VORTICITY BUDGET OVER MOUNTAINS, DIAGNOSED FROM HIRLAM ANALYSES AND FORECASTS

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Abstract

The analysis of vorticity equation offers a possibility for validation and diagnostics of a NWP model. The source terms of the equation are related to the resolved and subgrid-scale torque due to the orography and surface properties. The terms resolved by the model can be estimated from three-dimensional numerical analyses and forecasts. In the vertically integrated equation, these terms sum up to give an estimate of required torque due to the subgrid-scale surface stress, which can be compared by the corresponding value given by the physical parametrizations of the model. Diagnosis of the complicated terms of the equation requires special attention to the discretization and numerical approximations. In this study, the vorticity method is applied to the diagnosis of High Resolution Limited Area Model (HIRLAM).

1 INTRODUCTION

Holopainen and Oort (1981a,b) applied vorticity and enstrophy (vorticity squared) equations to estimate surface stress over ocean, based on observed upper air wind statistics. Their results were of the same order of magnitude and direction as the independent estimates based on surface winds. According to the vorticity equation, the change of vorticity is due to a local change in time, horizontal and vertical advection, stretching, tilting induced by the nonuniform vertical velocity, influence of baroclinicity over an uneven surface and the effects of friction. In the vertically integrated form, over long enough time period, an approximate balance is expected to exist between the parametrized surface stress, horizontal advection and the resolved drag while the contribution of all other terms should be minor (Holopainen and Oort, 1981a).

The different terms causing the change of the resolved-scale absolute vorticity may as well be diagnosed from three-dimensional analyses and forecasts of a NWP model. In the model, the term related to baroclinity appears in interaction of the flow with the resolved-scale mountains. The frictional torque is related to the parametrized momentum fluxes due to subgrid-scale features of the surface. Formally, it is written as a curl of the surface stress.

In the present study, components of the vorticity equation will be estimated from numerical analyses. Data from the High Resolutionan Limited Area Model (HIRLAM), documented by Undén et al. (2002), from January-February 2004, will be used. The three-dimensional and vertically integrated vortically equation will be applied. A first attempt will be made to estimate the curl of surface stress as a residual term of the vorticity equation. The problems related to the method are discussed.

2 VORTICITY EQUATION

Applying the gradient operator $\vec{k}\cdot\nabla_\zeta\times$ to the equation of horizontal motion (Eq. (1)) along a horizontal surface with a generalized vertical coordinate ζ[1] gives us an equation (Eq. (2)) for the absolute vorticity η as a sum of relative ξ and planetary f components, $\eta = \xi + f$ (see e.g. Haltiner and Williams (1980); Holopainen and Oort (1981a)):

$$\frac{\mathrm{d}\tilde{\mathrm{v}}}{\mathrm{dt}} = -\alpha\nabla_\zeta p - \nabla_\zeta\Phi - f\vec{k}\times\vec{v} + \vec{F} \qquad (1)$$

[1]Note that the pressure gradient term consists of two parts including both pressure and geopotential gradients when a generalised vertical coordinate is used.

$$\frac{\partial \xi}{\partial t} + \vec{v} \cdot \nabla_\zeta \eta + \dot{\zeta} \frac{\partial \eta}{\partial \zeta} + \eta \nabla_\zeta \cdot \vec{v} + \vec{k} \cdot \nabla_\zeta \dot{\zeta} \times \frac{\partial \vec{v}}{\partial \zeta} + \frac{1}{p_s} \frac{\partial J(p, \Phi)}{\partial \zeta} = \vec{k} \cdot \nabla_\zeta \times \vec{F} \tag{2}$$

(a) (b) (c) (d) (e) (f) (g)

Here, $\vec{v}$ is the horizontal wind, $\dot{\zeta} = \frac{\mathrm{d}\zeta}{\mathrm{dt}}$ vertical velocity, α=1/ρ, ρ is density, $\Phi = gz$ is geopotential, g is gravitational acceleration, p is pressure, p_s surface pressure and $\vec{F}$ denotes frictional forces. The term $J(a, b)$ denotes a Jacobian along the coordinate surface: $J(a, b)=\frac{\partial a}{\partial x}\frac{\partial b}{\partial y} - \frac{\partial b}{\partial x}\frac{\partial a}{\partial y}$. The hydrostatic equation $\alpha = -\frac{\partial \Phi}{\partial p}$ was applied in derivation of Eq. (2). The terms of the vorticity equation represent, the local time change of vorticity (a), horizontal (b) and vertical (c) advection, stretching (d), tilting induced by the nonuniform vertical velocity (e), change of vorticity due to baroclinicity (f) and effects of friction (g). Only the two last terms represent true sources and sinks of vorticity, the rest describe its redistribution.

The frictional forces are represented by the vertical divergence of subgrid-scale vertical momentum fluxes

$$\vec{F} = -g\frac{\partial \vec{\tau}}{\partial p}, \quad \vec{\tau} = -\rho \overline{\vec{v}' w'}, \tag{3}$$

where w is the vertical velocity, an overline denotes gridbox average and a prime $'$ subgrid-scale deviation. At the upper boundary $\vec{\tau}$=0, at the surface $\vec{\tau}=\vec{\tau}_s$. Integrating over an atmospheric column from the surface $p = p_s$ to the top of atmosphere $p = 0$ and denoting the integral by a hat, $\hat{\varphi}=\int_o^{p_s} \varphi \frac{dp}{g}$, gives

$$\widehat{\frac{\partial \xi}{\partial t}} + \widehat{\vec{v} \cdot \nabla_\zeta \eta} + \widehat{\dot{\zeta} \frac{\partial \eta}{\partial \zeta}} + \widehat{\eta \nabla_\zeta \cdot \vec{v}} + \widehat{\vec{k} \cdot \nabla_\zeta \dot{\zeta} \times \frac{\partial \vec{v}}{\partial \zeta}} + J(h, p_s) = \vec{k} \cdot \nabla_\zeta \times \vec{\tau}_s, \tag{4}$$

where h is the terrain height. The term $J(h, p_s)$ results from vertical integration of the baroclinic term (Eq. (2), term (f)) and represents the joint effect of baroclinicity and orography (in oceanology, an analoguous term called JEBAR was introduced by Sarkisyan and Ivanov (1971)). Over a level surface this term disappears. It arises when using a terrain-following (hybrid) vertical coordinate and is explicitly resolved by the model.

Over a long-time average of Eq. (2) the term $\widehat{\frac{\partial \xi}{\partial t}}$ and the integrated divergence term are expected to be close to zero. Also, the vertically integrated stretching and terms including vertical velocities should be small (Holopainen and Oort, 1981a). Thus an approximate balance is expected to exist between the terms representing the torque due to the subgrid-scale surface stress (which should be parametrized by the model), horizontal advection and resolved drag:

$$\vec{k} \cdot \nabla_\zeta \times \vec{\tau}_s \approx \widehat{\vec{v} \cdot \nabla_\zeta \eta} + J(h, p_s) \tag{5}$$

In the following, *all* terms of the three-dimensional Eq. (2) are calculated in the model grid and then numerically integrated in vertical to obtain an estimate for the required parametrized subgrid-scale torque $\vec{k} \cdot \nabla_\zeta \times \vec{\tau}_s$. Comparison with the results of Eqs. (4) - (5) allows to estimate the accuracy of the method and justify the possible simplifications.

3 DATA AND METHODS

Data produced for the International HIRLAM project by running the reference HIRLAM data assimilation and forecast cycle at Finnish Meteorological Institute are used in this study. A model version 6.3 is applied over an Atlantic-European area with a horizontal resolution of 22 km and 40 levels in vertical. Lateral boundary conditions for HIRLAM are provided by operational forecasts of European Centre for Medium Range Forecasts (ECMWF) with an interval of six hours. HIRLAM uses a hybrid vertical coordinate approaching to sigma coordinate (p/p_s) close to the surface and pressure coordinate in the stratosphere. Upper boundary of the model is defined at the pressure level of 10 hPa. A staggered Arakawa C-grid is used in HIRLAM so that wind components are shifted by half a gridlength in the direction of x-axis (u-component) or y-axis (v-component), with respect to the scalar variables like pressure, temperature and humidity.

The HIRLAM reference system produces three-dimensional numerical analyses in a continuous data assimilation cycle with an interval of 3 hours, i.e. 8 analyses daily. Observational data used by the present reference system includes only the conventional sources: soundings (TEMP, PILOT), aircraft reports (AIREP) and surface observations (SYNOP, SHIP, buoy). Note that no sounding data are available during the intermediate (03, 09 etc. UTC) analysis times. Numerical analyses are provided by a multivariate three-dimensional variational data assimilation system (Undén et al., 2002). Analysis is based on the first guess by the +03h forecast from the perious cycle. The analysed variables include horizontal wind components, temperature and humidity as well as the surface pressure. (Initial values for the two additional three-dimensional prognostic variables of HIRLAM, cloud condensate content and kinetic energy of turbulence, are based on the first guess information only.) The upper-air analysis is initialized by using an incremental digital filter (Huang and Yang, 2002), i.e. by integrating the forecast model backward and forward around the analysis time in adiabatic and full (including physical parametrizations) modes.

Special routines based mostly on optimal interpolation (Rodriguez et al., 2003) take care of assimilation of the near-surface variables like the two-metre or sea surface temperature, snow and ice cover, which influence in the forecasts through the physical parametrization schemes. Forecasts with a lead time up to 54 hours are produced four times a day, based on 00, 06, 12 and 18 UTC analyses. A fourth-order horizontal diffusion scheme is used for smoothing during the model integrations.

Calculation of the complicated and sensitive terms of the vorticity equation (Eq. (2) requires particular attention to the discretization and numerical approximations. In this study, terms of the vorticity equation (Eq. (2) are calculated by postprocessing analysed and predicted three-dimensional horizontal wind components u and v, using also model's surface pressure and surface elevation. All calculations are done at the hybrid model levels without vertical or horizontal interpolations. Vorticity, horizontal divergence and horizontal and vertical advection as well the tilting term are calculated according to the HIRLAM discretization in the staggered grid. Vertical velocity $\dot{\zeta}$ is obtained by vertical integration of the horizontal divergence. Horizontal derivatives needed in calculation of the Jacobian and the torque due to the stress are approximated by centered differences, as are also all vertical derivatives. Vertical integrations are done by summing values weighted by the layer thicknesses $\Delta p(x, y, \eta)$.

Operational HIRLAM analyses with an interval of 3 hours during January and February 2004, 472 analyses in total, are used to calculate the terms of vorticity equation. For comparison, these terms are calculated also from 236 +48 hour forecasts made with an interval of 6 hours. An estimate of the required surface flux is calculated as a residual term both from the analyses and forecasts. Values of parametrized surface turbulent momentum fluxes accumulated during the +48 hour forecasts are obtained from the standard output of HIRLAM and used for calculation of the parametrized torque due to the surface stress $\vec{k} \cdot \nabla_\zeta \times \vec{\tau}_s$. A third estimate for the curl of surface stress may be obtained based on the analysed wind components at the lowest model level, by using a bulk formulation for the surface momentum fluxes in accordance to the HIRLAM surface layer parametrizations (Undén et al., 2002).

4 RESULTS

Preliminary results will be presented concerning the observed and predicted circulation in January-February 2004 and analysed an predicted vorticity budget. The mean curl of surface stress estimated with different methods will be compared.

5 PRELIMINARY DISCUSSION

The vorticity method applied in this study has several potential advantages. Required by the model torque due to the subgrid-scale stress can be estimated based on full three-dimensional information given by the numerical analysis. The present-day analyses of the free atmosphere large scale flow can be considered quite reliable - perhaps in constrast to the analyses of boundary-layer variables. In the future, improving analyses will provide also improving estimates of the required stress. The vorticity method avoids the problems connected with direct diagnostics of the (vector) momentum equations. In the present formulation, the source terms of atmospheric momentum arising in the interaction with the underlying surface, are fully described. The vorticity method is applicable also in limited area models, not only in the global ones, where the analysis of the zonal atmospheric angular momentum budget provides an alternative (Kim and Hogan, 2004).

A three-dimensional comparison of the predicted and analysed vorticity and the components of vorticity equation offers an additional possibility for validation and understanding interactions within the model. In principle, analysis of the vorticity budget allows to estimate the three-dimensional distribution of the curl of the subgrid-scale stress, and to compare the magnitude of the analysed and parametrized by the forecast model values both at the surface and in the atmosphere. However, the differences between the parametrized and derived from analysis stress do not reveal differences between various processes responsible for the total stress. An estimate of the torque due to the (surface) stress also does not directly give the stress components in the way they are formulated in the forecast model.

The evident model-dependency of the results is both an advantage and disadvantadge. Depending on the model's horizontal resolution, processes of different scales are resolved and parametrized. Comparing results obtained from analyses and forecasts of different resolutions might help in understanding these scale-dependencies and related physical processes. However, the accuracy of the numerical analyses does not necessarily improve with improving resolution. Observation coverage varies over different areas and during different times of day. Consequently, the quality of the analyses varies from time to time, and the model's short forecasts, used as background for the analyses, play a different role.

Several model-specific problems may influence the results. Formulation of model's vertical coordinate depends in a way or another on surface elevation and influences in the accuracy of the results especially close to steep orography. All NWP models use specific methods to smooth the analysed initial state to be suitable for the forecast model (digital filter, normal mode initialization, smoothing inherent to four-dimensional data assimilation methods etc). In a limited-area model like HIRLAM, quality of the forecasts and analyses close to the model's lateral boundaries may be be worse than over the central areas, due to boundary relaxation problems.

Clear disadvantadges of the method are related to the (numerically) complicated nature of the terms of vorticity equation. Required by the model stress is estimated as a residual, sum of these complicated terms. Accurate calculation of e.g. the horizontal gradients of derived from the analysis vertical velocity over uneven surface may be practically impossible. Calculated terms need to be filtered in order to obtain meaningful results. Smoothing is obtained in several ways. All calculations are based on initialized by the model analyses, that provides a model-specific smoothing of the basic data. Natural filtering results from averaging over time and area as well as calculating vertically integrated values. In addition, spectral or grid-space smoothing of the individual terms of Eqs. (2) and (4) may be needed.

6 CONCLUSIONS

The analysis of the vorticity budget is expected to lead to a better understanding of surface related processes influencing the momentum fluxes in the model, not to provide exact guidance for tuning of parametrizations.

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