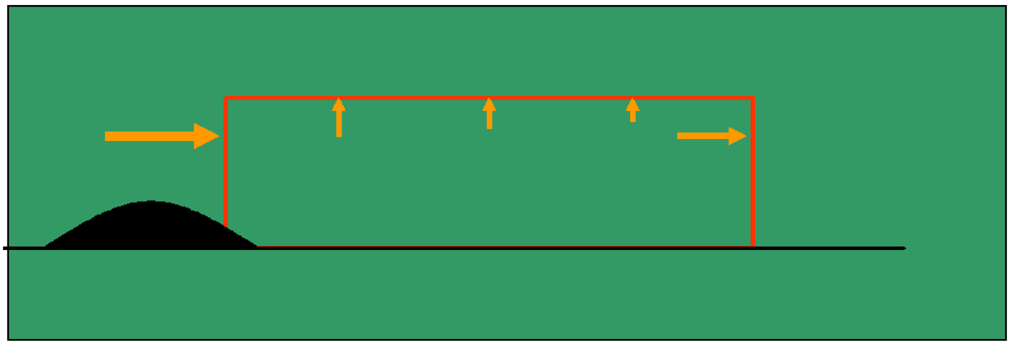

Figure 2. A cross section of the flow in the box in Fig. 1

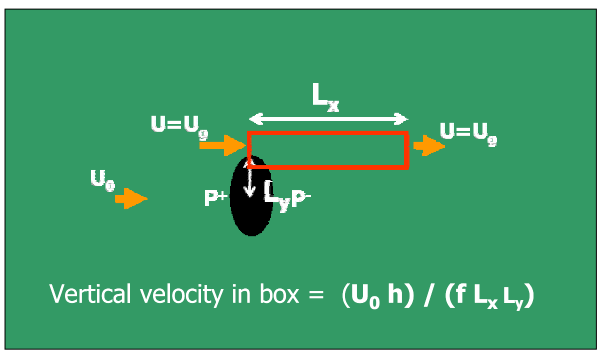

Figure 3. The variables in the equations in Fig. 5. The flow is the same as in Fig. 1 and Fig. 2

$$\frac{\partial p}{\partial z} = -\rho g \quad \text{(Hydrostatic eq.)}$$

$$\frac{P+}{\rho} = -\frac{1}{2} u_0^2 \quad \text{(Bernoulli eq.)}$$

$$Ug = -\frac{1}{f\rho}\frac{\partial p}{\partial y} \quad \text{(Geostrophic wind)}$$

$$(\nabla \bullet \overline{V})_k = 0 \quad \text{(Continuity eq.)}$$

$$V_g = V_{box} = 0 \qquad U_{p+} = 0 \qquad P+ = -P-$$

V_g: Geostrophic wind to the north

Figure 4. The equations of the flow. P is pressure, z is height, Uo is upstream wind speed, f is the Coriolis parameter, U and V are components of the horizontal flow. P+ and P- are the pressure anomalies at the mountain.

3. APPLICATIONS TO REAL FLOW

Collecting cases where the observed flow in SW-Iceland at 12 UTC is from the SSE at 850h gives a dataset of 234 days during a period of 13 years. Assuming a mean wind of 10 m/s and dimensions of boxes as shown in Fig. 6, applying the equation in Fig. 3 gives a vertical velocity of 2 cm/s. This is not very much, but may be enough to have a detectable impact on the climate and even on individual weather events. Counting the number of days with precipitation in the two boxes, the box on the left hand side has about twice as often precipitation than the easternmost box. Yet, the southerly flow at 850 hPa has to pass higher mountains to enter the western box than to enter the eastern box.

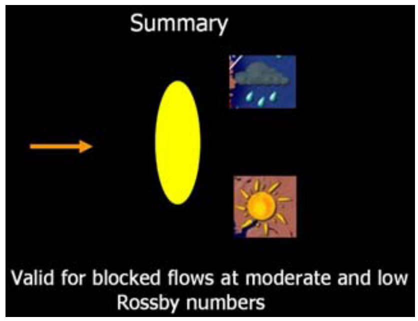

Figure 5. A qualitative summary of the impact of the vertical motion derived in section 2

4. DISCUSSION

In the derivation of the equation in Fig. 4, several approximations that need a further discussion have been made. Firstly, the pressure distribution upstream and downstream of the mountain are considered to be symmetric, the pressure gradient is assumed to be linear and the magnitude of the pressure anomaly is found by simple energy calculations on a streamline. All of these approximations can be criticized, particularly the symmetry approximation. In linear flow, the pressure anomalies are symmetric (Smith, 1980), but being blocked, the flow in the present problem is highly non-linear. In fact, the lee-side pressure anomaly can be expected to be weaker than the upstream anomaly. Assuming geostrophic adjustment at the edges of the mountain requires a very high Rossby number, higher than in the meoscale flows considered here. At Rossby numbers close to 1, the winds at the northern corner can in fact be expected to be super-geostrophic. Assuming no winds in the meridional direction, which is certainly crude, compensates for a possible underestimation of the flow into the box.

In the cases from Iceland, several factors may contribute to the difference in the frequency of precipitation in the two boxes, including wind veering with height and local topography. The Iceland case should therefore be looked upon as an illustration of the theory, rather than a proof of the importance of wake asymmetry for the climatology of precipitation.

5. CONCLUSIONS

A simple expression for vertical velocities on each side of a mountain wake has been derived. The expression and its derivation give a physical insight into which factors determine the vertical velocity and it is suitable for textbooks and curriculum in mesoscale and/or mountain meteorology.

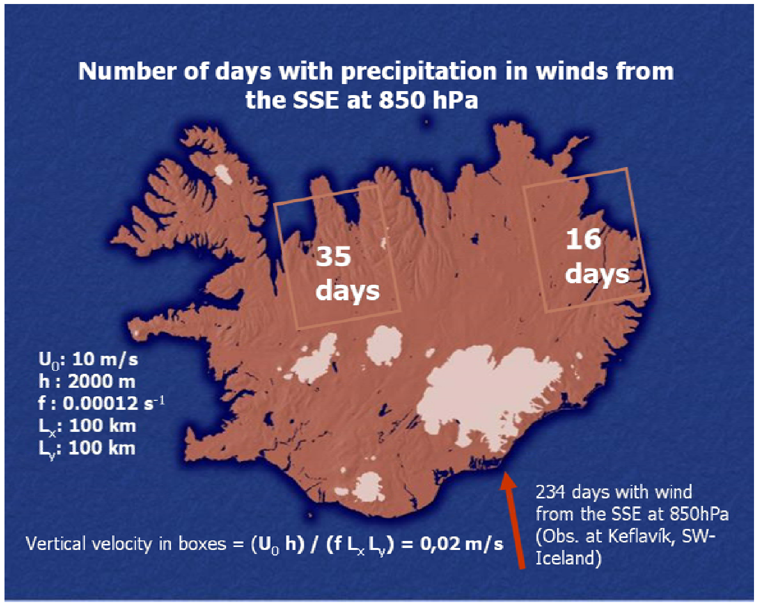

Figure 6. The variables in the equations in Fig. 5. The flow is the same as in Fig. 1 and Fig. 2

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